MECH3460 Dynamically Loaded Bolts Part 2

Fastener from a bridge

Connecting rod bolts from Rolls Royce Merlin, Main bearing bolt ALCO from 1940s

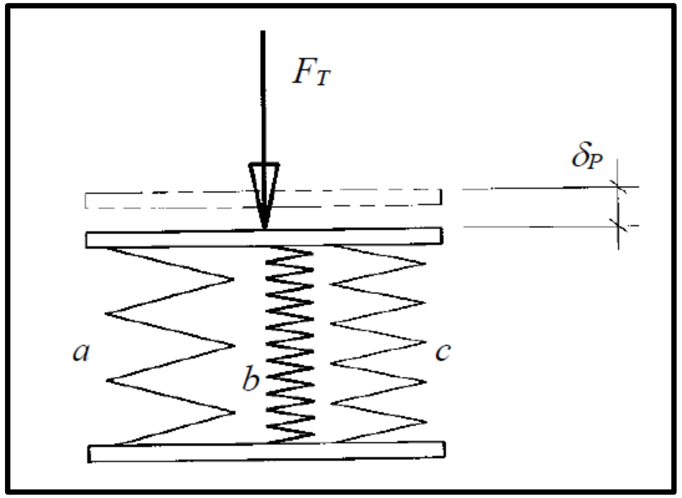
Main bearing bolt ALCO from 1940s diesel engine 1970

An Introductory lecture on Dynamically Loaded Bolts MECH3460: Mechanical Design 2 By Paul Briozzo and Dr. Andrei Lozzi

Springs in Parallel and in Series (Review)

In order to arrive at a model of how the external load is divided between increased bolt tension and decreased member's compression, we will examine how springs in parallel share a load, we can then arrive the fractions 'r' of the external load 'P' experienced by the bolt.

$$F_b = F_I + rP \qquad \text{Eq 3}$$



Elements in Parallel

Assuming that the bolt and its surrounding flange members both behave as linear springs elements, which is quite reasonable while they are stressed within the yield condition:

$$F = k \,.\, \delta \qquad \text{Eq 4}$$

In the Figure alongside F_T is equal to the sum of individual forces provided by the springs, and since the deflection δ_P is the same for all springs:

$$F_T = F_a + F_b + F_c \qquad \text{Eq 5}$$

$$= k_a\,\delta_P + k_b\,\delta_P + k_c\,\delta_P$$

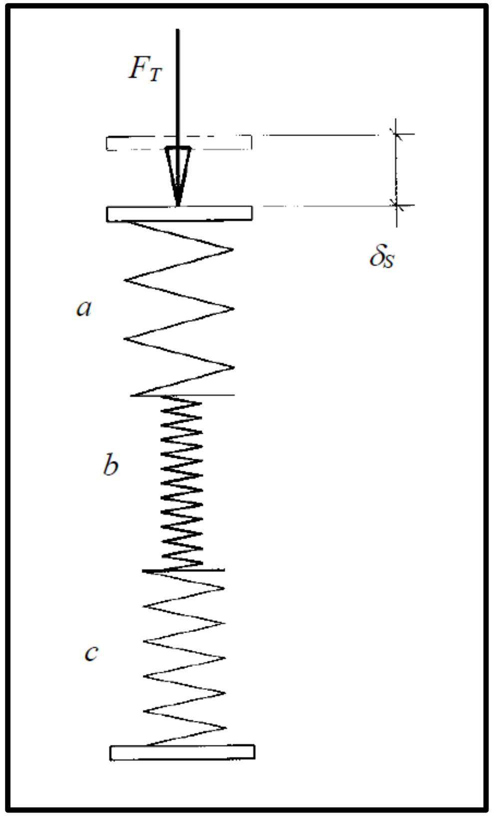
$$\therefore F_T = \delta_P\,(\Sigma_i\, k_i)$$

$$F_T = \delta_P\,.\,k_P$$

$$\text{Where: } k_P = \Sigma k_i \qquad \text{Eq 6}$$

We conclude **that the stiffness of elements in parallel is equal to the sum of their individual stiffness.**

Springs in Parallel and in Series (Review)



Elements in Series

The image to the left demonstrates springs in series where now the whole force F_T is transmitted undiminished through each spring. The total deflection is the sum of each individual spring's deflection:

$$\delta_S = \delta_a + \delta_b + \delta_c \qquad \textbf{Eq 7}$$

$$\therefore F_T = k_S\,(\Sigma_i\,\delta_i) \qquad \textbf{Eq 7a}$$

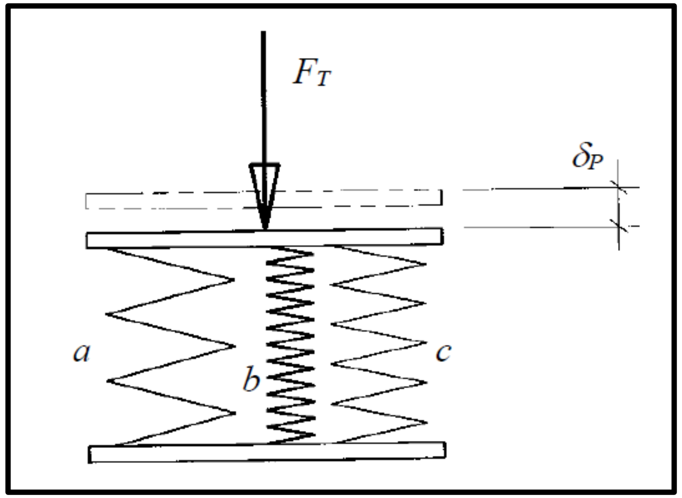
$$k_S = F_T / (\delta_a + \delta_b + \delta_c) \qquad \textbf{Eq 7b}$$

$$1/k_S = \Sigma\,(1/k_i) \qquad \textbf{Eq 7c}$$

$$k_S = 1/(\Sigma\,1/k_i) \qquad \textbf{Eq 8}$$

We conclude that for elements in series, it seems easiest to state that the inverse if the total stiffness is the sum of the inverses of the individual elements' stiffness i.e., Eq 7c.

Elements in Parallel

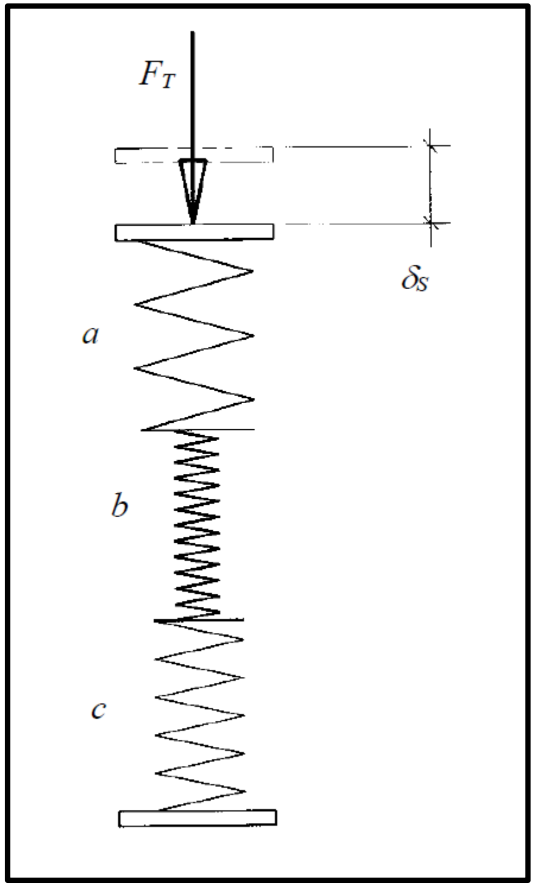


Elements in Parallel

For a system of elements in parallel, the total stiffness will be greater than the stiffness of any one element.

If there is a big disparity between the stiffness of the elements, the stiffest element will effectively dominate.

Elements in Series



Elements in Series

With elements in series, the opposite is true, the total stiffness will be less than the least stiff element, and if there is a significant disparity in their stiffness, then the softest element will dominate.

You can probably come up with a few everyday examples of these situations, like if you put your pillow on a solid floor, it will not matter much if the floor is from hardwood, concrete or steel, the softness of the pillow will dominate.

Elements in Parallel - Bolted Flange

We note that when the external force *P* was applied to the flange assembly, the bolts and flanges both expanded by the same additional amount δ_P. The bolt and flange here acted in parallel. Therefore, the force on the bolt is the **preload** plus that required to

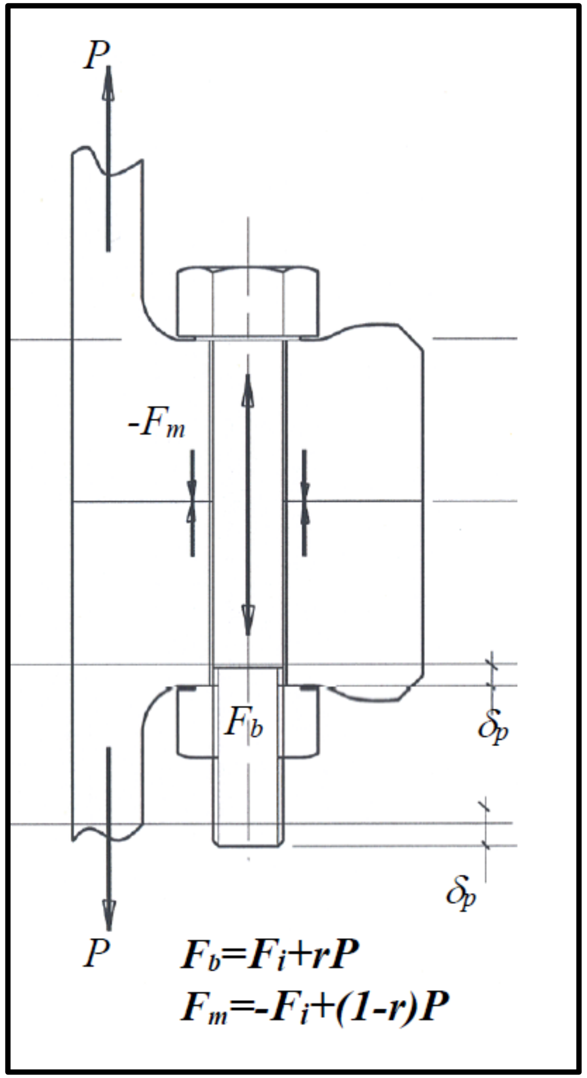
extend the bolt by δ_P $\quad F_b = F_i + \delta_P * k_b$ **Eq 9**

since P was applied to both the flanges and bolt, $\quad F_b = F_i + P/\,k_P * k_b$ **Eq 9a**

and the stiffness of the parts in parallel k_P using Eq 6. $F_b = F_i + P * \mathbf{k_b} \,/\, (k_b + k_m)$ **Eq 9b**

The fraction of the external load that the bolt experiences, Eq 3 is:

$$r = k_b \,/\, (k_b + k_m)$$ **Eq 10**



An obvious observation of Eq 10 is that when dealing with an alternating external force ($\pm P$), **the way to reduce the alternating stresses on the bolt, is to design a joint with relatively more elastic bolts (low k_b) and relatively stiffer members (high k_m).**

Stiffness of Bolts

Assuming that the shank of the bolt, that exists between the faces of the flanges, can be characterised as a bar of one diameter, then the elongation of that

bar will vary proportionally to the force and the bar's length, $\delta_P = Fl/AE$ **Eq 11**

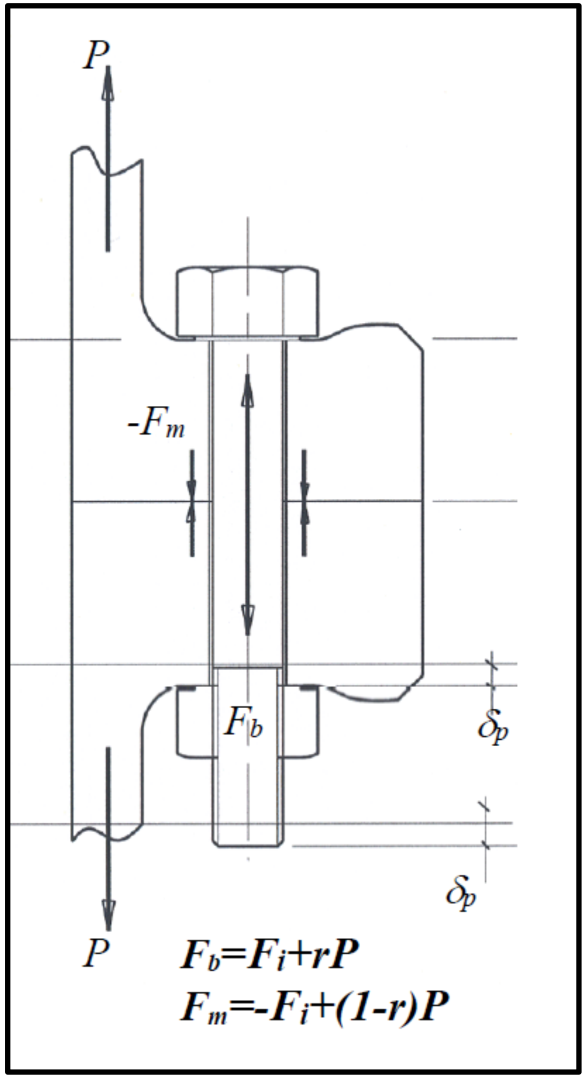
inversely with its cross-sectional area and its modulus of elasticity

Rearranging equation 4 as applied to the bolt : $\delta_P = Fb/k_b$

substituting into Eq 11, we get that the stiffness of the shank of the bolt:

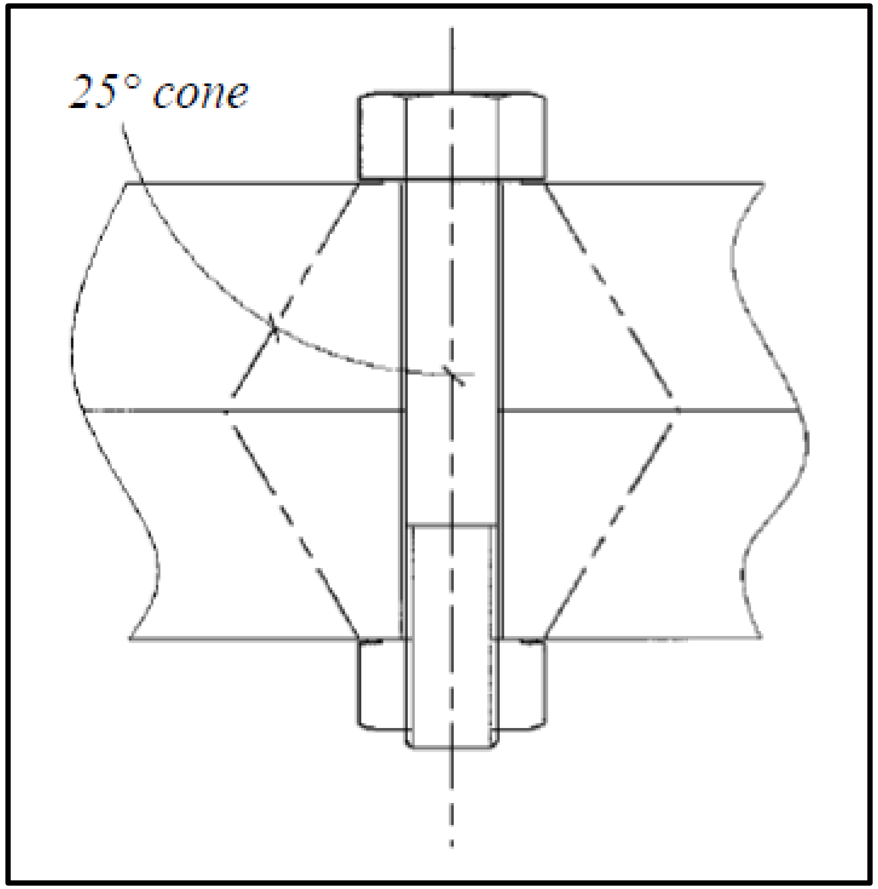
$k_b = AE/l$ **Eq 12**

Note that the shank rather than the core or stress diameter is referred to as the bolt's shank diameter plays a more dominant role through the thickness of the two members.



Stiffness of Flange Members

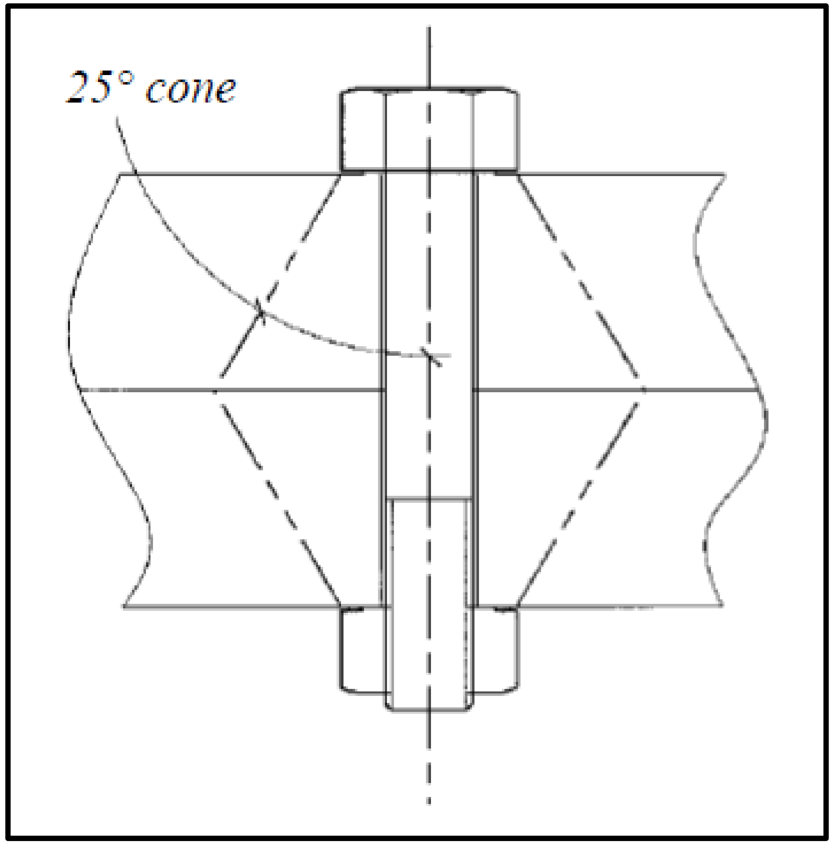
The stiffness of flanges is a much more interesting. Before so much science was applied to stress analysis, there was a simple rule that said, that within a solid body the stress distribution coming away from a point load, is contained within a cone of 60° included angle. Finite Element Analysis (FEA) and experimental observations support that rule.

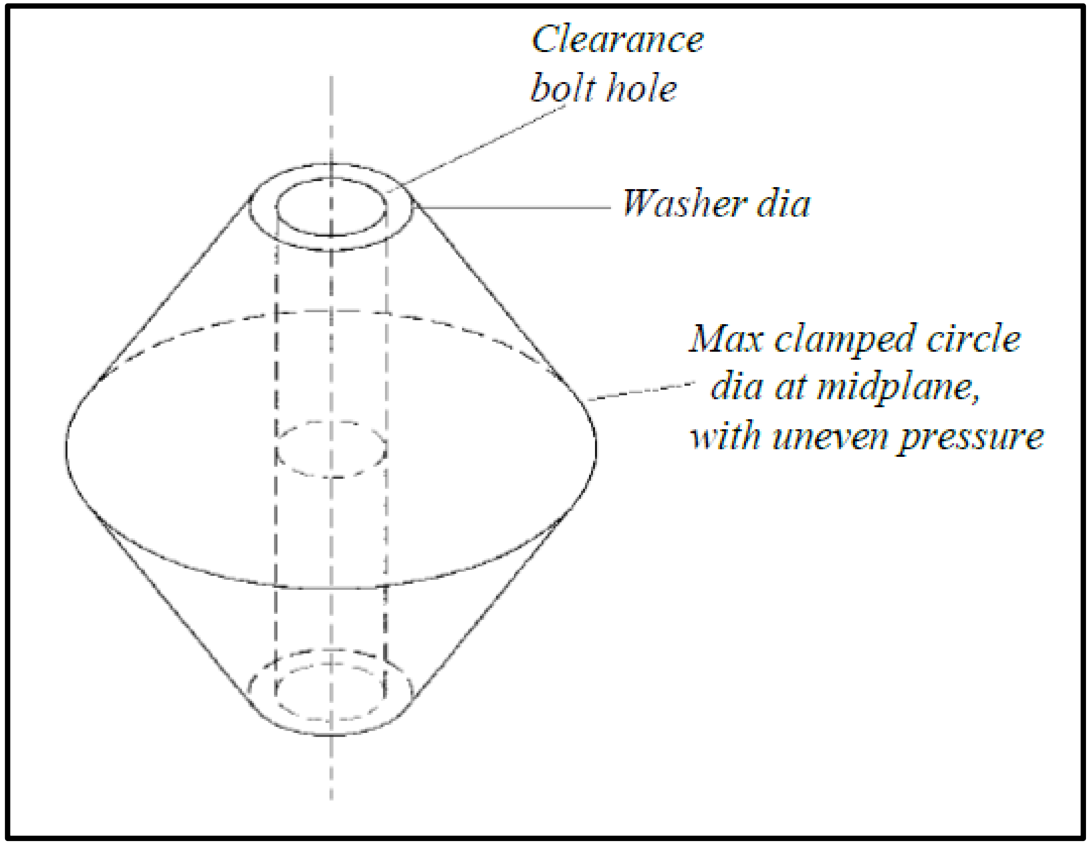


The bolt only compresses, somewhat non uniformly, the material contained within 2 cones of about 25°, surrounding the CL, starting from both ends of the bolt. At present the 25° exaggerates k_m by possibly 5%.

It is suggested that an FS of 1.2 minimum be used.
In the past 30°, 33° and even 45° have been used.

Stiffness of Flange Members

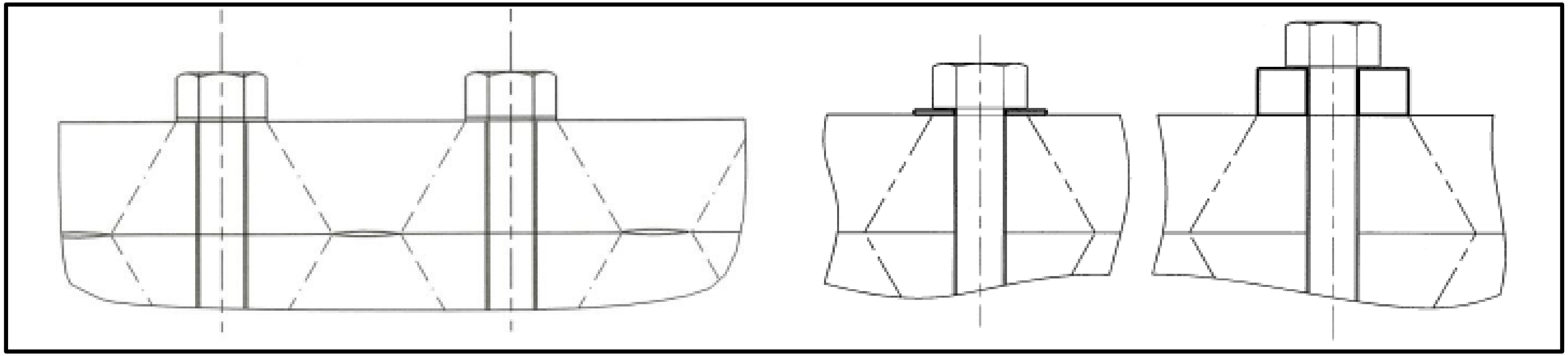




The volume of the flanges that is compressed by a bolt can be described as 2 hollow truncated cones, sometimes referred to as frustums. This is shown in the left image and in the pictorial view image on the right.

Apart from the conical surfaces the boundaries are the clearance bolt hole and the washer diameters under the head of the bolt and of the nut.

Stiffness of Flange Members



If common bolts are used, the effective washers diameters are the circular sections under the hexagonal bolt head and nut. The washer diameters are approximately 1.5 to 2.0 the bolt shank diameter.

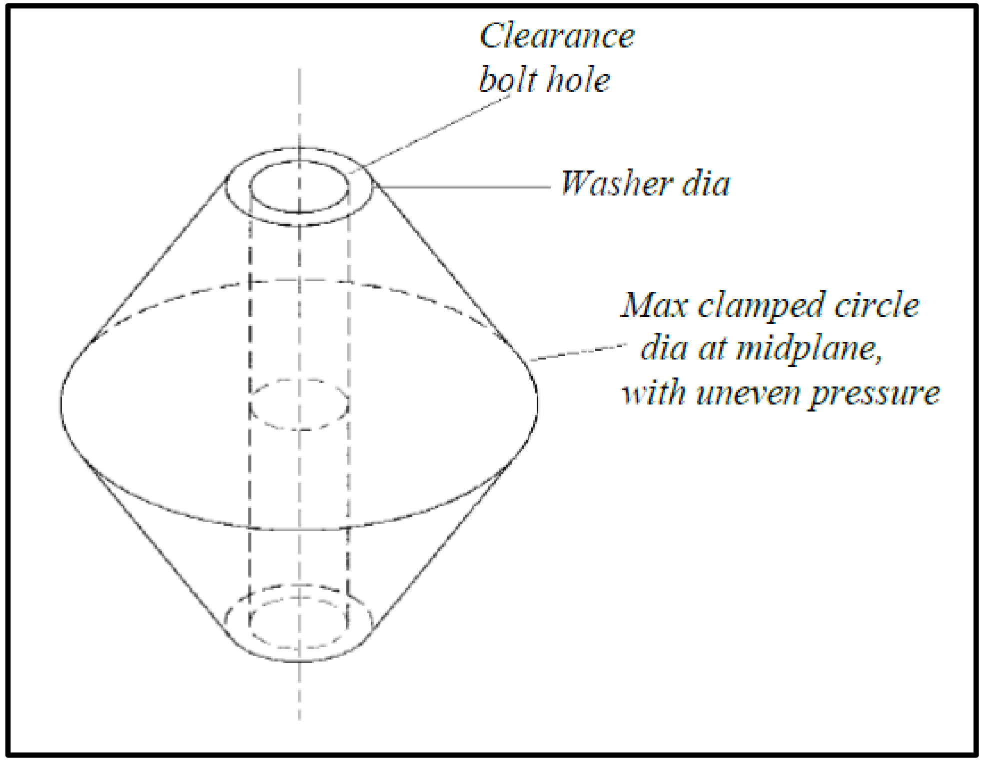
Larger diameter washers may be used, but to contribute to stiffness of the truncated cones they have to be considerably thicker than the usual washers as shown alongside.

Stiffness of Flanges

The stiffness of these hollow cones can be calculated, but one must be careful that these values be used only if the flanges are sufficiently wide to contain the full cones.

Note that there are situations where lighter and more economic assemblies are obtainable by using narrower flanges and bigger bolts.

The stiffness of the cones shown alongside can be evaluated by applying the differential version of Eq 11, applied per unit force.



$$\delta_b = \frac{Fl}{AE} \qquad \textbf{Eq 11}$$

Where dx is the thickness of a circular disc slice through the cone and $A(x)$ is the area of that slice.
$d\delta$ is integrated for one cone, then summed for

$$d\delta = \frac{Fdx}{E \cdot A(x)} \qquad \textbf{Eq 13}$$

Stiffness of Flanges

For two cones, using Eq 8, we get:

$$k_m = \frac{\pi E d \tan\alpha}{2\ln\dfrac{(l\tan\alpha + d_w - d)(d_w + d)}{(l\tan\alpha + d_w + d)(d_w - d)}} \qquad \textbf{Eq 14}$$

using *l* as the bolt grip length.

α the cone angle of 25°, *d* bolt hole dia. And

for common bolts, washer dia. $d_w = 1.5d$, we get:

$$k_m = \frac{0.466 \cdot \pi E d}{2\ln\left(5 \cdot \dfrac{(0.466 \cdot l + 0.5 \cdot d\ )}{(0.466 \cdot l + 2.5 \cdot d\ )}\right)} \qquad \textbf{Eq 15}$$

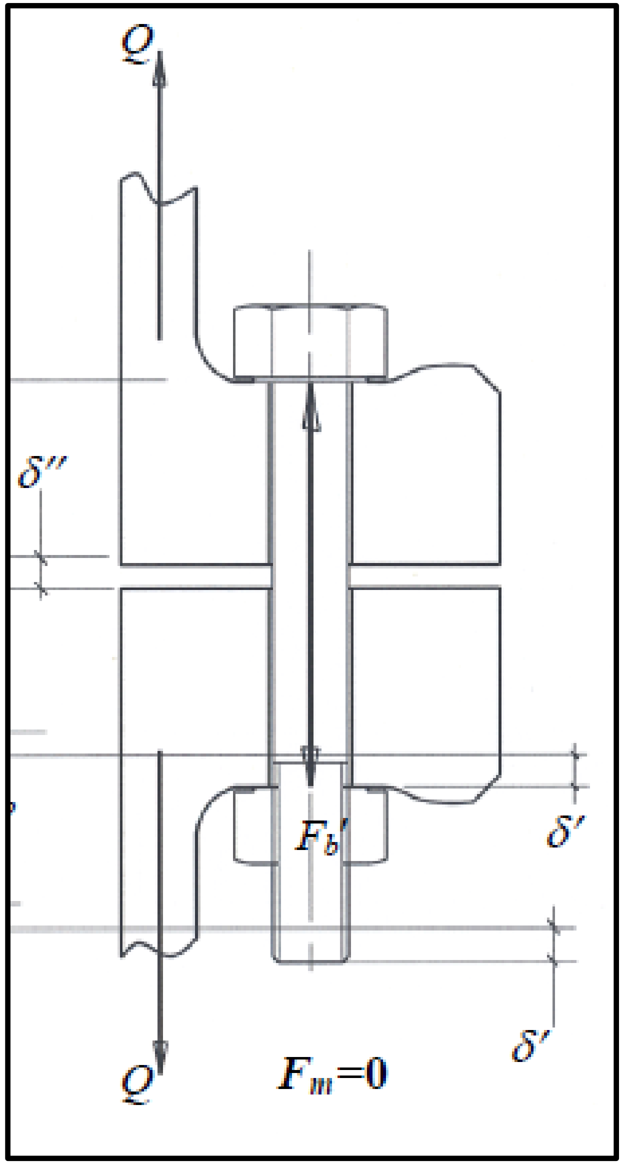
Fraction of External Load Added to the Bolt

$$r = \frac{k_b}{k_b + k_m} \qquad \textbf{Eq 10}$$

Eq 10 gave the ratio of the external load transmitted to the bolt.

As noted this ratio can be made small if we can design a joint with bolts of relatively low stiffness and flanges of relatively high stiffness.

We must also keep in mind that we will need sufficient cross-section of bolt or bolts to provide a preload (with a safety margin) that cannot be overcome by the largest reasonably expected external load, as represented by the figure alongside.



Fraction of External Load Added to the Bolt

$$k_b = \frac{AE}{l} \qquad \textbf{Eq 12}$$

Eq 12 indicates that we can decrease the stiffness of bolts if we make them longer and or reduce their diameter.

The flanges will also become less stiff as we increase their thickness (a fact that may not be obvious from Eq 14), but their stiffness will fall off much less quickly than the stiffness of the bolts.

Consequently, if we use more but thinner bolts in thicker flanges, r of Eq 10 will decrease, improving the bolts' fatigue life. These have been seen in aircraft machinery, but which has of late been more common in well-made ground-based vehicles as well.

$$k_m = \frac{\pi E d \tan\alpha}{2\ln\dfrac{(l\tan\alpha + d_w - d)(d_w + d)}{(l\tan\alpha + d_w + d)(d_w - d)}} \qquad \textbf{Eq 14}$$

$$k_m = \frac{0.466 \cdot \pi E d}{2\ln\left(5 \cdot \dfrac{(0.466 \cdot l + 0.5 \cdot d)}{(0.466 \cdot l + 2.5 \cdot d)}\right)} \qquad \textbf{Eq 15}$$

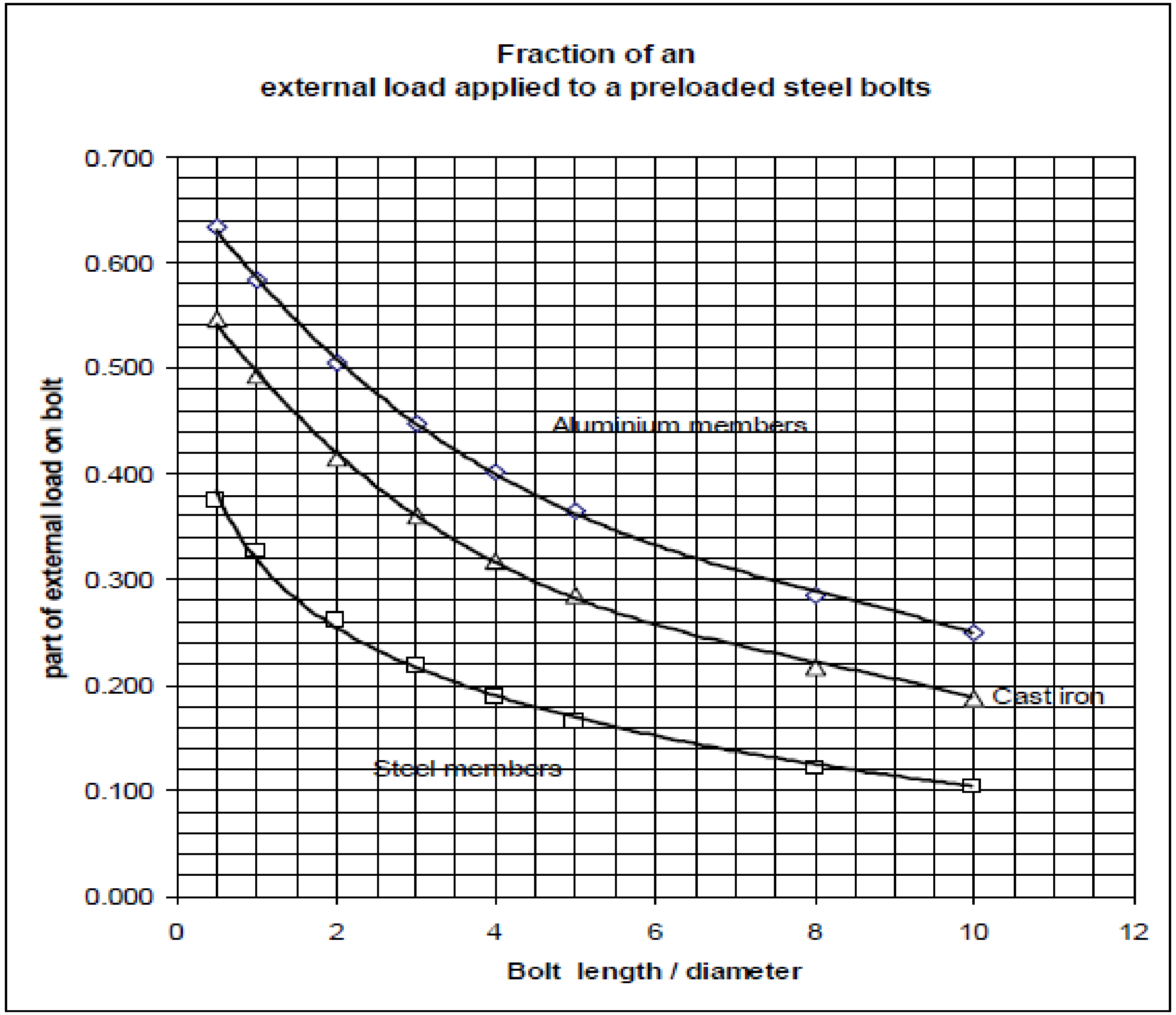
$$r = \frac{k_b}{k_b + k_m} \qquad \textbf{Eq 10}$$

Fraction of External Load Added to the Bolt

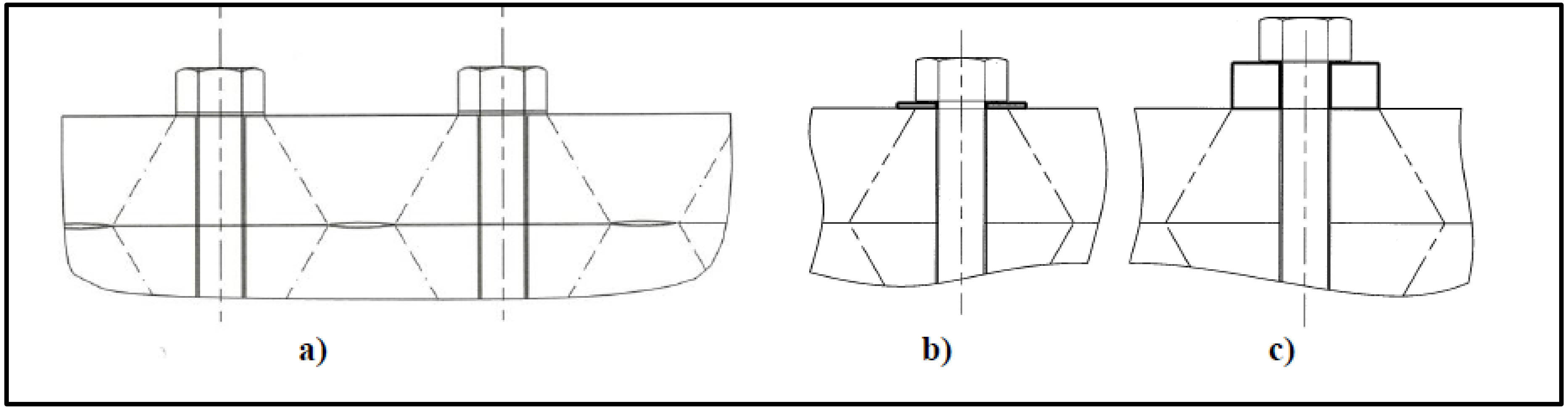
The graph alongside evaluates Eq 10 using equations 12 & 15, to provide an overview of the advantages of relatively long bolts. It also shows the detrimental effect of using flanges material of Young's modulus less than that of steel, i.e., aluminum and cast iron.

Since k_m varies as the modulus of elasticity of the flange material, there are deleterious consequences when softer materials are used for the flanges with steel bolts. In rare and smart designs, light alloy bolts are used in aluminum alloy assemblies (Porsche).
Using steel, a bolt of length equal to its diameter, it will experience ~33% of the external load.
A 10 times longer bolt only sees 9% of that load.

If cast iron or aluminum flanges are used, the bolt sees about 2 to 2.5 times those percentages. The graphs shown alongside must be taken as only indicative, in part because the modus E for steel and Al alloy can vary by ~5%.



Gaskets and Flat Washers



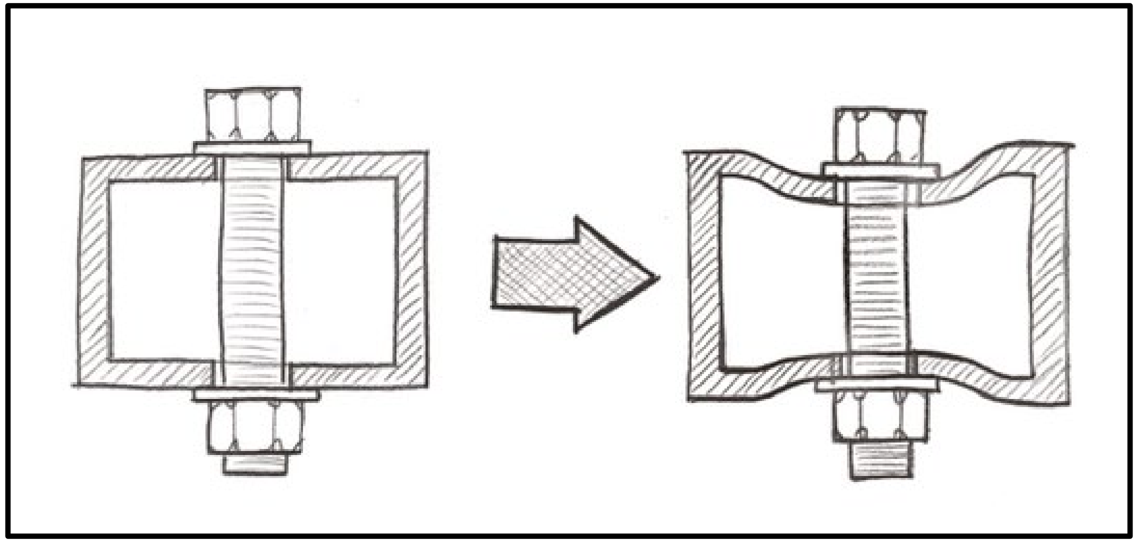
Since compressive stress exits only within the hollow cones, the clamping pressure at the flanges interface in:

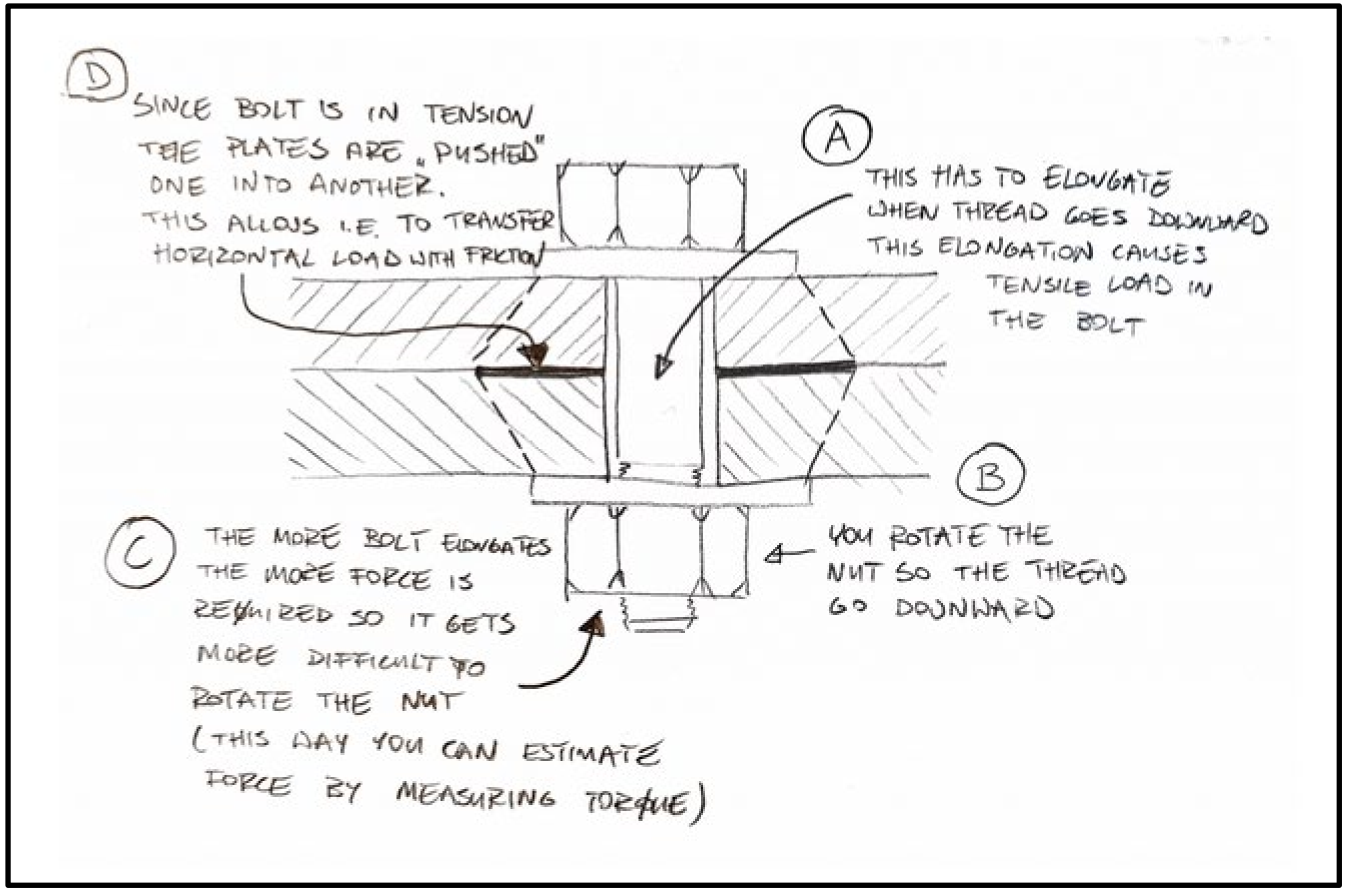
In the Figure above, a) Cannot be uniform. In fact, after some **preload** is applied to those bolts the flanges will separate and a gap between the flanges will open, between the compression cones. This indicates that if the bolted joint contains high fluid pressure and a compression gasket is used, then the cones may have to merge, by using more bolts or thicker flanges.

In the Figure above, b) Highlights the fact that the normal steel washers do not expand the top cone diameter, because they are too thin. Normal steel washers protect the flanges from scoring and provide smooth surfaces and relatively low coefficient of friction for the nut to turn on.

In the Figure above, 12 c) A thick steel washer effectively expands the cone and at the same time extends the bolt.

Bolt Preloading - the what !





Images Sourced from Enterfea 2021

An interesting overview of what Bolt Preloading is from a structural rather than a machine perspective Is discussed at https://enterfea.com/bolt-preload-tutorial/

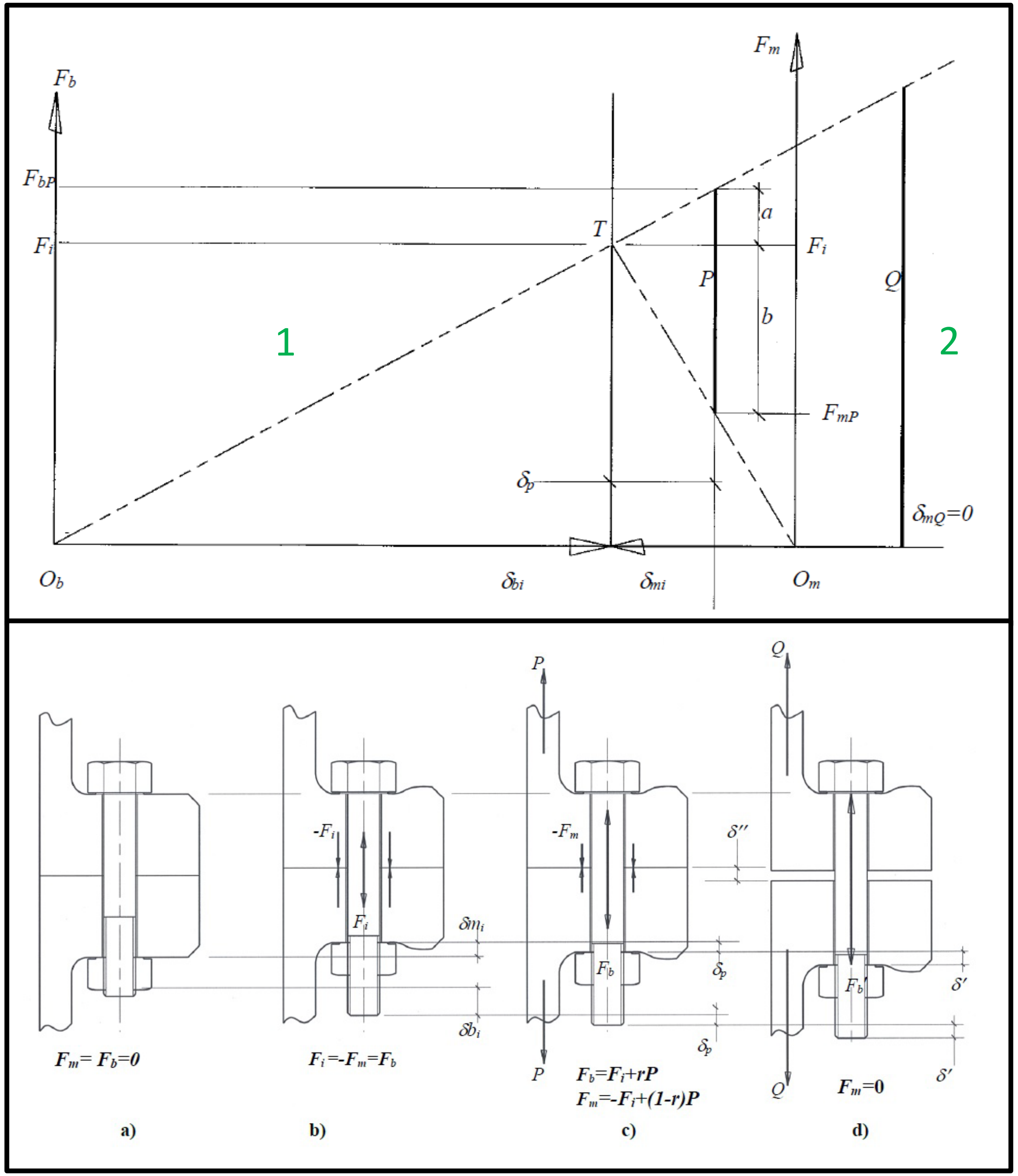
Force Deflection Characteristics of a Bolted Joint

The figure alongside represents diagrammatically the 4 states of the bolted joint shown below. This diagram is a little unusual because it is made up of 2 diagrams, (1 and 2), which are almost mirror images of one another.

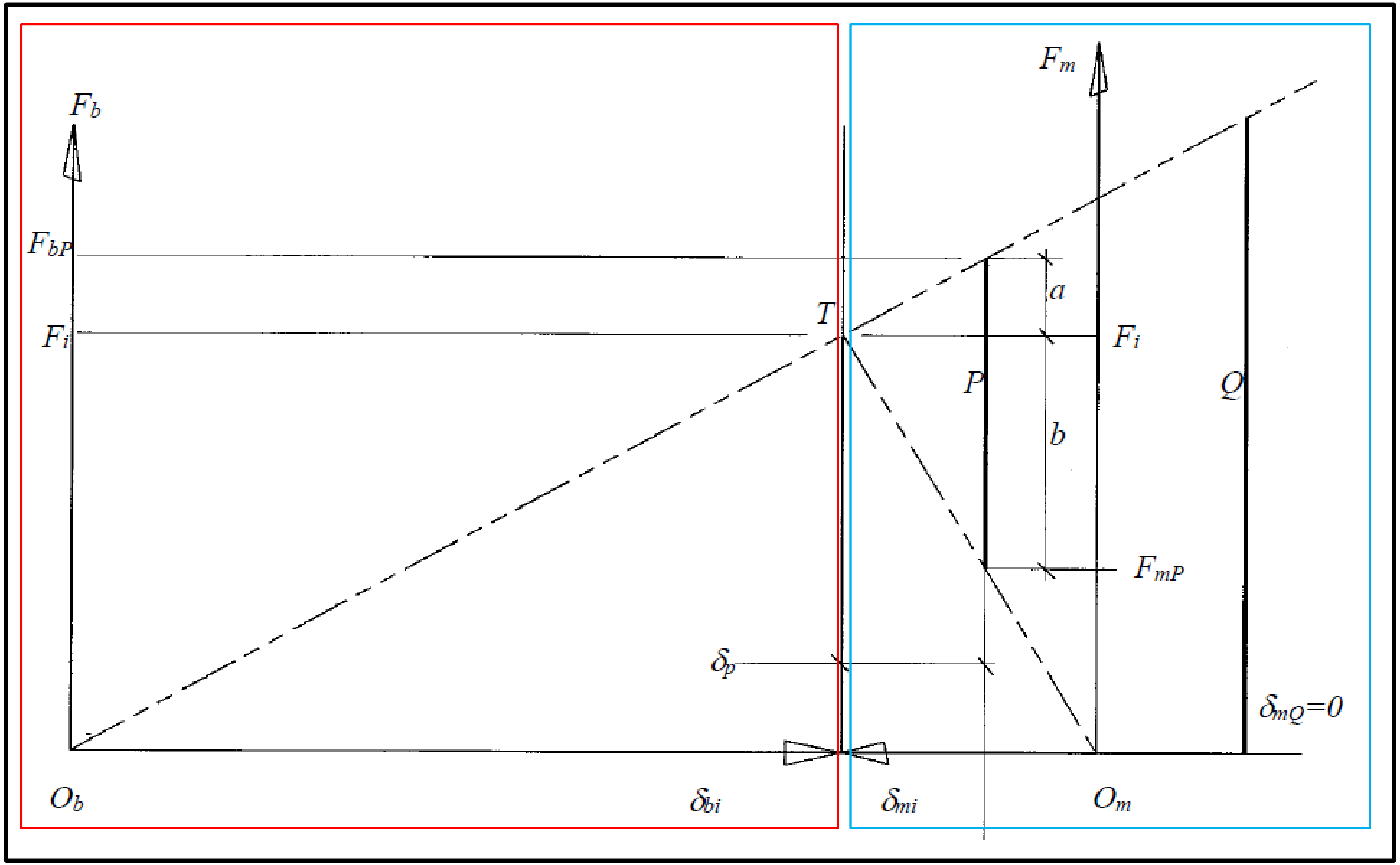
On the left of the vertical line through ***T*** is the tension extension diagram for the bolt, on the right of ***T*** is the compression contraction diagram for the members.

The broken line from the left origin **O*_b_*** through ***T*** shows the linear response of the bolt, on the right of ***T*** the broken line from origin **O*_m_*** is the linear response of the flanges.

As you should expect from Eq 7, i.e., $\delta_S = \delta_a + \delta_b + \delta_c$ the line **O*_b_* *T*** has a gradient of $\boldsymbol{k_b}$ and the line **O*_m_* *T*** has a gradient of $\boldsymbol{k_m}$.



Force Deflection Characteristics of a Bolted Joint



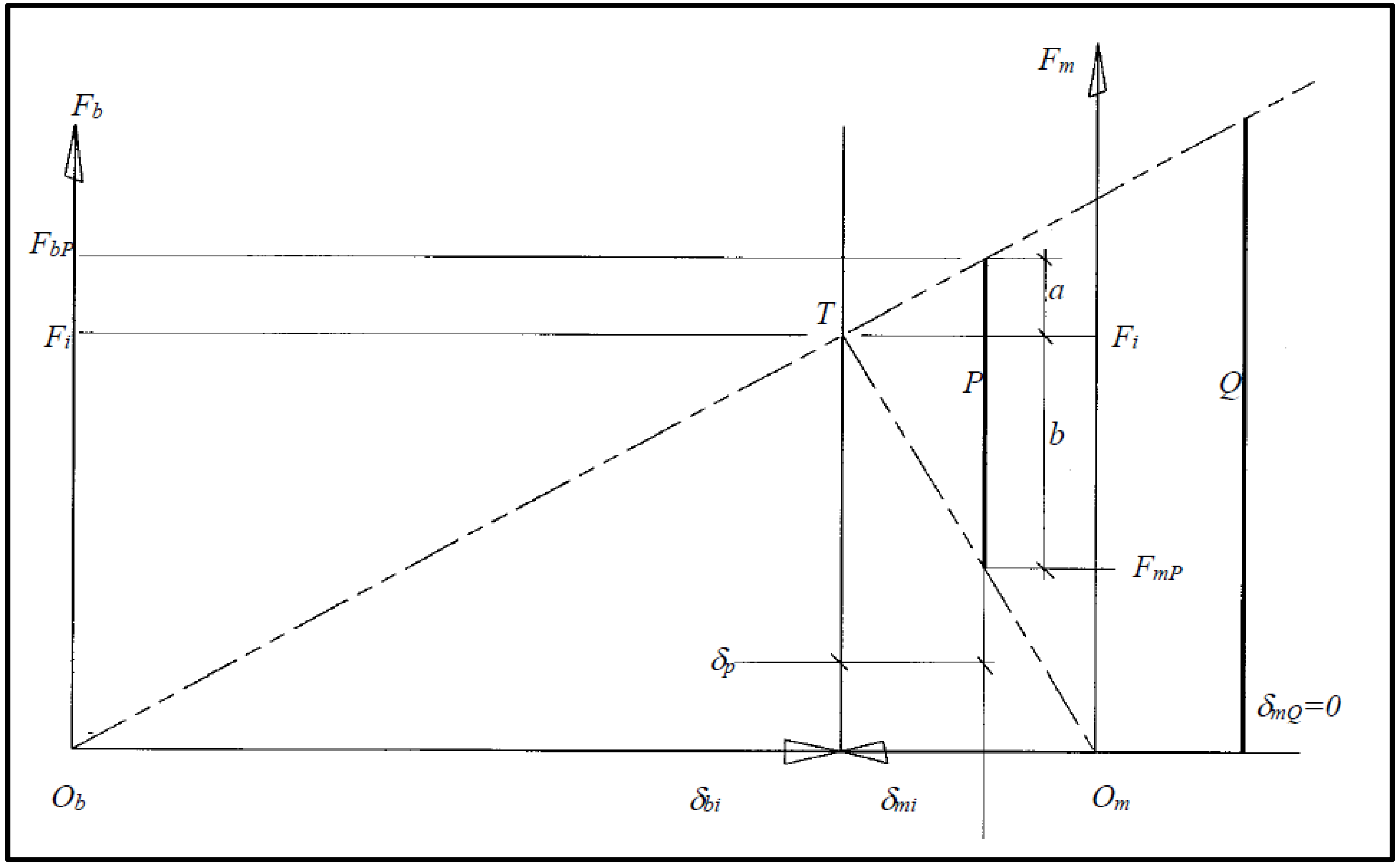
Force deflection graph for a bolted joint in the states portrayed in the previous Figures a) to d). The above diagram is in principle 2 similar graphs mirrored about the vertical line through T. On the left (red box) of T are the bolt tension-extension axes, on the right (blue box) is the flanges compression-contraction axes.

Force Deflection Characteristics of a Bolted Joint

The line P is proportional to the external force that is transmitted to the flanges.

P increases the bolt tension and decreases the members compression. Note that *P* together with the new bolt tension F_bP and members compression F_mP are of course in equilibrium.

Because of the gradients of the dotted lines we can argue that portion *a* of *P* is proportional to k_b, and portion *b* of *P* is proportional to k_m, therefore the equations for both the bolt and members (on the right) reflect this.



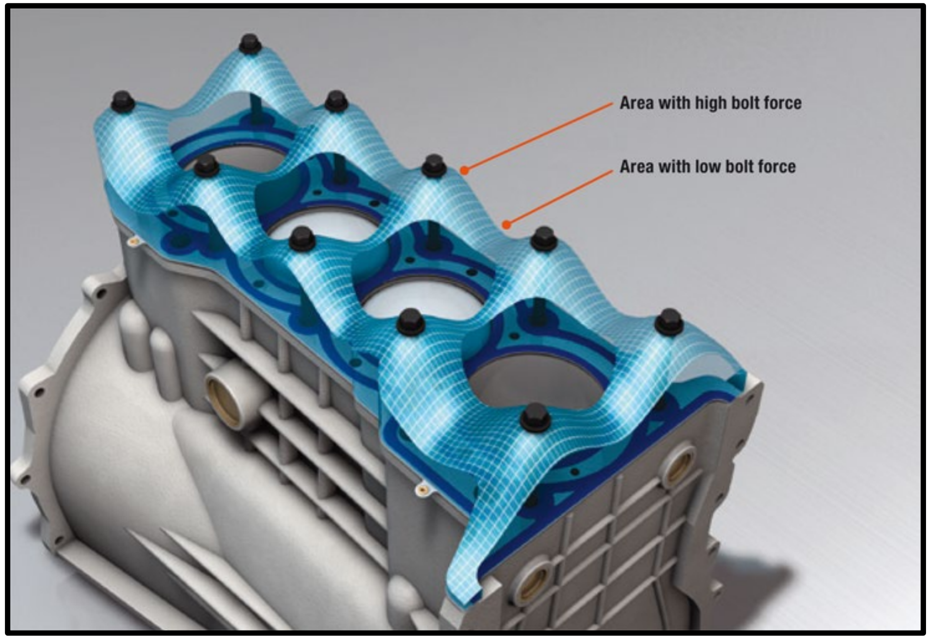
For the bolt: $$F_{bP} = F_i + \frac{a}{a+b} \cdot P = F_i + \frac{k_b}{k_b + k_m} \cdot P$$ **Eq 16**

and for the members: $$F_{mP} = F_i - \frac{b}{a+b} \cdot P = F_i - \frac{k_m}{k_b + k_m} \cdot P$$ **Eq 17**

Force Deflection Characteristics of a Bolted Joint - Synopsis

If P is a variable load then to improve the fatigue life of the bolt, apart from just making everything bigger and heavier, we may design a relatively more elastic bolt and stiffer flanges.

The fraction of P on the bolts is proportional to k_b and inversely to k_m. If a high preload is required then it can be achieved by simply using more bolts.

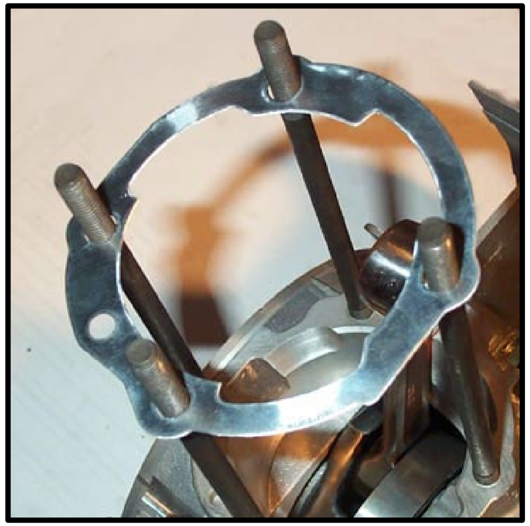


Reduced Clamping Force

One shortcoming is that having a more elastic bolt leads to decreasing the clamping force between the flanges, as given by Eq 17. This has to be considered in your designs. Some level of residual clamping force is absolutely necessary.

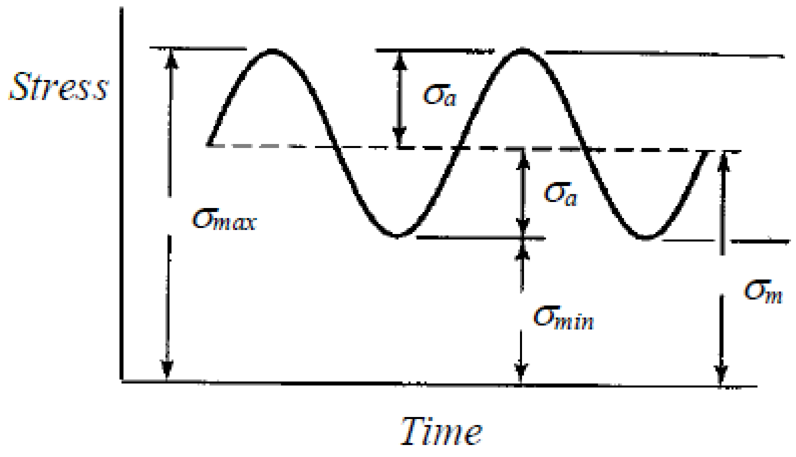
Clamped mated faces tend to wear and embed together when subjected to vibration, thereby reducing the bolt elongation and consequently of course their preload.
Furthermore mating faces tend to oxidize, undergo electrolytic reactions and otherwise erode.

If an open gasket is used, then be warned that they tend to pack, crush or otherwise loose stiffness thereby becoming thinner, of course also reducing bolt preload. For better longevity it is better to use a compression gasket (continuous O-ring) that is contained in a grove. Thereby changes in the gasket will have minimal effect on bolt tension.



Stress and Strains

The analysis of the fatigue life of a bolt is carried out under the assumption that the load imposed on the bolt can be represented as a combination of a mean load (constant in time) and an alternating sinusoidal load that is added to it.
The figure alongside represents this view.



The external load could vary sinusoidally between limits e.g.: -P to +P, 0 to +P, or –P to 0. **Eq 18**

The upper and lower limits substituted in Eq 16 to give: F_{bmax} and F_{bmin} **Eq19**

The mean and alternating force on the bolt stem is then:

$$F_{bm} = \frac{F_{b\max} + F_{b\min}}{2}$$ **Eq 20**

$$F_{ba} = \frac{F_{b\max} - F_{b\min}}{2}$$ **Eq 21**

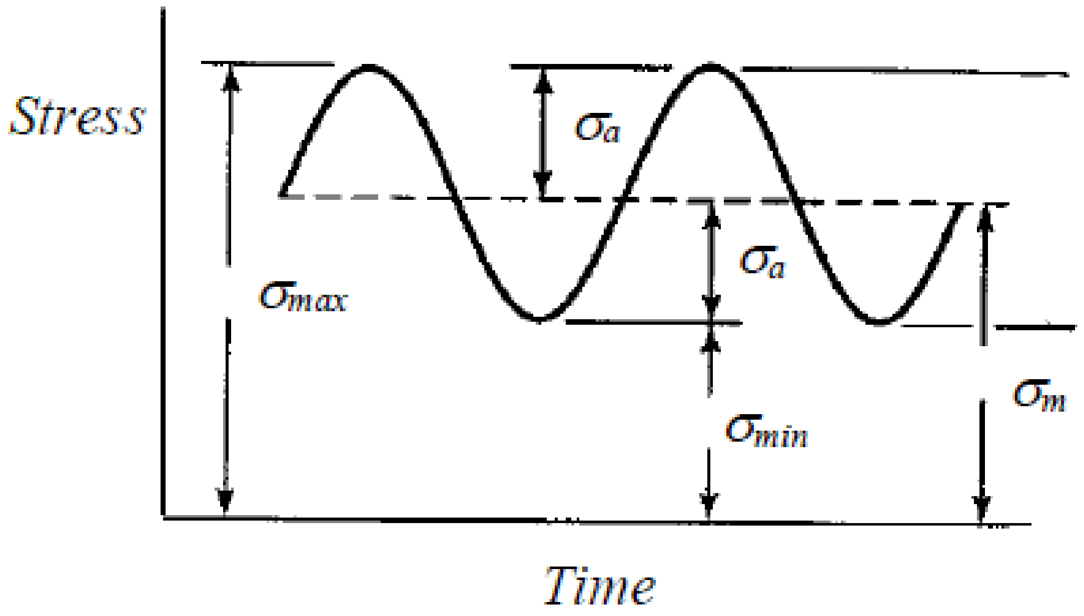
The mean and alternating stresses at the threads, shown in Fig 11 (slide 12):

$$\sigma_m = \frac{F_{bm}}{A_s}$$ **Eq 22**

$$\sigma_a = \frac{F_{bt}}{A_s}$$ **Eq 23**

Where, A_s is the stress cross-sectional area of the thread, based upon D_{stress} shown in Fig 6. A_s is available from tables, can be calculated or can be estimated.

Stress and Strains



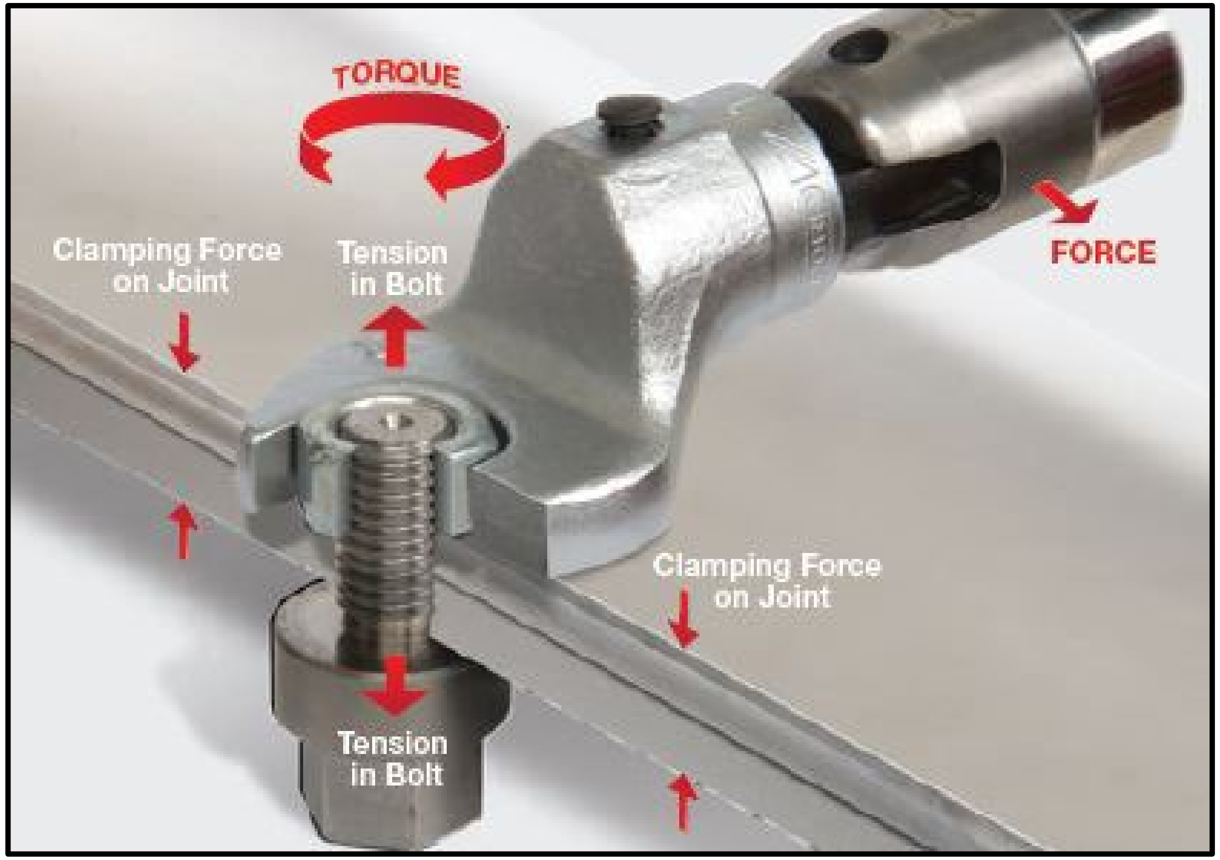
Focusing on the figure above which shows the stress - time variation in a preloaded bolt, to which a sinusoidal external load is applied. Note that σ_m may be +ve, zero or –ve but σ_a is always +ve.

A more complex $\sigma_a(t)$ can be approximated by a number of sinusoidal components, i.e. by a Fourier transformation. Each of these components may be in turn examined in a similar manner to that outlined here.

Preload

It is suggested that a preload of 75% of proof stress be used for the typical reusable connection and up to 90% for those joints that will remain permanent connected or rebuilt very few times, when possibly the fasteners will discarded.

The following slides cover some typical methods of preloading bolted joints:



Preload – Torque Wrench

Preload or pre-tensioning of bolts is most commonly done with the use of torque wrenches. Unfortunately, studies of the resulting tension induced in bolt stems, with unlubricated threads, exhibit about a 15% standard deviation from the mean tension.

This reduces to about 8% for lubricated bolts using oil. This leaves us realistically at best with an uncertainty of about +/- 20% (+/- 2.5 sd) in the preload of the bolts if a torque wrench is used.

The use of graphite greases or surface binding lubricants like molybdumenn disulfide can greatly reduce this uncertainty.

Obviously, torque wrenches are very practical but, only give a moderate level of confidence unless used by knowledgeable personnel.

Preload – Turn Angle

A more reliable means of determining preload can be achieved when the components that are clamped together are well made, with flat parallel faces and with no foreign material trapped between them.

Here the nuts are turned by hand until all clearance is removed and a rapid rise in resistance to further turning can be detected.

The nuts are usually then torqued to a low preliminary level, followed by turning the nuts by a prescribed angle (~180° to 360°) to provide the required bolt stem elongation.

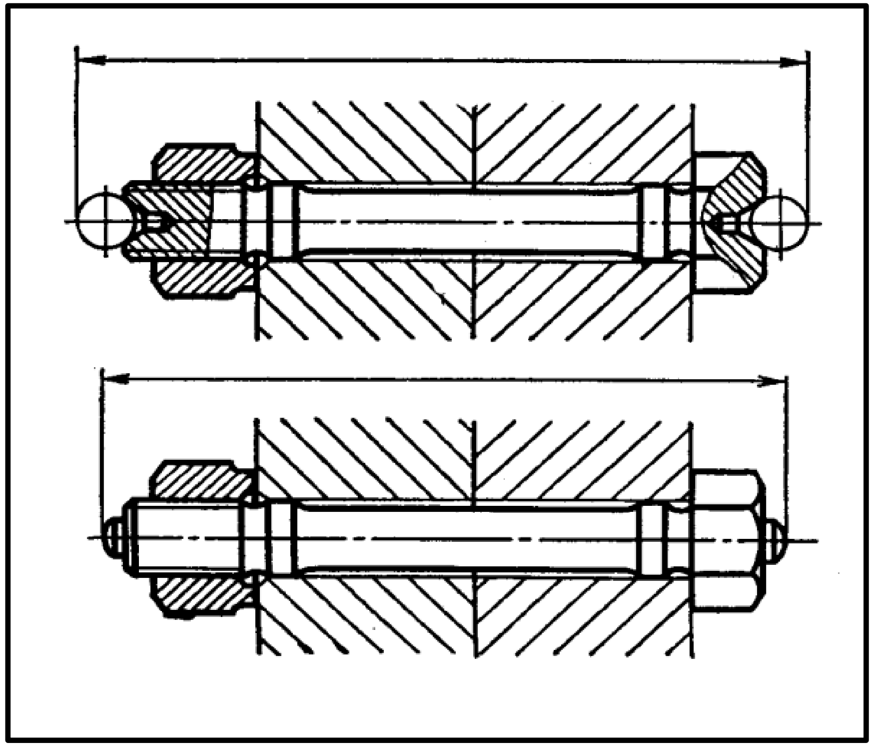
Preload – Direct Length Measurements

Possibly the most accurate means to determine the bolt preload is by direct measurement of the bolt length before and while the nut is turned.

The procedure usually begins with the low preliminary torqued condition as described in the figure alongside. This is taken to be 0 preload and the length of the bolt is measured.

The nut is then turned progressively while the length of the bolt is checked, when the required extension is reached the bolt is deemed preloaded.

This procedure can best be done if a bolt has ground faces at each end, or if a conical recess is machined at the ends. With the use of steel balls clamped into these recesses, the overall length can be measured with a gauge quite precisely. Obviously, this technique is very time consuming and is carried out on only the more critical fasteners in relatively critical machines.



Above are 2 common means of directly measuring bolt length and consequently bolt elongation. These diagrams show the nut's and the stem's lower threads to begin at the same place.
This is impossible to ensure since tolerances have to be applied to all dimensions.
The only safe means is to have a longer than necessary nut and locate the stem's threads always within it.

Bolts Strengths

Shown in the table below are the material properties for a range of ASTM high grade commercially bolts and screws. This American Society for Testing of Metals standard (A574 M) note that 99% of fasteners exceed the stated strengths. Fastener manufacturers provide properties within and beyond the range shown below, e.g. 6.6, 14.9, 18.8 etc.

Class (bolt grade)	Min proof strength S_P N/mm^2	Min tensile strength S_u N/mm^2	Min yield strength S_y N/mm^2	Endurance strength S_e N/mm^2
4.8	310	420	340	65
5.8	380	520	420	81
8.8	600	830	660	129
9.8	650	900	720	140
10.8	830	1040	940	162
12.9	970	1220	1100	190

Data from Shigley et al, showing minimum strengths for a range of commercial grade bolts.

The relatively high strength steel used in these bolts do not show a clear yield condition, resulting is some deformation at stresses levels below but near yield.

Therefore, the proof stress (S_P) is used which is about 85% of S_Y.

The fully corrected endurance limit takes into account all the normal mass production manufacturing processes for fasteners with rolled thread. Note also that S_e is approximately 15.5% of S_u.

Fatigue Diagrams

We will use here 3 lines or curves to determine safe bolt designs under fatigue conditions: the alternating and mean stresses have to be below: (Where FS = Factor of Safety)

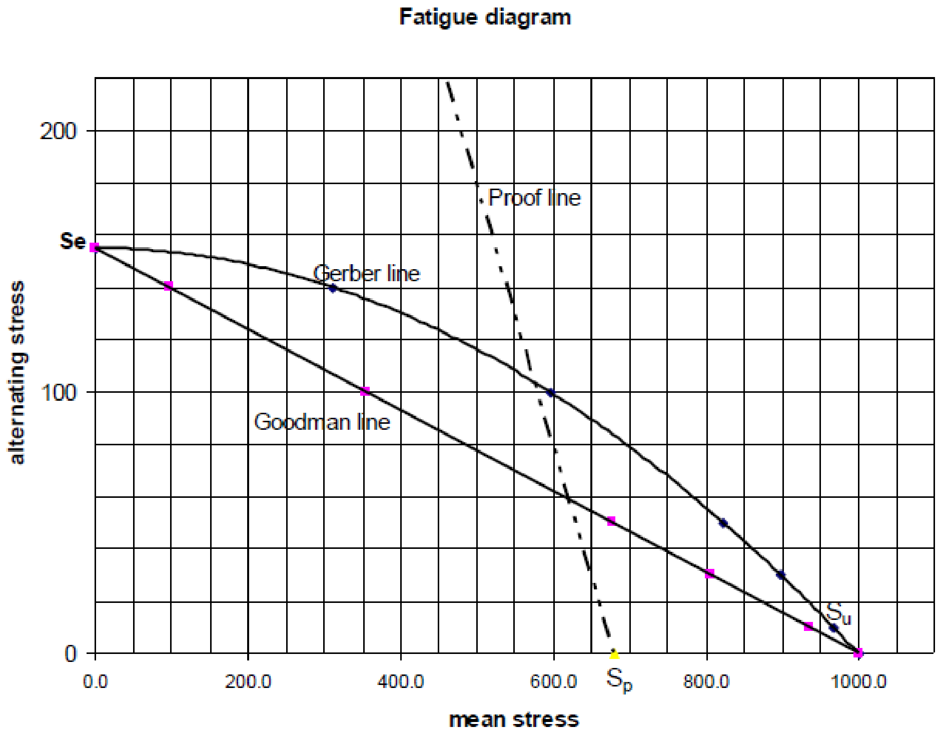
1. The proof line: $$\frac{\sigma_a}{S_P} + \frac{\sigma_m}{S_P} = \left(\frac{1}{FS}\right) \qquad \textbf{Eq 24}$$

2. and either the Gerber parabola: $$\frac{FS\sigma_a}{S_e} + \left(\frac{FS\sigma_m}{S_u}\right)^2 = 1 \quad \textbf{Eq 25}$$

3. or the ASME ellipse: $$\left(FS\frac{\sigma_a}{S_e}\right)^2 + \left(FS\frac{\sigma_m}{S_y}\right)^2 = 1 \quad \textbf{Eq 26}$$

The functions in Eq 24 and 25 plus the Goodman line are plotted alongside for a hypothetical material of S_u = 1000 Nmm^2 and S_p and S_u as per the discussion and the Table from the previous slide.

The ASME Ellipse has not been plotted.



Fatigue Diagrams

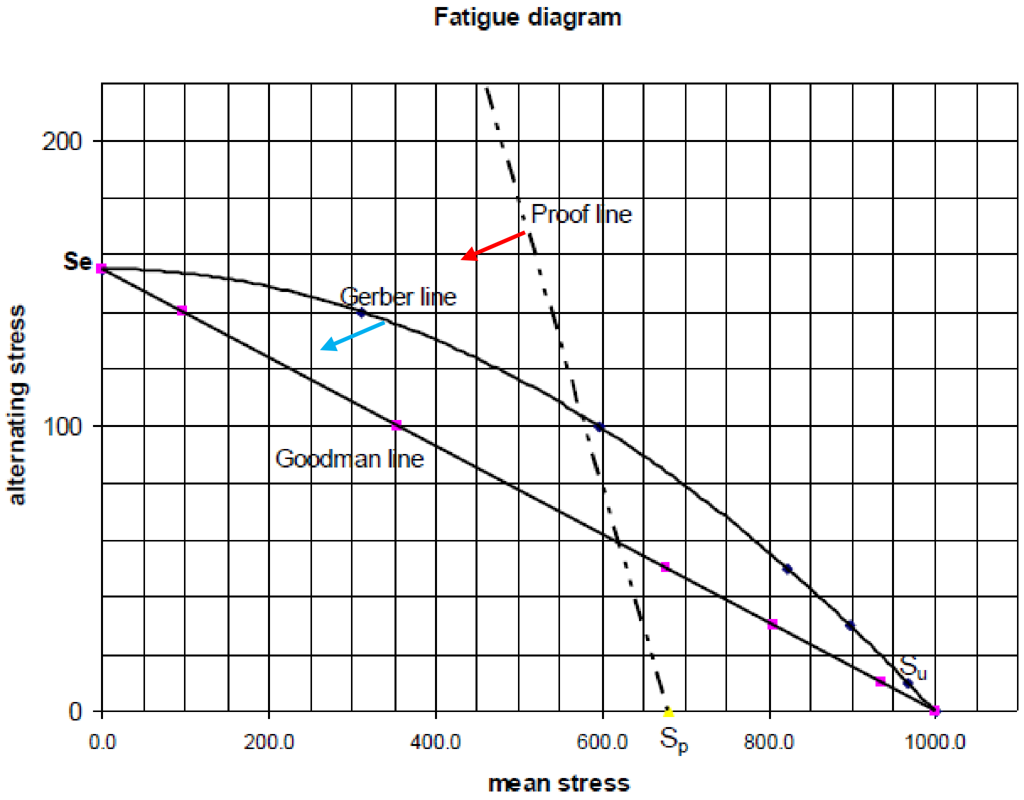
Alongside are plots of Goodman, Gerber and Proof relations.

Any combination of σ_a and σ_m stresses represented by a point to the left of the Proof line ( ) indicates that no yielding will take place.

Any combination of σ_a and σ_m below the Gerber line, indicates that any bolt made to the ASTM standard described in slide 27, will have a likelihood of less than 1% of experiencing a fatigue failure ( ).

Hence any bolt that can be represented by a point below both the proof and Gerber lines can be deemed safe.

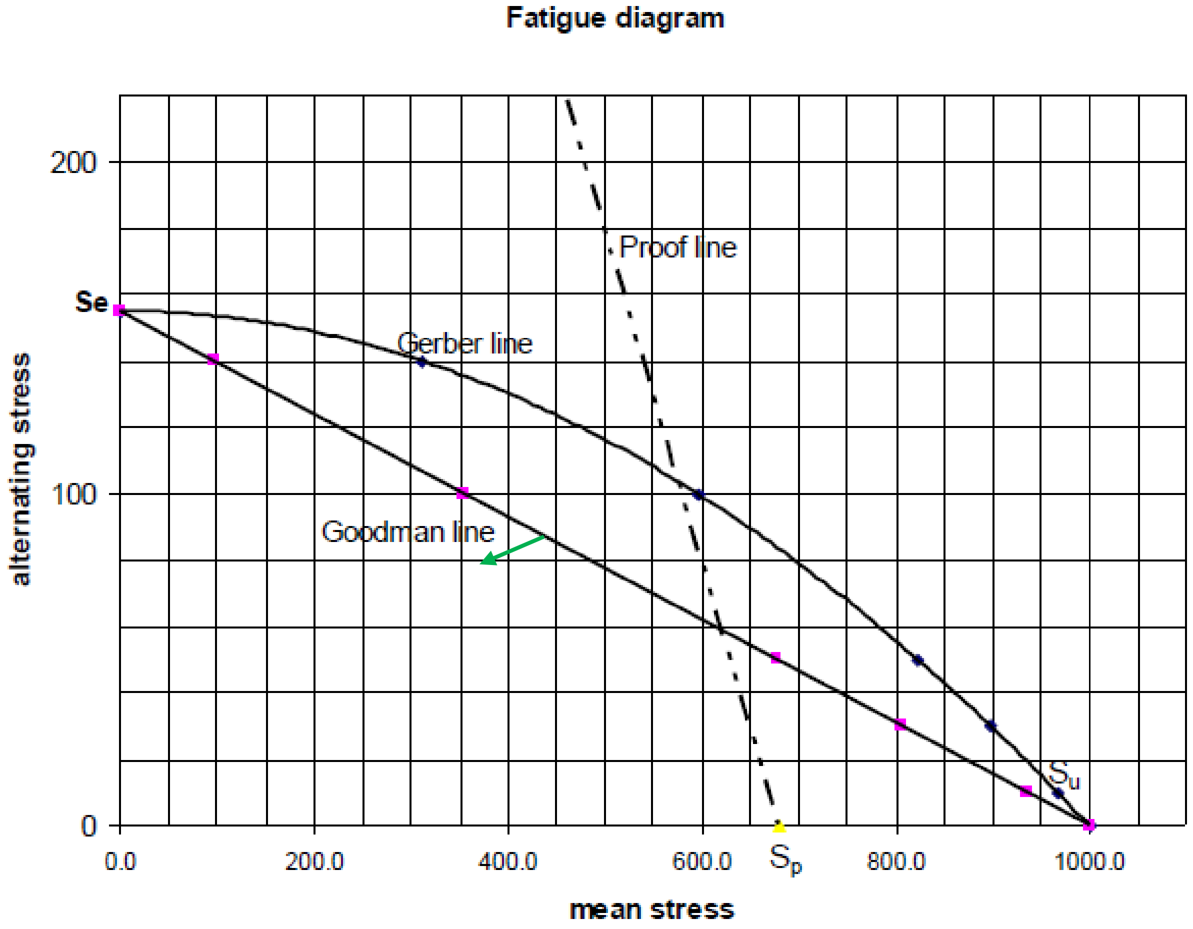
Some texts propose that the ASME ellipse may be used as an alternative to the Gerber parabola.



Fatigue Diagrams

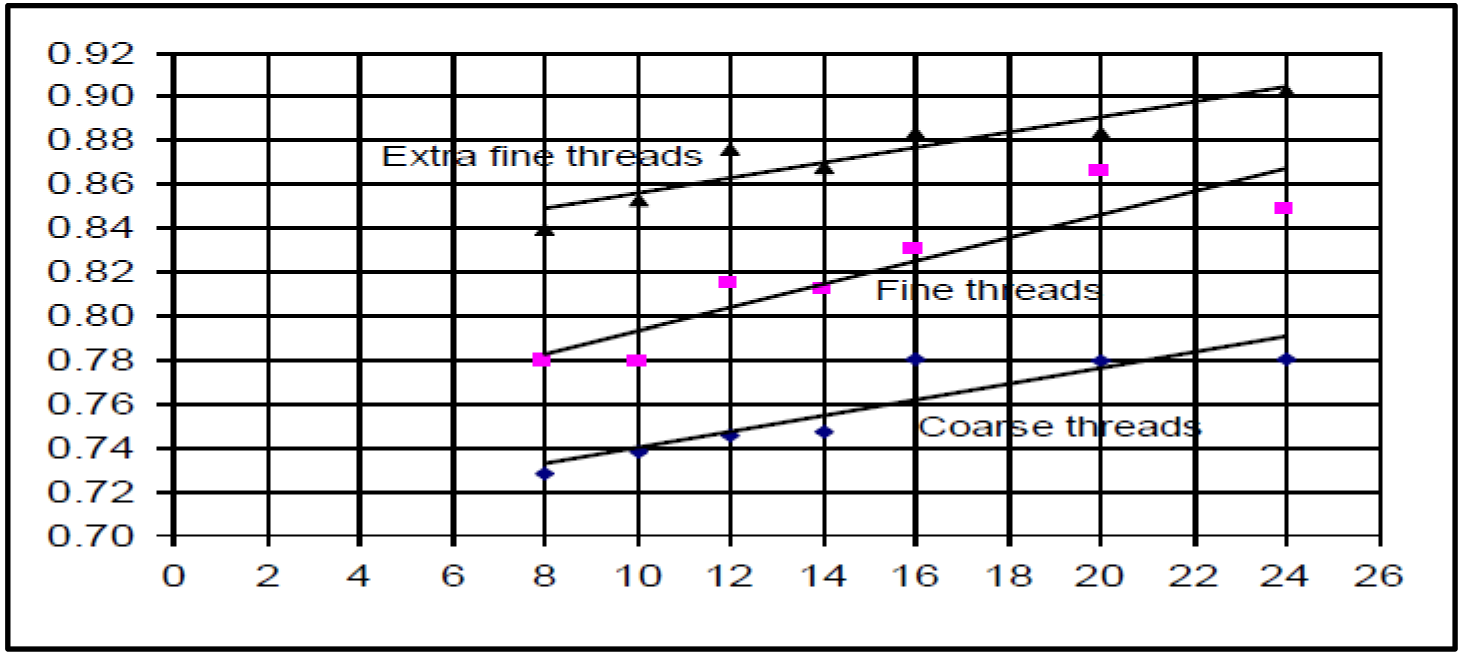
The Goodman line ( ) has often been used in the past to indicate the safe design condition as we now use the Gerber parabola, prior to the adoption of PCs, simply because it has been easier to calculate, being just a straight line.

But, one can see at a glance, that using the Goodman line as a criterion discounts 10 to 30% of available fatigue strength for no good reason, when one has available the sort of numerical solvers that built currently into engineering programs and even Excel.



Loss of Stem Cross Sectional Area Due to Thread Depth

Plot of cross-section of stem remaining, allowing for depth of threads, plotted against major diameter in mm.



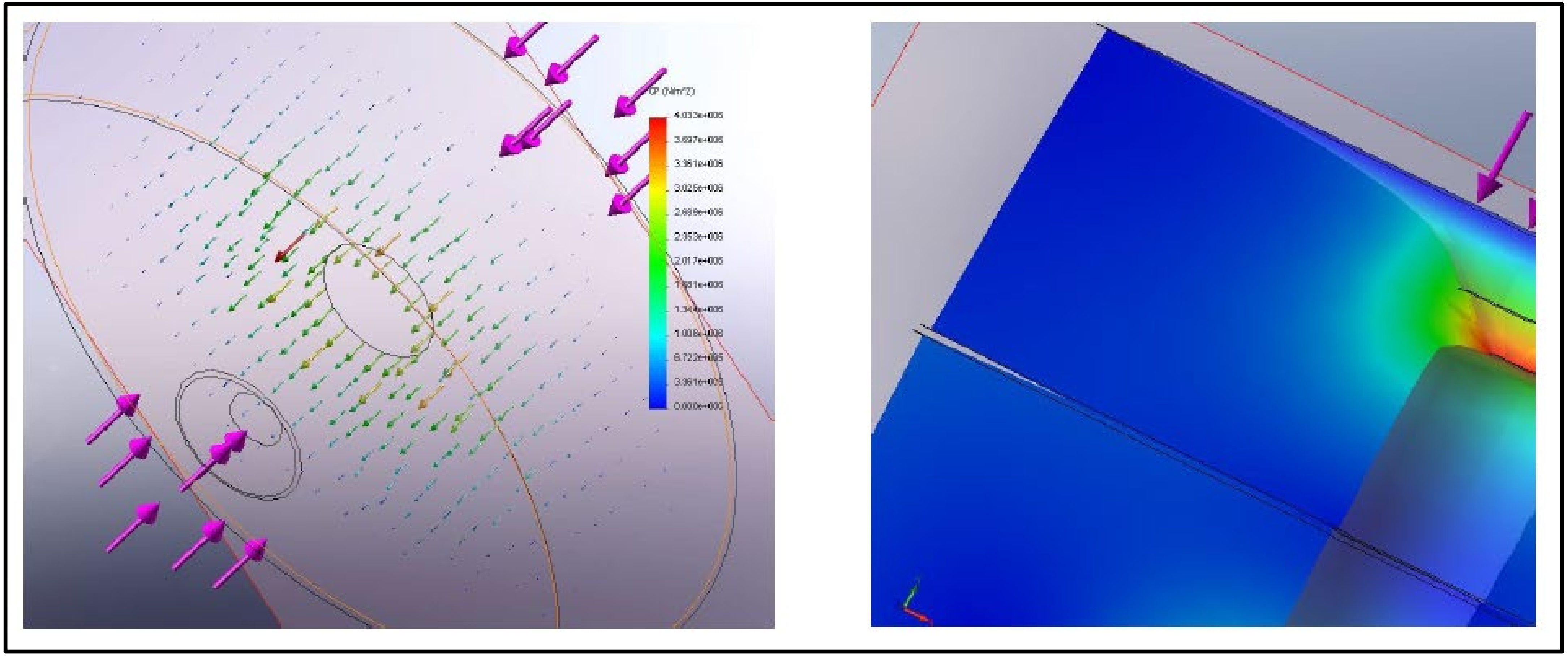
Major Bolt Diameter (mm)

The graph above describes the fraction of cross sectional lost to the thread for coarse, fine and extra fine threads.

At right is a table of the gradients and Y intercepts for the straight line regressions through the points above.

	Extrafine	fine	coarse
m	0.00363	0.00531	0.00406
yintercpt	0.703	0.741	0.817

FEA Results of the Compression Between Two Flat Plates Clamped by a Bolt



Note that the faces of the plates have stretched under compression and have therefore become convex at their interface.