ELEC2104 – Lecture 08

BJT Amplifiers, MOSFET



4-resistor biasing: example



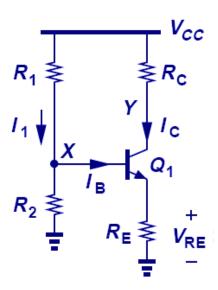
Design of Four-Resistor Bias Network: Example

- Design 4-resistor bias circuit with given parameters: $I_C = 750 \, \mu A$, $\beta_F = 100$, $V_{CC} = 15 \, V$, $V_{CE} = 5 \, V$. Assume we need to operate in forward-active region, $V_{BE} = 0.7 \, V$
 - Let $V_E = 5 V$, $V_C = 10 V$
 - Divide $(V_{CC} V_{CE})$ between R_E and R_C .

$$\alpha = \frac{\beta_F}{\beta_F + 1} = 0.99 \quad \longrightarrow \quad I_E = \frac{I_C}{\alpha} = 757 \ \mu A$$

$$R_E = \frac{V_E}{I_E} = 6.60 \ k\Omega$$

$$R_C = \frac{V_{CC} - V_C}{I_C} = 6.67 \ k\Omega$$



Design of Four-Resistor Bias Network: Example

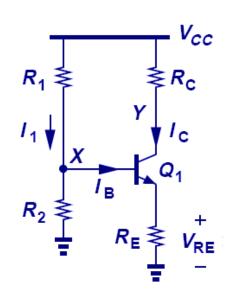
- Design 4-resistor bias circuit with given parameters: $I_C = 750 \, \mu A$, $\beta_F = 100$, $V_{CC} = 15 \, V$, $V_{CE} = 5 \, V$. Assume we need to operate in forward-active region, $V_{BE} = 0.7 \, V$
 - Set $I_1 = 10I_B$ and $I_2 = 9I_B$

$$I_B = \frac{I_C}{\beta_F} = 7.5 \mu A$$

$$V_B = V_E + V_{BE} = 5.7 V$$

$$I_1 = 10I_B = 75 \mu A \longrightarrow R_2 = \frac{V_B}{I_2} = 84.4 k\Omega$$

$$I_2 = 9I_B = 67.5 \mu A \longrightarrow R_1 = \frac{V_{CC} - V_B}{I_1} = 124 k\Omega$$

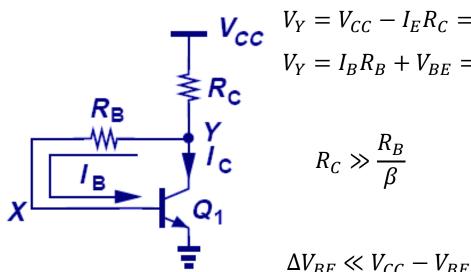


Two-resistor biasing



Two-resistor biasing (with collector-to-base feedback resistor)

- We can self-bias a transistor to force it into forward-active operation (guaranteed)
 - With R_B , the collector must be at a higher potential than the base
 - If BE junction is off (hence BJT is in cutoff), then $V_X = V_Y = V_{CC}$, hence $V_{BE} = V_X > V_{on}$
 - The transistor hence must be in forward-active operation.
- The collector voltage provides the necessary V_x and I_B.



$$V_{CC} V_{Y} = V_{CC} - I_{E}R_{C} = V_{CC} - I_{C}(1 + 1/\beta)R_{C}$$

$$V_{Y} = I_{B}R_{B} + V_{BE} = \frac{R_{B}I_{C}}{\beta} + V_{BE}$$

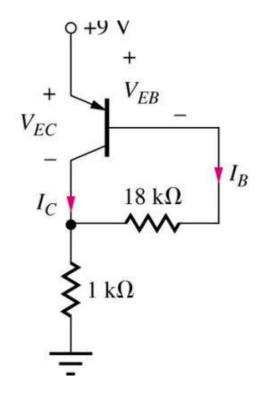
$$I_{C} = \frac{V_{CC} - V_{BE}}{(1 + 1/\beta)R_{C} + R_{B}/\beta}$$

$$R_C \gg \frac{R_B}{\beta}$$
 provides insensitivity of Q-point (I_C) to β

 $\Delta V_{BE} \ll V_{CC} - V_{BE}$ provides insensitivity of Q-point (I_C) to variation in ΔV_{BE}

Two-Resistor Bias Network Analysis: Example

• Find Q-point for pnp transistor in a two-resistor bias circuit with β_F = 50, V_{CC} = 9 V. Assume it is in forward-active operation, so V_{EB} = 0.7 V.



Analysis is the same as for a npn transistor

Apply Kirchoff's Voltage Law:

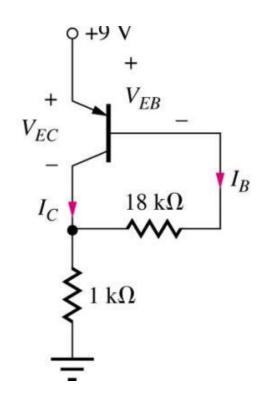
$$-V_{CC} + V_{EB} + V_{BC} + V_C = 0$$

$$V_{EB} + 18\ 000I_B + 1000(I_C + I_B) = 9$$

$$V_{EB} + 18\,000I_B + 1000(51)I_B = 9$$

Two-Resistor Bias Network Analysis: Example

• Find Q-point for pnp transistor in a two-resistor bias circuit with β_F = 50, V_{CC} = 9 V. Assume it is in forward-active operation, so V_{EB} = 0.7 V.



$$V_{EB} + 18\,000I_B + 1000(51)I_B = 9$$

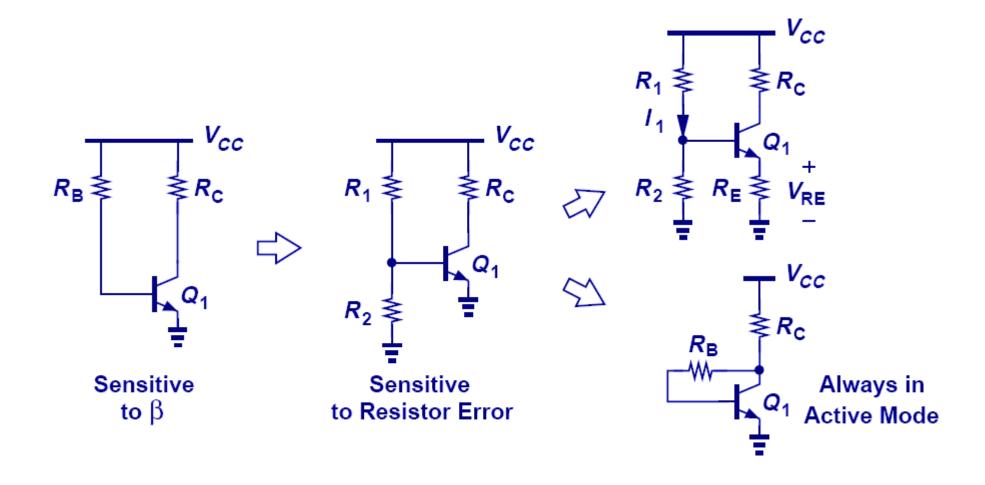
$$I_B = \frac{9\,V - 0.7\,V}{69\,000\,\Omega} = 120\,\mu A$$

$$I_C = 50I_B = 6.01\,mA$$

$$V_{EC} = 9 - 1000(I_C + I_B) = 2.88\,V$$

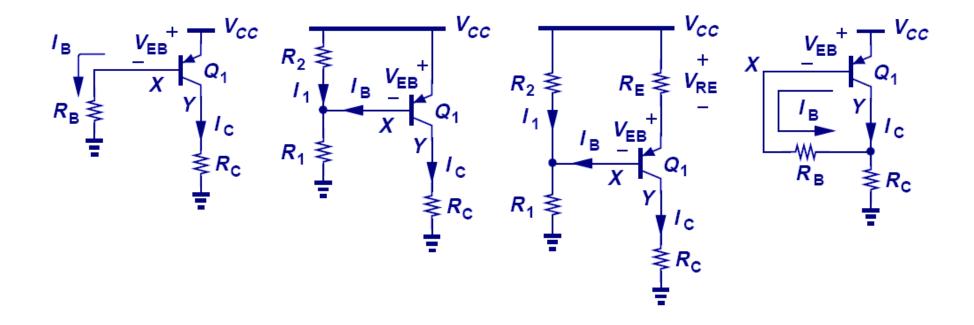
Q-point is: (6.01 mA, 2.88 V)

Summary of Biasing Techniques



PNP Transistor Biasing

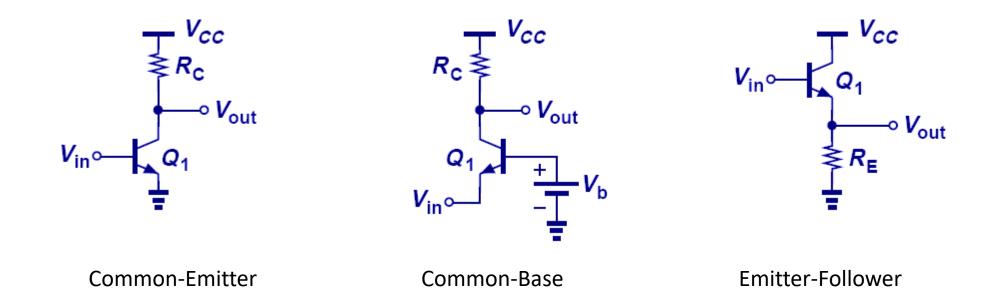
- Exact same principles, exact same design techniques
- Pay attention to direction of voltage and current



BJT amplifier topologies



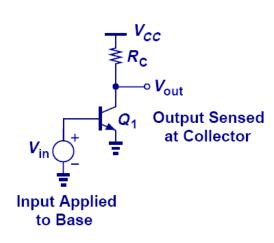
BJT Amplifier Topologies



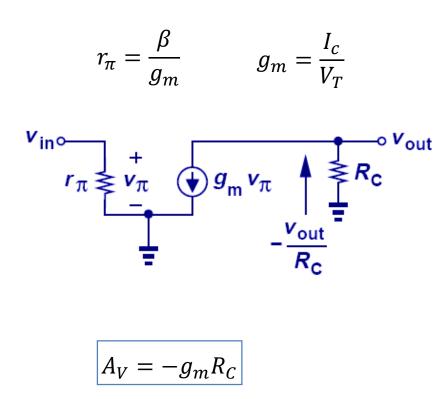
- These are the most useful configurations
- All of v_{out} , v_{in} are small signals (on top of V_C , V_B etc.). Assuming we have biased the BJT correctly.

Common-Emitter Amplifier

Gain of amplifier – Voltage gain

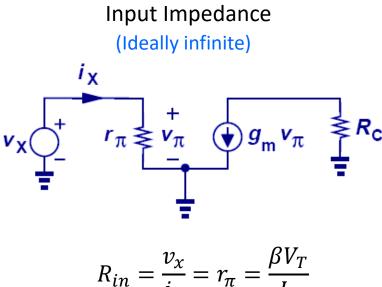


$$A_V = \frac{v_{out}}{v_{in}}$$
$$-\frac{v_{out}}{R_C} = g_m v_{\pi} = g_m v_{in}$$



Common-Emitter Amplifier

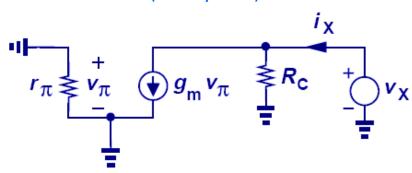
- Input/Output Impedance
 - Determines the capability to interface with preceding and following stages
 - When measuring the output impedance, the input port has to be grounded so that V_{in} is grounded, i.e. we don't care whatever circuit was connected to B. (again, this is the small signal circuit)



$$R_{in} = \frac{v_x}{i_x} = r_\pi = \frac{\beta V_T}{I_C}$$



(Ideally zero)

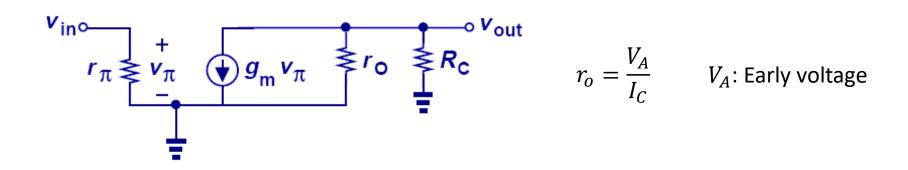


$$R_{out} = \frac{v_x}{i_x} = R_C$$
 (since $v_\pi = 0$)

Tradeoff between output impedance and voltage gain

Common-Emitter Amplifier with Early effect

• Early effect will lower the gain of the CE amplifier, as it appears in parallel with R_C.

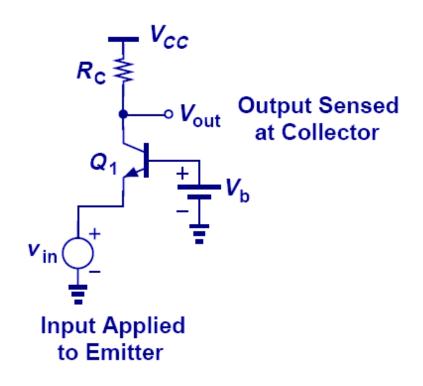


$$A_V = -g_m(R_C||r_0)$$

$$R_{out} = R_C || r_0$$

Common-Base Amplifier

• In common base topology, where the base terminal is biased with a fixed voltage, emitter is fed with a signal, and collector is the output.

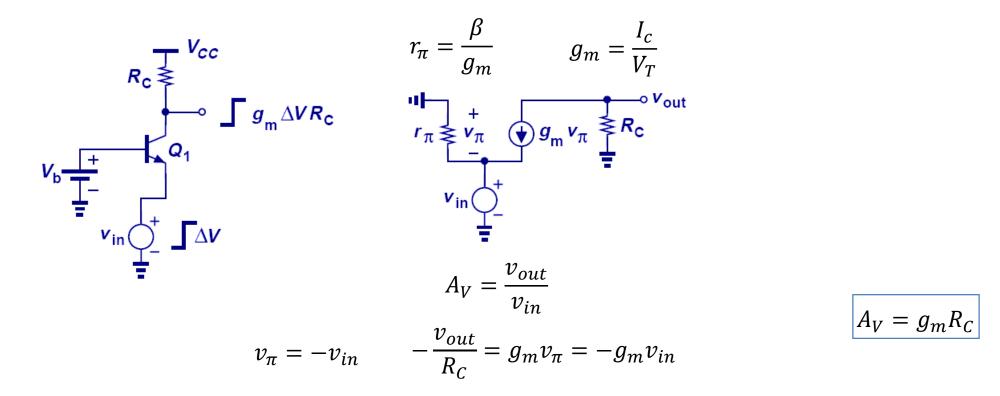


Why does this work?

Change in V_E changes V_{BE} , hence I_C

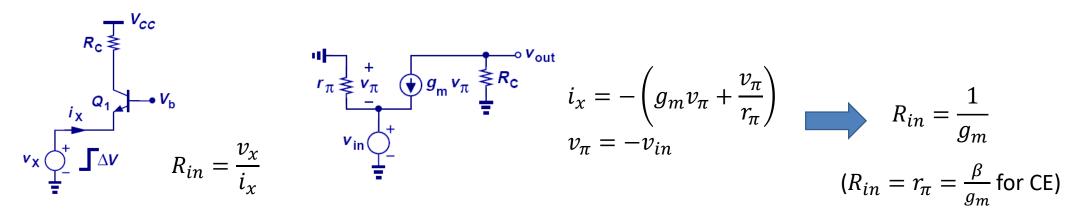
Common-Base Amplifier

- Voltage gain
 - The voltage gain of CB stage is $g_m R_C$, which is identical to that of CE stage in magnitude and opposite in phase (positive V_{in} means negative V_{BE}).



Common-Base Amplifier

• The input impedance of CB stage is much smaller than that of the CE stage.



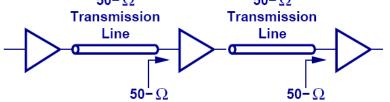
- CB stage's low input impedance can be used to create a match with $50~\Omega$ transmission line (no reflection to the transmission line)
 - · However, low input impedance could mean input signal is attenuated

$$\frac{v_i}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in}}$$

 v_{sig} : incoming signal

 v_{in} : input received by the circuit

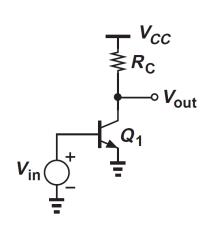
 R_{sig} : impedance of the signal (output impedance from previous stage)

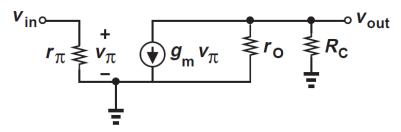


- Output impedance:
 - Same as CE stage

BJT Amplifier: Example

• The circuit below is biased with a collector current of 1 mA and $R_C = 1 \text{ k}\Omega$. If $\beta = 100$, $V_T = 26 \text{ mV}$, and $V_A = 10 \text{ V}$, determine the small-signal voltage gain and the I/O impedances.





Early effect reduces the voltage gain

$$g_{m} = \frac{I_{c}}{V_{T}} = \frac{1 \times 10^{-3}}{26 \times 10^{-3}} = (26 \,\Omega)^{-1}$$

$$r_{\pi} = \frac{\beta}{g_{m}} = 100(26) = 2.6k\Omega$$

$$r_{0} = \frac{V_{A}}{I_{C}} = \frac{10 \, V}{1 \, mA} = 10k\Omega$$

$$A_{V} = -g_{m}(R_{C}||r_{o})$$

$$= -\frac{1}{26} (1k\Omega||10k\Omega) \approx -35 \quad \text{If no early effect:}$$

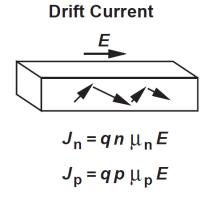
$$A_{V} \approx -38$$

$$R_{in} = r_{\pi} = 2.6k\Omega$$

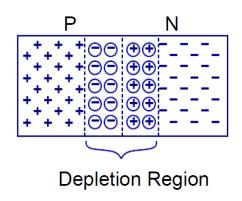
$$R_{out} = (R_{C}||r_{o}) = 909\Omega$$

Recap

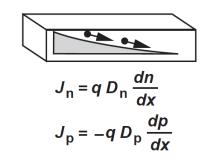
• P-type and N-type semiconductors

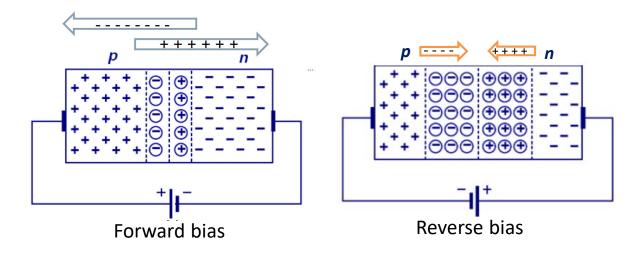


• PN Junctions as Diodes



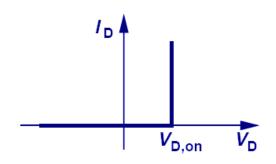
Diffusion Current





Recap

• Diode Models

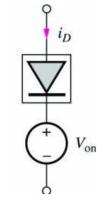


$$V_{D} < V_{D,on}$$

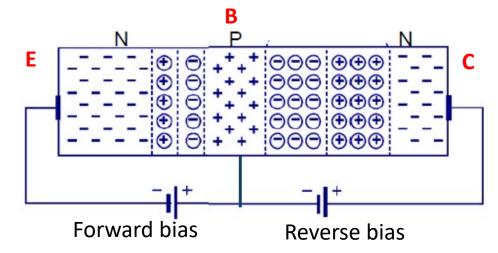
$$- > V_{D,on}$$

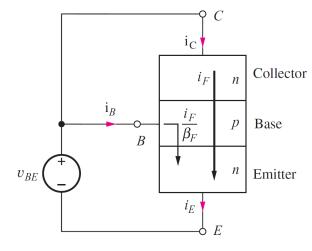
$$- > V_{D,on}$$

$$- > V_{D,on}$$



Append two PN junctions together to create BJT





Active mode of operation

ELEC2104 – Week 8: Part II

MOSFET – Theory of Operation

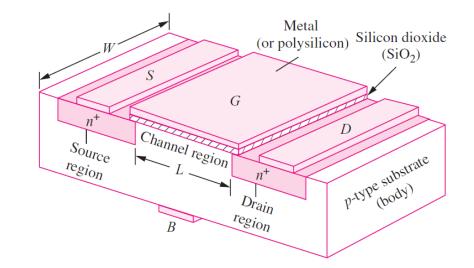
Textbook Chapter Reading:

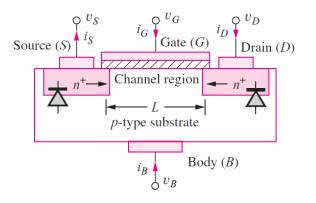
Chapter 4.1 – 4.4

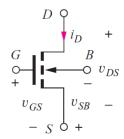


Metal-Oxide Semiconductor

- MOSFET: Metal-oxide-semiconductor Field Effect Transistor
- Four terminals device
- Gate (G): Top conductive plate (e.g. metal) residing on a thin dielectric (oxide insulator) layer
- Two heavily doped regions (n⁺ or p⁺ type):
 - Source (S) provides carriers
 - **Drain (D)** receive carriers
- Substrate or Body: p- or n-type semiconductor
- NMOS or PMOS transistors
 - NMOS: electrons flow in the channel







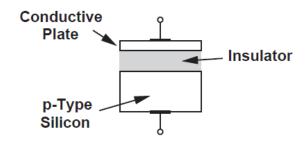
 i_G usually 0

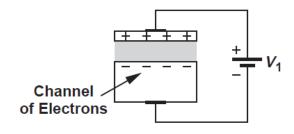
MOS Capacitor Operation

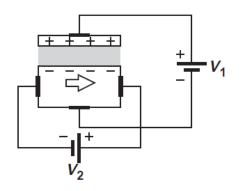
- First, understand the MOS operation
- Consider a simple geometry consisting of a conductive plate (Metal), an insulator (Oxide insulator) and a lightly doped piece of silicon (Semiconductor)
- Apply V₁ on the top plate
 - Attracts negative charges (electrons)
 - Creates a channel of free electrons

$$Q = CV_1$$

- Forms a good conductive path with least resistance
- Apply V₂ across the silicon
 - Electrons drift from left to right
 - Current flows through the channel

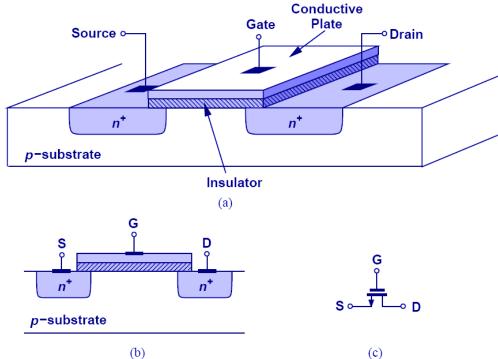






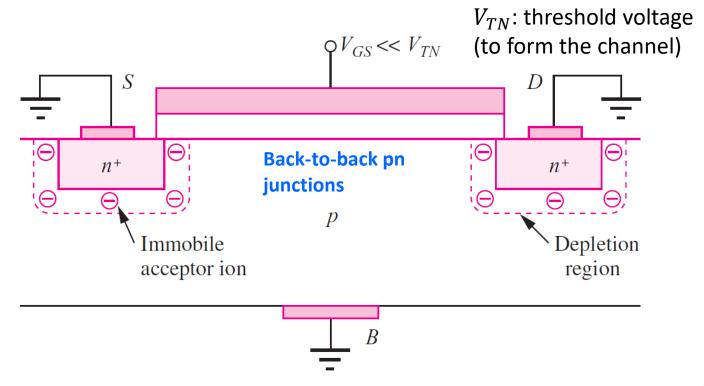
MOSFET Structure

- N-type MOS (NMOS) Transistor
- Central region is the MOS cap
- N-type source/drain and p-type substrate
- Two n-well regions on either side of the MOS capacitor can supply electrons
 - Symmetric. Unlike BJT

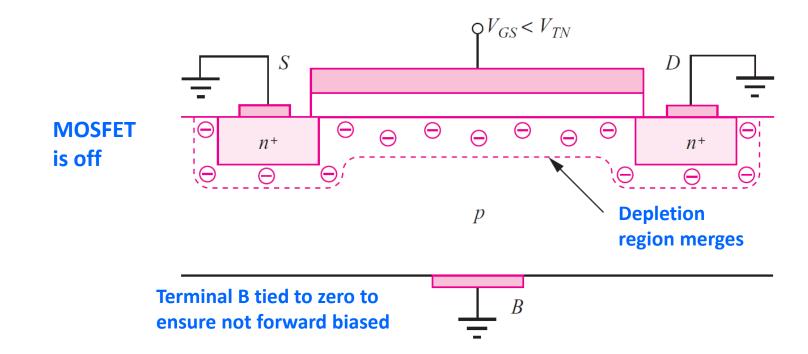


(c)

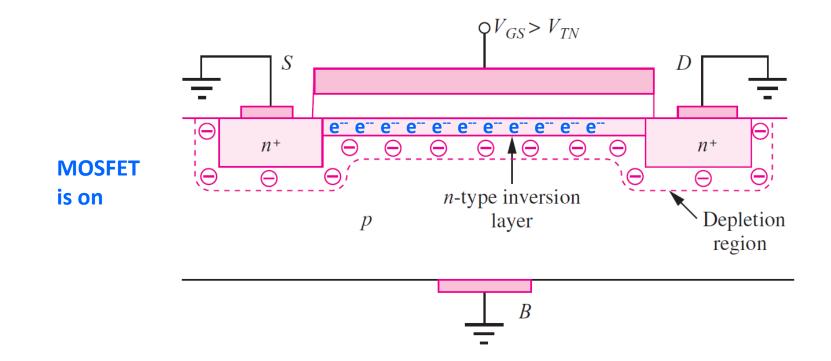
- First, consider the arrangement where:
 - Source (S) and Drain (D) are grounded
 - Gate (G) voltage is near zero
 - No current can flow



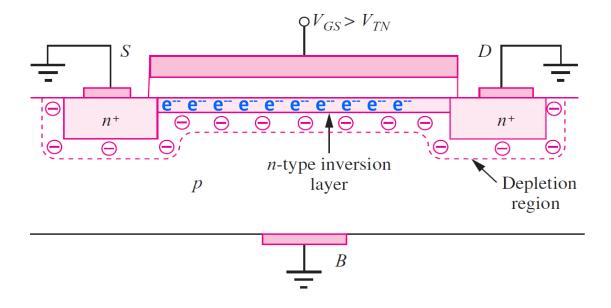
- $V_{GS} < V_{TN}$
 - \bullet As V_G rises, positive charge on the gate repels the holes in the substrate, exposes negative ions, and creates depletion region
 - Depletion region is devoid of free carriers,
 - No channel of mobile charges formed yet, sill no current



- $V_{GS} > V_{TN}$
 - V_G becomes sufficiently positive, free electrons are attracted to the oxide-silicon interface
 - An inversion layer is formed, a conductive channel connecting S to D
 - The voltage at which this layer just forms -> "threshold voltage" V_{TN}



• The conductive channel between S and D can be viewed as a voltagedependent resistor

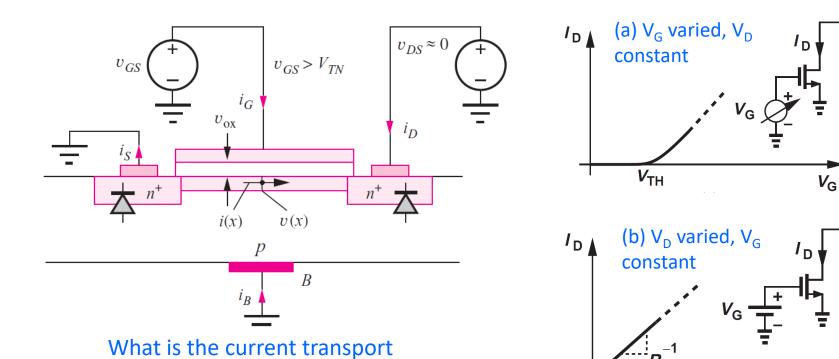


Now, with a voltage applied between S and D, current can flow.

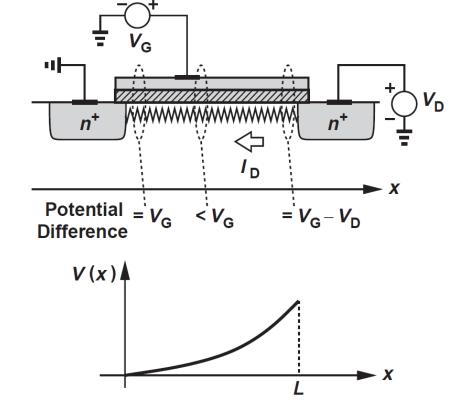
- If positive voltage is applied between D and S, electrons in the channel inversion layer will flow -> I_D > 0
- Gate terminal is insulated from the channel, $i_G = 0$.

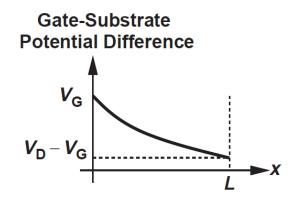
mechanism in a MOSFET?

Drift.

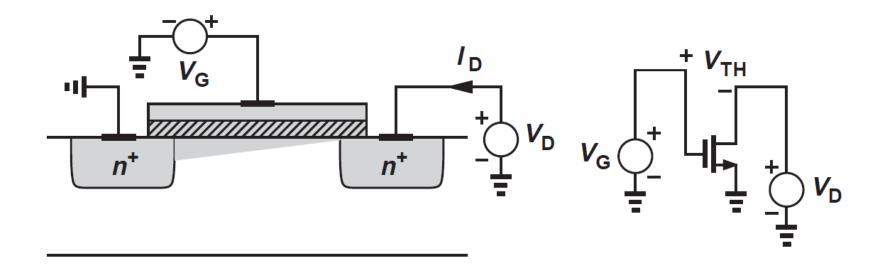


- If the drain voltage (V_D) remains higher than source voltage (V_S) , the voltage at each point along the channel with respect to ground increases as we go from S to D
- Potential difference between gate and substrate is reduced close to drain
- Density of electrons falls to minimum at x = L.

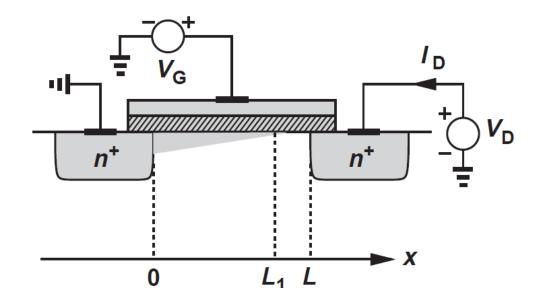




- If V_D is high enough, $V_G V_D \le V_{TN}$, the channel ceases to exist near the drain.
 - Gate-substrate potential difference is not sufficient to attract electrons
- Channel is pinched off

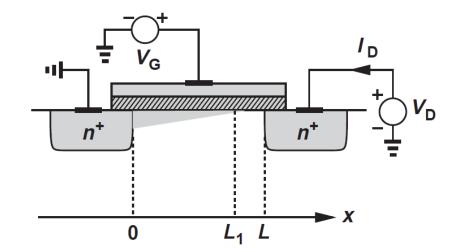


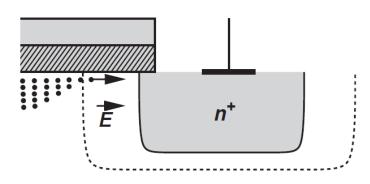
- If V_D rises further, $V_D > V_G V_{TN}$ at x = L
- Voltage difference between gate and substrate falls to V_{TN} at $L_1 < L$
- Device contains no channel between L₁ and L.



Can the device still conduct?

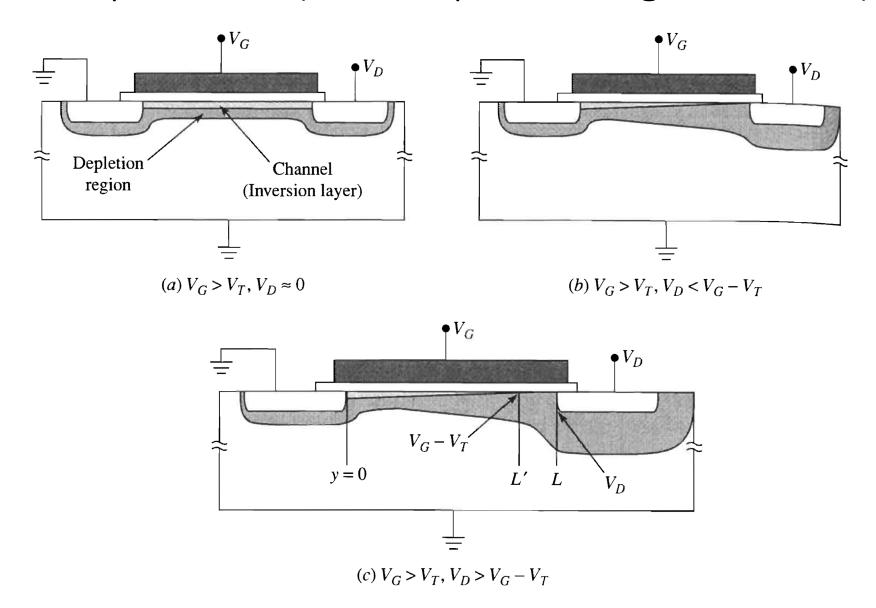
- The end of the channel at L_1 is in the depletion region.
- Electrons are rapidly swept to D due to the built-in E field in depletion region.
- When $V_D > V_G V_{TN}$, V_D no longer affects current significantly
- MOSFET acts as a constant current source independent of V_{DS}



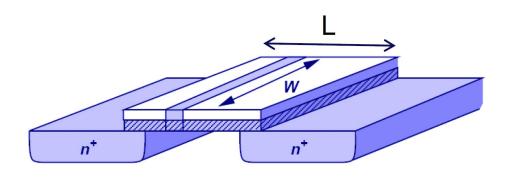


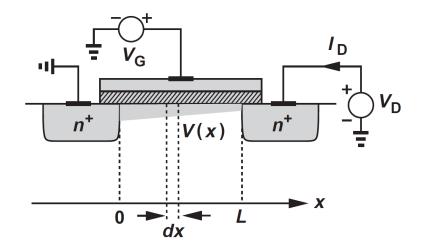
Electrons injected into depleted region gets swept onto the drain

Channel pinch-off (with depletion region shown)



Channel current related to channel charge





Starting out with Q = CV, as channel voltage varies along x:

Q: charge density in the channel per area *W*: width of the channel

 C_{ox} : capacitance in the oxide per area

$$Q(x) = C_{ox} (V_{GS} - V_{TN} - V(x))$$

$$I_{drift} = -WQ(x)\mu_n E = WQ(x)\mu_n \frac{dV}{dx}$$

$$\int_0^L I_D dx = WC_{ox}\mu_n \int_{V_S} (V_{GS} - V_{TN} - V(x))dV$$

$$= WC_{ox}\mu_n \left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS}$$

On the other hand, I_D should be constant because of current conservation, hence $\int_0^L I_D dx = I_D L$

$$I_D = \beta \left(V_{GS} - V_{TN} - \frac{V_{DS}}{2} \right) V_{DS} = \frac{1}{2} \beta \left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2 \right]$$
 where $\beta = \mu_n C_{ox} \frac{W}{I}$

MOSFET Regions of operations

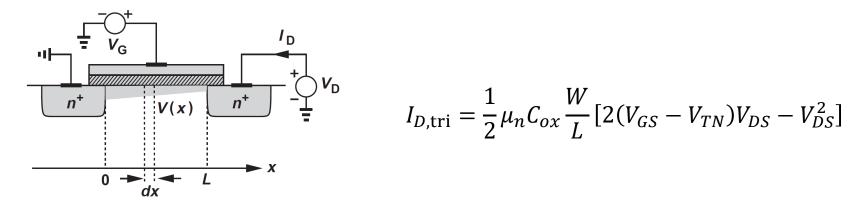


Regions of Operations

Linear or Triode Region

$$V_{GS} - V(x) \ge V_{TN}$$
 for $0 \le x \le L$

Resistive channel directly connects the source and drain



• For small drain voltages: $\frac{V_{DS}}{2} \ll (V_{GS} - V_{TN})$

$$I_D \approx \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN}) V_{DS}$$

Acts like a resistor connected between drain and source

On Resistance

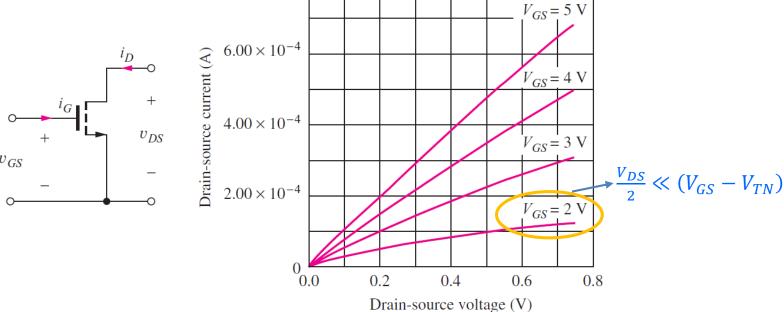
Resistor value can be controlled by the gate-source voltage

$$R_{on} = \frac{V_{DS}}{I_D} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN})}$$

 8.00×10^{-4}

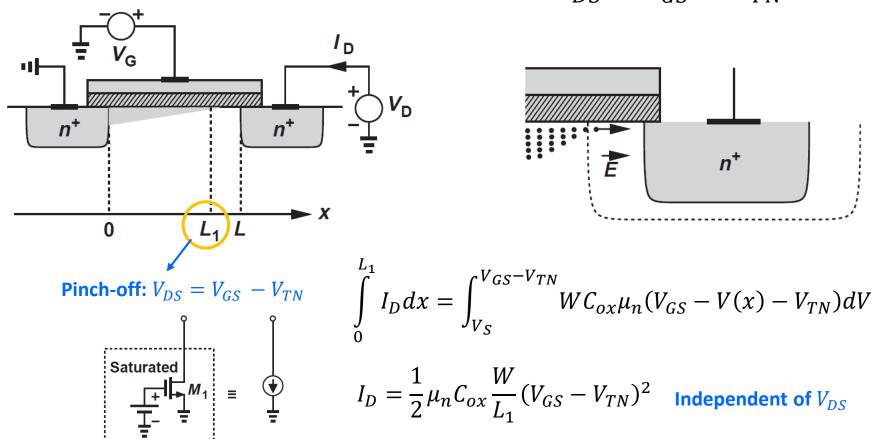
Near the origin, the i-v curves can be approximated by straight lines having

different slopes



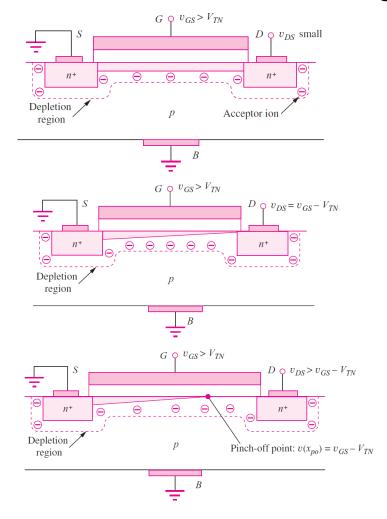
Regions of Operations

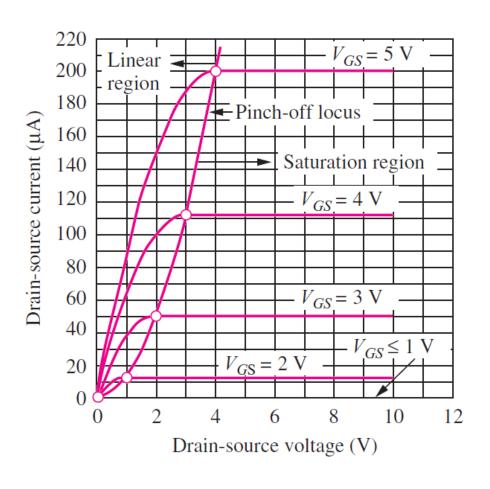
- Saturation Region
 - Drain current saturates at a constant value for $V_{DS} > V_{GS} V_{TN}$



Regions of Operations

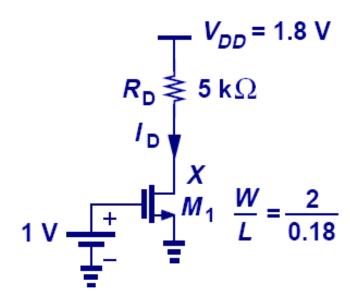
• Linear/Triode and Saturation Regions





Determining Region of Operation

- What region of operation is this transistor operating in?
- Circuit analysis is much easier than with BJTs, because current can only go in one direction
- For the example below, it's triode or saturation: plug numbers into equations to verify



MOSFET Regions Of Operation

• NMOS:

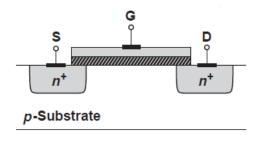
- Cut-off :: $V_{GS} = 0$
- Saturation :: $V_{GS} > V_{TH} \& V_{DS} > V_{GS} V_{TH}$
- Triode :: $V_{GS} > V_{TH} \& V_{DS} < V_{GS} V_{TH}$

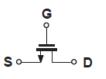


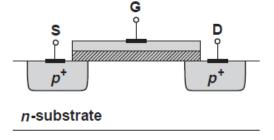
- Cut-off :: $V_{GS} = 0$
- Saturation :: $V_{GS} < V_{TH} & V_{DS} < V_{GS} V_{TH}$
- Triode :: $V_{GS} < V_{TH} \& V_{DS} > V_{GS} V_{TH}$

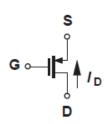


$$I_{D,\text{sat}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS}) \qquad I_{D,\text{tri}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$









$$I_{D,\text{tri}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2 \right]$$

Bipolar Transistor versus MOSFET Summary

Comparison of Bipolar Transistor and MOSFET characteristics:

Bipolar Transistor	MOSFET
Exponential Characteristic Active: $V_{\text{CB}} > 0$ Saturation: $V_{\text{CB}} < 0$ Finite Base Current Early Effect	Quadratic Characteristic Saturation: $V_{\rm DS} > V_{\rm GS} - V_{\rm TH}$ Triode: $V_{\rm DS} < V_{\rm GS} - V_{\rm TH}$ Zero Gate Current Channel-Length Modulation
Diffusion Current –	Drift Current Voltage-Dependent Resistor

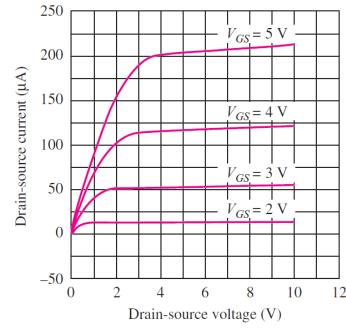
• Bipolar devices have a higher g_m than MOSFETs for a given bias current due to its exponential IV characteristics

MOSFET channel-length modulation



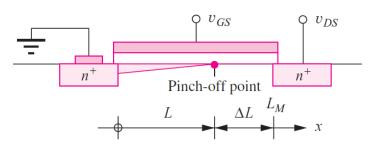
Channel-Length Modulation

- Drain current is not quite constant in the saturation region
- Drain current does increase slightly as drain-source voltage increase



$$L = L_M - \Delta L$$

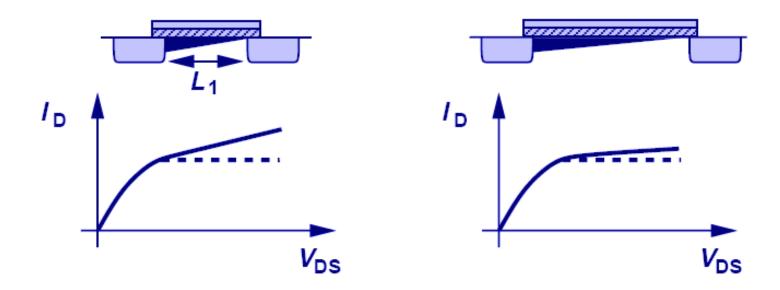
Actual length L of resistive channel decreases as ΔL increases with V_{DS}



$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_1} (V_{GS} - V_{TN})^2 \qquad \Rightarrow \qquad I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS}) \qquad \lambda \text{: Channel-length modulation parameter}$$

Channel Length Modulation

- Unlike the Early voltage in BJT, the channel- length modulation factor can be controlled by the circuit designer.
 - We cannot control base width in a BJT, but we can control gate length of a MOSFET
 - For long L, the channel-length modulation effect is less than that of short L.

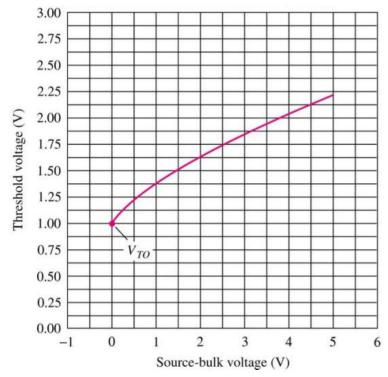


Body effect

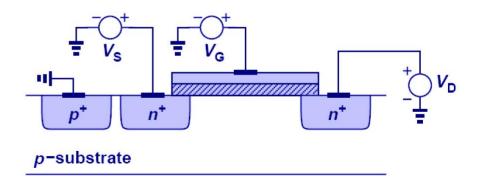


Body Effect in Strong Inversion

- So far, we have assumed both source and substrate are tied to ground.
- What happens if the substrate is at a different voltage from the source?
- Threshold voltage can change: $V_{Th} = V_{Th0} + \gamma \left(\sqrt{V_{SB} + 2\psi_B} \sqrt{2\psi_B} \right)$



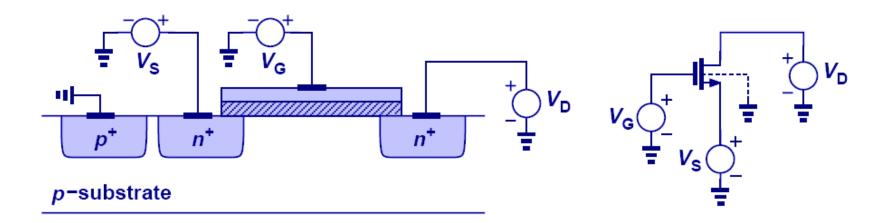
 V_{Th0} = threshold voltage for V_{SB} = 0 γ = body-effect parameter = $\frac{\sqrt{qN_A 2\varepsilon_s}}{C_{ox}}$ (\sqrt{V}) $2\psi_B$ = surface potential (V)



Body Effect

- For most circuits in this course, the substrate will be connected to the same potential as the source.
- Then, no effect.

$$V_{Th} = V_{Th0} + \gamma \left(\sqrt{V_{SB} + 2\psi_B} - \sqrt{2\psi_B} \right)$$

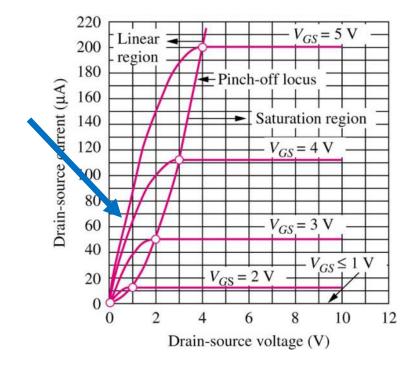


MOSFET circuit models



MOSFET Circuit Models

- A MOSFET can be represented using different circuit models depending on its region of operation
- When it is operating in the triode region, but V_{DS} is much smaller than $(V_{GS} V_{Th})$, the I-V curve is linear
- The MOSFET is a resistor independent of V_{DS}

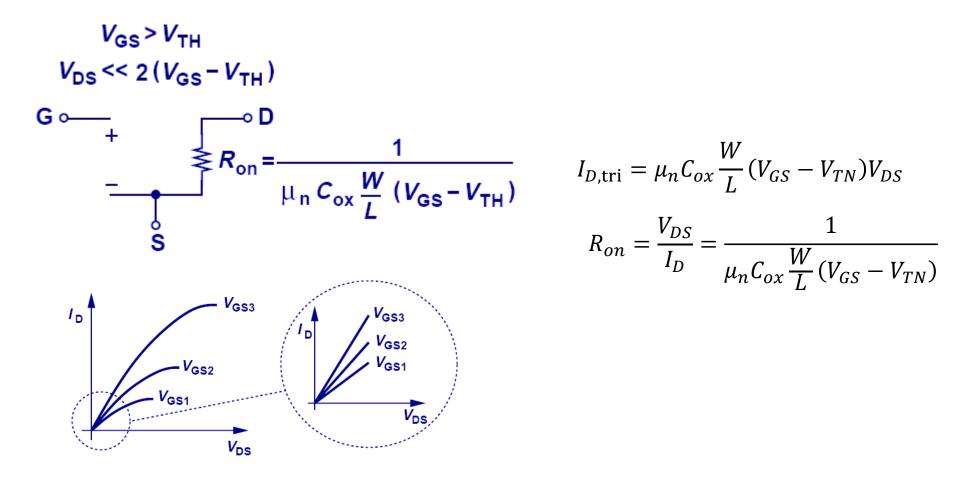


$$I_{D,\text{tri}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

$$\approx \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN})V_{DS}$$

Linear Triode Circuit Model

 When it is operating in the triode region, but V_{DS} is much smaller than $(V_{GS} - V_{Th})$, the I-V curve is linear

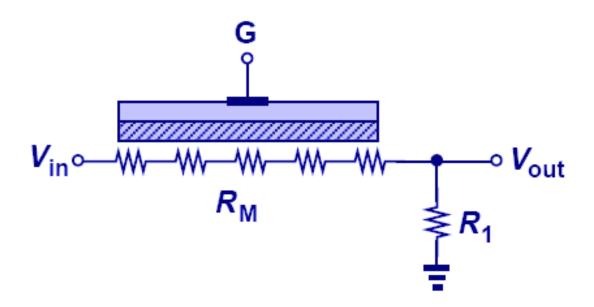


$$I_{D,\text{tri}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN}) V_{DS}$$

$$R_{on} = \frac{V_{DS}}{I_D} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TN})}$$

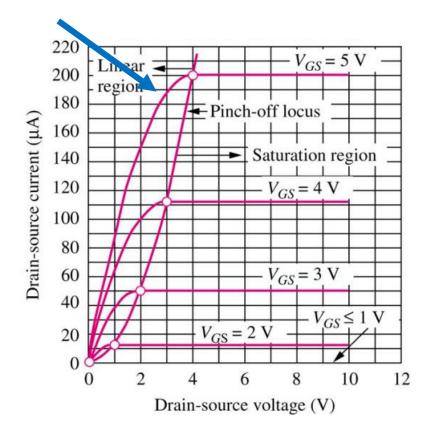
Triode Operation

 This type of gain control finds application in cell phones to avoid saturation near base stations



Non-linear Triode Circuit Model

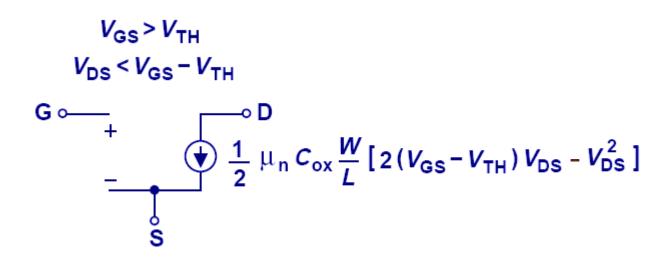
- When operating in the triode region, and V_{DS} approaches $(V_{GS} V_{Th})$, the I-V curve is non-linear
- The MOSFET is a voltage-controlled current source.



$$I_{D,\text{tri}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$$

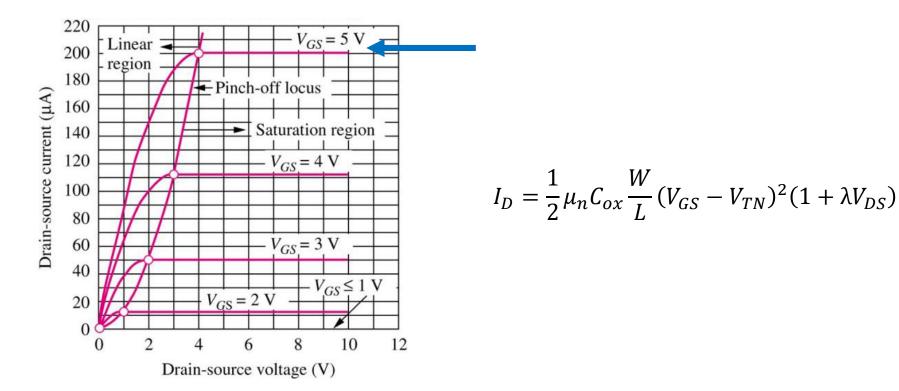
Non-linear Triode Circuit Model

- When operating in the triode region, and V_{DS} approaches $(V_{GS} V_{Th})$, the I-V curve is non-linear
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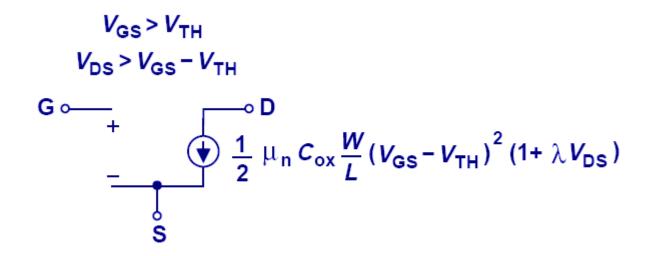
Saturation Circuit Model

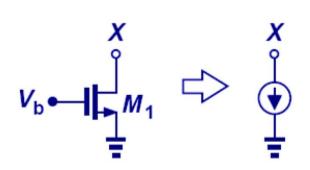
- When operating in saturation, $V_{GS} > V_{TH}$ and $V_{DS} > V_{GS} V_{TH}$
- Ideally the drain current would be only dependent on V_{GS}
- If we want to take channel-length modulation into account, it must be a voltagecontrolled current source as well



Saturation Circuit Model

The MOSFET in saturation is a voltage-controlled current source



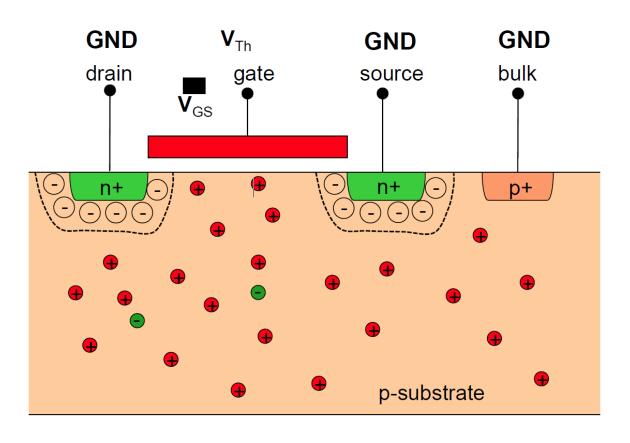




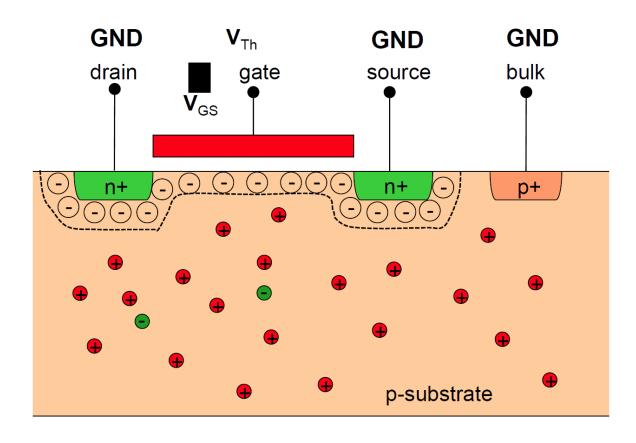
Backup slides - MOSFET



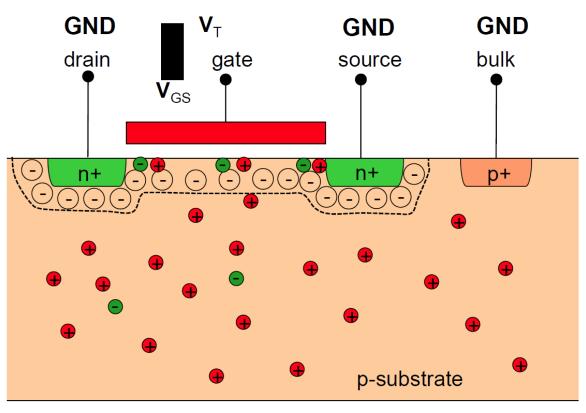
Inversion Channel And Gate Voltage



Inversion Channel And Gate Voltage

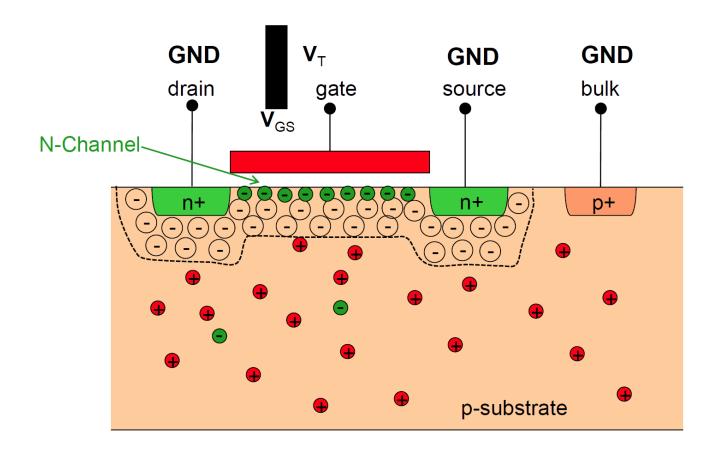


Inversion Channel at Threshold



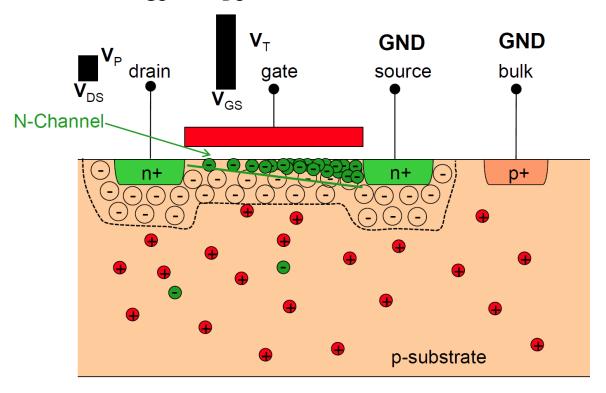
 $V_G = V_{Th} =$ #electrons = #holes in the channel

Inversion Channel



Channel Current

- V_{DS} > 0, potential at oxide silicon interface rises from S to D
- Potential difference between gate and oxide silicon interface decreases as it approaches the drain $(V_{GS} V_{DS})$, electron density decreases accordingly



Channel Pinch-Off

- At $V_{GS} V_{DS} \le V_{TN}$ -> channel ceases to exist
- Pinch-off voltage: $V_P = V_{GS} V_{TN}$

