Incertitude paramétrique des réseaux de neurones en tarification IARD

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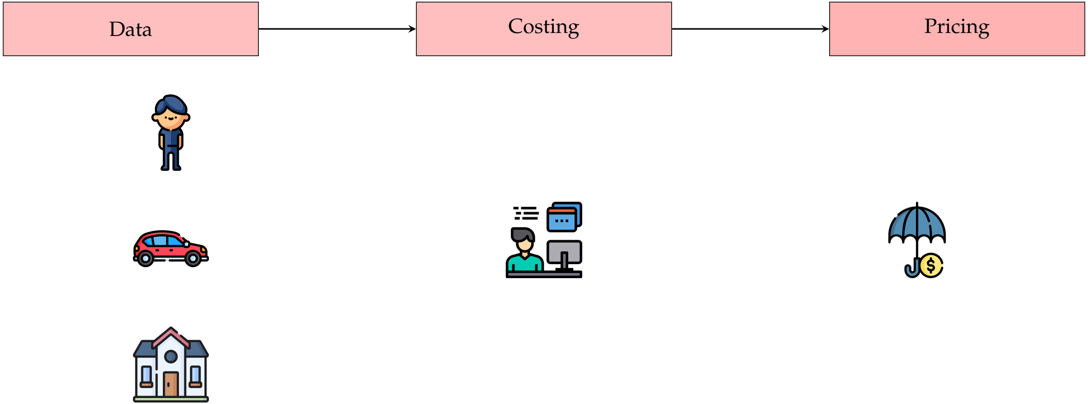
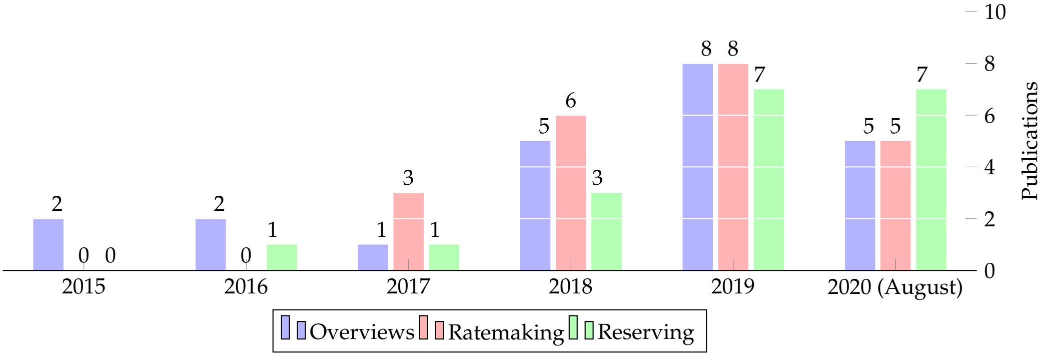
Motivation



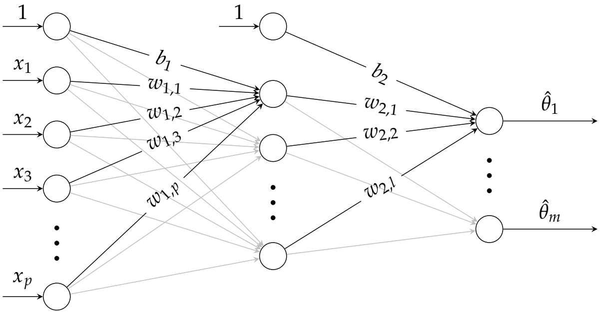
Motivation

Publications in P&C actuarial science using machine learning



Source : [Blier-Wong et al., 2021]

Neural networks



Neural networks for insurance pricing:

[Lowe and Pryor, 1996, Dugas et al., 2003, Noll et al., 2018, Ferrario et al., 2018, Schelldorfer and Wuthrich, 2019, Wuthrich, 2019b, Wuthrich, 2019a, Denuit et al., 2019, Richman and Wüthrich, 2020, Gao et al., 2021, Kuo and Richman, 2021, Shi and Shi, 2021, Delong et al., 2021]

Neural networks

- Let Y be the response variable, $\boldsymbol{x}$ be covariates
- Shape of a GLM:
$$E[Y] = g(\boldsymbol{x}\boldsymbol{\beta} + \beta_0),$$
where g is the inverse link function
- Shape of a neural network with one hidden layer:
$$E[Y] = g_2(\boldsymbol{h}\boldsymbol{W} + \boldsymbol{b}),$$
where
$$\boldsymbol{h} = g_1(\boldsymbol{W}_1\boldsymbol{x} + \boldsymbol{b}_1)$$
- The vector $\boldsymbol{h}$ contains *intermediate representations* of the input variables $\boldsymbol{x}$

Motivation

Objective of the paper

Explore the effects of parameter uncertainty within neural networks for insurance pricing

- How does one model parameter uncertainty within neural networks?
- How does this affect prior insurance pricing?
- How does this affect posterior insurance pricing?

Overview

Model uncertainty

- Let $\boldsymbol{\Omega}$ be the model parameters
- Let the likelihood be of the shape $f_Y(y) = E_{\boldsymbol{\Omega}}[f_Y(y|\boldsymbol{\Omega})]$
- Law of total variance

$$Var(Y) = E_{\boldsymbol{\Omega}}[Var(Y|\boldsymbol{\Omega})] + Var_{\boldsymbol{\Omega}}(E[Y|\boldsymbol{\Omega}])$$

- Most machine learning models: process variance only
  - Implicit assumption: parameters are certain

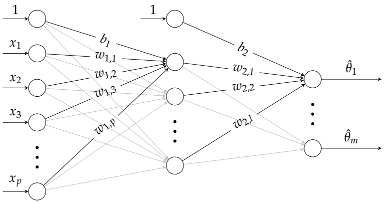
$$Var(Y) = E_{\boldsymbol{\Omega}}[Var(Y|\boldsymbol{\Omega})]$$

- Explored in mortality modeling [Cairns, 2000, Cairns et al., 2006]
- Is this assumption viable for insurance pricing ?

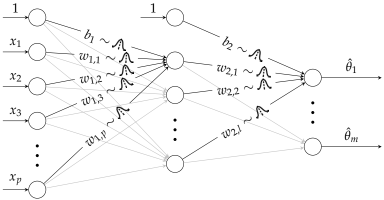
Neural networks

Adding model uncertainty

Neural network



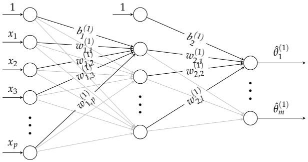
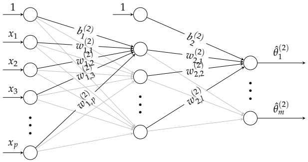
Bayesian neural network



- Bayesian GLM: [Frees et al., 2016], section 14.3
- Bayesian neural network: [Graves, 2011, Blundell et al., 2015, Gal, 2016]
- Similar to chapter 2 of [Denuit et al., 2019]
- Applications in recommender systems, computer vision, medical imaging, natural language processing, time series forecasting [Wang and Yeung, 2020, Abdar et al., 2021]

Bayesian neural networks

Sample $\boldsymbol{\omega}^{(1)}$ Sample $\boldsymbol{\omega}^{(2)}$ Sample $\boldsymbol{\omega}^{(T)}$

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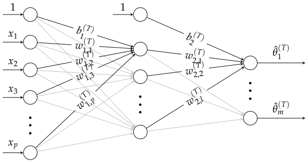


Predictions from Bayesian neural networks:

$$\hat{\theta}_j = \frac{1}{T}\sum_{t=1}^{T}\hat{\theta}_j^{(t)}, \quad j = 1, \ldots, m$$

Consider the following notation

- $\boldsymbol{y}$: Response vector
- $\boldsymbol{X}$: Design matrix
- $\mathcal{D}$: Training dataset $\{x_i, y_i; i = 1, \ldots, n\}$
- $\boldsymbol{\omega} \in \boldsymbol{\Omega}$: Set of all parameters
- $f(\boldsymbol{y}|\boldsymbol{\omega}, \boldsymbol{X})$: likelihood
- $\pi(\boldsymbol{\omega})$: posterior distribution of parameters

- Instead of gradient descent, one trains the model by finding the *posterior distribution of parameters*

$$
\begin{aligned}
\pi(\boldsymbol{\omega}|\mathcal{D}) &= \frac{f(\boldsymbol{Y}|\boldsymbol{\omega}, \boldsymbol{X})\pi(\boldsymbol{\omega})}{f(\boldsymbol{Y}|\boldsymbol{X})} \\
&= \frac{\prod_{i=1}^{n} f(\boldsymbol{y}_i|\boldsymbol{\omega}, \boldsymbol{x}_i)\pi(\boldsymbol{\omega})}{\int \cdots \int_{\Omega} \left[\prod_{i=1}^{n} f(\boldsymbol{y}_i|\boldsymbol{x}_i, \boldsymbol{\omega})\pi(\boldsymbol{\omega})\right] d\boldsymbol{\omega}}
\end{aligned}
$$

- Denominator: intractable

Variational function

- Distribution $\pi(\boldsymbol{\omega}|\mathcal{D})$ is too complex to identify
- Integrating over conditionally independent weights & products of rvs
- Alternative: compute a *variational function* that approximates $\pi(\boldsymbol{\omega}|\mathcal{D})$
- Example variational function: independent weights

$$q_\psi(\boldsymbol{\omega}) = \prod_{i=\text{layers}} \prod_{j=\text{neurons}} \left(q_{\mu_{ij0},\sigma^2_{ij0}}(b_{ij}) \times \prod_{k=\substack{\text{next}\\\text{neurons}}} q_{\mu_{ijk},\sigma^2_{ijk}}(w_{ijk}) \right)$$

- Variational inference: train parameters of $q_\psi(\boldsymbol{\omega})$ such that it approximates $\pi(\boldsymbol{\omega}|\mathcal{D})$
- [Graves, 2011, Blei et al., 2017]

Evidence lower bound

- Loss function for variational inference: evidence lower bound

$$\mathcal{L}_{VI}(\psi) = E_{q_\psi(\boldsymbol{\omega})}\left[\log f(\boldsymbol{Y}|\boldsymbol{X},\boldsymbol{\omega})\right] - KL(q_\psi(\boldsymbol{\omega})||\pi(\boldsymbol{\omega})).$$

- Minibatch S of size M approximation:

$$\hat{\mathcal{L}}_{VI}(\psi) = \frac{n}{M}\sum_{i\in S} E_{q_\psi(\boldsymbol{\omega})}\left[\log f(\boldsymbol{y}_i|\boldsymbol{x}_i,\boldsymbol{\omega})\right] - KL(q_\psi(\boldsymbol{\omega})||\pi(\boldsymbol{\omega}))$$

Maximizing the ELBO

Update rule for all variational distribution parameters (mean and variance)

Input: Dataset $\boldsymbol{X}, \boldsymbol{Y}$, learning rate η;
Initialize ψ parameter randomly;
repeat

Sample a minibatch S of size M $\boldsymbol{x}_i, \boldsymbol{y}_i, \hat{\boldsymbol{\omega}}$;
Compute the stochastic derivative with respect to ψ:

$$\widehat{\Delta\psi} \leftarrow \frac{N}{M} \sum_{i \in S} \frac{\partial}{\partial \psi} \log f(\boldsymbol{y}_i | \boldsymbol{x}_i, \hat{\boldsymbol{\omega}}) + \frac{\partial}{\partial \psi} KL(q_{\boldsymbol{\psi}}(\hat{\boldsymbol{\omega}}) || \pi(\hat{\boldsymbol{\omega}}))$$

Update ψ:

$$\psi \leftarrow \psi - \eta \widehat{\Delta\psi}$$

until *convergence of* ψ.;

Implications for prior pricing

Prediction

- Consider an arbitrary function $h(Y, \Omega)$ such that

$$E[h(Y^*, \Omega)|\boldsymbol{x}^*, \Omega, \mathcal{D}] = \int h(y, \omega) f(y|\boldsymbol{x}^*, \Omega = \omega) dy$$

- Estimation unconditional to Ω

$$E[h(Y^*, \Omega)|\boldsymbol{x}^*, \mathcal{D}] \approx E[h(Y^*, \Psi)|\boldsymbol{x}^*, \mathcal{D}] \underset{T \to \infty}{\longrightarrow} \frac{1}{T} \sum_{t=1}^{T} E[h(Y^*, \Psi)|\boldsymbol{x}^*, \Psi = \tilde{\psi}_t, \mathcal{D}]$$

Examples with the Poisson distribution

Example with a Poisson response variable, i.e. $Y|\Omega \sim Poisson$

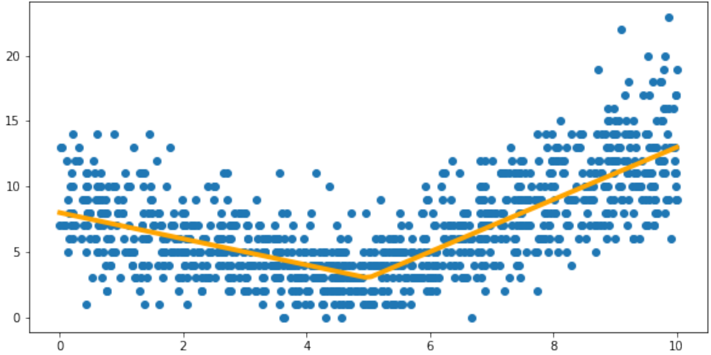
- Let variables for new customer be $\boldsymbol{x}^*$, objective: predict response variable Y^*
- Let $\tilde{\lambda}_t$ be a random sample from the variational distribution $\Lambda|\mathcal{D}$

$$E[Y^*|\mathcal{D}] = E[E[Y^*|\Omega, \mathcal{D}]] \approx E[E[Y^*|\Lambda, \mathcal{D}]] \approx \frac{1}{T}\sum_{t=1}^{T} E[Y^*|\Lambda = \tilde{\lambda}_t] = \frac{1}{T}\sum_{t=1}^{T}\tilde{\lambda}_t = \bar{\lambda}.$$

$$\begin{aligned} Var(Y^*|\mathcal{D}) &= E[Var(Y^*|\Omega, \mathcal{D})] + Var(E[Y^*|\Omega, \mathcal{D}]) \\ &\approx E[Var(Y^*|\Lambda, \mathcal{D})] + Var(E[Y^*|\Lambda, \mathcal{D}]) \\ &= \bar{\lambda} + \frac{1}{T-1}\sum_{t=1}^{T}\left(\tilde{\lambda}_t - \bar{\lambda}\right)^2. \end{aligned}$$

Simulation data

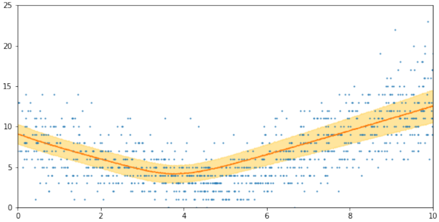
- See Example 2.1 from [Charpentier et al., 2021]
- Consider a single covariate $X \sim Unif(0, 10)$
- Let $Y \sim Pois(\mu(x))$, with $\mu(x) = 8 - x + 3\max(x - 5, 0)$

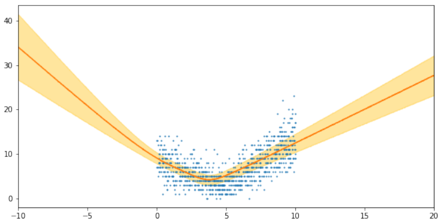


Simulation data

Confidence intervals for the rate parameter

- Train BNN with two hidden layers
- 20 hidden neurons in each hidden layer
- ReLU activations
- Confidence interval of the rate parameter: $\widehat{\mu(x)} \pm \sqrt{Var(\widehat{\mu(x)})}$

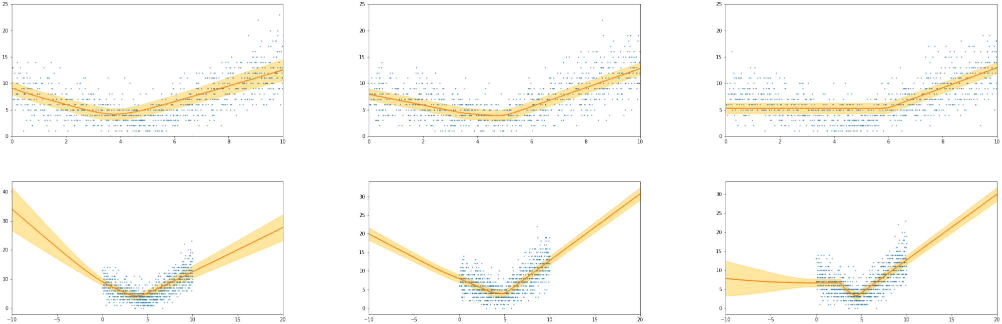




Simulation data

Confidence intervals for the rate parameter

2×20 $\quad$ 2×3 $\quad$ 2×3



Implications for posterior pricing

Bayesian neural networks and posterior pricing

Given a model prediction μ and n periods of experience $\overline{Y}$, how should one combine the two quantities within a BLUP to minimize mean squared error ?

Define

- Hypothetical mean $\mu(\boldsymbol{\omega}) = E[Y^* | \boldsymbol{x}^*, \mathcal{D}, \boldsymbol{\Omega} = \boldsymbol{\omega}]$
- Process variance $v(\boldsymbol{\omega}) = Var(Y^* | \boldsymbol{x}^*, \mathcal{D}, \boldsymbol{\Omega} = \boldsymbol{\omega})$

Also,

$$\mu = E[\mu(\boldsymbol{\Omega})], \quad v = E[v(\boldsymbol{\Omega})], \quad a = Var(\mu(\boldsymbol{\Omega}))$$

Implications for posterior pricing

One has

$$E[Y^*] = \mu, \quad Var(Y^*) = v + a, \quad Cov(Y_1^*, Y_2^*) = a$$

Best weighted average of model prediction and n periods of actual experience:

$$Z\overline{Y} + (1 - Z)\mu, \quad Z = \frac{n}{n + k}, \quad k = \frac{E[Var(Y|\boldsymbol{\Omega})]}{Var(E[Y|\boldsymbol{\Omega}])}$$

Poisson case: Approximate with

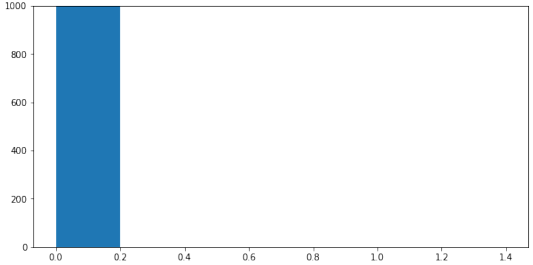
$$k \approx \frac{\sum_{t=1}^{T} \tilde{\lambda}_t}{\frac{1}{T-1}\sum_{t=1}^{T}(\tilde{\lambda}_t - \overline{\lambda})^2},$$

where $\tilde{\lambda}_t, t = 1, \ldots T$ are samples from the variational distribution $\Lambda|\mathcal{D}$

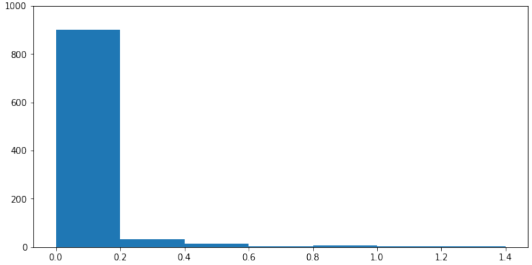
Implications for posterior pricing

- MTPL frequency data [Dutang and Charpentier, 2020]
- Poisson neural network
- Histograms of $1/k = \frac{Var(E[Y|\mathbf{\Omega}])}{E[Var(Y|\mathbf{\Omega})]}$

2×10



2×50



Conclusions

- Bayesian neural networks capture parameter uncertainty
- Can identify where models are confident, where models fail
- Implications for prior pricing: increased predictive variance
- Implications for posterior pricing: update rules for BLUP predictions
- Empirical finding: well calibrated neural networks do not introduce much parameter uncertainty

Conclusion

- Thanks for your attention!
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