Problemas

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1)

a) $\dfrac{dy}{dx} = \dfrac{x^2 y^2}{1+x}$

Solución:

$\int \frac{1}{y^2}\,dy = \int \frac{x^2}{1+x}\,dx$

$\int y^{-2}\,dy = \int \frac{x^2-1+1}{1+x}\,dx$

$\frac{y^{-1}}{-1} + c = \int \frac{(x-1)\cancel{(x+1)}}{\cancel{1+x}} + \frac{1}{1+x}\,dx$

$\frac{-1}{y} + c_1 = \int \left(x-1+\frac{1}{1+x}\right)dx$

$\frac{-1}{y} + c_1 = x^2 - x + \ln(1+x) + c$

$\frac{1}{y} = -x^2 + x - \ln(1+x) - c$

$\boxed{y = \frac{1}{x^2 + x - \ln(1+x) - c}}$

Mediante el algoritmo de la división

$x^2 \;\{\; \underline{x+1}$

$-x^2 - x \;\{\; x-1$

$-x$

$+\; x+1$

1

$\frac{x^2}{1+x} = \frac{\cancel{(x+1)}(x-1)+1}{\cancel{1+x}} \quad \frac{}{1+x}$

$\frac{x^2}{1+x} = x - 1 + \frac{1}{1+x}$

2) $e^x \cdot y\,\frac{dy}{dx} = e^{-y} + e^{-2x-y}$

Solución

$e^x \cdot y\,\frac{dy}{dx} = e^{-y} + e^{-2x} \cdot e^{y}$

$e^x \cdot y \cdot \frac{dy}{dx} = e^{-y}(1+e^{-2x})$

$\frac{y \cdot dy}{e^{-y}} = \frac{(1+e^{-2x})}{e^x}\,dx$

$\int y e^{y}\,dy = \int \left(\frac{1}{e^x} + \frac{e^{-2x}}{e^x}\right)dx$

$\int y e^{y}\,dy = \int (e^{-x} + e^{-3x})\,dx$

Integración por partes.

	D	I
+	y	e^y
−	1	e^y
+	0	e^y

$y e^{y} - 1 e^{y} = \frac{e^{-x}}{-1} + \frac{e^{-3}}{-3} + c$

$y e^{y} - e^{y} + e^{-x} + \frac{1}{3}e^{-3x} = c$

3) $\dfrac{dy}{dx} = \dfrac{xy + 2y - x - 2}{xy - 3y + x - 3}$

Solución:

$\frac{dy}{dx} = \frac{y(x+2) - (x+2)}{y(x-3) + (x-3)}$

$\frac{dy}{dx} = \frac{(x+2)(y-1)}{(x-3)(y+1)}$

$\int \frac{y+1}{y-1}\,dy = \int \frac{x+2}{x-3}\cdot dx$

$\int \frac{y-1+1+1}{y-1}\,dy = \int \frac{x-3+3+2}{x-3}\,dx$

$\int \left(\frac{\cancel{y-1}}{\cancel{y-1}} + \frac{2}{y-1}\right)dy = \int \left(\frac{\cancel{x-3}}{\cancel{x-3}} + \frac{5}{x-3}\right)dx$

$y + 2\ln(y-1) = x + 5\ln(x-3) + c$

$y + 2\ln(y-1) - x - 5\ln(x-3) = c$

Algoritmo de la multiplicación

$D \,\underline{|\, d}$

$R \quad q$

$D = d \cdot q + R$

Resuelvan la Ecuación Diferencial sujeta a la condición Inicial que se indica.

a) $\dfrac{dx}{dy} = 4(x^2+1) \,.\; x\left(\frac{\pi}{4}\right) = 1$

$\boxed{\int \frac{1}{1+u^2}\,du = \arctan(u) + c}$

Solución:

$\int \frac{1}{x^2+1}\,dx = \int 4\,dy$

$\arctan(x) = 4y + c$

$x\left(\frac{\pi}{4}\right) = 1 \quad y = \pi/4, \; x = 1$

$\arctan(1) = 4\left(\frac{\pi}{4}\right) + c$

$\arctan(1) = \pi + c$

$\frac{\pi}{4} = \pi + c$

$c = -\frac{3\pi}{4}$

Rpta: en (x)

$\arctan(x) = 4y - \frac{3\pi}{4}$

$4y = \frac{3}{4}\pi + \arctan(x)$

$y = \frac{3}{16}\pi + \frac{1}{4}\arctan(x)$

b) Resuelva: $\dfrac{dy}{dx} = (x+y+1)^2$

Sol:

Sea $u = x + y + 1 \;\rightarrow\; \boxed{\frac{dy}{dx} = u^2}$

$du = dx + dy$

$\frac{du}{dx} = \frac{dx}{dx} + \frac{dy}{dx}$

$\frac{du}{dx} = 1 + u^2$

$\int \frac{1}{1+u^2}\,du = \int dx$

$\arctan(u) = x + c$

$\arctan(x+y+1) = x + c$

$x + y + 1 = \tan(x+c)$

$y = \tan(x+c) - x - 1$

$\tan(\theta) = \frac{\sqrt{3}}{3}$

$\theta = \arctan\left(\frac{\sqrt{3}}{3}\right)$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = 30^\circ = \frac{\pi}{6}$ radian

4) Resuelva: $\dfrac{dy}{dx} = \cos(x+y)$

Solución:

Para Puntos

1) Resuelva $\dfrac{dy}{dx} = 1 + e^{y-x+5}$

2) $x^2 y^2\,dy = (y+1)\,dx$

Solución:

$\int (u-1)^2\,du$

4) Resolva: $\frac{dy}{dx} = \cos(x+y)$

Solución:

↳ Sea $u = x+y \sim \frac{dy}{dx} = \cos(u)$

$du = dx + dy$

$\frac{du}{dx} = \cancel{\frac{dx}{dx}} + \frac{dy}{dx}$

$\frac{du}{dx} = 1 + \cos(u)$

$\int \frac{1}{1+\cos(u)} du = \int dx$

$\int \frac{1}{1+\cos(x)} dx = \operatorname{cosec}(x) - \operatorname{cotg}(x) + c$

$\int \frac{1}{(1+\cos(x))} \cdot \frac{(1-\cos x)}{(1-\cos x)} \cdot dx = \int \frac{1-\cos x}{1-\cos^2 x} dx = \int \frac{1-\cos x}{\operatorname{sen}^2 x} dx$

$\int \frac{1}{\operatorname{sen}^2(x)} dx - \int \frac{\cos(x)}{\operatorname{sen}^2(x)} dx$

$u = \operatorname{Sen} x$

$du = \cos x \, dx$

$\int \operatorname{cosec}^2(x) dx - \int \frac{1}{u^2} du$

$-\operatorname{cotg}(x) - \int u^{-2} du$

$-\operatorname{cotg}(x) - \frac{u^{-1}}{-1} + c$

$-\operatorname{cotg}(x) + \frac{1}{\operatorname{sen}(x)} + c = \operatorname{cotg}(x) + \operatorname{cosec}(x) + c$

$\operatorname{Cosec}(u) - \operatorname{cotg}(u) = x + c$

$\operatorname{cosec}(x+y) - \operatorname{cotg}(x+y) = x + c$

Para Puntos

1) Resolva $\frac{dy}{dx} = 1 + e^{y-x+5}$

2) $x^2 y^2 dy = (y+1) dx$

Solución:

$\int \frac{y^2}{(y+1)} dy = \int \frac{1}{x^2} dx$

$\int \frac{(u-1)^2}{u} du$

$\int u - 2 \frac{1}{u} du = -\frac{1}{x} + C$

$\int u du - \int 2 du + \int \frac{1}{u} du = -\frac{1}{x} + C$

$\frac{u^2}{2} - 2u + \ln(u) = -\frac{1}{x} + C$

$\frac{(y+1)^2}{2} - 2(y+1) + \ln(y+1) + c = -\frac{1}{x} + C$

$\frac{y^2 + 2\ln(y+1) + 1}{2} - 2 + \frac{1}{x} = C$