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Nise cap 2

1-a) $\mathcal{L}(u(t)) = \int_0^{\infty} e^{-st} \cdot 1 \cdot dt = -\frac{1}{s} e^{-st} \Big|_0^{\infty} = \frac{1}{s}$

b-) $\mathcal{L}(t u(t)) = \int_0^{\infty} e^{-st} t \, dt = \frac{1}{s^2}$

c-) $\mathcal{L}(\sin(wt) u(t)) = \int_0^{\infty} \sin(wt) u(t) e^{-st} dt$

$= \frac{e^{-st}}{s^2+w^2} (s \sin(wt) - w \cos(wt)) \Big|_0^{\infty}$

$= \frac{w}{s^2+w^2}$

2-a) $f(t) = e^{-at} \sin(wt) u(t) \Rightarrow \mathcal{L}(f(t)) = \frac{w}{(s+a)^2 - w^2}$

b-) $g(t) = e^{-at} \cos(wt) u(t) \Rightarrow \mathcal{L}(g(t)) = \frac{s+a}{(s+a)^2 - w^2}$

8-) $y''' + 3y'' + 5y' + y = x''' + 4x'' + 6x' + 8x$

$\Rightarrow (s^3 + 3s^2 + 5s + 1) y(s) = (s^3 + 4s^2 + 6s + 8) x(s)$

$\Rightarrow \frac{y(s)}{x(s)} = \frac{s^3 + 3s^2 + 5s + 1}{s^3 + 4s^2 + 6s + 8}$

9.) a) $\frac{x(s)}{F(s)} = \frac{7}{s^2 + 8s + 10}$

$\Rightarrow (s^2 + 8s + 10)\,x(s) = 7F(s) \Rightarrow x'' + 8x' + 10x = 7F$

c) $\frac{x(s)}{F(s)} = \frac{s+3}{s^3 + 11s^2 + 12s + 18}$

$\Rightarrow (s^3 + 11s^2 + 12s + 18)(x(s)) = F(s)(s+3)$
$\Rightarrow x''' + 11x'' + 12x + 18 = F' + 3F$

11.) $\frac{s^4 + 2s^3 + 5s^2 + s + 4}{s^5 + 3s^4 + 2s^3 + 4s^2 + 5s + 2}$ $C(s)$, $R(t) = t^3$

$= (s^5 + 3s^4 + 2s^3 + 4s^2 + 5s + 2)\,C(s) = (s^4 + 2s^3 + 5s^2 + s + 4)(R(s))$
$= c''''' + 3c'''' + 2c''' + 4c'' + 5c' + 2c = r'''' + 2r''' + 5r'' + 5r' + 4r$

$\Rightarrow 188(t) + (36 + 90t + 9t^2 - 3t^3)\,u(t)$

Frankling cap 3

3.2. b) $f(t) = 3 + 7t + t^2 + \delta(t)$
$\mathcal{L} = \frac{3}{s} + \frac{7}{s^2} + \frac{2}{s^3} + 1$

d) $f(t) = t^2 + 2t + 1$
$\mathcal{L}(f(t)) = \frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}$

a.) $f(t) = \operatorname{sen}(t)$

$\mathcal{L}(f(t)) = \dfrac{1}{s^2+1^2}$

3.5) a.) $f(t) = 3\cos(6t)$

$\mathcal{L}(f(t)) = \dfrac{3s}{s^2+6^2} = \dfrac{3s}{s^2+36}$

b.) $f(t) = \operatorname{sen}(2t) + 2\cos(2t) + e^{-t}\operatorname{sen}(2t)$

$\mathcal{L}(f(t)) = \dfrac{1}{s^2+4} + \dfrac{2s}{s^2+4} + \dfrac{2}{(s+1)^2+4}$

c.) $f(t) = t^2 + e^{-2t}\operatorname{sen}(3t)$

$\mathcal{L}(f(t)) = \dfrac{2}{s^3} + \dfrac{3}{(s+2)^2+9}$

3.5) a.) $f(t) = \operatorname{sen}(t)\operatorname{sen}(3t) \Rightarrow \dfrac{1}{2}\left(-\cos(t+3t) + \cos(t-3t)\right)$

$\mathcal{L}(f(t)) = -\dfrac{1}{2}\mathcal{L}(\cos(4t)) + \dfrac{1}{2}(\cos 2t)$

$\mathcal{L}(f(t)) = -\dfrac{1}{2}\dfrac{s}{s^2+16} + \dfrac{1}{2}\dfrac{s}{s^2+4} = \dfrac{6s}{(s^2+4)(s^2+16)}$

3.7) a.) $f(s) = \dfrac{2}{s(s+2)}$ $\qquad 2 = A(s+2) + (s)B$

$\mathcal{L}^{-1}(f(s)) = \dfrac{A}{s} + \dfrac{B}{s+2}$ $\qquad s = -2 \Rightarrow B = -1/2$

$s = 0 \Rightarrow A = 1$

e.) $F(s) = \dfrac{1}{s^2+4}$

$\mathcal{L}^{-1} = \dfrac{1}{2} \operatorname{sen}(2t)$

f.) $F(s) = \dfrac{2(s+2)}{(s+1)(s^2+4)}$

$\mathcal{L}^{-1} = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+4}$ $\quad 2s+4 = As^2 + Bs^2 + Bs + Cs + 4A + C$

$A + B = 0$ $\quad A = 2/5$

$B + C = 2$ $\quad B = -2/5$

$4A + C = 4$ $\quad C = 12/5$

$\Rightarrow \mathcal{L}^{-1}\{F(s)\} = \dfrac{2}{5} e^{-t} - \dfrac{2}{5} \cos(2t) + \dfrac{6}{5} \operatorname{sen}(2t)$

3.9 e) $y'' + 2y' = e^t$ $\quad y(0) = 1$ $\quad y'(0) = 2$

$\Rightarrow s^2 y(s) - s - 2 + 2(s\,y(s) - 1) = \dfrac{1}{s-1}$

$y(s) = \dfrac{s^3 + 3s + 2}{s(s-1)(s+2)} = \dfrac{1}{3(s-1)} + \dfrac{3}{2s} - \dfrac{5}{6(s+2)}$

$\mathcal{L}^{-1}\{y(s)\} = \dfrac{1}{3(s-1)} + \dfrac{3}{2s} - \dfrac{5}{6(s+2)} \Rightarrow y(t) = \dfrac{e^t}{3} + \dfrac{3}{2} - \dfrac{5e^{-2t}}{6}$