ELEC2104 – Week 5

Diode circuit models, rectifiers

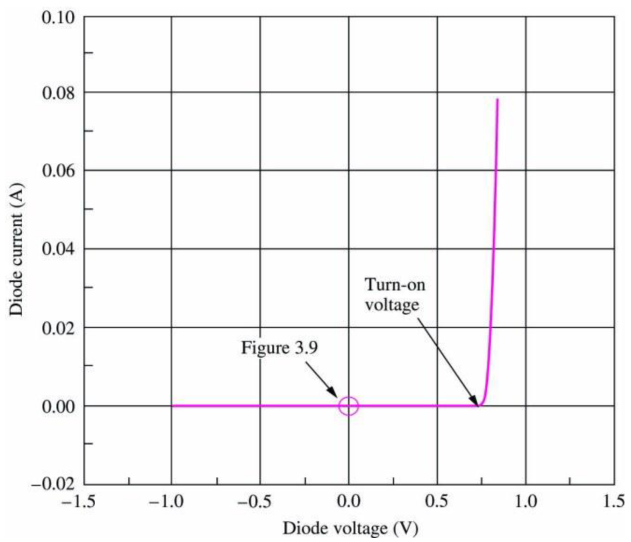
THE UNIVERSITY OF SYDNEY

Diode Circuit Analysis

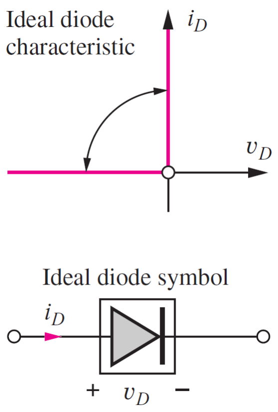
- Several techniques can be used to solve for I_D and V_D
  - Mathematical Analysis using Exponential Model
  - Graphical Analysis using Load Line Approach
  - Ideal Diode Model
  - Constant-Voltage Model

Constant-voltage Diode (CVD) Model

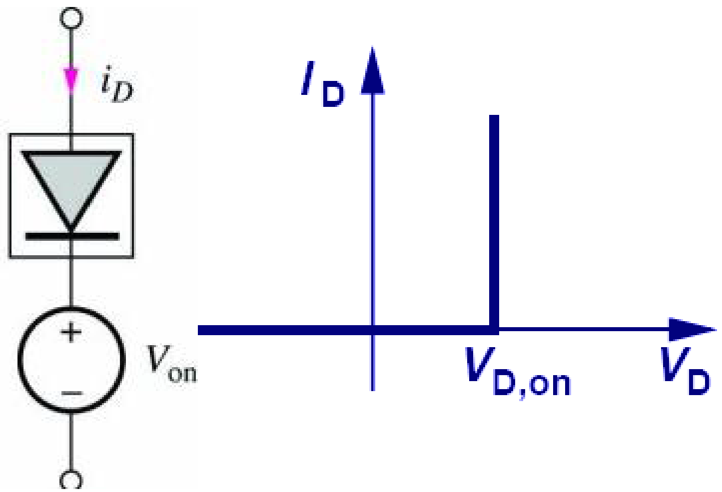
- The ideal diode model is useful for determining operating region, but does not give a very accurate estimate of Q-point and how the rest of the circuit functions
- Constant-voltage diode model takes into account the voltage drop across the diode when it is on
- Diode operates as an open circuit if $V_D < V_{D,on}$ and a constant voltage source of $V_{D,on}$ if VD exceeds $V_{D,on}$



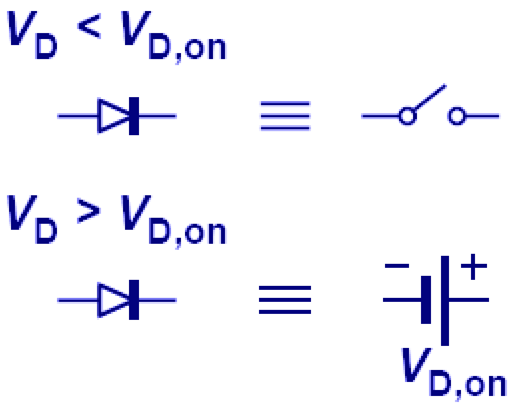
I-V on linear scale



ideal



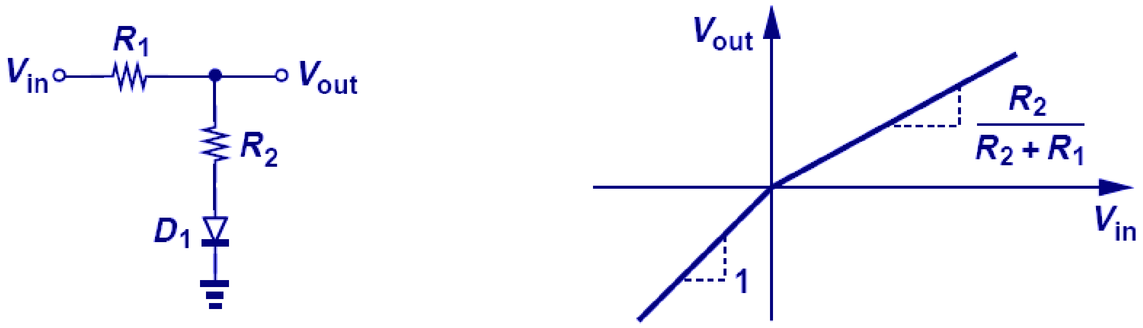
CVD



Try and error with the two regimes

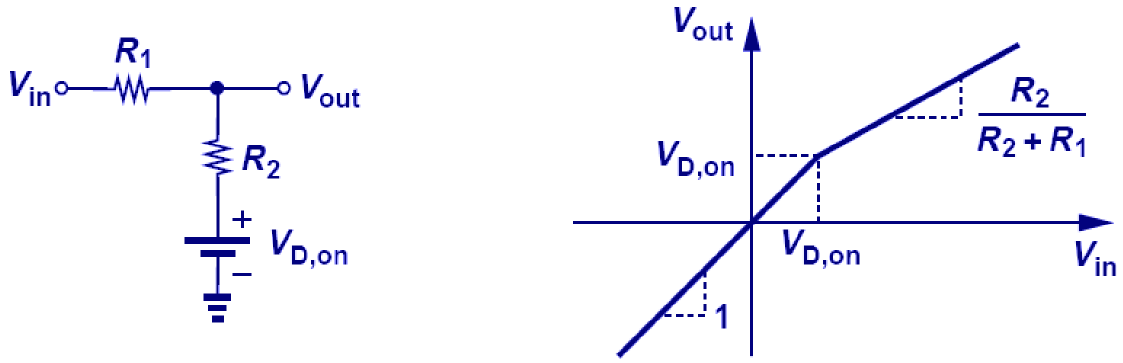
Constant-voltage Diode (CVD) Model

- The constant-voltage model yields a different break point in slope when compared to the ideal model



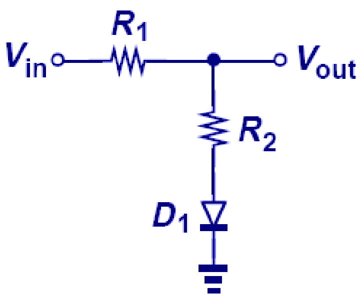
(a) (b)

Ideal diode

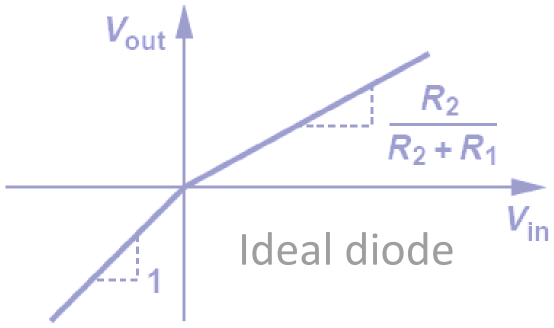


Constant-voltage diode

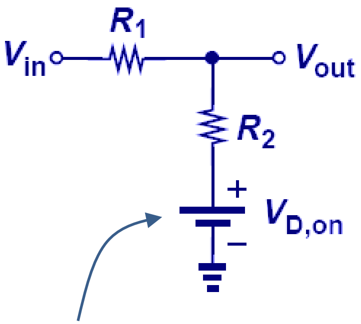
Constant-voltage Diode (CVD) Model



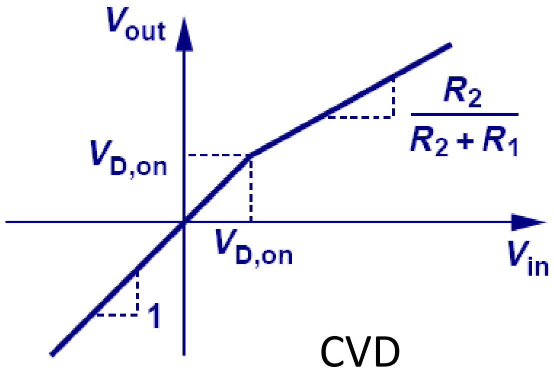
(a)



(b)



"on" state is just like a ideal constant-voltage source (fixed voltage drop, zero internal resistance)

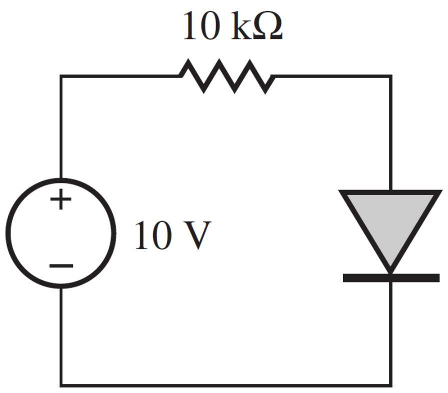


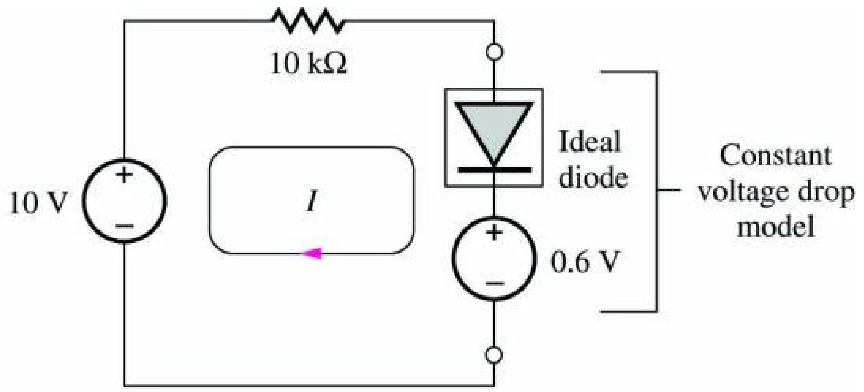
CVD

- For $V_{in} < V_{D,on}$:
  - Diode is off,
  - No current flows
  - $V_{out} = V_{in} = V_D$
- For $V_{in} > V_{D,on}$:
  - If assume diode off
  - Then $V_D = V_{out} = V_{in} > V_{D,on}$. Can't be.
  - Diode must be on.
  - $V_D = V_{D,on}$
  - Current: $\frac{V_{in} - V_{D,on}}{R_1 + R_2}$
  - $V_{out} = \frac{R_2}{R_1 + R_2}(V_{in} - V_{D,on}) + V_{D,on}$

Constant-voltage Diode Model

- Consider the example from earlier. Compare a constant-voltage analysis with the ideal diode model.





Using ideal diode model,

$$I_D = \frac{(10-0)V}{10k\Omega} = 1mA$$

Using constant-voltage model,

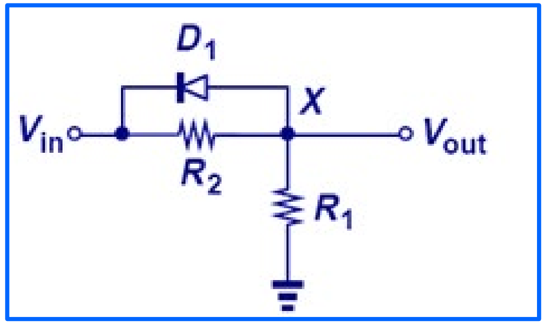
$$I_D = \frac{(10-V_{on})V}{10k\Omega}$$

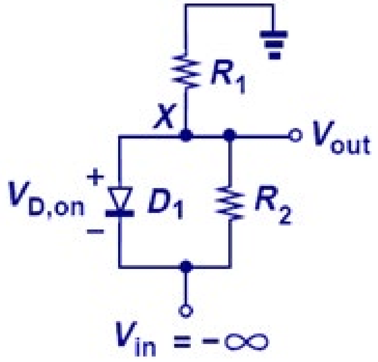
$$= \frac{(10-0.6)V}{10k\Omega} = 0.940\ mA$$

Q-point is closer to the value found by load-line analysis (0.95 mA, 0.6 V)

From now on, use constant-voltage model by default.
V_{on} is usually given in each problem. If not, use 0.6V.

Constant-voltage Diode Model

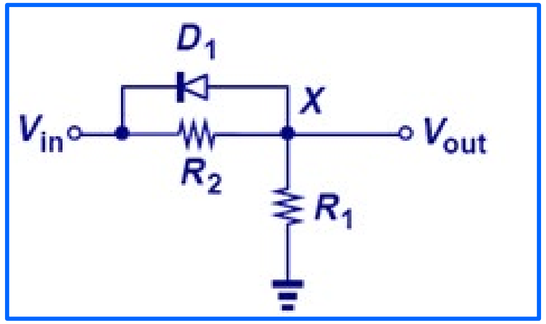




- Qualitative analysis:
  - If $V_{in} = -\infty$, then D1 has to be on. Current flows from right to left through D1. Current flows through R1 and R2.
  - If $V_{in} = \infty$, then D1 has to be off. Current flows through R1, R2.

- Where is the transition point?
  - Assume D1 is on, but no current flows through it (i.e. $V_{in} - V_X$ just about enough to turn it on)
  - From this point, increase V_{in} by any small amount will turn off D1.

Constant-voltage Diode Model

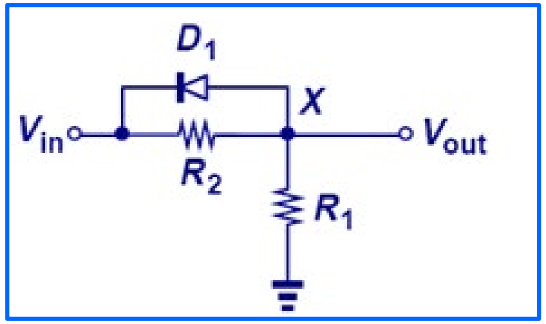


Transition point: D1 is on, but no current flow through it

D1 is on: $V_X = V_{out} = V_{in} + V_{d,on}$

No current flowing through D1: $I_{R1} = I_{R2}$

Constant-voltage Diode Model



We compute the current flowing through R_2 and R_1:

$$I_{R1} = \frac{-V_{out}}{R_1} = \frac{-(V_{D,on}+V_{in})}{R_1}$$

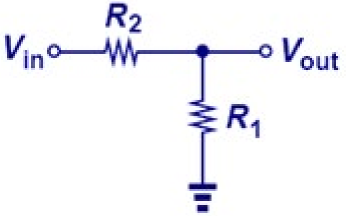
$$I_{R2} = \frac{V_{out} - V_{in}}{R_2} = \frac{V_{D,on}}{R_2}$$

Find the point where the D_1 turns off ($I_{R1} = I_{R2}$)

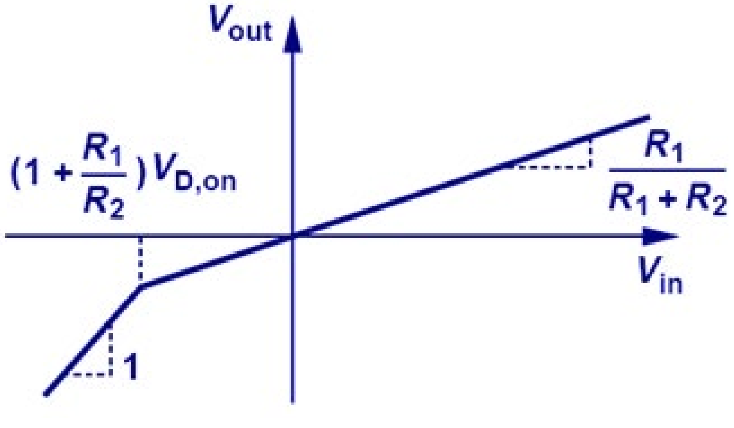
$$\frac{-(V_{D,on}+V_{in})}{R_1} = \frac{V_{D,on}}{R_2}$$

$$V_{in} = -\left(1 + \frac{R_1}{R_2}\right) V_{D,on}$$

When D1 off:

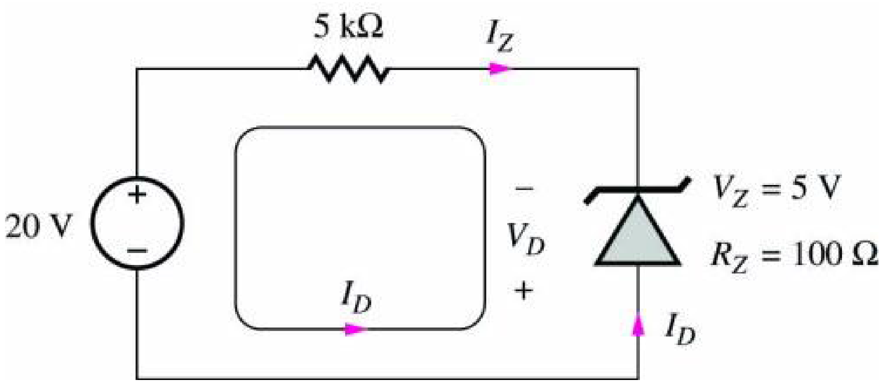


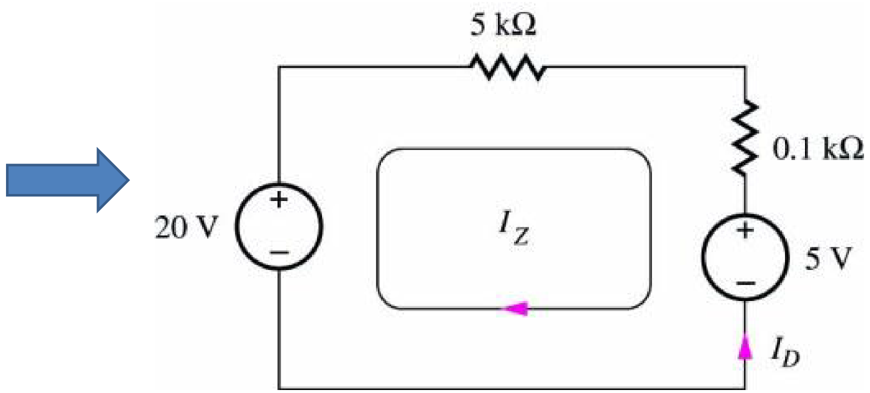
$$V_{out} = \frac{R_1}{R_1+R_2} V_{in}$$



Piecewise-Linear Model

- Zener Diode
  - Similar to constant-voltage model, but now include reverse breakdown
  - The Zener diode will have a voltage drop V_Z as well as a resistance R_Z

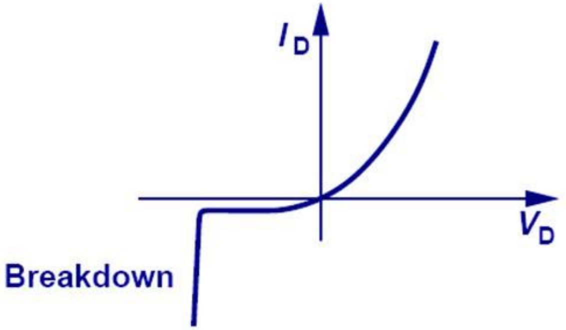




$$I_Z = -I_D > 0$$

$$20 - 5100 I_Z - 5 = 0$$

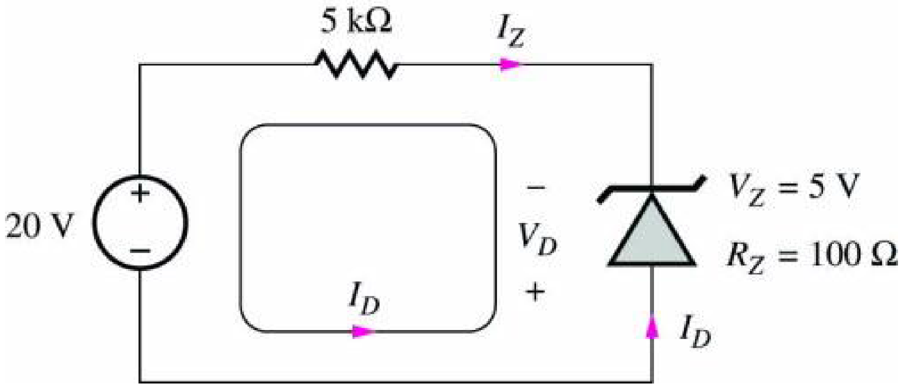
$$I_Z = \frac{(20-5)\text{V}}{5100\Omega} = 2.94\ \text{mA}$$

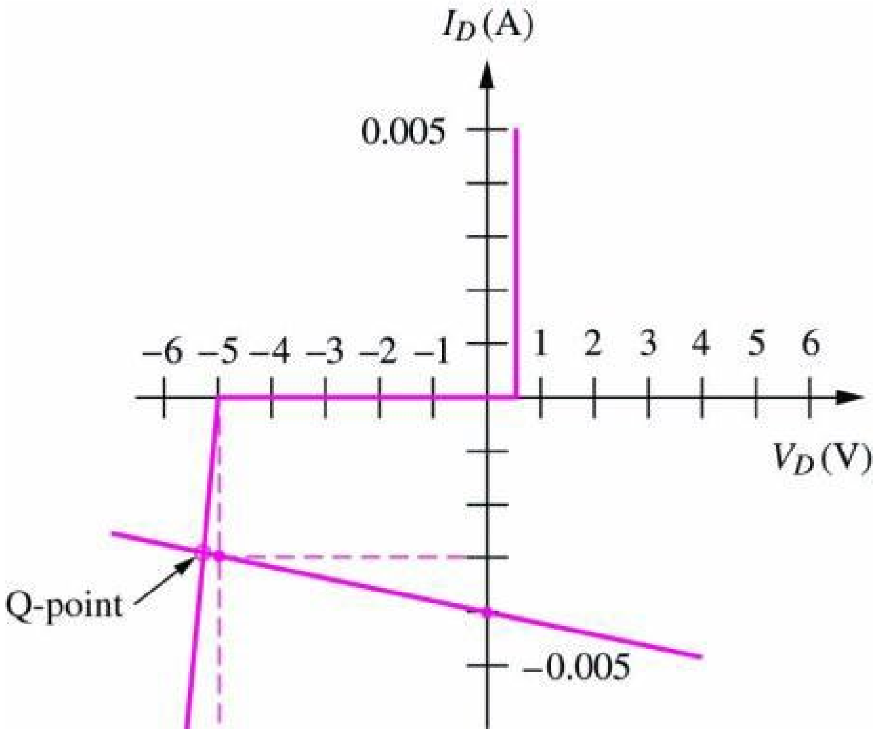


Reverse Breakdown Analysis

- Model Zener diode with a piecewise linear model
- We can verify the Q-point by using load-line analysis
  - Choose 2 points (0 V, -4 mA) and (-5 V, -3 mA) to draw the load line
  - It intersects the I-V characteristic at the Q-point: (-2.9 mA, -5.2 V)

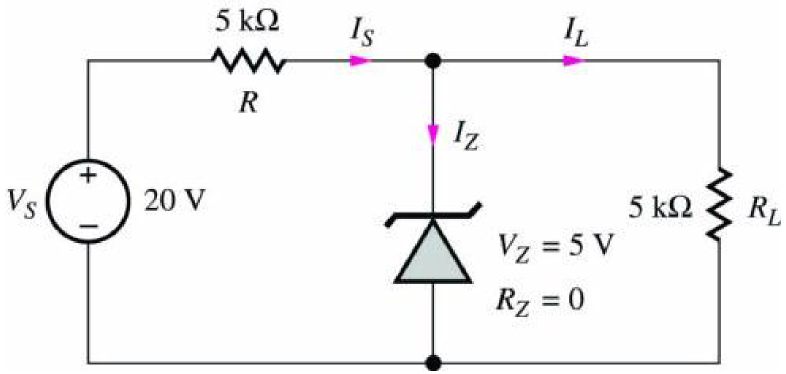
Load-line equation $-20 = V_D + 5000 I_D$





Zener Voltage Regulator

- A Zener diode can serve as a voltage regulator in breakdown operation, where $I_Z > 0$
- The Zener diode keeps the voltage across load resistor R_L constant



$$I_S = \frac{V_S - V_Z}{R} = 3\text{mA}$$

$$I_L = \frac{V_z}{R_L} = 1\text{mA} \;\; | \;\; I_Z = I_S - I_L = 2\text{mA}$$

If $R_L = 50\text{k}\Omega$

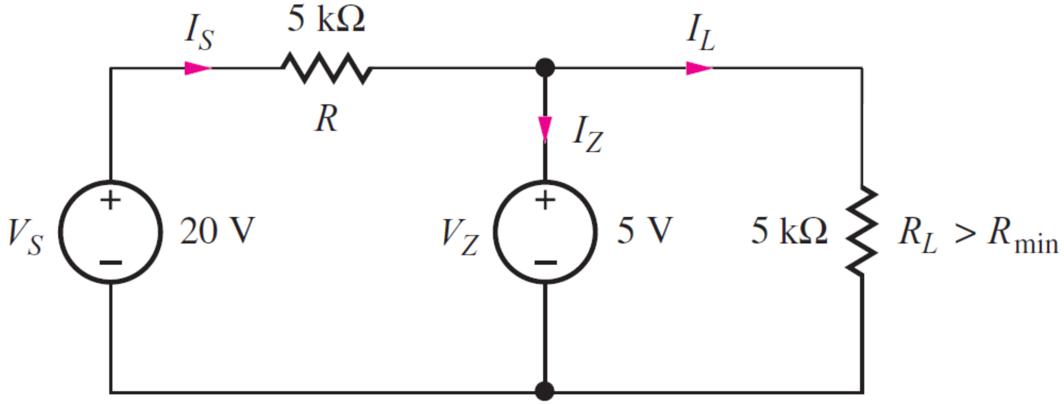
$$I_S = \frac{V_S - V_Z}{R} = 3\text{mA}$$

$$I_L = \frac{V_z}{R_L} = 0.1\text{mA} \;\; | \;\; I_Z = I_S - I_L = 2.9\text{mA}$$

Zener Voltage Regulator

- The minimum value of load resistance for the Zener diode to continue to act as a voltage regulator

What R_L is needed for the diode to be in Zener breakdown?



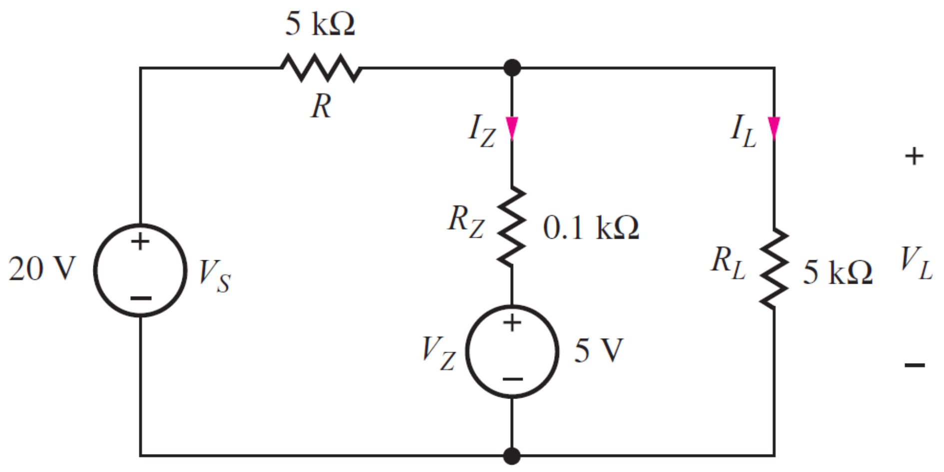
$$I_Z = \frac{V_S - V_z}{R} - \frac{V_Z}{R_L} > 0$$

$$\Rightarrow R_L > \frac{R}{\left(\frac{V_S}{V_Z} - 1\right)} = R_{min}$$

For proper regulation, I_z must be positive. If $I_z < 0$, the diode no longer controls the voltage across the load resistor.

Example

- Find the output voltage and the Zener diode current for the following Zener-diode based voltage regulator circuit



$$\frac{V_S - V_L}{R} - \frac{V_L}{R_L} - \frac{V_L - V_Z}{R_Z} = 0$$

$$V_L = 5.19\ V$$

$$I_Z = \frac{5.19\ V - 5\ V}{100} = 1.9\ mA$$

What if $R_L = 50\text{k}\Omega$?

Line and Load Regulation

- **Line regulation** characterizes how sensitive the output voltage is to input voltage changes

$$\frac{dV_L}{dV_S} mV/V$$

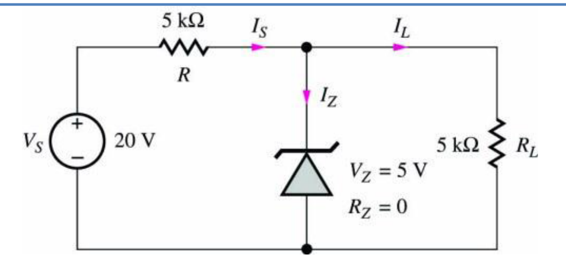
This is defined without knowing what the load is. The picture on the right is just an example.

  - For a fixed load current (assume you can fix it), line regulation = $\frac{R_Z}{R+R_Z}$

- **Load regulation** characterizes how sensitive the output voltage is to changes in load current

$$\frac{dV_L}{dI_L}\Omega$$

  - For changes in load current (e.g. by changing R_L), load regulation = $-(R_Z \| R)$
  - Load regulation is the Thévenin equivalent resistance looking back into the regulator from the load terminals



$$\frac{V_L - V_S}{R} + \frac{V_L - V_Z}{R_Z} + I_L = 0$$

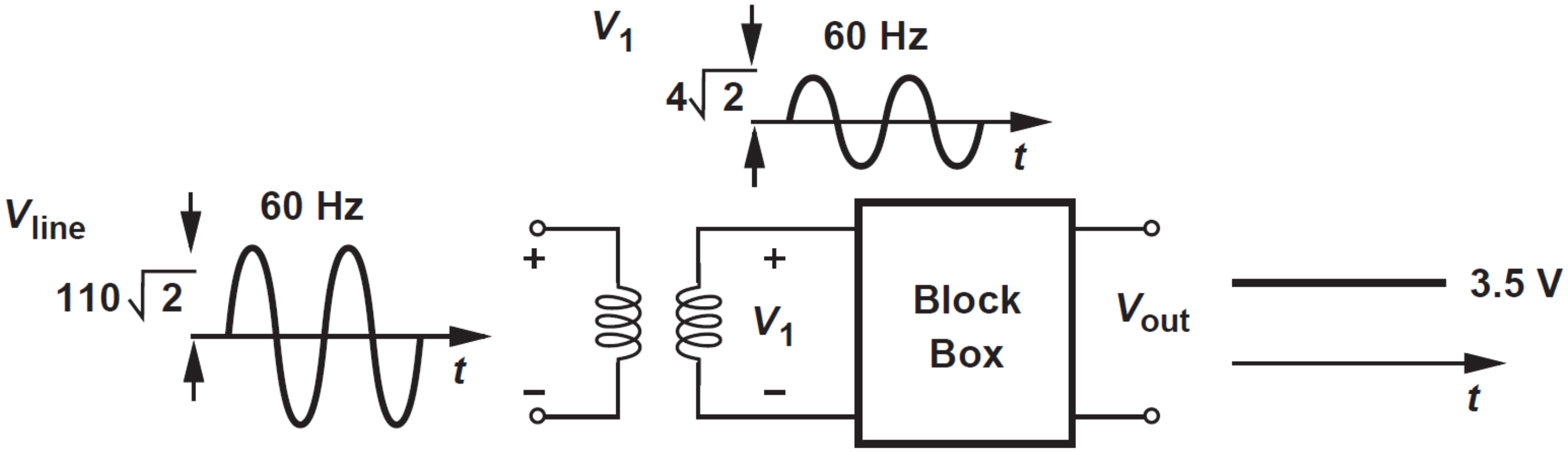
Summary of Diode Circuits

- Diode circuits can be analyzed using several different models
  - Ideal diode model is good for quickly determining what mode the diode is operating in
  - Constant-voltage model is useful to get a reasonable estimate of the Q-point
  - For precise numbers, either use simulation, iterative mathematical analysis, or draw a load line

Half-wave rectifier

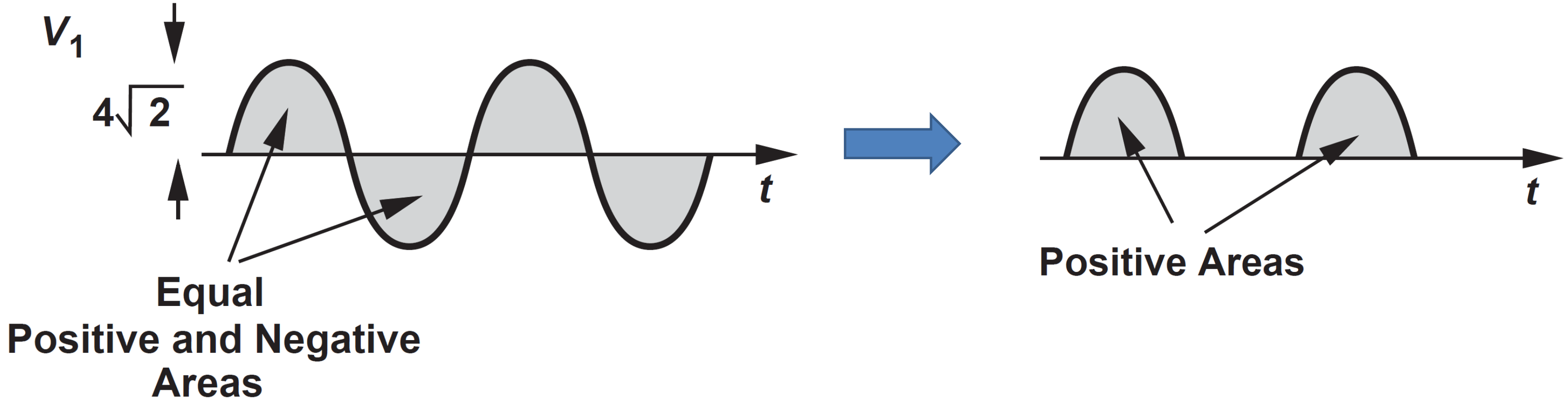
Overview of a Diode Half-Wave Rectifier

- An important application of diode is DC chargers.
- The half-wave rectifier is the black box (after transformer) that passes only the positive half of the stepped-down sinusoid



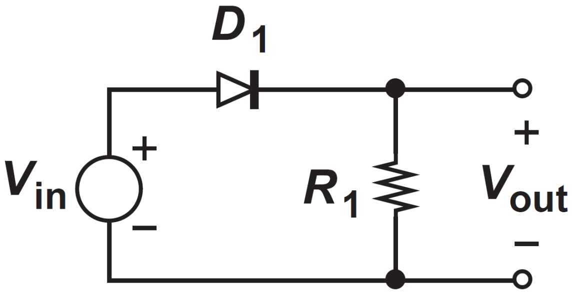
DC Chargers

- A basic rectifier first converts an AC voltage to a positive half-wave
- A filter then eliminates AC components of the waveform to produce a nearly constant dc voltage output.
- Rectifier circuits are used in virtually all electronic devices to convert the 240-V 50-Hz AC power outlet source to the DC voltages



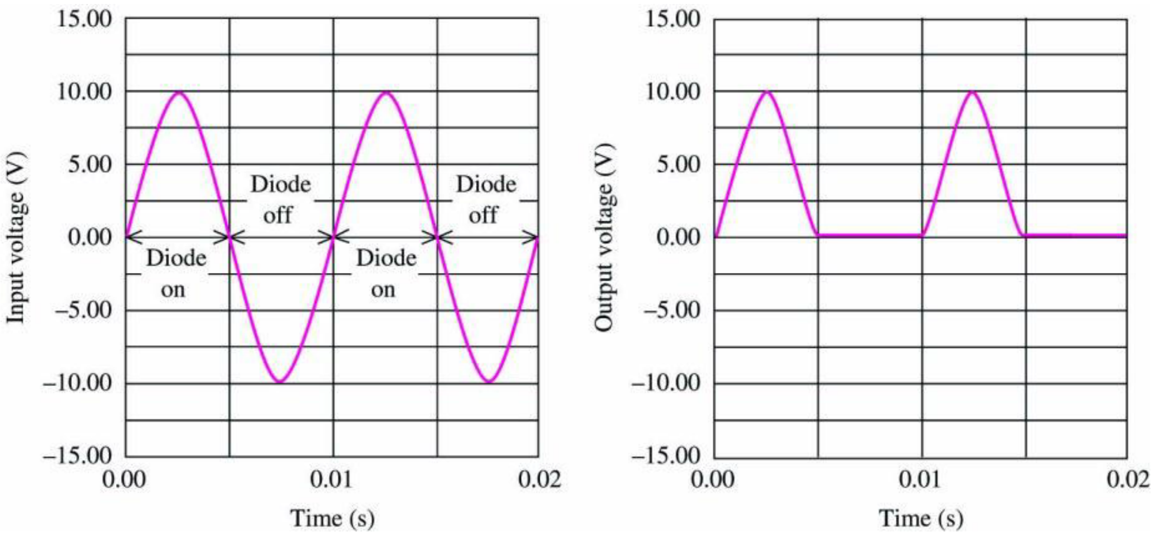
Diode Half-Wave Rectifier

- A basic rectifier converts an ac voltage to a pulsating dc voltage
- A very common application of diodes is half-wave rectification, where either the positive or negative half of the input is blocked

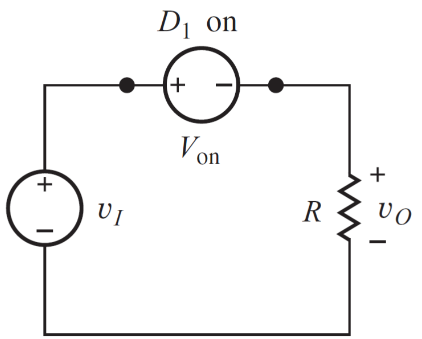


Ideal model:
Diode is on during the positive half-cycle

Diode is off during negative half-cycle



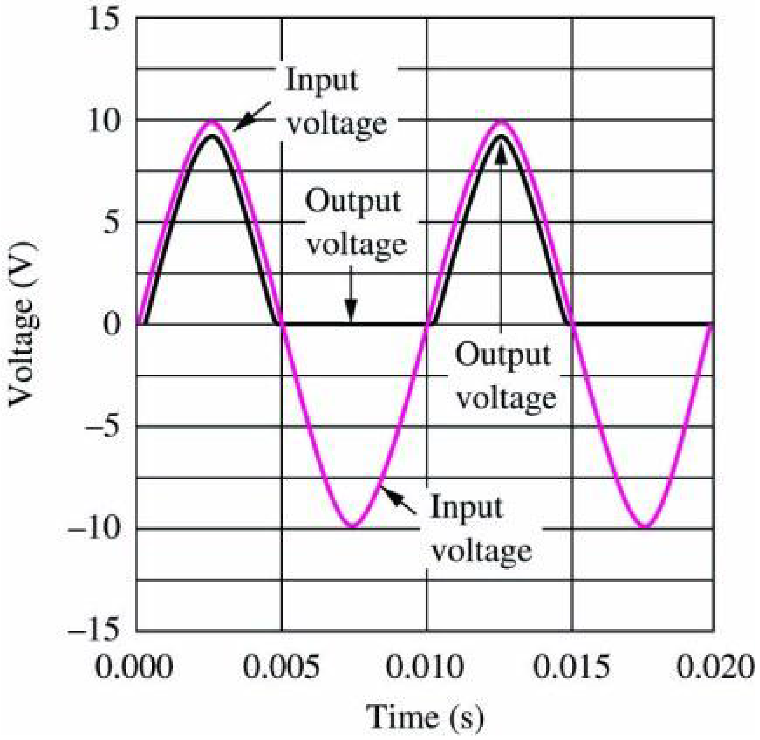
Diode Half-Wave Rectifier



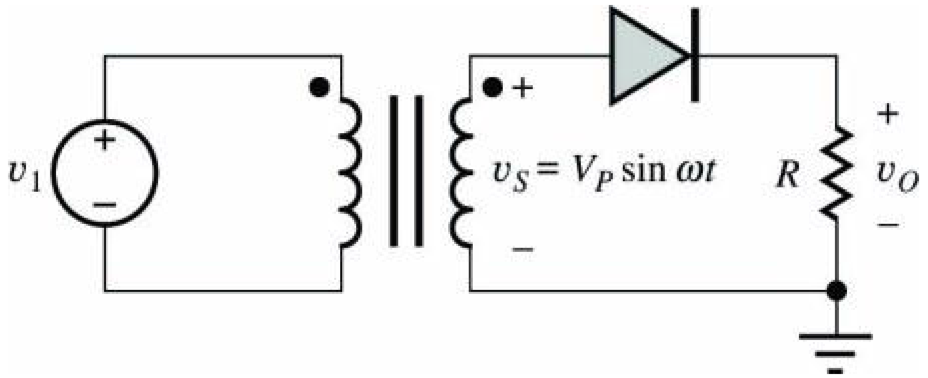
- **Constant-voltage model**
  - During the on-state of the diode:

$$V_{out} = V_{in} - V_{on} = (V_p \sin(wt) - V_{on})$$

- The output voltage is still zero when the diode is off

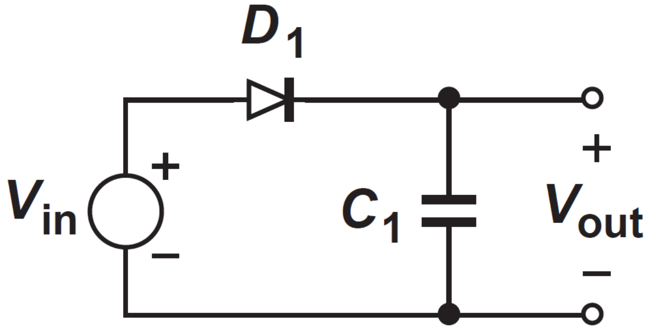


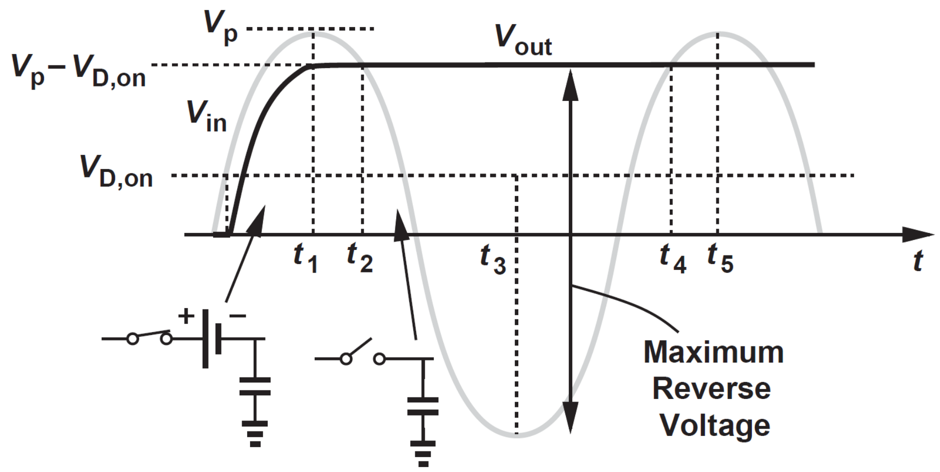
Often a step-down transformer is used to convert the 240-V, 50-Hz voltage available from the power line to the desired AC voltage level as shown.



Diode Half-Wave Rectifier

- But, how do we generate a *constant* output?
- If the resistor in half-wave rectifier is replaced by a capacitor, a fixed voltage output is obtained since the capacitor (assumed ideal) has no path to discharge.

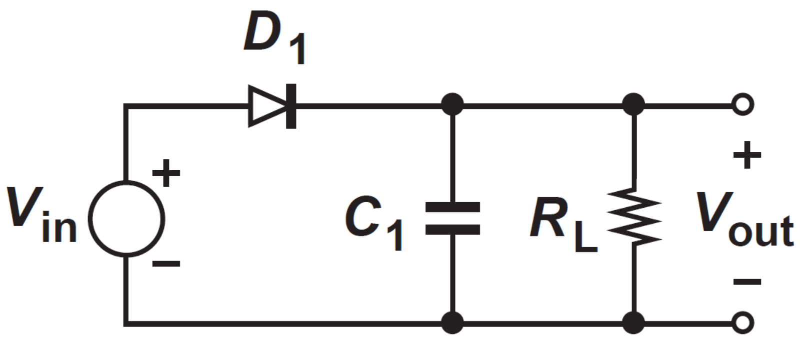




- At the peak of the input voltage, the diode current tries to reverse, and the diode cuts off.
- There is no circuit path to discharger, capacitor retains a constant voltage

Diode Half-Wave Rectifier

- This circuit can be dangerous!
- The capacitor can retain a lethal charge after the power source is removed.
  - Especially if connected to 240V AC
- A practical circuit should include a way to discharge the capacitor safely.
- The resistor should consume a current large enough to discharge the capacitor in a reasonable time, but small enough to minimize unnecessary power waste.

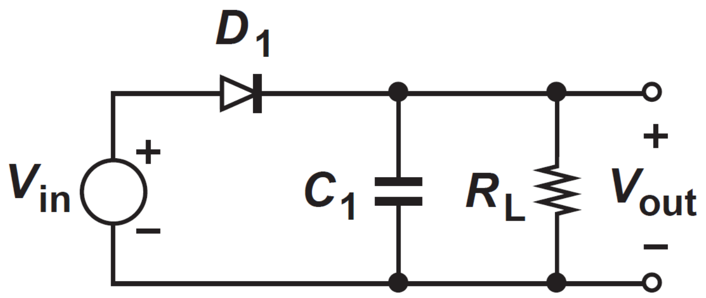


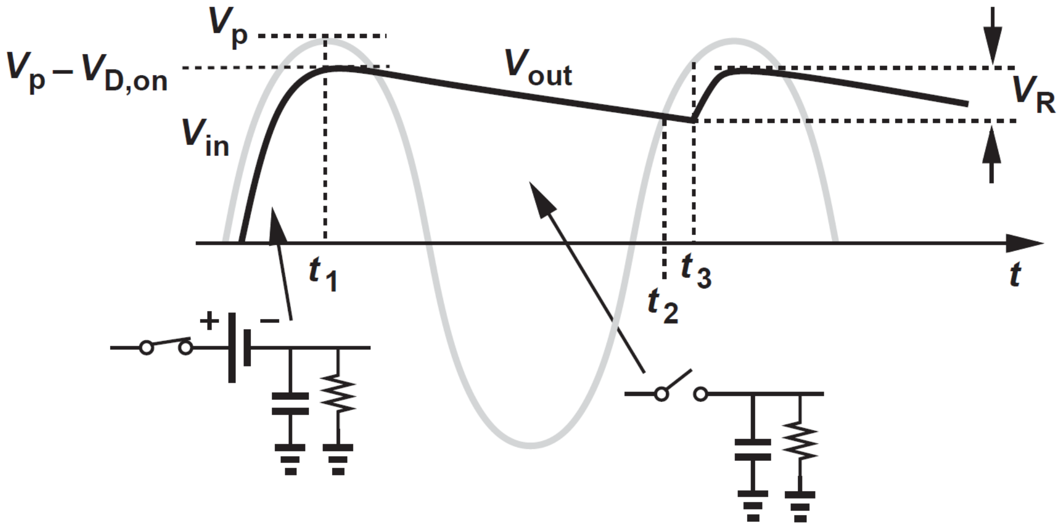
Normally, this circuit would be connected to an output impedance, or load resistor.

Otherwise, the circuit should include a bleeder resistor connected as close as practical across the capacitor.

Diode Half-Wave Rectifier

- A resistor in parallel allows a path for the capacitor to discharge
- During the first quarter cycle, the diode is on and the capacitor charges up to the peak value.



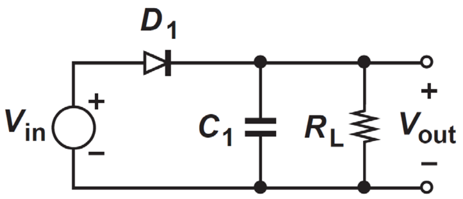


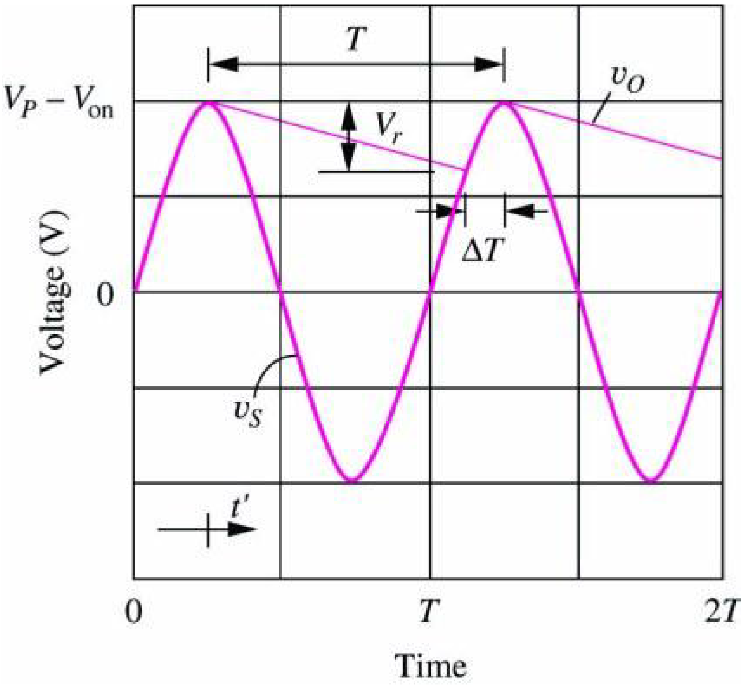
- At the peak input, the diode cuts off and the capacitor discharges exponentially through R.

Rectifier Design Considerations

Capacitor voltage in RC circuit: $V_c = V_i e^{-\frac{t}{RC}}$

- Peak-to-peak Amplitude of Ripple
  - The ripple amplitude is the decaying part of the exponential
  - Ripple voltage becomes a problem if it goes above 5 to 10% of the output voltage





V_{in}: source
V_P: peak
V_{on}: diode turn-on
V_r: ripple
V_o: output

$$V_r = (V_P - V_{on})\left[1 - \exp\left(-\frac{T - \Delta T}{RC}\right)\right]$$

ΔT: conduction interval (i.e. charging time)

Linearize exp(-x) for small x using 1 – x:

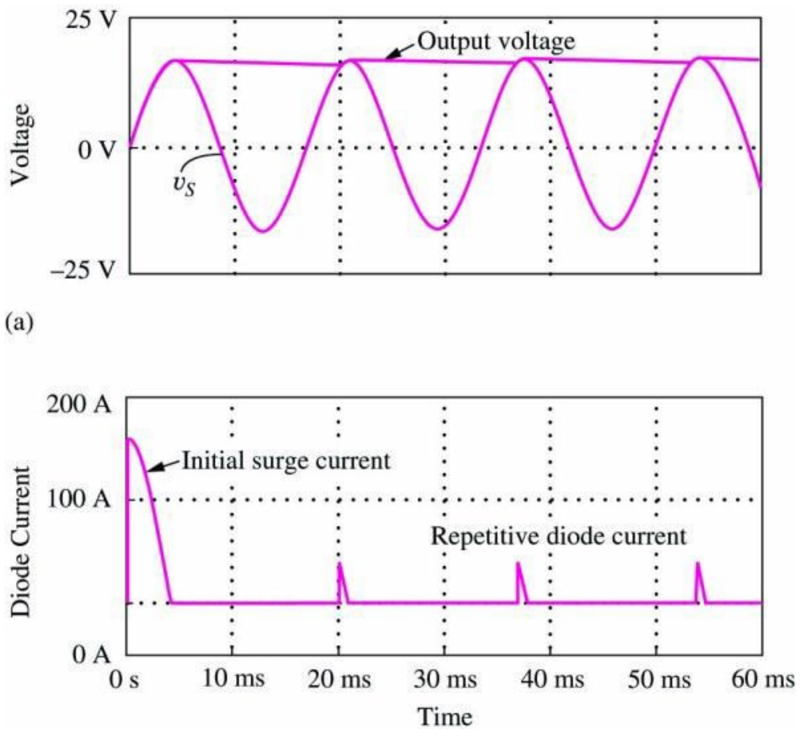
$$V_r \cong (V_P - V_{on})\frac{T}{RC}\left(1 - \frac{\Delta T}{T}\right)$$

$$V_r \cong \frac{(V_P - V_{on})}{R}\frac{T}{C}$$

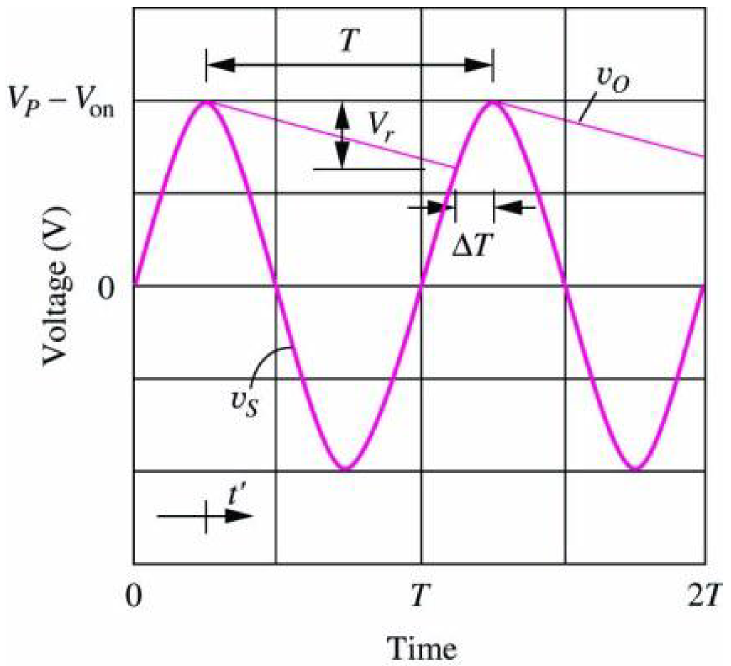
If ΔT is small (for a good rectifier, this is true)

Rectifier Design Considerations

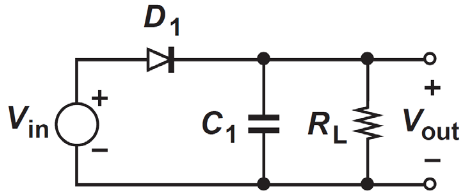
- Conduction Interval
  - The diode conducts for a short time ΔT called the **conduction interval**
  - Its angular equivalent is called the **conduction angle** $\theta_C = 2\pi \frac{\Delta T}{T}$



Expanded View for One Cycle



Rectifier Design Considerations



$$V_r \approx \frac{V_P - V_{on}}{R}\frac{T}{C}$$

- Finding ΔT
  - Voltage at $t = \frac{5}{4}T - \Delta T$:

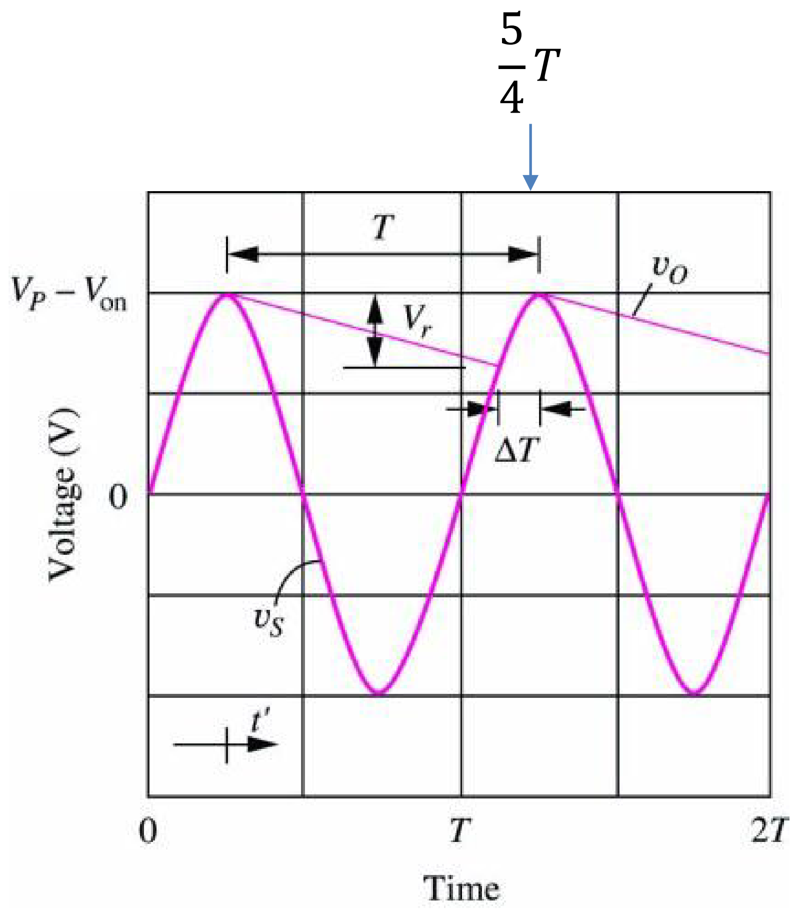
(Input is a sin wave) $V_p \sin\left(2\pi \frac{\left(\frac{5}{4}T - \Delta T\right)}{T}\right) = V_p - V_r$

$$V_p \sin\left(\frac{5}{2}\pi - \theta_C\right) = V_p - V_r$$

$$V_p \cos(\theta_C) = V_p - V_r$$

For small θ_C , $\cos\theta_C \approx 1 - \frac{{\theta_C}^2}{2}$

$$\theta_C^2 = \frac{2V_r}{V_p} \qquad \theta_C = 2\pi\frac{\Delta T}{T} = 2\pi f \Delta T$$



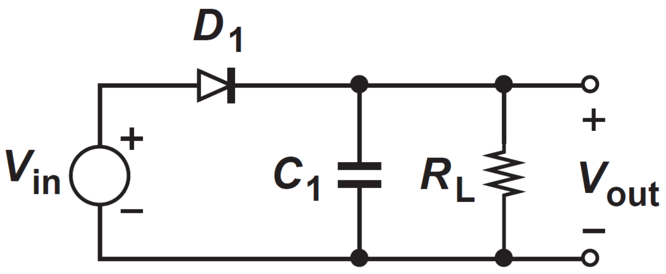
Rule of thumb approximation

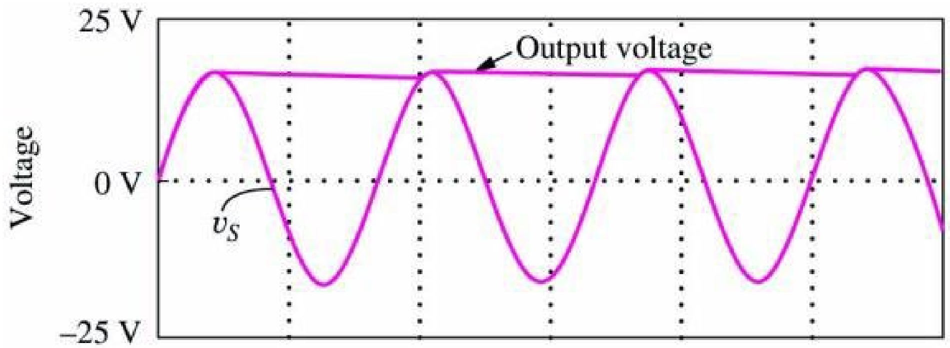
$$\Delta T \cong \frac{1}{2\pi f}\sqrt{\frac{2V_r}{V_p}} = \frac{1}{2\pi f}\sqrt{\frac{2T}{RC}\frac{(V_P - V_{on})}{V_P}}$$

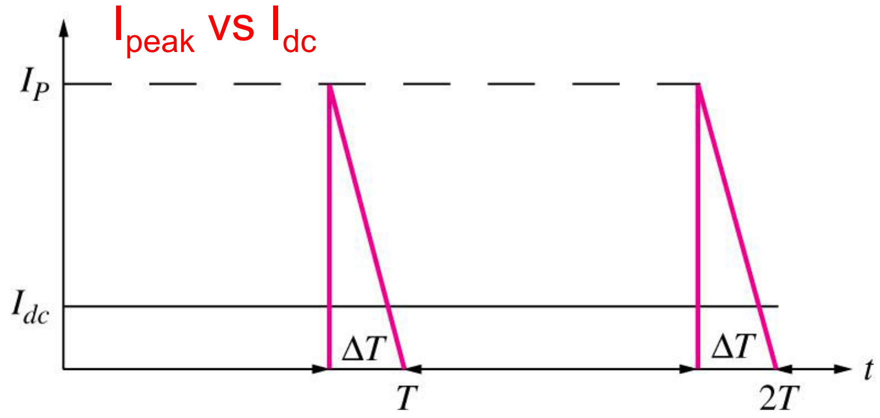
Rectifier Design Considerations

- Diode Current
  - We can approximate the DC current being drawn while discharging the capacitor as:

$$I_{dc} = \frac{V_P - V_{on}}{R}$$





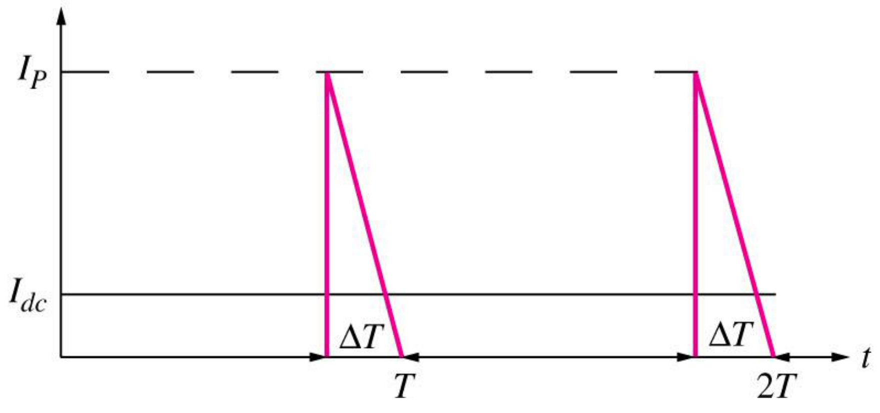


The current in the short conduction interval must be large to charge the capacitor

Rectifier Design Considerations

- Diode Current
  - The peak current is supplied by the diode to replenish the total charge lost from the filter capacitor in each cycle.

$$Q = I_p \frac{\Delta T}{2} = I_{dc} T \Rightarrow I_P = I_{dc} \frac{2T}{\Delta T} = \frac{V_p - V_{\text{on}}}{R} \frac{2T}{\Delta T}$$



Area of triangle:

$$Q = I_p \frac{\Delta T}{2}$$

Integration of current over time = charge

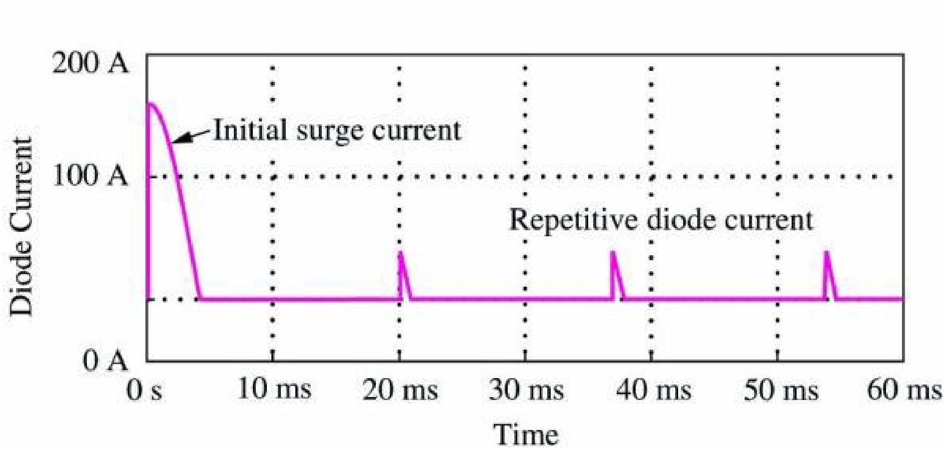
Rectifier Design Considerations

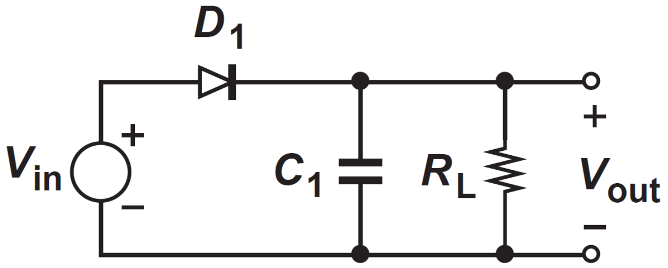
Impedance of capacitor:

$$Z_C = \frac{1}{j\omega C}$$

- Maximum diode current
  - The diode has its maximum current (**surge current**) at t=0
  - Capacitor is completely discharged
  - Largest forward bias across the diode
- This current has to be controlled so it does not damage the device.
  - In reality, surge current will not reach max value once circuit series resistances are taken into account

$$I = \frac{V}{Z_C} = Vj\omega C \qquad I_{SC} \approx C2\pi f V_p$$



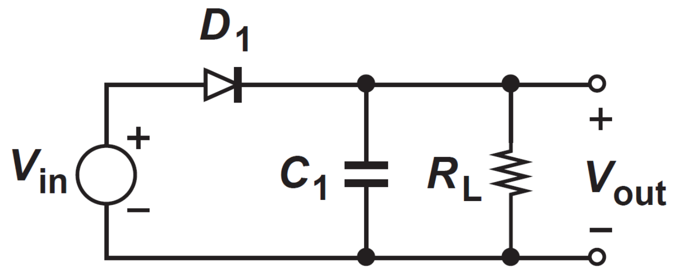


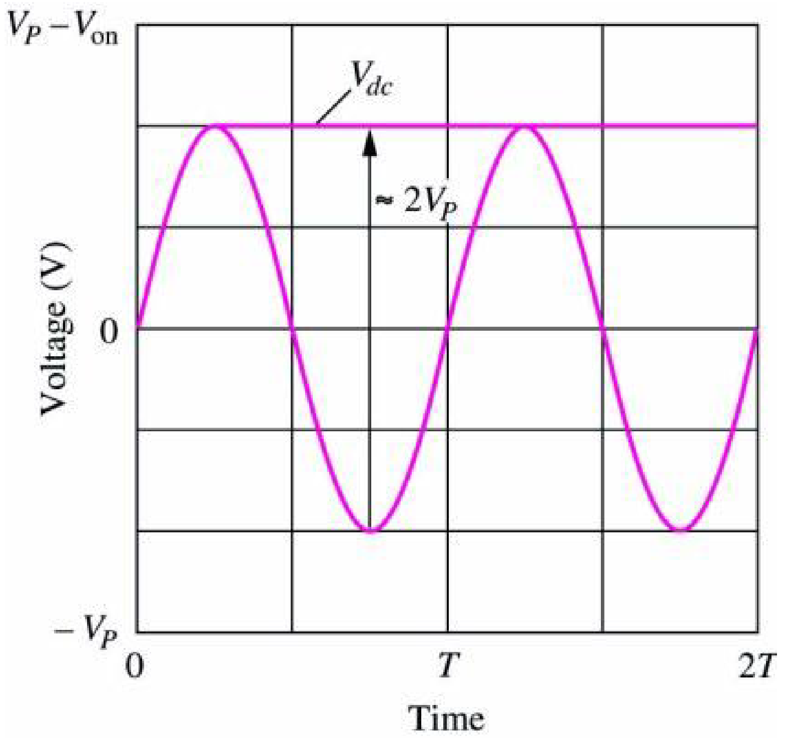
Rectifier Design Considerations

- Peak Inverse Voltage Rating (PIV)
  - PIV of the rectifier diode is lower bound of the diode breakdown voltage.
  - When the diode is off, the reverse-bias across the diode is V_{out} - V_{in}.
  - V_{in} at its negative peak:

$$\mathrm{PIV} \geq V_{out} - v_{in}^{\min} = V_P - V_{on} - (-V_P) \cong 2V_P$$

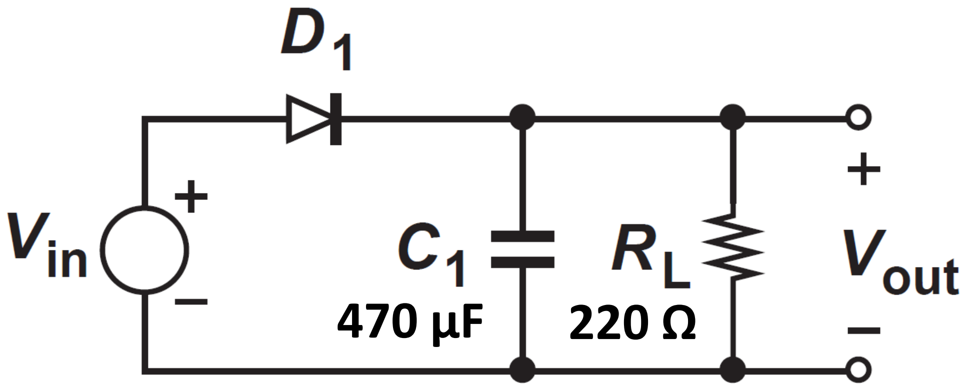
This is the minimum Zener breakdown voltage required for the diode





Example

- V_{in} is a sine wave with peak voltage of 15V @ 60 Hz. What is the voltage output ripple? Assume diode turn-on voltage is 0.65 V



$$V_{max} = V_{in} - V_{D1} = 14.35\ V$$

$$V_{C1} = V_{max}\exp\left(-\frac{T-\Delta T}{R_L C_1}\right) \approx V_{max}\left(1 - \frac{T}{R_L C_1}\right)$$

$$V_r \approx V_{max}\frac{T}{R_L C_1}$$

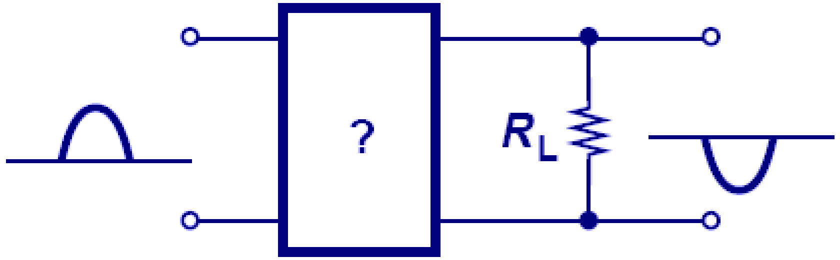
$$V_r = (14.35\ V)\frac{1}{60\ \times 220 \times 470 \times 10^{-6}} = 2.31\ V$$

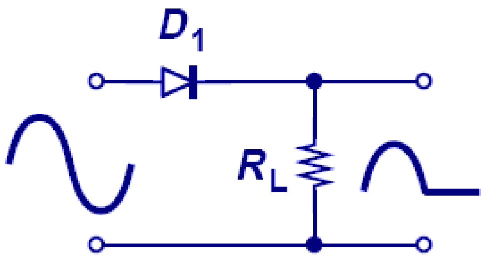
This rectifier is not so great

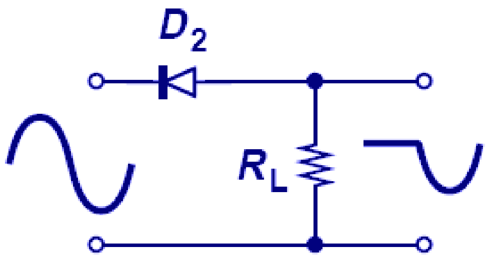
Full-wave rectifier

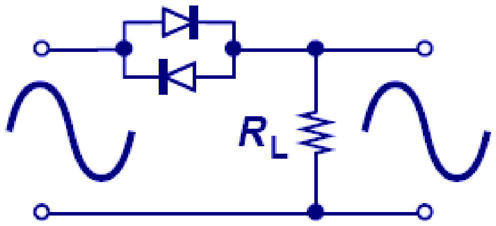
Full-Wave Rectifier

- We want to build a circuit that can invert the negative half of the input cycle





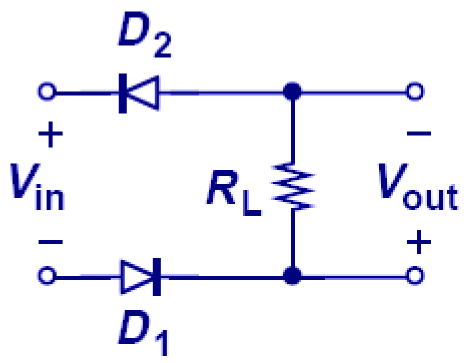




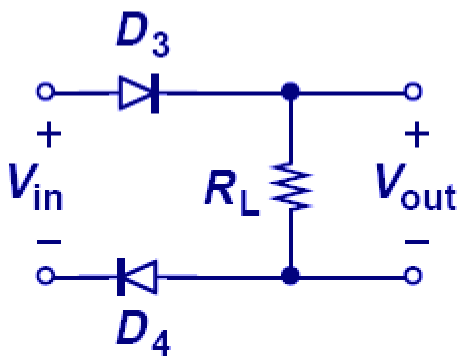
Nope.

Full-Wave Rectifier

- What is the output of these two?



Blocking positive half

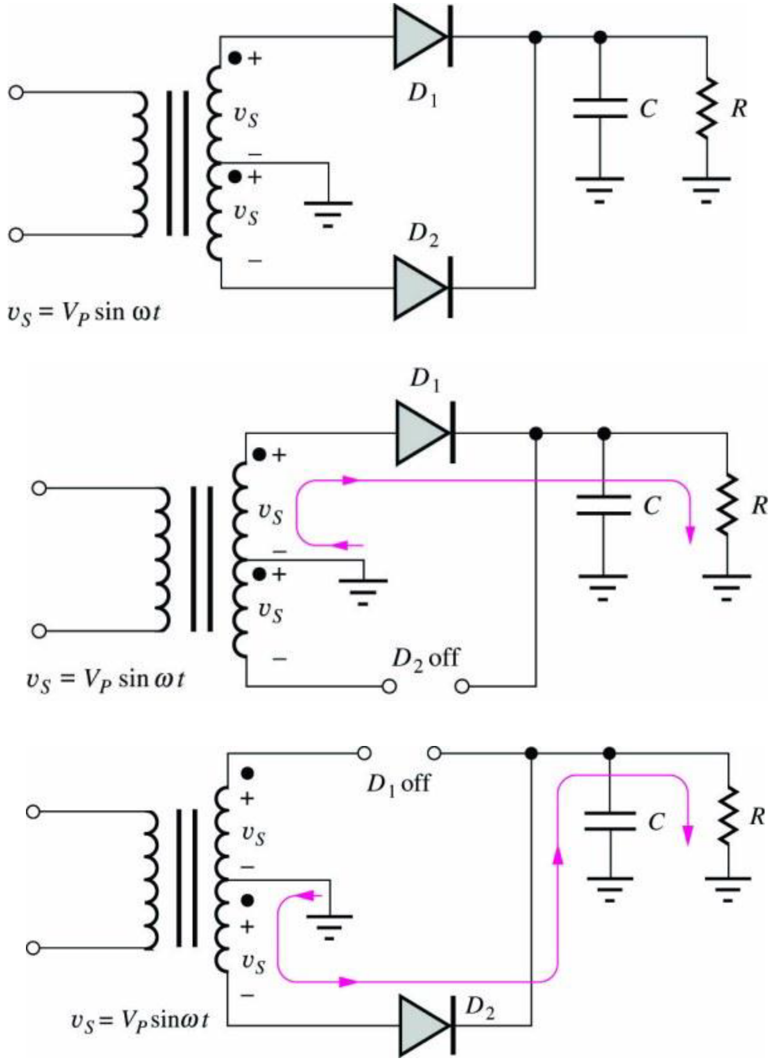


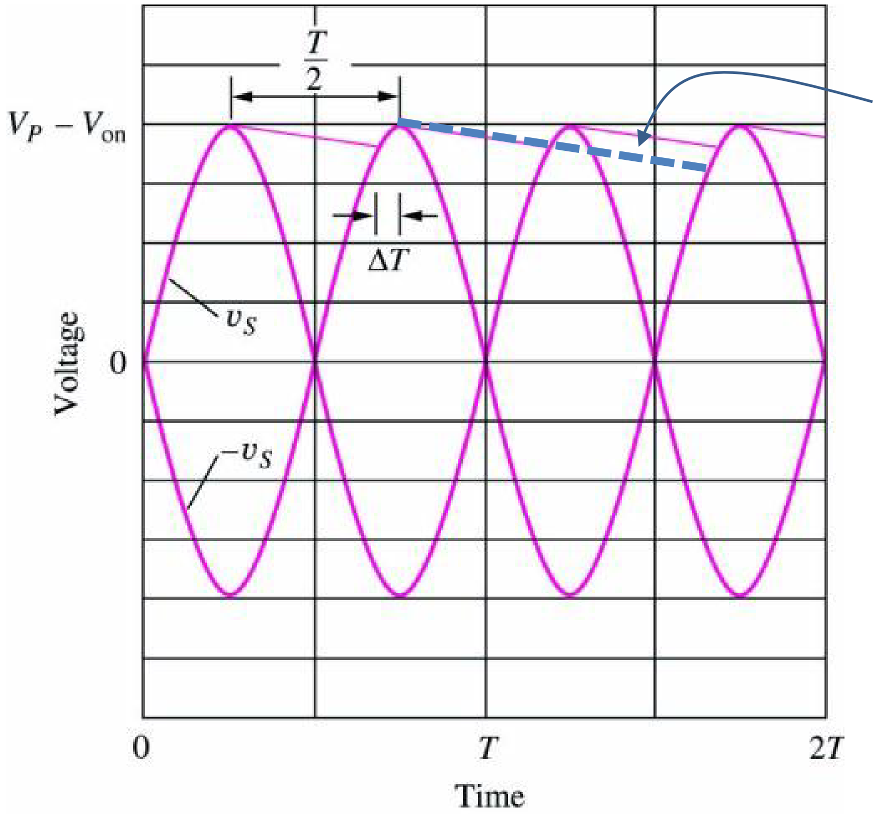
Blocking negative half

Nope.

Centre-Tapped Full-Wave Rectifier

Commonly grounded at the center



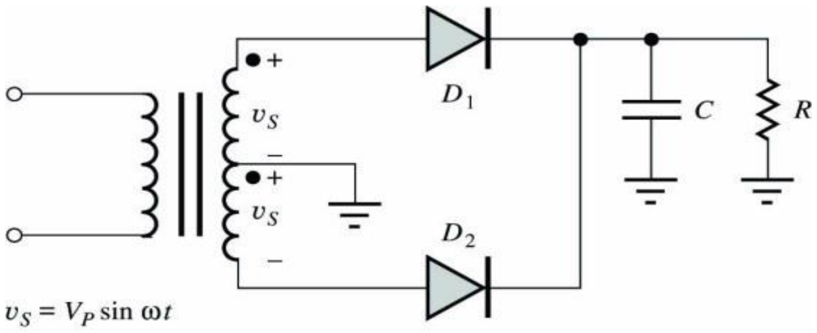


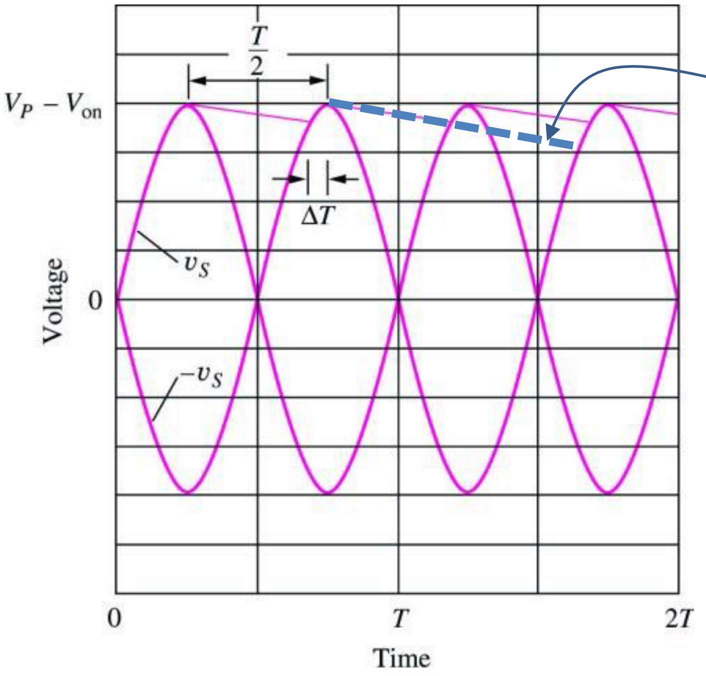
Discharge for half-wave rectifier

Centre-Tapped Full-Wave Rectifier

- Full-wave rectifiers cut capacitor discharge time in half
- Require half the filter capacitance to achieve a given ripple voltage.
- All specifications are the same as for half-wave rectifiers, except I_p halved.
- Ripple amplitude:

$$V_R \cong \frac{(V_P - V_{on})}{2RCf_{in}}$$

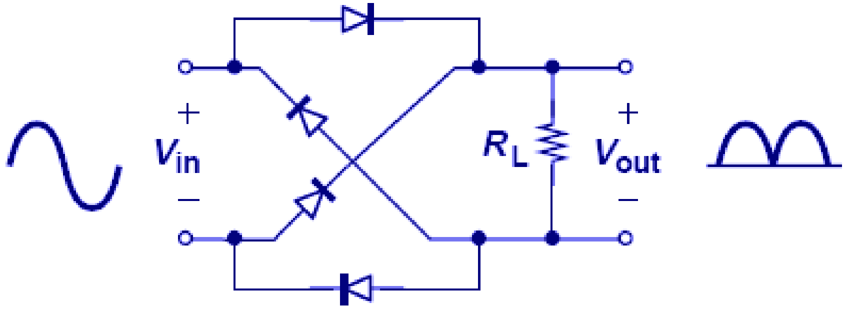




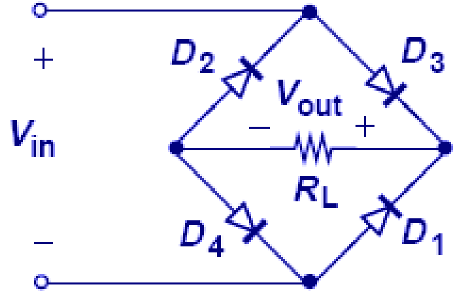
Discharge for half-wave rectifier

Full-Wave Bridge Rectifier

- D1 and D2 pass/invert the negative half cycle of input
- D3 and D4 pass the positive half cycle.



(a)



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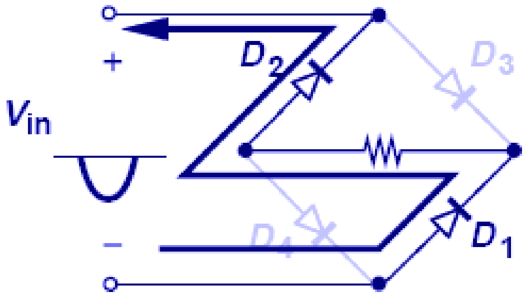
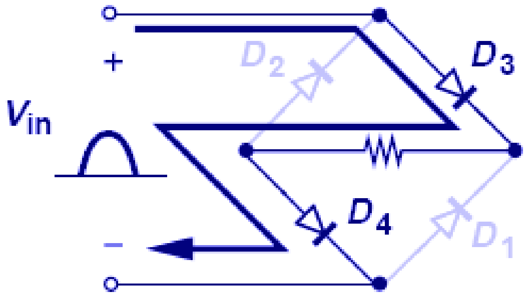
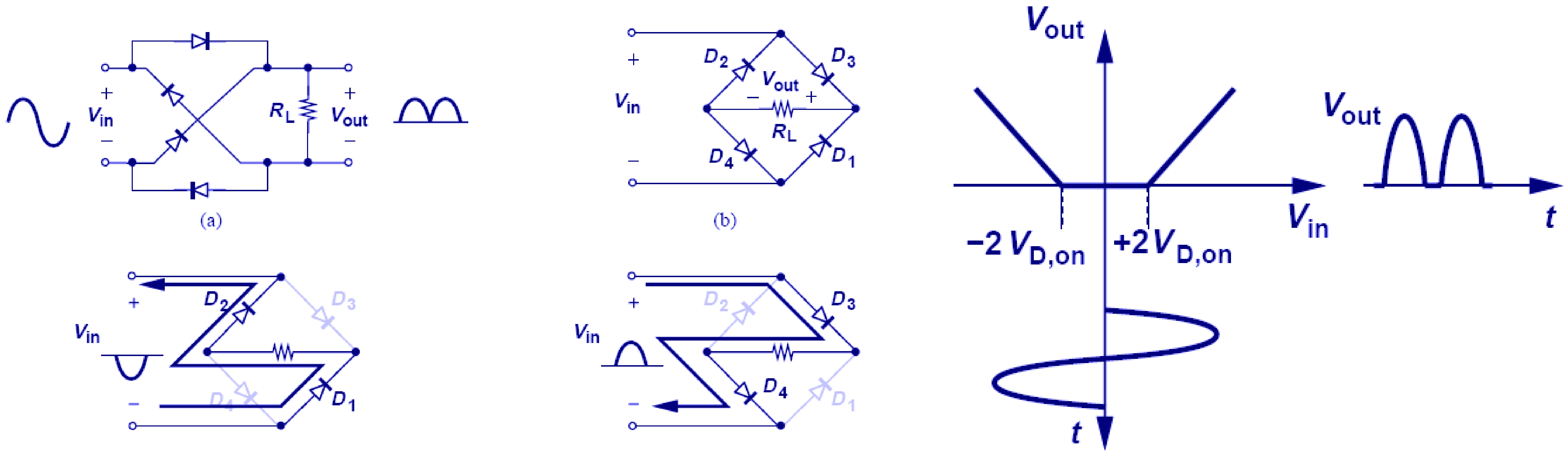




Full-Wave Bridge Rectifier

- Full-Wave bridge rectifier using constant-voltage model
- The dead-zone around V_{in} arises because V_{in} must exceed 2 $V_{D,ON}$ to turn on the bridge

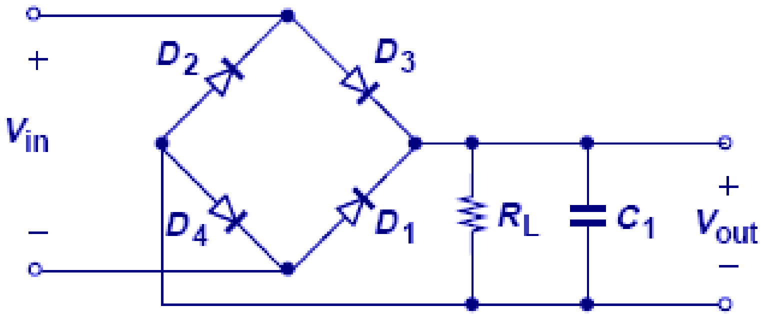


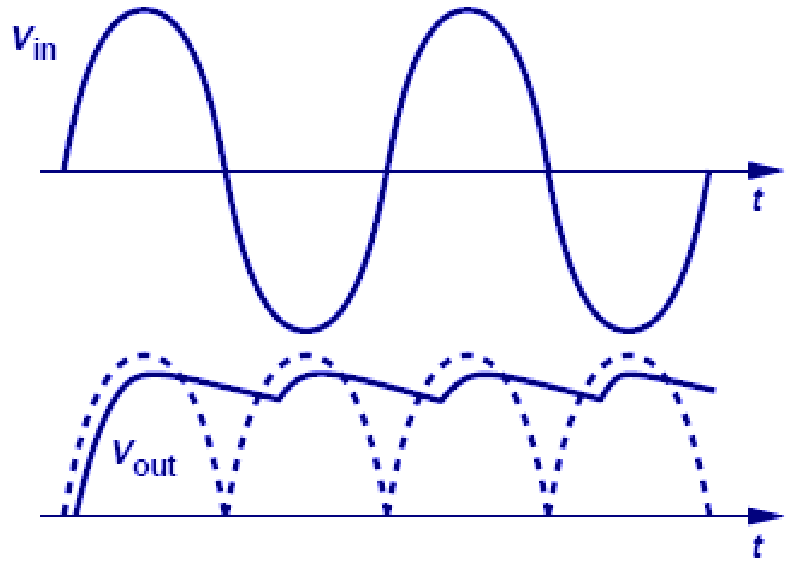
Full-Wave Bridge Rectifier with Capacitor

- This is the complete bridge rectifier.
- Since C1 only gets ½ of period to discharge, ripple voltage is decreased by a factor of 2.

$$V_R \cong \frac{V_P - 2V_{D,on}}{2R_L C_1 f}$$

Or, only need smaller capacitor to achieve the same ripple voltage





Full-Wave Bridge Rectifier Peak Inverse Voltage

- Each diode is subjected to approximately one V_p peak inverse voltage (PIV) (versus $2V_p$ in half-wave rectifier)

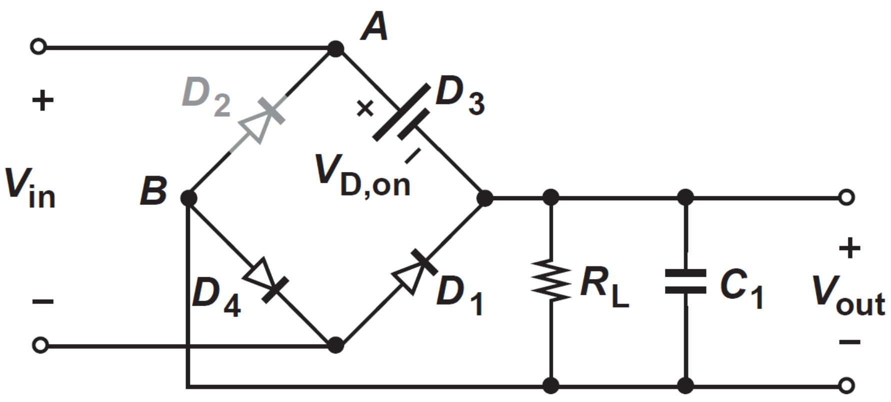
$$V_{out} = V_P - 2V_{D,on}$$

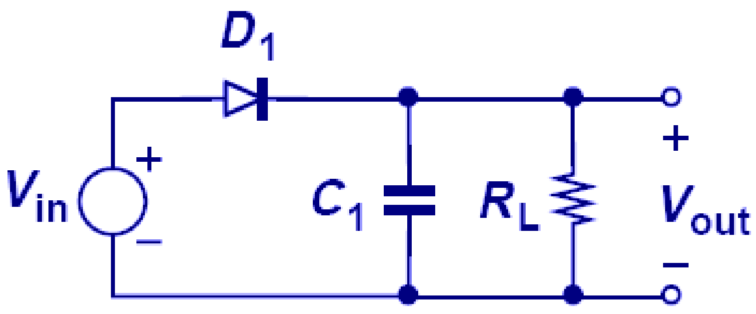
When $V_{in} = V_P$:

$$V_{AB} = V_{D,on} + V_{out}$$

$$PIV \geq V_P - V_{on} \cong V_P$$

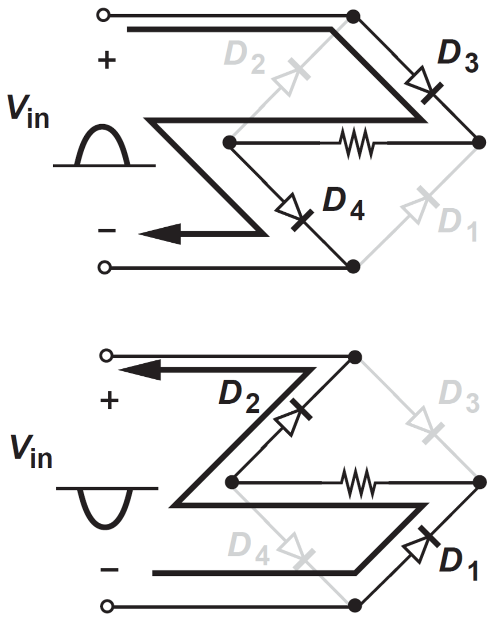
$$PIV \geq 2V_P$$

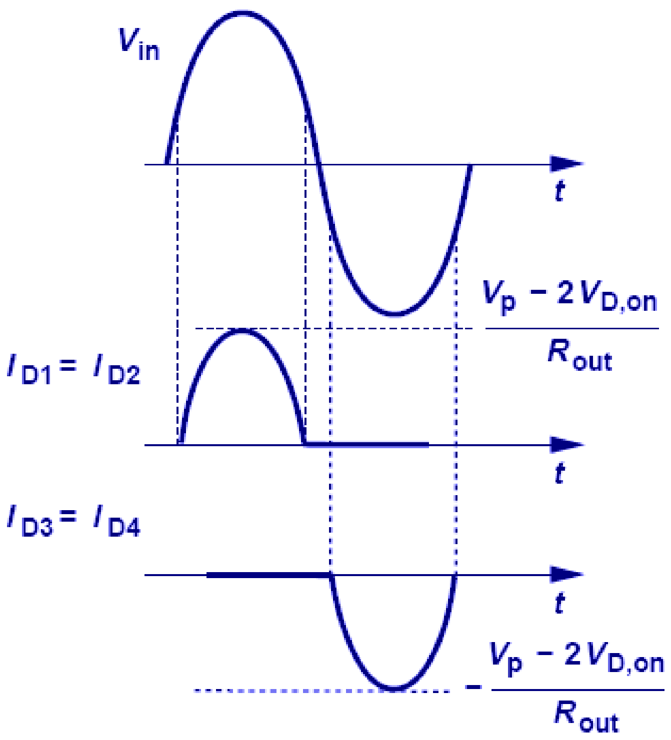




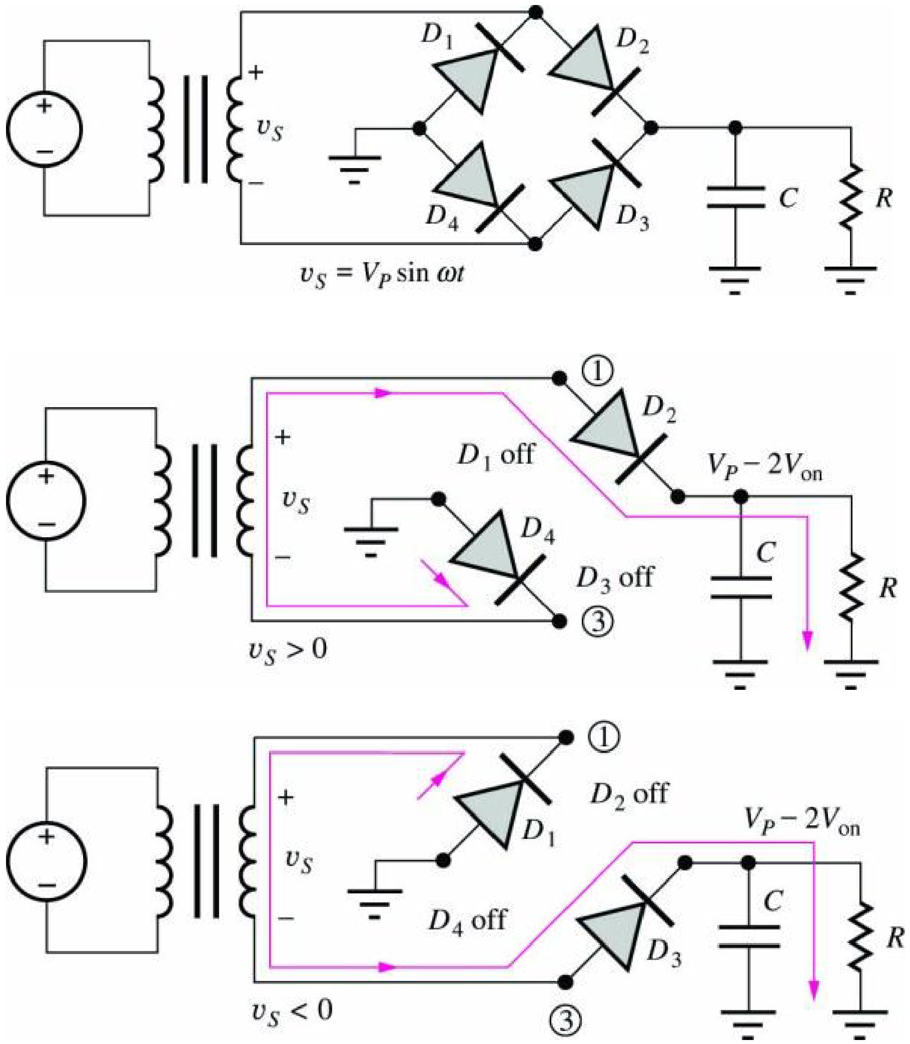
Full-Wave Bridge Rectifier Peak Current

- Peak current reduced (charging more frequently)
- Surge current when circuit is first powered on is reduced proportional to the smaller capacitor





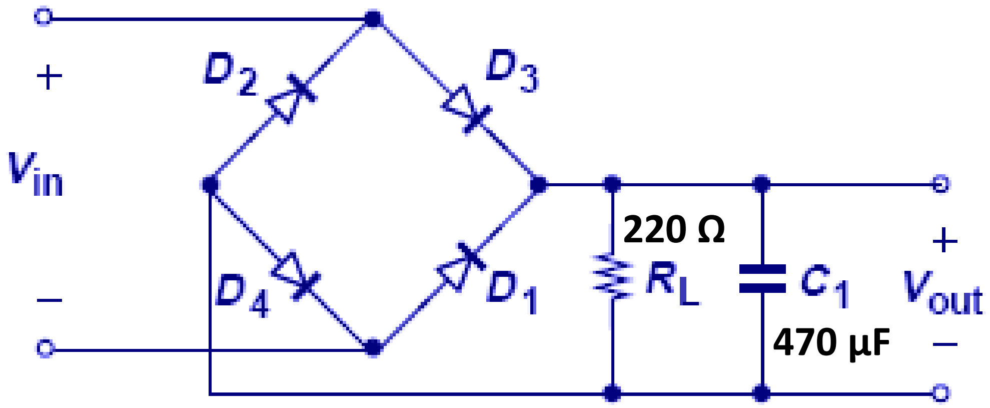
Full-Wave Bridge Rectifier Summary



Specifications are the same as for a half-wave rectifier except PIV = V_P and I_P halved

Example

- Consider the example from before where is a sine wave with peak voltage of 15V @ 60 Hz. What is the voltage ripple? Assume diode turn-off voltage 0.65 V.



$$V_{max} = V_{in} - 2V_D = 13.7\ V$$

$$V_r = \frac{V_{in} - 2V_D}{2R_L C_1 f}$$

$$V_r = (13.7\ V)\frac{1}{2 \times 60\ \times 220 \times 470 \times 10^{-6}} = 1.1\ V$$

Rectifier Summary

Comparison of Rectifiers with Capacitive Filters

RECTIFIER PARAMETER	HALF-WAVE RECTIFIER	FULL-WAVE RECTIFIER	FULL-WAVE BRIDGE RECTIFIER
Filter capacitor	$C = \frac{V_P - V_{on}}{V_r} \frac{T}{R}$	$C = \frac{V_P - V_{on}}{V_r} \frac{T}{2R}$	$C = \frac{V_P - 2V_{on}}{V_r} \frac{T}{2R}$
PIV rating	$2V_P$	$2V_P$	V_P
Peak diode current (constant V_r)	Highest I_P	Reduced $\frac{I_P}{2}$	Reduced $\frac{I_P}{2}$
Surge Current	Highest	Reduced ($\propto C$)	Reduced ($\propto C$)
Comments	Least complexity	Smaller capacitor Requires center-tapped transformer Two diodes	Smaller capacitor Four diodes No center tap on transformer