

ELEC2104 – Week 4

PN Junction current, circuit models



THE UNIVERSITY OF  
SYDNEY

## Fermi level LEVELs in equilibrium

$$w = \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_{bi} + V_R)}$$

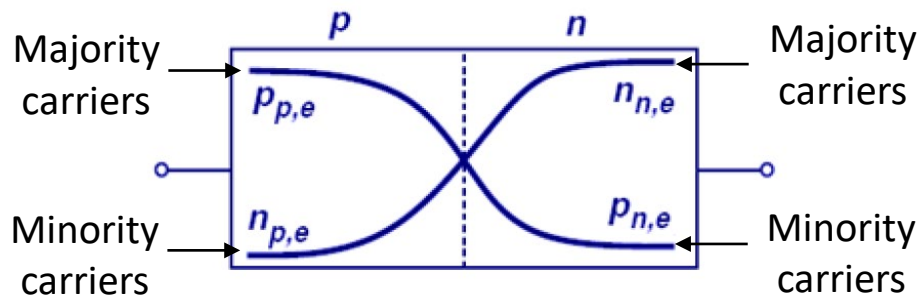
$$V_{bi} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

$w$	depletion region width
$\varepsilon_s$	permittivity of semiconductor
$q$	elementary charge
$N_A, N_D$	acceptor and donor density
$V_{bi}$	built-in potential
$V_R$	external bias
$k_B$	Boltzmann constant
$n_i^2$	intrinsic carrier density

pn junction currents and  $V_{bi}$

# Carrier concentration in pn Junction in equilibrium

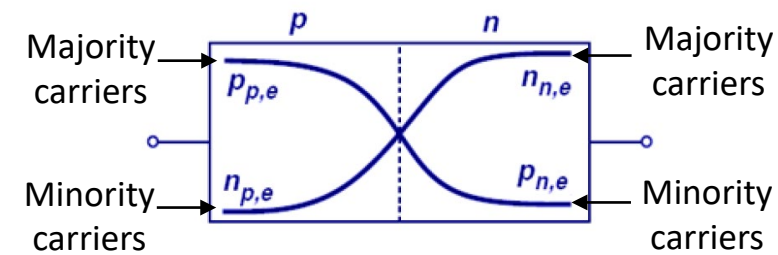
- In equilibrium
  - No external connections (open terminals)



$n_n$  : Concentration of electrons on n side  
 $p_n$  : Concentration of holes on n side  
 $p_p$  : Concentration of holes on p side  
 $n_p$  : Concentration of electrons on p side

- p side contains large excess of holes and n side contains large excess of electrons
  - Sharp concentration gradient (several orders of magnitude over  $\mu\text{m}$ )
  - Large diffusion currents
  - Electrons flow from n side to the p side
  - Holes flow from p side to the n side
  - The shape of the  $p(x)$  and  $n(x)$  is unknown (we don't need to know them)

# PN Junction Currents



Concentration gradient (Diffusion current)



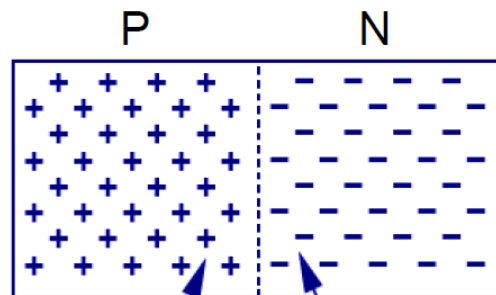
Strong E-Field (drift current)

**JUNCTION REACHES EQUILIBRIUM**

$$I_{drift,p} = I_{diff,p} \text{ and } I_{drift,n} = I_{diff,n}$$

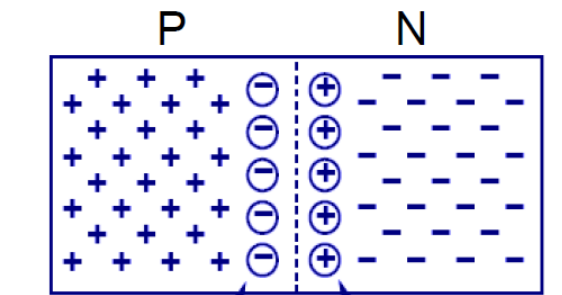
Hole drift and diffusion currents must be equal and opposite at equilibrium.

Holes Diffuse  $\longrightarrow$

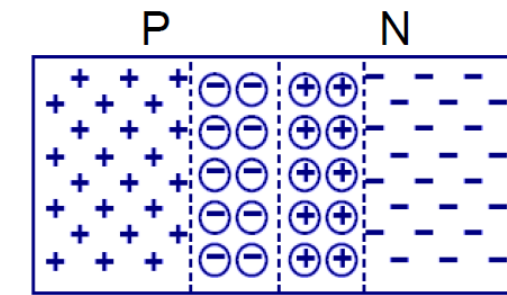


Free Holes  
Free Electrons

$\longleftarrow$  Holes Drift



Negative Acceptor Ions  
Positive Donor Ions



Depletion Region

$\longleftarrow$  Electrons Diffuse

Electrons Drift  $\longrightarrow$

Electron drift and diffusion currents must be equal and opposite at equilibrium.

# Calculating the built-in potential by balancing currents

- We don't know  $n(x), p(x)$  in the junction region
- We don't know  $V(x)$
- No need to know these

E-field is defined as  $E = -\frac{dV}{dx}$

$$I_{drift,p} = I_{diff,p}$$

$$q\mu_p p E = qD_p \frac{dp}{dx}$$

$$-q\mu_p p \frac{dV}{dx} = qD_p \frac{dp}{dx}$$

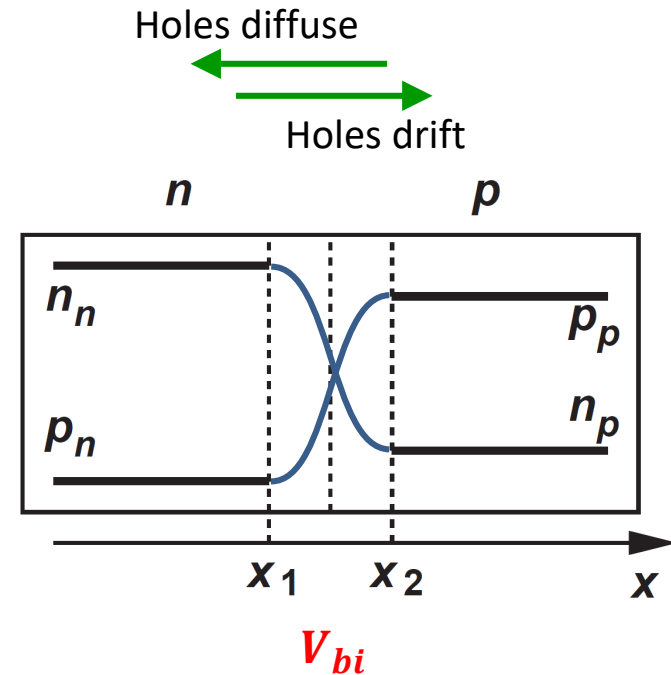
$$-\mu_p \int_{x_1}^{x_2} dV = D_p \int_{p_n}^{p_p} \frac{dp}{p}$$

$$V(x_2) - V(x_1) = -\frac{D_p}{\mu_p} \ln \frac{p_p}{p_n}$$

Einstein's relation:  $\frac{D_p}{\mu_p} = \frac{kT}{q}$

$$|V_{bi}| = \frac{kT}{q} \ln \frac{p_p}{p_n}$$

$$p_p = N_A, p_n = \frac{n_i^2}{N_D}$$



$$|V_{bi}| = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

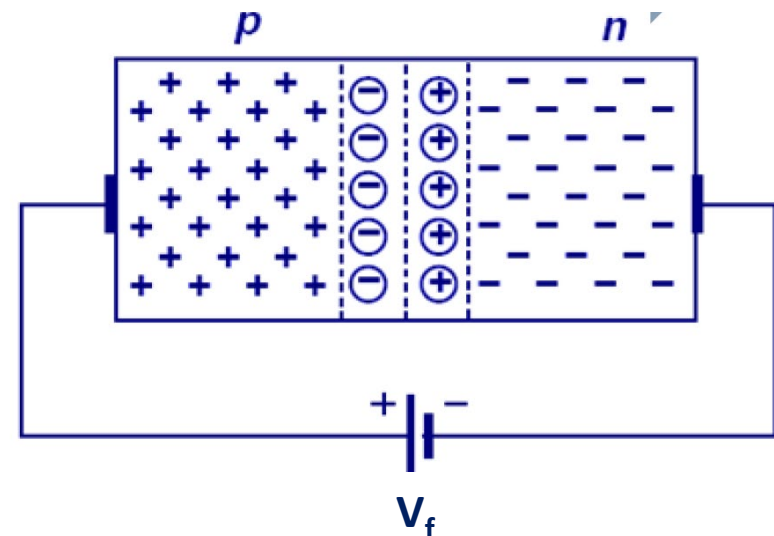
**Built-in potential barrier**

Recall  
 $np = n_i^2$

pn junction current under forward bias

# PN Junction under Forward Bias

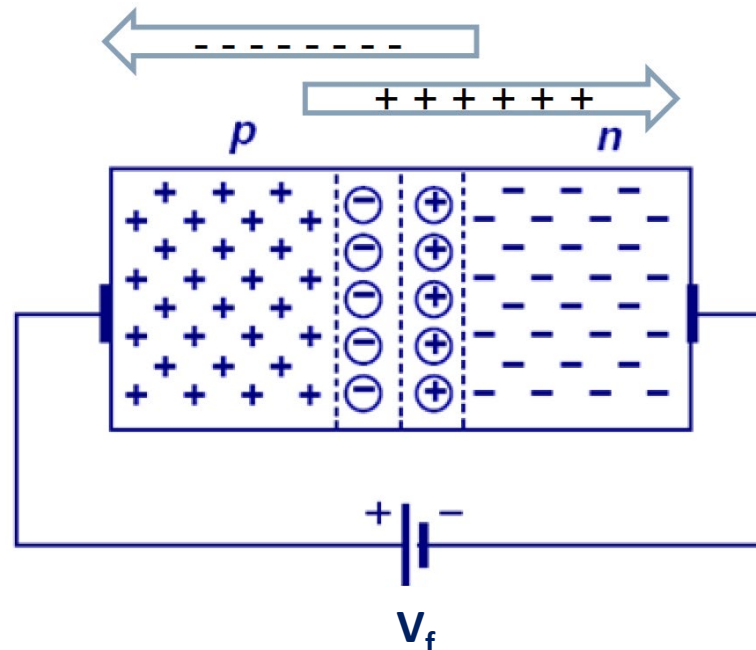
- Consider now an external voltage source  $V_F$  is applied across the junction to make  $p$  side more positive than the  $n$  side
- The connection of the positive voltage to the  $p$  terminal is called forward bias
- $V_F$  creates an E-field directed from  $p$  to  $n$  side
  - Opposes the built-in field
  - Potential barrier reduces due to weakening of the field

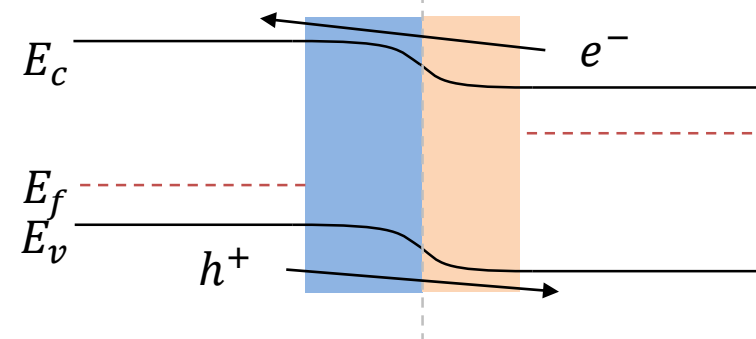
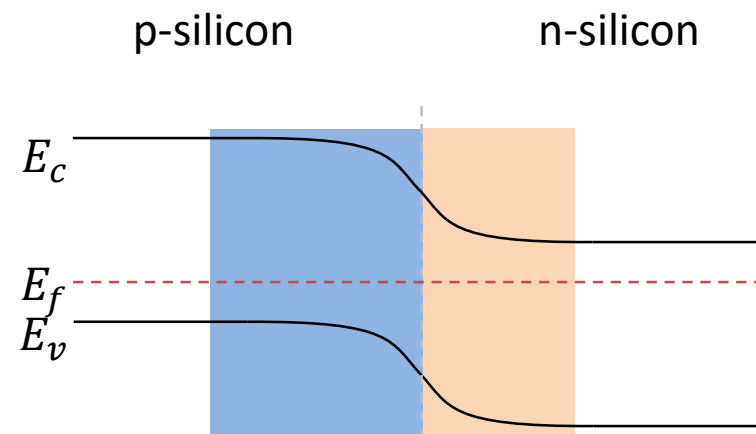
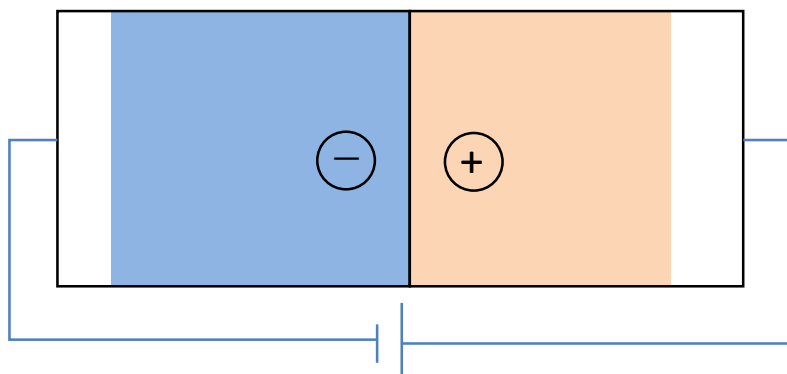
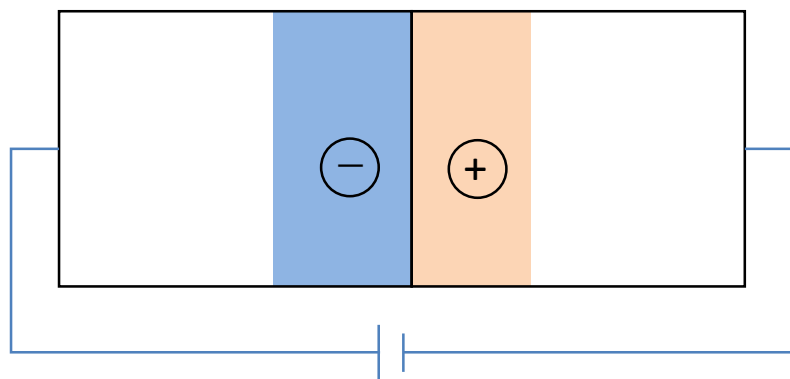
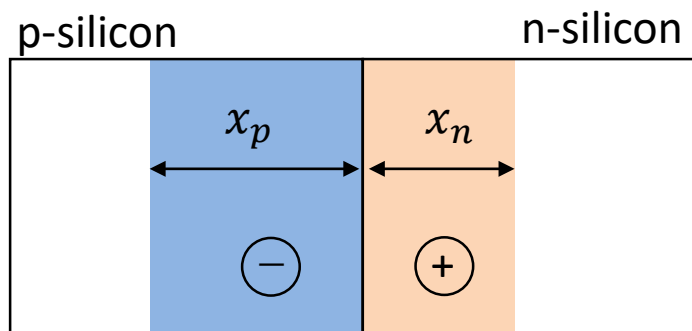




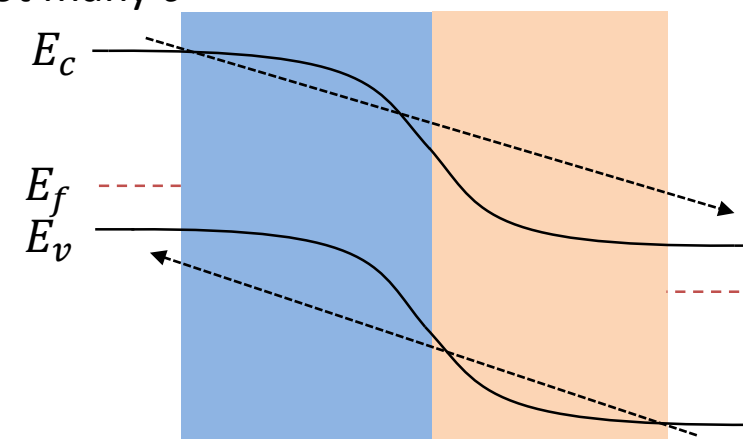
# PN Junction under Forward Bias

- With the built-in potential lowered, it is easier for holes to cross into the n region and electrons to cross into the p region
- This generates a forward diode current
  - Increases the diffusion current

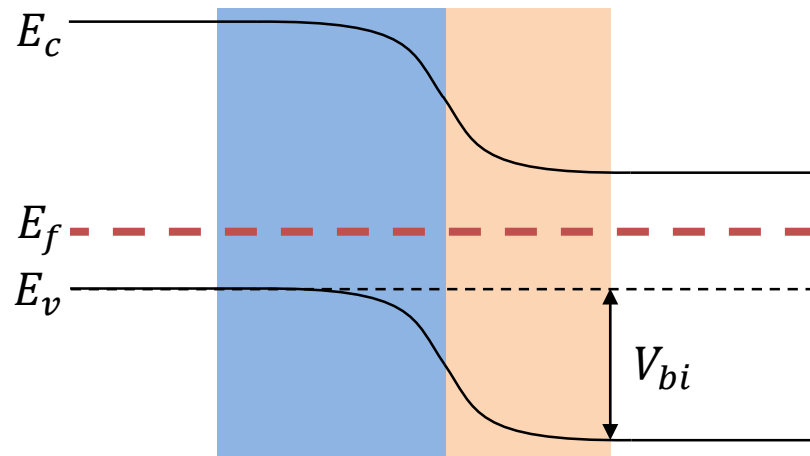
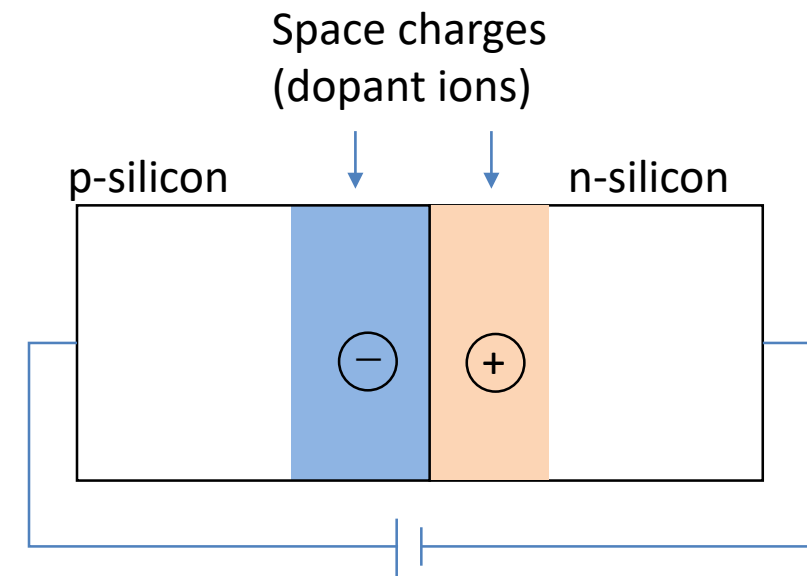




Not many  $e^-$

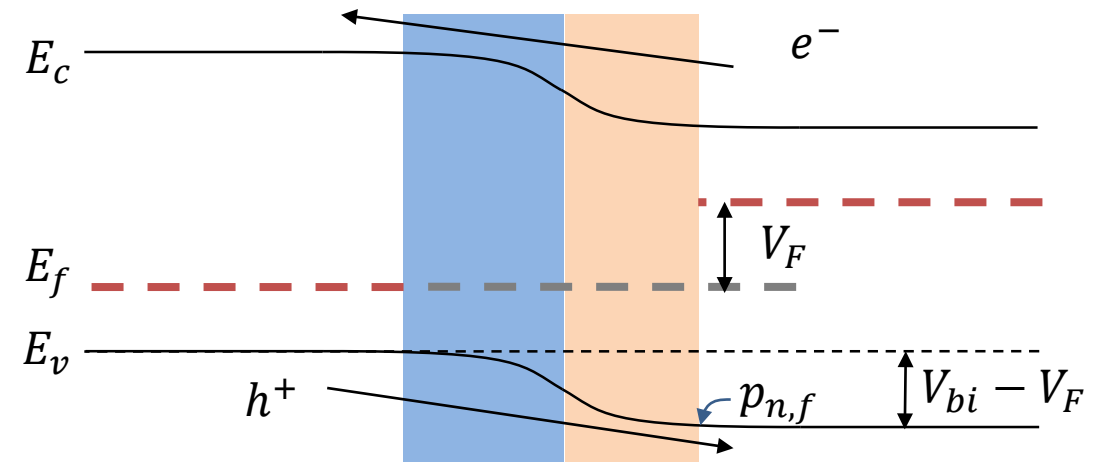
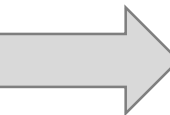


- $V_T = \frac{k_B T}{q}$  thermal voltage
- $N_A, N_D$  acceptor and donor density
- $p_p$  hole density in p side (majority)
- $p_{n,e}$  hole density in n side (minority) in equilibrium
- $p_{n,f}$  hole density in n side (minority) under forward bias



'e' for equilibrium

$$V_{bi} = V_T \ln \frac{N_A N_D}{n_i^2} = V_T \ln \frac{p_p}{p_n} \Rightarrow p_{n,e} = p_p \exp\left(-\frac{V_{bi}}{V_T}\right)$$



'f' for forward bias

$$p_{n,f} = p_p \exp\left(-\frac{V_{bi} - V_F}{V_T}\right)$$

# Minority Carriers in Forward Bias

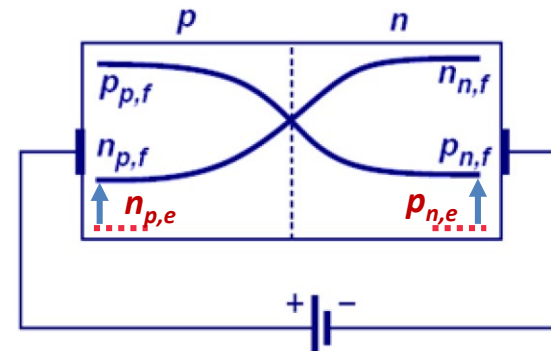
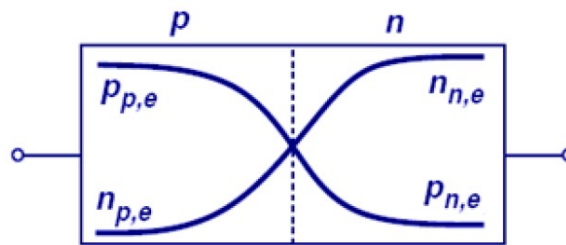
- Minority carriers (holes) on the n-side depletion region boundary under forward bias

$$p_{n,e} = p_p \exp\left(-\frac{V_{bi}}{V_T}\right) \quad \text{'e' for equilibrium}$$

- Potential barrier is lowered by an amount equal to  $V_F$

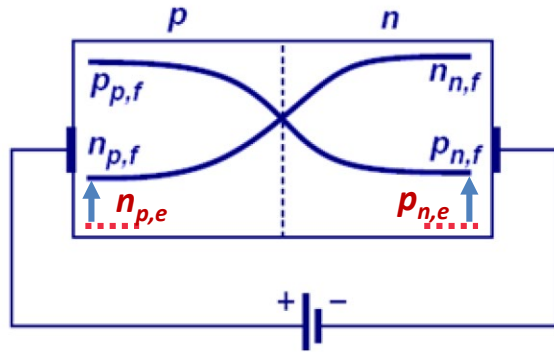
$$p_{n,f} = p_p \exp\left(-\frac{V_{bi} - V_F}{V_T}\right) \quad \text{'f' for forward bias}$$

- The minority carrier concentration rises rapidly while the majority carrier concentration remains relatively constant



# I/V Characteristics in Forward Bias

- Change in the hole concentration (minority carriers) on the  $n$  side

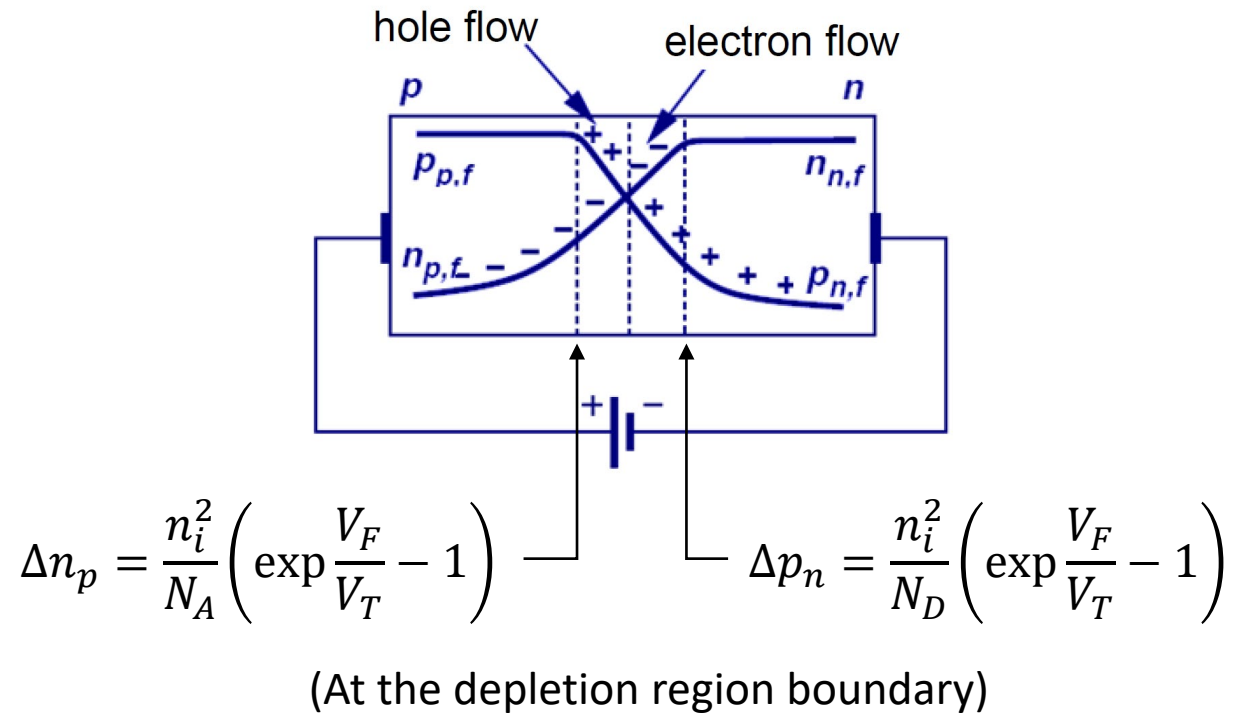


$$\begin{aligned}\Delta p_n &= p_{n,f} - p_{n,e} = p_p e^{-\frac{V_{bi}-V_F}{V_T}} - p_p e^{-\frac{V_{bi}V_F}{V_T}} \\ &= \frac{N_A}{\exp \frac{V_{bi}}{V_T}} \left( \exp \frac{V_F}{V_T} - 1 \right) & p_p &= N_A \\ &= \frac{n_i^2}{N_D} \left( \exp \frac{V_F}{V_T} - 1 \right) & \exp \frac{V_{bi}}{V_T} &= \frac{N_A N_D}{n_i^2}\end{aligned}$$

- Change in the electron concentration (minority carriers) on the  $p$  side

$$\Delta n_p = \frac{n_i^2}{N_A} \left( \exp \frac{V_F}{V_T} - 1 \right)$$

- These changes of minority carrier concentration is calculated at the depletion region boundary



In the bulk, minority carrier concentration gradually returns to the original

Since current is proportional to  
gradient in carrier concentration

$$I_{hole} \propto \frac{n_i^2}{N_D} \left( \exp \frac{V_F}{V_T} - 1 \right), \quad I_{elec} \propto \frac{n_i^2}{N_A} \left( \exp \frac{V_F}{V_T} - 1 \right)$$

# I/V Characteristics in Forward Bias

$$I_{hole} \propto \frac{n_i^2}{N_D} \left( \exp \frac{V_F}{V_T} - 1 \right), \quad I_{elec} \propto \frac{n_i^2}{N_A} \left( \exp \frac{V_F}{V_T} - 1 \right)$$

- It can be proved that  $I_{tot} = I_S \left( \exp \frac{V_F}{V_T} - 1 \right)$  Derivation is not required; need to understand what each term means.

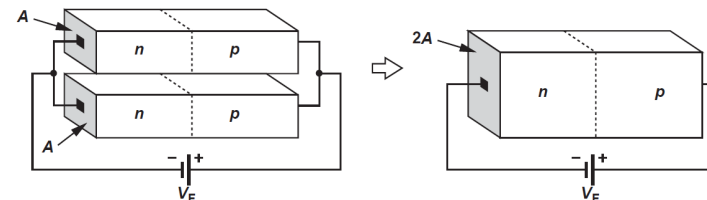
- $I_S$  is called the “reverse saturation current and given by

$$I_S = Aqn_i^2 \left( \frac{D_n}{N_A L_n} + \frac{D_p}{N_D L_p} \right) \quad D_n, D_p: \text{diffusion constants}$$

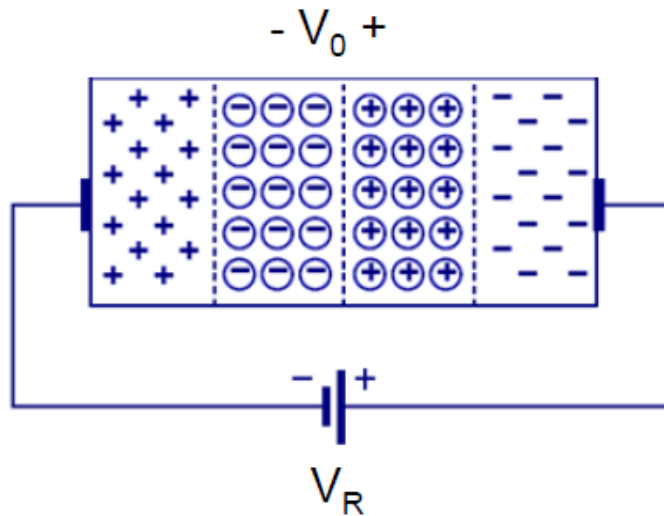
flow of electrons

flow of holes

- $L_{n,p}$  = electron and hole diffusion lengths
  - average length travelled before they recombine with the majority carriers
- $A$  = cross section area of the device



# Reverse saturation current



Reverse saturation current:

$$I_s = Aqn_i^2 \left( \frac{D_n}{N_A L_n} + \frac{D_p}{N_D L_p} \right)$$

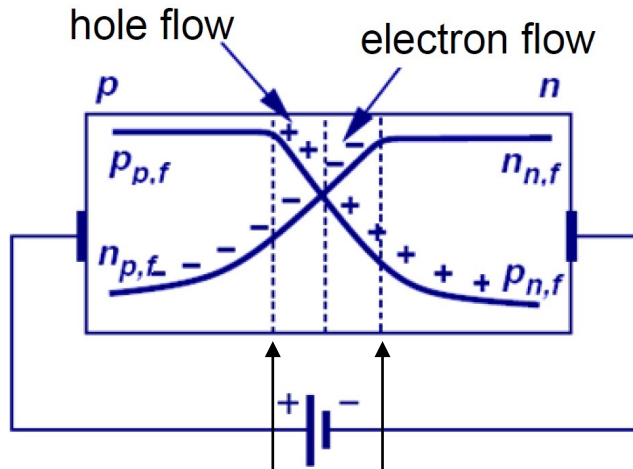
Remember:

$$D_n = \mu_n \frac{U_T}{q}, \quad D_p = \mu_p \frac{U_T}{q}$$

$U_T$  = Thermal Energy

- This reverse saturation current is the **drift current** of thermally generated minority holes and electrons near the junction
  - Does not depend on the potential barrier
  - The reverse saturation current is quite small but it increases with temperature.

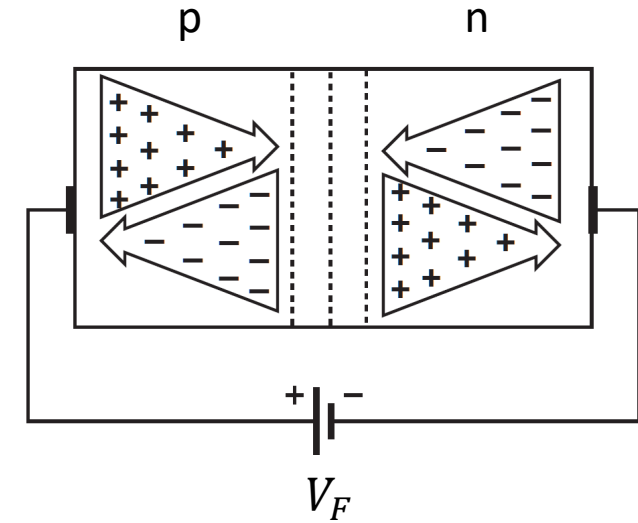




$$\Delta n_p \approx \frac{n_i^2}{N_A} \left( \exp \frac{V_F}{V_T} - 1 \right) \quad \Delta p_n = \frac{n_i^2}{N_D} \left( \exp \frac{V_F}{V_T} - 1 \right)$$

(At the depletion region boundary)

In the bulk, minority carrier concentration gradually returns to the original levels



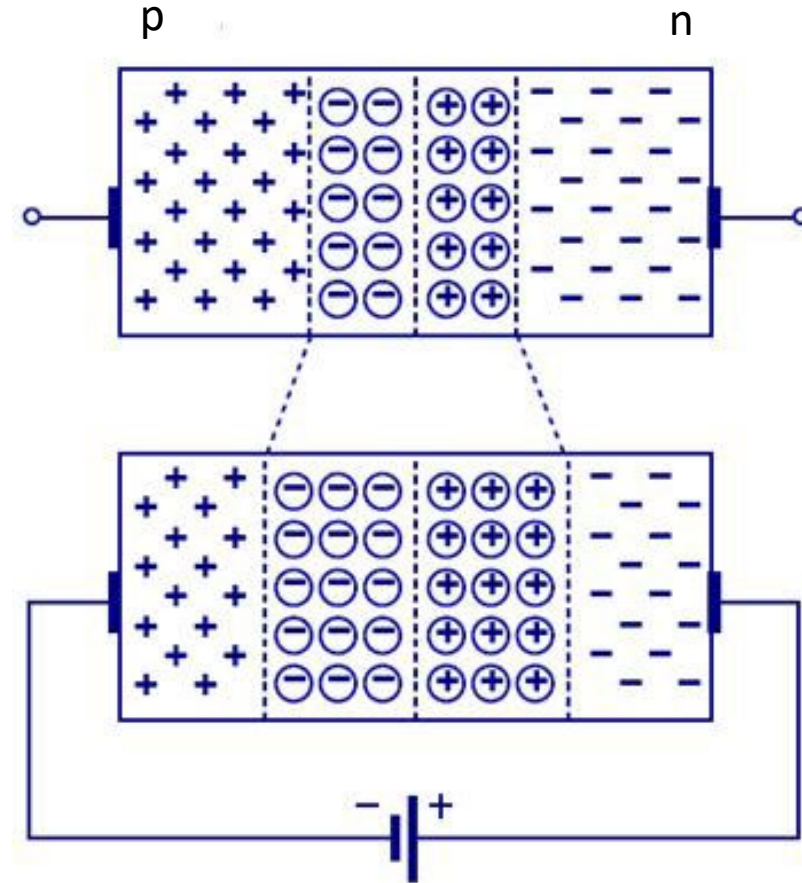
Minority carriers recombine with majority carriers as they diffuse in to the bulk

The majority carrier current and the minority carrier add to the same  $I_{tot}$  at any point

pn junction under reverse bias, capacitance

# PN Junction under Reverse Bias

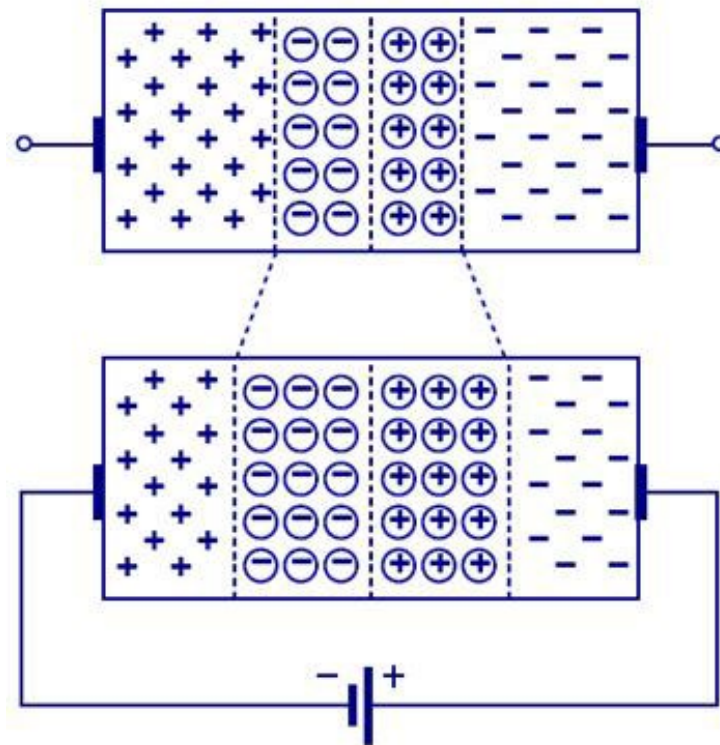
- An external voltage source  $V_R$  is now applied across the junction where the  $n$  side is made more positive than the  $p$  side
- The connection of the positive voltage to the  $n$  terminal is called reverse bias

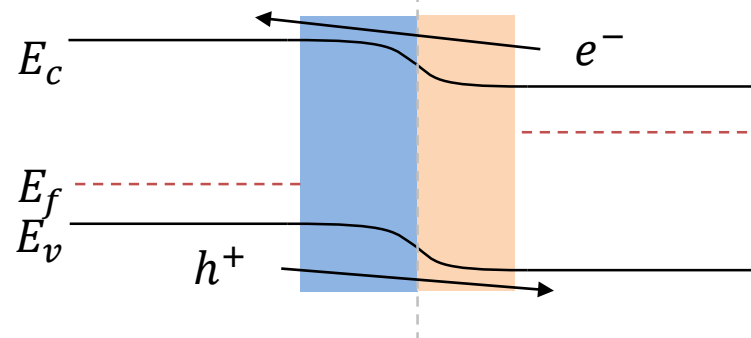
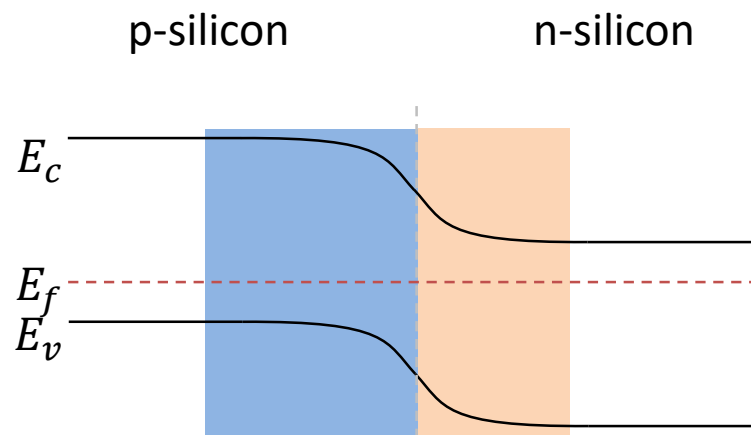
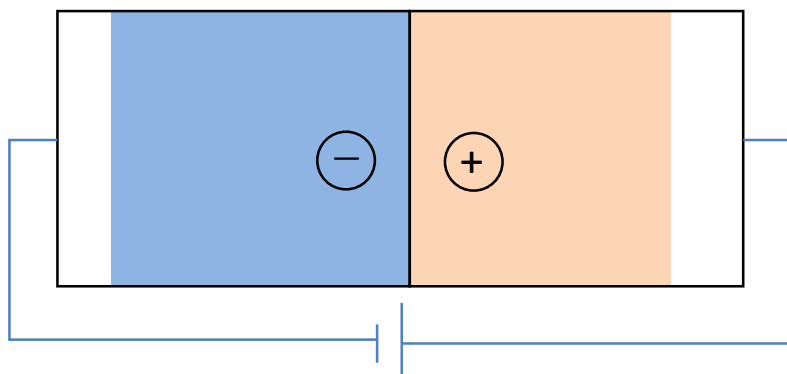
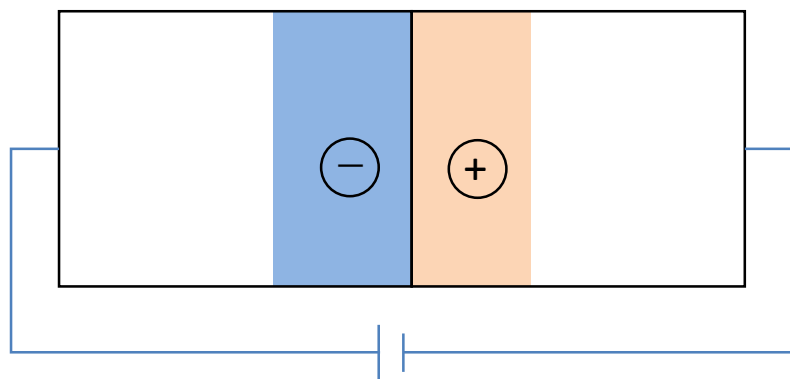
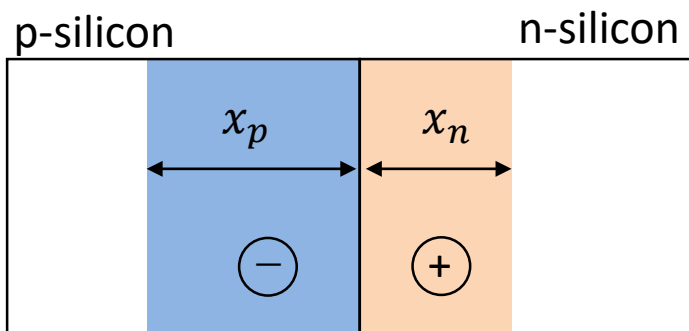


How is this useful?

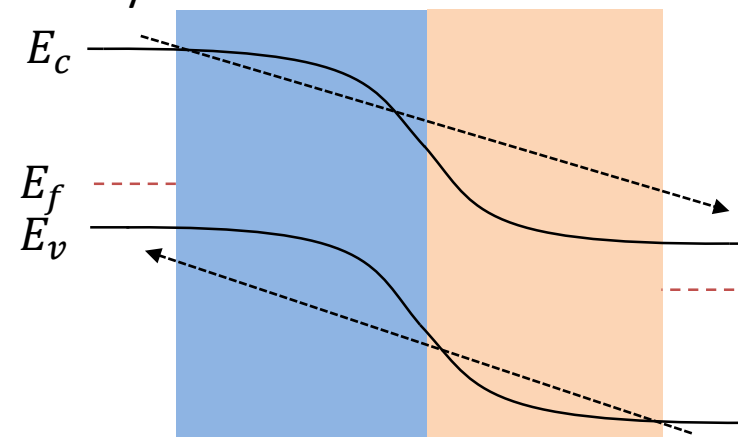
# PN Junction under Reverse Bias

- This reverse bias is magnifying the built-in potential of the diode
- It draws holes away on the p side, electrons away on the n side
- The depletion width is widened and the built-in electric field increased.



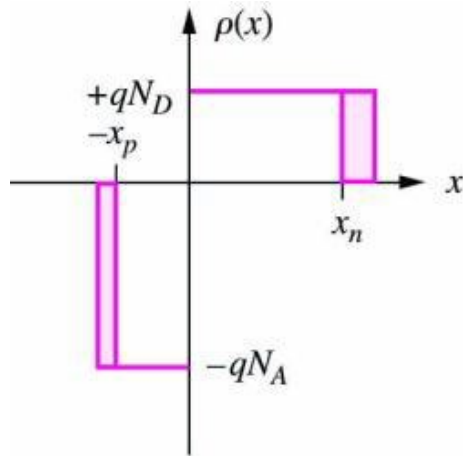


Not many  $e^-$

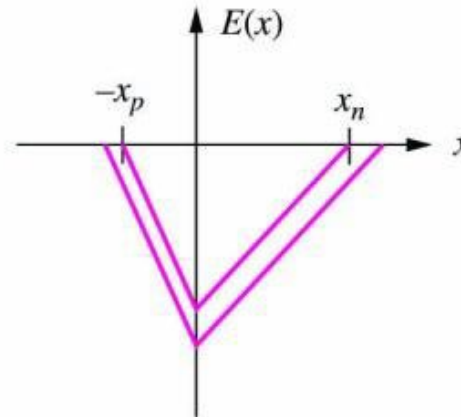


# PN Junction under Reverse Bias

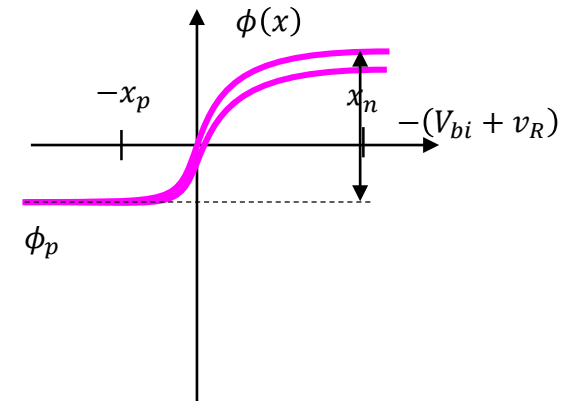
- External reverse bias adds to the built-in potential of the pn junction. The shaded regions below illustrate the increase in the characteristics of the space charge region due to an externally applied reverse bias,  $V_R$



(a) Space charge density



(b) Electric field



(c) Electrostatic potential

# PN Junction under Reverse Bias

$$V_{bi} = \frac{k_B T}{q} \ln \frac{N_A N_D}{n_i^2}$$

- External reverse bias increases the width of the depletion region
- The larger electric field must be supported by additional charge.

$$w_d = x_n + x_p = \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_{bi} + V_R)}$$

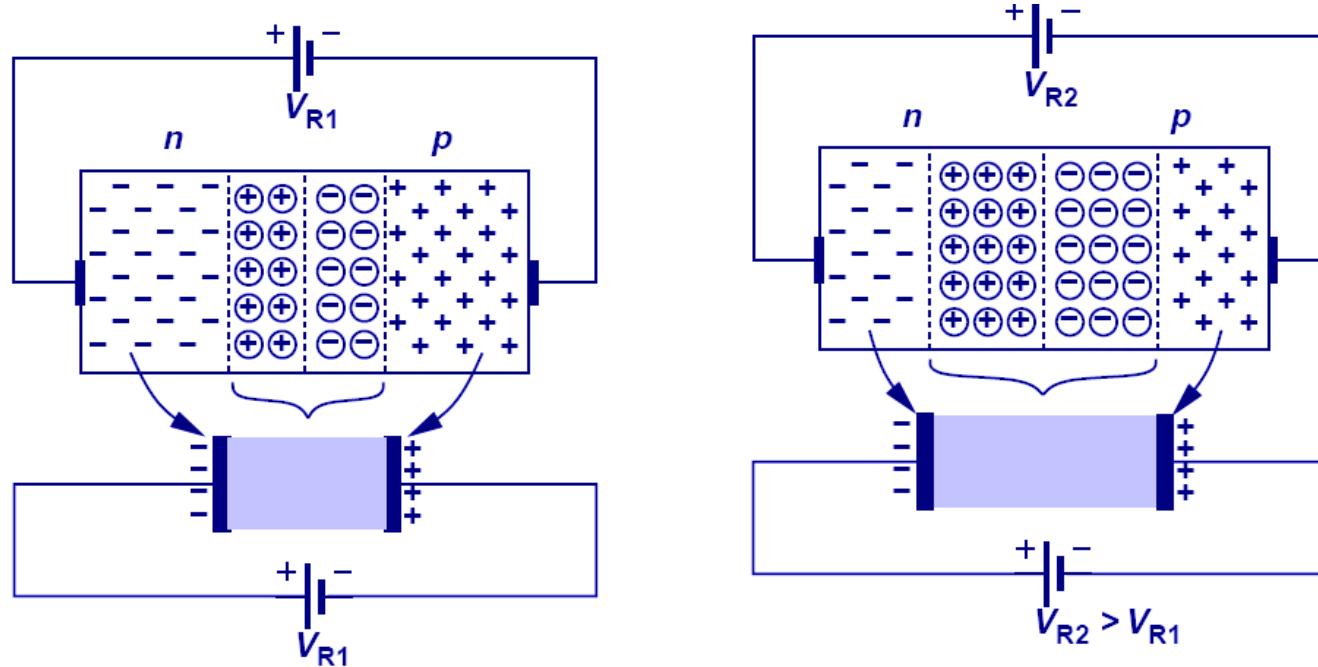
$$w_{d0} = \sqrt{\frac{2\varepsilon_s}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) V_{bi}}$$

$$w_d = w_{d0} \sqrt{1 + \frac{V_R}{V_{bi}}}$$

$w$	depletion region width
$\varepsilon_s$	permittivity of semiconductor
$q$	elementary charge
$N_A, N_D$	acceptor and donor density
$V_{bi}$	built-in potential
$V_R$	external bias
$k_B$	Boltzmann constant
$n_i^2$	intrinsic carrier density

# Junction capacitance

- The  $n$  and  $p$  sections can be viewed as two plates of a capacitor



- As  $V_R$  increases, the width of the depletion region increases
- Capacitance decreases as the two plates move away from each other

$$C = \frac{\epsilon A}{d}$$

$\epsilon$       permittivity  
 $A$       area  
 $d$       width

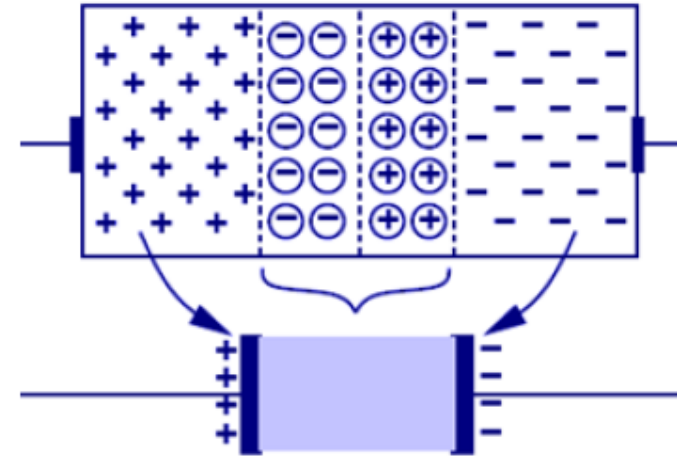


# Junction capacitance

$C_{j0}$  is the zero bias junction capacitance per unit area in equilibrium

$$C_{j0} = \frac{\epsilon_s}{w_{d0}} \text{ F/cm}^2$$

$$= \sqrt{\frac{\epsilon_s q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_{bi}}} \text{ F/cm}^2$$



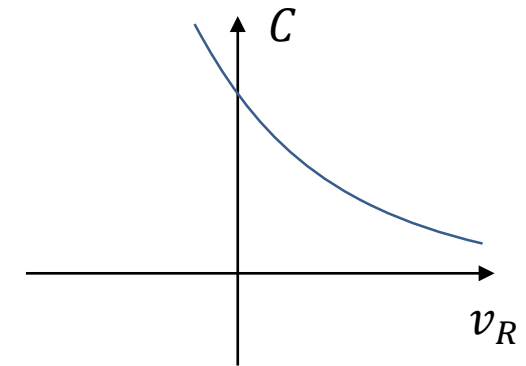
# Junction capacitance

- By varying  $V_R$ , the depletion width changes, changing its capacitance value
- The PN junction is therefore, actually a voltage-dependent capacitor

Capacitance vs. bias voltage:

$$C_j = \frac{\epsilon_s}{w_d} \text{ F/cm}^2$$

$$C_j = \frac{\epsilon_s}{w_{d0} \sqrt{1 + \frac{v_R}{V_{bi}}}} = \frac{C_{j0}}{\sqrt{1 + \frac{v_R}{V_{bi}}}} \text{ F/cm}^2$$



# Junction capacitance

- Capacitance of the junction per unit area (F/cm<sup>2</sup>)

$$C_j = \frac{C_{j0}}{\sqrt{1 + \frac{v_R}{V_{bi}}}} \longleftarrow \text{Important to remember}$$

where  $V_{bi}$  = built-in potential

$v_R$  = reverse bias voltage

$C_{j0}$  = capacitance at zero bias ( $V_R=0$ )

$$C_{j0} = \sqrt{\frac{\epsilon_{Si} q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_{bi}}}$$

where  $\epsilon_{Si}$  = dielectric constant of Si ( $11.7 \times 8.85 \times 10^{-14} \text{ F/cm}^{10}$ )

# Example

A *pn* junction is doped with  $N_A = 2 \times 10^{16} \text{ cm}^{-3}$  and  $N_D = 9 \times 10^{15} \text{ cm}^{-3}$ . Determine the capacitance of the device with (a)  $V_R = 0$  and  $V_R = 1\text{V}$ .

① Obtain the built-in potential

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = 0.73 \text{ V}$$

② For  $V_R = 0$ , solve for  $C_{j0}$

$$\begin{aligned} C_{j0} &= \sqrt{\frac{\epsilon_{si} q}{2} \frac{N_A N_D}{N_A + N_D} \frac{1}{V_{bi}}} \\ &= 2.65 \times 10^{-8} \text{ F/cm}^2 = 0.265 \text{ fF}/\mu\text{m}^2 \end{aligned}$$

③ For  $V_R = 1\text{V}$ ,

$$\begin{aligned} C_j &= \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_{bi}}}} \\ &= 0.172 \text{ fF}/\mu\text{m}^2 \end{aligned}$$

# Diode I-V characteristics

# Diode I-V Characteristics

- PN Junction is a diode
  - For both forward and backward

$$i_D = I_S \left[ \exp\left(\frac{qv_D}{kT}\right) - 1 \right] = I_S \left[ \exp\left(\frac{v_D}{V_T}\right) - 1 \right]$$

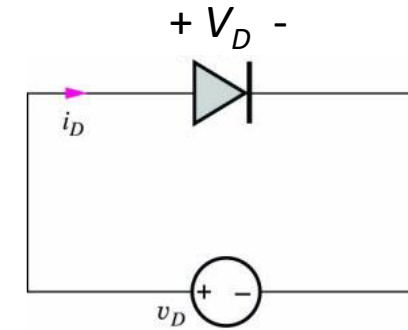
$U_T$  = Thermal Energy

$V_T = \frac{U_T}{q}$  = Thermal Voltage

where

$I_S$	=	reverse saturation current (A)
$v_D$	=	voltage applied to diode (V)
$q$	=	electronic charge ( $1.60 \times 10^{-19}$ C)
$k$	=	Boltzmann's constant ( $1.38 \times 10^{-23}$ J/K)
$T$	=	absolute temperature (Kelvins)
$V_T$	=	$kT/q$ = thermal voltage (V) (25 mV at room temp.)

$I_S$  is typically between  $10^{-18}$  and  $10^{-9}$  A, and is strongly temperature dependent due to its dependence on  $n_i^2$ .



# Diode I-V Characteristics

› Reverse bias:

$$i_D = I_S \left[ \exp\left(\frac{v_D}{V_T}\right) - 1 \right] \cong I_S [0 - 1] \cong -I_S$$

› Zero bias:

$$i_D = I_S \left[ \exp\left(\frac{v_D}{V_T}\right) - 1 \right] \cong I_S [1 - 1] \cong 0$$

› Forward bias:

$$i_D = I_S \left[ \exp\left(\frac{v_D}{V_T}\right) - 1 \right] \cong I_S \exp\left(\frac{v_D}{V_T}\right)$$

# PN Junction Summary

## › Junction equation

$$I_D = I_S \left( \exp \frac{V_D}{V_T} - 1 \right)$$

where  $I_D$  = diode current

$V_D$  = diode voltage

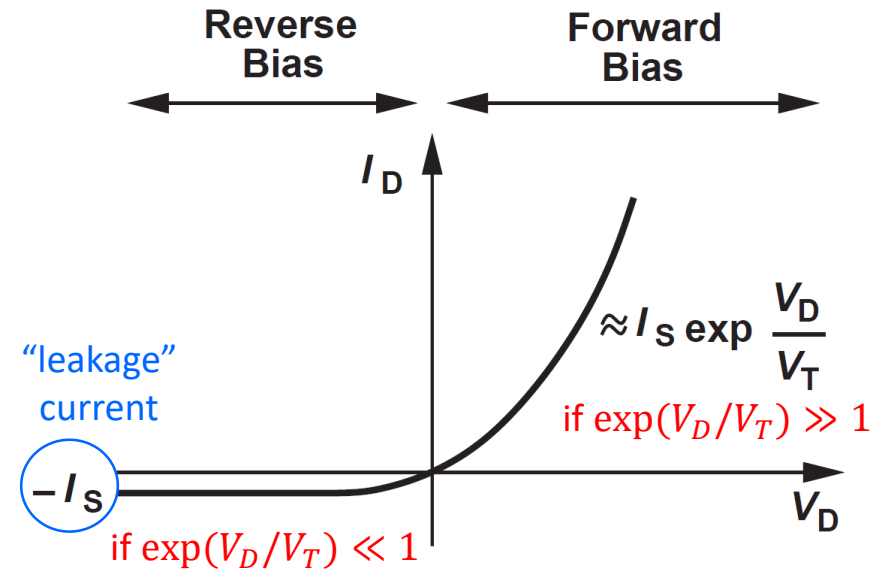
## › Forward bias

- Applied voltage opposes the built-in potential
- Diffusion current is raised substantially

## › Reverse bias

- Applied voltage enhances the field
- Current flow is prohibited

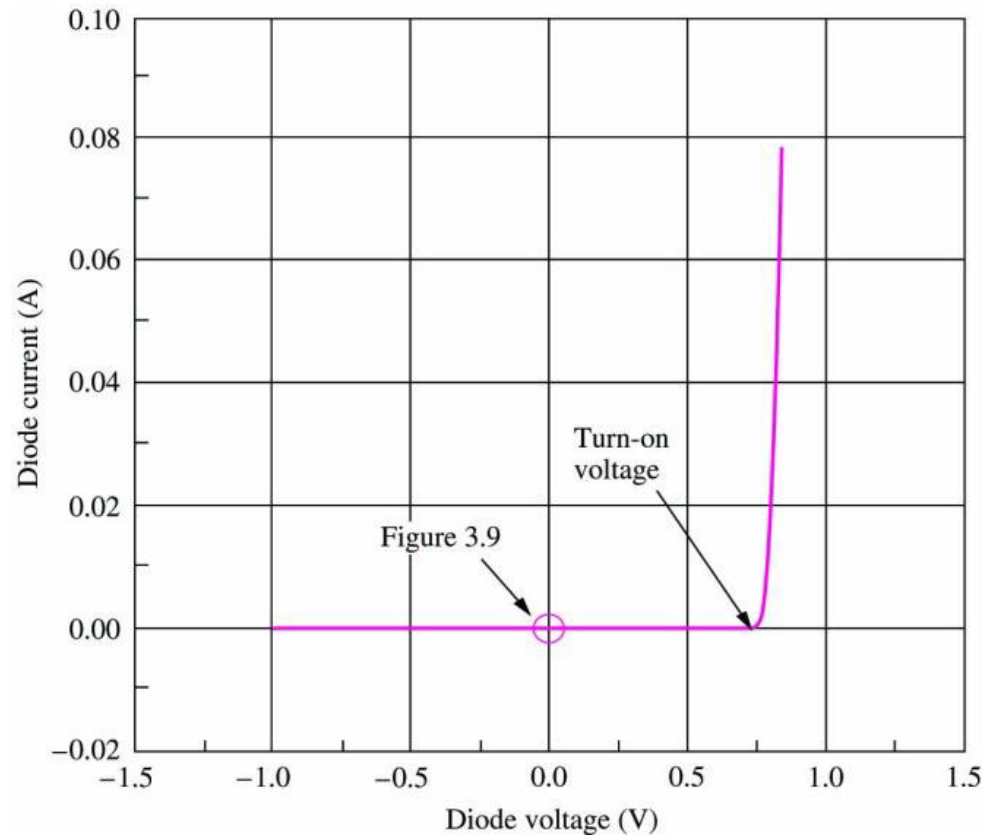
$$I_S = Aqn_i^2 \left( \frac{D_n}{N_A L_n} + \frac{D_p}{N_D L_p} \right)$$





# Diode I-V Characteristics

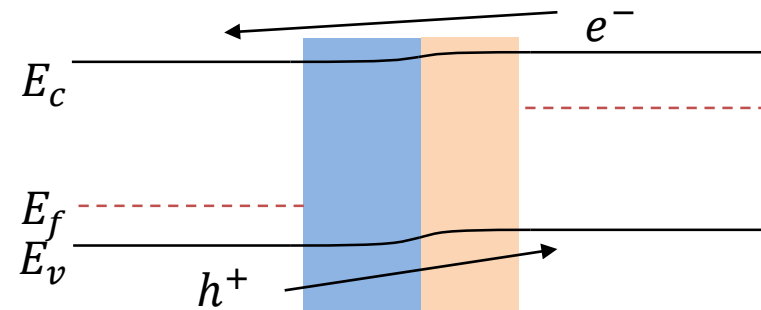
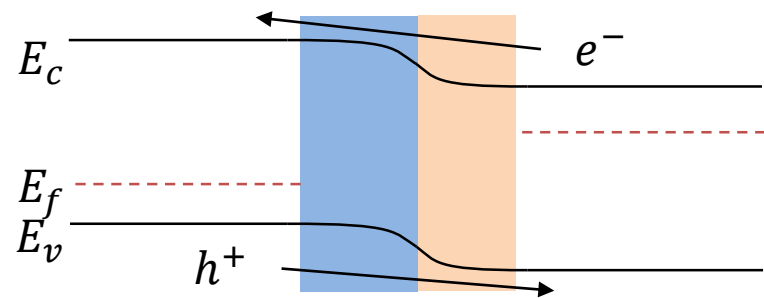
- We can plot the flow of current in response to the applied voltage
  - This is called an I-V curve



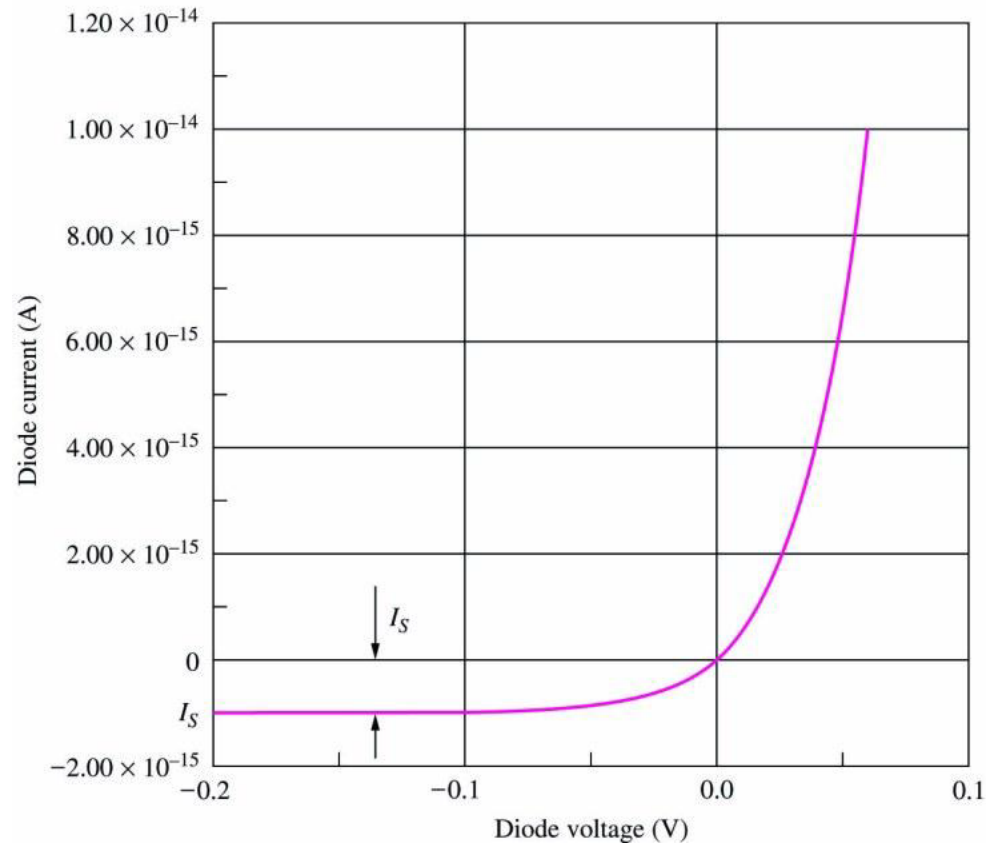
Note that the applied voltage must exceed the built-in potential before significant diffusion current can flow

This is called the **“turn-on voltage”**

# Turn-on

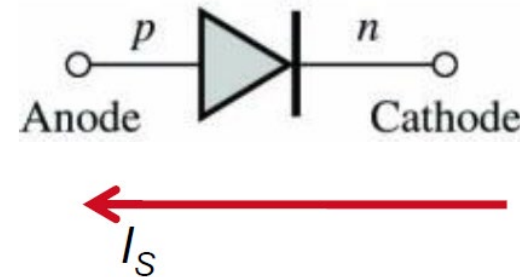


# Diode I-V Characteristics



A very small amount of current  $I_S$  does flow across the diode in the reverse direction when the diode is reverse biased.

This is called the  
**“reverse saturation current”**



# Example

- A diode operates in the forward bias region with a typical current level. Suppose we wish to increase the current by a factor of 10. How much change in diode voltage is required?

① Express diode voltage as a function of its current

$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

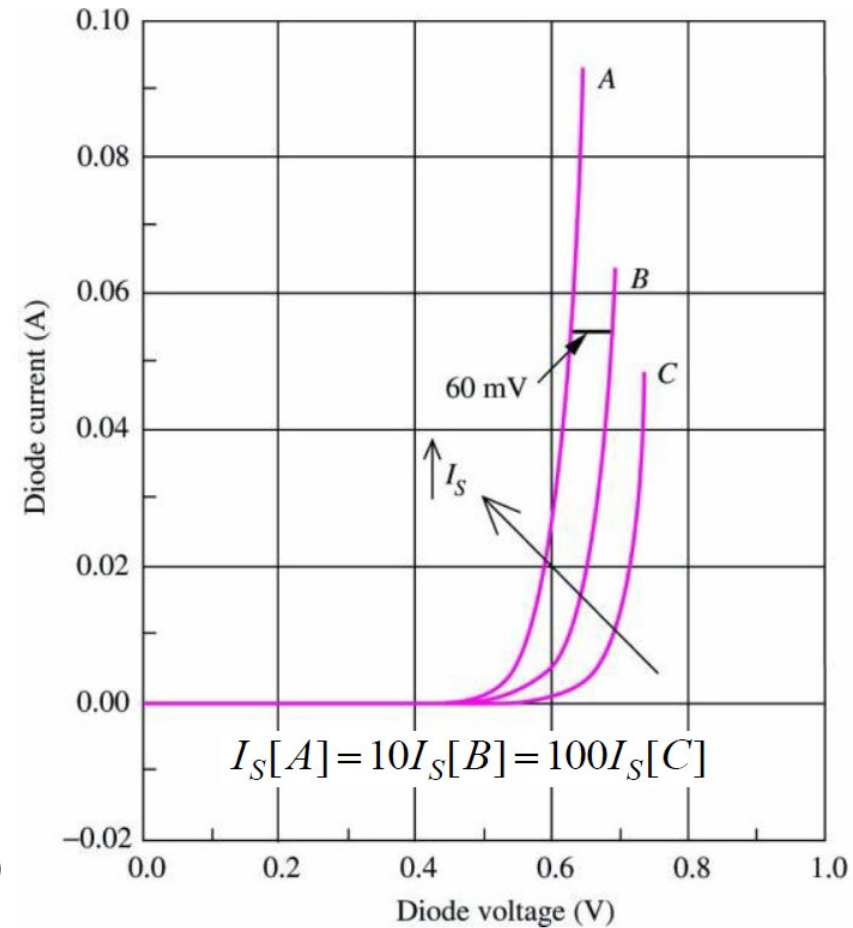
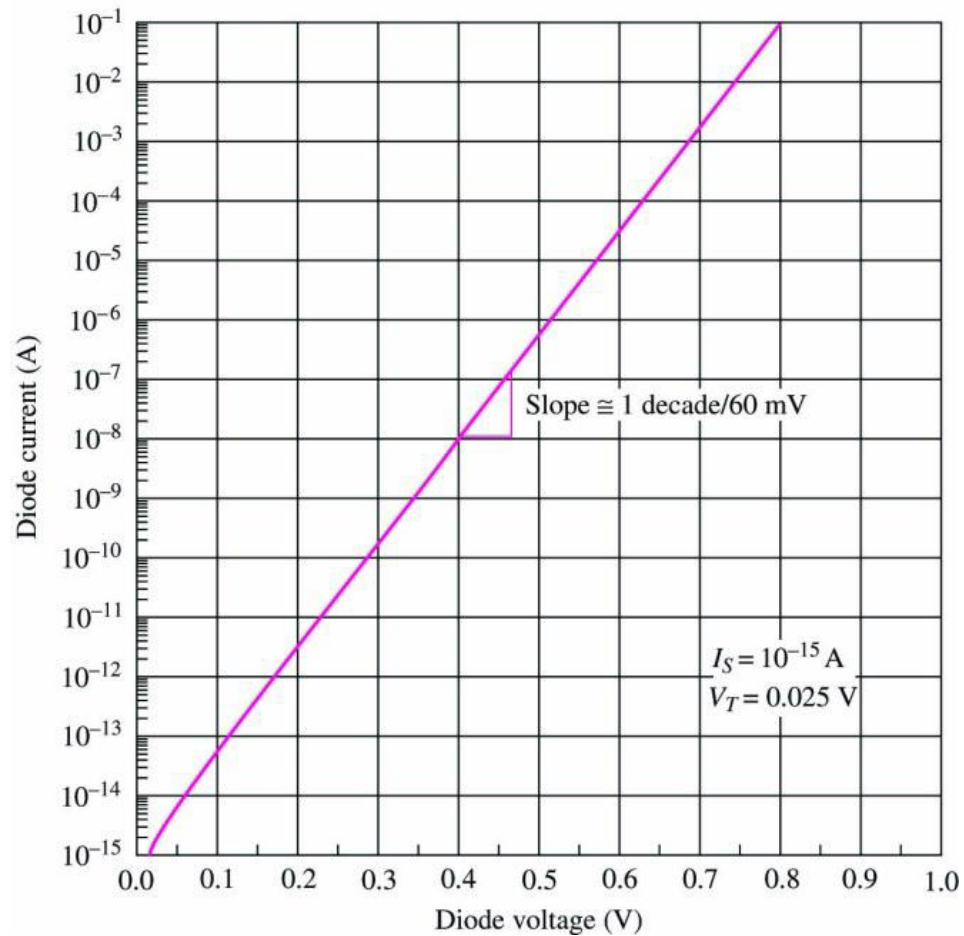
② Define  $I_1 = 10 I_D$

$$\begin{aligned} V_{D1} &= V_T \ln\left(\frac{10I_D}{I_S}\right) \\ &= V_D + V_T \ln(10) \end{aligned}$$

Thus, the diode voltage must rise by  $V_T \ln 10 \approx 60\text{mV}$  (at  $T = 300\text{ K}$ )

# Diode I-V Characteristics

$I_S$  effect on diode current  $i_D = I_S \left[ \exp\left(\frac{v_D}{V_T}\right) - 1 \right] \cong I_S \exp\left(\frac{v_D}{V_T}\right)$



# Example

Find diode voltage for diode with given specifications, assuming the diode is operating at room temperature with  $V_T = 0.025 \text{ V}$

$$(I_S, I_D) = (0.1 \text{ fA}, 300 \text{ }\mu\text{A}), (10 \text{ fA}, 300 \text{ }\mu\text{A}), (0.1 \text{ fA}, 1 \text{ mA})$$

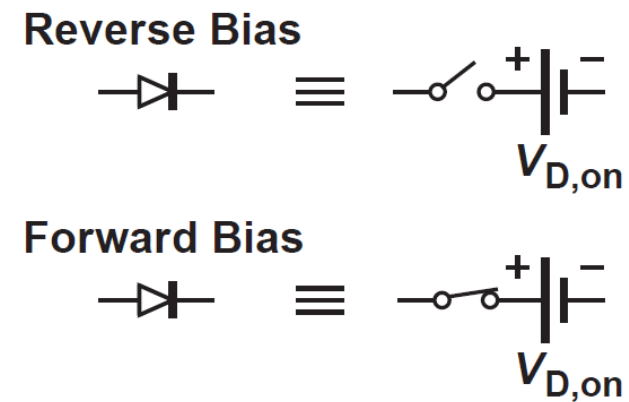
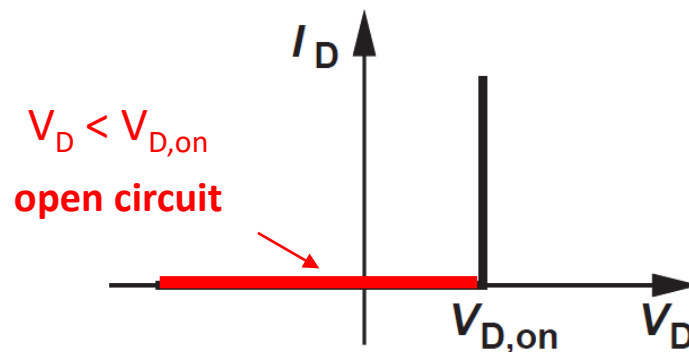
$$\text{With } I_S = 0.1 \text{ fA: } V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 1(0.025\text{V}) \ln\left(1 + \frac{3 \times 10^{-4} \text{ A}}{10^{-16} \text{ A}}\right) = 0.718 \text{ V}$$

$$\text{With } I_S = 10 \text{ fA: } V_D = (0.025\text{V}) \ln\left(1 + \frac{3 \times 10^{-4} \text{ A}}{10^{-14} \text{ A}}\right) = 0.603 \text{ V}$$

$$\text{With } I_S = 0.1 \text{ fA}, I_D = 1 \text{ mA: } V_D = (0.025\text{V}) \ln\left(1 + \frac{10^{-3} \text{ A}}{10^{-16} \text{ A}}\right) = 0.748 \text{ V}$$

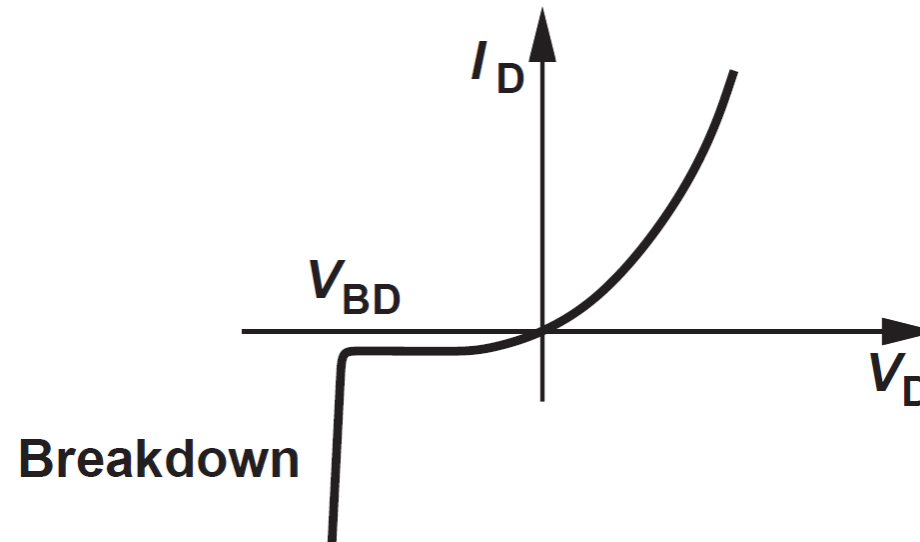
# Constant-Voltage Model

- Nonlinear equation of the exponential I/V diode model makes the analysis of circuits difficult
- Approximate forward bias voltage by a constant value
- Device is considered fully off if below this constant value
- Acts as an ideal voltage source
- $V_D < V_{D,on}$ 
  - open circuit



# Reverse breakdown

- As the reverse bias increases, “breakdown” occurs
- A sudden enormous current is observed



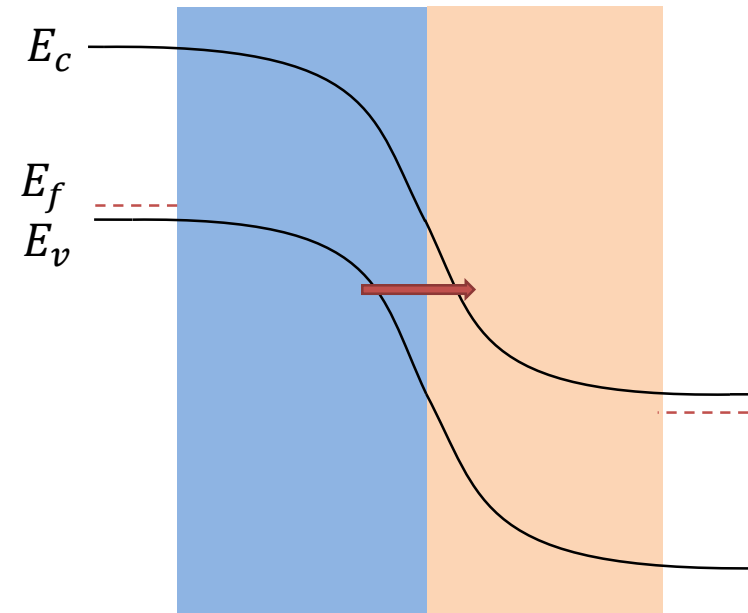
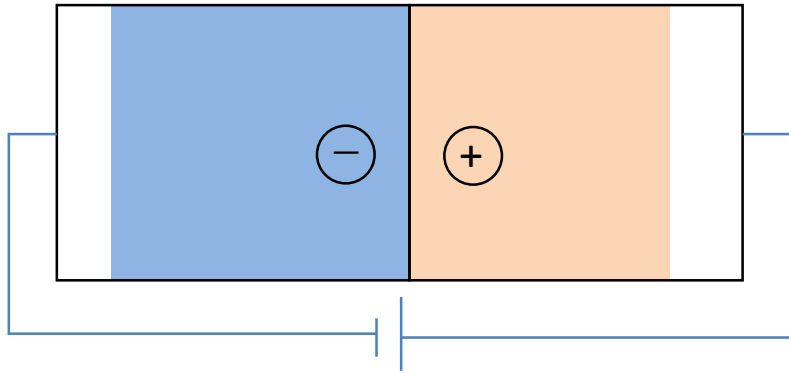
- Breakdown resulting from high voltage (high E-field) can occur in any material
- The voltage at which this occurs is the **breakdown voltage**,  $V_{BD}$



# Zener breakdown

- Occurs in heavily doped diodes
- A high E-field ( $10^6$  V/cm) may impart enough energy to the remaining covalent electrons to tear them from their bonds
- Freed electrons are accelerated by the field and swept to the  $n$  side

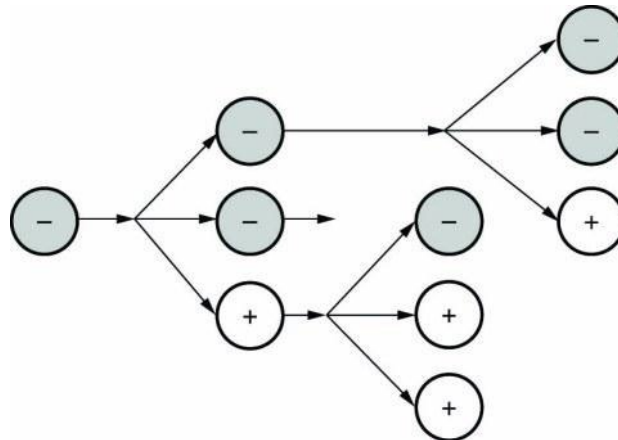
# Zener breakdown



Quantum  
tunnelling

# Avalanche breakdown

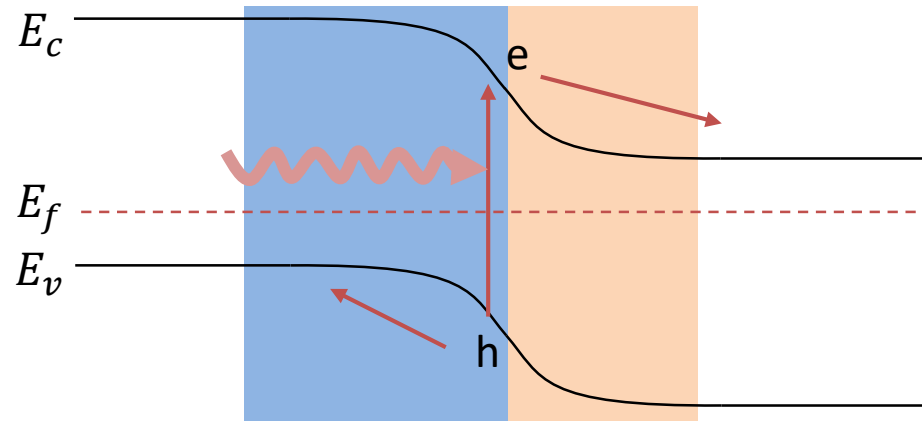
- Each carrier entering the depletion region experiences high E-field
- Large acceleration causes the carriers to gain enough energy to break the electrons from their covalent bonds
  - IMPACT IONISATION
- Each freed electron by the impact may speed itself up so much in the field as to collide with another atom with sufficient energy, thereby freeing one more covalent bond electron
  - More ionising collisions, rapidly raising the number of free carriers
- Application:
  - Avalanche photodetector (APD) for single photons (SAPD) in quantum optics



# Diode optoelectronic devices

# Diode photodetector

- If the depletion region of a  $pn$  junction diode is illuminated with light with sufficiently high frequency, photons can provide enough energy to cause electrons to jump the semiconductor bandgap to generate electron-hole pairs

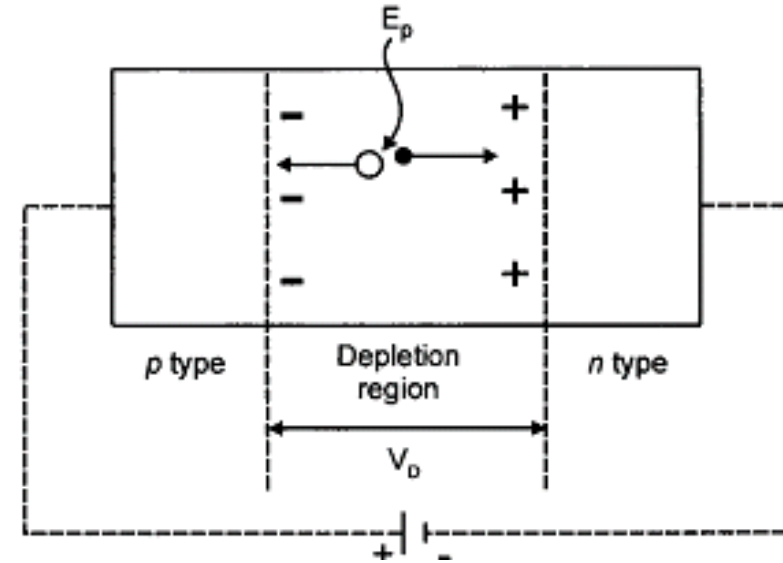
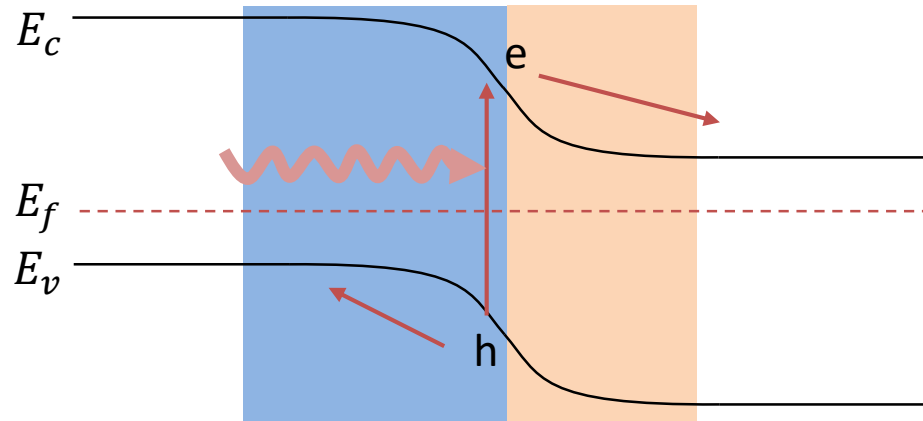


# Diode photodetector

- Light absorption and emission in a semiconductor is heavily dependent on the detailed band structure of the semiconductor.
- Semiconductors for which the minimum of the conduction band occurs at the same wavevector as the maximum of the valence band have a stronger absorption of light (known as a direct band-gap versus an in-direct band-gap: Si has an indirect band-gap).

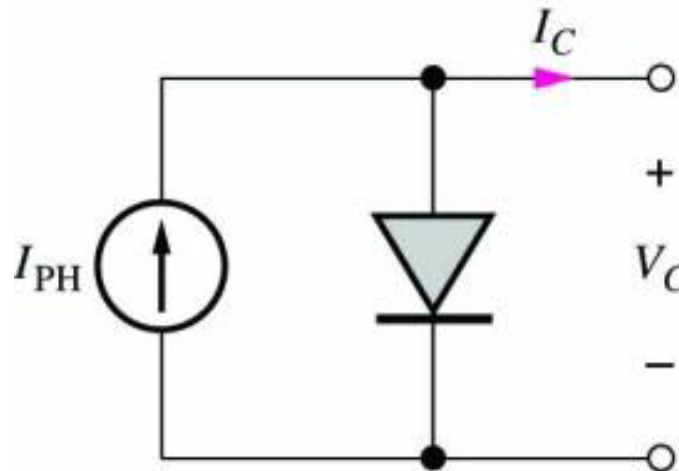
# Light and Diodes

- When electron-hole pairs are generated in the depletion region, this creates a reverse current



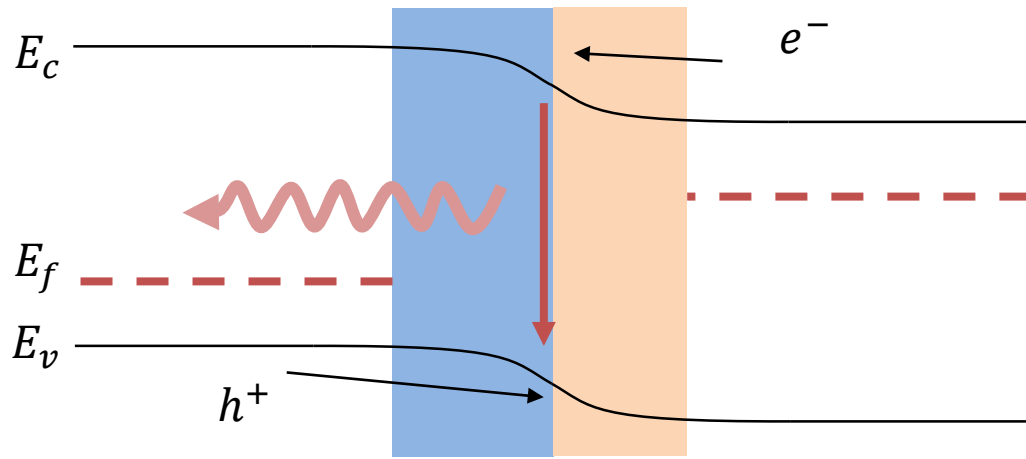
# Photodiodes

- Photodiodes are operated in reverse bias because the hole-electron pairs are generated in the depletion region.
- In photodiode applications, a dc current is generated through the diode
- Photodiodes are used for cameras, remote control receivers, CD players, solar cells, pulse oximeters, and many other applications

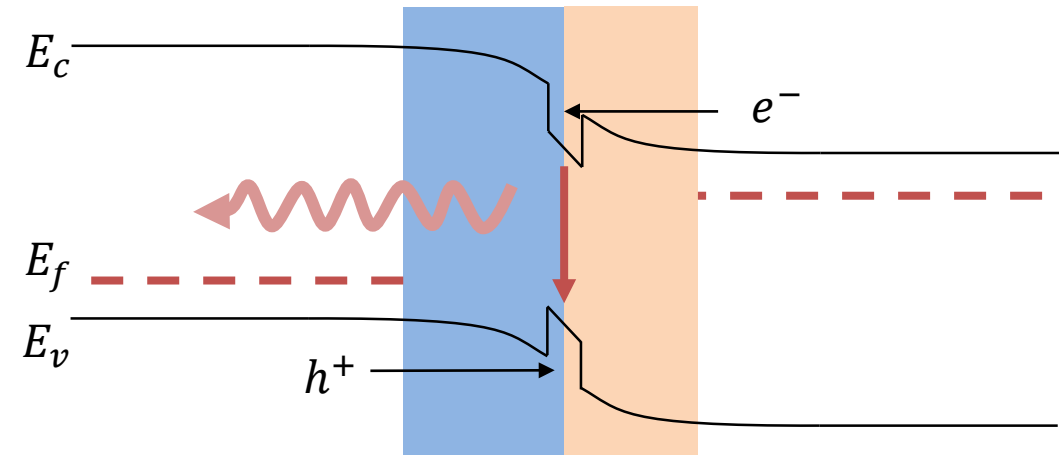




# Light-emitting Diodes



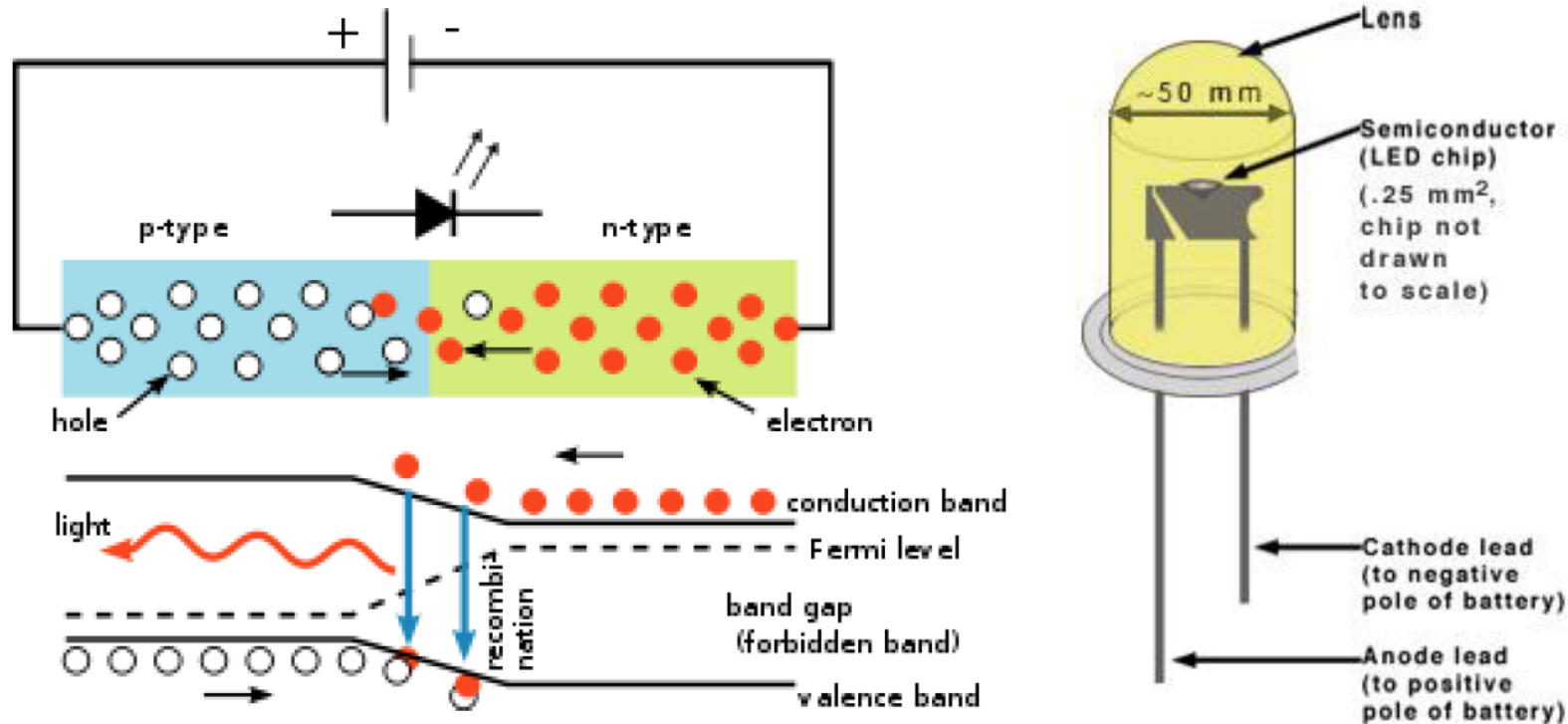
Forward biased pn-junction  
Electrons and holes flow into the junction region  
They can recombine to emit light



Efficiency is much improved by engineering quantum wells  
Nowadays they usually have  $\sim 5$  quantum wells

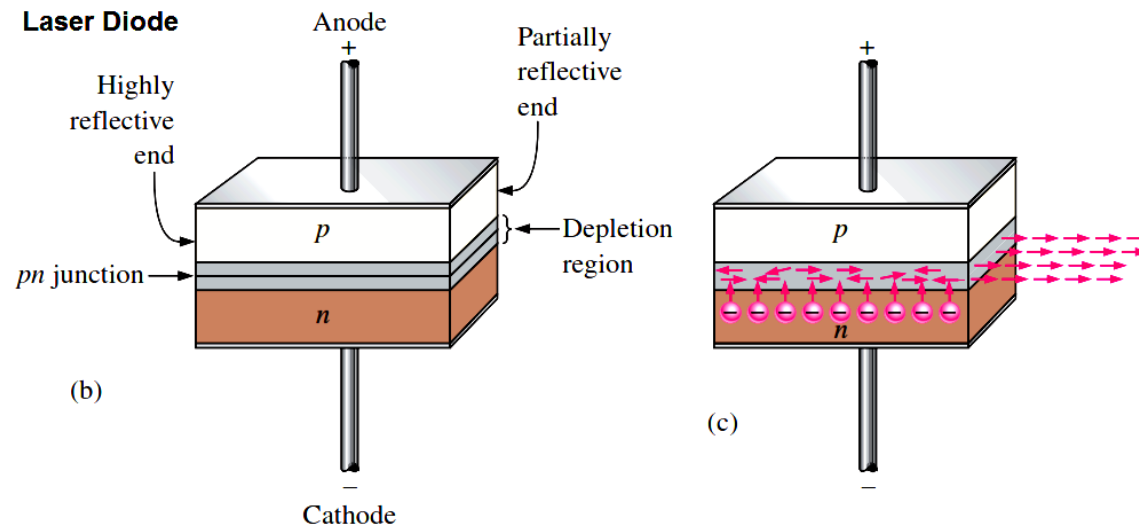
# Light-Emitting Diodes (LEDs)

- Light-Emitting Diodes (LEDs) use recombination processes in the forward-biased  $pn$  junction diode to produce light.
- When a hole and electron recombine, an energy equal to the bandgap of the semiconductor is released as a photon.



# Lasers

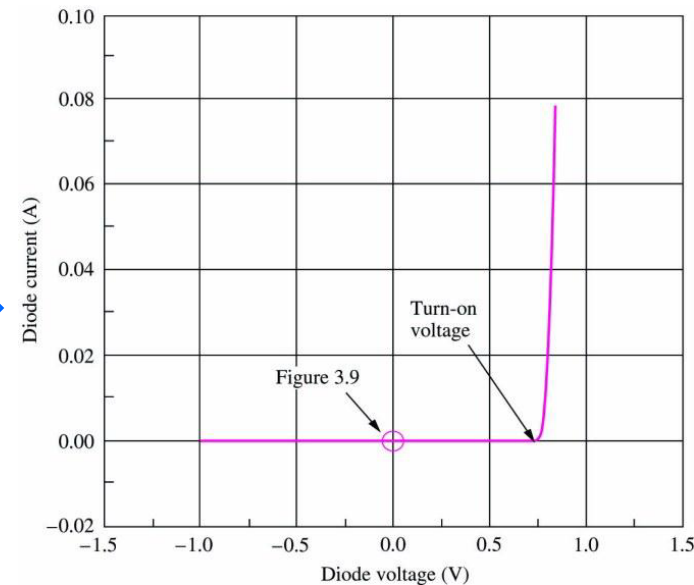
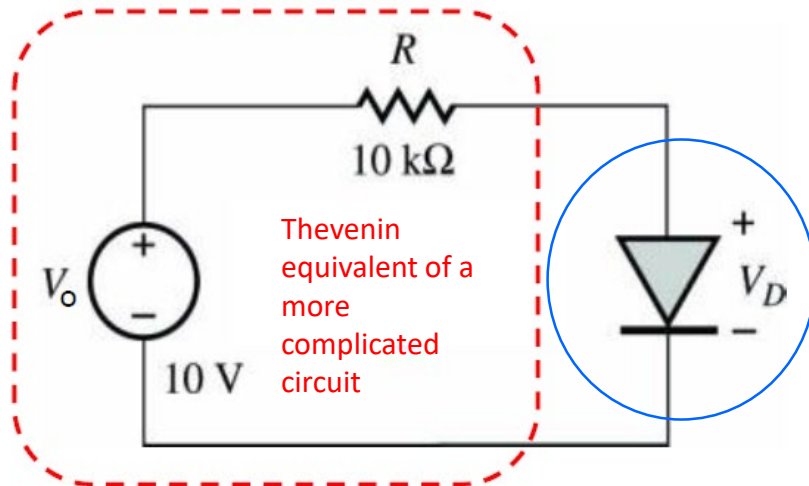
- Laser diodes also consist of a p-n diode with an active region where electrons and holes recombine resulting in light emission.
- However, a laser diode also contains an optical cavity
- The laser cavity consists of a waveguide terminated on each end by a mirror.
- Photons, which are emitted into the waveguide, can travel back and forth in this waveguide provided they are reflected at the mirrors.



# Diode circuit models

# Diode Circuit Analysis

- Consider the following simplified circuit model:



Diode I-V characteristic

- Analyse loop equation:

$$V = I_D R + V_D$$

- Goal - to find the **quiescent operating point (Q-point)** or **bias point** for the diode
- Solving these equations gives us the Q-Point = ( $I_D$ ,  $V_D$ )

# Diode Circuit Analysis

- Several techniques can be used to solve for  $I_D$  and  $V_D$ 
  - Mathematical Analysis using Exponential Model
  - Graphical Analysis using Load Line Approach
  - Ideal Diode Model
  - Constant-Voltage Model

# Mathematical Analysis

- It is difficult to solve exponential equations by hand

$$V_0 = I_D R + V_D \qquad I_D = I_S \left[ \exp \left( \frac{V_D}{V_T} \right) - 1 \right]$$

$$V_0 = I_S \left[ \exp \left( \frac{V_D}{V_T} \right) - 1 \right] R + V_D$$

- We now need to find the zero point solution of the function

$$f = V_0 - I_S \left[ \exp \left( \frac{V_D}{V_T} \right) - 1 \right] R - V_D$$

- A numerical answer can be found by using Newton's iterative method

# Newton's Iterative Method

- Iterative solutions

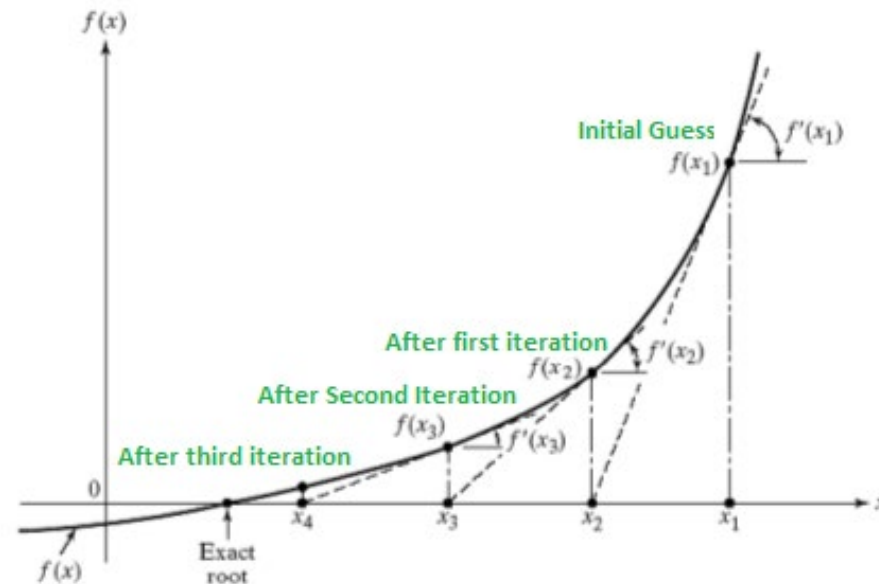
$$f = V_0 - I_S \left[ \exp\left(\frac{V_D}{V_T}\right) - 1 \right] R - V_D$$

## Steps

1. Make an initial guess  $V_D^0$
2. Evaluate  $f$  and its derivative  $f'$  for this value of  $V_D$
3. Calculate new guess for  $V_D$  using

$$V_D^1 = V_D^0 - \frac{f(V_D^0)}{f'(V_D^0)}$$

4. Repeat steps 2 and 3 till convergence



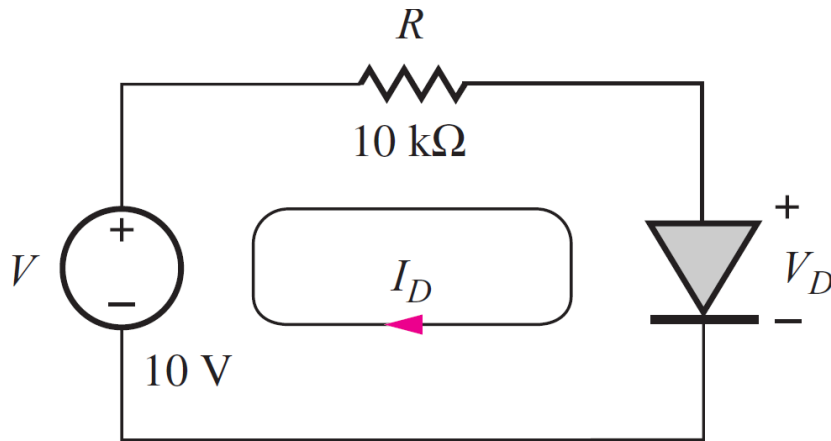
- Numerically compute using MATLAB to make things easier



# Load-line Analysis

- When the I-V characteristics is given in graphical form
  - Use load-line analysis (graphical approach) to solve

$$V = I_D R + V_D \quad (\text{load line for the diode})$$



## Step 1:

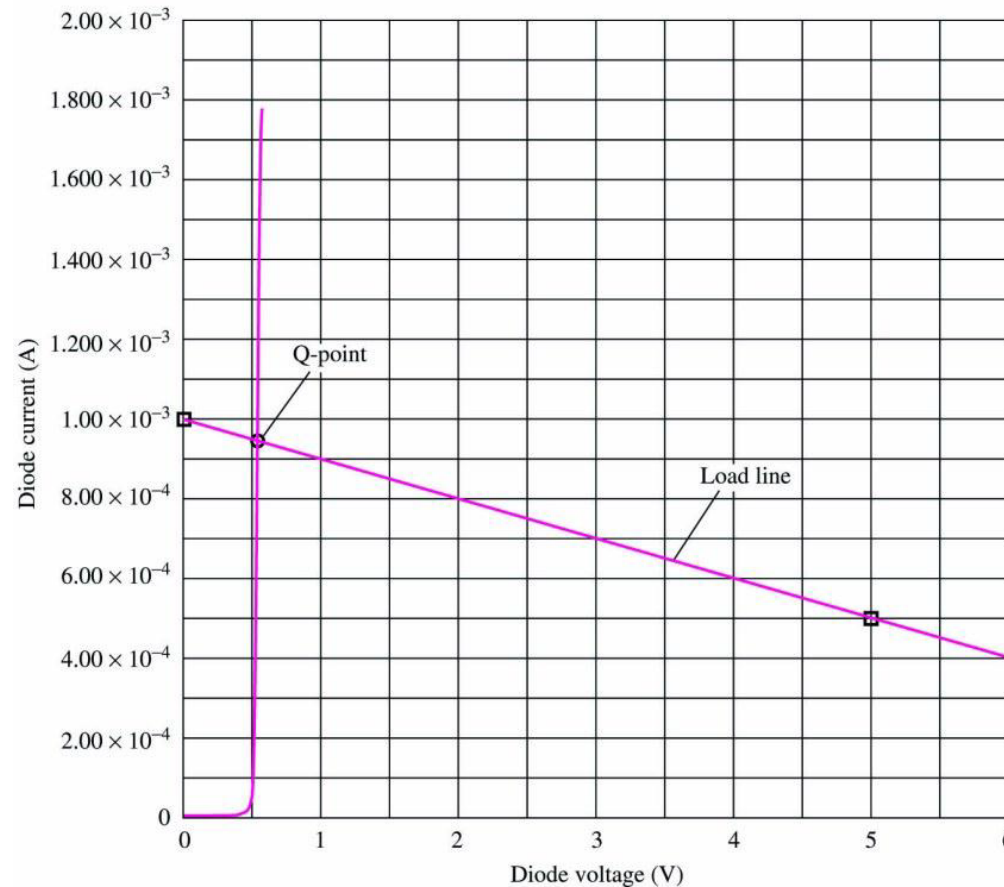
Transform circuit to a Thévenin equivalent with the diode as the output load

## Step 2:

Write the load line equation for  $V_D$  in terms of  $I_D$

$$V_D = V - I_D R$$

# Load-line Analysis



**Q-point = (0.95 mA, 0.6V)**

## Step 3:

Find 2 points to plot the load line

## Step 4:

Plot the diode I-V curve

$$I_D = I_S \left[ \exp \left( \frac{V_D}{V_T} \right) - 1 \right]$$

## Step 5:

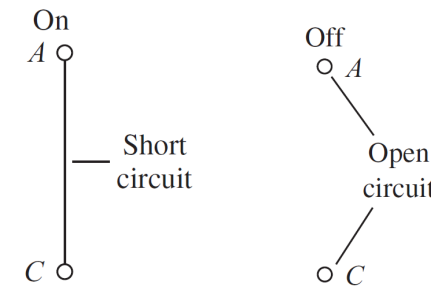
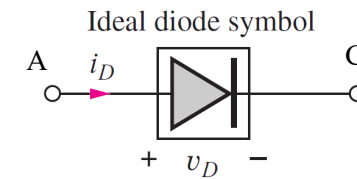
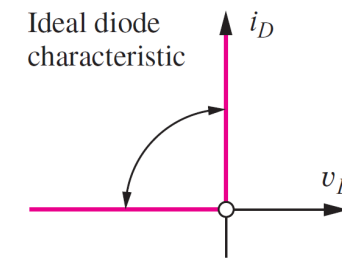
The Q-point is the intersection of the load line and I-V curve

# Ideal Diode Model

- Significant tolerances exist for sources and passive components.
  - We need answers precise to only 2 or 3 significant digits
- Many circuits can be analyzed using a simpler model
- We can simplify the model further to just a switch

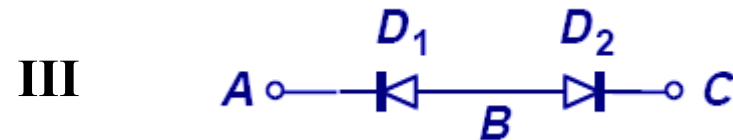
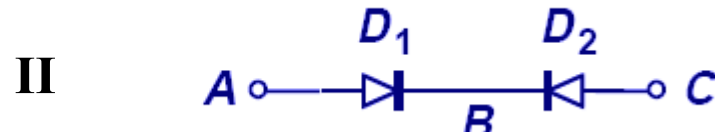
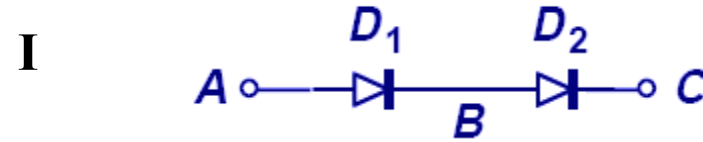
# Ideal diode model

- The  $I$ - $V$  characteristic for the **ideal diode** consists of two straight-line segments
- Forward biased
  - Positive current,  $V_D$  is zero
$$V_D = 0 \text{ for } I_D > 0$$
- Reverse biased
  - $V_D < 0$ , then the current  $I_D$  is zero
$$I_D = 0 \text{ for } V_D \leq 0$$
- Two states
  - Conducting – ON – short circuit
  - Non-conducting – OFF – open circuit



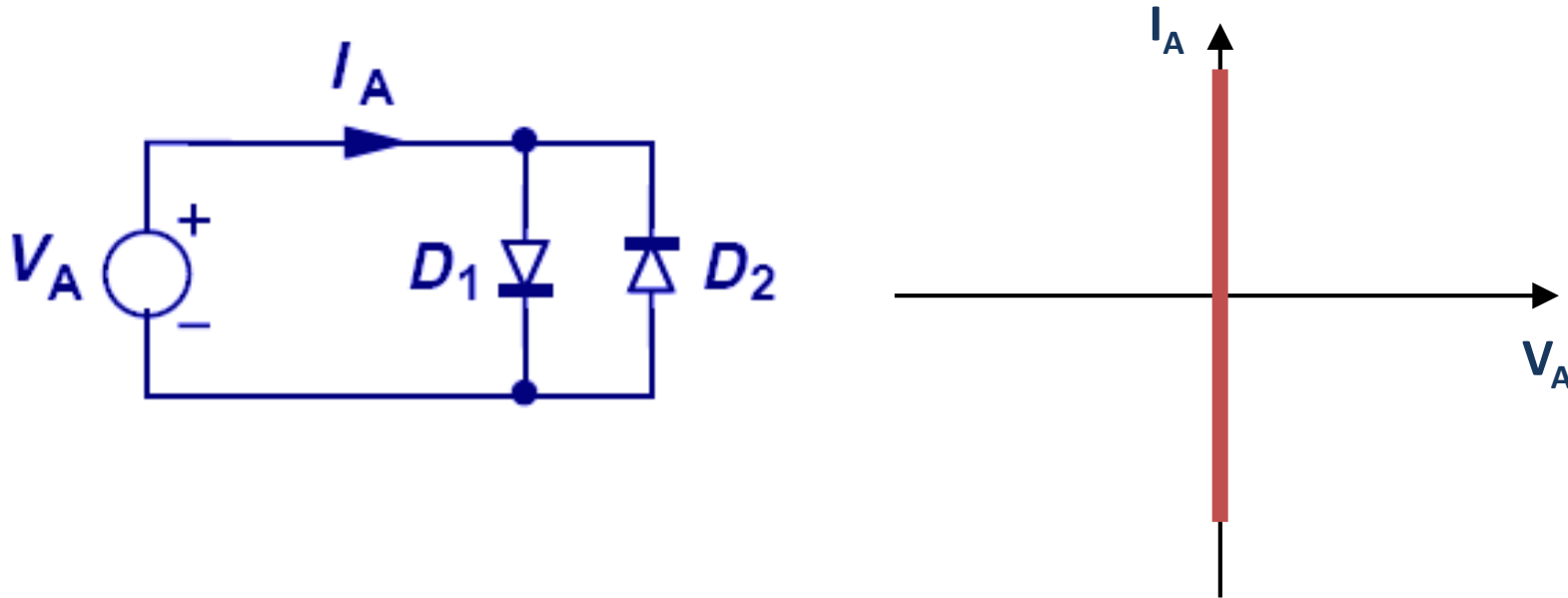
# Exercise

- Diodes can be placed in series (or in parallel). Determine which one of the configurations in Fig. 3.4 can conduct current



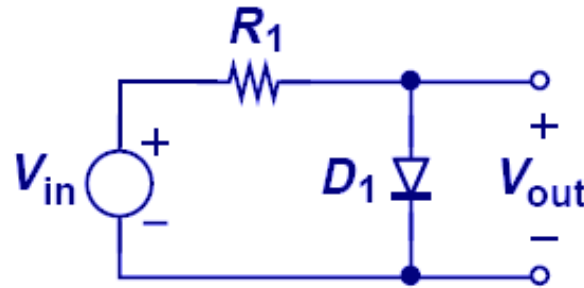
# Exercise

- Plot the I/V characteristic for the following diode configuration

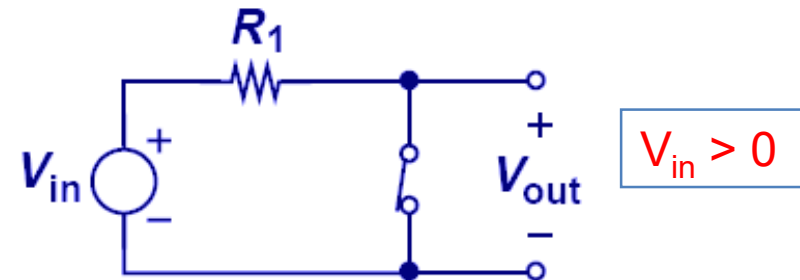
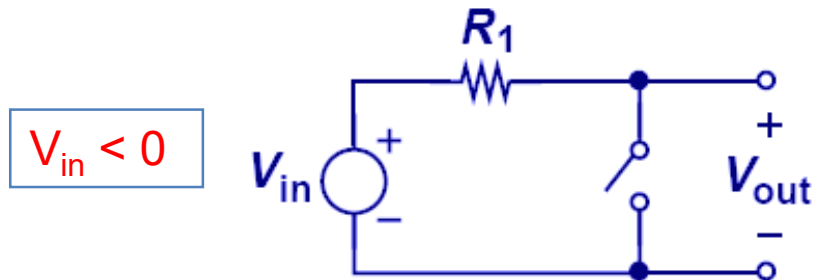


# Ideal Diodes in Resistor Circuits

- The ideal diode model assumes zero voltage drop across the diode when it is on, and zero current through the diode when it is off

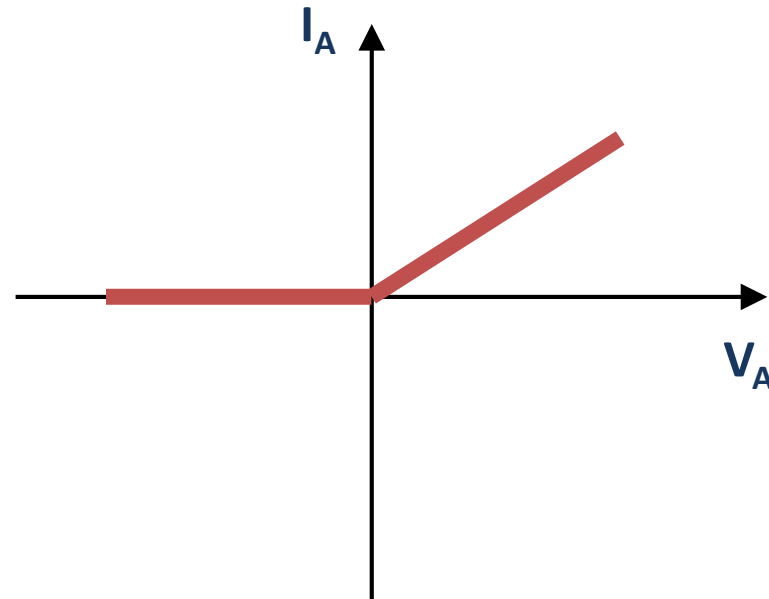
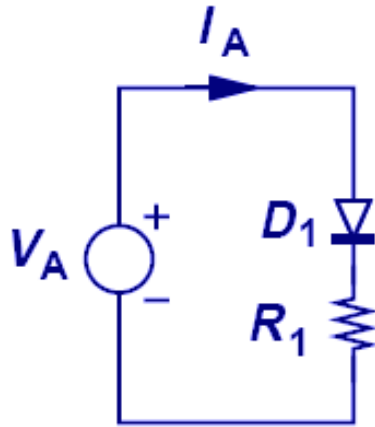


- When  $V_{in}$  is less than zero, the diode opens, so  $V_{out} = V_{in}$
- When  $V_{in}$  is greater than zero, the diode shorts, so  $V_{out} = 0$



# Exercise

- Plot the I/V characteristic for the following diode-resistor configuration





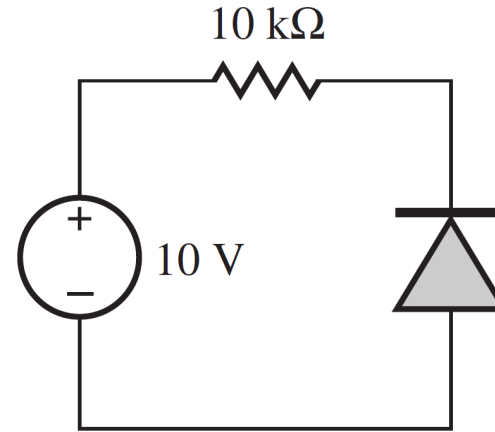
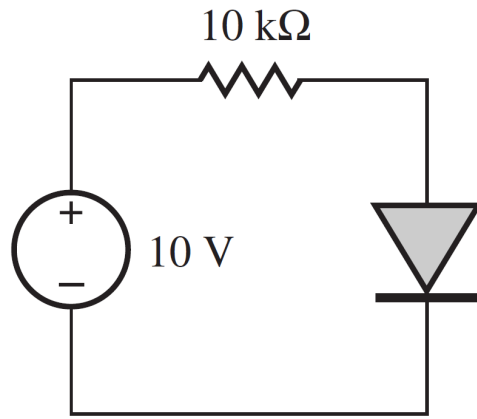
# Analysis using Ideal Diode Model

## Steps

1. Identify anode and cathode of the diode and label  $V_D$  and  $I_D$
2. Guess diode's region of operation from circuit.
  - Which diode is forward or reverse biased?
3. Analyze circuit using ideal diode model to verify assumed region of operation.
  - Redraw the circuit diagram based on assumption
4. Check results to check consistency with assumptions.
  - Does your assumption makes sense? If not, go back to Step 2.
5. Calculate Q-point using the constant-voltage or exponential diode models.

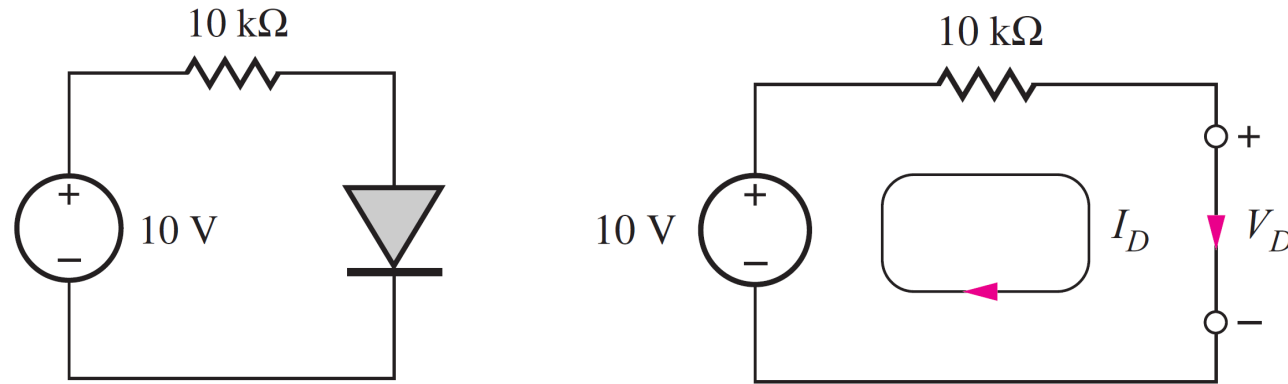
# Example

- Find the Q-point for the following circuits by assuming an ideal diode model.



# Example

- Find the Q-point for the following circuits by assuming an ideal diode model.



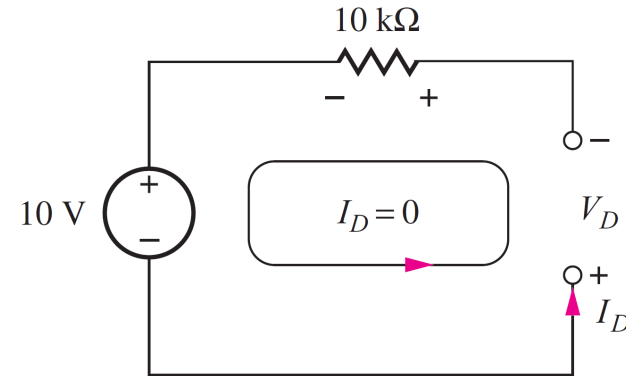
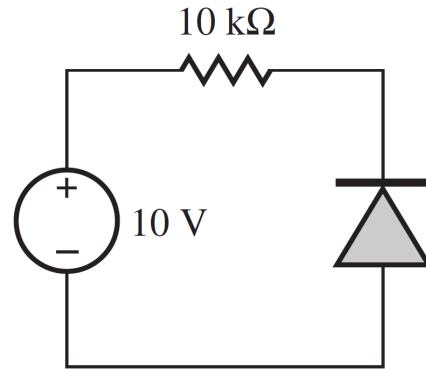
- Since source appears to force positive current through diode, assume diode is on

$$I_D = \frac{(10 - 0)V}{10k\Omega} = 1mA$$

- Because  $I_D$  is greater than 0, the assumption was correct
- Q-point = (1mA, 0V)

# Example

- Find the Q-point for the following circuits by assuming an ideal diode model.



- Since source is forcing current backward through diode, assume diode is off

$$10 + V_D + 10^4 I_D = 0$$

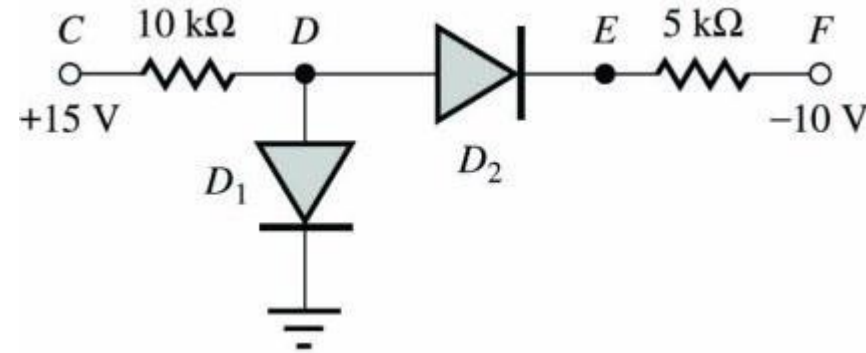
$$V_D = -10V$$

- Because  $V_D < 0$ , the assumption was correct
- Q-point = (0, -10V)

# Two-Diode Analysis

- The ideal diode model can be used to analyze circuits with more than one diode

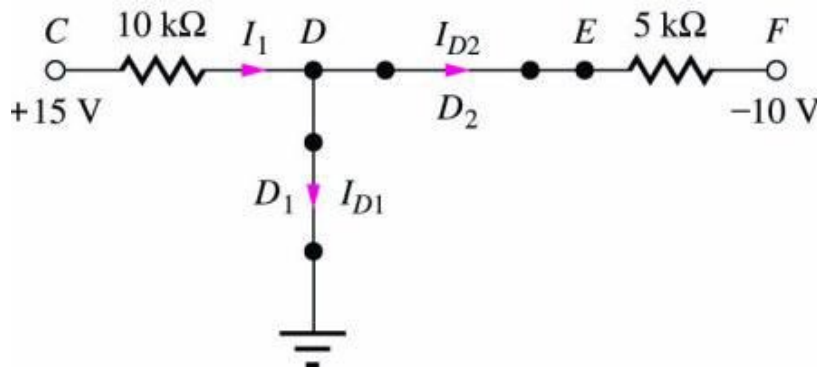
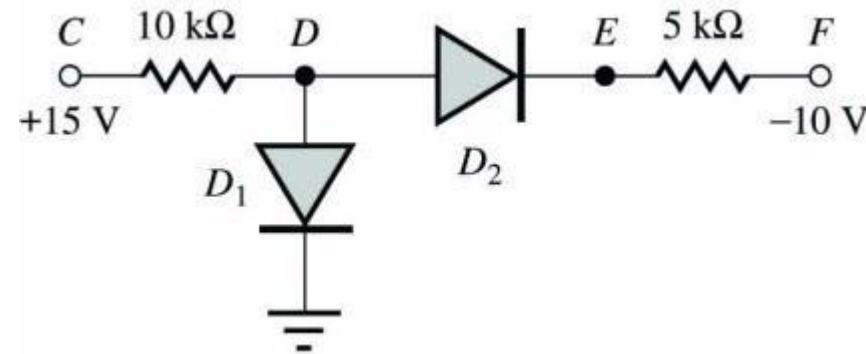
**Example:** The 15-V source appears to force positive current through  $D_1$  and  $D_2$ , and the -10-V source is forcing positive current through  $D_2$ . Assume both diodes are on.



# Two-Diode Analysis

- The ideal diode model can be used to analyze circuits with more than one diode

**Example:** The 15-V source appears to force positive current through  $D_1$  and  $D_2$ , and the -10-V source is forcing positive current through  $D_2$ . Assume both diodes are on.



$$I_1 = I_{D1} + I_{D2}$$

$$I_1 = \frac{(15 - 0)V}{10k\Omega} = 1.50 \text{ mA}$$

$$I_{D2} = \frac{0 - (-10V)}{5k\Omega} = 2.00 \text{ mA}$$

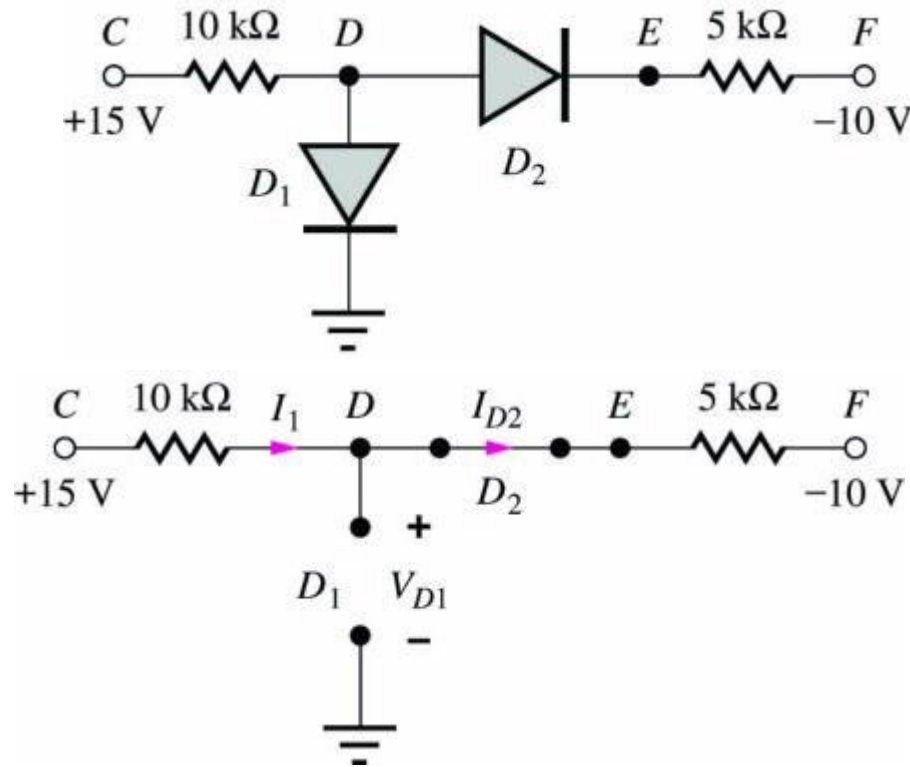
$$I_{D1} = 1.50 - 2.00 = -0.500 \text{ mA}$$



# Two-diode Analysis

**Let's try again!**

$I_{D1} < 0$  is not allowed. Thus, assume  $D1$  off and  $D2$  on.



Then,  $I_{D2} = I_1$ .

$$15 - 10,000I_1 - 5,000I_{D2} - (-10) = 0$$

$$I_1 = \frac{25\text{ V}}{15,000\Omega} = 1.67\text{ mA}$$

$$V_{D1} = 15 - 10,000I_1 = 15 - 16.7 = -1.67\text{ V}$$

**Q-Points**

**$D1$  : (0 mA, -1.67 V): off**

**$D2$  : (1.67 mA, 0 V): on**

Results are consistent with the assumptions.