Again, by this theorem, if X and Y are independent, then $\boldsymbol{E(XY) = E(X)E(Y)}$. As we know from the discrete case, the converse of this fact is not *necessarily* true. (See Example 8.12.)

EXERCISES

A

1. Let the joint probability mass function of random variables X and Y be given by

$$p(x,y) = \begin{cases} \dfrac{1}{25}(x^2+y^2) & \text{if } x = 1, 2, \quad y = 0, 1, 2 \\ 0 & \text{elsewhere.} \end{cases}$$

Are X and Y independent? Why or why not?

2. Let the joint probability mass function of random variables X and Y be given by

$$p(x,y) = \begin{cases} \dfrac{1}{7}x^2y & \text{if } (x,y) = (1,1), (1,2), (2,1) \\ 0 & \text{elsewhere.} \end{cases}$$

Are X and Y independent? Why or why not?

3. Let X and Y be independent random variables each having the probability mass function

$$p(x) = \frac{1}{2}\Big(\frac{2}{3}\Big)^x, \qquad x = 1, 2, 3, \ldots.$$

Find $P(X = 1, Y = 3)$ and $P(X + Y = 3)$.

4. From an ordinary deck of 52 cards, eight cards are drawn at random and without replacement. Let X and Y be the number of clubs and spades, respectively. Are X and Y independent?

5. What is the probability that there are exactly two girls among the first seven and exactly four girls among the first 15 babies born in a hospital in a given week? Assume that the events that a child born is a girl or is a boy are equiprobable.

6. Suppose that the number of claims received by an insurance company in a given week is independent of the number of claims received in any other week. An actuary has calculated that the probability mass function of the number of claims received in a random week is

$$p(n) = \Big(\frac{1}{5}\Big)\Big(\frac{4}{5}\Big)^n, \qquad n \geq 0.$$

Find the probability that the company will receive exactly 8 claims during the next two weeks.

7. Let X and Y be two independent random variables with distribution functions F and G, respectively. Find the distribution functions of $\max(X, Y)$ and $\min(X, Y)$.

8. A fair coin is tossed n times by Adam and n times by Andrew. What is the probability that they get the same number of heads?

9. The joint probability mass function $p(x, y)$ of the random variables X and Y is given by the following table. Determine if X and Y are independent.

x \ y	0	1	2	3
0	0.1681	0.1804	0.0574	0.0041
1	0.1804	0.1936	0.0616	0.0044
2	0.0574	0.0616	0.0196	0.0014
3	0.0041	0.0044	0.0014	0.0001

10. Let the joint probability density function of random variables X and Y be given by

$$f(x, y) = \begin{cases} 2 & \text{if } 0 \leq y \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Are X and Y independent? Why or why not?

11. Suppose that the amount of cholesterol in a certain type of sandwich is $100X$ milligrams, where X is a random variable with the following probability density function:

$$f(x) = \begin{cases} \dfrac{2x+3}{18} & \text{if } 2 < x < 4 \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability that two such sandwiches made independently have the same amount of cholesterol.

12. Let the joint probability density function of random variables X and Y be given by

$$f(x, y) = \begin{cases} x^2 e^{-x(y+1)} & \text{if } x \geq 0, \quad y \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Are X and Y independent? Why or why not?

13. Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} 8xy & \text{if } 0 \leq x < y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine (Verifique) si

Determine if $E(XY) = E(X)E(Y)$.

14. Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} 2e^{-(x+2y)} & \text{if } x \geq 0, \quad y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X^2Y)$.