Collaborative Insurance Sustainablity And Network Structure

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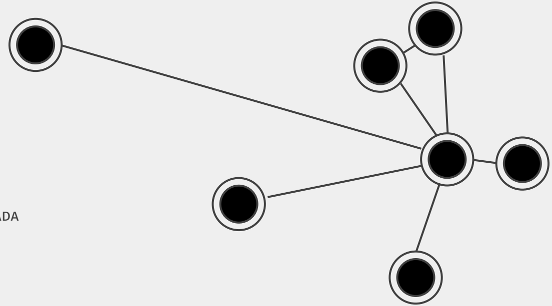

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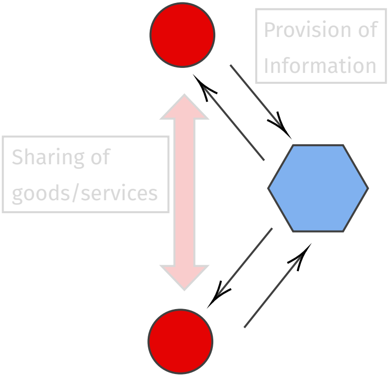
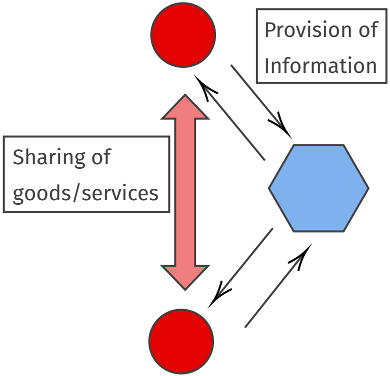
The Main Idea: Sharing deductibles with friends



- In a nutshell - we will consider different risk sharing models between peers in a network and analyze the robustness of the mechanisms



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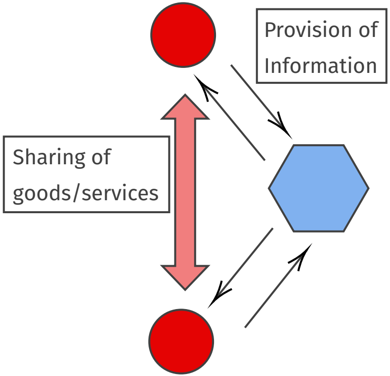
- A (very) short primer on graph-terminology
  - Degree distribution and random graphs



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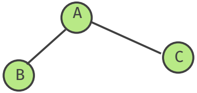
- A (very) short primer on graph-terminology
  - Degree distribution and random graphs
- Risk sharing on a network
  - Reciprocal engagements & a simple example
  - Extensions for perceived fairness, solutions for graphs with high degree variance and extensions to friends-of-my-friends

Definition of a graph

A graph $\mathcal{G}$ is an ordered pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ represents a set of vertices (or nodes) and $\mathcal{E}$ a set of edges (or links) such that $\mathcal{E} \subseteq \{\{x, y\} | x, y, \in \mathcal{V}, x \neq y\}$. That is, an edge links two vertices together.

Adjacency matrix

The adjacency matrix A for the set of vertices $\mathcal{V} = \{v_1, ..., v_n\}$ is of size $n \times n$ and such that $A_{i,j} = 1$ if there is a edge from vertex v_i to v_j. In the following we will also assume that $A_{i,i} = 0$ (ie. a node does not have a connection to itself).

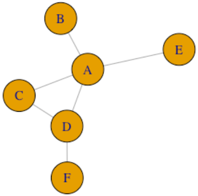


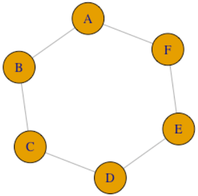
$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array}$$

Degree distribution

The degree of a vertex is the number of edges that are incident to it. The average degree of a graph is denoted as $\overline{d}$. The degree vector for each vertex can be calculated by taking $A\mathbb{1} = d$. Note that the average degree of a network is:

$$\mathbb{E}(D) = \frac{1}{|\mathcal{V}|}\sum_{v\in\mathcal{V}} d(v) = \frac{2|\mathcal{E}|}{|\mathcal{V}|} = \frac{1}{|\mathcal{V}|}\|d\|_1$$





Erdős–Rényi

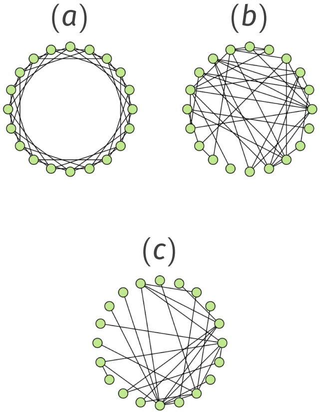
Each edge $i, j \in \mathcal{V} \times \mathcal{V}$ is included in the network with probability p independent from other edges. The Network then has a degree distribution that follows a binomial distribution $\mathcal{B}(n-1, p)$. Also, if p not too large but n_V:
$p \sim \lambda / n_V, d(v) \sim \mathcal{P}(\lambda)$ See [5] for details.

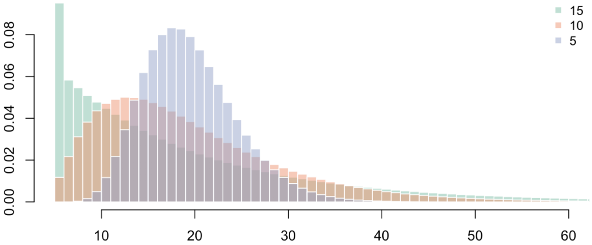
Preferential attachment

To start with, consider a small network, and create a new node at every time-step. During every time-step, the new node is connected to the existing nodes with a probability p that is proportional to the degree of an existing node d_i. This way, every new node is more likely to connect to existing "popular" nodes. See [1] or [6] for details. Here the degree distribution follows a power law.

For a discussion on the nature of networks see eg. [3]

Random Network Generation





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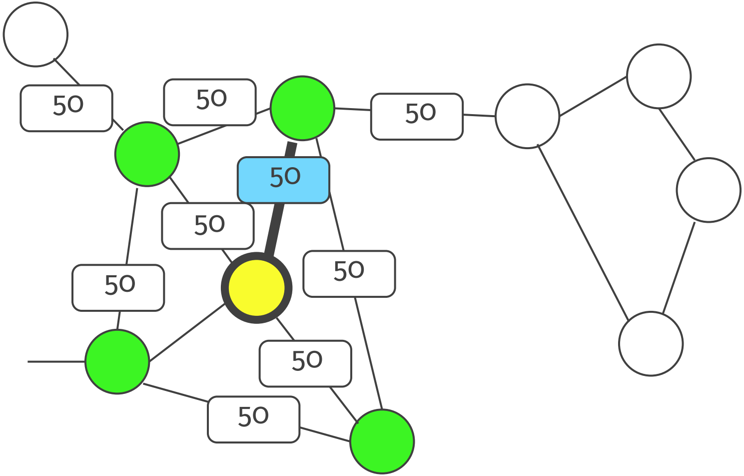
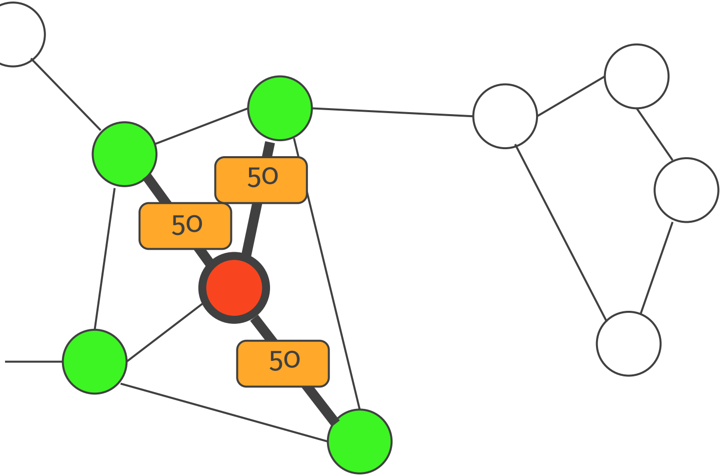
The random cost for a member after one period can then be summarized as:

$$X_i = Z_i \cdot \min\{s, Y_i\} = \begin{cases} 0 \text{ if no claim occurred } (Z_i = 0) \\ \min\{s, Y_i\} \text{ if a claim occurred } (Z_i = 1) \end{cases}$$

Risk Sharing on a network

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First, assume fixed commitments for the whole network set to $\gamma = s/\bar{d}$. Ie. each node i will pay γ to all its "friends" that claim a loss. Assume all edges are used.

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Risk and reciprocal engagements

Let $\mathcal{V}_i$ denote the set of friends (the set of nodes that are connected to node i). Also assume fixed contributions γ. The cost for node i then becomes:

$$\xi_i = Z_i \cdot \min\{s, Y_i\}$$

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which can be shown to be a risk sharing principle (for a definition of a risk-sharing principle see eg. [4])

Generate network with $n = 5000$ nodes, with degree distribution

$$D \stackrel{\mathcal{L}}{=} \min\{5 + [\Delta], n - 1\},$$

where $[\Delta]$ has a rounded Gamma distribution with mean $\overline{d} - 5$ and variance σ^2.
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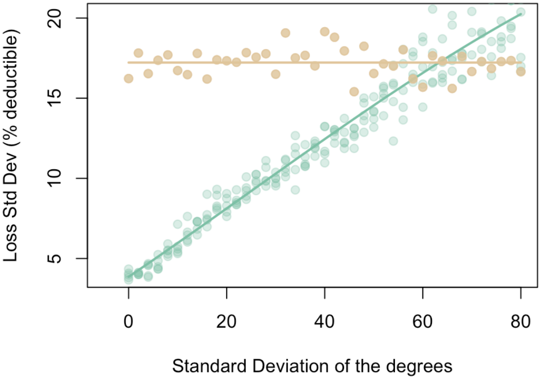
Consider a risk sharing model where the reciprocal engagements weigh $\gamma = 50$ and $\mathbb{E}(X) = 4.5\%$ of s. The losses are distributed over the network via the Bernoulli mechanism

A simple example



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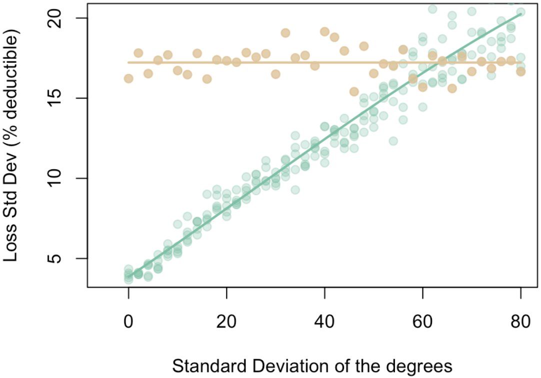
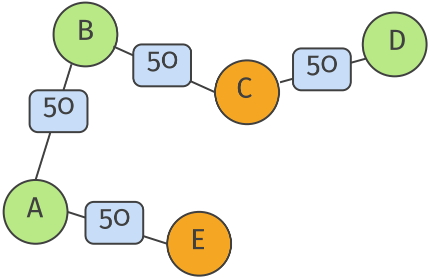
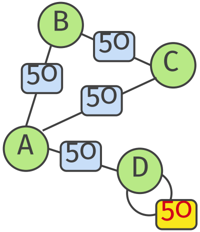
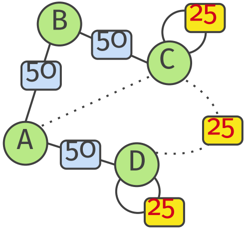


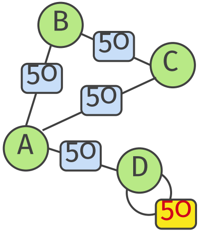


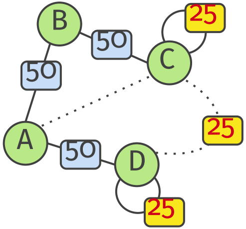




We can consider some extensions

- Introducing self-contribution to ensure fairness in the mechanism and alleviate the variance issue





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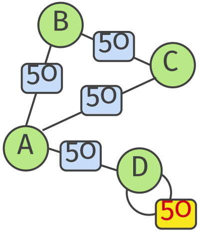

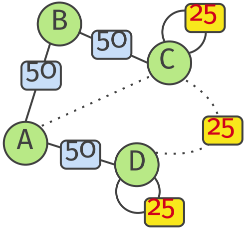

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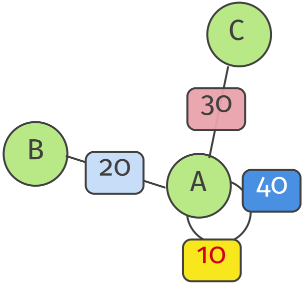
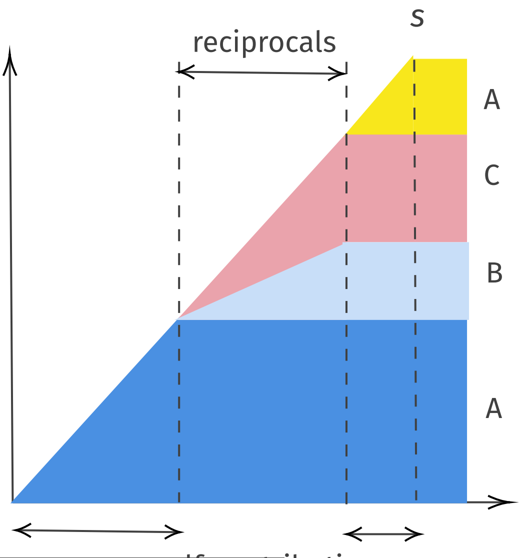
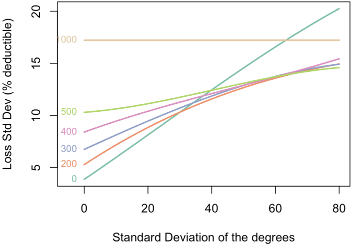
- Extending the reciprocal engagements to friends-of-friends to expand the set of possible edges for nodes with only a few links

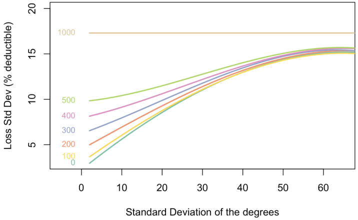
Reciprocal engagement with self-contribution







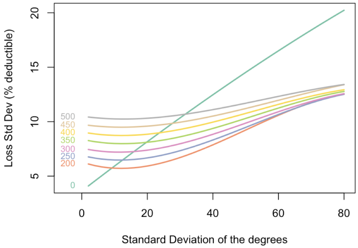
- It can be shown that in many cases, it is likely that the nodes who actually claim a loss will end up paying *less* than its adjacent vertices. The system seems fair ex-ante but ex-post (after the occurrence of the claim) it might seem unfair
- Self contribution makes a "fair" ex-post outcome more likely



Instead of a fixed γ we can also formulate the γ-vector as a linear programming optimization problem. This can lead to smaller total contributions, especially for "popular" nodes:

$$\gamma_1^* = \operatorname{argmax} \left\{ \sum_{(i,j)\in\mathcal{E}} \gamma_{(i,j)} \right\}$$
$$\text{s.t. } \gamma_{(i,j)} \in [0, \gamma],\ \forall (i,j) \in \mathcal{E}$$
$$\sum_{j\in\mathcal{V}_i} \gamma_{(i,j)} \leq s,\ \forall i \in \mathcal{V}$$

In words, this means we have a limited liability s and our objective is to maximize the overall coverage for our network. Sparsity of the solution can also be achieved by reformulating the problem into a mixed integer problem.



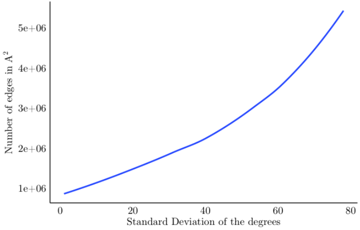
A further simple extension is to consider not only the adjacent nodes but also their adjacent nodes (friends-of-my-friends). This will naturally increase the number of connections. After the first stage (optimal reciprocal engagements) nodes not yet fully covered can satisfy their demand with the friends of their friends (typically γ_2 can be smaller than γ_1)

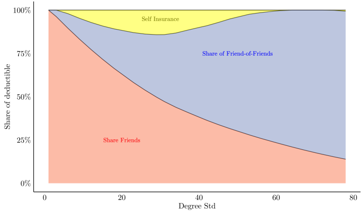
$$\gamma_2^\star = \text{argmax}\left\{\sum_{(i,j)\in\mathcal{E}^{(2)}} \gamma_{(i,j)}\right\}$$

$$\text{s.t. } \gamma_{(i,j)} \in [0,\gamma_2],\ \forall (i,j)\in\mathcal{E}^{(2)},\ \mathcal{E}^{(2)} \text{ from } A^2$$

$$\sum_{j\in\mathcal{V}_i^{(1)}} \gamma^\star_{1:(i,j)} + \sum_{j\in\mathcal{V}_i^{(2)}} \gamma_{(i,j)} \leq s,\ \forall i\in\mathcal{V}$$

Risk sharing with Friends-of-Friends





In this setup, the computational cost increases drastically with the degree variance. First, because more is left over from the first part, but also because the number of edges in A^2 increases.

But, depending of the level of γ_2, this property actually ensures that the network mechanism is still able to allocate most of the risk (as the number of edges A^2 runs in the opposite direction as the amount of risk "sharable" in A^1)

- Using Networks/Graphs allows us to tap into some hidden and interesting information (eg. [2]) and can be employed with novel digital techniques to enhance transparency
- An insurance product based on networks can be considered a risk sharing mechanism
- The proposed basic mechanism can be extended in many ways

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But...

- The proposed model is inherently static
- The model (as of yet) also neglects some locality properties such as higher-level groups

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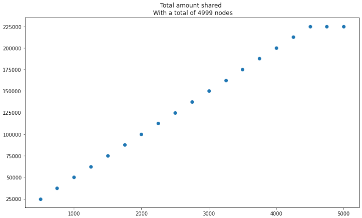
Sparsification of results



By changing the problem from its linear form to its mixed-integer form, we can achieve the desired level of sparsity. M comes naturally here and actually makes the problem slightly simpler in terms of memory usage. The important part here part here is the m parameter which restricts the number of nonzero edges.

$$\begin{cases} \boldsymbol{z}^\star = \underset{\boldsymbol{z} \in \mathbb{R}_+^m}{\operatorname{argmax}} \left\{ \mathbf{1}^\top \boldsymbol{z} \right\} \\ \text{s.t. } \boldsymbol{T}^\top \boldsymbol{z} \leq s\mathbf{1}_m \\ \quad z_i \leq M y_i \\ \quad \sum_i y_i \leq m \end{cases}$$

, where $y_i \in \{0, 1\}$

GitHub Repo

- Code on Github
- Paper on arXiv