

# ELEC2104 – Week 3

## Schottky contact, PN Junctions

Reference for Fermi levels, Schottky contact:

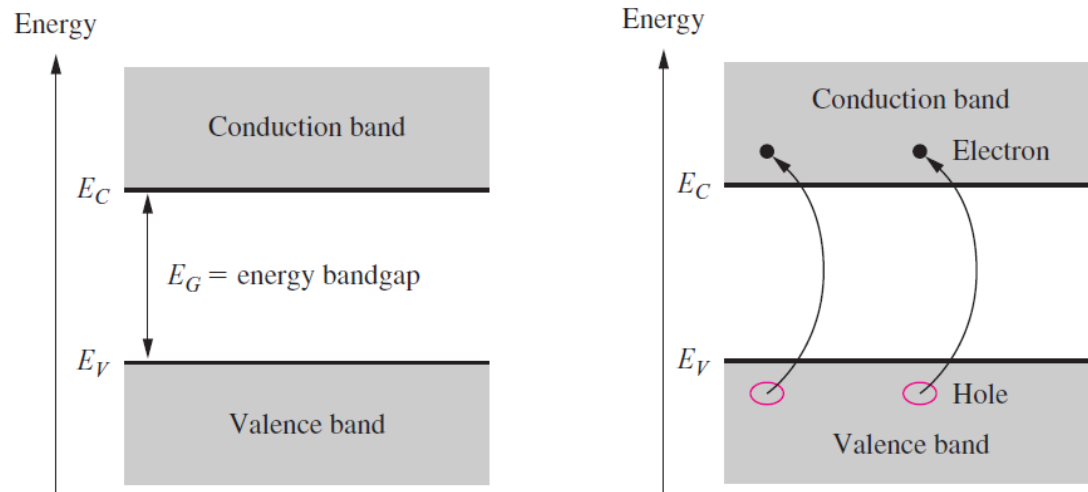
*Device Electronics for Integrated Circuits* by Richard S. Muller, Theodore I. Kamins, Mansun Chan



THE UNIVERSITY OF  
SYDNEY

# What have we learnt so far?

- › Silicon is a semiconductor with
  - $n$  is the number of (free) electrons in the conduction band
  - $p$  is the number of holes in the valence band



- › For both intrinsic and extrinsic semiconductor

$$pn = n_i^2$$

# What have we learnt so far?

- › Doping produces free electrons or holes in a semiconductor
- › In n-type semiconductor  $N_D \gg N_A$  and  $n \gg p$ , so:

$$n \approx N_D \text{ and } p \approx n_i^2/N_D$$

- › In p-type semiconductor  $N_A \gg N_D$  and  $p \gg n$ , so

$$p \approx N_A \text{ and } n \approx n_i^2/N_A$$

- › The sum of all charges is zero

$$p + N_D = n + N_A$$

# What have we learnt so far?

- › An electric field or a concentration gradient leads to the movement of these charge carriers
- › Drift current

$$j_n^{drift} = Q_n \mathbf{v}_n = (-qn)(-\mu_n \mathbf{E}) = qn\mu_n \mathbf{E} \quad \text{A/cm}^2$$

$$j_p^{drift} = Q_p \mathbf{v}_p = (+qp)(+\mu_p \mathbf{E}) = qp\mu_p \mathbf{E} \quad \text{A/cm}^2$$

- › Diffusion current

$$j_p^{diff} = (+q)D_p \left( -\frac{dp}{dx} \right) = -qD_p \frac{dp}{dx} \quad \text{A/cm}^2$$

$$j_n^{diff} = (-q)D_n \left( -\frac{dn}{dx} \right) = +qD_n \frac{dn}{dx} \quad \text{A/cm}^2$$

- › Total current

$$\mathbf{j}^T = \mathbf{j}^{drift} + \mathbf{j}^{diff}$$

# Fermi level



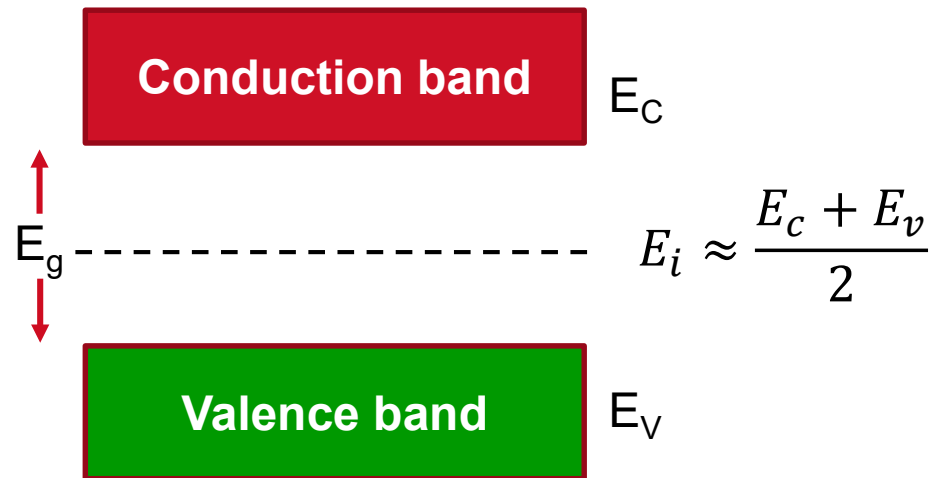
# Fermi energy level

## › Fermi level

- The energy level where the probability of finding a (free) electron is one-half

## › Intrinsic semiconductor

- $E_i$  is the undoped or intrinsic Fermi level (for semiconductors without doping)
  - $E_i = \frac{E_c + E_v}{2}$  (mid-gap)
- Generally, the Fermi level is denoted  $E_f$



$$n_i = p_i = \text{intrinsic carrier density}$$

# Fermi level

## › Definition of the Fermi level $E_f$ in the general case

Fermi-Dirac distribution

$$f = \frac{1}{1 + e^{(E-E_f)/k_B T}}$$

Possibility of a state with energy  $E$  being occupied by an electron

$$E = E_f: f = \frac{1}{2}$$

$$n = N_C \frac{1}{1 + e^{(E_C-E_f)/k_B T}} \approx N_C e^{-(E_C-E_f)/k_B T}$$

$$p = N_V \left( 1 - \frac{1}{1 + e^{(E_V-E_f)/k_B T}} \right) \approx N_V e^{-(E_f-E_V)/k_B T}$$

$n, p$ : electron and hole concentration

$N_C, N_V$ : effective density of states for the Conduction band and the Valance band

$k_B$ : Boltzmann constant

$n_i$ : intrinsic carrier concentration

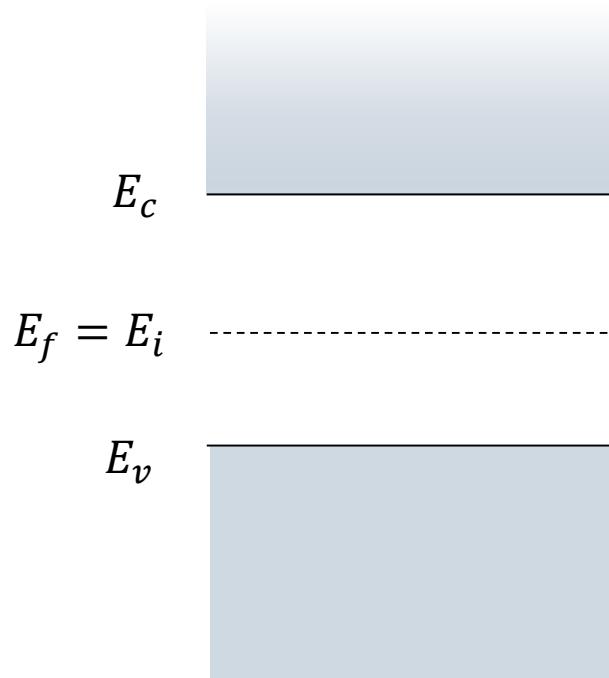
$$\text{since } n_i = N_C e^{-(E_C-E_i)/k_B T} = N_V e^{-(E_i-E_V)/k_B T}$$

$$\text{We have } n = n_i e^{(E_f-E_i)/k_B T} \quad p = n_i e^{(E_i-E_f)/k_B T}$$

$$(np = n_i^2)$$

# Fermi level and carrier concentration

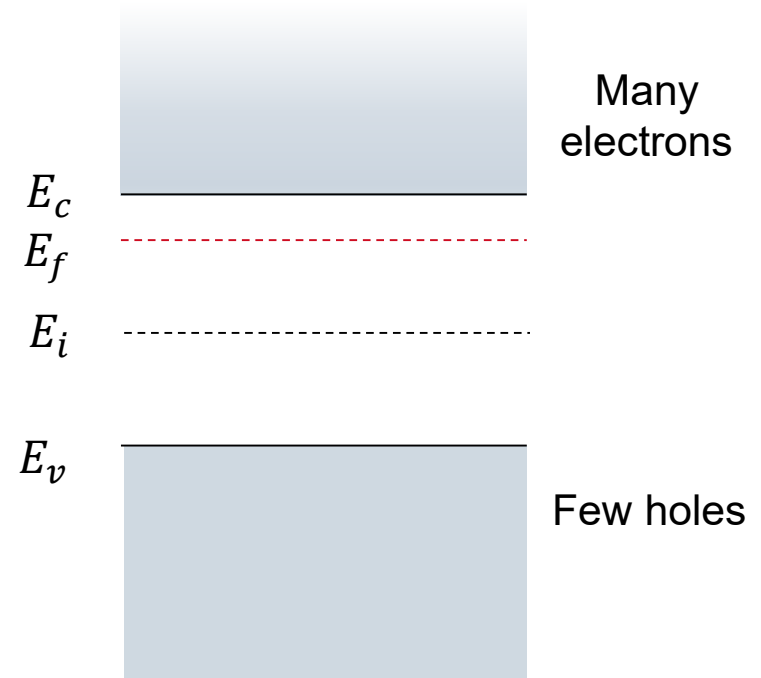
Intrinsic



$$n = n_i e^{(E_f - E_i)/k_B T}$$

$$p = n_i e^{(E_i - E_f)/k_B T}$$

N-doped



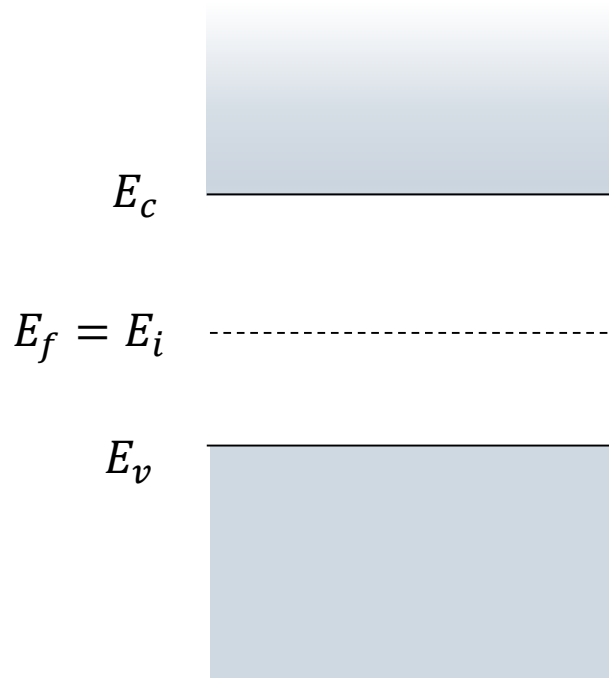
$$N_D \approx n = n_i e^{(E_f - E_i)/k_B T}$$

$$p = n_i e^{(E_i - E_f)/k_B T}$$



# Fermi level and Carrier concentration

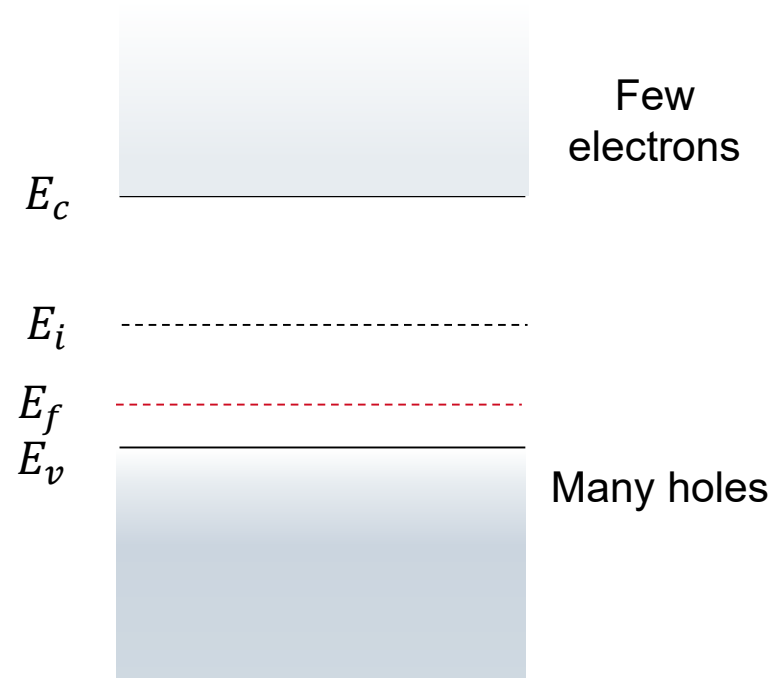
Intrinsic



$$n = n_i e^{(E_f - E_i)/k_B T}$$

$$p = n_i e^{(E_i - E_f)/k_B T}$$

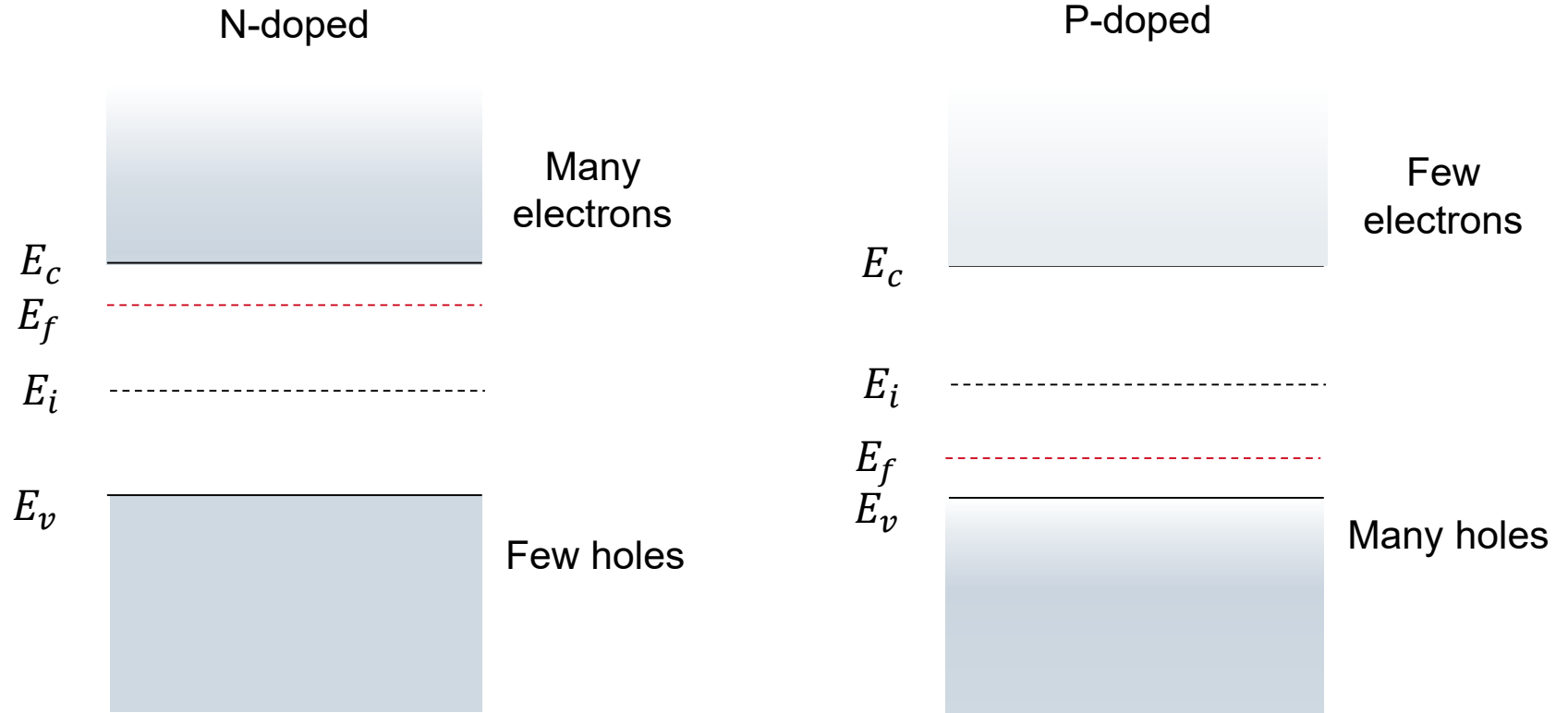
P-doped



$$n = n_i e^{(E_f - E_i)/k_B T}$$

$$N_A \approx p = n_i e^{(E_i - E_f)/k_B T}$$

# Fermi level and Carrier concentration



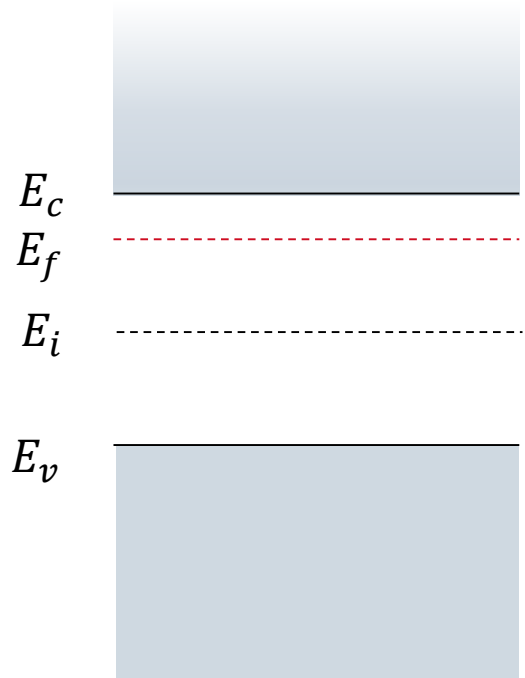
$$n = n_i e^{(E_f - E_i)/k_B T}$$

$$p = n_i e^{(E_i - E_f)/k_B T}$$

$$E_f \nearrow, n \nearrow, p \searrow$$

Remember this

# Fermi level and Carrier concentration



Given  $N_D$ , ask  $E_f$ ?

since  $n = n_i e^{(E_f - E_i)/k_B T} \approx N_D$

We have  $(E_f - E_i) = k_B T \ln \frac{N_D}{n_i}$

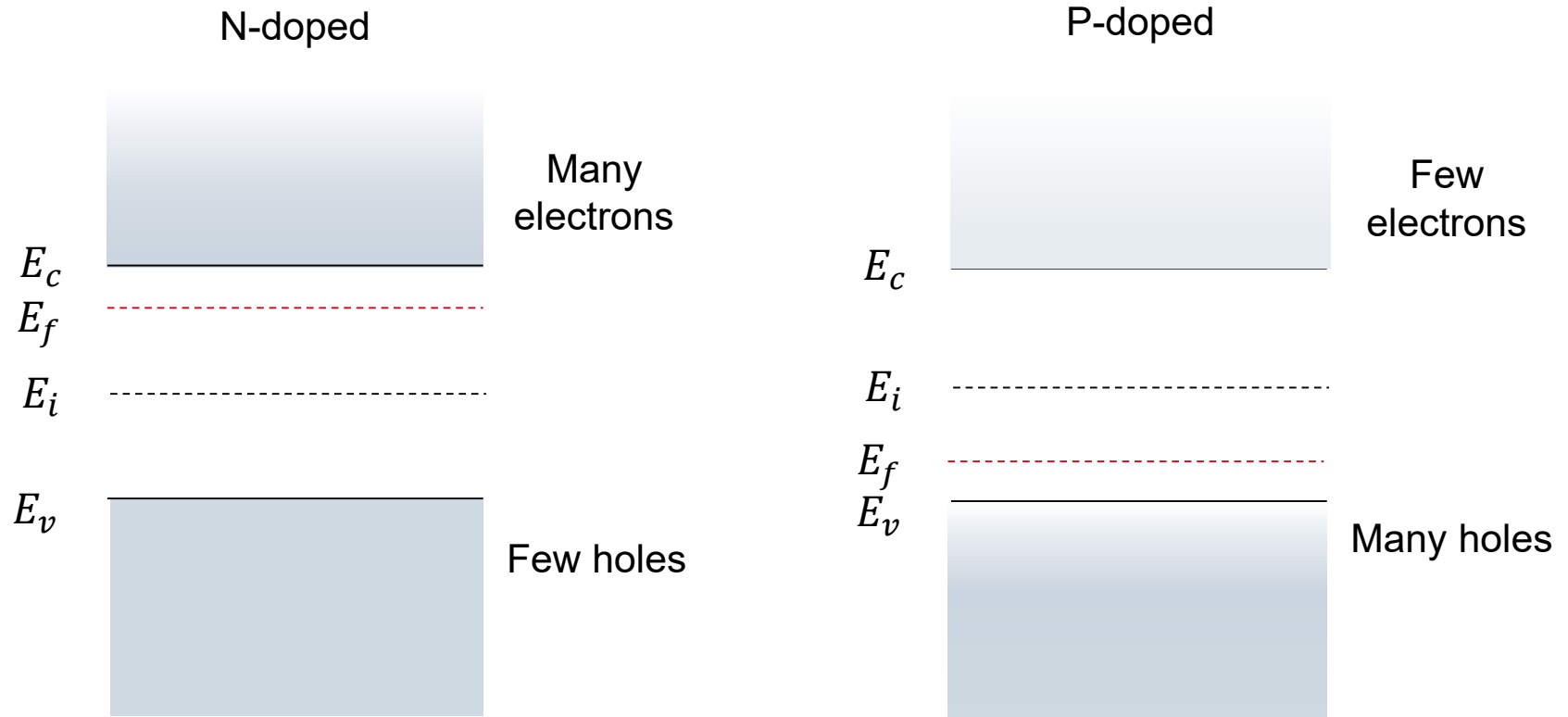
Example:

$$N_D = 10^{16} \text{cm}^{-3}, n_i = 10^{10} \text{cm}^{-3}, k_B T = 26 \text{meV}$$

$$(E_f - E_i) = 0.026 \text{eV} \times \ln \frac{10^{16}}{10^{10}} = 0.36 \text{eV}$$

$$N_D \approx n = n_i e^{(E_f - E_i)/k_B T}$$

# Fermi level



Fermi level is a token for carrier concentration.

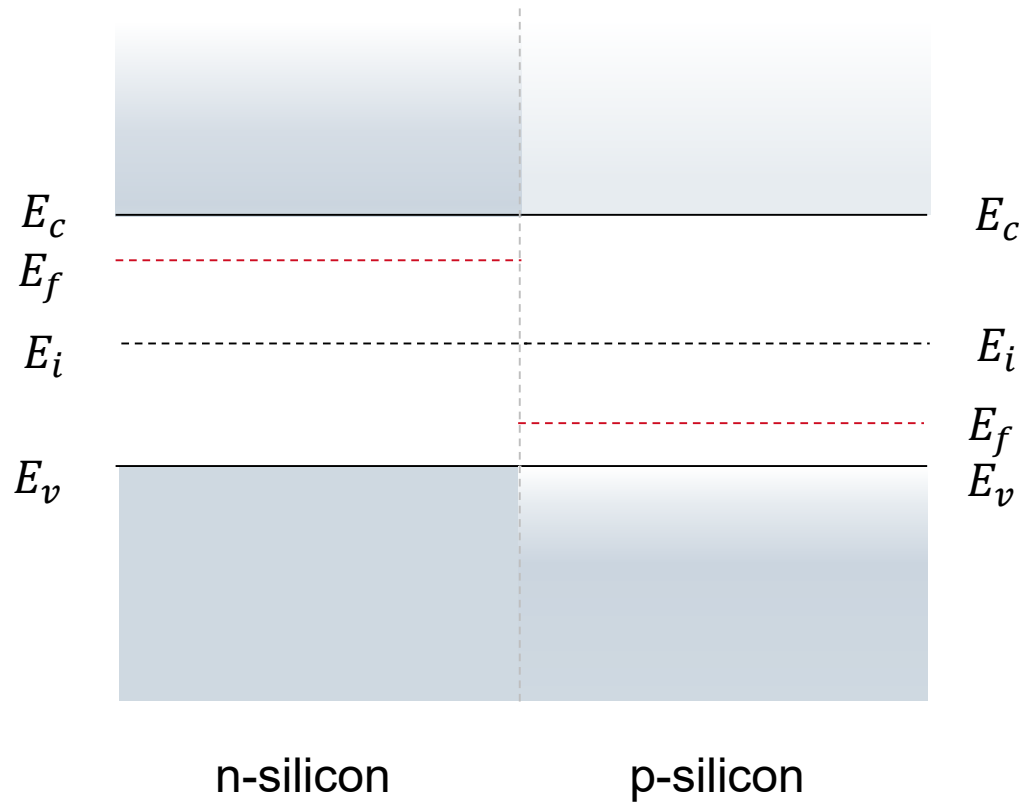
It is where the electron filling probability *would be*  $\frac{1}{2}$ .

The higher  $E_f$ , the more electrons, less holes. Vice versa.

Fermi level must levels



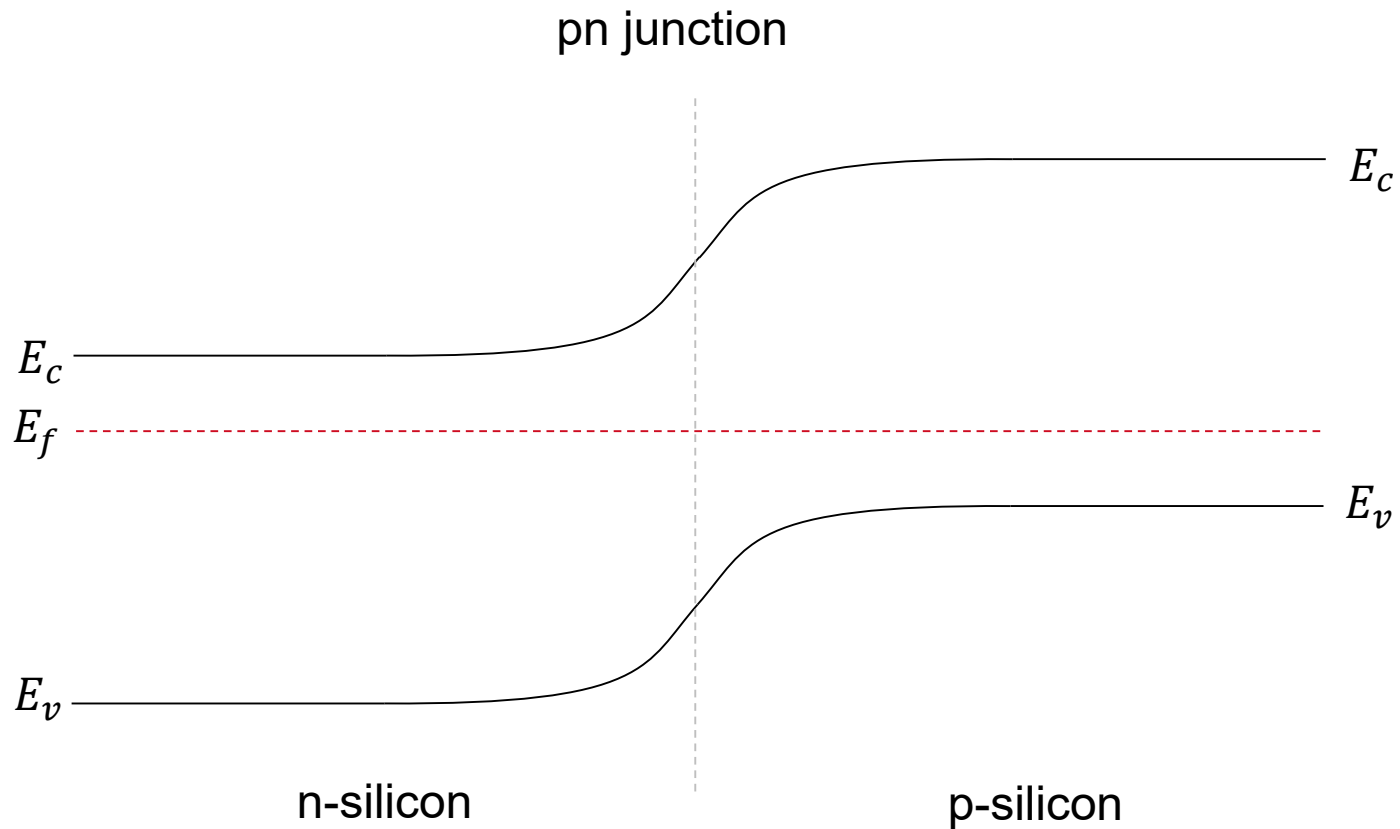
In a piece of any material in equilibrium, the Fermi level must  
levels  
through out the structure.



What happens now?

How does the Fermi level LEVEL?

# pn junction



Note:  $E_c$ ,  $E_v$  can move up and down together (with  $E_c - E_v = E_g$  fixed), just like the voltage at a node in a circuit. If there are electric fields in the space, these “voltage levels” can shift up and down.



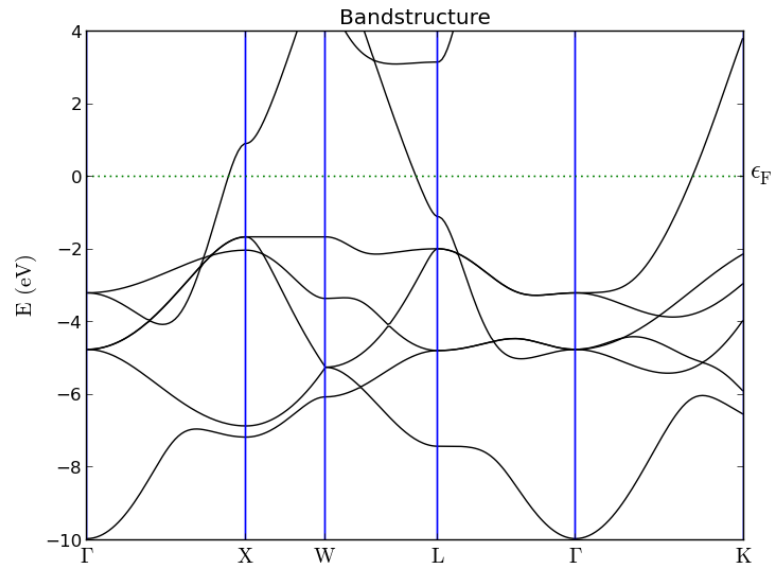
# Metal-semiconductor contact (Schottky diode)

# Metal

## › Band structure and Fermi level of metal

- No bandgap, Fermi level crosses some bands
- Many carriers  $10^{23}\text{cm}^{-3}$
- Fermi level won't move with respect to the bands
  - Even if you could “dope” it, the extra carriers are negligible compared to intrinsic

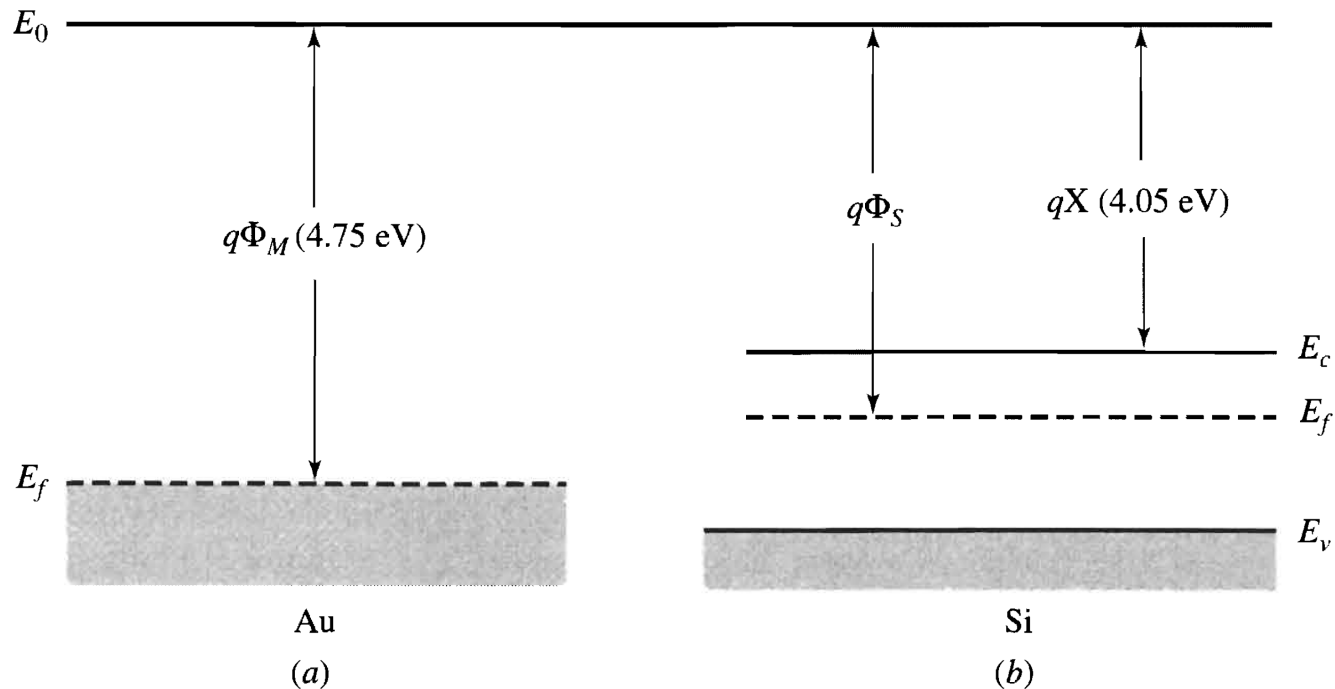
### Gold



# Metal vs Semiconductor

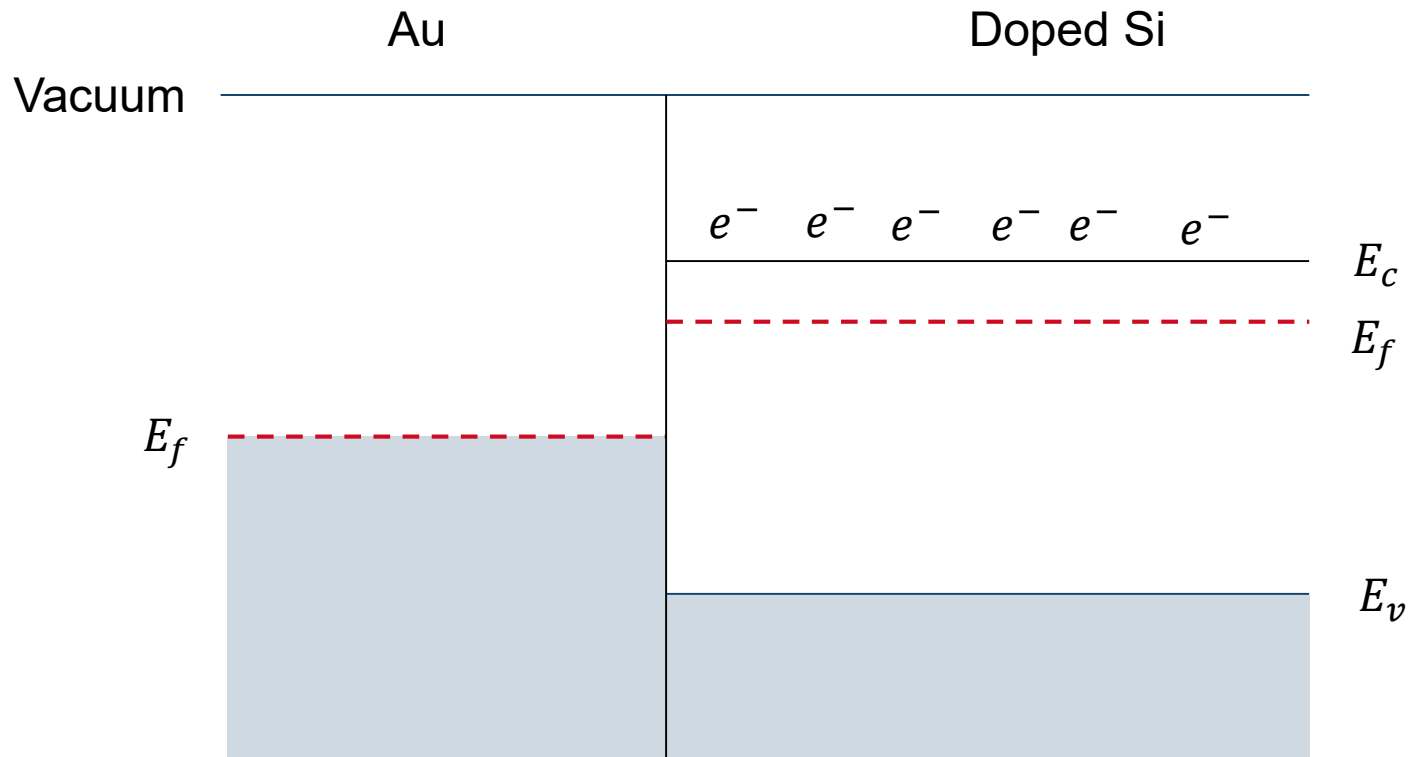
- ›  $\phi$ : work function (energy needed to kick electrons out to vacuum)

Without electric field, the energy levels are flat over space



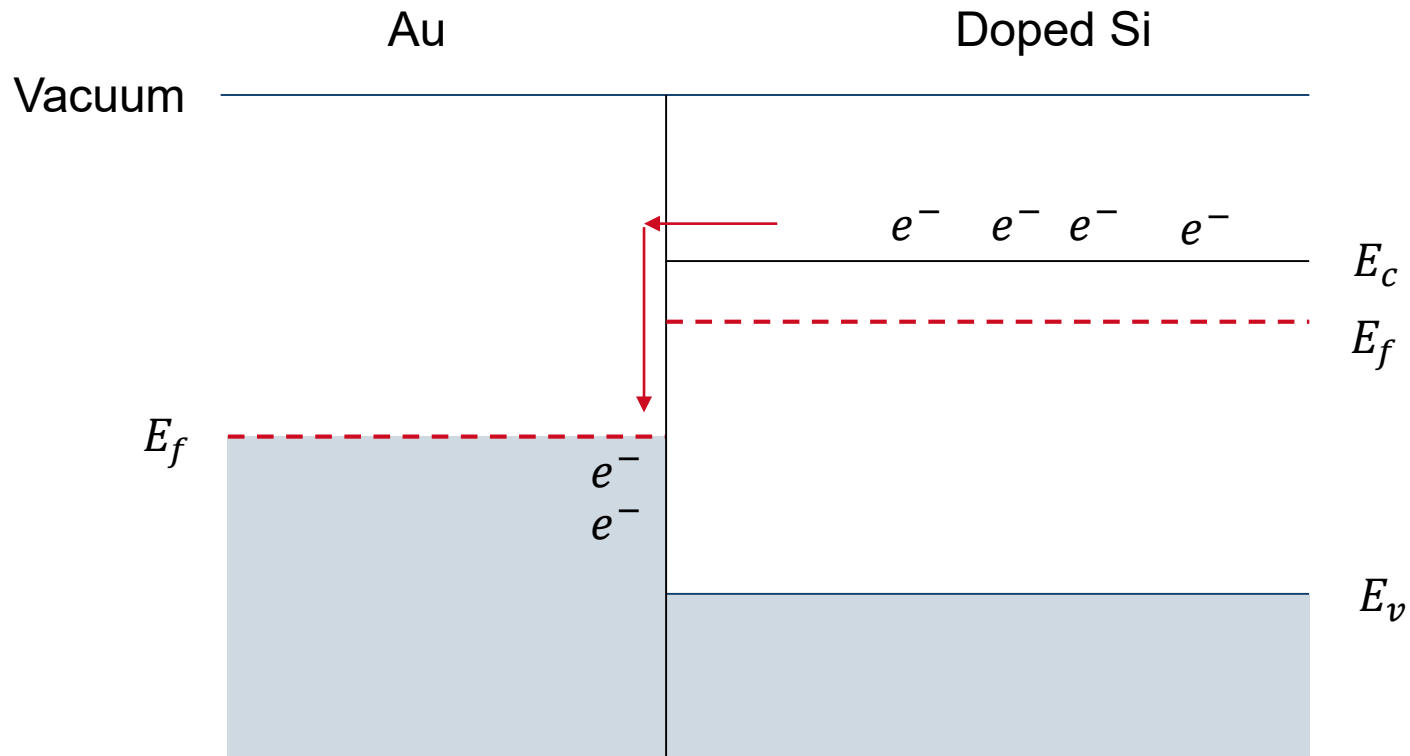
$\phi_m$  metal work function  
 $\phi_s$  semiconductor work function  
 $\chi$  electron affinity

# Metal-Semiconductor contact

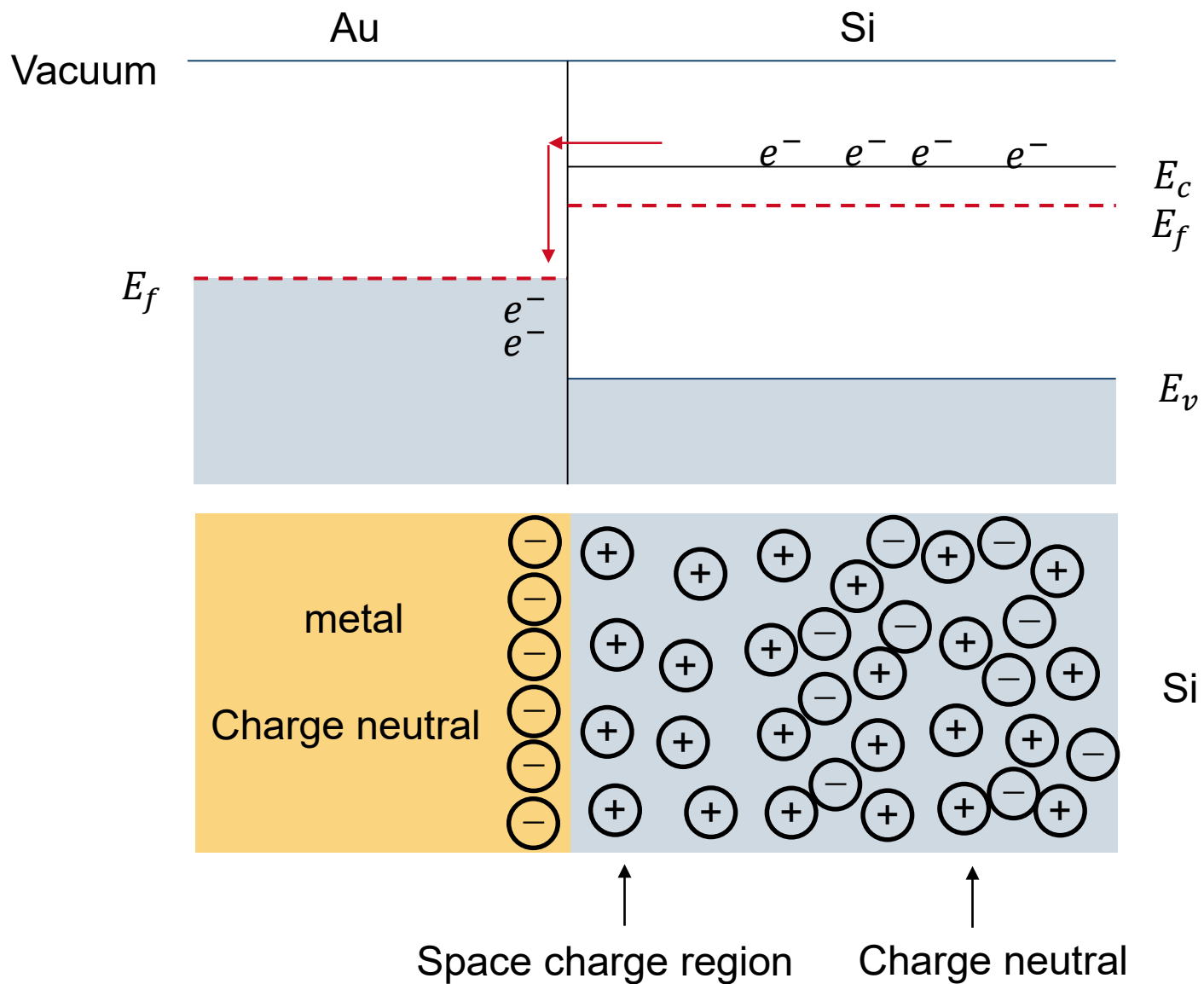


Fermi level: Si > Au  
“water level”

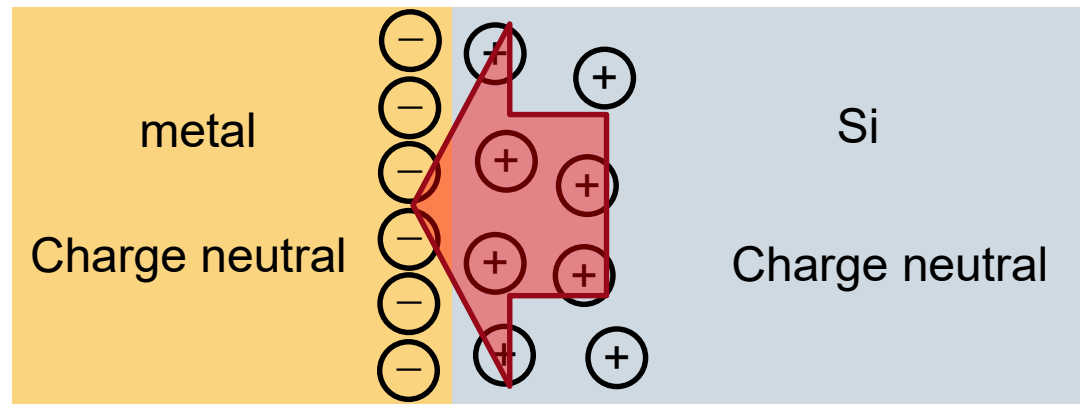
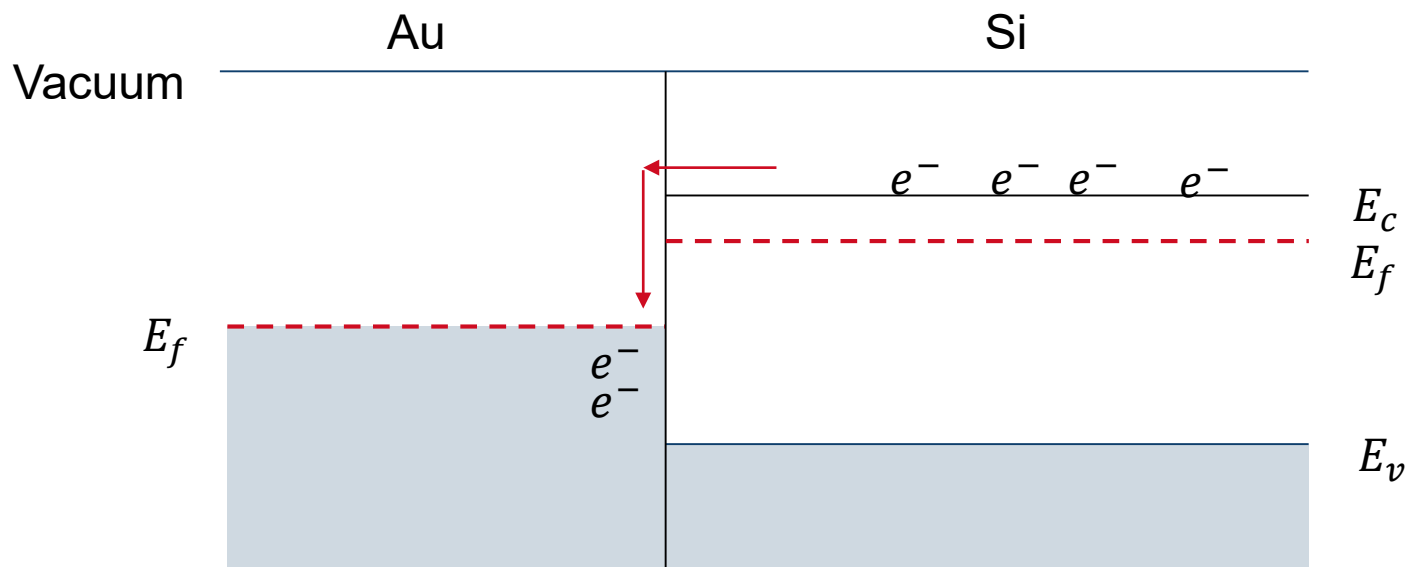
# Metal-Semiconductor contact



Electrons move from semiconductor to metal surface  
Leaving behind positively charged region (positively charged ion lattice)  
Electric field is created



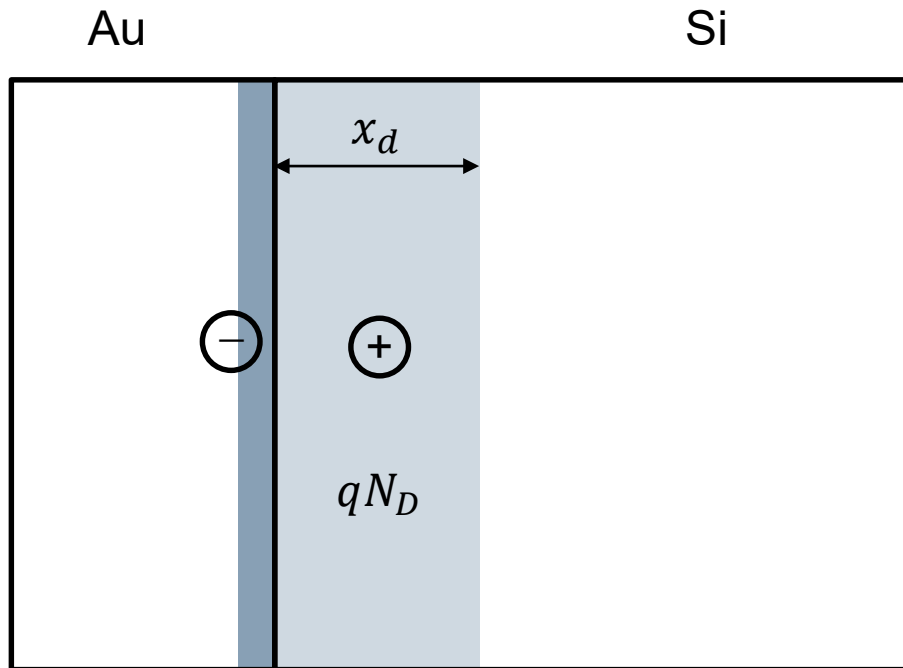
The charge can't flow forever. What stops them eventually?



↑  
Space charge region

Electric field established

# Charge in depletion region



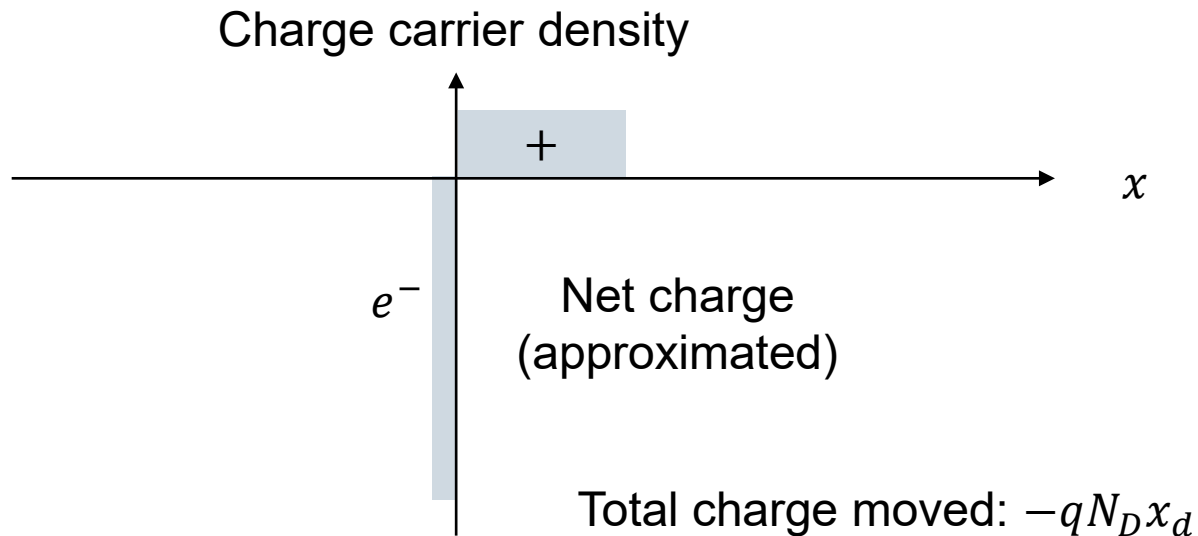
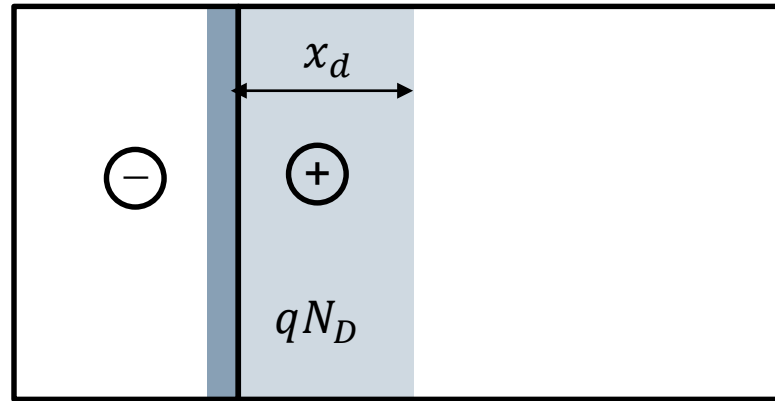
For simplicity, assume the depletion region has no free carriers (the free carrier density is orders of magnitude smaller than original)

Hence, the space charge region has a uniform charge density:  $qN_D$

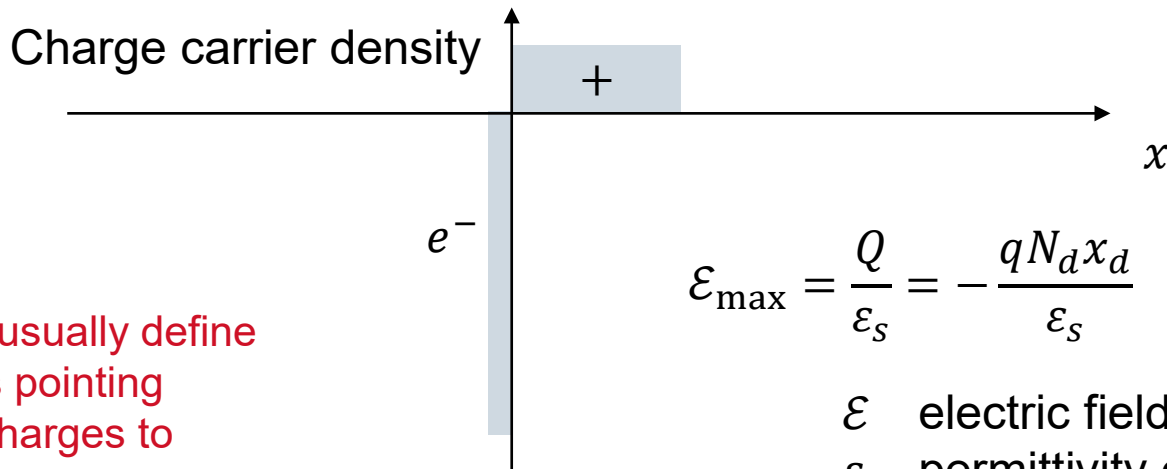
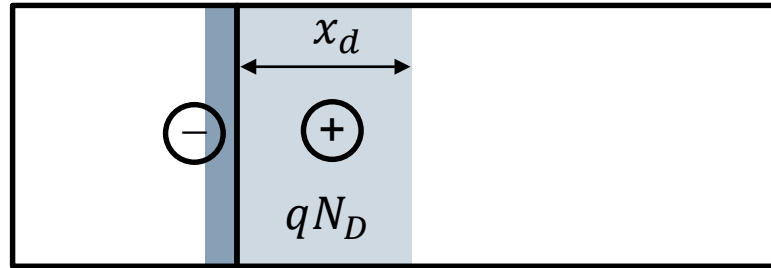
Total charge moved:  $-qN_D x_d$



# Charge in depletion region



# Electric field in depletion region

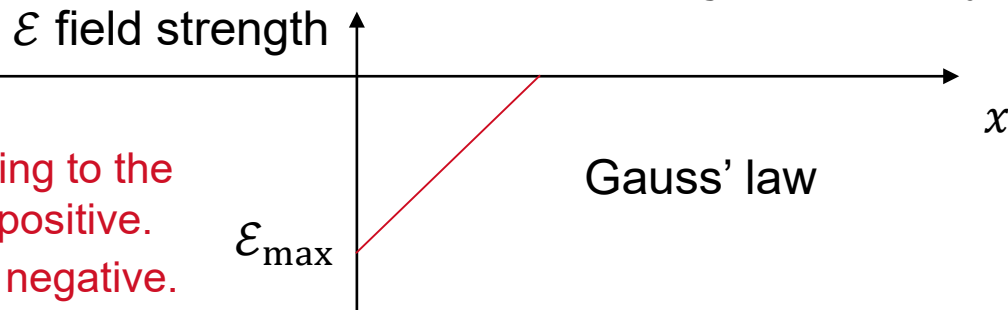


$$\mathcal{E}_{\max} = \frac{Q}{\epsilon_s} = -\frac{qN_d x_d}{\epsilon_s}$$

Gauss' law:

$$\mathcal{E} = \frac{1}{\epsilon_s} \int \rho dx$$

$\mathcal{E}$  electric field strength  
 $\epsilon_s$  permittivity of semiconductor

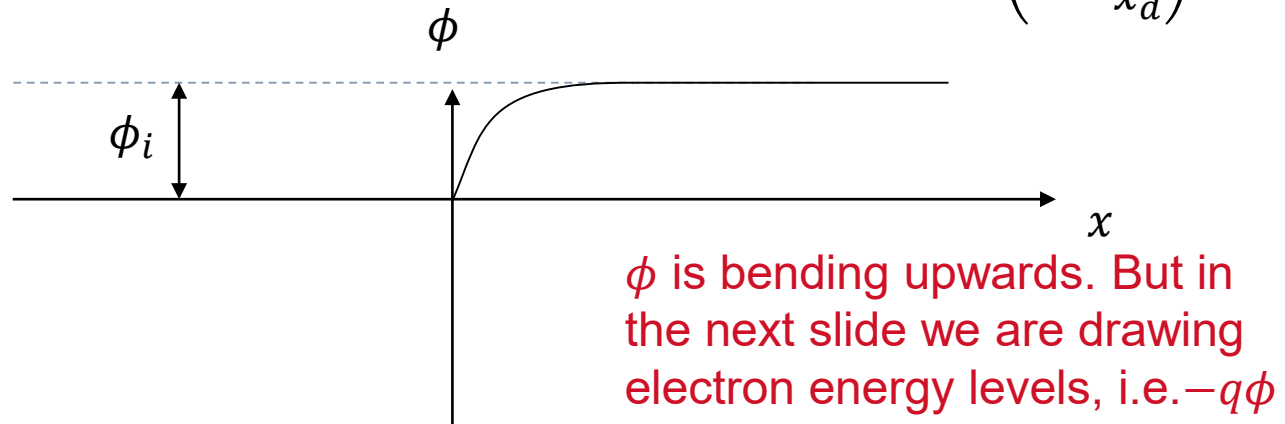
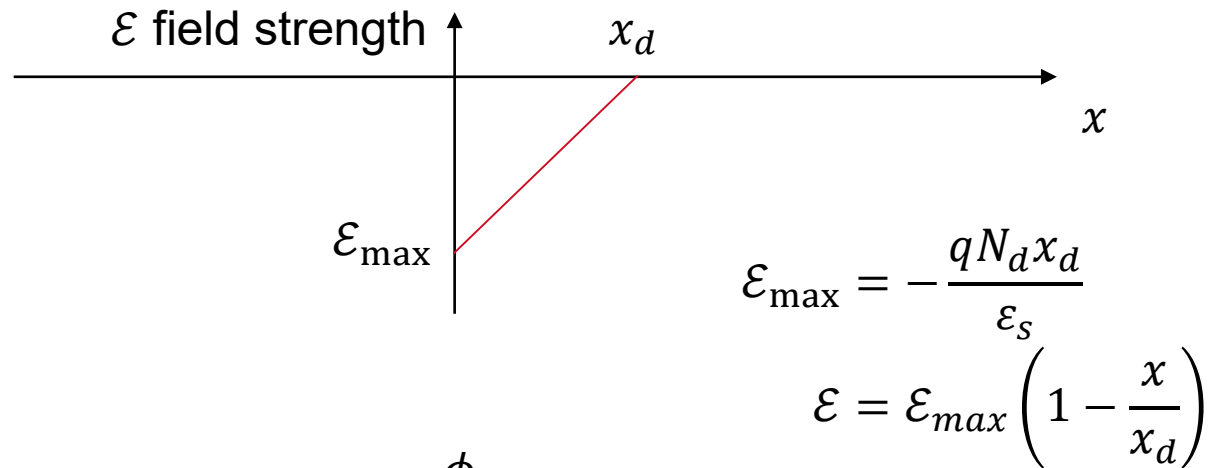


Gauss' law

Physically, we usually define electric field as pointing from positive charges to negative charges

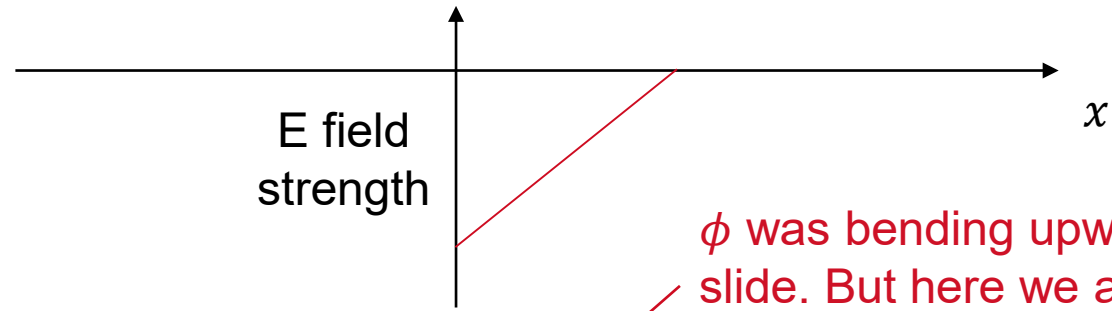
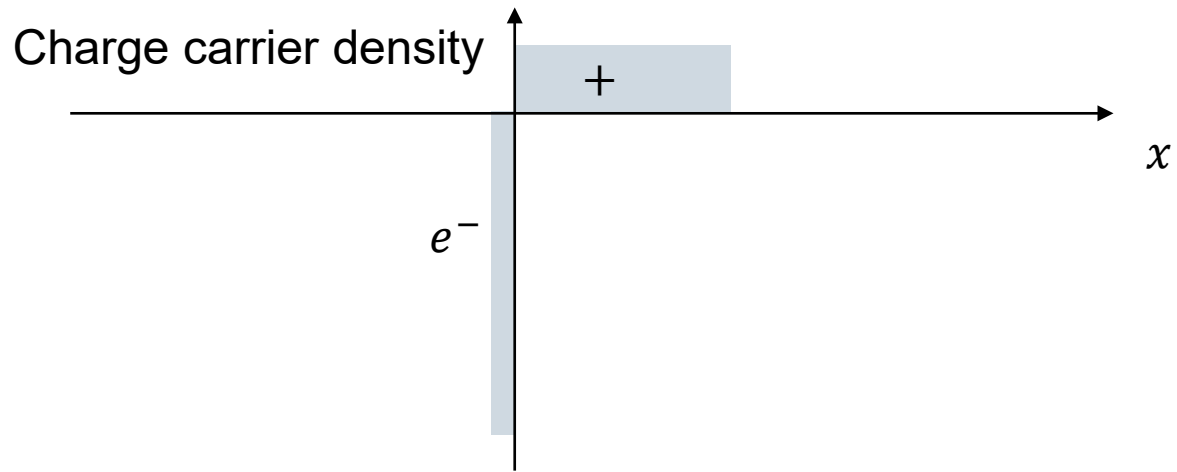
Here, we define  $\vec{E}$  pointing to the right (i.e. positive  $x$ ) as positive. Hence the value of  $\vec{E}$  is negative.

# Electric field and potential

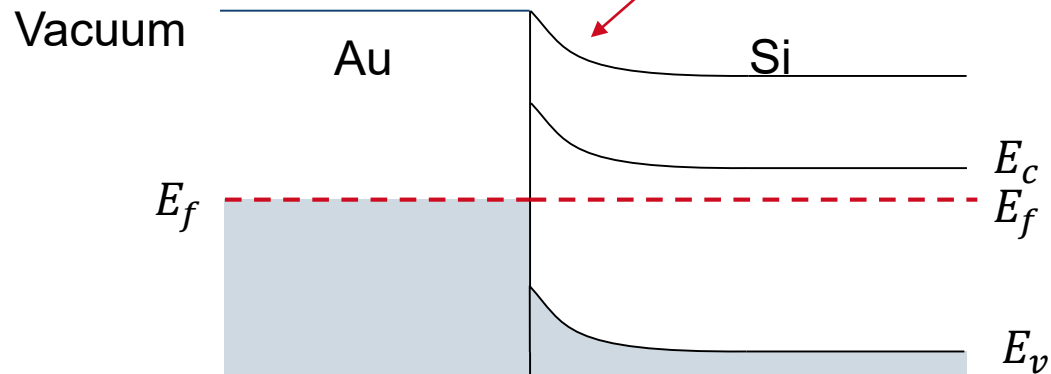


$$\phi(x) = -\int \varepsilon dx = \frac{qN_d}{2\varepsilon_s} (x_d^2 - (x_d - x)^2), \quad (x < x_d)$$

potential difference: 
$$\phi_i = -\frac{1}{2} \varepsilon_{\max} x_d = \frac{1}{2\varepsilon_s} qN_d x_d^2$$

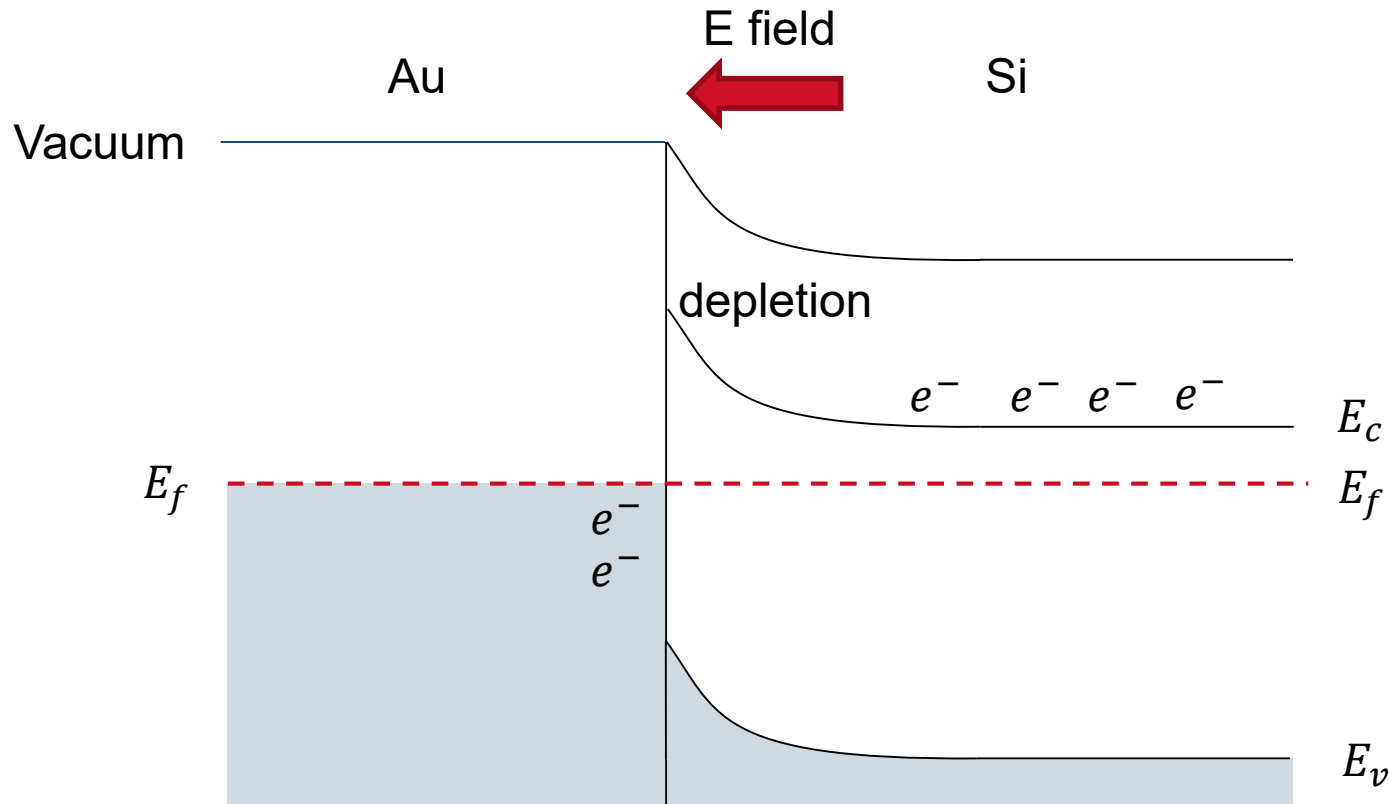


$\phi$  was bending upwards in last slide. But here we are drawing electron energy levels, i.e.  $-q\phi$



*When does charge stop moving?*

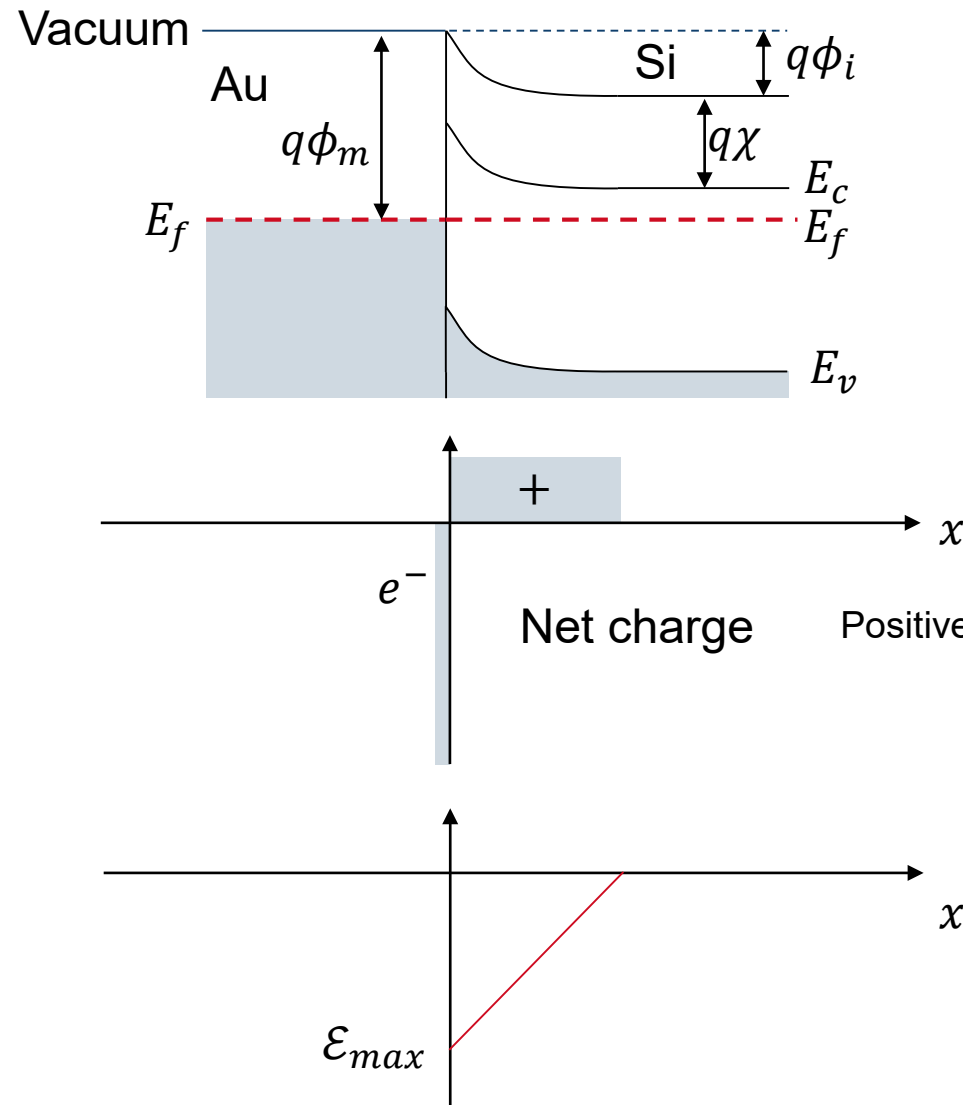
# Metal-Semiconductor contact



Principle: Fermi level is horizontal throughout

Note:  $E_c$ ,  $E_v$  can move together (with  $E_c - E_v = E_g$  fixed). Just like voltage at a node in a circuit, if there are electric fields in the space, these “voltage levels” can shift up and down.

# Depletion region length



$$Q = qN_d x_d$$

$Q$  total transferred charge

$x_d$  depletion region length

$N_d$  electron density in Si

$$\mathcal{E}_{max} = -\frac{Q}{\epsilon_s} = -\frac{qN_d x_d}{\epsilon_s} \quad (\text{Gauss'})$$

$\mathcal{E}$  electric field strength

$\epsilon_s$  permittivity of semiconductor

$$\phi_i = -\frac{1}{2} \mathcal{E}_{max} x_d = \frac{1}{2} \frac{qN_d x_d^2}{\epsilon_s} \quad \left. \vphantom{\phi_i} \right\} x_d$$

$$q\phi_i = q\phi_m - q\chi - (E_c - E_f)$$

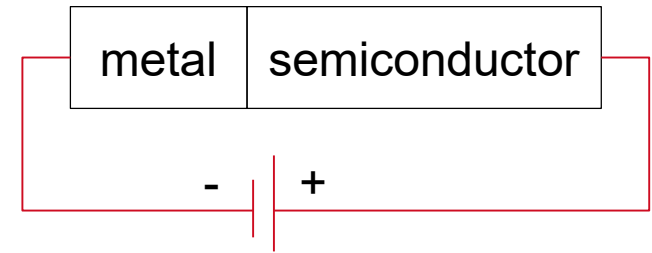
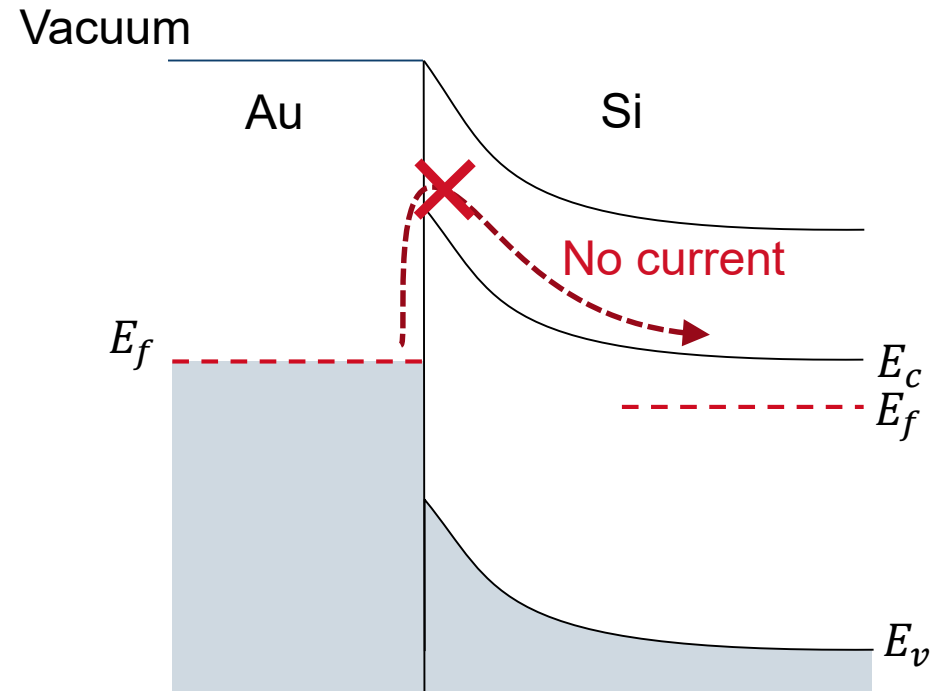
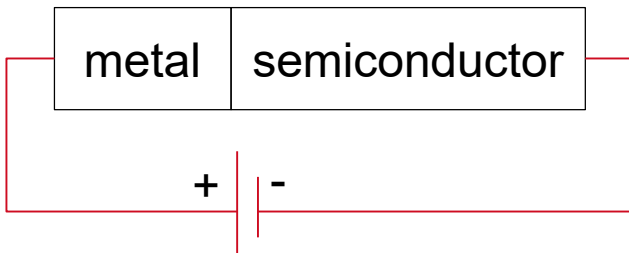
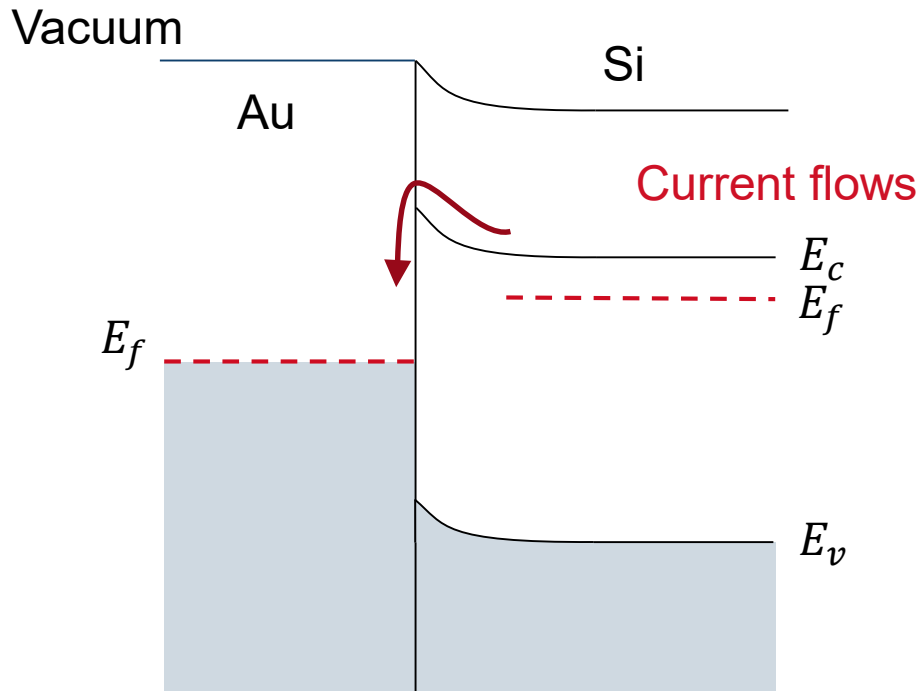
$\phi_i$  voltage drop over depletion

$\phi_m$  metal work function

$\chi$  electron affinity

Will have an example calculation of  $x_d$  in tutorial

# Schottky contact (Schottky diode)



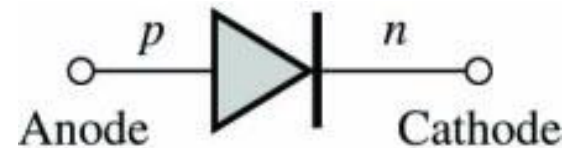
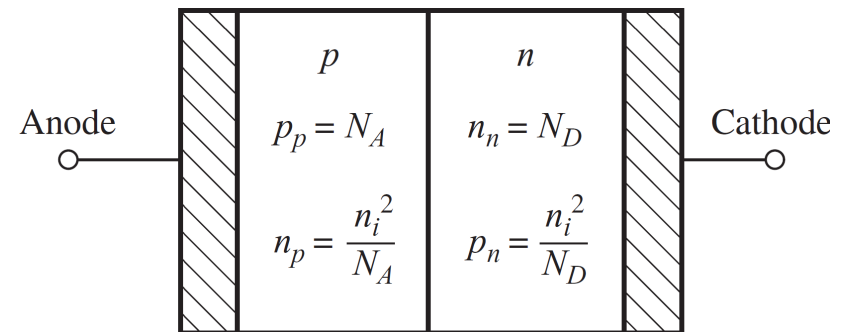
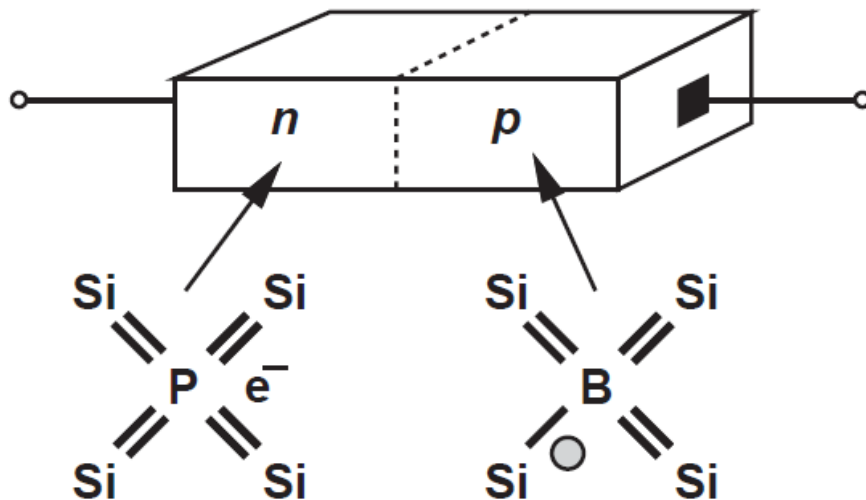
# pn junctions





# PN Junctions

- › If we join a  $p$ -type semiconductor to a  $n$ -type semiconductor, we can create a PN junction
- › This is a semiconductor diode
- › The  $p$ -side is called the “anode” and the  $n$ -side is called the “cathode”

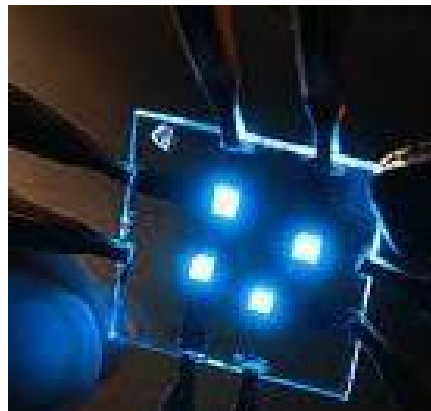


**Diode symbol**

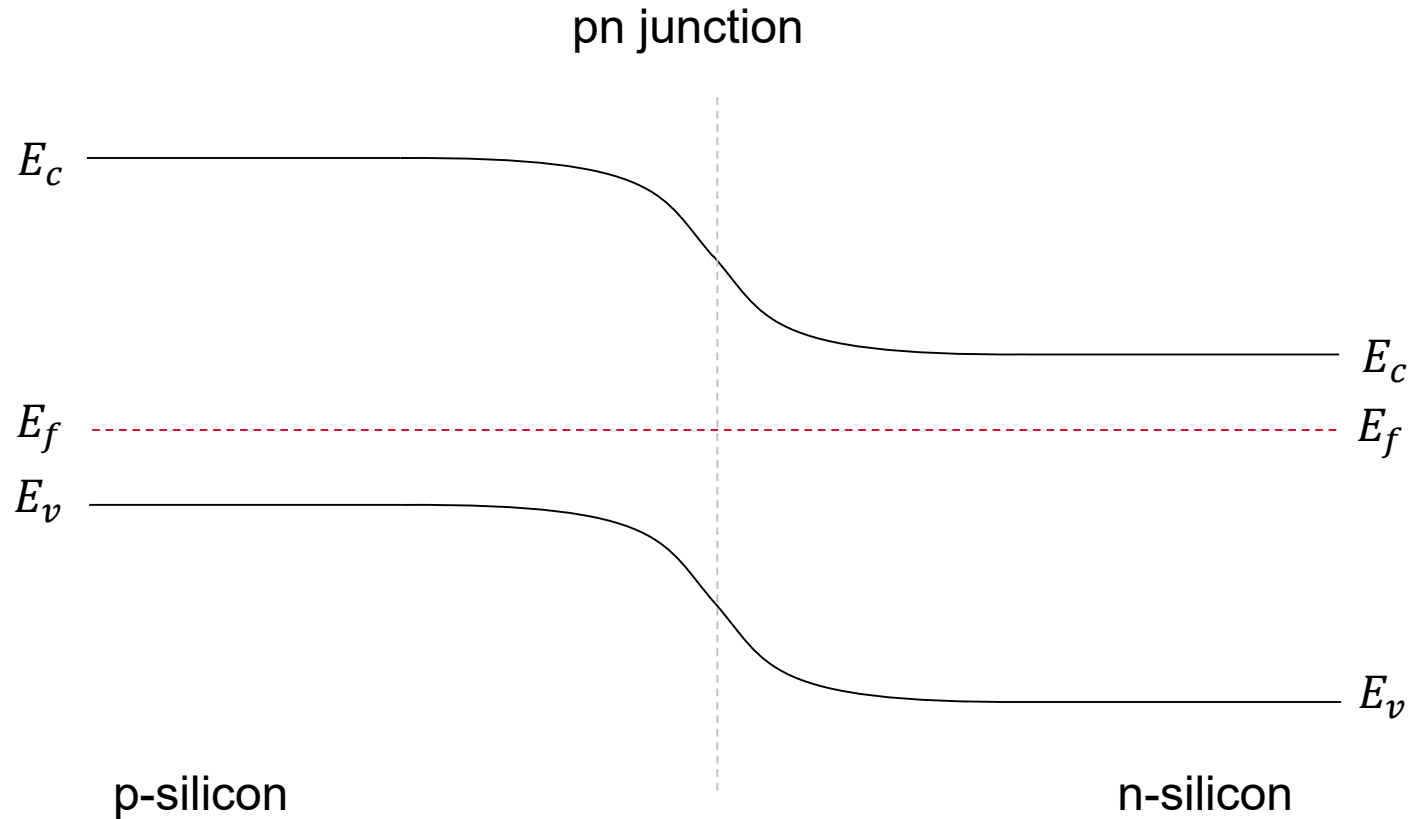
# PN Junction Diodes

## › Diode Uses

- PN junctions are the fundamental building blocks of almost all semiconductor devices
- It is critical to understand how these junctions work if you want to understand anything about transistors
- Diodes are used for rectifiers, voltage protectors, lasers, radiation detectors

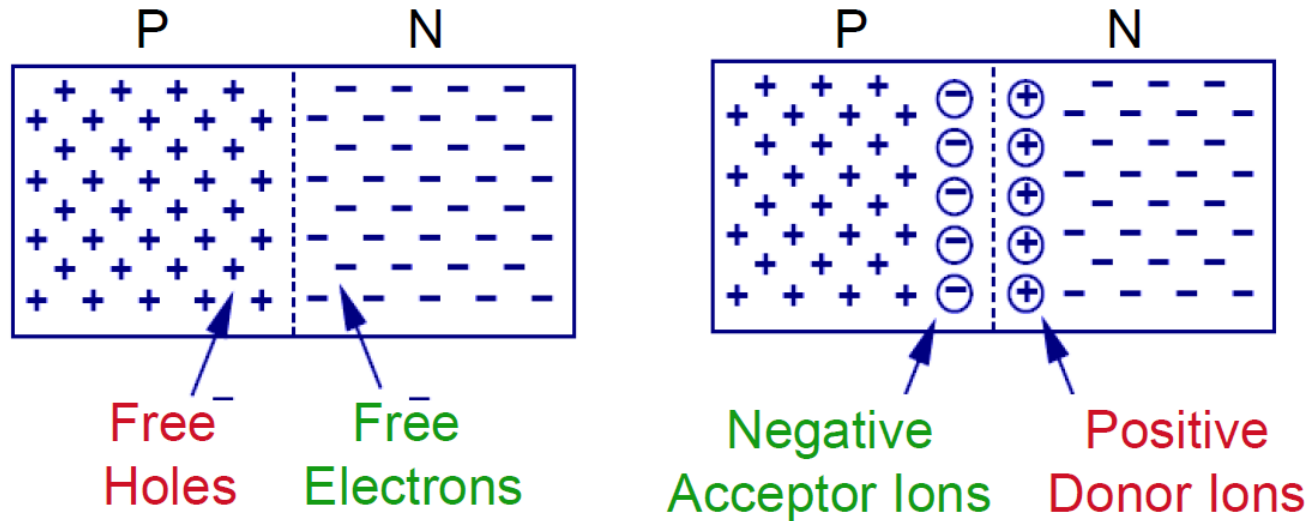


# PN junction in equilibrium



This is the only way the Fermi level can level

# PN junction in equilibrium

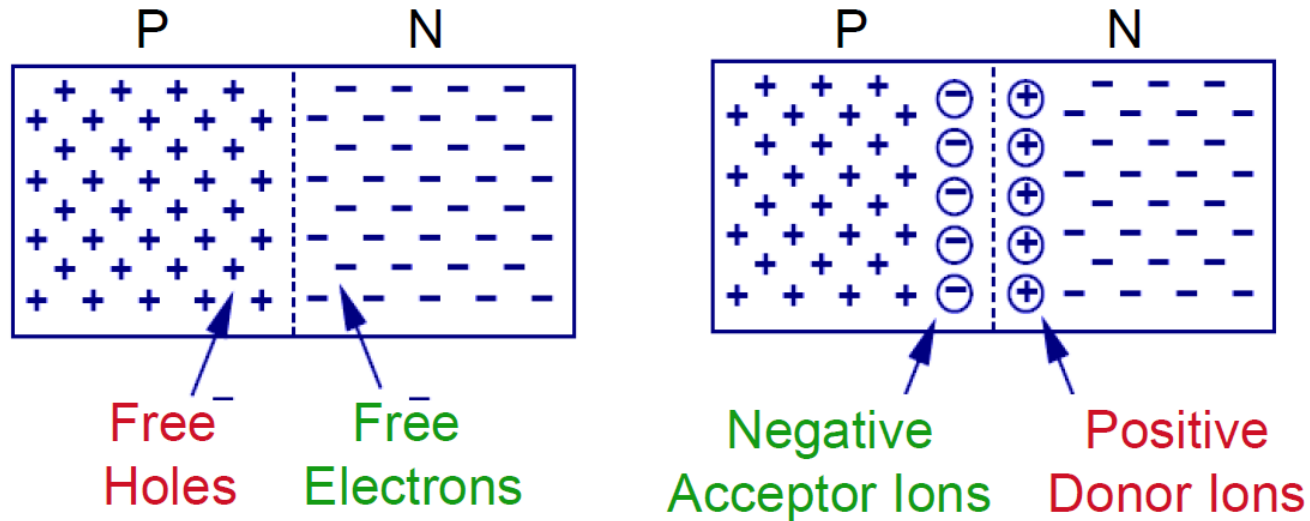


Electron diffuse from n to p  
Holes diffuse from p to n

Electrons from the n region recombine with holes in the p region  
Holes from the p region recombine with electrons in the n region

What stops them from flowing indefinitely?

# PN junction in equilibrium

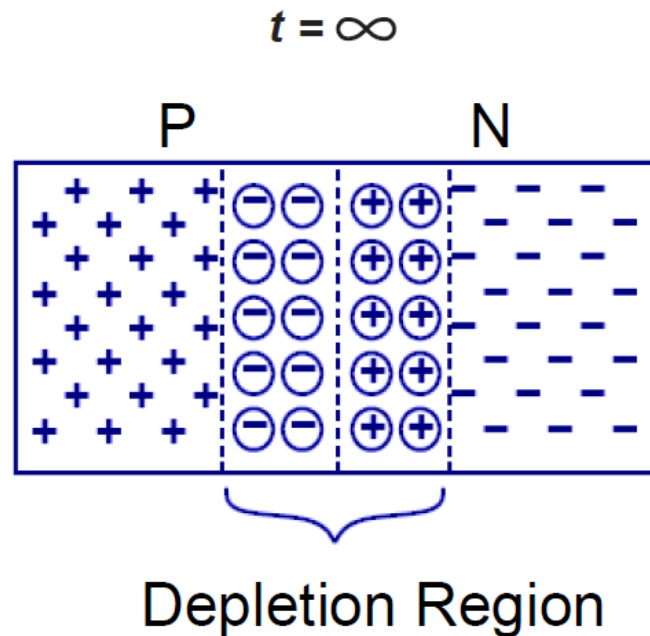


When an electron is taken from the n region or a hole is taken from the p region, a charged donor or acceptor is left behind.

The fixed charged ions create electric fields

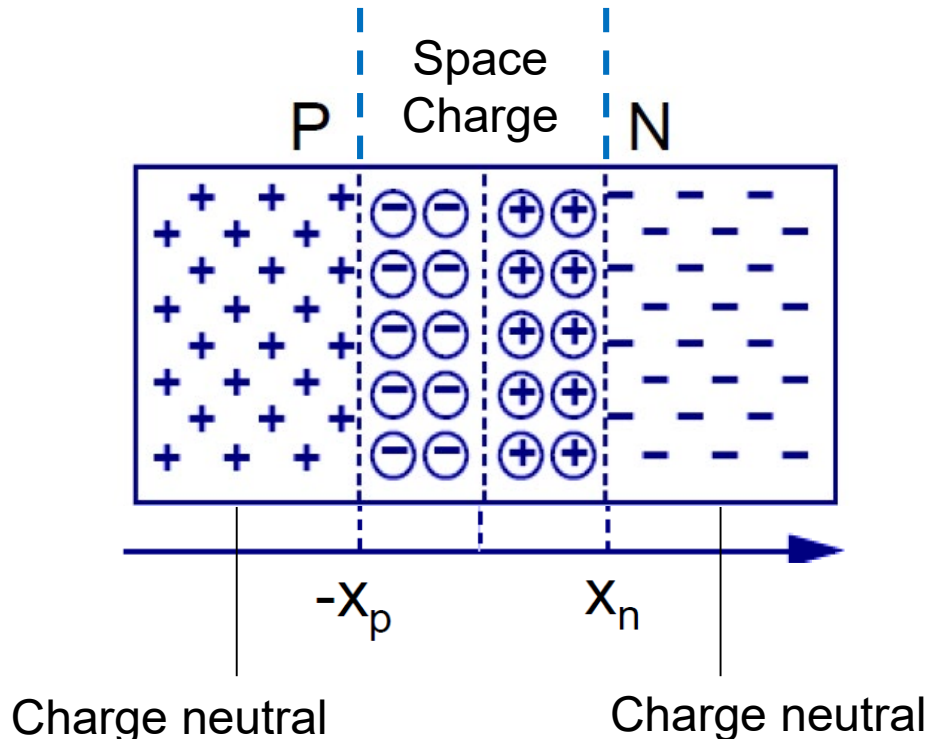
# Depletion Region

- › As free electrons and holes diffuse across the junction, a region of fixed ions is left behind
- › This region is known as the “depletion region” because it is 'depleted' of charge carriers



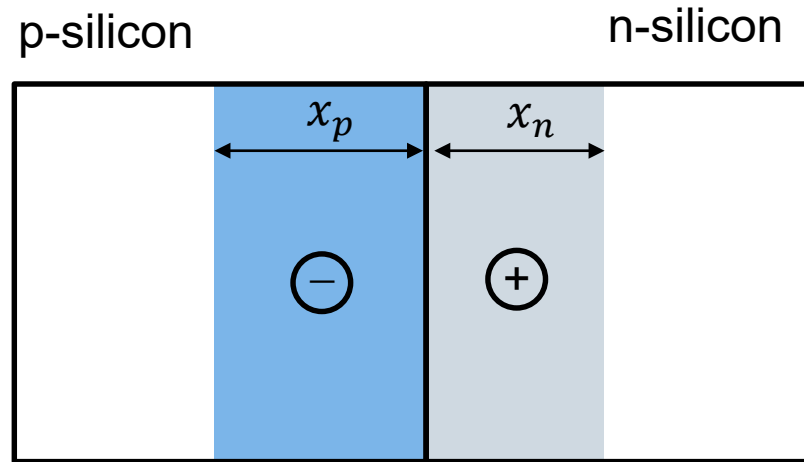
# Depletion Region

- › The depletion region is made up of fixed ions due to diffusion of carrier to opposite sides of the junction
  - also known as space-charge region



- › Depletion region is not charge-neutral
  - Creates a built-in voltage
- › The regions outside of the depletion region are *still charge-neutral*

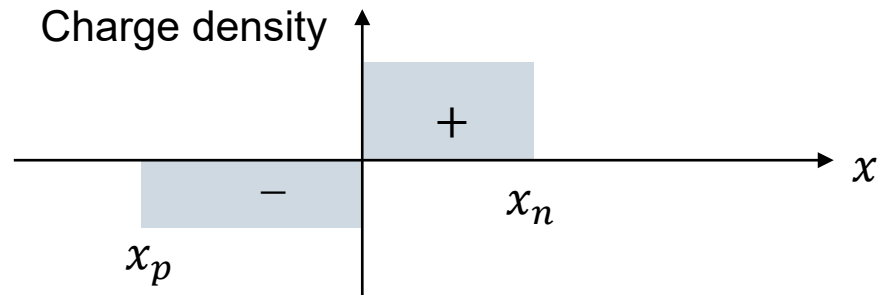
# Electric field in depletion region



For simplicity, assume the depletion region has no free carriers (the free carrier density is orders of magnitude smaller than original)

Hence, the space charge region has a uniform charge density

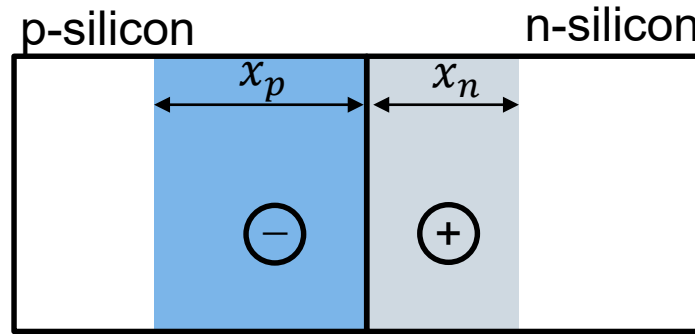
p-side:  $qN_A$       n-side:  $qN_D$



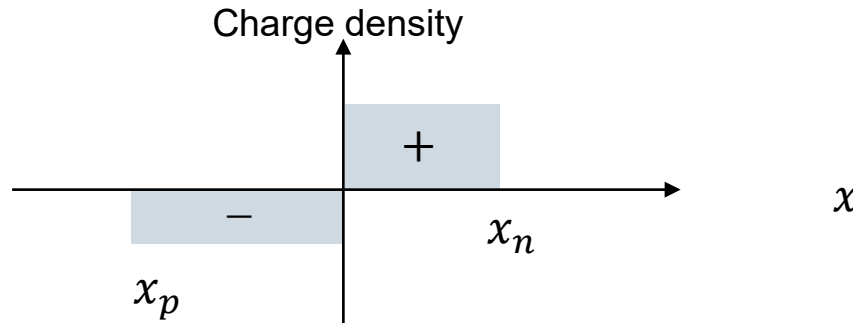
Charge neutral condition  $qN_Ax_p = qN_Dx_n \Rightarrow \frac{x_p}{x_n} = \frac{N_D}{N_A}$



# Electric field in pn junction



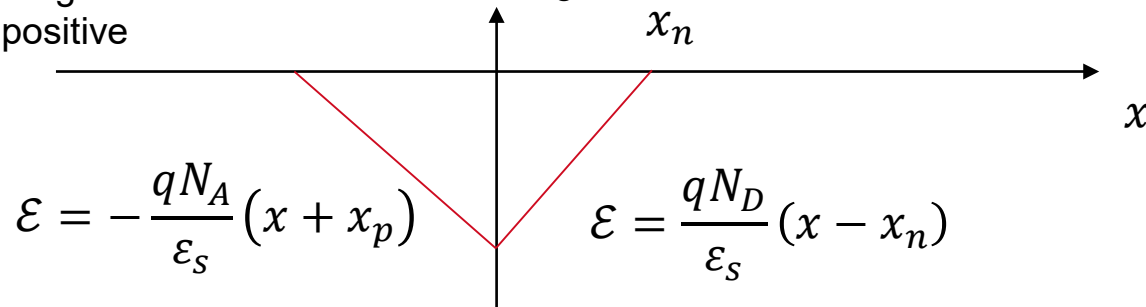
$$\frac{x_p}{x_n} = \frac{N_D}{N_A}$$



Define  $\mathcal{E}$  pointing to  $+x$  as being positive

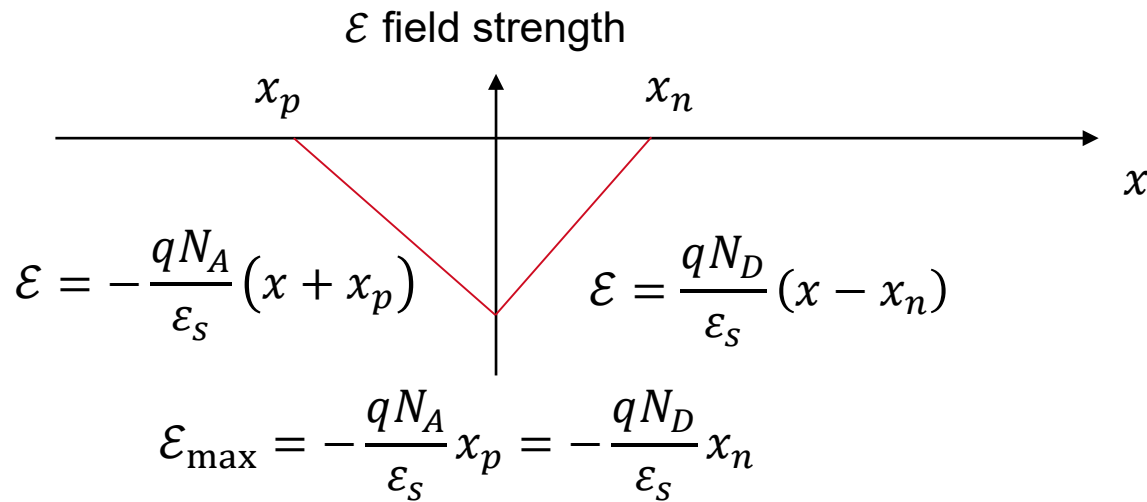
$\mathcal{E}$  field strength

Gauss' law:  $\mathcal{E} = \frac{1}{\epsilon_s} \int \rho dx$

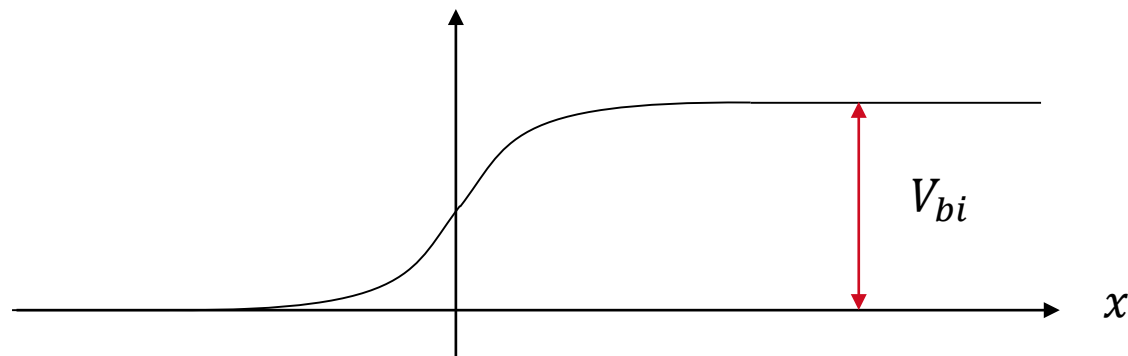


$$\mathcal{E}_{\max} = -\frac{qN_A}{\epsilon_s}x_p = -\frac{qN_D}{\epsilon_s}x_n$$

# Build-in Potential in pn-junction

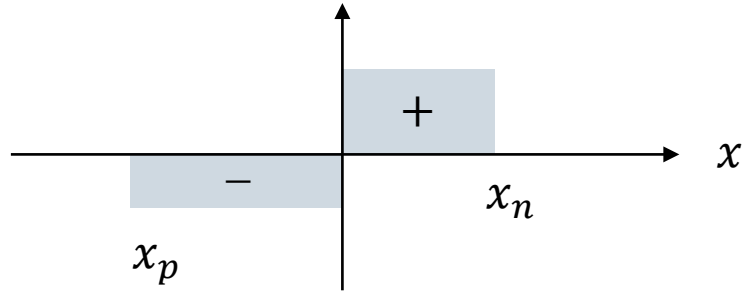


Build-in Potential  $V = -\int \mathcal{E} dx$



$$V_{bi} = \frac{1}{2} \frac{qN_A}{\epsilon_s} x_p^2 + \frac{1}{2} \frac{qN_D}{\epsilon_s} x_n^2 = \frac{1}{2} \mathcal{E}_{\max} w \quad w = x_p + x_n$$

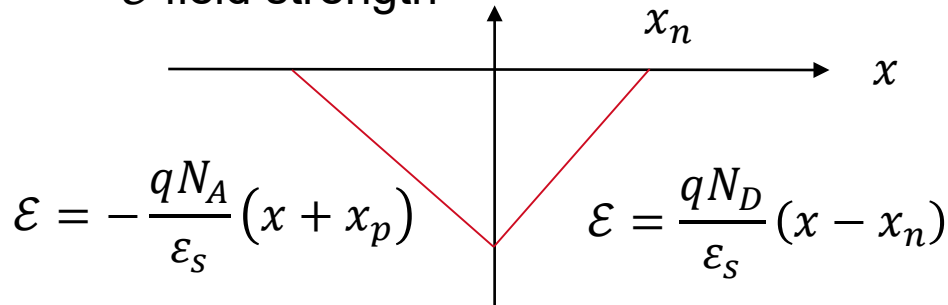
# Depletion region width $w$



$$\text{Let } w = x_p + x_n$$

$$\frac{x_p}{x_n} = \frac{N_D}{N_A}$$

$\mathcal{E}$  field strength



$$\mathcal{E}_{\max} = -\frac{qN_A}{\epsilon_s}x_p = -\frac{qN_D}{\epsilon_s}x_n$$

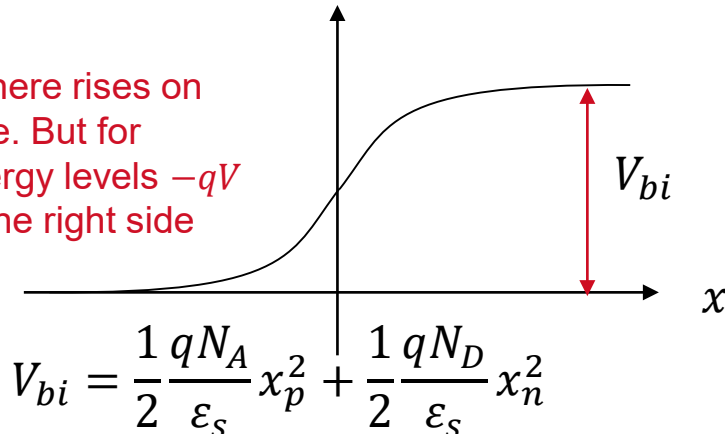
$$\text{Hence: } x_p = \frac{N_D w}{N_A + N_D}$$

$$x_n = \frac{N_A w}{N_A + N_D}$$

$$V_{bi} = \frac{1}{2} \frac{qN_A}{\epsilon_s} x_p^2 + \frac{1}{2} \frac{qN_D}{\epsilon_s} x_n^2$$

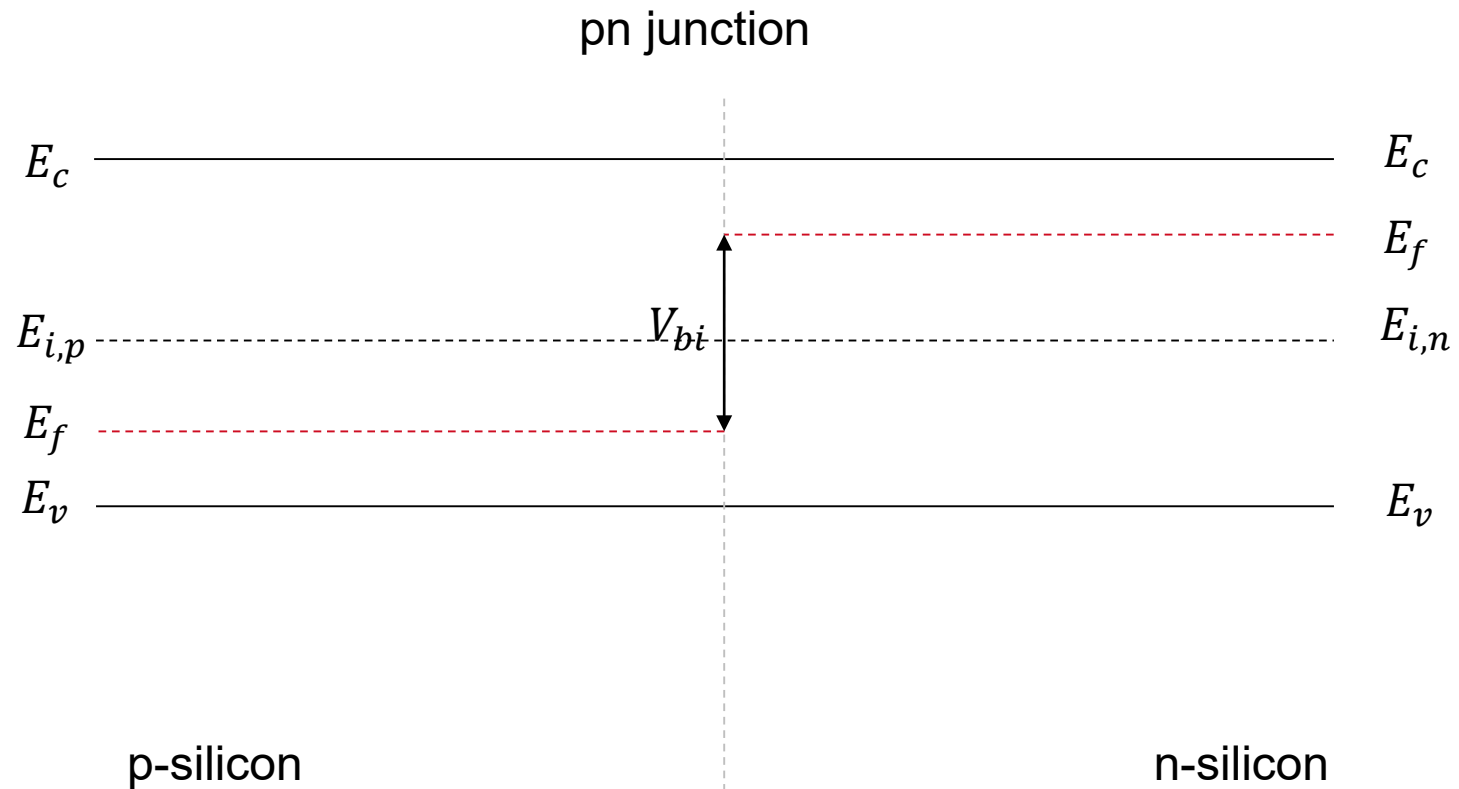
$$= \frac{1}{2} \frac{q}{\epsilon_s} \frac{N_A N_D}{N_A + N_D} w^2$$

Potential  $V$  here rises on the right side. But for electron energy levels  $-qV$  it drops on the right side



$$\text{i.e. } w = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_{bi}}$$

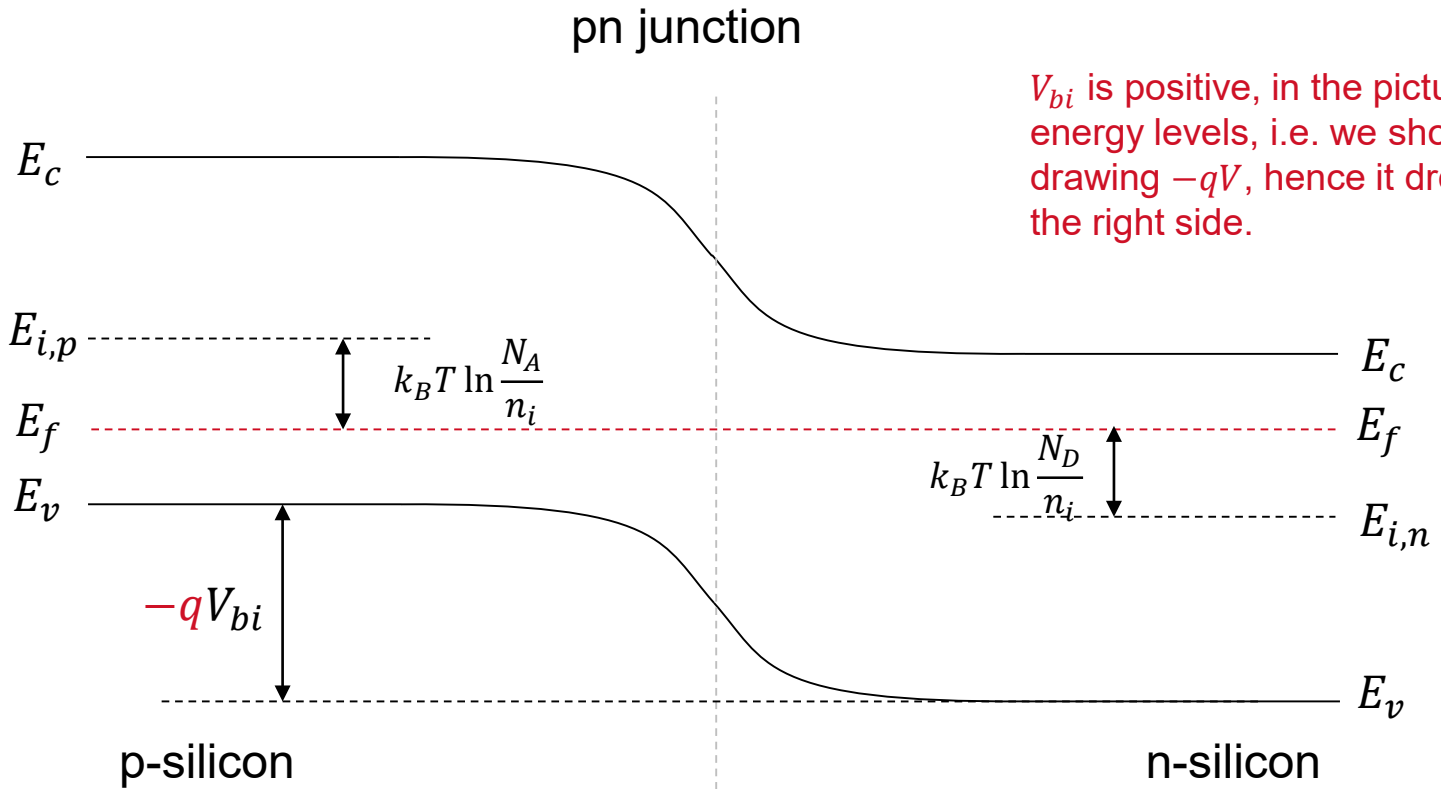
# How to calculate $V_{bi}$ ? Fermi level have to LEVEL



$$E_{i,p} - E_f = k_B T \ln \frac{N_A}{n_i}$$

$$E_f - E_{i,n} = k_B T \ln \frac{N_D}{n_i}$$

# How to calculate $V_{bi}$ ? Fermi level have to LEVEL



$$E_{i,p} - E_f = k_B T \ln \frac{N_A}{n_i}$$

$$E_f - E_{i,n} = k_B T \ln \frac{N_D}{n_i}$$

$$V_{bi} = \frac{1}{q} (E_{c,p} - E_{c,n}) = \frac{1}{q} (E_{v,p} - E_{v,n}) = \frac{1}{q} (E_{i,p} - E_{i,n}) = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

# Depletion width in pn-junctions

$$w = \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_{bi}}$$

$$V_{bi} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

# Example

Find built-in potential and depletion-region width for a diode with the following charge densities.

$$N_A = 10^{17} \text{cm}^{-3}$$

$$N_D = 10^{20} \text{cm}^{-3}$$

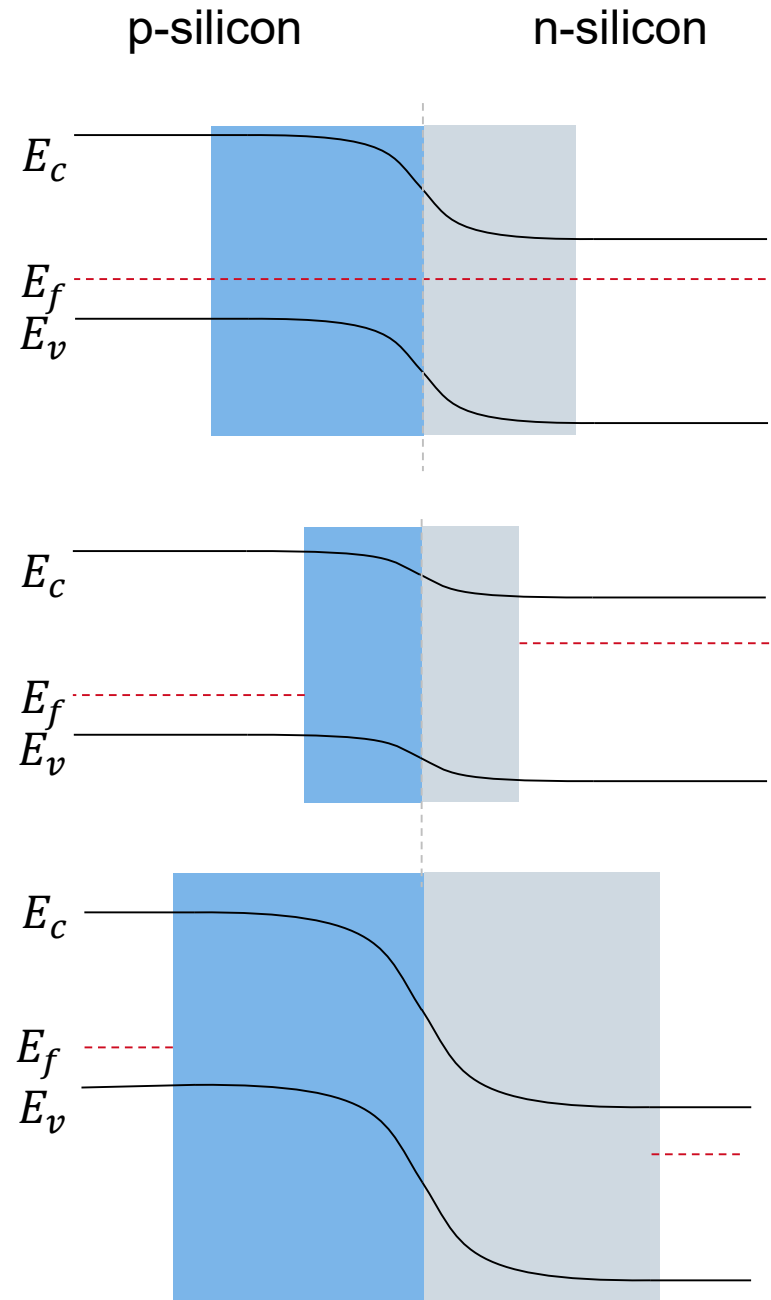
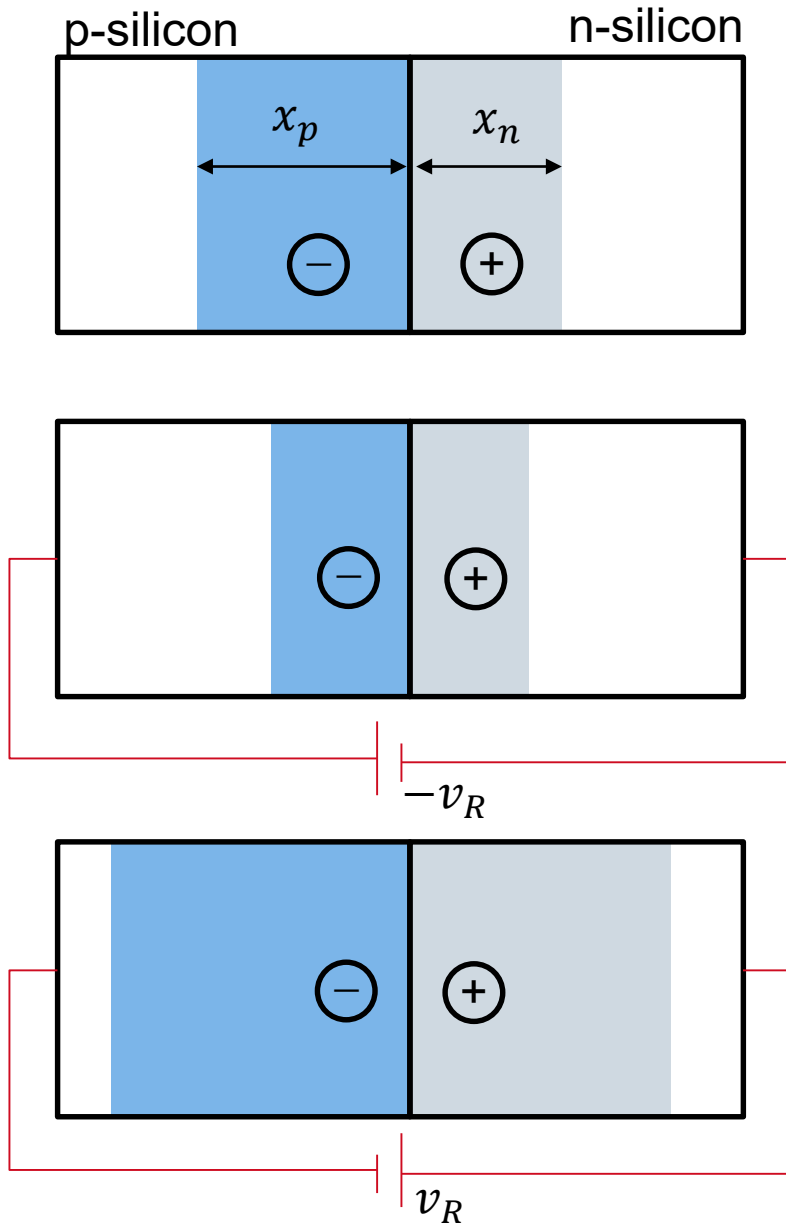
$$k_B T = 0.025 \text{eV}$$

$$\begin{aligned} V_{bi} &= \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} \\ &= 0.025 \ln \frac{10^{17} 10^{20}}{10^{20}} = 0.979 \text{ V} \end{aligned}$$

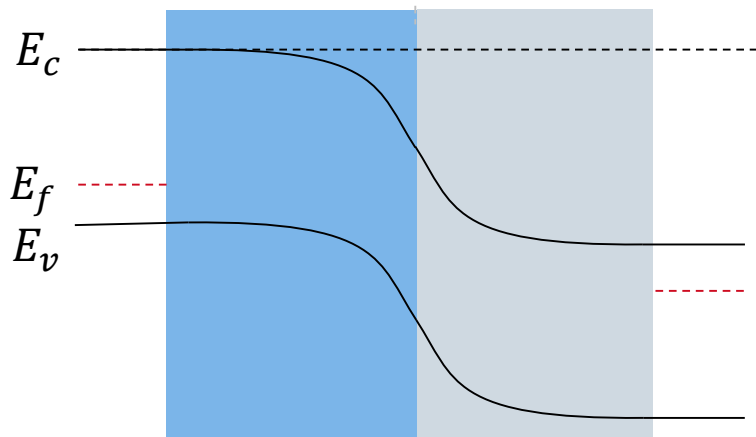
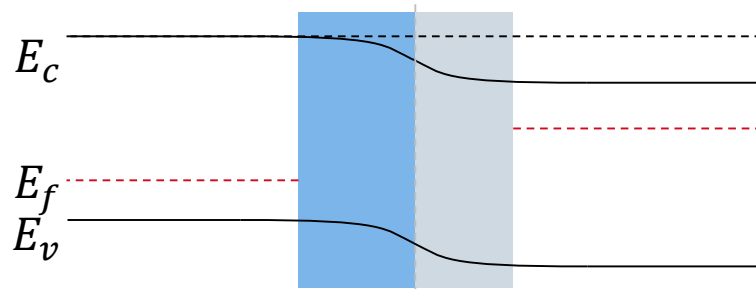
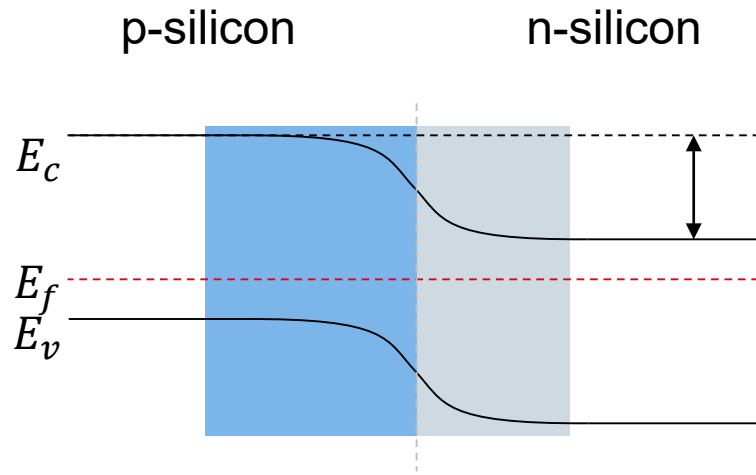
$$w = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_{bi}} = 0.113 \mu\text{m}$$

$$\mathcal{E}_{max} = \frac{2V_{bi}}{w} = 173 \text{ kV/cm}$$

# pn-junction with bias







Depletion region charge  
Electric field  
potential difference

$$V_{bi} = -\frac{1q}{2\epsilon_s} \frac{N_A N_D}{N_A + N_D} w^2$$

$$V_{bi} + v_R = -\frac{1q}{2\epsilon_s} \frac{N_A N_D}{N_A + N_D} w^2$$

$v_R$ : external applied voltage

$$w = \sqrt{\frac{2\epsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_{bi} + v_R)}$$

# Batteries actually move Fermi levels!

