ELEC2104 – Week 7

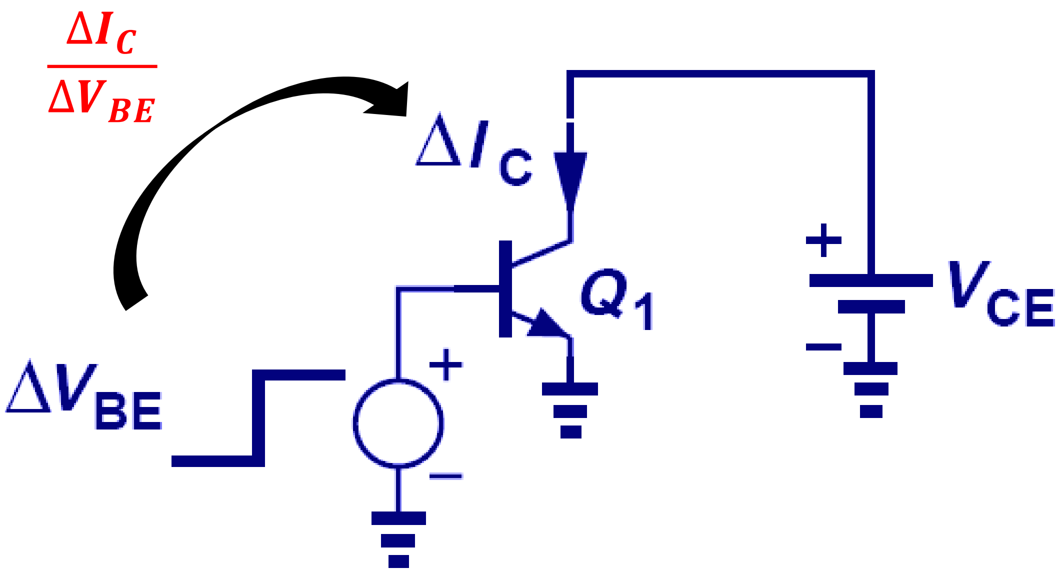
BJT and circuits, other modes of operation

THE UNIVERSITY OF SYDNEY

BJT small signal models

BJT Transconductance

- Transconductance, g_m is a small-signal measure of how well the transistor converts voltage to current.
- **g_m is one of the most important parameters** in circuit design.
- Remember, for large-signal analysis, conductance G = 1/R



For small changes:

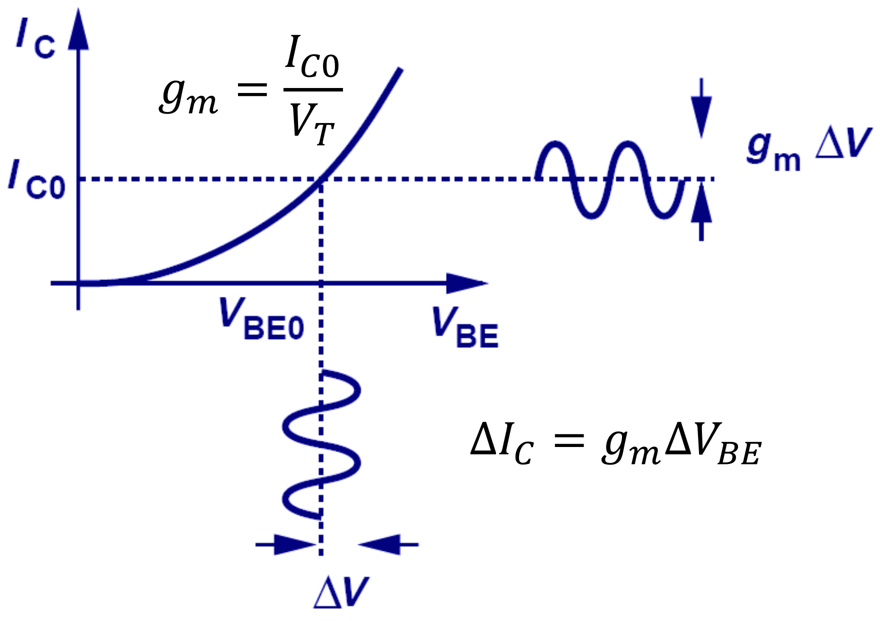
$$g_m = \frac{dI_C}{dV_{BE}} = \frac{d}{dV_{BE}}\left(I_S \exp\frac{V_{BE}}{V_T}\right)$$

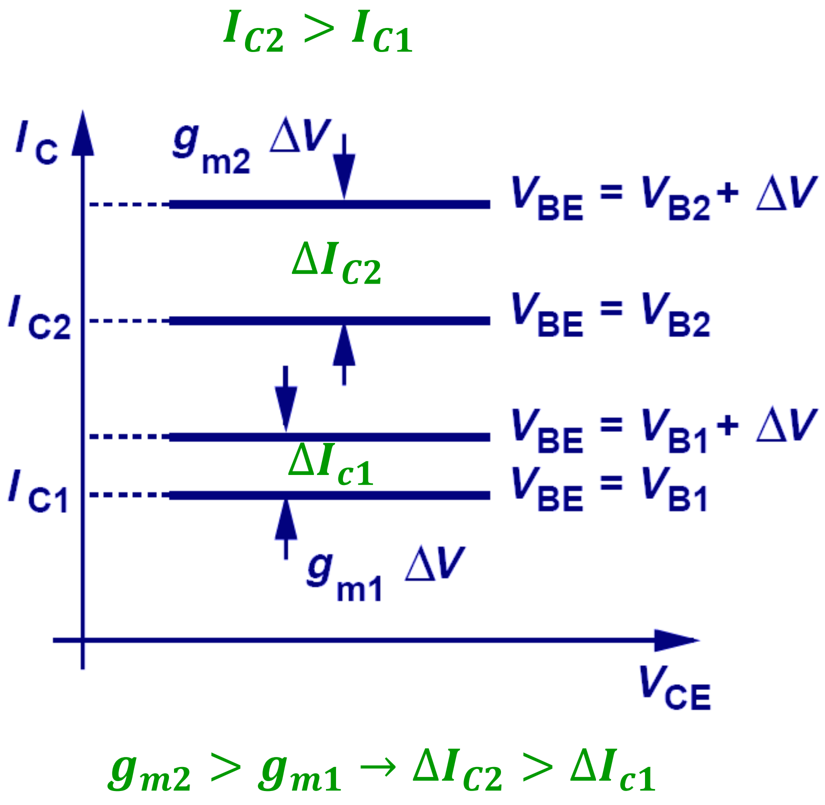
$$g_m = \frac{1}{V_T}\left(I_S \exp\frac{V_{BE}}{V_T}\right)$$

$$\boxed{g_m = \frac{I_C}{V_T}}$$

BJT Transconductance

- g_m can be visualized as the slope of I_C versus V_{BE} characteristics at a given collector current I_{C0} and base-emitter voltage V_{BE0}
- A large g_m will cause a large change in I_C for the same change in ΔV_{BE}

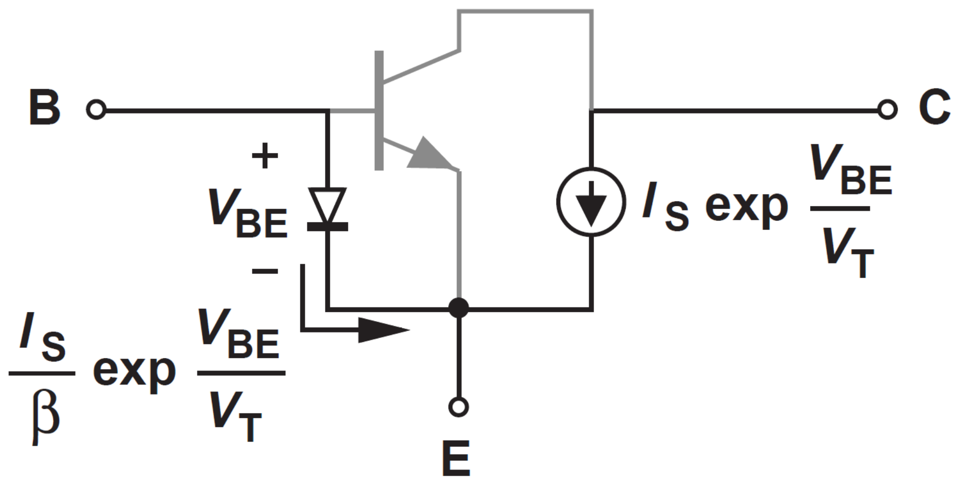




BJT Transconductance

Question:

- If I_C remains constant, but β varies, does g_m change?



$$g_m = \frac{I_C}{V_T}$$

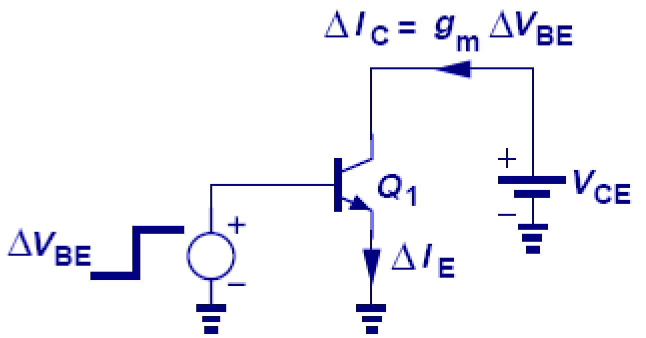
So, no.

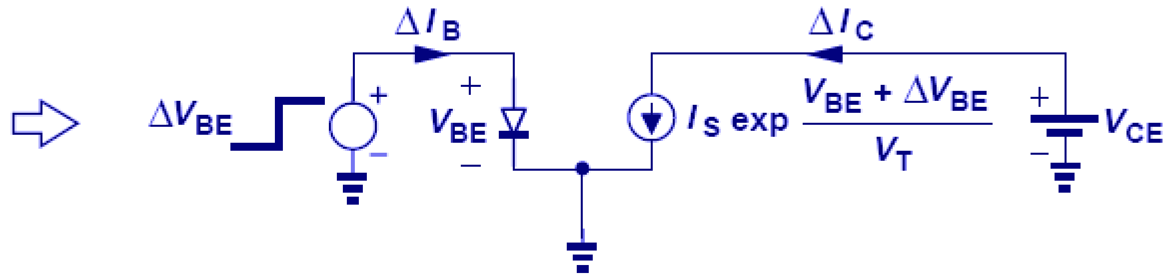
$\beta \nearrow, I_B \searrow$

- Collector bias (or quiescent) current plays an important role in the design of BJT amplifiers

BJT Small-Signal Model

- A small-signal model can be derived from the large-signal model by applying a small change to the input rather than a DC signal
- We can then propagate the ΔV_{BE} term and analyze subsequent changes in current in the three terminals





Large signal model

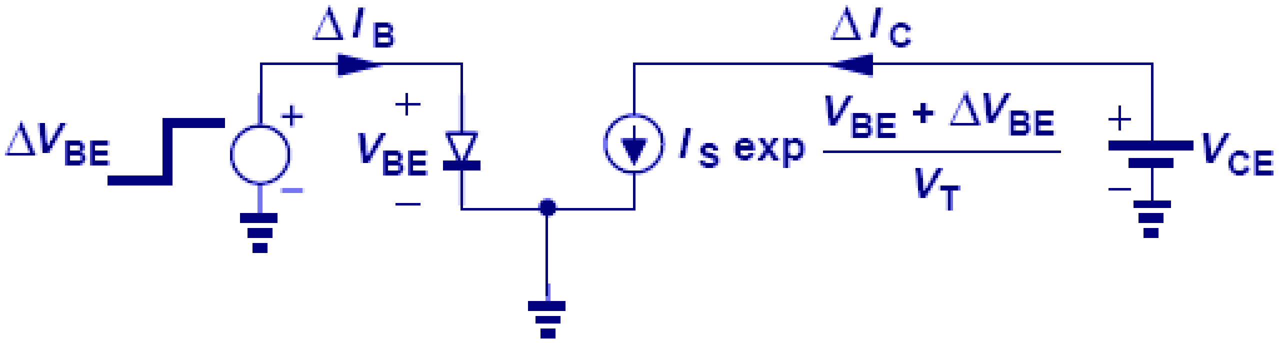
1. Change in collector current: $\Delta I_C = g_m \Delta V_{BE}$
2. Change in base current: $\Delta I_B = \dfrac{g_m \Delta V_{BE}}{\beta}$

$$\frac{\Delta V_{BE}}{\Delta I_B} = \frac{\beta}{g_m} = r_\pi$$

Small-signal resistance
(differential resistance)

BJT Hybrid-Pi Model

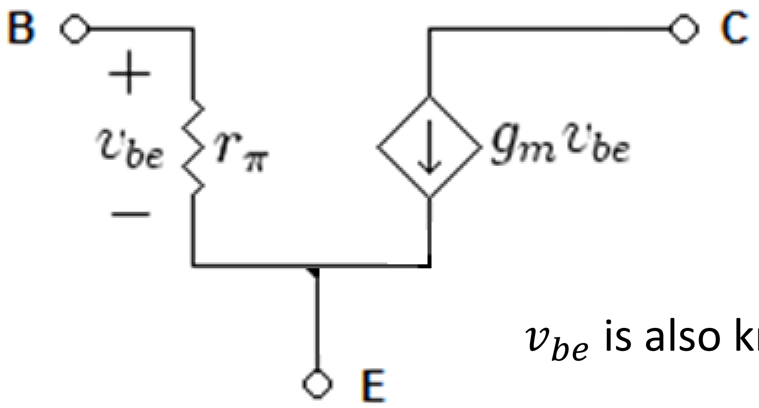
- Small signal resistance r_π - placed between base and emitter



Large signal model

$$r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

$$g_m = \frac{I_C}{V_T}$$



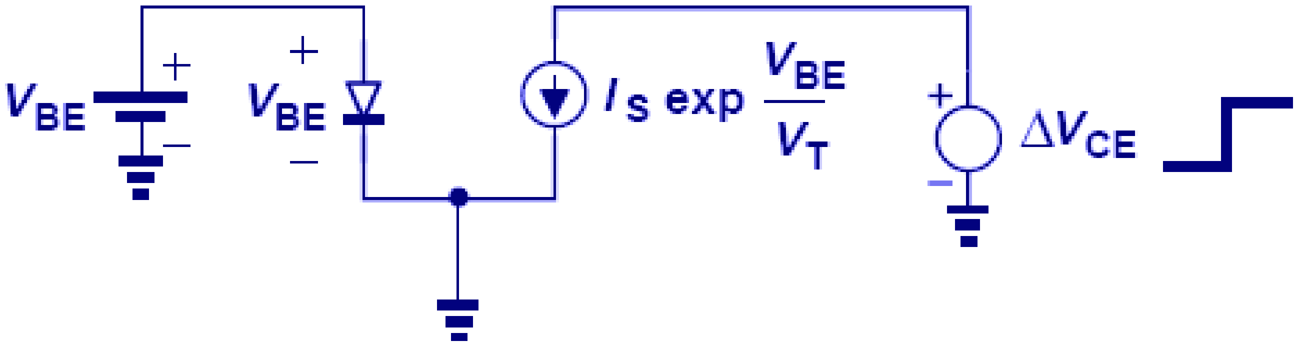
Small signal model
(This looks like an inverted π)

v_{be} is also known as v_π

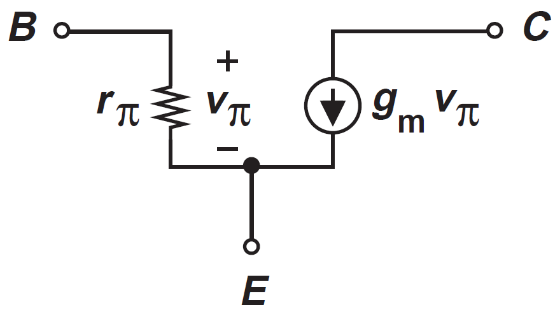
Only works at a certain Q-point
For a different Q-point, g_m is different

BJT Hybrid-Pi Model

- What about ΔV_{CE}?
- In forward-active operation, ΔV_{CE} has no effect on the collector current or base current
- ΔV_{CB} has no effect on the small signal model, either.



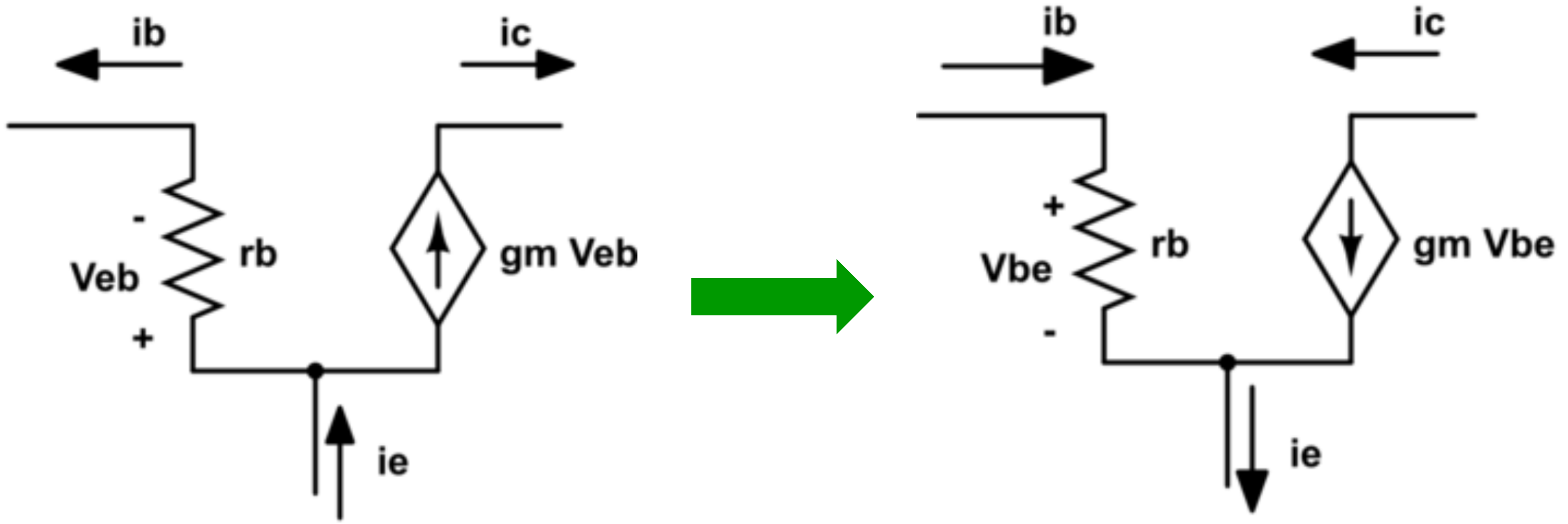
- ΔV_{CE} and ΔV_{CB} does not affect the model



Hybrid π model for small signals

Hybrid-Pi Model for PNP Transistor

- The small signal model for PNP transistor is exactly the same as that for an NPN transistor.

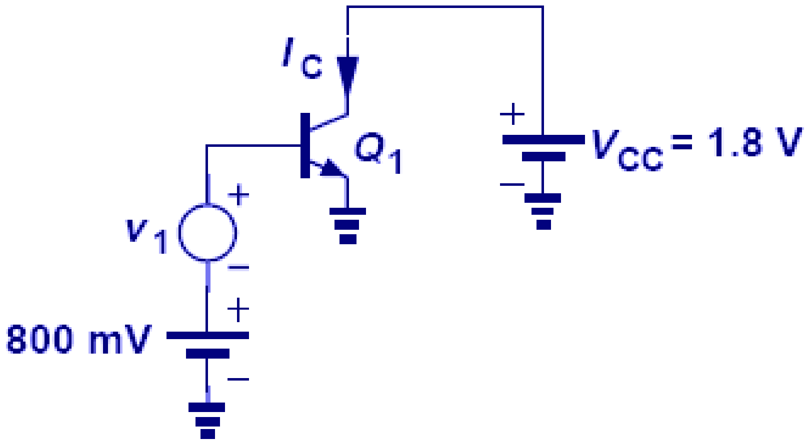


- Remember, ΔV_{BE}, i.e. v_{be} is not directional
- Small signal studies the changes in the circuit

Example: Small-Signal Analysis

- Consider the following circuit where v_1 is the signal generated by a microphone, $I_S = 3 \times 10^{-16} A,\ \beta = 100$, and $V_T = 26\, mV$.

  (a) Determine the small signal parameters of Q_1



Need to find g_m, r_π

Here, small signal parameters are calculated from DC operating point

1. Find the collector bias current when $v_1 = 0$

$$I_c = I_s \exp \frac{V_{BE}}{V_T}$$
$$= 6.92\, mA$$

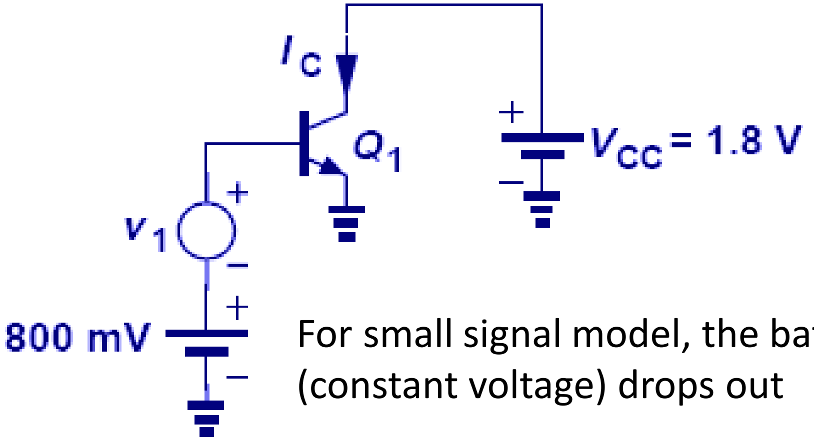
2. Calculate g_m and r_π

$$g_m = \frac{I_C}{V_T} = 0.267\Omega^{-1}$$
$$r_\pi = \frac{\beta}{g_m} = 375\Omega$$

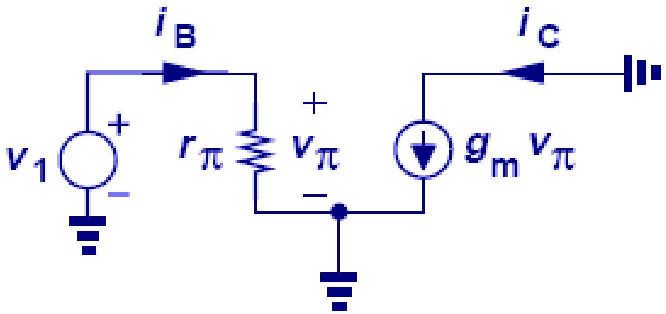
Example: Small-Signal Analysis

- $I_C = 6.92\ mA$, $g_m = 0.267\Omega^{-1}$, and $r_\pi = 375\ \Omega$.

  (b) If microphone generates 1 mV signal, how much change is observed in the collector and base currents?



For small signal model, the battery (constant voltage) drops out



Since $v_\pi = v_1$,

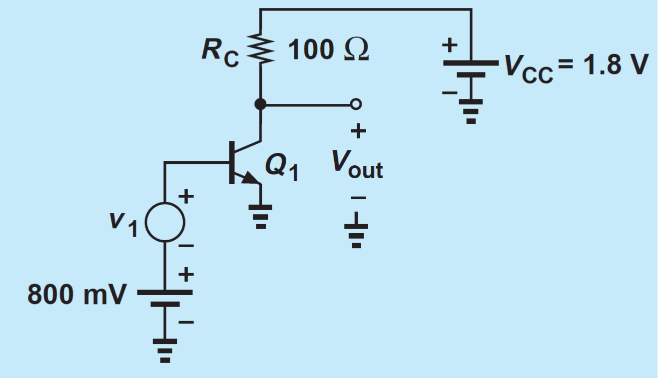
$$\Delta I_C = g_m v_1 = 0.267mA$$

$$\Delta I_B = \frac{v_1}{r_\pi} = 2.67\mu A = \frac{\Delta I_C}{\beta}$$

Example 2: Small-Signal Analysis

- $I_C = 6.92\ mA, g_m = 0.267\Omega^{-1}$, and $r_\pi = 375\ \Omega$.

(c) The circuit is now modified by connecting a resistor R_C to convert the collector current to a useful output voltage. Verify the transistor is still operating in the forward active mode.



For active mode: $V_C > V_B > V_E$

$$V_C = V_{out}$$
$$= V_{CC} - I_C R_C$$
$$= 1.108\ V > V_B > V_E = 0$$

∴ Device in active mode

(c) What is the output signal level for a 1 mV input signal from the microphone

$$\Delta I_C = 0.267\ mA$$

Upon flowing through R_C,

$$\Delta V_{out} = \Delta I_C R_C = 26.7\ mV$$

Amplifies the input by a factor of 26.7

Rc ↑ , Gain ↑

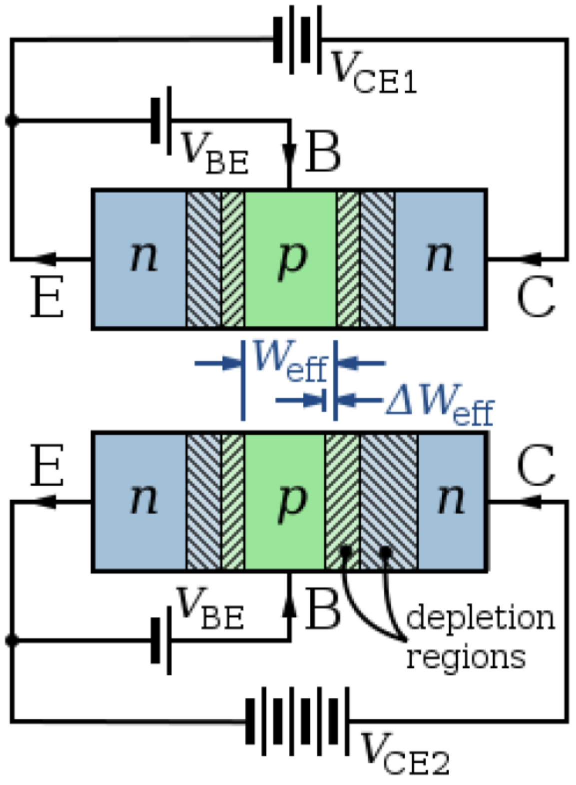
What if Rc → ∞?

BJT need to be in forward active mode.

Early effect

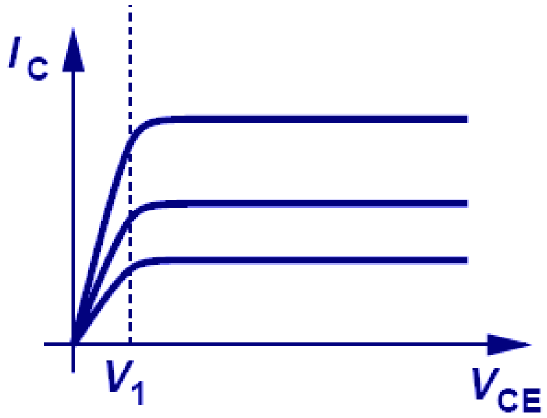
BJT Non-Ideal Transistors: Early Effect

- Maximum gain in amplifiers can be limited by the nonideality in the device
- *Assumption at question*: collector current does depend on V_{CE}
- As V_{CE} increases, the depletion region between base and collector increases.
  - Higher reverse-bias across base-collector junction
- The *effective base width* decreases
  - Remember that base is itself thin
  - Narrower base means higher gain
- This is called the Early Effect.

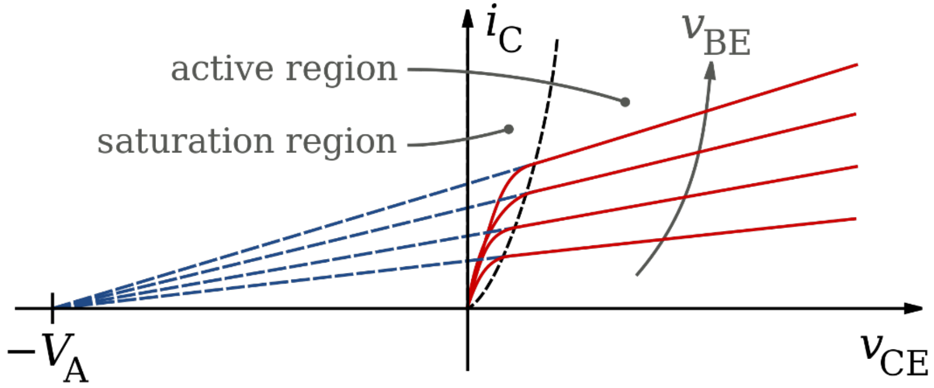


BJT Non-Ideal Transistors: Early Effect

- Ideal model:

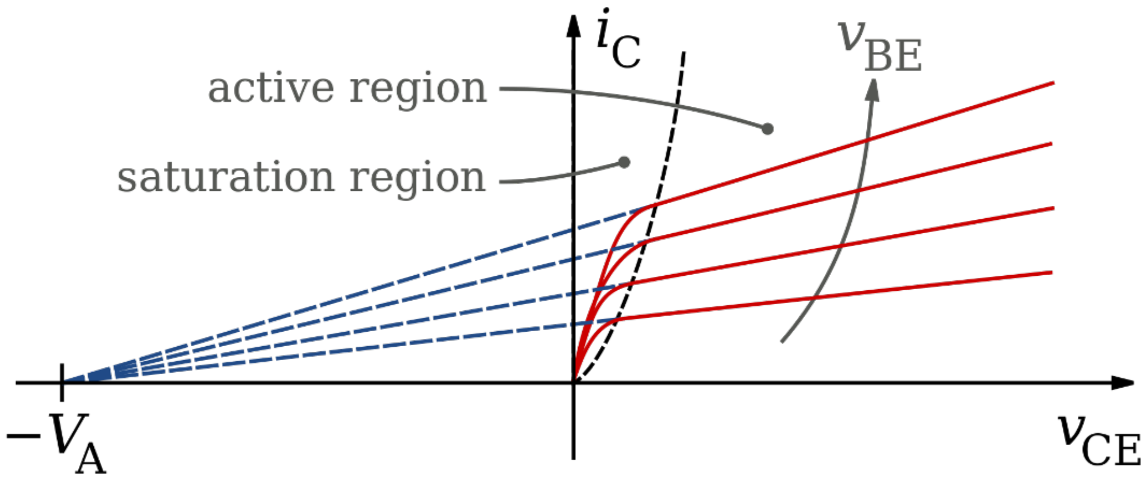


- More accurate transistor I-V characteristics:



BJT Non-Ideal Transistors: Early Effect

- When output characteristics are extrapolated back to point of $I_c = 0$, curves intersect (approximately) at a common point $V_{CE} = -V_A$
- This is called the Early Voltage
- Typically between 15 V and 150 V
- If not stated, assume it is infinite (no Early)



Rise in I_c can be expressed by a multiplicative factor

$$I_C = I_S\left(\exp\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$$

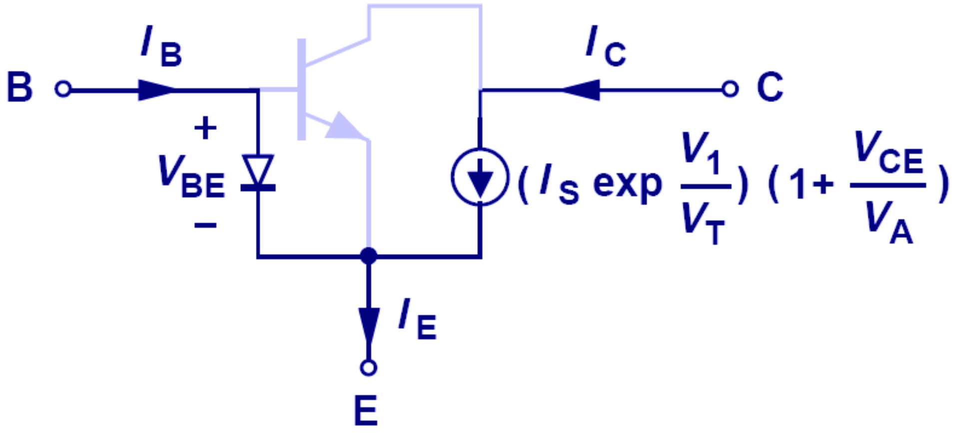
$$\beta_F = \beta_{F0}\left(1+\frac{V_{CE}}{V_A}\right)$$

$$I_B = \frac{I_S}{\beta_{F0}}\left(\exp\frac{V_{BE}}{V_T}\right)$$

Note, $I_B = \frac{I_C}{\beta_F} \neq \frac{I_C}{\beta_{F0}}$

Modelling the Early Effect

- Early effect can be accounted for in large-signal models by simply applying a correction factor to the collector current.
- Base current *does not change*, independent of V_{CE}



$$\left(I_S \exp\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)$$

$$g_m = \frac{dI_c}{dV_{BE}} = \left(I_S \exp\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE}}{V_A}\right)\frac{1}{V_T} = \frac{I_C}{V_T}$$ Same as before

$$\frac{dI_c}{dV_{CE}} = \left(I_S \exp\frac{V_{BE}}{V_T}\right)\left(\frac{1}{V_A}\right) \approx \frac{I_C}{V_A}$$ If $V_{CE} \ll V_A$ (usually true)

Modelling the Early Effect

- In small-signal models, a resistor is added to the output to account for the slope of the collector current

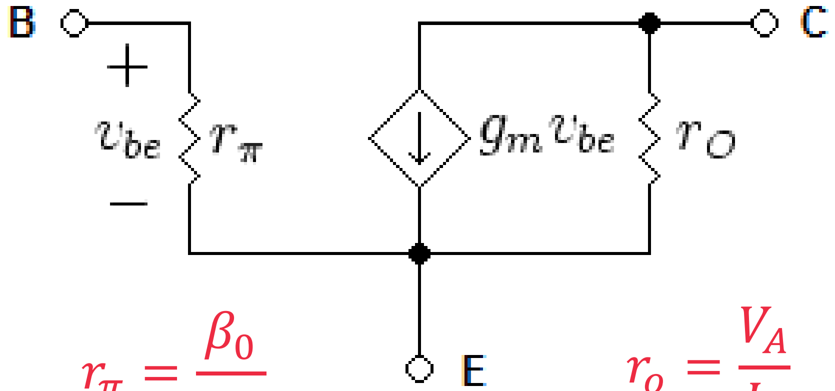
$$I_C + \Delta I_C = \left(I_S \exp\frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE} + \Delta V_{CE}}{V_A}\right)$$

$$\Delta I_C = \left(I_S \exp\frac{V_{BE}}{V_T}\right)\left(\frac{\Delta V_{CE}}{V_A}\right)$$

$$r_o = \frac{\Delta V_{CE}}{\Delta I_C} = \frac{V_A}{I_S \exp\frac{V_{BE}}{V_T}} \approx \frac{V_A}{I_C}$$

output resistance r_0 represent the Early effect

$$g_m = \frac{I_C}{V_T}$$



$$r_\pi = \frac{\beta_0}{g_m} \qquad r_o = \frac{V_A}{I_C}$$

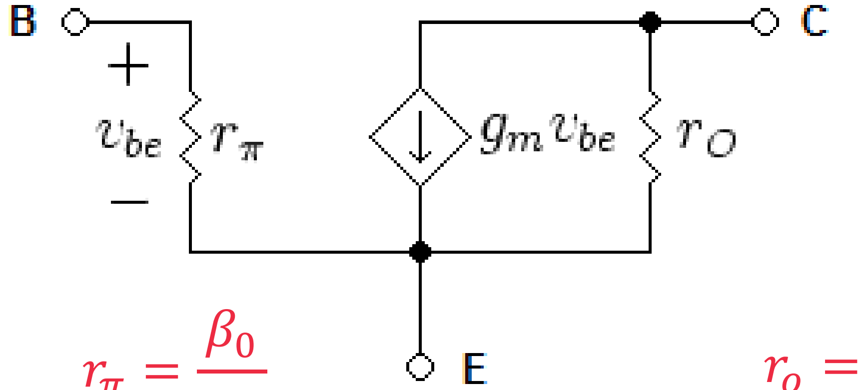
Why is r_o parallel to the current source?

β_0 is the small-signal common-emitter current gain. $\beta_0 \approx \beta_F$.

$\beta_F = \beta_{F0}\left(1 + \frac{V_{CE}}{V_A}\right) \approx \beta_{F0}$ as usually $V_{CE} \ll V_A$

Hybrid-pi model with Early effected included

$$g_m = \frac{I_C}{V_T}$$



$$r_\pi = \frac{\beta_0}{g_m}$$

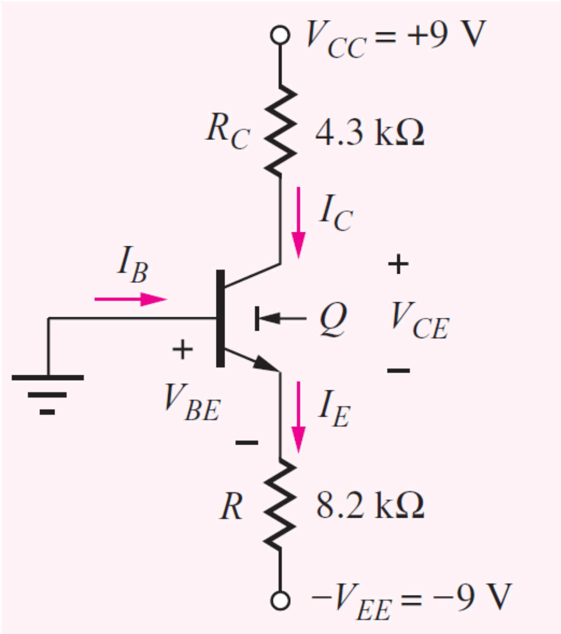
$$r_o = \frac{V_A}{I_C}$$

Other modes of operation

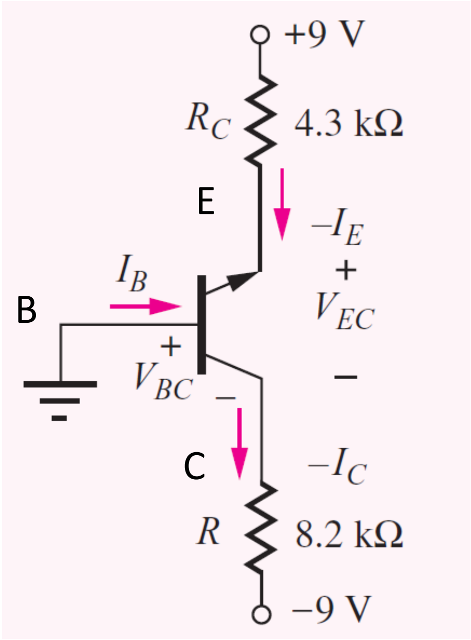
Reverse BJT

- What happens if we plug the terminals of the BJT upside down?

Forward active



Inverted



Reverse active

$V_{BE} > 0$ → Forward biased

$V_{BC} < 0$ → Reverse biased

$V_{BE} < 0$ → Reverse biased

$V_{BC} > 0$ → Forward biased

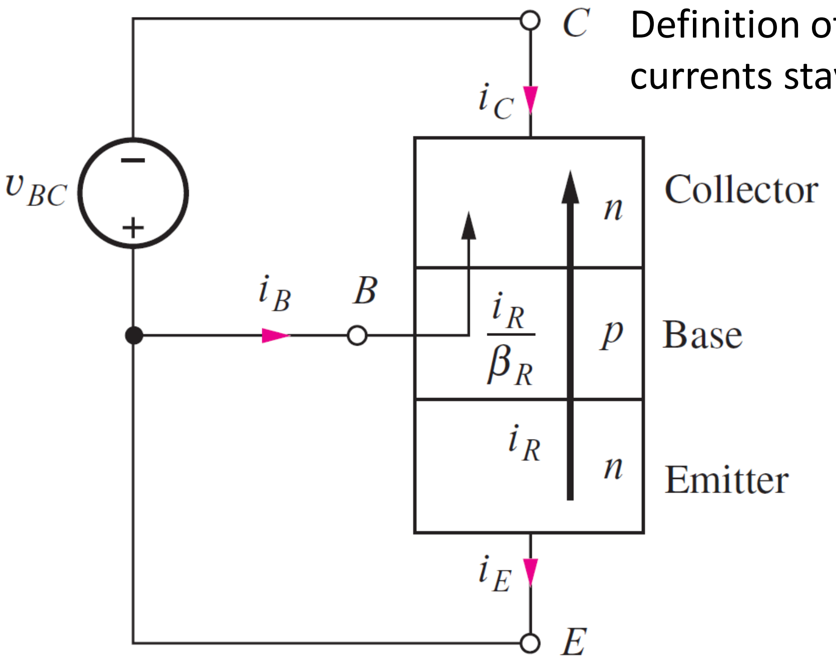
NPN BJT Reverse Characteristics

- A BJT is not quite symmetrical, because the emitter is much more heavily doped than the collector
- The reverse-transport current enters the emitter, travels across the narrow base, and exits the collector terminal. Similar to forward-transport current:

$$i_R = -i_E = I_S \left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right]$$

  - controlling voltage is now V_{BC}
- Emitter is acting as the collector
- Collector is acting as the emitter

  › Base-collector voltage establishes the collector current, i_C



Definition of the sign of currents stay the same

NPN BJT Reverse Characteristics

- A fraction of current i_R must still be supplied as base current:

$$i_B = \frac{i_R}{\beta_R} = \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right]$$

› Reverse common-emitter current gain:

$$0 \le \beta_R \le 10$$

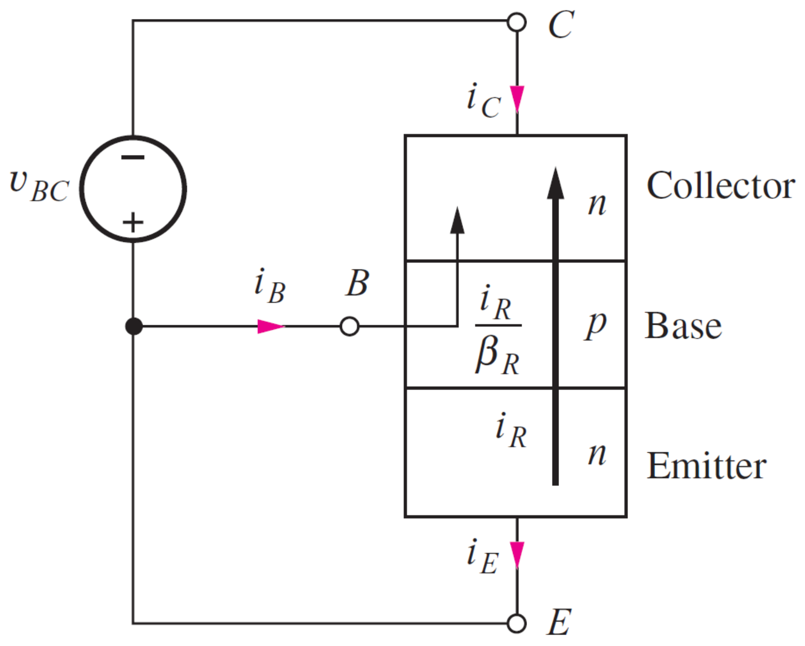
› NPN reverse collector current -($i_B + i_R$):

$$i_C = -\frac{\beta_R + 1}{\beta_R} I_S\left[\exp\left(\frac{V_{BC}}{V_T}\right) - 1\right]$$

$$\frac{1}{\alpha_R}$$

› Reverse common-base current gain:

$$0 \le \ \alpha_R = \frac{\beta_R}{\beta_R + 1} \le 0.95 \qquad \beta_R = \frac{\alpha_R}{1 - \alpha_R}$$



Can we use reverse-active BJT as an amplifier?

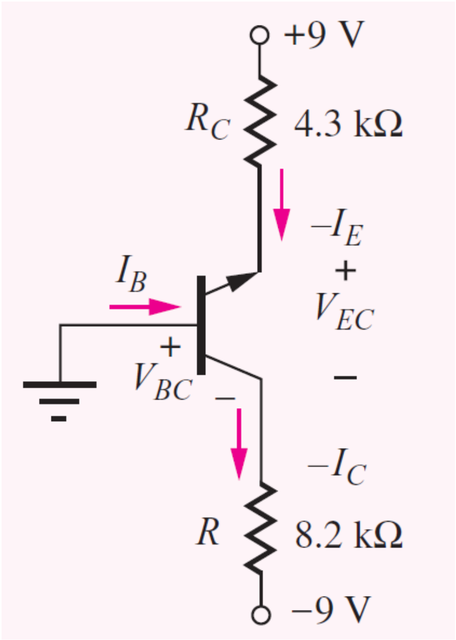
Yes but poor performance

Reverse BJT Analysis

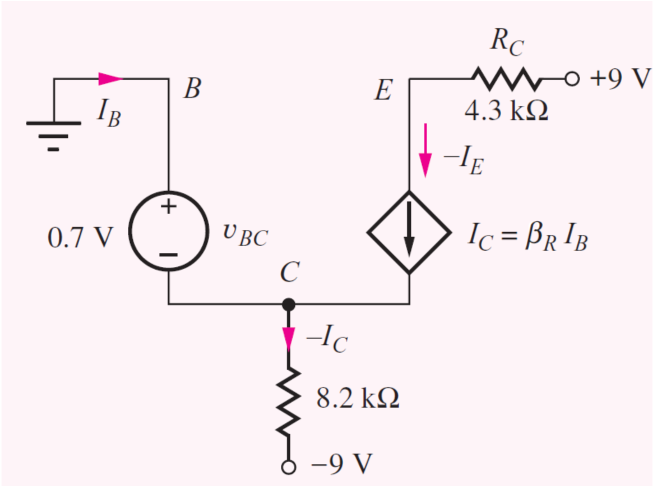
- Find the new Q-point for the transistor with collector and emitter terminals interchanged. Assume $\beta_R = 1$.

Observations:

- Base-emitter junction is reverse-biased by the +9 V source
- -9 V source will pull current out of the collector through the 8.2 kΩ resistor
- Base-collector junction is forward-biased
- Reverse active region of operation

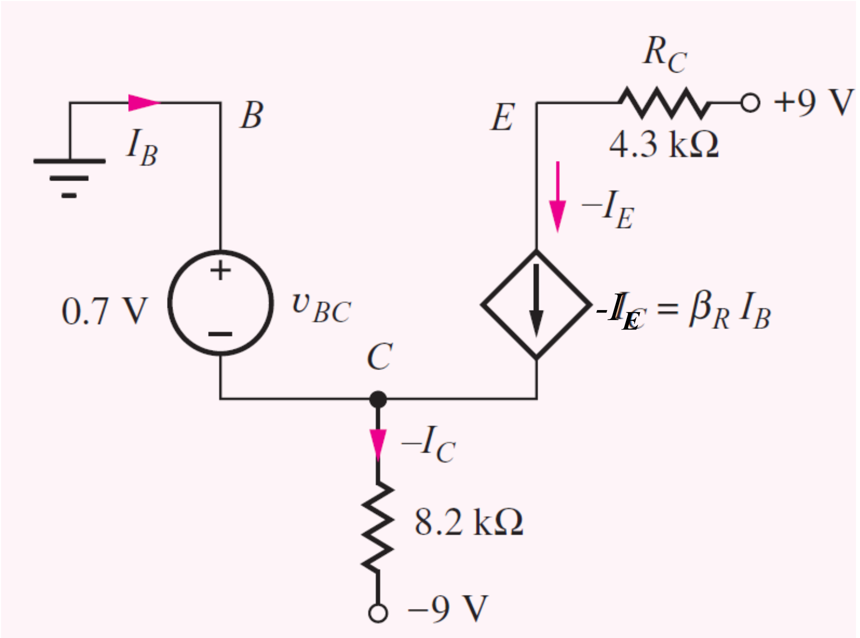


Use simplified model and assume v_{BC} = 0.7 V



Reverse BJT Analysis

- Find the new Q-point for the transistor with collector and emitter terminals interchanged.



Reverse active currents are very different from forward active currents

$$-I_C = \frac{-0.7-(-9)}{8200} = 1.01\ mA$$

$$-I_C = (\beta_R + 1)I_B \longrightarrow I_B = 0.505\ mA$$

$$-I_E = \beta_R I_B = 0.505\ mA$$

$$V_E = 9 - 0.505mA(4.3k\Omega) = 6.83V$$

$$V_{CE} = V_C - V_E = -0.7V - 6.83V = -7.5V$$

The Q-point is (-1.01 mA, -7.5 V).

I_C V_{CE}

If one day in your lab you find your circuit doesn't show high gain, check the connection of the terminals.

PNP Reverse Active Region

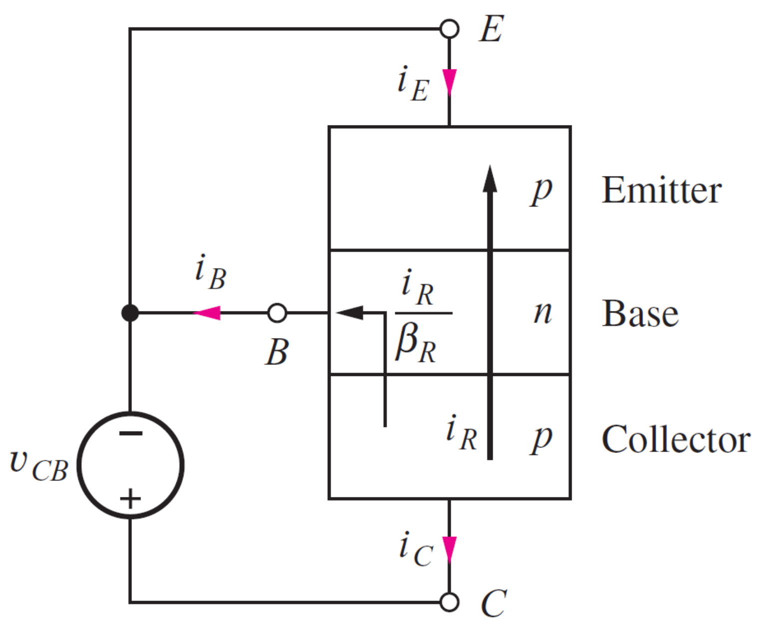
- PNP reverse active operation

$$i_R = -i_E = I_S\left[\exp\left(\frac{V_{CB}}{V_T}\right) - 1\right]$$

$$i_B = \frac{i_R}{\beta_R} = \frac{I_S}{\beta_R}\left[\exp\left(\frac{V_{CB}}{V_T}\right) - 1\right]$$

$$i_C = -I_S\left(1 + \frac{1}{\beta_R}\right)\left[\exp\left(\frac{V_{CB}}{V_T}\right) - 1\right]$$

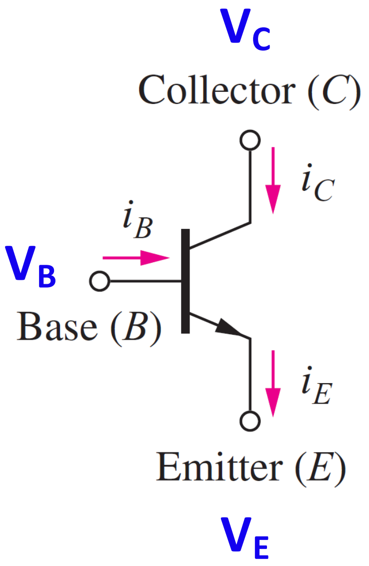
$$i_C = i_E - i_B$$



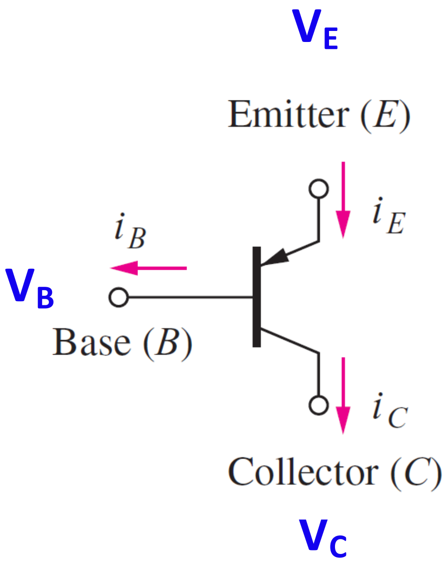
Other Regions of Operation

- **BJT cut-off region**:
  - Both the base-collector and base-emitter junctions are reverse-biased

npn



$\boxed{V_C > V_B < V_E}$



pnp

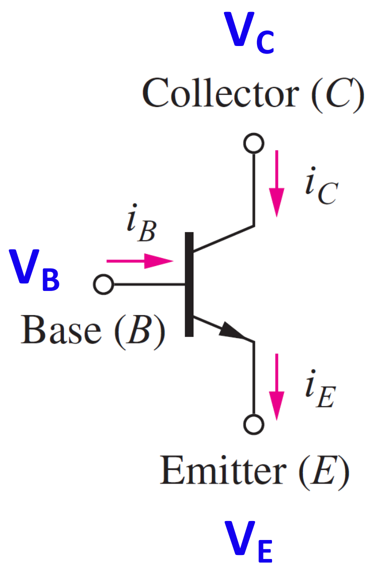
$\boxed{V_C < V_B > V_E}$

- Similar to turning off a switch
  - No carriers can go from emitter to collector
  - Say in a circuit you have C-E currents (npn). Now, apply a very negative V_B will kill this current.

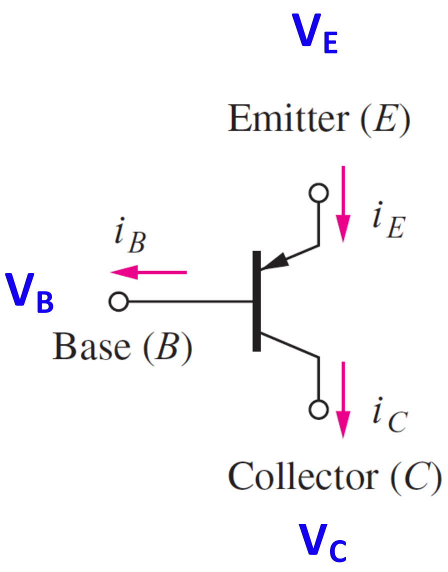
Other Regions of Operation

- **BJT saturation region**:
  - Both the base-collector and base-emitter junctions are forward-biased

npn



$$V_C < V_B > V_E$$
$$V_C \approx V_E$$



pnp

$$V_C > V_B < V_E$$
$$V_C \approx V_E$$

- Similar to turning on a switch
  - Increasing the base current will no longer increase the collector current

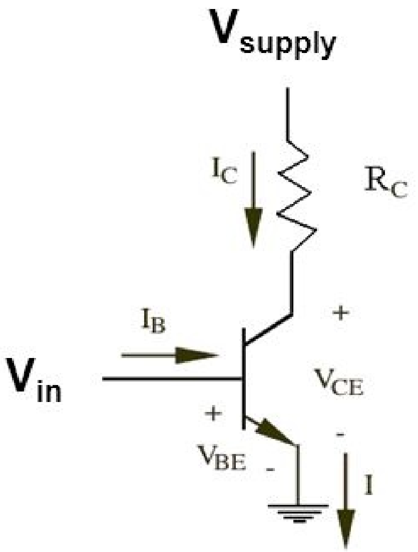
BJT Saturation and Cut-off

- Saturation and cut-off regions *not* generally used for BJTs
- BJTs are not good switches – use a MOSFET if you want a switch
- Saturation usually occurs when base current has become so large that the collector current can no longer increase proportionately
  - BJT transport equations no longer applicable

BJT Operation Regions

Base-Emitter Junction	Base-Collector Junction	
	Reverse Bias	**Forward Bias**
Forward Bias	**Forward Active Region** Good amplifier	**Saturation Region** Closed switch
Reverse Bias	**Cut-off Region** Open switch	**Reverse Active Region** Poor Amplifier

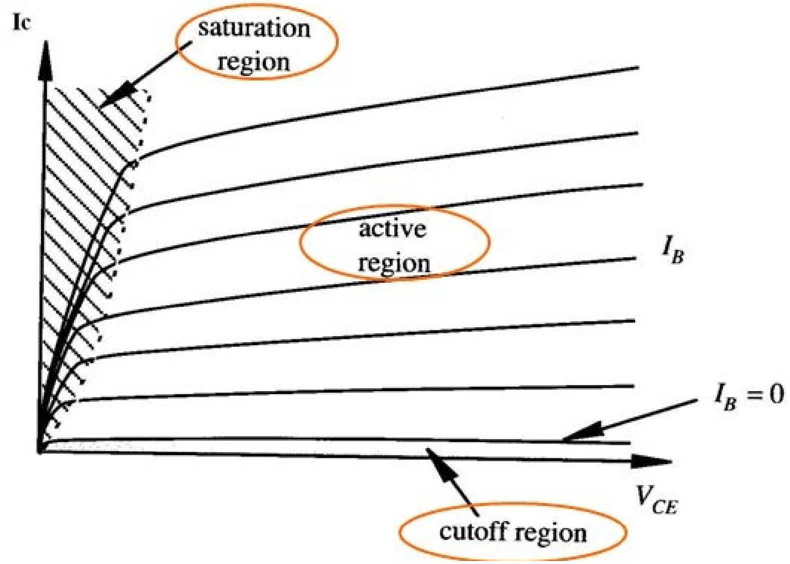
BJT Operation Regions



1) **Cutoff Region:**
   - $V_{BE} < V_{cut\text{-}in}$, $i_B = 0$
2) **Active / Linear Region:**
   - $V_{BE} = V_{cut\text{-}in}$, $i_B > 0$
3) **Saturation Region:**
   - $V_{BE} = V_{cut\text{-}in}$, $i_B > i_{C,max}$

V_{cut-in} say 0.7V

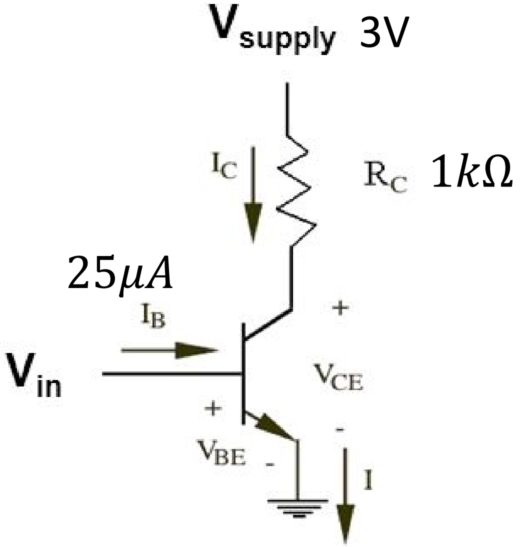
V_C not high enough



V_B not high enough

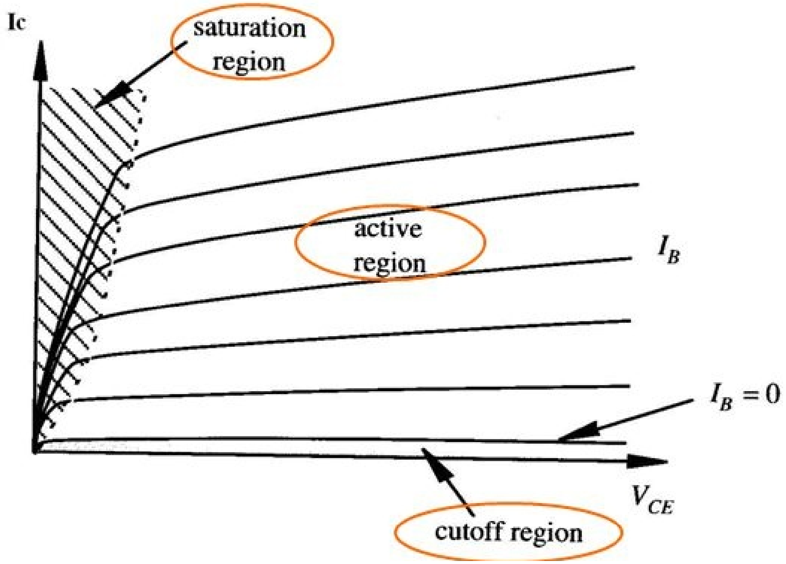
BJT Operation Region example

$\beta = 100$



$I_C = 100 \times 25\mu A = 2.5mA$
$V_{CE} = 3V - 2.5mA \times 1k\Omega = 0.5V$
$V_B = 0.7V$
$V_{BC} = 0.2V$

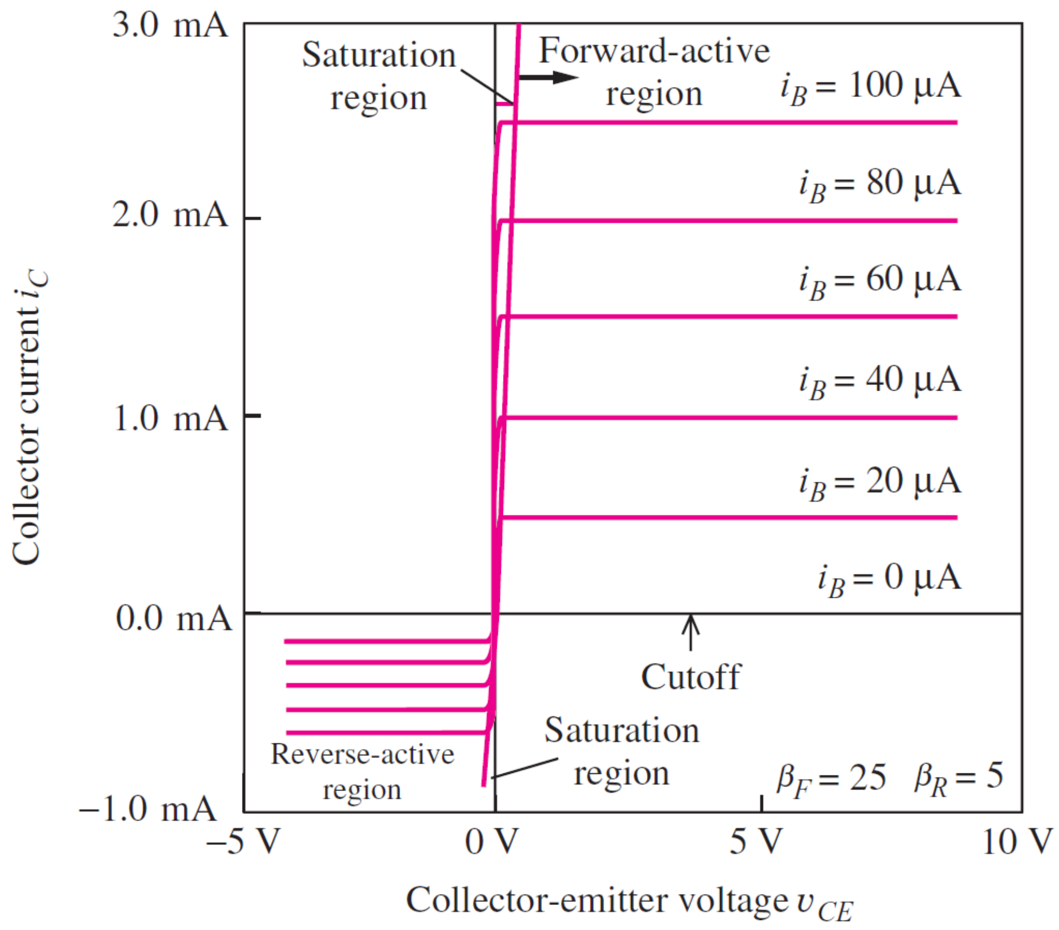
BC is forward biased, so, saturation.
(Note, here can't use $I_C = \beta I_B$ to begin with, but the operation region is correct)



BJT I-V Characteristics

- For $I_B = 0$, transistor is in cutoff
  - If $I_B > 0$, I_C also increases.
- For $V_{CE} > V_{BE}$, npn transistor is in forward-active region, $I_C = \beta_F I_B$
  - I_B is independent of V_{CE}
- For $V_{CE} < V_{BE}$, transistor is in saturation
- For $V_{CE} < 0$, roles of collector and emitter reverse

- Q-point is represented by V_{CE} and I_C
  - Simply pinpoint it on the I-V chart



Biasing the BJT

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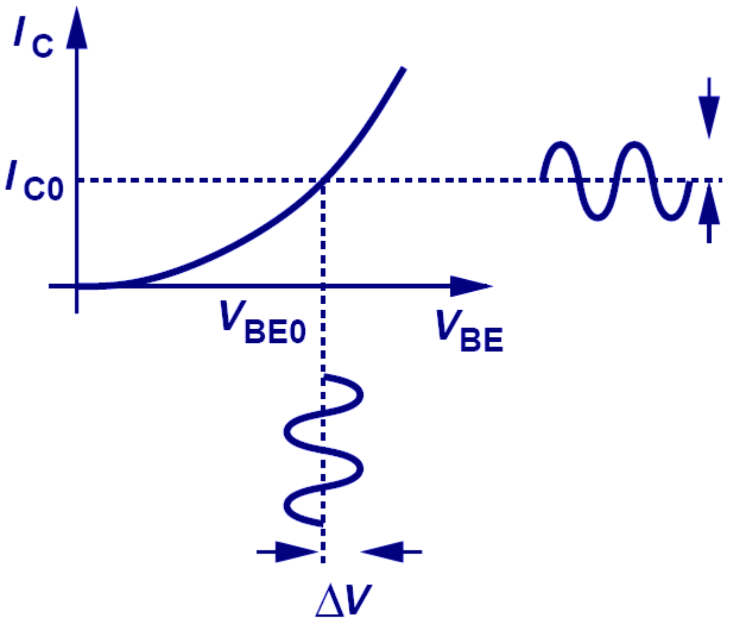
Biasing a BJT

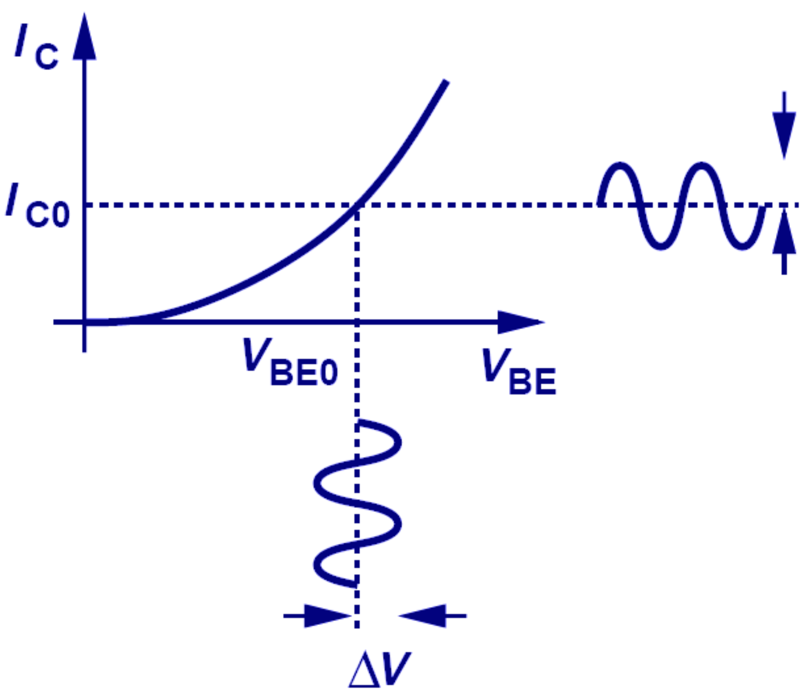
- The purpose of biasing a transistor is to establish Q-point
  - Establish operating region of the transistor
- Recall, BJTs operates as amplifiers if it is biased in the active mode
  - base-emitter: forward biased and base-collector: reverse biased

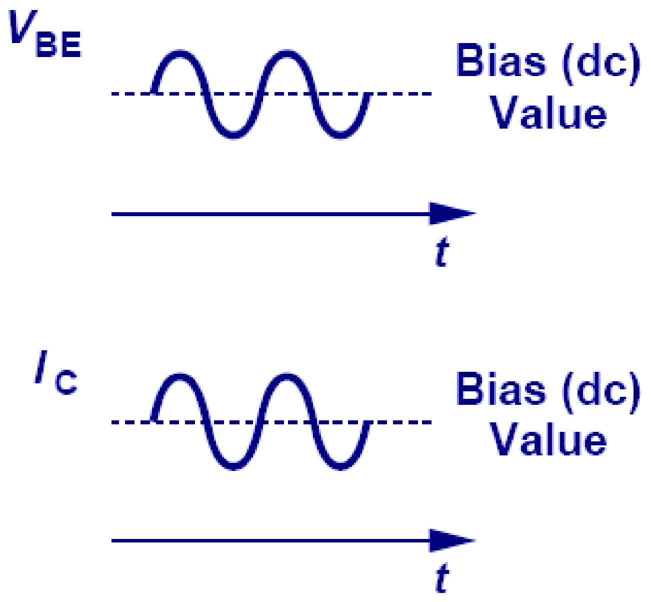


Then, small-signal operation will occur at the Q-point

Biasing a BJT

- Amplification properties ie. small-signal parameters such as $g_m = I_C/V_T$, $r_\pi = \beta/g_m$, and $r_o = V_A/I_C$ depend on the bias conditions.
- The bias point is a DC value

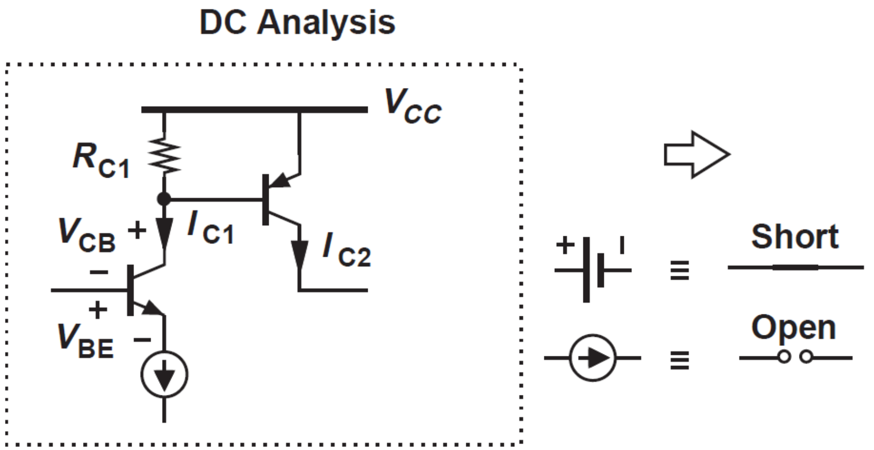




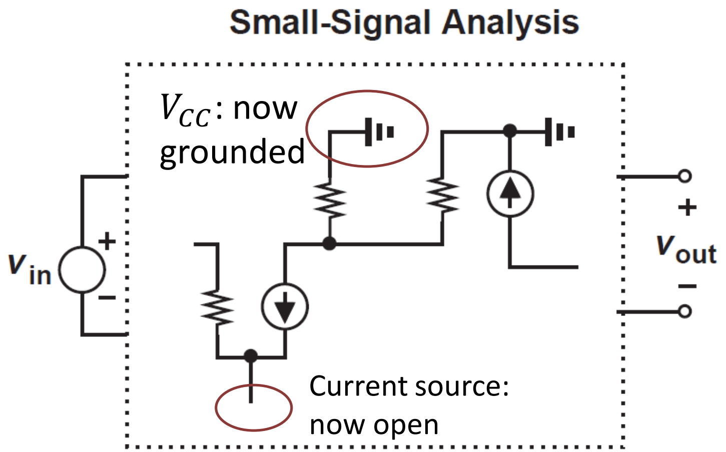
Small changes in voltages and currents around the bias values

Large Signal Vs Small Signal Analysis

- First, we need to determine the DC operating point in the absence of signals
- **DC Analysis**: perform large-signal analysis to determine the region of operation and small-signal parameters
- Second, we study the response of the circuit to small signals
- **Small Signal Analysis**: compute quantities such as voltage gain



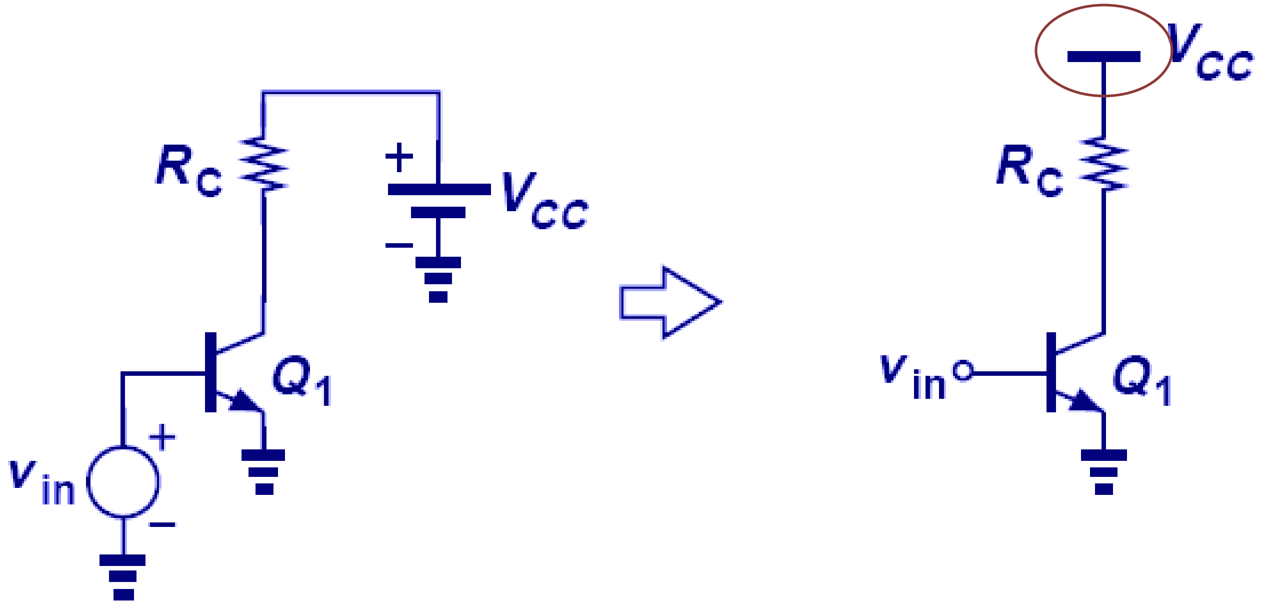
Supply voltage sources establish bias points



Constant sources that do not vary with time are set to zero

Notation

- LARGE SIGNAL and small signal notations
  - Lowercase v_{in} indicates a small-signal input voltage
  - DC supplies shown as solid power supplies – sometimes represented with a horizontal bar

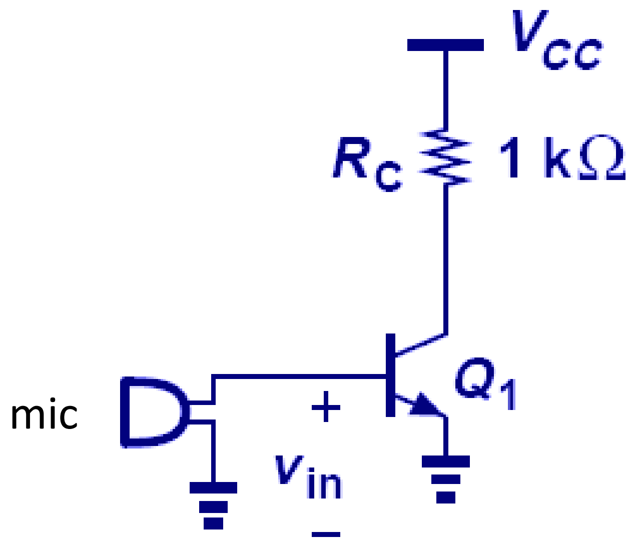


Small Signal vs Large Signal

- Under what conditions can we represent devices with small signal models?
  - If signal perturbs bias point negligibly -> operates in small signal regime
  - g_m and r_π vary negligibly (considered constants) -> linear representation holds
  - Rule of thumb : 10% variation in the collector current

Small Signal Amplification: Bad Biasing

- The microphone provides a small-signal input
  - Connected to the amplifier to amplify the small output signal of the microphone
- This is an example of bad biasing:



WHY?

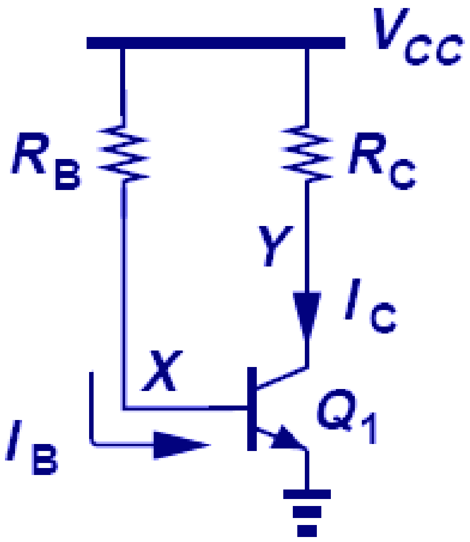
- Not biased to operate in forward-active region
- There is no DC bias current running through the transistor to set the transconductance
- This is a bad design

DC Biasing of BJTs

- Four-resistor biasing
- Two-resistor biasing with collector-to-base feedback resistor

Base resistor fixed biasing

- Use the existing V_{CC} to bias the base (natural thinking)
  - Base is tied to V_{CC} through a relatively large resistor R_B, as we expect I_B to be small
- Assuming a constant value for V_{BE}, solve for both I_B and I_C and determine the terminal voltages of the transistor
- Forward-active operation still not guaranteed



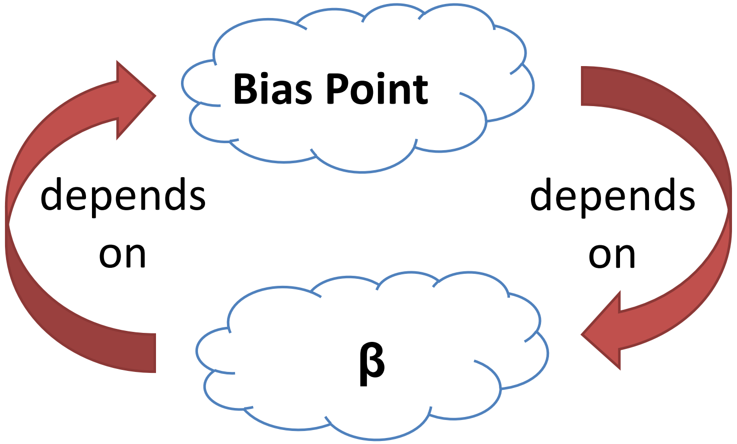
$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

$$I_C = \beta I_B$$ ← Assume forward active

$$V_{CE} = V_{CC} - I_C R_C$$

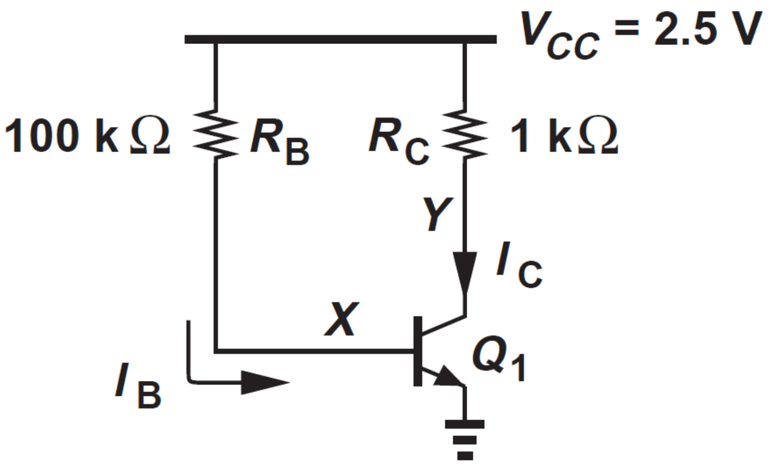
V_{CE} determines whether device in active mode or not

V_{CE} must be larger than V_{BE}



Base resistor fixed biasing

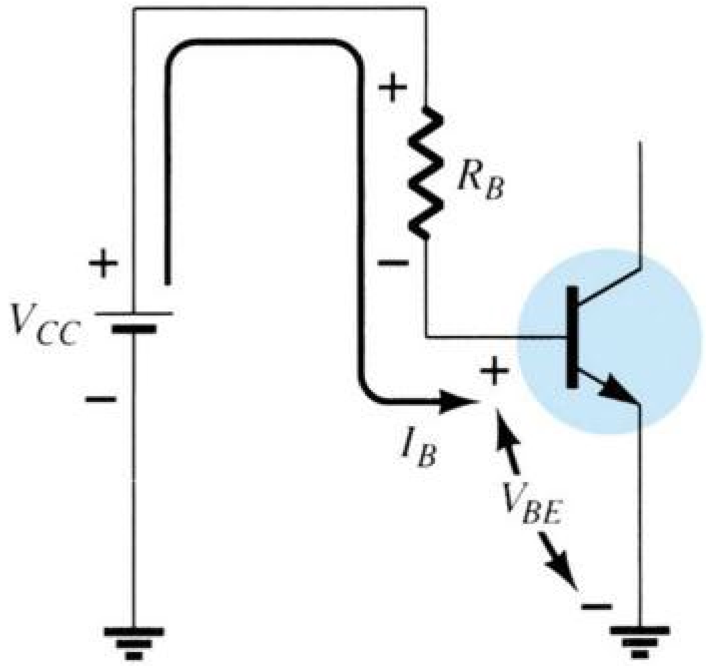
- Example: Determine the collector bias current for the circuit shown below. Assume $\beta = 100$ and $I_S = 10^{-17}$.



Base resistor fixed biasing

- Base-Emitter Loop
- From Kirchhoff voltage law (KVL):

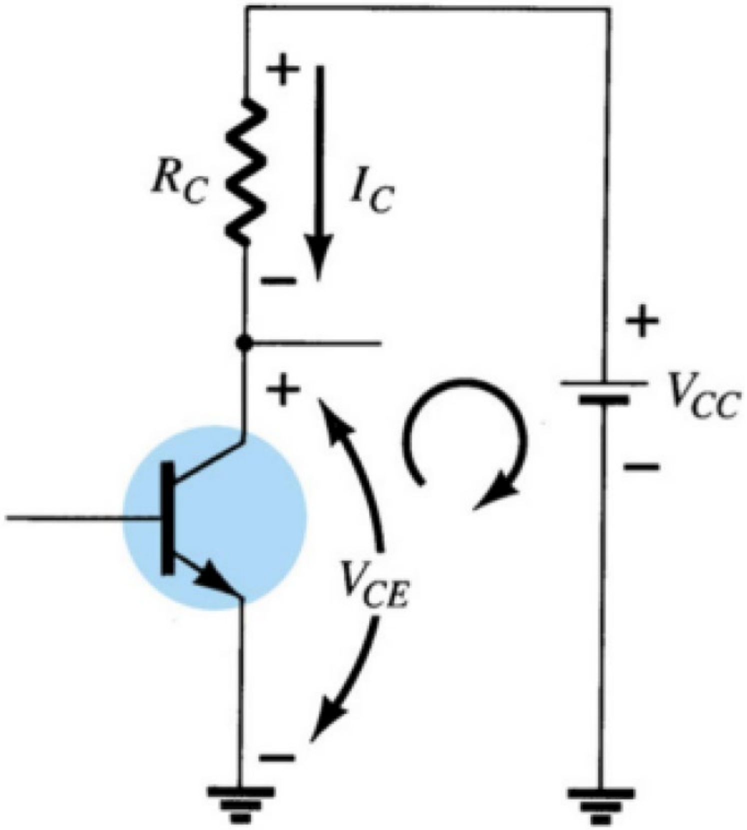
$$+V_{CC} - I_B R_B - V_{BE} = 0$$



- Solving for base current:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = 18\mu A$$

Base resistor fixed biasing



- Collector-Emitter Loop
- Find collector current from base current
  - Assuming forward active

$$I_C = \beta I_B = 1.8mA$$

- Solving for collector-emitter voltage:

$$V_{CE} = V_{CC} - I_C R_C = 0.7V$$

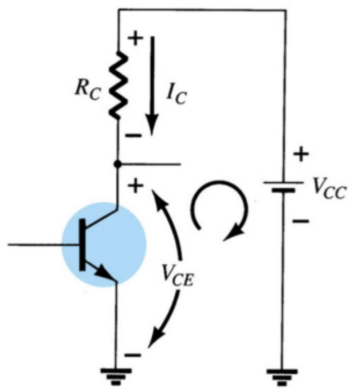
V_{BC} close to 0

- Saturation: when the transistor is operating in saturation, current through the transistor is not controlled by V_{BE}

$$I_{C,\text{sat}} = \frac{V_{CC}}{R_C}$$

V_{CE} small

Base resistor fixed biasing



- Load-line Analysis (consider V_{CC}, R_C, BJT, ground):
  - The end points of the load line are:

Saturation:

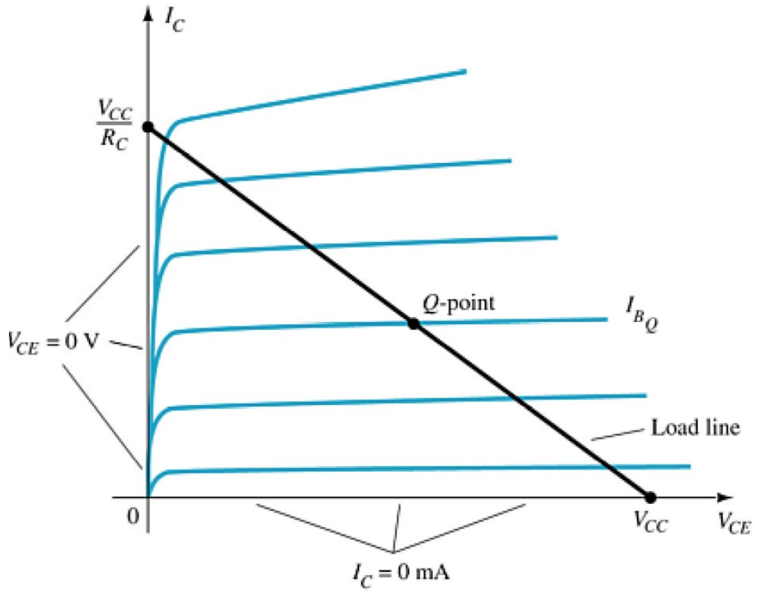
$$I_{C,\text{sat}} = \frac{V_{CC}}{R_C}$$

$$V_{CE} = 0V$$

Cut-off:

$$I_C = 0\,mA$$

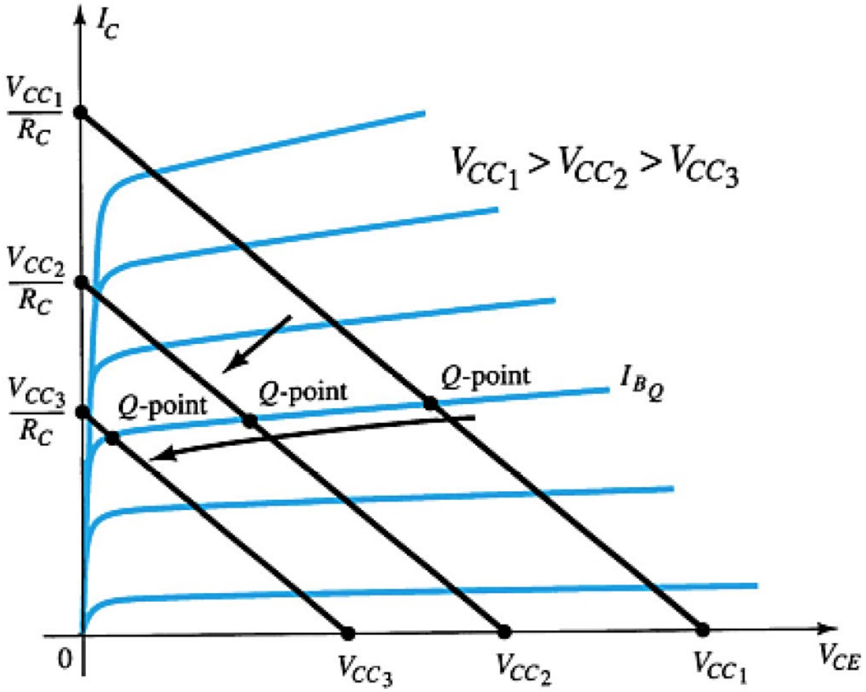
$$V_{CE,\text{cutoff}} = V_{CC}$$



- The Q-point is the operating point
  - where the value of R_B sets the value of I_B
  - The corresponding fixed I_B curve intersects with load-line,
  - setting the values of V_{CE} and I_C

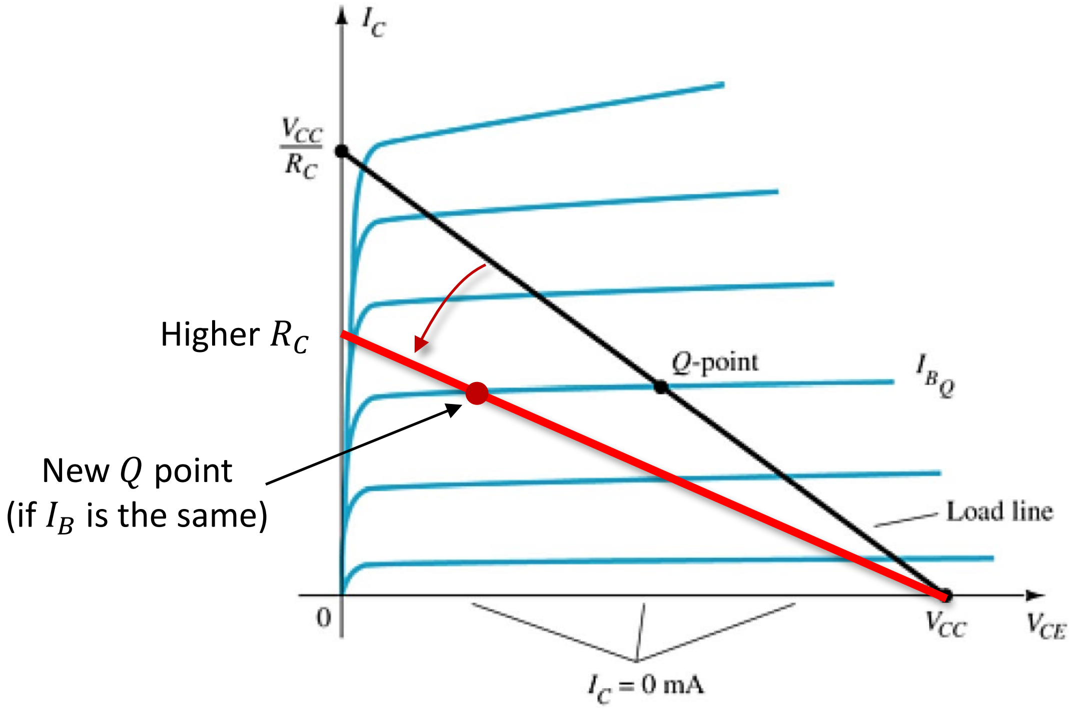
Base resistor fixed biasing

- Load-line Analysis: $V_{CE} = V_{CC} - I_C R_C$
  - Changing V_{CC}:



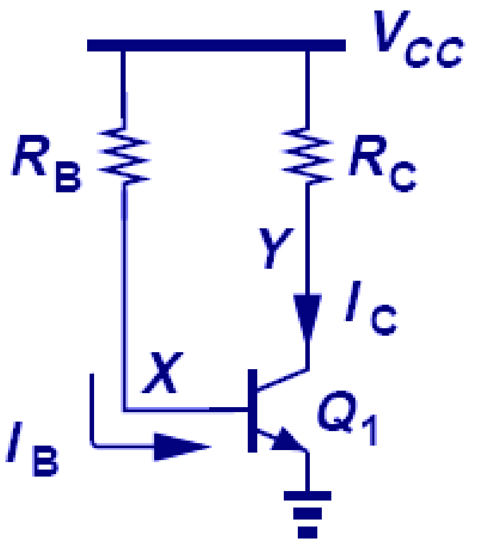
Base resistor fixed biasing

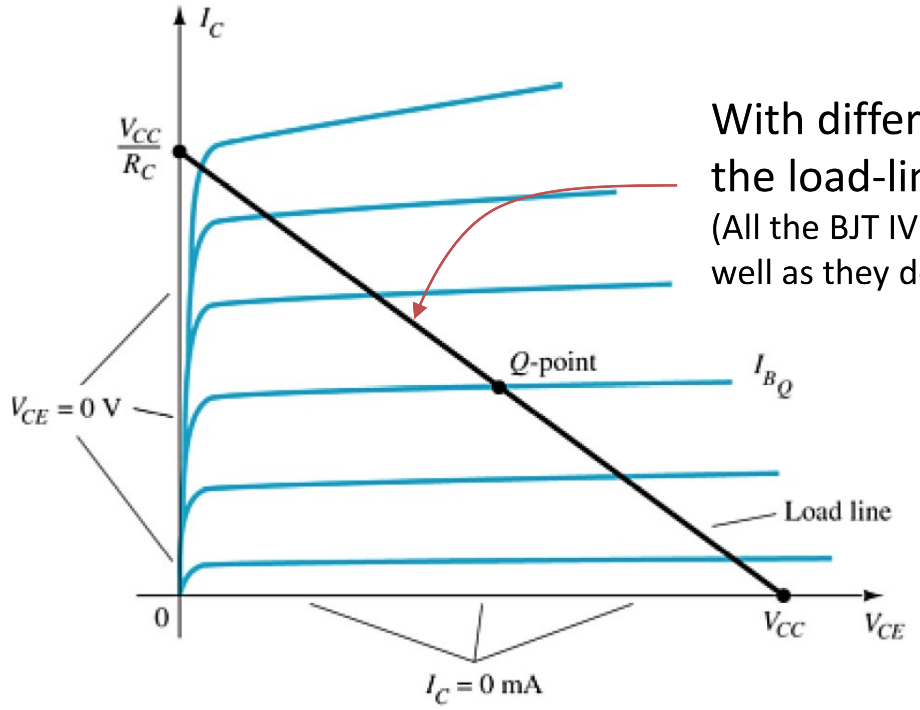
- Load-line Analysis: $V_{CE} = V_{CC} - I_C R_C$
  - Changing R_C changes the load-line



Base resistor fixed biasing

- β might vary due to thermal stability issues
  - temperature changes
  - different transistors used
- Since I_B *is fixed*, I_C depends on β, which changes and shifts the Q-point
- Poor stability



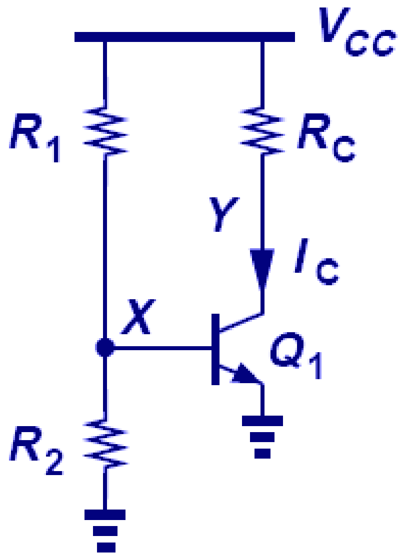


With different β, you slide along the load-line

(All the BJT IV curves in blue color shifts as well as they depend on β)

Improved base resistor fixed biasing

- Instead of fixed I_B, now fix V_{BE}
- To reduce dependence of I_C on β, I_C must be set by applying a well-designed *constant* V_{BE}: $I_C = I_S \exp(V_{BE}/V_T)$
- The resistor divider sets V_{BE}
    - Forward-active operation can be controlled
- I_C is independent of β if base current is small



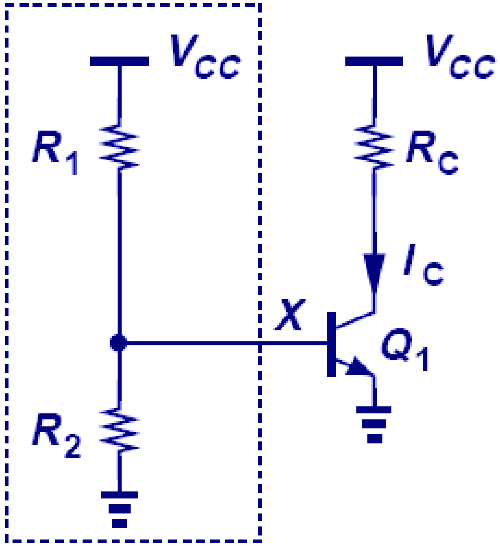
$$V_x = \frac{R_2}{R_1 + R_2} V_{CC}$$

$$I_C = I_S \exp\left(\frac{R_2}{R_1 + R_2}\frac{V_{CC}}{V_T}\right) \longleftarrow \text{independent of } \beta$$

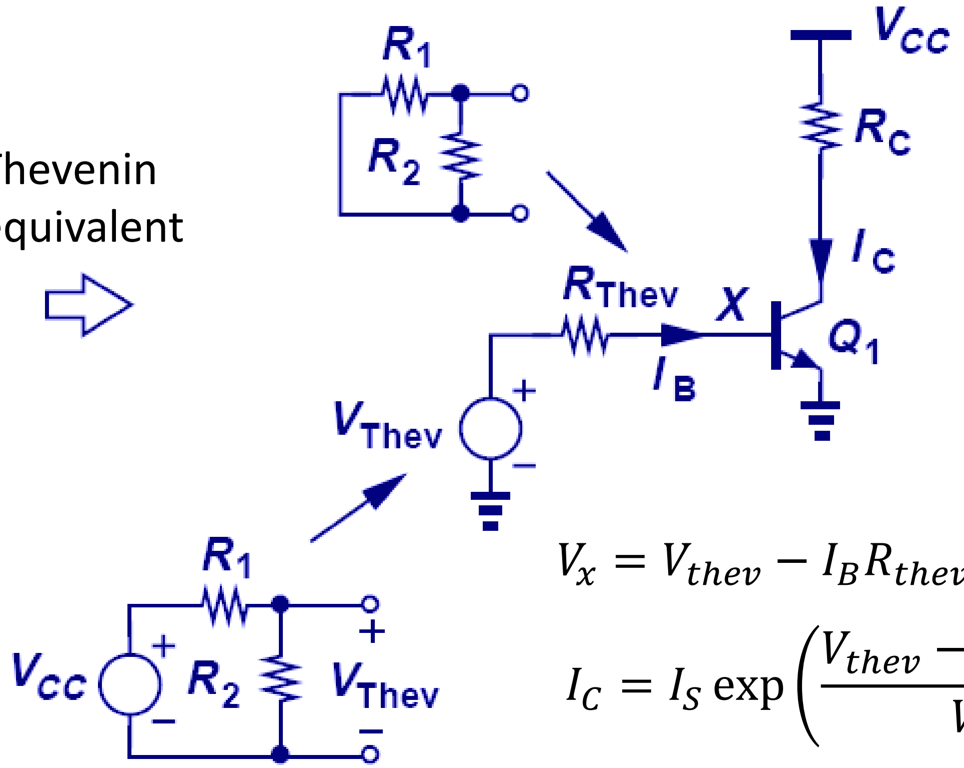
This assumes the base current is negligible

Base current issues

- What if I_B is not negligible?
- What is V_x if $I_B > 0$?



Thevenin equivalent



$$V_{thev} = \frac{R_2}{R_1 + R_2} V_{CC}$$

$$R_{thev} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$V_x = V_{thev} - I_B R_{thev}$$

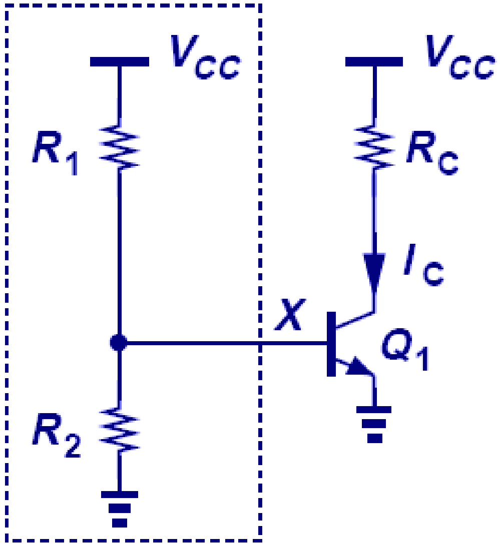
$$I_C = I_S \exp\left(\frac{V_{thev} - I_B R_{thev}}{V_T}\right)$$

Exponential dependence of I_C on voltage generated by resistive divider

We want I_C to not change much against variations in the circuit.

Base current issues

- Example: If R_2 is 1% higher than its nominal value, what will be the error in the collector current?



$$V_x = V_{thev} - I_B R_{thev} = \frac{R_2}{R_1 + R_2} V_{CC} - I_B \frac{R_1 R_2}{R_1 + R_2}$$

If R_2 is 1% higher than its nominal value, so is V_X.

$$I_C = I_S \exp\left(\frac{V_x}{V_T}\right)$$

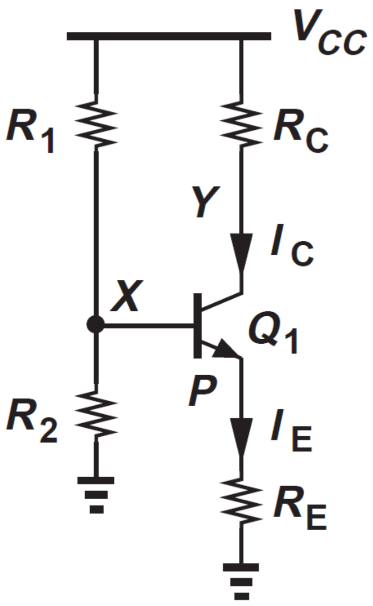
Multiplying the collector current by

$\exp\left(\frac{0.01 V_{BE}}{V_T}\right) \approx 1.36$ (for V_{BE} = 800 mV)

A 1% error in one resistor value introduces a 36% error in I_C

Four-resistor bias network

- We don't want I_C to be affected too much by the bias resistors
- Adding R_E



Why is this better?

1. Say we have chosen R_1, R_2, R_C, R_E such that the BJT is in forward active mode.
2. Now, when we set up the circuit in the lab, it turns out R_2 is larger than designed value
3. This raises V_x
4. In turn, this would increase V_{BE}
5. Now I_C would increase, so would I_E
6. Hence $V_E = I_E R_E$ also increases, reducing V_{BE}. "counter acting" - feedback
7. You can repeat this analysis for the other resistors.
8. In reality, we will see that in the four-resistor Bias, V_{CE} and I_C won't have exponential dependence on resistors.

Four-resistor bias network

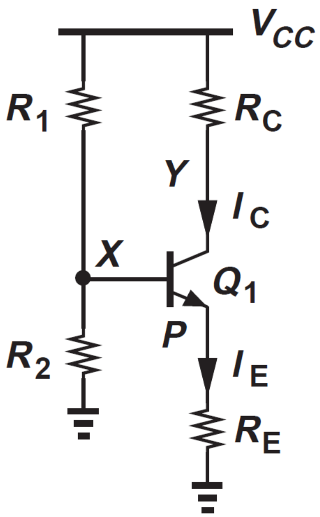
- What is I_B now?

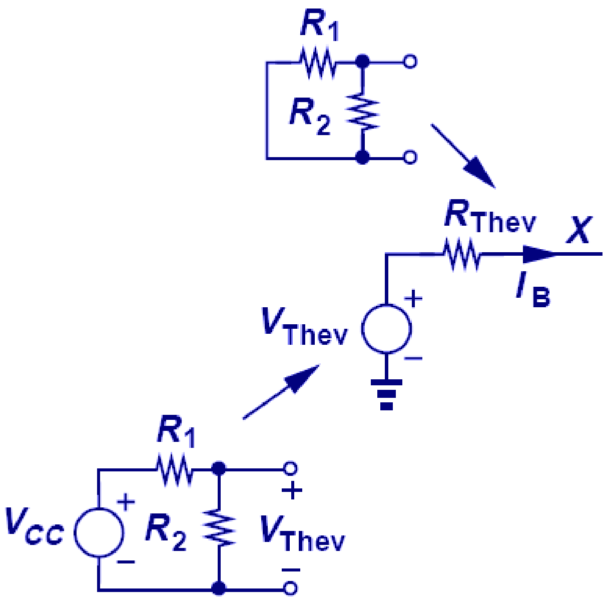
$$V_{thev} = I_B R_{thev} + V_{BE} + I_E R_E$$

$$I_E = (\beta_F + 1) I_B$$

$$I_B = \frac{V_{thev} - V_{BE}}{R_{thev} + (\beta_F + 1) R_E}$$

(V_{BE} not known yet, but is roughly at 0.7V)





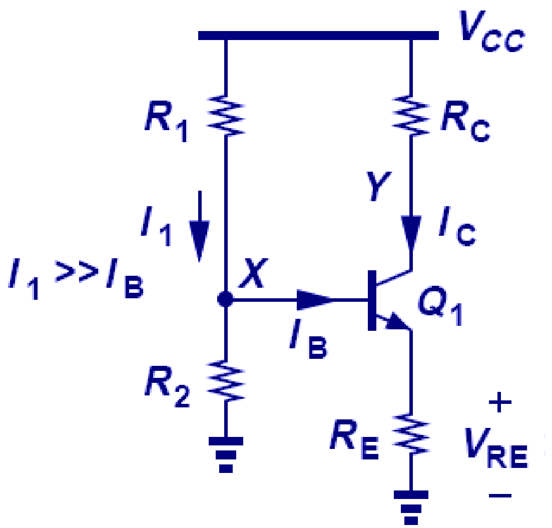
Four-resistor bias network

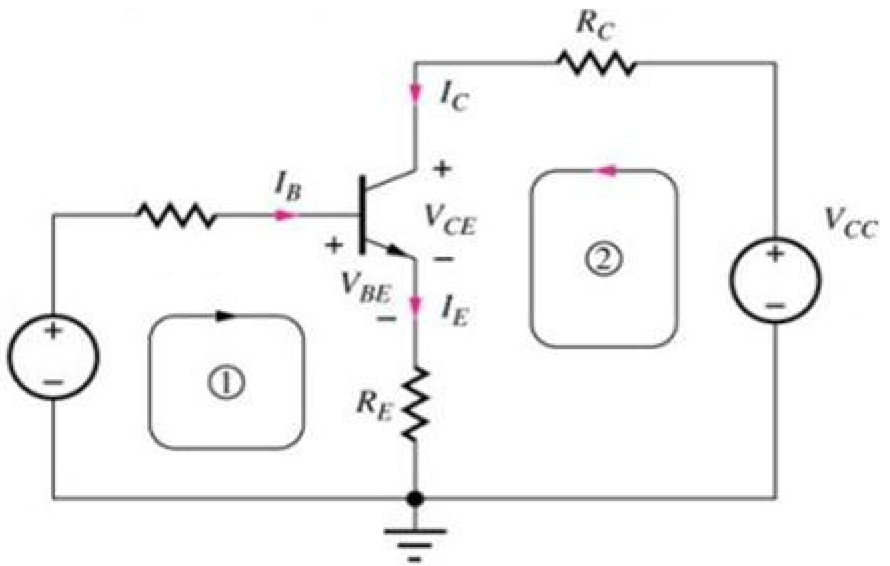
- What is I_C now?

$$I_E = \frac{V_{thev} - V_{BE} - I_B R_{thev}}{R_E} \cong \frac{V_{thev} - V_{BE}}{R_E}$$

(V_{BE} not known yet but is roughly at 0.7V)

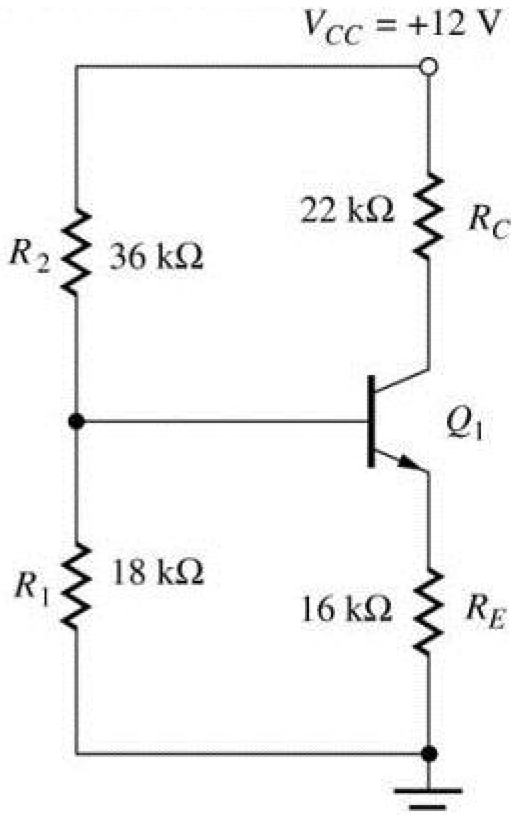
$I_C \approx I_E$ Dependance on resistors is not exponential anymore





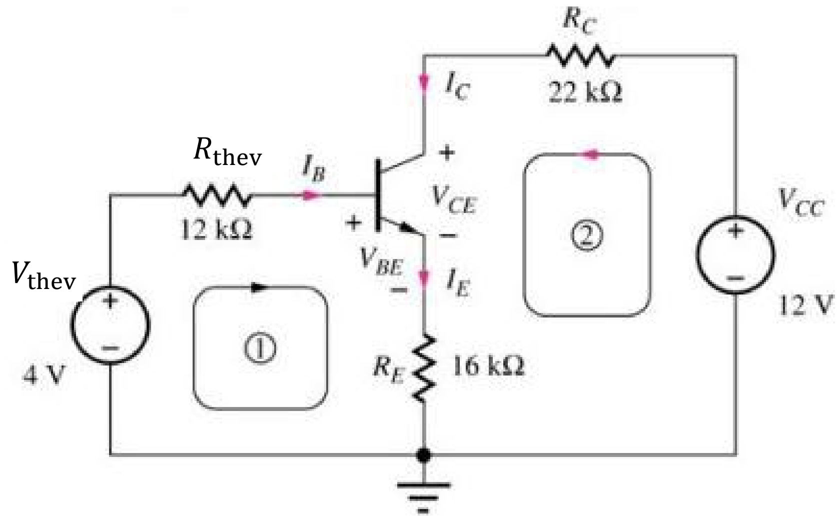
Four-resistor bias network example

- Find the Q-point of this transistor. Let $\beta_F = 75$.
  - First, find the Thévenin equivalent of the base input network:



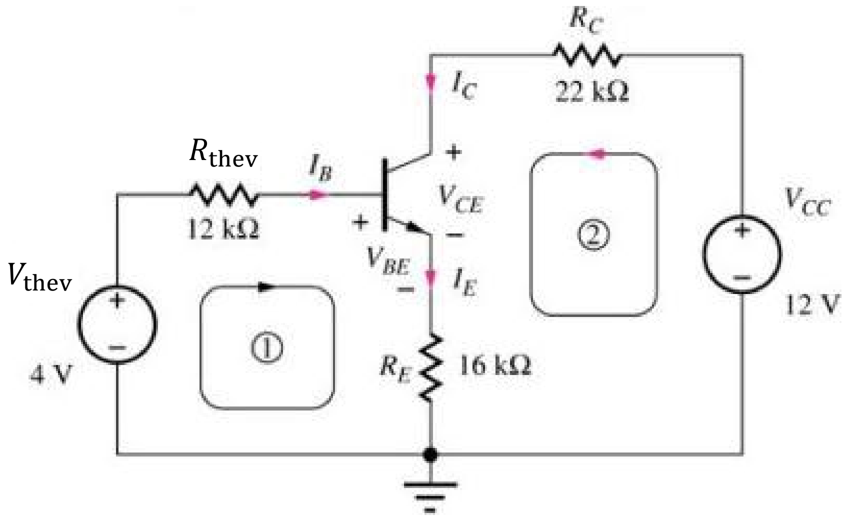
$$V_{thev} = \frac{R_1}{R_1 + R_2} V_{CC} = 4V$$

$$R_{thev} = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2} = 12\Omega$$



Four-resistor bias network example

- Find the Q-point of this transistor. Let $\beta_F = 75$.
  - Second, solve for all terminal currents:



$$V_{thev} = I_B R_{thev} + V_{BE} + I_E R_E$$

$$4 = 12000 I_B + 0.7 + 16000(\beta_F + 1) I_B$$

Assuming forward active

$$I_B = 2.68\ \mu A$$

$$I_C = \beta_F I_B = 201\ \mu A$$

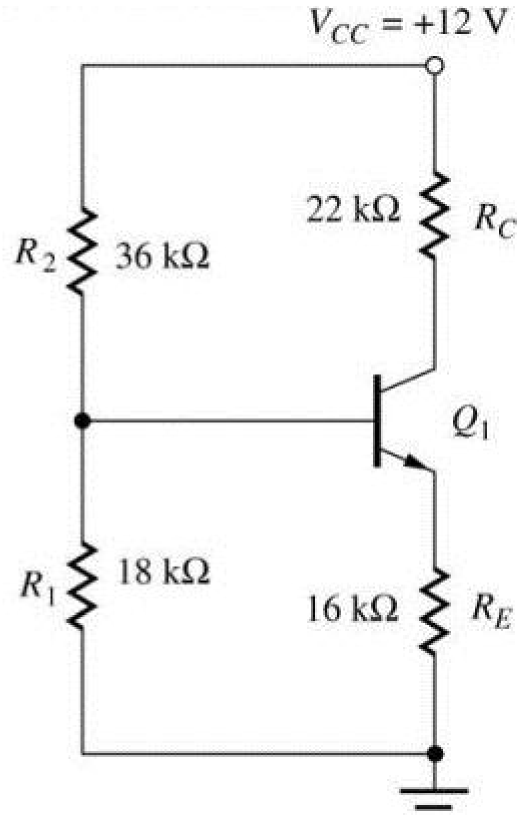
$$I_E = (\beta_F + 1) I_B = 204\ \mu A$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = 4.32\text{V}$$

$V_{CE} > V_{BE}$ hence it is forward active

Four-resistor bias network: load line analysis

- We can plot a load line against the I-V curves to verify our analysis.
- Load-line equation:

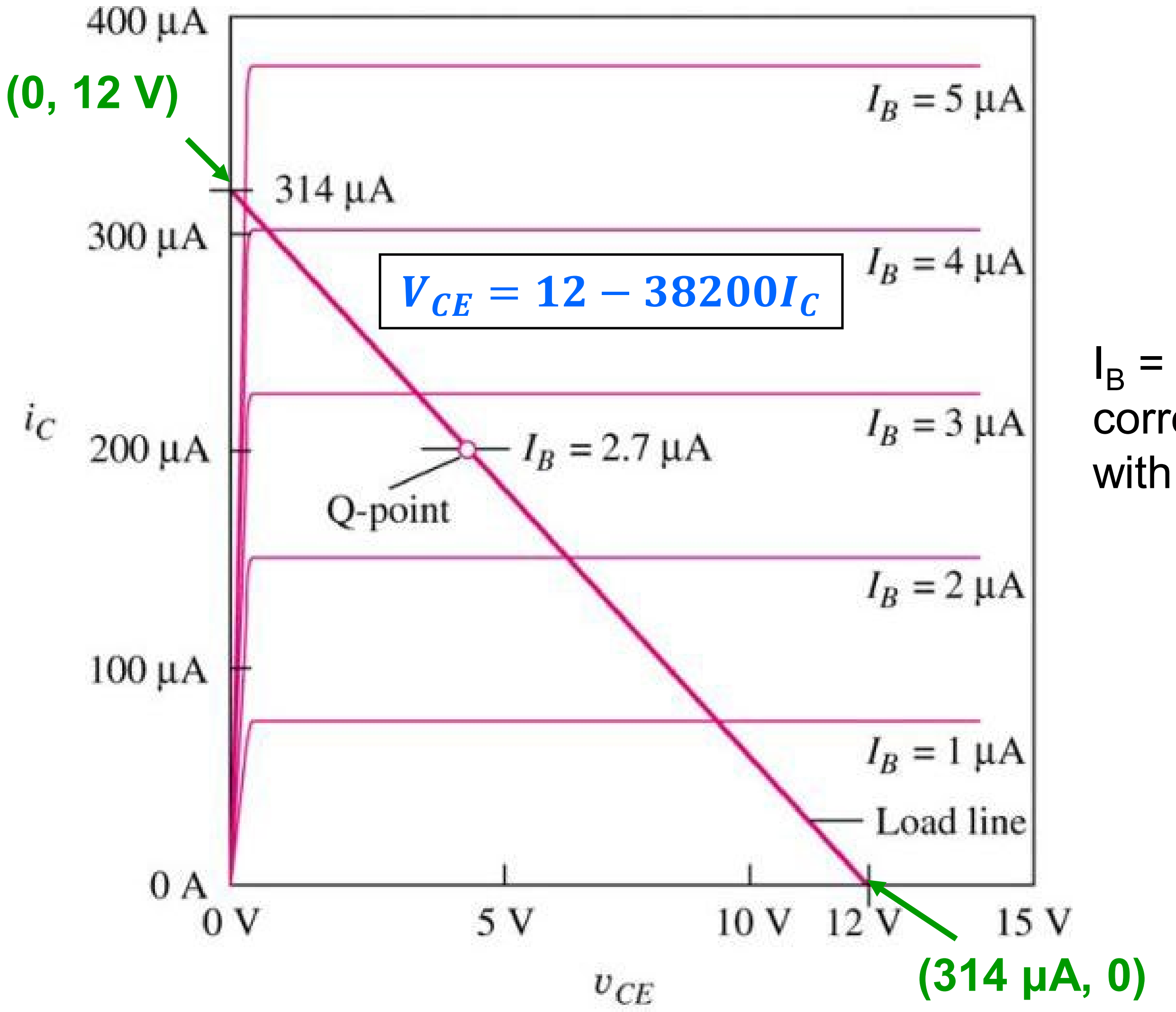


$$V_{CE} = V_{CC} - I_C R_C - I_E R_E$$

$$V_{CE} = V_{CC} - \left(R_C + \frac{R_E}{\alpha_F}\right) I_C$$

$$V_{CE} = 12 - 38200 I_C$$

Four-resistor bias network: load line analysis



I_B = 2.7 µA, intersection of corresponding characteristic with load line gives Q-point

$I_C = 201\ \mu A$

$V_{CE} = 4.32\text{V}$

Design principles of four-resistor bias network

- Transistor needs to work in forward-active mode by setting V_{BE}
- Design I_C to provide the small signal parameters, g_m, r_{be}, etc.
- Choose Thévenin equivalent base voltage

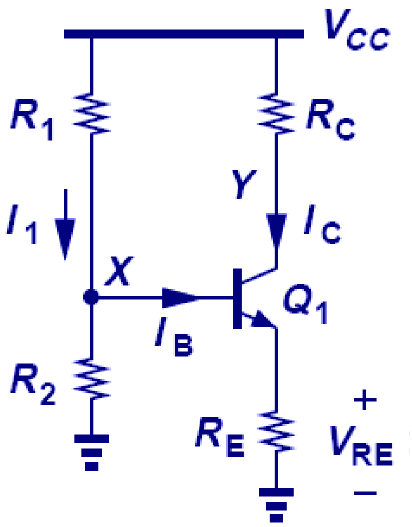
$$\frac{V_{CC}}{4} \leq V_{Thev} \leq \frac{V_{CC}}{2}$$

- Select R_1 to set $I_1 = 10I_B$

$$R_1 = \frac{V_{CC} - V_{thev}}{10I_B}$$

- Select R_2 to set $I_2 = 9I_B$

$$R_2 = \frac{V_{thev}}{9I_B}$$



These aren't fixed values, but they are a good starting point for design.

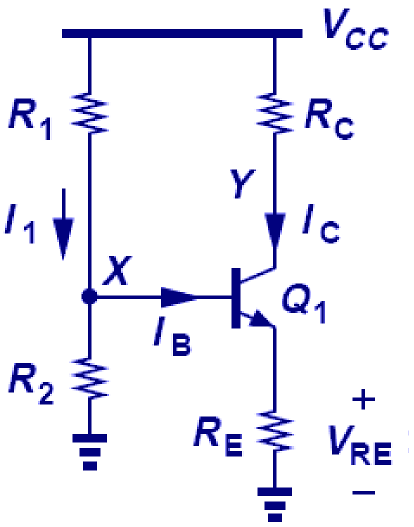
Design of four-resistor bias network

- R_E is determined by V_{Thev} and desired I_C

$$R_E \cong \frac{V_{thev} - V_{BE}}{I_C}$$

- R_C is determined by desired V_{CE}

$$R_C \cong \frac{V_{CC} - V_{CE}}{I_C} - R_E$$



You can also set R_c and R_E first, then figure out what R_1 and R_2 is needed.