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EJEMPLOS

Sea (X, Y) un vector aleatorio discreto con función de probabilidad dada por la siguiente tabla.

X\Y	0	1	2	3	total
0	1/10	1/10	2/10	1/10	5/10
1	1/10	2/10	1/10	1/10	5/10
total	2/10	3/10	3/10	2/10	1.00

CALCULE LA COVARIANZA COV[X,Y] Y E[X| Y=1]

Solución:

$$E[X|y=1] = \sum_x x\, p_{X|y}(x|y=1)$$

v.a discreta

$$p_{X|y}(x=0|y=1) = 1/3 \; ; \; p_{X|y}(x=1|y=1) = 2/3$$

X	$p_{X/y}(x/y=1)$	$xP(x/y=1)$
0	1/3	$0(1/3) = 0$
1	2/3	$1(2/3) = 2/3$
	1.00	

$$E[X|y=1] = 0\, p_{X/y}(0/y=1) + 1\, p_{X/y}(1/y=1)$$

$$= 0(1/3) + 1(2/3) = \frac{2}{3}$$

$$\to E[X|y=0]; \dots, E[X|y=3]$$

La covarianza: $Cov[X,Y] = \underbrace{E[XY]}_{p(x,y)} - \underbrace{E[X]}_{P_X(x)}\underbrace{E[Y]}_{P_Y(y)}$

$X = 0, 1 \} \; Y = 0, 1, 2, 3$

$(0,0), (0,1), (0,2), (0,3)$
$(1,0), (1,1), (1,2), (1,3)$

$$E[XY] = \sum_x \sum_y xy\, p(x,y)$$

$$= \cancel{0(0)\,p(0,0) + 0(1)p(0,1) + \dots + 0(3)\,p(0,3)}$$
$$+ \cancel{1(0)\,p(1,0)} + 1(1)\,p(1,1) + 1(2)\,p(1,2) + 1(3)\,p(1,3)$$
$$= p(1,1) + 2\,p(1,2) + 3\,p(1,3)$$
$$= 2/10 + 2 \cdot 1/10 + 3(1/10) = 7/10$$

$$E[XY] = 7/10$$

$$E[X] = \sum_x x\, p_X(x) = \cancel{0\, p_X(0)} + 1\, p_X(1) = 5/10$$

$$E[Y] = \sum_y p_Y(y) = \cancel{0\, p_Y(0)} + 1\, p_Y(1) + 2\, p_Y(2) + 3\, p_Y(3)$$

$$= \frac{3}{10} + 2\left(\frac{3}{10}\right) + 3\left(\frac{2}{10}\right) = \frac{15}{10}$$

$$Cov[X,Y] = E[XY] - E[X]E[Y]$$

$$= \frac{7}{10} - \frac{5}{10}\cdot\left(\frac{15}{10}\right) = -\frac{1}{20} = -0.05$$

Sea (X, Y) un vector aleatorio continuo con función de densidad dada por

$$f_{X,Y}(x,y) = \begin{cases} x+y & \text{si } \ 0 < x, y < 1, \\ 0 & \text{en otro caso.} \end{cases}$$

CALCULE LA COVARIANZA COV[X,Y] Y E[X|Y=y]

Solución:

$$E[X|Y=y] = \int_{-\infty}^{+\infty} x\, f_{X|Y}(x|y=y)\, dx$$

$$f_{X|Y}(x|y) = \begin{cases} \dfrac{2(x+y)}{1+2y} \ ; & x \in (0,1) \ ; \ \boxed{y \in (0,1)} \\ 0 \ ; & c.c. \end{cases}$$

$$E[X|Y=y] = \int_0^1 x\, \frac{2(x+y)}{1+2y}\, dx = \frac{1}{1+2y}\int_0^1 2x(x+y)\, dx$$

$$= \frac{1}{1+2y}\left(\frac{2}{3}x^3 + x^2 y\right)\Big|_0^1 = \frac{(2/3 + y)}{1+2y}$$

$$E[X|Y=y] = \frac{2/3 + y}{1+2y}$$

$$Cov[X,Y] = E[XY] - E[X]E[Y]$$

$$E[XY] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy\, f(x,y)\, dy\, dx$$

$$E[X] = \int_{-\infty}^{+\infty} x\, f_X(x)\, dx$$

$$E[Y] = \int_{-\infty}^{+\infty} y\, f_Y(y)\, dy$$

$$E[XY] = \int_0^1\int_0^1 xy(x+y)\, dy\, dx$$

$$f_X(x) = \int_0^1 f(x,y)\, dy = \int_0^1 (x+y)\, dy$$

$$= xy + \frac{y^2}{2}\Big|_0^1 = x + \frac{1}{2}$$

$$f_X(x) = x + \frac{1}{2} \ ; \ f_Y(y) = y + \frac{1}{2}$$

$f_X(x) = x + \frac{1}{2}$; $f_Y(y) = y + \frac{1}{2}$

$E[X] = \int_0^1 x\left(x+\frac{1}{2}\right) dx$; $E[Y] = \int_0^1 y\left(y+\frac{1}{2}\right) dy$

Propiedades:

1. $\text{Cov}(X,Y) = E(XY) - E(X)E(Y)$.
2. $\text{Cov}(X,Y) = \text{Cov}(Y,X)$.
3. $\text{Cov}(X,X) = \text{Var}(X)$.
4. $\text{Cov}(c,Y) = 0$.
5. $\text{Cov}(cX,Y) = c\,\text{Cov}(X,Y)$.
6. $\text{Cov}(X_1+X_2,Y) = \text{Cov}(X_1,Y) + \text{Cov}(X_2,Y)$.
7. Si X y Y son independientes, entonces $\text{Cov}(X,Y) = 0$.
8. En general, $\text{Cov}(X,Y) = 0 \not\Rightarrow X,Y$ independientes.

Coeficiente de Correlación:

$$\rho(X,Y) = \frac{\text{Cov}[X,Y]}{\sqrt{\text{Var}[X]\,\text{Var}[Y]}} \quad ; \text{Var}[X] \neq 0 \text{ y } \text{Var}[Y] \neq 0$$

Var. aleatorias!

- $-1 \leq \rho(X,Y) \leq 1$ $\quad$ $-1 \leq r \leq 1$ → Muestra.
- Si X, Y son v.a's independientes
$$\rho(X,Y) = 0$$
- Si c = cte:
$$\rho(cX,Y) \neq c\,\rho(X,Y), \text{ en general}$$
- $\rho(X_1+X_2,Y) \neq \rho(X_1,Y) + \rho(X_2,Y)$
- $|\rho(X,Y)| = 1$ si y solo si
$Y = aX + b$; a y b constantes. $\quad y = a + bx$
  - Si $a > 0$, $\rho(X,Y) = 1$
  - Si $a < 0$; $\rho(X,Y) = -1$

Nota 1

$\text{Var}[X] = E[X^2] - (E[X])^2$ $\quad$ $E[X^2] = \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx$

$E[X^2] = \sum_x x^2 p_X(x)$

$$E[x^2] = \sum_{x} x^2 p_x(x)$$

Nota 2:

Sea (x,y) un vector aleatorio y sea $h(x,y)$ una función de (x,y)

$$\left\{ \begin{array}{l} h(x,y) = x^2+y^2 \\ h(x,y) = \sqrt{x} + \sqrt{y} \end{array} \right.$$

- $E[h(x,y)] = ?$ es una v.a.
  - **Discreto:**

$$E[h(x,y)] = \sum_{x}\sum_{y} h(x,y)\, p(x,y)$$

  - **Continuo:**

$$E[h(x,y)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(x,y) f(x,y)\, dy\, dx$$

Varianza: $(X_1, X_2, X_3, \dots, X_n)$

matriz varianza - covarianza.

$$Var[X_1, X_2, \dots, X_n] = \begin{bmatrix} Var[X_1] & Cov[X_1,X_2] & \cdots & Cov[X_1,X_n] \\ Cov[X_2,X_1] & Var[X_2] & \cdots & Cov[X_2,X_n] \\ \vdots & & & \\ Cov[X_n,X_1] & Cov[X_n,X_2] & \cdots & Var[X_n] \end{bmatrix}_{n\times n}$$

$$E[X_1, \dots, X_n] = (E[X_1], \dots, E[X_n])$$

Coeficiente de Correlación:

$$\rho(X_1, X_2, \dots, X_n) = \begin{bmatrix} \rho(X_1,X_1) & \rho(X_1,X_2) & \cdots & \rho(X_1,X_n) \\ \rho(X_2,X_1) & \rho(X_2,X_2) & \cdots & \rho(X_2,X_n) \\ \vdots & & \ddots & \\ \rho(X_n,X_1) & \rho(X_n,X_2) & \cdots & \rho(X_n,X_n) \end{bmatrix}$$

Matriz de correlación!