

$1/f$ noise and other slow, nonexponential kinetics in condensed matter

M. B. Weissman

Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

Fluctuations in resistivity with an approximately $1/f$ spectrum are found in most conducting materials. Some of the detailed mechanisms, which turn out to be quite diverse, are explored in this review. Several new techniques are explained. Various theories are critically examined. Some connections with other widespread phenomena, such as highly nonexponential dielectric relaxations, are briefly investigated.

CONTENTS

I. Introduction	537
A. The scope of this review	537
B. General features of the phenomenon	537
C. Open questions and previous reviews	539
II. Theory	539
A. The Hooge approach	539
B. Temperature fluctuations	540
C. The McWhorter model	540
D. The Dutta-Horn approach	541
E. Transport effects	544
F. The relation to universal conductance fluctuations	544
G. General smeared kinetics	545
H. Series kinetics	546
III. New Developments in Basic Techniques	548
A. Non-Gaussian effects	548
B. Symmetry measurements	550
C. Other recent techniques	551
IV. Results	551
A. Spatial cross correlation	551
B. Smeared activated kinetics	552
C. Semiconductors	554
D. Metals and semimetals	559
E. Possible problems for the defect-motion model	563
F. Internal friction	564
G. SQUID's	566
H. Magnetic noise	566
I. Putting the nuisance to work	567
V. Conclusions	568
Acknowledgments	568
References	568

I. INTRODUCTION

A. The scope of this review

Almost any resistor through which current is flowing exhibits voltage fluctuations with a power spectral density inversely proportional to frequency, or nearly so (see Fig. 1). This $1/f$ noise, which in general has not been well understood, is superposed on the Johnson noise that is present for well understood thermodynamic reasons in equilibrium.

Two features of $1/f$ noise are immediately striking. First, its near ubiquity suggests that at least some feature of the explanation should not depend on detailed models of particular materials, since the form of the noise spec-

trum is about the same in metals, semiconductors, semimetals, three-dimensional materials, quasi-one-dimensional materials, and even in more exotic electrical devices such as SQUID's. Second, the spectral form is nearly scale invariant—a tape recording of the noise sounds the same no matter at what speed it is played back. Especially in recent years, such scale invariance, together with the material independence, has inevitably stimulated thoughts of models in which scale invariance enters in a fundamental way.

This review will describe the current state of theoretical and experimental work on $1/f$ noise, with particular regard to the question of how much of the explanation is universal and how much fits into any clear theoretical picture. The path to that rather general goal, however, will take us through some highly specific discussions of different materials and experiments. I will include thoughts on the connections between $1/f$ noise and some related phenomena, some material on $1/f$ noise in devices other than resistors, and some ideas on using $1/f$ noise as a tool for studying other phenomena. Essentially nothing will be included concerning applications of established models to practical devices. Nothing is included on those advances in experimental techniques which do not require explanation for an outsider wishing to understand the data. I shall also avoid discussing $1/f$ noise from other fields, such as astrophysics and geophysics. The references cited will not be exhaustive, but should be sufficient for tracking down the original works. Thanks to a relatively recent review in this journal (Dutta and Horn, 1981) it will be possible (and, for other reasons, necessary) for this review to put relatively little emphasis on introductory material, as compared with current research issues.

B. General features of the phenomenon

Despite some very fundamental disagreements among workers in the $1/f$ noise field, a number of basic features of the phenomenon are agreed upon (see van der Ziel, 1979; Dutta and Horn, 1981; Hooge, Kleinpenning, and Vandamme, 1981; Weissman, 1981a; Nelkin, 1983; Bell, 1985; Kogan, 1985). These are summarized here.

In nearly all resistors, several lines of evidence indicate that resistance fluctuations are present in the absence of a

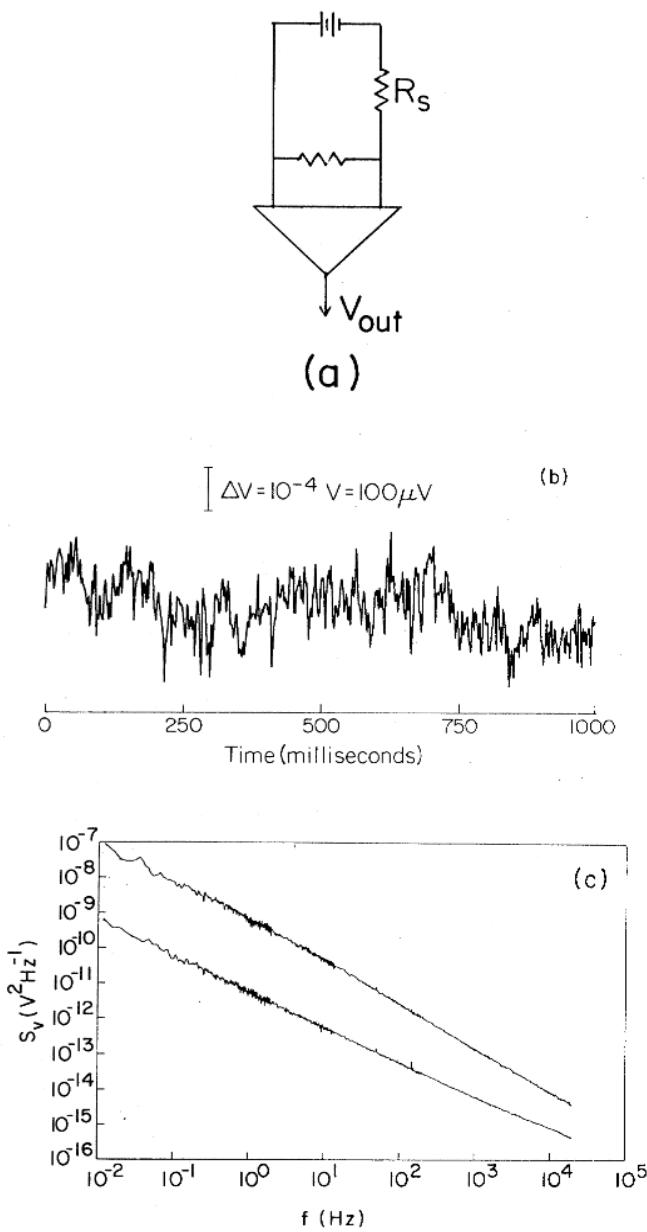


FIG. 1. (a) The basic experimental configuration and typical observations of $1/f$ noise. Schematic diagram of the simplest measuring apparatus for $1/f$ noise. R_s is a large, constant resistor. The unlabeled resistor is the sample. Various modifications, such as the use of ac currents with phase-sensitive detection, bridge circuits, and multiprobe samples, are common. (b) An actual fluctuating voltage from a silicon resistor with about $100 \mu\text{A}$ of current (1 V average bias), measured in a setup like that shown in part (a). (c) Noise spectra from two thick-film resistors, shown over a very broad range of frequencies. The upper plot is taken from an IrO_2 -based film at $T = 556 \text{ K}$, the lower from a ruthenate-based film at $T = 300 \text{ K}$. Each point in each spectrum represents the average square of the Fourier transforms of 1200 1024 point traces, such as that in part (b). Several such spectra, taken at different sampling rates, are stitched together for each broad-band spectrum shown (from Pellegrini, Saletti, Terrini, and Prudenzati, 1983).

driving current. Thus, for small currents, the spectral density of the measured voltage fluctuations is proportional to the square of the current, since $\delta V = I \delta R$. The same resistance fluctuations are obtained regardless of whether they are measured with dc or ac current probes. For some resistors with particularly large $1/f$ noise, the spectral density of Johnson noise (equilibrium noise required by the fluctuation-dissipation theorem) may be used as a sufficiently accurate and fast ohm-meter to allow determination of the $1/f$ fluctuation with no applied bias (Voss and Clarke, 1976; Beck and Spruit, 1978).

Webb and Gershenfeld (1987) have shown that for typical $1/f$ noise sources there are no traces of any deterministic dynamics. Works on how broadband noise appears in strongly driven, deterministic nonlinear systems are not then directly relevant, and would be more properly dealt with in a review on chaos and turbulence.

Thus, except in a few atypical cases, the question becomes why should a material in equilibrium or quasiequilibrium have resistance fluctuating with a $1/f$ spectrum? Furthermore, since there is nothing special about resistance other than its ease of measurement, the question can be generalized to other electrical and mechanical properties.

Although at the time of Dutta and Horn's review in this journal (1981) there was still some evidence for long-range correlations in the noise in one system (Bi) (Voss and Clarke, 1976), even then most evidence (such as the size scaling of the noise spectral density and the independence of the spectral form on geometry) pointed toward essentially local noise sources. In particular, leaving aside disputes over surface versus volume effects, it is agreed that the spectral density ordinarily scales inversely with system size, as expected for local independent sources. More recent evidence indicates local sources hold in general, as will be discussed later.

The actual observed spectra are ordinarily of the form $S(f) \propto f^{-\alpha}$, $0.8 < \alpha < 1.4$ over an extensive frequency range (see Fig. 1). The integral over all frequencies of such a spectrum would diverge. In practice, the absolute magnitude of the spectrum is usually small enough so that the integral of the extrapolated spectrum would not become large enough to be troublesome unless the range of the integral were well outside the observed frequency range, especially when $\alpha = 1$, for which only a logarithmic divergence appears. The high-frequency divergence is not generally troublesome, since finite scattering times, intrinsic capacitative rolloffs, etc., cause a natural high-frequency cutoff regardless of the specific noise mechanism. No such general argument avoids the low-frequency divergence for $\alpha \geq 1$. Thus in evaluating any theory of $1/f$ noise one must keep in mind that some low-frequency cutoff must be present. Such cutoffs can arise from some inherent feature of the model, from limitations imposed by finite system size, or because the theory describes some nonequilibrium system in which things actually do break down on a sufficiently long time scale (e.g., a glass may crystallize).

C. Open questions and previous reviews

Most works on $1/f$ noise address some standard questions: (1) Is there a general formula to describe at least some portion of $1/f$ noise? (2) Is the noise a bulk effect or a surface effect? (3) Are the fluctuations in carrier number or mobility?

The reviews of Hooge (1976), Van der Ziel (1979), and Hooge, Kleinpenning, and Vandamme (1981) are largely organized around these questions. To some extent, the reviews of Dutta and Horn (1981) and Bell (1985) also focus on these questions.

Since, as we shall see, our experimental and theoretical answer to the first question is an unambiguous no, the second and third questions lose the special significance that they take on when one believes that the properties of a single fundamental mechanism are being studied. In fact, we shall see that neither of these questions has a generally applicable answer.

Several reviews (Hooge, 1976; Hooge, Kleinpenning, and Vandamme, 1981) from the Eindhoven group have set forth evidence for the famous Hooge formula

$$S_R(f)/R^2 = \alpha_H/N_c f, \quad (1)$$

with $\alpha_H \approx 2 \times 10^{-3}$. N_c is the number of charge carriers in a homogeneous sample, R is resistance, and $S_R(f)$ is the power spectral density of the resistance fluctuations at frequency f . While this relation is surprising and even somewhat paradoxical, as we shall see, from a theoretical viewpoint, it does give a useful rule-of-thumb estimate of $S_R(f)$ in many materials.

The original data fitting Eq. (1) were taken in Au films doped with enough impurities to lower significantly the mean free path (Hooge and Hoppenbrouwers, 1969). Subsequent data taken on semiconductors could be fit only if a correction were made for the fraction of the scattering due to nonphonon sources, on the assumption that only phonon scattering fluctuated (Hooge and Vandamme, 1978). Although it was not noted, such a correction would have made the original Au data *not* fit the formula. Numerous other data that did not fit the original formula led to several other *ad hoc* corrections and finally to the proposal that α_H be made an adjustable parameter (Vandamme, 1983).

The hypothesis that a fundamental $1/f$ noise source obeying Eq. (1) exists is compatible with the existence of other, perhaps larger, noise sources with similar spectral properties. However, the hypothesis is hard to reconcile with very quiet resistors, particularly those for which none of the *ad hoc* corrections apply. Such resistors are not hard to find; for example, an ordinary commercial silicon wafer with moderately clean surfaces has an α_H value 3 orders of magnitude smaller than the canonical 2×10^{-3} (Black, Weissman, and Restle, 1982). Films of Pt (Scofield, 1985; Scofield, Mantese, and Webb, 1985) and Nb (Scofield and Webb, 1985) are also much quieter than predicted by even modified versions of Eq. (1). Many samples of semiconductors and metals do not show

the $f^{-1.0}$ power law (for examples, see summaries in Fleetwood and Giordano, 1983a; Kogan, 1985; Scofield, 1985). Experimentally, it has become even clearer since the time of the Dutta-Horn review that the Hooge formula has no fundamental significance. This conclusion was also tentatively reached by Van der Ziel (1979).

This review is to a large extent a natural extension of the previous work of Dutta and Horn (1981), which established much of the groundwork for a nonfundamental theory. The questions most actively considered by them will be dealt with briefly, leaving more room for two questions with which they could not deal extensively at that time. These are the following.

(1) What are the detailed mechanisms of the conductivity fluctuations?

(2) Why should these mechanisms, which turn out to be quite diverse, yield such similar spectra?

In addition to the review of Dutta and Horn (1981) and that of Van der Ziel (1979), which focuses on more applied questions connected with semiconductors, a useful review very similar in spirit to this one has recently appeared in Russian (Kogan, 1985). A brief review by Nelkin (1983) anticipated some of the current interest in the connection of multistep kinetics to $1/f$ noise. Bell's chapter on $1/f$ noise (1985) contains some important mathematical and historical background, as well as other material not covered here. An entertaining, informal review by Press (1978) helps to put condensed matter $1/f$ noise in the context of other types of $1/f$ noise. Press also anticipated the importance of higher-order statistics in analyzing $1/f$ noise, although he incorrectly indicated that filtered Gaussian noise should have some filter-dependent properties other than its spectrum. My previous review (Weissman, 1981a) contained some critical analysis of earlier theories, most of which is not included here. One of the best introductions to the processes that dominate the noise in metals is the book on internal friction by Nowick and Berry (1972), which does not actually mention noise.

II. THEORY

We may begin by considering several well-known but incorrect or inapplicable theories. Readers who are unfamiliar with the $1/f$ noise literature are advised to skip to Sec. II.C.

A. The Hooge approach

A large number of theories have been developed in response to the Hooge formula. As we have seen, this formula does not give an accurate picture of the data in any major class of system. Nevertheless, it would not be unheard of for an oversimplified experimental result to inspire a correct or useful theory. In this case, however, one feature of the Hooge formula has led to a set of theories that cannot account for $1/f$ noise. That is, the

factor $1/N_c$ suggests that some independent fluctuations are occurring on each of the mobile carriers—the interpretation of the Hooge group (Hooge, Kleinpenning, and Vandamme, 1981) which has been incorporated into the theories. Precisely this property, however, would be inconsistent with the $1/f$ spectrum itself.

Any fluctuations tied to individual mobile carriers cannot persist for times longer than the carrier remains in the sample. One may easily check that, for almost all metal and semiconductor samples under ordinary operating conditions, the carrier transit time ($\tau_T = l^2/V\mu$) and often also the diffusion time ($\tau_0 = l^2q/kT\mu$) are in the microsecond to millisecond range, which would require the spectrum to flatten out below characteristic frequencies well within the observed range. (Here k is Boltzmann's constant, T is temperature, l is the sample length, μ is the carrier mobility, and q is the carrier charge.) Combining the requirements $f > V\mu/l^2$ and $V^2\alpha_H/Nf > 4kTR$ (the Johnson noise spectral density) gives $V > 4kT/q\alpha_H$, which means that independent-carrier noise with a typical α_H could not produce observable $1/f$ noise at room temperature at any frequency for $V \lesssim 50$ V, a condition that applies in all the experiments described in this review. Thus, although a simple $1/f$ spectrum does not contain enough information to discriminate between many different models, it does contain enough information to rule out easily this common form of general explanation based on independent mobile carriers.

Some of the theories that try to find a fundamental explanation of the Hooge formula are well enough known to make it worthwhile to look at some of their particulars, despite the general argument against such theories as an explanation for $1/f$ noise. The theory of Handel (1980) and the related theory of Ngai (1980) both explain $1/f$ noise as arising from quantum beats between elastically scattered and weakly inelastically scattered carriers, with different inelastic scattering mechanisms used by the two theories. In order to obtain the low-frequency beats, however, monochromatic incident quantum waves are assumed—a completely unrealistic assumption in view of the fact that thermal frequencies are almost always 6–15 orders of magnitude greater than the frequencies in the observed $1/f$ range. Severe problems also arise for the theory when one remembers that the wave packet is localized within the sample, so that the beats, if any, integrate to zero.

The theory of Jindal and Van der Ziel (1981) is related to temperature fluctuation theories in that fluctuations in phonon occupation numbers are the source of the noise. However, the Hooge independence property is preserved by an argument that each carrier interacts with only a small subset of the phonon modes. Low fluctuation frequencies are obtained by extrapolating phonon scattering times as a function of wave vector down to very low wave vectors. Unfortunately the relevant wavelengths are typically larger than the sample size. Since after each carrier scattering event (requiring $\sim 10^{-12}$ s) any carrier

would interact with a different set of phonon modes (within the model), the lowest actual characteristic time would be a carrier scattering time, even shorter than a transit time.

B. Temperature fluctuations

One of the fundamental thermodynamic variables that has to fluctuate in any ordinary experimental ensemble is the energy or, at constant pressure, the enthalpy. So long as the important internal degrees of freedom equilibrate rapidly compared with the heat exchange with the environment, these fluctuations are equivalent to temperature fluctuations, as they are often called. In metals, the integrated spectral density over several decades is not far from that expected for these spontaneous fluctuations, and several experimental observations at one point suggested that enthalpy fluctuations might cause $1/f$ noise in metals, although the spectrum was unexplained (Voss and Clarke, 1976). A number of imaginative theories attempted to show how enthalpy fluctuations caused $1/f$ noise (Liu, 1977; Putterman, 1977; Mikulinsky and Starobinets, 1980; Miller, 1981). However, the temperature dependence of the spectral density ruled out equilibrium enthalpy fluctuations (Dutta and Horn, 1981 and references therein), and the absence of space-time correlations ruled out any enthalpy fluctuations (Black, Weissman, and Fliegel, 1981; Scofield, Darling, and Webb, 1981). A straightforward thermodynamic prediction shows that enthalpy fluctuations are small compared to Johnson noise at all frequencies in the near-linear operating range of a resistor, except when unusually large temperature coefficients of resistivity are present (Weissman, 1979). In such unusual cases this noise is observable, fits the thermodynamic prediction, and is not of a $1/f$ form (Ketchen and Clarke, 1978; Weissman and Dollinger, 1981).

C. The McWhorter model

One obvious way to produce a $1/f$ spectrum is to superimpose a large number of Lorentzian spectra [defined by $S(f) \propto f_c/(f_c^2 + f^2)$] with an appropriate distribution of corner frequencies f_c (Bernamont, 1937; du Pré, 1950; Van der Ziel, 1950). This distribution is $D(f_c) \propto 1/f_c$, where $D(f_c)\Delta f_c$ is the net variance contributed by Lorentzians with corner frequencies between f_c and $f_c + \Delta f_c$.

In some cases this distribution of corner frequencies can be derived from a more physically motivated distribution of some variable on which f_c depends. The first specific version of such a theory is the McWhorter model of charge fluctuations at a semiconductor-oxide interface (McWhorter, 1957). If there are electron traps distributed through the oxide and if the rate-limiting step for the trapping-detrappling process is simple quantum tunneling from the bulk or from fast interface states to the traps,

the distribution of corner frequencies $D(f_c)$ is determined by the distribution of distances l from the traps to the interface, $D(l)$. (I shall not specify different names for these distribution functions when their arguments make the notation unambiguous.)

Since the tunneling rates are of the form $f_c = f_0 e^{-l/l_0}$, one finds

$$D(f) \propto \left| \frac{\partial l}{\partial f} \right| D(l) \propto l D(l)/f .$$

Thus a constant $D(l)$ —i.e., a uniform distribution of traps—leads to $1/f$ noise. Furthermore, physically reasonable values of the parameters, $f_0 \approx 10^{12}$ Hz and $l_0 \approx 10^{-8}$ cm, would require that $D(l)$ be approximately constant for only $10^{-7} < l < 4 \times 10^{-7}$ cm to produce the entire observed spectrum.

By elementary Fermi statistics, only states with energies near the Fermi level have much fluctuation in their occupancy. Therefore the absence of a pronounced temperature dependence in the $1/f$ noise in semiconductors would require that over a range of temperatures about the same number of traps have depths near the Fermi level. Thus a realistic McWhorter model requires that the trapping states have not only a range of positions but also a range of energies. Furthermore, the distributions of these two variables cannot be highly correlated. Otherwise, at a given temperature, since only a narrow range of trap depth would be seen in the fluctuations, only a narrow range of f_c would appear (McWhorter, 1957).

Despite the physical plausibility of the McWhorter model we shall see that few cases are actually explained by it.

In a recent version of the McWhorter model, Pendry, Kirkman, and Castano (1986) have used a rather artificial one-dimensional (1D) disordered potential to calculate a distribution of energy widths of Anderson localized states, from which an occupation fluctuation spectrum was calculated, essentially using tunneling kinetics. An extraordinarily large voltage fluctuation spectrum of a $1/f$ form was predicted. There is little reason to suspect that the prediction would survive conversion to a fully 3D model or that it has any connection with the thermally activated kinetics of the actual trapping states described in the Results section (Sec. IV).

D. The Dutta-Horn approach

More generally, but following the McWhorter argument, one can obtain $1/f$ noise when some parameter appearing exponentially in a rate expression has a flat distribution (Bernamont, 1937). The obvious candidates besides tunneling distances are activation energies in rate expressions with an Arrhenius form, $f_0 e^{-E^\pm/kT}$ (du Pré, 1950). Dutta, Dimon, and Horn (1979), following du Pré, proposed that the nearly $1/f$ spectrum in metals was due to a broad distribution of activation energies for some processes and that this explanation might extend to other

materials. There has been a surprising amount of misunderstanding of this simple proposition, which I shall try to clarify.

The picture underlying the Dutta-Horn interpretation may be introduced by considering a single two-state system. It can be characterized by two energies—the energy difference between the states, ΔE , and the thermal activation energy for making the transition, E^\pm , which is inferred from the temperature dependence of the transition rates (see Fig. 2). In a collection of two-state systems each of these energies is distributed, with joint distribution function $D(E^\pm, \Delta E)$.

Each of these two-state systems will have an exponential autocorrelation function and a corresponding Lorentzian spectrum. The corner frequency $\omega_c = 2\pi f_c$ will be given by the sum of the two switching rates between the two states (Machlup, 1954). The ratio of the time spent in the two states is just the ratio of these two switching rates.

More precisely, the two switching rates between the states will be $2\pi f_0 e^{-(E^\pm + \Delta E/2)/kT}$ and $2\pi f_0 e^{-(E^\pm - \Delta E/2)/kT}$, where we have picked E^\pm to be the energy difference between the transition state and the average of the two metastable states. The corner frequency f_c will be $f_0(e^{-(E^\pm + \Delta E/2)/kT} + e^{-(E^\pm - \Delta E/2)/kT})$, whose temperature dependence is not quite of the Arrhenius form, but nearly so, as long as ΔE is much smaller than E^\pm . The variance in occupancy of either metastable state is $\text{sech}^2(\Delta E/2kT)/4$.

The spectral density of the occupancy fluctuations from one site is given by

$$S(f, T) = \text{sech}^2 \left[\frac{\Delta E}{2kT} \right] \left[\frac{f_c}{f_c^2 + f^2} \right] / 2\pi .$$

The variance or net noise magnitude gives a temperature dependence to $S(f, T)$ from one site via $\text{sech}^2(\Delta E/2kT)$. Kinetic factors give a temperature dependence of $S(f, T)$ via the temperature dependence of f_c . For $f = f_c$, however, $dS(f, T)/df_c = 0$.

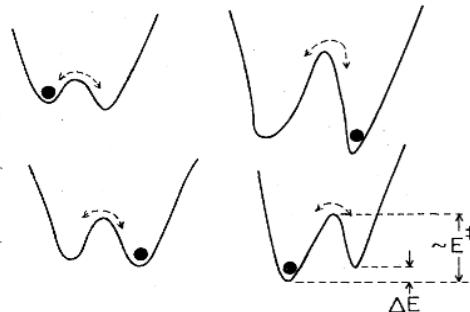


FIG. 2. The distinction between the two energies ΔE and E^\pm for a two-level system in the classical regime, shown for a small collection of such systems. In the quantum regime the configuration coordinate is no longer entirely localized in either well, but the resulting eigenstates are still separated in energy by an amount close to ΔE , so long as the barrier is large.

Particularly for charge-trapping transitions, especially in nondegenerate semiconductors, one must allow for an entropy difference as well as an energy difference between the two states. In such cases ΔF , where F is the Helmholtz free energy, is more relevant than ΔE . If the entropy difference ΔS differed significantly from two-state system to two-state system, then a distribution $D(E^\pm, \Delta E, \Delta S)$ would have to be considered, further complicating matters, but that usually is not the case, since the main term in ΔS is determined by how close the electron bath is to degeneracy. Depending on the details of the mechanical constraints, ΔH (enthalpy) or some intermediate potential could be more appropriate than ΔE , but that distinction will not be important in the following discussion.

In order to obtain much variance from a site, one needs $\text{sech}^2(\Delta F/2kT)$ to be not too much less than one, i.e., $|\Delta F| \lesssim 2kT$. For the McWhorter case $\Delta F = \Delta E - T\Delta S$, where ΔS is not negligible, and therefore different ΔE 's are required for different T .

In most cases of interest for metals, unlike the McWhorter case, the different states have approximately the same entropy, and therefore the usual condition for occupancy fluctuations to occur is $\Delta E \lesssim 2kT$. To obtain $1/f$ noise, one needs a fairly flat $D_1(E^\pm)$ for $E^\pm \sim 20 \pm 2kT$, where

$$D_1(E^\pm) = \int_{-\infty}^{\infty} D(E^\pm, \Delta E) \text{sech}^2 \left[\frac{\Delta E}{2kT} \right] d\Delta E ,$$

the integral essentially selecting only those transitions that are thermally allowed. If the total noise power depends weakly on temperature for some range $T_1 < T_2$, then $D(E^\pm, \Delta E)$ must not vanish for $\Delta E \lesssim kT$, since that would lead to an exponential freezing out of the fluctuations. Overall, there can be little connection between the distributions of E^\pm and ΔE if the spectral form is approximately independent of temperature. The relevant energy ranges for $E^\pm (\gtrsim 10kT)$ and for $\Delta E (\lesssim 2kT)$ do not overlap for $1/f$ noise at any one temperature.

While it is easy to recognize thermally activated kinetics when f_c is unique or narrowly distributed, simply by measuring its temperature dependence, $1/f$ noise presents a more difficult case. When there are features in the spectrum, the temperature dependences of their characteristic frequencies can directly show thermal activation (see Fig. 3). However, the objection is sometimes raised that such features might be unrelated to a featureless underlying $1/f$ noise.

Usually, even when spectral features cannot be resolved at any one temperature, the spectral exponent α is not quite one and is temperature dependent. If the spectral shape results from a broad but not arbitrarily broad distribution of E^\pm and/or f_0 , just such an effect would be expected, since at any temperature T and observation band around some frequency f the spectrum comes mainly from sites with $E^\pm/kT \approx \ln(f_0/f)$. For different values of T , these are different sites and give different spectral slopes.

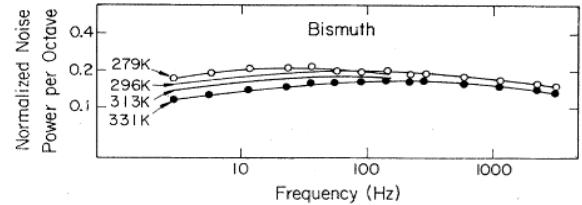


FIG. 3. The spectral density, integrated by octave, shown for a Bi sample at four different temperatures. On such a power-per-octave plot, pure $1/f$ noise gives a horizontal line. Small deviations are thus evident. The normalization used gives a value of 1.0 when $S_R(f) = R^2/(fN_A)$, where N_A is the number of atoms in the sample (from Black, Restle, and Weissman, 1983b).

Dutta, Dimon, and Horn (1979) suggested further that under some realistic conditions these changes in spectral slope should be related to the temperature dependence of the spectral density itself. Qualitatively, if $\alpha > 1$, there is an excess of slow sites (as compared with a flat logarithmic distribution of rates), so raising the temperature, which speeds up the rates, will increase the spectral density. Of course, in order to observe this temperature-dependent spectral density, it is necessary that it not be obscured by the dependence of $D(E^\pm, \Delta E)$ on ΔE or by a strongly temperature-dependent coupling of the underlying transitions to the observed resistivity.

More quantitatively, Dutta, Dimon, and Horn (1979) showed that under somewhat restrictive conditions a specific relation between the spectral slope and the temperature dependence of the spectral density can be derived. They explicitly assumed that the fluctuations have (1) thermally activated kinetics, (2) a single $f_0 \gg f$ (observational), (3) a smooth $D_1(E^\pm)$ and implicitly, (4) $D(E^\pm, \Delta E) = D_1(E^\pm)\delta(\Delta E)$, and (5) no temperature dependence of the effect of the transitions on the logarithm of R , such as can arise from temperature-dependent scattering mechanisms. These conditions imply

$$\int_f^{\infty} S(f') df' \propto \int_0^{kT \ln(f_0/f)} D_1(E^\pm) dE^\pm . \quad (2)$$

Here $S(f) = S_R(f)/R^2$. Differentiating twice, we obtain the Dutta-Horn relation

$$-\frac{\partial \ln S(f, T)}{\partial \ln f} = 1 + \frac{1}{\ln(f_0/f)} \left[\frac{\partial \ln S(f, T)}{\partial \ln T} - 1 \right] . \quad (3)$$

Of the four conditions, (1) and (3) are likely to be well satisfied in practice. Condition (2), as we shall see, is not generally met. However, even in the extreme limit that E^\pm is not distributed, but f_0 is, we obtain a similar equation:

$$-\frac{\partial \ln S(f, T)}{\partial \ln f} = 1 + \frac{kT}{E^\pm} \frac{\partial \ln S(f, T)}{\partial \ln T} . \quad (4)$$

Scofield, Mantese, and Webb (1986) have shown that Eq.

(4) holds exactly, regardless of the distribution of the f_0 , unlike Eq. (3), which requires a smooth distribution of E^\pm . This relation is not easily distinguished from Eq. (3) unless data are taken over a wide temperature range (Weissman, 1981a; Scofield, Mantese, and Webb, 1986). Equation (3) does give a linear dependence of $S(f, T)$ on T for pure $1/f$ noise, which results from the narrowing of the distribution of characteristic times as the temperature is increased. Such an effect is of course absent for distributed temperature-independent prefactors. However, due to the poor accuracy with which condition (4) is likely to hold, such relatively weak temperature dependences are not likely to be easily observed.

[Scofield, Mantese, and Webb (1986) have shown, however, that for noise from H diffusion in Nb, Eq. (4) gives a distinctly better fit than Eq. (3), which is to be expected since the diffusion constant has a single activation energy and the shape of the spectrum is determined by temperature-independent geometrical factors. However, they caution that the relatively sharp corner in the diffusion spectrum made this distinction much easier than for typical $1/f$ spectra.]

Conditions (4) and (5) are the weak point of the assumptions leading to the Dutta-Horn (DH) relation. In general, the coupling of the fluctuations to the measured parameter will have some temperature dependence. For example, in metals, impurity scattering is likely to fluctuate more than phonon scattering, and the different temperature dependences of these two scattering rates would lead to an additional temperature dependence of $S(f, T)$. For nondegenerate semiconductors with carrier trapping noise, the dependence of $D(E^\pm, \Delta E)$ on ΔE gives another temperature dependence to $S(f, T)$ quite independent of $D_1(E^\pm)$. As we shall see, the experimental results for the two sides of Eq. (3) commonly are offset by a quantity that varies slowly with temperature but still allows one often to detect common structure to both terms.

Nevertheless, very strong deviations from the DH relation are rare. Since many processes in condensed matter, such as defect creation-annihilation, inherently involve $\Delta E \gg kT$, such processes would give rise to noise not even approximately obeying the DH relation. The offset between the two sides of Eq. (3) for a process with an activated net noise magnitude (i.e., $\Delta E > kT$) is $\sim \Delta E/kT$, which grows rapidly as the temperature is lowered. The persistence of $1/f$ noise roughly obeying the DH equation over a wide range of T then implies that the transitions involved either have very small ΔE for some fundamental reason (i.e., symmetry) or by coincidence (unlikely for many systems) or at least have a spread of ΔE extending to zero. Some confusion on this point has arisen, in part because, in a paper preceding that of Dutta, Dimon, and Horn (1979), Eberhard and Horn (1978) had suggested that the temperature dependence of the spectral density might be due to ΔE . This suggestion was superseded by the later interpretation.

As recognized for some time, then, the Dutta-Horn relation is not quite a sufficient condition to imply that the

$1/f$ spectrum in a particular system arises from $D_1(E^\pm)$. However, its applicability to a wide number of systems, which we shall discuss under Results, strongly supports the thermally activated kinetics picture, generalized to allow some spread in f_0 . The Dutta-Horn relation is also very far from being a necessary condition for distributed thermally activated transitions, and its prevalence thus has the strong additional implications for the dependence of $D(E^\pm, \Delta E)$ on ΔE discussed above. We now explore these additional implications.

There is a well-known collection of excitations that have a fairly flat spread of ΔE around zero. These are the two-level systems (TLS) familiar from the heat capacity and other measurements of amorphous materials (Anderson, Halperin, and Varma, 1972; Phillips, 1972), which have also been suggested as sources of $1/f$ noise (Black, Restle, and Weissman, 1983a, 1983b; Kogan and Nagaev, 1984). In general, one expects some coupling between these atomic motions and any measured parameter, including ρ . Thus any material with amorphous regions should show some resistivity fluctuations. Although those TLS with $\Delta E \gg kT$ will freeze out into their low-energy state, the remainder will fluctuate freely, and their number decreases only linearly in T . At low temperatures, where the TLS kinetics are tunneling dominated, a distribution of tunneling parameters should give $1/f$ noise (Ludviksson, Kree, and Schmid, 1984). Similar arguments suggest that just as ΔE is distributed, so should E^\pm be, in the thermally activated range, again giving $1/f$ noise (Black, Restle, and Weissman, 1983a; Kogan and Nagaev, 1984). Of course, the reduction of the $1/f$ problem to the amorphous TLS problem in some systems would not be a complete solution, since the nature of the TLS in various systems is not well known.

As we shall discuss, there are cases (most clearly, Cr and Bi) in which $1/f$ noise almost certainly comes from crystalline, though not defect-free, regions. In these cases both the spread in E^\pm and the existence of noise-generating processes with small ΔE require some discussion. Carrier trapping and defect creation-annihilation do not generally have small ΔE , and thus are not capable of explaining $1/f$ noise in metals. However, given the existence of impurities or other defects, the translational and rotational symmetries of a crystal require that there be energetically degenerate sites for the defect (Kogan and Nagaev, 1982). If we can account for both the spread in E^\pm and the inequivalence of these sites in their effect on ρ , then the Dutta-Horn conditions follow naturally.

In general we can expect that simple translational diffusion is not particularly relevant to much $1/f$ noise because the same symmetry that allows one to find a set of degenerate states also causes them to have the same ρ . Only if the diffusion occurred over distances comparable to the sample size—a special case to be discussed—would translational diffusion affect R in a good crystal. If defects are densely enough located for defect-defect interaction or interference terms to show up in ρ , transla-

tional diffusion may again matter (Weissman, 1978, 1981a; Yakota, 1980), but in this case the energy degeneracy may also be lifted, so that the picture is in between that for good crystals and that for amorphous regions. In such relatively dirty crystals, a spread in E^\pm is not especially surprising. Scofield and Webb (1985) have emphasized that defects come in various sizes, so that a substantial spread in characteristic frequencies can arise for defect diffusion due to the range of length scales.

Rotational diffusion of defects, on the other hand, would affect ρ even in a good crystalline region, since many defects are local symmetry breakers. In the simplest case, first discussed by Kogan and Nagaev (1982), exactly the same motions responsible for the Snoek mechanism of internal friction (see, for example, Nowick and Berry, 1972) would cause low-frequency noise that would persist almost to $T=0$ and that would have activated kinetics at ordinary temperatures (see Fig. 4). Such noise would have a distinctive symmetry signature, which we shall discuss later. The most puzzling aspect of such low-frequency noise, however, would be the $1/f$ spectrum itself, since each similar defect should have the same rotational diffusion rate. This sort of physical realization of the Dutta-Horn picture requires postulating of some inhomogeneities (e.g., local strains) to smear out $D_1(E^\dagger)$. Some such smearing is, however, routine in internal friction results (Nowick and Berry, 1972), so that we at least do not require an entirely new postulate for the noise.

E. Transport effects

One familiar class of noise that has occasionally been invoked to account for $1/f$ noise is transport noise, caused by the exchange of some conserved quantity between the system and an external reservoir. Energy ("temperature") fluctuations are one example, already discussed. Carrier number fluctuation noise can be a form of transport noise, when Coulomb interactions do not effectively suppress charge exchange with the outside. In the special limit where the kinetics of this exchange are dominated by electromigration rather than by thermal diffusion, a form of shot noise results. Another transport noise would result from defect motion, such as H diffusion and electromigration in Nb (Scofield and Webb, 1985).

Long-range defect motion has several properties in common with the sorts of defect rotations and TLS transitions that fit the DH picture. It would not freeze out at low temperatures, but would slow down due to activated kinetics. However, the spectral shape of such transport noise may be determined less by the range of activation energies than by the sample geometry. In order to move very far, a defect must make many individual activated steps, so that one sees an averaged diffusion coefficient even if there are local variations in the activation energy or prefactor. However, the spectral shape for diffusion kinetics is not a simple Lorentzian, so that $1/f$ noise

from diffusion processes cannot be ruled out *a priori*.

The spectra for number fluctuations by diffusion in and out of a rectangular solid with three widely separated lengths ($l_1 \gg l_2 \gg l_3$) have four distinct regimes (see, for example, van Vliet and Fassett, 1965), ranging from f^0 to $f^{-3/2}$. Transport noise over distances comparable to sample dimensions would be detectable by space-time cross correlations, which are rarely found, and, so far as I know, never found in systems with good $1/f$ spectra. Furthermore, it would be very surprising for a near-universal phenomenon such as the $1/f$ spectrum to depend on sample geometry. As mentioned, diffusion over short distances can cause noise when translational symmetry is broken in the material. While $1/f$ -like spectra could be obtained for geometrical reasons for some defect-defect interaction terms (Weissman, 1977, 1978), the spectral range would be severely limited by some scale-breaking dimensions such as mean free paths and grain sizes. In practice, defect-defect interaction transport noise is mostly easily thought of as a variant of DH noise in which geometrical factors as well as $D(E^\dagger)$ contribute to smearing the spectrum (see, for example, Scofield and Webb, 1985).

F. The relation to universal conductance fluctuations

Recently, Feng, Lee, and Stone (1986) have pointed out an interesting connection between $1/f$ noise and the "universal conductance fluctuations" observed in the magnetoresistance of materials with strong impurity scattering at very low temperature. The random component of the magnetoresistance results from the details of the arrangement of the impurity scatterers. In 1D and 2D materials each individual scatterer contributes to the random term in the resistance a non-negligible but random amount, regardless of the density of inelastic scatterers. Thus if a single scatterer moves a distance of an inverse Fermi wave vector, the entire random term in the conductance ($\sim e^2/\hbar$) in each inelastic scattering region in 1D and 2D can be substantially changed.

Interesting effects can occur in the spectrum if there are actually enough mobile sites for a significant fraction

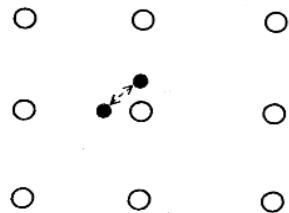


FIG. 4. The motion of an interstitial impurity between two energetically equivalent sites that have different effects on the tensor ρ . Such mechanisms are also responsible for Snoek internal friction, due to the inequivalence of effects of the two sites on the strain tensor.

of the universal conductance fluctuations to appear on an experimental time scale (Weissman, 1987). A small number of fast-relaxing sites are sufficient to scramble the interference terms that give rise to the random conductance. Slower-relaxing sites are then unlikely to contribute any more to the decay of the conductance autocorrelation function. Quantitatively, a distribution of single-site relaxation rates that would give $1/f$ noise if the sites contributed independently to the random conductance will give $S_R(f) \propto f^{-\alpha}$ with $\alpha < 1$ and with $1-\alpha$ being proportional to a normalized density of mobile sites (Weissman, 1987). Distributions of single-site relaxation rates that would give $S_R(f) \propto f^{-\alpha}$, $\alpha > 1$, for independently contributing sites give stretched exponential autocorrelation functions.

In the high-temperature, 3D limit, the noise extrapolated from the low-temperature regime appears to be smaller than the typical observed noise. Noise due to rotation of single symmetry-breaking defects, to pairwise interference terms in the scattering between defects, and to local motions in the asymmetrical environment of extended defects does not fall off as rapidly with increasing temperature as does the "universal" term, which depends on many elastic scattering events. Calculations by Pelz and Clarke (1987) show that the "universal" term is much smaller than the other terms, which they dub "local interference," except in samples that are unusually cold and dirty. Hershfield (1987) has calculated that even for randomly located pointlike scatterers, ordinary interference terms (Born approximation) give larger noise than does the "universal" term for ordinary room-temperature metals. Since the "universal" term arises from multiple elastic scattering events, which are rare when phonon scattering dominates, it virtually disappears in ordinary room-temperature metals.

G. General smeared kinetics

Given the diversity of systems that have $1/f$ spectra, it is tempting to search for some general argument, perhaps analogous to the central limit theorem, which would give the correct distribution of characteristic rates. One possible connection (Weissman, 1981a) is that any rate determined by a large number of independent factors should, with the usual caveats, have a logarithm-normal distribution. That is, since the logarithm of the net rate is the sum of the logarithms of the separate factors, it will have a normal distribution independent of the details of the individual distributions. If the breadth of this normal distribution is large enough, many octaves of $1/f$ noise will appear, corresponding to the flat top of the logarithm-normal distribution. Of course an infinite number of octaves of white noise and of f^{-2} noise also occur, corresponding to the tails of the distribution. In fact, the derivative of $\partial \ln S(f) / \partial \ln f$ with respect to $\ln f$ is greatest for such a logarithm-normal distribution precisely when $\partial \ln S(f) / \partial \ln f = 1$. Thus there are more octaves with $\partial \ln S(f) / \partial \ln f \approx -\alpha$ for any α with $0 < \alpha < 2$ and

$\alpha \neq 1$ than for $\alpha = 1$. A single such logarithm-normal distribution, with its center randomly located with respect to the observed frequency range, would be unlikely to give $1/f$ noise, even though most of its noise power would be found in regions of the spectrum with $\partial \ln S(f) / \partial \ln(f) \approx -1$.

One could avoid this problem by postulating a superposition of many distributions for different processes with different mean logarithm rates, so that the tails of each distribution are covered by the others. Then only the relatively flat parts of the distributions would be observable. The problem then would just become one of accounting for the overall distribution of mean logarithm rates of the separate processes.

The distribution of characteristic logarithm rates should then have at least one of the following properties:

- (1) It should be broad, with a center rate somewhere not too far from the accessible observation range, or
- (2) it should have tails that decrease less rapidly than exponentials, e.g., as power laws.

The first property has no connection with the multiple-factor central-limit-theorem approach discussed above; the second contradicts it to the extent that the central limit theorem is applicable.

Montroll and Shlesinger (1982) considered similar multiple-factor models that might appear in some sequential processes. They pointed out that anomalous tails of otherwise normal distributions are well known in the social sciences, and that such tails might be connected with $1/f$ noise.

Bak, Tang, and Wiesenfeld (1987) have suggested that systems which cease equilibrating at some minimally stable configuration will resemble percolation clusters in that they will have dilation symmetries rather than characteristic length scales. They suggest that spectra of the form $f^{-\alpha}$ then arise via a standard power-law relation between length and time. Whether or not such considerations are relevant in other fields, they seem not to be related to most $1/f$ noise in condensed matter, for which the spectral shape is remarkably weakly dependent on whether a material is amorphous or crystalline, how much it has been annealed, whether it is a true equilibrium state, etc. The only experimental evidence cited by Bak, Tang, and Wiesenfeld is the observation of some frequency-dependent spatial correlations in Bi films (Voss and Clarke, 1976). They suggest that more experiments of that type be undertaken. As discussed in Sec. IV.A, many such experiments have already been made, and all produced negative results, including those made on Bi.

The $1/f$ spectrum may be viewed as an extreme limiting case of many types of broadened kinetics, including those with power-law autocorrelation functions and stretched exponential autocorrelation functions, $e^{-(t/t_0)^\beta}$ with $0 < \beta < 1$ (Montroll and Bender, 1984). For any spectrum to be normalizable, it must have a low-frequency regime that is flatter than $1/f$ and a high-frequency regime that is steeper. All the standard broadened kinetic forms have a smooth, monotonically

increasing spectral slope, which must then pass through a $1/f$ regime, which has an extent that depends on the parameter, such as β , describing how much the spectrum is broadened. Such kinetics, especially stretched exponentials, arise in a variety of phenomena, including dielectric relaxation in insulators (Jonscher, 1977), viscous damping in glasses (Wong and Angell, 1976; Brawer, 1984), and magnetization relaxation in spin glasses (Chamberlin, Mozurkewich, and Orbach, 1984). They have inspired a number of theoretical approaches to explanations in which the scaling behavior of the kinetics rather directly reflects some underlying scale invariance of the physical system, not simply an unspecified broad distribution of parameters entering exponentially into rates.

The extent of the similarity between $1/f$ kinetics and those of low-frequency dielectric relaxation is not widely appreciated. It was brought out by Ngai (1980), but unfortunately in the context of a theory which, as discussed in Sec. II.A, was untenable. The majority of the dielectric relaxations cited by Jonscher (1977) have extended high-frequency tails corresponding to fluctuation power spectra of the form $f^{-\gamma}$, $1 \leq \gamma \leq 1.2$. The principle difference from $1/f$ noise kinetics lies in the presence either of low-frequency rolloffs or of a supervening conduction process at low frequencies. From measurements of $S(f, T)$ over a wide temperature range, Dutta and Horn (1981) inferred the presence of low-frequency rolloffs in a number of $1/f$ noise sources. The presence of a masking long-range conduction process in some cases is irrelevant to the spectrum of local rearrangements.

I shall describe theories of stretched exponentials, etc., since they may prove useful in understanding some cases of $1/f$ noise. More importantly, experimental findings and techniques from the $1/f$ noise field have direct bearing on the popular hypothesis of Jonscher (1977)—that these smeared kinetics must come from some many-body dynamical effects rather than from quasistatic inhomogeneities in single-site environments.

The TLS picture and the rotational diffusion picture both involve transitions between a small number of possible states. Only by adding noise from many independent systems with somewhat different parameters is $1/f$ noise obtained. Scaling theories in general involve series, or at least coupled processes to generate the spectral form rather than independent, parallel, but distributed processes, such as in the TLS or rotational diffusion variants of the Dutta-Horn picture. At least at a fixed temperature, spectral form cannot be used to distinguish between series and parallel models. As we shall see, other experimental tests can be used to make that distinction.

Two rather specific physical models have been found to imply stretched exponential relaxation. If a spin system is relaxed by dipole-dipole interactions with dilute spins of another type randomly located in a three-dimensional space, one obtains by simple argument $\beta = \frac{1}{2}$ (Tse and Hartmann, 1968). Several variants of this basic pattern have been observed experimentally (e.g., Compaan, 1972). If some sites are relaxed by short-range in-

teractions with randomly located diffusing objects in d dimensions, one obtains $\beta = d/2$ for $d < 2$ (Bordewijk, 1975). Even in three dimensions, a power-law distribution of waiting times between hops for the diffusing objects can produce stretched exponential relaxations (Shlesinger and Montroll, 1984). In this last case, the fact that relaxation occurs as a result of a series of steps gives a narrower distribution of relaxation times than is found for the steps themselves. This pattern may be suggestive for other cases in which $1/f$ noise, or a corresponding relaxation with an extended logarithmic regime, is found for some parameters but stretched exponential relaxation for other parameters in the same system.

The dipole-dipole and contact-diffusion stretched exponentials arise by very similar mathematics, which may be easily generalized. Relaxing centers relax increasing volumes of space around them as a function of time. For the dipoles this volume $B(\tau)$ increases as $r^3(\tau)$ where $r(\tau) \propto \tau^{1/6}$, giving $B(\tau) \propto \tau^{1/2}$, since the dipole-dipole matrix element squared is proportional to r^{-6} . For the contact diffusers, $B(\tau) \propto \tau$ for $d \leq 2$ and $B(\tau) \propto \tau^{d/2}$ for $d > 2$. The probability of a particular site's not being relaxed is proportional to $e^{-n_R B(\tau)}$ where n_R is the concentration of relaxers, since the relaxed volumes are randomly located and may overlap. Clearly the essential content is simply that the relaxed volume around each of the random initial sites grows as a power β of time.

The essential difference between $1/f$ kinetics arising from parallel processes at different localized defects in crystals and stretched exponential kinetics from similar processes may just be that, while in the former neighboring defects affect the logarithm of a relaxation rate that scales as a power law in the distance, in the latter the rate itself scales as a power of that distance.

H. Series kinetics

Series kinetics models are not nearly so simple. By series kinetics, I mean processes in which many steps with comparable rates must occur before a particular transition that affects the measured variable happens. The intervening steps may themselves be transitions of the type detected in the measurement or they may be unobserved. Such models share the property that one cannot isolate discrete sites contributing only to a narrow range of the fluctuation spectrum, but beyond that there is much diversity. I shall discuss only specific models that have already appeared in the literature, to give some sampling.

One obvious generalization of a TLS model is to allow for n -state systems. For small n , no important qualitative changes result except for samples so small that one can observe individual sites, or at least the statistical properties of small numbers of sites. Marinari, Parisi, Ruelle, and Windey (1982) have considered a model in which n becomes infinite. The measured variable is the position of a random walker on a suitable 1D random potential,

so the model will be called a random walk in a random potential (RWRP). When the roughness of the random potential scales as a positive power of the distance scale, a $1/f$ spectrum results at low frequencies. Although the model is very different from those considered by Dutta and Horn, it still obeys the same relation between $\partial \ln S(f, t)/\partial \ln f$ and $\partial \ln S(f, T)/\partial \ln T$. The typical configuration "distance" required to find a bump in the potential of height E^\dagger is given by some monotonic function $l(E^\dagger)$, so that the typical time-distance scaling is given by $\tau(l) \approx eE^\pm(l)/kT \times \tau_0$ or $l(\tau) \approx l(kT \ln \tau/\tau_0)$. The original version assumed a power-law form for $l(E)$, but we need not restrict ourselves to that case. The mean-square distance traveled after time τ is then $l^2(\tau) \approx l^2(kT \ln \tau/\tau_0)$. This is precisely the DH form where $l^2(kT \ln \tau/\tau_0)$ replaces $\int_0^{kT \ln \tau/\tau_0} D(E^\pm) dE^\pm$ —simply a change of notation as far as the mathematics are concerned. Therefore the DH equation holds for this RWRP picture (Restle, Hamilton, Weissman, and Love, 1985).

In the RWRP there is no distinction between the transition states and the ensemble, although that distinction is a crucial element of the simple TLS realization of the DH equations. The temperature independence of the net noise magnitude is preserved because the accessible low-temperature ensemble is just as spread out in configuration space as the accessible high-temperature ensemble, although the density of coverage decreases at low temperatures.

At any rate the simpler versions of the DH picture cannot be distinguished from the more elegant, if less clearly physical, RWRP by measurements of the spectrum as a function of temperature. Experimental methods for distinguishing between such models will be discussed in the non-Gaussian effect section (III.A).

An obvious disadvantage of the RWRP theory is that there is no known physical quantity that can play the role of the configuration coordinate in most systems, while numerous defect transitions could fit the simpler versions of the smeared activated kinetics models. The linking of many transitions into a one-dimensional random topography in configuration space introduces the scale-similar property of the noise spectrum at an early stage of the model at some cost in plausibility, particularly since, in configuration dimensions of two or more, the $1/f$ property does not follow from the RWRP model (Fisher, 1984).

Several groups have looked into various forms of nonexponential kinetics arising in Ising models that lack direct thermodynamic cooperativity between the spins but that do have kinetic cooperativity, in that the rate for any one spin flipping depends on the orientations of its neighbors. Such models may provide a simple picture for the many-body effects postulated by Jonscher (1977) to play a key role in kinetics with extended scaling regimes. Furthermore, in evoking only kinetic cooperativity, such models may shed light on the special features of glass transitions, in which some degrees of freedom freeze out

despite the absence of any latent heat or readily identified order parameter. The Ising spins are intended to represent all sorts of local transitions, not necessarily real spins.

Palmer, Stein, Abrahams, and Anderson (1984) suggested a model with hierarchically arranged spins, each of which would inhibit flipping of its neighbors in the next higher level if it were "down." For an infinite hierarchy, an individual site can be inhibited by an arbitrarily long chain of sites down the hierarchy and thus may have to wait arbitrarily long before it can flip. For some ranges of parameters, scaling behavior can be obtained. Depending on how many sites can inhibit a given site, one can either obtain ergodic behavior—any state reachable in a finite time—or nonergodic behavior. Possible connections with the glass transition were discussed, but not shown to be obtainable from any self-consistent assignment of rates and energies. The hierarchical picture itself is not clearly justified except in special cases.

Frederickson and Andersen (1984) proposed a nonhierarchical kinetic Ising model in which the flip rate for any site depends on the states of the neighbors. By assigning a lower energy to the "down," or flip-inhibiting, state, they obtained characteristic times that increase rapidly as the temperature is lowered. Although it was originally suspected that a sudden onset of nonergodic behavior might be found below some temperature, detailed Monte Carlo and analytic studies in two dimensions (Fredrickson and Brawer, 1986) show no such transition. Instead a dramatic finite increase in the typical characteristic times coupled with highly nonexponential kinetics was found. They observed stretched exponential parameters β as low as 0.10 in the temperature range in which the characteristic times slowed more rapidly than in an Arrhenius law. Such kinetics in a noise spectrum would easily pass as $1/f$ noise.

Despite the differences in these two kinetic Ising models, a common distinguishing trait emerges, which is likely to characterize most similar theories. Each individual site makes highly non-Lorentzian contributions to the equilibrium fluctuations because its characteristic relaxation rate is not fixed, but changes as the states of the neighbors change. Obviously one can check whether this changing of characteristic rates is occurring in any system in which the switching of individual two-state systems can be followed. We shall see that several systems that give $1/f$ noise show simple exponential behavior of their constituents and therefore are not to be described by any model in which variable rates are invoked. More subtle experimental implications will be discussed in the non-Gaussian effects section (III.A).

One interesting class of models has been considered that shows a continuous transition from essentially parallel kinetics to highly coupled kinetics as a function of a parameter describing the overall noisiness of the system. Nelkin and Harrison (1982) and Machta, Nelkin, Nieuwenhuizen, and Ernst (1985) have investigated the conductivity fluctuations in trapping models for nonin-

teracting charge carriers, a generalization of early work on noise in hopping conduction (Tunaley, 1976). When the carriers are mostly in the conducting state, one recovers a McWhorter-type model in which the net noise spectrum is just the sum of the spectra of fluctuations in occupancy of the individual traps. However, as the trap density is raised, the faster traps buffer the effects of the slower traps on the conducting state. The resulting spectrum of conductivity fluctuations is thus weighted more heavily toward high frequencies than is the spectrum obtained from the raw distribution of individual trap times. Although in a realistic semiconductor trap model at room temperature the trap density would not approach that required for these effects to show up directly, Coulomb interactions between sites can give some of the same buffering behavior. In general, such effects appear when a perturbation of the conductivity can relax by alternate routes—at high densities the fast mechanisms screen the slower ones.

Queisser (1985) discussed the logarithmic recovery of surface charge in GaAs devices, equivalent to a $1/f$ fluctuation spectrum, in terms of series kinetics models. However, the actual form of the recovery curve predicted was entirely determined by the use of a McWhorter distribution of trapping times. The introduction of series kinetics terminology did not lead to any experimental prediction.

III. NEW DEVELOPMENTS IN BASIC TECHNIQUES

A. Non-Gaussian effects

It is customary to characterize random noise by the power spectrum $S(f)$ alone. Clearly such characterization does not in general obtain the maximum possible information from a signal. One does not, for example, identify bird whistles, automobiles, and rustling leaves as separate contributions to a sound by averaging the intensity at several frequencies and then picking out characteristic frequencies. Similarly, the signal in Fig. 5 obviously differs in some important way from a more typical noise with the same Lorentzian spectrum.

The property that many noise sources share which makes it futile to measure properties other than their power spectrum is called Gaussianity. Mathematically, Gaussianity means that all multipoint correlation functions $\langle V(t)V(t+t_1)\cdots V(t+t_{N-2})\rangle$ can be expressed by decomposing them into the sum of all possible products of pairwise correlation functions (see, for example, Nelkin and Tremblay, 1981). Thus the pairwise correlation function (the Fourier transform of the power spectrum) contains all the available information. This property can arise in many ways, but the most common way is by the superposition of many independent sources, each of which contributes negligibly to the variance, in close analogy to the central limit theorem.

An extreme case of a non-Gaussian random signal is

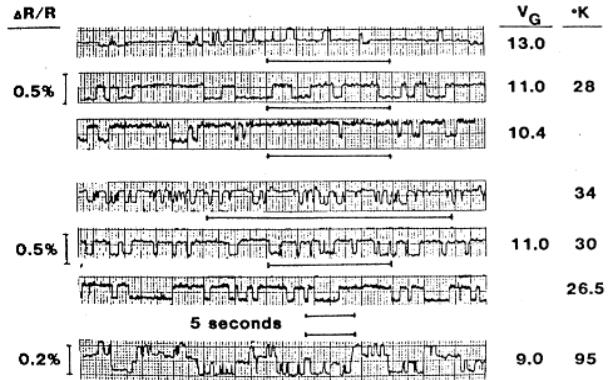


FIG. 5. Two-state switching in the voltage on small gated Si resistors, observed by Ralls *et al.* (1984). V_G is the gate voltage, and the right-hand column gives the temperature. Notice that at least two such switching sites are needed to explain the bottom trace.

the random telegraph shown in Fig. 5. If such a telegraph is Markovian, its power spectrum is a simple Lorentzian with characteristic frequency $\omega_c = k_1 + k_2$, the sum of the up and down switching rates (Machlup, 1954). For many such telegraphs superposed, ω_c is the only measurable rate. For a single one, k_1 and k_2 are directly measurable from switching time histograms, which also allow a check that the system is Markovian. This extreme case has been observed at low temperatures in very small samples of materials that show $1/f$ noise in larger samples and/or at higher temperatures (Ralls *et al.*, 1984; Rogers and Buhrman, 1984, 1985; Uren, Day, and Kirten, 1985; Wakai and Van Harlingen, 1986; Beutler, Meisenheimer, and Giordano, 1987). When such information is available, the noise sources can often be understood without resort to elaborate statistical analysis. These cases will therefore be discussed primarily in the Results section (Sec. IV).

Less extreme or more complicated cases require the use of a limited number of higher-order correlation functions and/or spectra, picked to be rapidly computable and sensitive to differences between different models of broadband noise. One such method has been to measure the variance of the noise power determined from a series of measurements within a given bandwidth and to compare this variance with that expected for Gaussian noise (Strasilla and Strutt, 1974). A recent extension of this technique suitable for digital data-taking devices is to measure both the variances and covariances of the noise power in a series of octave bands (Restle, Weissman, and Black, 1983). For Gaussian noise the covariances have expectation values of zero (see Table I).

Properly speaking, the spectral density, defined as an infinite-time average for a stationary process, cannot fluctuate. However, actual measurements of spectral density involve measuring the output of finite-bandwidth analog or digital filters over finite times. A time series of such measurements does, of course, have a variance. To be

precise, that variance depends on the shape of the frequency band selected, the time over which each measurement of the power is made, and the overall time of the record on which the variance is computed. However, except for cases in which the power varies over a very long time scale (in which case the record time is important), the ratio of the variance in noise power to that for a Gaussian noise with the same spectrum is nearly independent of such details.

Jones and Francis (1975) devised one of the first multi-bandwidth tests for non-Gaussian effects. They looked for correlations in the fluctuations in the noise power in narrow, harmonically related bands. In a variety of materials, they found none. They also looked for correlations between Johnson noise and resistance fluctuations using an ac technique to measure the latter, again finding none. These results were interpreted as being evidence against, respectively, nonlinear theories of $1/f$ noise generation and theories that suggested some close relation between Johnson noise and $1/f$ noise.

Another technique is to take power spectra and possibly cross spectra of the time series of noise power measurements in one or more bands, for which octaves are again convenient (Restle, Hamilton, Weissman, and Love, 1985). This power spectrum of the fluctuations in noise power within a band of the ordinary spectrum will be referred to as the second spectrum. For Gaussian noise, the second spectrum is nearly frequency independent, so long as the spectrum has no structure on the scale of a measurement bandwidth. This can readily be seen by treating the Gaussian noise as the superposition of an arbitrarily large number of smaller signals. What the second spectra of those component signals are is irrelevant, since they average to zero as the number gets large. The resulting second spectrum arises entirely from beat terms between the components, and the frequencies of these beats are simply all the differences of frequencies

TABLE I. A covariance matrix for a highly Gaussian noise source (Johnson noise from a 47-k Ω resistor at 300 K). The frequency range covered is 20–1810 Hz ("octave" 9.5 being only a half octave). These data are taken from 1000 Fourier-transformed sets of 1024 voltage readings. Ideally, Gaussian noise would give 1.0 for the variances of each of the records of noise power per octave, by definition of the normalization used. Due to the finite number of samples, a standard deviation of ~ 0.03 is expected. For pure Gaussian noise the cross correlations between the fluctuations in the noise power in the different octaves would be 0 ± 0.03 (Restle, Weissman, and Black, 1983).

Octave	4	5	6	7	8	9	9.5
4	0.99						
5	0.01	0.98					
6	0.00	0.04	1.02				
7	-0.02	0.01	0.02	0.99			
8	0.04	-0.05	-0.01	0.01	1.02		
9	-0.05	0.03	0.01	-0.07	-0.02	0.99	
9.5	0.01	0.05	-0.02	0.03	0.02	0.03	0.99

within the measured band. The detailed shape depends on the shape of the measurement band, but is nearly flat out to frequencies of a half bandwidth, which is higher than is measured in this technique.

Other techniques include taking histograms of noise power per octave (Restle, Weissman, and Black, 1983) and comparing average decays back to the mean from different starting deviations from the mean (Voss, 1978). (Although this latter technique was originally described as a test of "linearity" it was subsequently shown to be describable in terms of Gaussianity by Nelkin and Tremblay, 1981.) One test, of course, is to ignore time information and simply to compare the distribution of voltages with a Gaussian curve. These tests are particularly sensitive to large, sporadic bursts, usually from some extraneous source which must be distinguished from the $1/f$ components. The mean-decay method might, however, prove useful in analyzing deterministic or partially deterministic chaos.

The simplest sorts of $1/f$ noise models, in which the noise comes from a superposition of two-state systems with different characteristic times, give predictable deviations from Gaussian behavior. When a small number of random telegraphs (several per octave in f_c) are superposed, the net variance in the spectral density (using standard digital processing methods) is slightly higher than for Gaussian noise, according to Monte Carlo simulations (see Table II). There are peaks in the extra variance of spectral bands slightly above each f_c , when these are sufficiently far apart to resolve. The covariance falls to zero for octaves more than about four octaves apart. If the f_c 's are randomly picked, these are significant random variations in the spectral shape between samples when the number of contributors is small enough to give substantial non-Gaussian effects. Except when the duty cycle is very far from $\frac{1}{2}$, the second spectrum is white, as it obviously must be since neither of the two characteristic times is much longer than the inverse of f_c . The combination of sample-to-sample variation in spectral shape with non-Gaussian effects measurements allows a check of the two-state-systems picture in small samples, which

TABLE II. A covariance matrix from a simulation of a superposition of two-state systems, with randomly picked characteristic frequencies and duty cycles. The difference in the variance in the noise powers in the different octaves is reproducible for this particular set of two-state systems, but not if a new random set is drawn. The density of two-state systems in this simulation is at least ten times too large to allow individual systems to be clearly distinguished. From Restle *et al.*, 1985.

Octave	4	5	6	7	8	9
4	0.99					
5	0.03	1.03				
6	0.04	0.06	1.12			
7	-0.01	0.07	0.06	1.13		
8	0.04	0.01	0.03	0.11	1.27	
9	-0.02	0.04	0.01	0.09	0.11	1.36

are nonetheless too large to allow any particular two-state system to be identified.

The higher-order statistical methods should be particularly useful in identifying various types of series kinetics. For example, the RWRP shows very dramatic increases in the variance of the noise power for simulated samples large enough to show little sample-to-sample variation (see Table III). The covariances between octaves fall off much more slowly than for two-state systems. Essentially, the random walker is found in either sticky, quiet environments or slippery, noisy environments for periods much longer than the inverse of the bandwidth, on any time-frequency scale. Second spectra have not been simulated but are expected to be very far from white.

Likewise kinetic Ising models with smeared kinetics show qualitatively different non-Gaussian effects from those of two-state systems. The long characteristic times in the spectrum arise when spins get stuck, unable to flip. The second spectra, at least in small simulations, are similar to those of the spin itself (Alers, Weissman, Kinzig, and Israeloff, 1987). The similarity between the second spectrum and the ordinary spectrum may suggest a new sort of scale invariance connecting different orders of fluctuations, in systems with series kinetics.

B. Symmetry measurements

Once it is accepted that $1/f$ noise ordinarily is caused by local resistivity fluctuations, one still not only does not know the mechanism but also does not know in what mathematical space to locate the $\delta\rho$, that must be explained. In many materials exhibiting $1/f$ noise, ρ itself is a scalar for fundamental reasons, e.g., the material has cubic symmetry, or is isotropically amorphous, or is polycrystalline. However, noise is a deviation from the mean and may include a deviation from the mean symmetry. Such nonscalar effects need not imply any average deviation from isotropy of ρ or of its fluctuations. The instantaneous asymmetries of $\delta\rho$ are sensitive to the underlying fluctuation mechanism and are measurable.

In practice, measurements of the instantaneous symmetry have been possible only in planar samples, so that it is convenient to describe them in terms of a two-dimensional projection of $\delta\rho$, which we call $\delta\rho'$. A

TABLE III. A covariance matrix for an RWRP simulation. Huge non-Gaussian effects are evident, although the simulated sample-to-sample variations in the spectra were smaller than those for the system shown in Table II. From Restle *et al.*, 1985.

Octave	4	5	6	7	8	9
4	5.69					
5	0.82	9.30				
6	0.74	0.85	12.51			
7	0.71	0.79	0.90	10.67		
8	0.43	0.52	0.51	0.63	6.22	
9	-0.09	-0.05	-0.10	0.00	0.53	6.71

parameter $S \equiv 2\langle \det \delta\rho' \rangle / \langle \text{tr}[(\delta\rho')^2] \rangle$ conveniently characterizes the anisotropy of the fluctuations, with $S=1$ corresponding to scalars, $S=0$ to 2D dyads, and $S=-1$ to 2D traceless fluctuations.

Fairly complete explanations of the relations between S and experimental techniques and microscopic parameters, as well as of the relation between the properties of ρ' and of ρ have appeared elsewhere (Black, Snow, and Weissman, 1982; Weissman, Black, and Snow, 1982; Black, 1984). I shall summarize the key points. S may be measured on cross-shaped resistors (Fig. 6) for which one can simultaneously measure fluctuations in the resistivity along different paths through the same region. The sensitivity of the technique depends on the extent to which the different paths are orthogonal. When a sample is granular or otherwise full of large static inhomogeneities, the current at any one point may be constrained to a single axis of flow—in that case one finds $S \approx 0$ independently of the local mechanism (Black, Snow, and Weissman, 1982; Rammal, Tannous, and Tremblay, 1985).

One of the most interesting possibilities, because of its connection with a microscopic mechanism, is that $\delta\rho$ could be traceless—this corresponds to the rotation of an anisotropic defect in a higher-symmetry environment. If the orientation of the principle axes of $\delta\rho$ to the plane is random, $S = -\frac{5}{7}$ results, while if there is cubic symmetry with two of the principle axes in the sample plane, $S = -\frac{1}{2}$ results (Black, 1984).

The results of all the isotropy measurements to date are shown in Fig. 7. It is evident that the coupling to ρ is not universal. Specific implications will be discussed in the Results section.

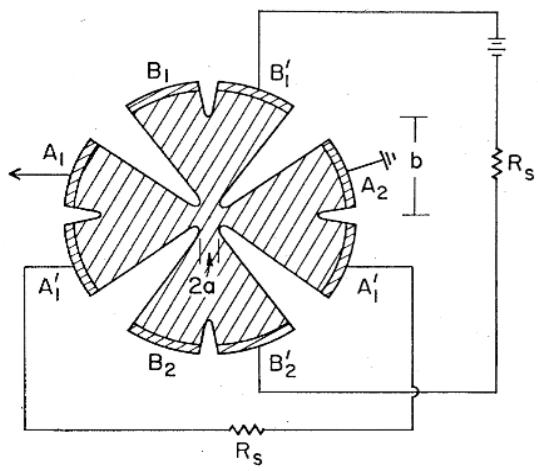


FIG. 6. A sample geometry suitable for measuring tensor properties of the noise. The distance a is typically tens of microns, the distance b should be more than $10a$. The particular circuit shown is that used for measuring voltage fluctuations on the A path with current flowing in the B path. The ratio of this quantity to the cross-correlated voltage fluctuations on the two paths, with current flowing in both, is sensitive to the isotropy parameter S (from Black, Snow, and Weissman, 1982).

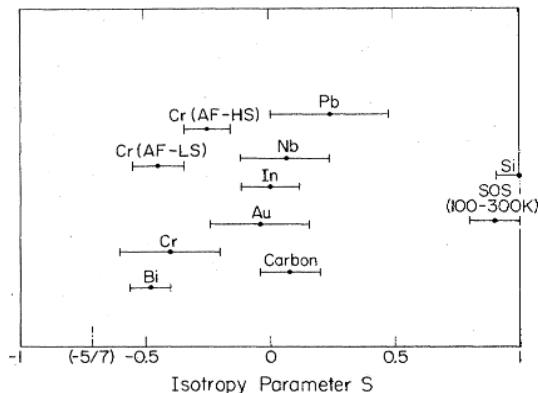


FIG. 7. Symmetry results for all materials on which the symmetry parameter S has been measured. (The vertical axis has no significance.) When temperature is not specified, it is ~ 300 K. Error bars for Bi, In, Nb, and SOS represent $\sim 95\%$ confidence limits. For Cr at 300 K, C, Pb, and Au, there were some indications of sample-to-sample variations or variations within a given sample as a function of time. The error bars represent the observed range of results. The result for Si is based on a single sample. The Cr (AF-LS) results are from samples in the temperature range 150–200 K on soda-lime glass, for which the tensile strain is approximately zero. The Cr (AF-HS) results are for similar samples on Si, for which there is significant tensile strain in the sample plane. In both cases, the samples were well within the apparent antiferromagnetic regime.

C. Other recent techniques

In recent years several techniques have been developed that look at fluctuations in properties other than the resistivity. Some of these techniques rely on the ease with which digital electronics allow one to take cross spectra between two signals.

Brophy (1957) was the first to measure fluctuations in the Hall coefficient of a semiconductor, Ge. The essential technique is to measure voltage noise on contacts orthogonal to the current path with an adjustable magnetic field out of the sample plane. The fluctuations were more or less as large as would be obtained for simple carrier number fluctuations, but not quite equal to the prediction of any model. However, geometrical factors entering into the measured magnitude of Hall fluctuations were not carefully taken into account. Kleinpenning (1980) and Vaes and Kleinpenning (1977) measured Hall coefficient noise in Ge as a function of field, finding slightly less noise than simple number fluctuation models would predict, but again without precise geometrical calibration.

Black, Restle, and Weissman (1983a) measured Hall noise in silicon-on-sapphire, with numerically computed geometrical calibration. They also measured the cross correlation between the Hall noise and the ordinary scalar resistivity noise. Allowing for geometrical factors, the local correlation coefficient between those two quanti-

ties was close to one, indicating that a single fluctuating parameter was likely to be involved.

Kleinpenning (1978) also measured fluctuations in thermo-EMF in point contacts in Si. The most significant result was that the thermo-EMF noise could not be reduced by more than about a factor of 0.6 by an applied voltage, indicating that the resistance noise and thermo-EMF noise were only partially correlated, suggesting a model in which more than one independent fluctuating parameter at each location was required. However, due to the different roles of surface effects in the resistance and the thermo-EMF, the results are also compatible with models in which only one parameter fluctuates locally, but not all locations enter with equal weightings into the two measured variables.

Krafft and Webb (1985) cross correlated the noise in junctions measured by dc and ac probes to check whether the fluctuations in junction capacitance and resistance had the same source, which they did. Measurements of the amplitude of fluctuations in the second harmonic coefficient showed that those too probably just came from the same source—barrier height fluctuations.

IV. RESULTS

A. Spatial cross correlation

At one time there appeared to be evidence for frequency-dependent spatial cross correlations over distances of several millimeters in the frequency range of ~ 1 Hz in both Bi and Cr films (Voss and Clarke, 1976; Zhigal'skiy and Karev, 1977). This result was one of the strongest pieces of evidence for a temperature-fluctuation model, since whatever was fluctuating seemed to diffuse through the film at a rate $\sim 1 \text{ cm}^2/\text{sec}$. Careful attempts to reproduce these results in even smaller samples turned up no correlations whatever (Black, Weissman, and Fliegel, 1981). No correlations were found between the noise in Au films separated by $\sim 0.6 \mu\text{m}$ of SiO_2 (Scofield, Darling, and Webb, 1981). More recent measurements on a variety of metal films show directly that no correlations are present down to a millihertz in frequency and $20 \mu\text{m}$ in length (Scofield, Mantese, and Webb, 1985). From the scaling of the noise spectral density with sample widths, the absence of correlations down to a distance scale of $0.5 \mu\text{m}$ was inferred (Scofield, Mantese, and Webb, 1985). Measurements on films of Pt as narrow as about 100 \AA indicate that any coherence length cannot be much larger than 100 \AA in those films (Fleetwood, Masden, and Giordano, 1983).

Other systems also show no extended spatial correlations. For example, a clever experiment by Koch (1983) on Josephson junctions in magnetic fields showed that correlations were absent to $\sim 1 \mu\text{m}$. The exceptions are the extreme low-temperature systems discussed in Sec. IV.H.

B. Smeared activated kinetics

In the years since evidence for the smeared-activation energy picture was found in metal films, the accumulation of evidence for such a picture in diverse systems has exceeded almost everyone's expectations. In one of the original film systems, Bi, the spectral peak has been directly observed to shift as a function of temperature (Black, Restle, and Weissman, 1983b), as shown in Fig. 3. The spectrum in silicon-on-sapphire (SOS) has been found to have features with activated characteristic frequencies (Black, Restle, and Weissman, 1983a; Weissman, Black, Restle, and Ray, 1983; see Fig. 8). Several granular thick-film resistors have been found to show very broad features with activated kinetics (Pellegrini, Saletti, Terreni, and Prudenziati, 1983; see Fig. 9). Two groups have looked for activated features in carbon resistors—although each resistor seems to have distinct features, the kinetics always seem to be activated (Black, Restle, and Weissman, 1983a; Fleetwood, Postel, and Giordano, 1984; see Fig. 10). Two different salts of the quasi-one-dimensional conductor TCNQ have been found to have activated features (not the same features; Jos, Zijlstra, and Ike, 1983; Rommelmann *et al.*, 1985). An activated kinetics model seems most compatible with $1/f$

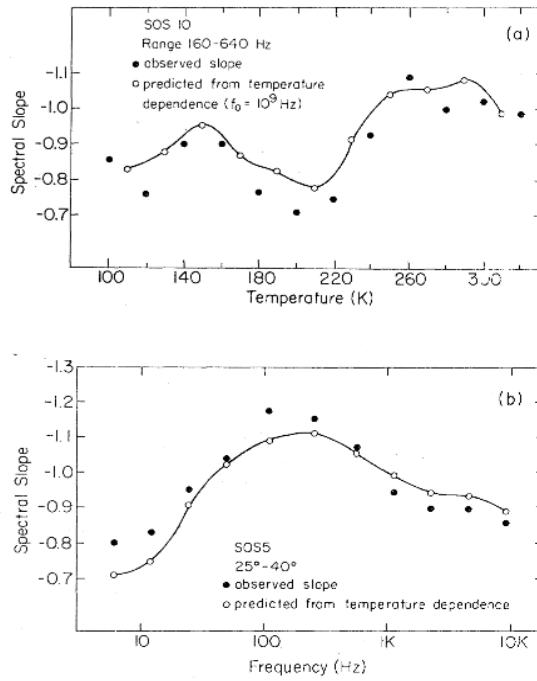


FIG. 8. Two forms of the Dutta-Horn relation for silicon-on-sapphire resistors. (a) A plot of the spectral slope $[\partial \ln S(f)/\partial \ln f]$ vs temperature for an SOS sample. The prediction (open circles, connected by a line that serves only to guide the eye) is based on the Dutta-Horn relation [Eq. (3)] (from Black, Restle, and Weissman, 1983a). (b) The same spectral slope and predicted slope, in another sample, shown as a function of frequency (Black, Restle, and Weissman, 1983a).

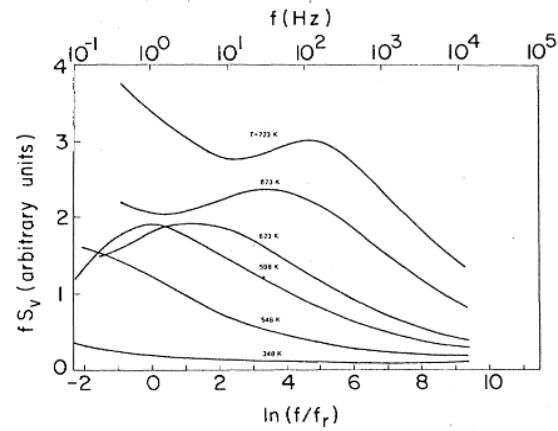


FIG. 9. A very broad, thermally activated feature in noise spectra over a broad frequency-temperature range in an IrO_2 -based thick-film resistor. Note that the spectrum has been weighted by frequency, giving plots similar to octave integral spectra. The reference frequency $f_r = 1 \text{ Hz}$. On a conventional plot these spectra look close to $1/f$ from 10^{-1} to $2 \times 10^4 \text{ Hz}$ (from Pellegrini, Saletti, Terreni, and Prudenziati, 1983).

noise results in charge-density-wave conductors (Bhattacharya, Stokes, Robbins, and Klemm, 1985). Even Josephson junctions operating in the 4-K range have been found to obey something like a Dutta-Horn relation (Koch, 1983). Further data on various metallic systems generally fit the Dutta-Horn relation (see Fig. 11; Fleetwood, 1985; Scofield, Mantese, and Webb, 1985), al-

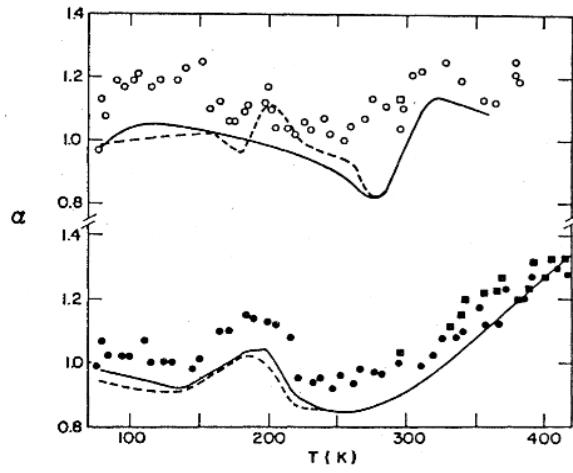


FIG. 10. The predicted Dutta-Horn spectral slope compared with the measured slope for two carbon resistors. Solid lines are the standard prediction. Dotted lines are based on the temperature dependence of $S_R(f, T)$ rather than on $S_R(f, T)/R^2$. Notice that some temperature dependence other than that predicted by the Dutta-Horn relation is clearly present, as indicated by the offset between the predicted and observed slopes (Fleetwood, Postel, and Giordano, 1984).

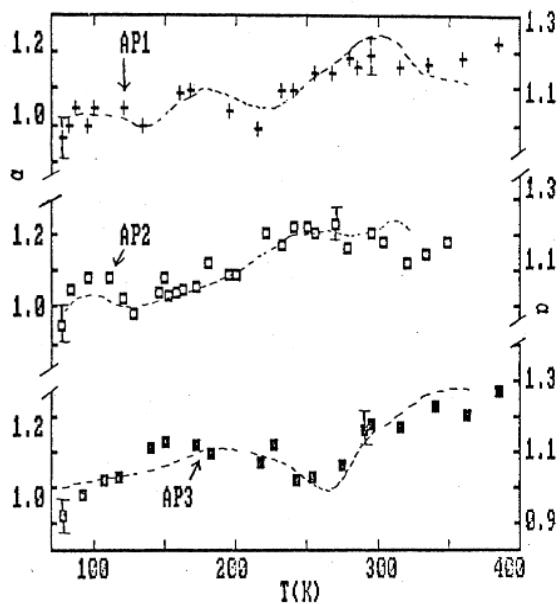


FIG. 11. The negative spectral slope $\alpha = -\partial \ln S(f)/\partial \ln f$ for three differently prepared AuPd samples. Dotted lines represent the predicted value based on the temperature dependence (Fleetwood, 1984).

though not always (Scofield, Mantese, and Webb, 1986). Both metal-insulator-metal (MIM) junctions (Rogers and Buhrman, 1984, 1985) and small silicon MOSFET's (Ralls *et al.*, 1984) have been found to have noise from a superposition of thermally activated two-state systems with a spread of activation energies (see Figs. 12 and 13).

The central question then is not whether there are activated low-frequency noises in many materials, but rather to what extent these account for the $1/f$ noise phenomenon. There might also be $1/f$ noise from similar processes with tunneling kinetics or from some very different fundamental mechanism with an intrinsic $1/f$ spectrum. These questions can be partly resolved by examining spectra over a wide temperature-frequency range.

The original temperature-dependent data of Eberhard and Horn (1978) showed reductions of spectral density of up to 2 orders of magnitude below typical room-temperature values for films near 100 K. These data give no reason to expect some temperature-independent $1/f$ noise minimum. The thick-film data also would require any such underlying "true" $1/f$ noise to be much smaller than the nearly $1/f$ noise found under normal conditions. Examination of the features in SOS shows that they cannot consist of a small set of Lorentzians—one needs a spread of activation energies and prefactors (Weissman, Black, Restle, and Ray, 1983). (Much more detailed evidence about this will be discussed in the semiconductor section, Sec. IV.C.) Although the features in one TCNQ salt were roughly fit with a discrete set of Lorentzians (Jos, Zijlstra, and Ike, 1983), detailed inspec-

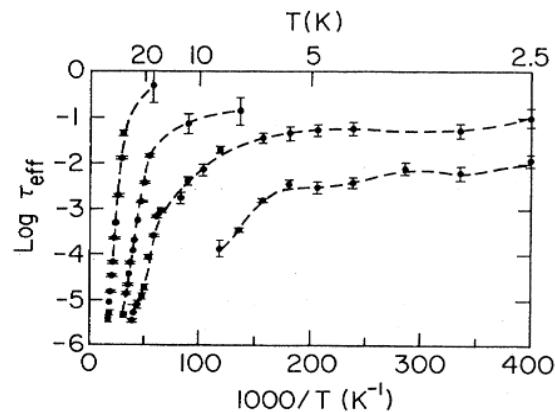


FIG. 12. The temperature dependences of the net switching rates for several sites in MIM junctions. Most clearly turn from an activated switching pathway to tunneling in the vicinity of $T = 15$ K (from Rogers and Buhrman, 1986).

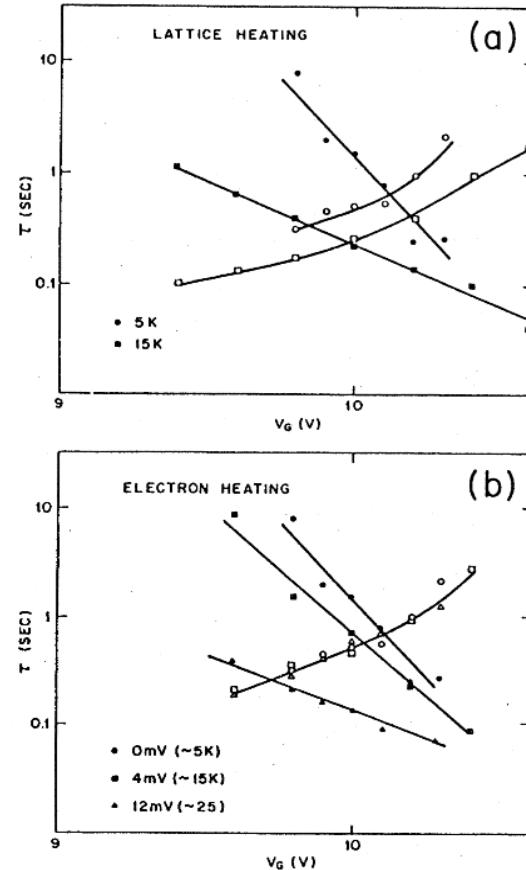


FIG. 13. The dependences of the kinetic parameters of a trap on temperature, voltage, and electron heating. (a) The mean lifetime for capture (solid symbols) and emission (open symbols) for a single trap site, shown as a function of gate voltage at two different temperatures. The nonexponential dependence of one of the rates on gate voltage is not typical of all sites (Jackel *et al.*, 1985). (b) The same parameters plotted for the same trap at three different current levels, which are roughly equivalent to different electron temperatures (Jackel *et al.*, 1985).

tion of the fit shows systematic residuals which point to either a large discrete set or a smear with features. The frequency-temperature dependence in Josephson junctions (Koch, 1983) is much broader than would be obtained with a single discrete relaxation feature.

The evidence then does not indicate the existence of some fundamental scale-invariant $1/f$ noise as accounting for the observed $1/f$ phenomena, which can be fully explained by smeared kinetics features known to be present. This view is buttressed by the absence of any plausible fundamental theory.

The question of how much of the kinetics may be due to tunneling is not as easy to answer at this point, because smeared tunneling rates are just as plausible as smeared activated rates. Silicon-on-sapphire is a good model system for McWhorter noise, but nonetheless the features, which are not minor, are activated. Further evidence, to be discussed, suggests that all the noise at silicon surfaces may be activated. Josephson junctions seem like good candidates for tunneling noise kinetics simply because of the low temperature of the superconducting transition, but most if not all the noise at 4 K has activated kinetics. Only low-temperature MIM junctions (Rogers and Buhrman, 1985) and SQUID's (Wakai and Van Harlingen, 1986) have been found to show temperature-independent features (see Fig. 12). One should not be too hasty in excluding contributions from tunneling at higher temperatures, since while thermal activation is easy to detect via temperature dependence, it is not so easy to get a handle on tunneling. Nevertheless, when temperature-independent features are not found, any tunneling contribution would have to be relatively featureless. Furthermore, when individual sites with tunneling kinetics have been found, the kinetics were found to become thermally activated above ~ 15 K (Rogers and Buhrman, 1985). The apparent requirement for atomic motion in the trapping of electrons by the states contributing to $1/f$ noise may be connected with the amorphous structure of the trap vicinity, in which the atomic configurations have many nearly degenerate metastable states, whose energies are shifted by the presence of a trapped electron.

C. Semiconductors

Very strong evidence has now accumulated to show that most if not all $1/f$ noise in semiconductors comes from some form of charge trapping-detrapping, one of the first mechanisms proposed. Much of the earlier evidence was summarized by Van der Ziel (1979). The kinetics, however, are more complicated than in the early versions of such models.

Some of the recent evidence is very simple. The interface density of states in MNOS devices can be varied and measured over many orders of magnitude, and it is proportional to the $1/f$ noise magnitude (Maes and Usmani, 1983; Maes, Usmani, and Groeseneken, 1985). The relative magnitudes of the Hall coefficient fluctuations and

resistivity fluctuations is the same as expected from surface-state perturbation measurements in SOS, and these two fluctuating quantities are almost perfectly correlated (Black, Restle, and Weissman, 1983a). Furthermore, the resistivity fluctuations are almost perfectly scalar (Black, Restle, and Weissman, 1983a).

The traditional debate over whether number fluctuations or mobility fluctuations are involved turns out to have a trivial resolution, in that the trapping changes both variables, to the extent that they are separable (see, for example, Black, Restle, and Weissman, 1983a; Ralls *et al.*, 1984).

Since trapping affects ρ both by changing carrier number (intrinsically a scalar effect) and by changing mobility (which need not be scalar), the expected symmetry parameter S is not obviously one. However, in semiconductors with charged sites located more than a mean free path apart, the scattering induced by each site is also scalar, so $S=1$ would still hold. While the density of fluctuating trap sites, integrated over a large bandwidth, would not give enough sites to violate this condition, reasonable extrapolations to include very deep acceptors and very shallow donors would indicate that there could be a number of charged sites within a mean free path of each fluctuating trap. However, even in metals, anisotropic resistivity terms from interscatterer interference are very small when the scatterers are more than a wavelength apart (Martin, 1972; Pelz and Clarke, 1987). In nondegenerate semiconductors, the broad range of carrier wavelengths further dilutes any interference effects. Although detailed calculations have not been made, there is little reason to expect significant nonscalar effects from trapping in semiconductors, particularly nondegenerate semiconductors.

In SOS, it has also been possible to measure both the kinetics of the slow oxide surface states and their coupling to the conductivity, so that the $1/f$ spectrum and the magnitude of the oxide trapping noise could be demonstrated with no major adjustable parameters (Garfunkel and Weissman, 1985a). In this case, most of the noise did not come from oxide traps, but SOS contains another surface with many extended defects which probably accounts for the remainder.

There continue to be many reports (VandeVoorde, Iddings, Love, and Halford, 1979; Black, Weissman, and Restle, 1982) of the sensitivity of $1/f$ noise in semiconductors to surface treatment. Proper surface treatments can give α_H of 10^{-6} or even less. While a demonstration that most $1/f$ noise in some semiconductors comes from the surface is not necessary to show that trapping is involved, it is nearly sufficient, in that no other special surface mechanism is known. Occasional comments in the literature (e.g., Hooge, Kleinpenning, and Vandamme, 1981) that a bulk noise is suppressed by some surface treatments have never been supported by any model in ordinary resistors, and it is extremely unlikely that they will be.

The surface origin of much of the noise helps explain

how semiconductors with moderately high doping can show an apparent $\alpha_H \approx 10^{-3}$ in point-contact measurements, which has often been reported from the Eindhoven group (Hooge, Kleinpenning, and Vandamme, 1981). Such large values of α_H in a bulk sample would imply a large temperature coefficient of resistivity if the trapping model held (Weissman, 1981b). However, due to the very large current density near the rim of a point contact, ordinary surface noise gives large noise which scales with contact radius in the same way as bulk noise (Black, Restle, and Weissman, 1983a).

Given the likelihood that trap occupation fluctuations are the cause of $1/f$ noise in semiconductors, the troubling question remains why the spectrum is $1/f$. An obvious and simple explanation was supplied by McWhorter (1957)—there is a uniform distribution of tunneling distances to the traps. However, the spectral features that have been found show thermal activation with a range of activation energies and prefactors. One then must ask if the features are from an activated fraction of the noise or whether it is all activated.

The best evidence on this question, as well as the most conclusive evidence for the origin of the noise in localized transitions, comes from the work of Ralls *et al.* (1984), Uren, Day, and Kirton (1985), and Kirton and Uren (1986) on tiny gated silicon resistors and of Rogers and Buhrman (1984, 1985) on MIM junctions, as well as of Rogers, Buhrman, Kroger, and Smith (1986) on amorphous Si. These systems are small enough for the trapping-detrappling of single charges to make a detectable step in the resistance. By varying the temperature and gate voltage, Ralls *et al.* could determine on and off activation energies, the trap depth, and trap location (see Table IV). About a half-dozen traps with measurable steps were found for which the steps were not mixed up with those from other traps. For each observed trap both trapping and detrappling were thermally activated. The impression given by this small number of traps is one of random depths and activation energies. Ralls *et al.* found no evidence of any traps with tunneling kinetics.

These data on single traps also allow some inferences

to be made about the kinetic pathway for trapping. Since the traps are only a few angstroms from the semiconductor-oxide interface, simple electronic tunneling would dominate unless motions of some atoms were required for the transition. [A similar conclusion had previously been reached from less convincing evidence (Black, Restle, and Weissman, 1983a).] The simplest picture would be one in which there were two states for the trap atoms, one giving a deeper trap. There would then be two transition states—one with the deep trap formed, but no electron in it, and the other with an electron in the shallow trap. In either case the electric dipole moment of the transition state virtually coincides with one of the stable states, so that either the on or off rate should show very little electric field dependence (Restle, Hamilton, Weissman, and Love, 1985). If both transition states were important, both rates could be field dependent, but with nonexponential dependences. In fact, Ralls *et al.* found that both rates show strong, nearly exponential dependences, indicating a single transition state with the electron partway between the bulk and the trap (see Fig. 13). Thus the transition seems to be adiabatic, with the electron and the lattice moving in concert.

Further data (Jackel *et al.*, 1985) on the dependence of the switching rates on temperature and on an effective temperature of heated mobile electrons also help define the kinetic pathway (see Fig. 13). As might be expected, the rate of electron detrappling depends only on lattice temperature, not on electron heating. Furthermore, the rate of electron trapping is much less sensitive to the electron effective temperature than to the lattice temperature. This result fits well with the idea that both the electron and the atoms near the trap are partially moved from their untrapped positions in the transition state. The slightly nonexponential dependence of the ratio of the trapping to detrappling rates on gate voltage presumably reflects the dependence of the untrapped wave function on electric field, but this effect has not yet been fully analyzed.

From the single-trap data of Ralls *et al.*, it would be tempting to conclude that $1/f$ noise in silicon, at least,

TABLE IV. Properties of individual traps observed in small MOSFET's by Ralls *et al.* (1984). V_G is the gate voltage. The polarity (Pol.) gives the sign of ΔR for the untrapped state. ΔF is the derivative of the logarithm of a switching rate (on or off) with respect to gate voltage, in volts. The distance d_0 is the estimated distance of the trap in the oxide from the Si interface. dE_T/dV_G is a measure of the displacement of the charge between the conduction state and the trapped state. The activation energies for the rates, E^\pm , are determined from temperature dependences, except for those in parentheses, for which estimated electron heating was taken into account.

T (K)	V_G (V)	Observation range	Pol.	Gate voltage dependence				Activation energy	
				$\Delta R/R$ %	ΔF_{off} (1/V)	ΔF_{on} (1/V)	dE_T/dV_G (meV/V)	d_0 (nm)	E_{off}^\pm (meV)
101–111	3.8–5.3	—	—	0.3	−1.4	2.2	28	1.9	210
48–56	2.2–5.5	+	+	0.1	0.47	−1.0	6	0.3	70
26–34	10–13	+	+	0.2	0.57	−1.0	3	0.1	26
(12–20)	3.5–4.5	—	—	0.7	−2.9	4.3	10	0.5	(16)
4.2	10.3–10.9	+	+	0.2	4.7	−7.5	5	0.2	(9)

simply results from traps with a range of activation energies and other parameters. The weakness of the argument, of course, is the small number of samples and traps studied, which leaves open the possibility that these are not the main source of $1/f$ noise. However, a number of studies (Rogers and Buhrman, 1984, 1985; Restle, Hamilton, Weissman, and Love, 1985; Uren, Day, and Kirton, 1985) have shown that, when one averages over samples, or increases sample size, or increases temperature, the very highly featured spectra turn into $1/f$ spectra (see Fig. 14).

The data of Rogers and Buhrman (1984) on MIM junctions fit the same general picture. These Nb-Nb₂O₅-PbBi tunneling junctions, with areas of $\sim 0.1 \mu\text{m}^2$, showed noise that consisted almost entirely of a few discrete Lorentzian components below 80 K. The temperature dependences of the frequencies and magnitudes of these components can be fit by assuming that each comes from a single TLS with activated kinetics. Over the ensemble of samples, there appears to be a smooth $D(E^\pm, \Delta E)$, as well as a significant spread in $\ln f_0$. As had been surmised earlier, no particular correlation could be found between the ΔE , E^\pm , and $\ln f_0$ values for the traps. As usual, the activated kinetics (as opposed to tunneling) would be hard to account for with simple electronic trapping, so some atomic motion seems to be involved. Above ~ 100 K, too many Lorentzians are present to resolve. The spectra on the different samples become similar and nearly $1/f$ above 200 K.

More recent data from Rogers and Buhrman (1985; Fig. 12) show that below 15 K tunneling kinetics are dominant, as would be expected if the atomic displacements are only several angstroms. This result is important both in establishing that tunneling $1/f$ noise can ex-

ist and in supporting the connection between the tunneling TLS inferred from low-temperature heat capacity measurements and the activated TLS inferred from $1/f$ noise.

Recent data of Wakai and Van Harlingen on dc SQUID's (1986) show resolvable two-state transitions, no doubt from events in the junctions. Figure 15(a) shows the histogram of lifetimes of the two states of one such system. The exponential lifetime distribution indicates a Markovian two-state system, with no complicated series effects. Figure 15(b) shows a transition with one nonexponential distribution. These data appear to come from a three-state system, in which, for example, an electron might be free, weakly trapped, or, following a nonadiabatic lattice relaxation, deeply trapped.

An intermediate case—samples too large to allow direct observation of steps, but too small to give Gaussian noise if the noise is made of discrete steps—has also been studied in SOS (Restle, Hamilton, Weissman, and Love, 1985). For samples with area $\lesssim (1 \mu\text{m})^2$ the spectra vary significantly between samples while retaining an approximate $1/f$ shape. This spectral variation is about as expected for the trap density calculated from the noise spectral density, assuming randomly distributed activation energies for the traps. The sizes of the spectral features show strong random temperature dependence, as expected if each is due to a small number of traps, which would ordinarily be smoothly distributed about the Fermi level.

Small but significant non-Gaussian effects were found over most of the temperature-frequency range studied (Restle, Hamilton, Weissman, and Love, 1985). The extra variance of the spectral density and interoctave covariance were very similar to those found in simulations of random two-state systems with sample-to-sample variation in spectral shape similar to that of the actual samples (see Table V).

There are also some substantial non-Gaussian effects (Restle, Hamilton, Weissman, and Love, 1985), with strong structure in their frequency dependence. Extra variance of the spectral density appears in a few narrow frequency-temperature windows. Each window probably corresponds to the Lorentzian contribution of a particular trap. However, the second spectrum is not compatible with a superposition of two-state systems but requires some low-frequency modulation of the magnitude of the noise from each such system. The magnitude of the modulation (of the order of 10%) is not necessarily large enough to have been detected in the single-trap experiments. The picture that emerges is of some traps in an environment sufficiently amorphous not only to give the $1/f$ spectrum but also to provide other slow processes in their vicinity.

The detailed dynamics of these complicated multistate contributors to the noise are now beginning to be mapped out by measurements on extremely small systems. Rogers (1986) reported observing pairs of traps, only one of which could switch at any given time, in MIM junc-

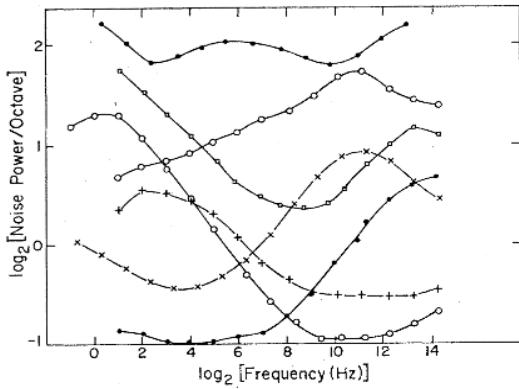


FIG. 14. The spectra of a number of similarly fabricated SOS resistors, each with $\sim 1 \mu\text{m}^2$ area. The vertical displacements are for visual clarity only; the amplitudes of the noise in the different devices were roughly equal. The deviations from an average $1/f$ spectrum were uncorrelated with any known parameter and were of about the right magnitude to be caused by random selection of trapping times for each device (Restle, Hamilton, Weissman, and Love, 1985).

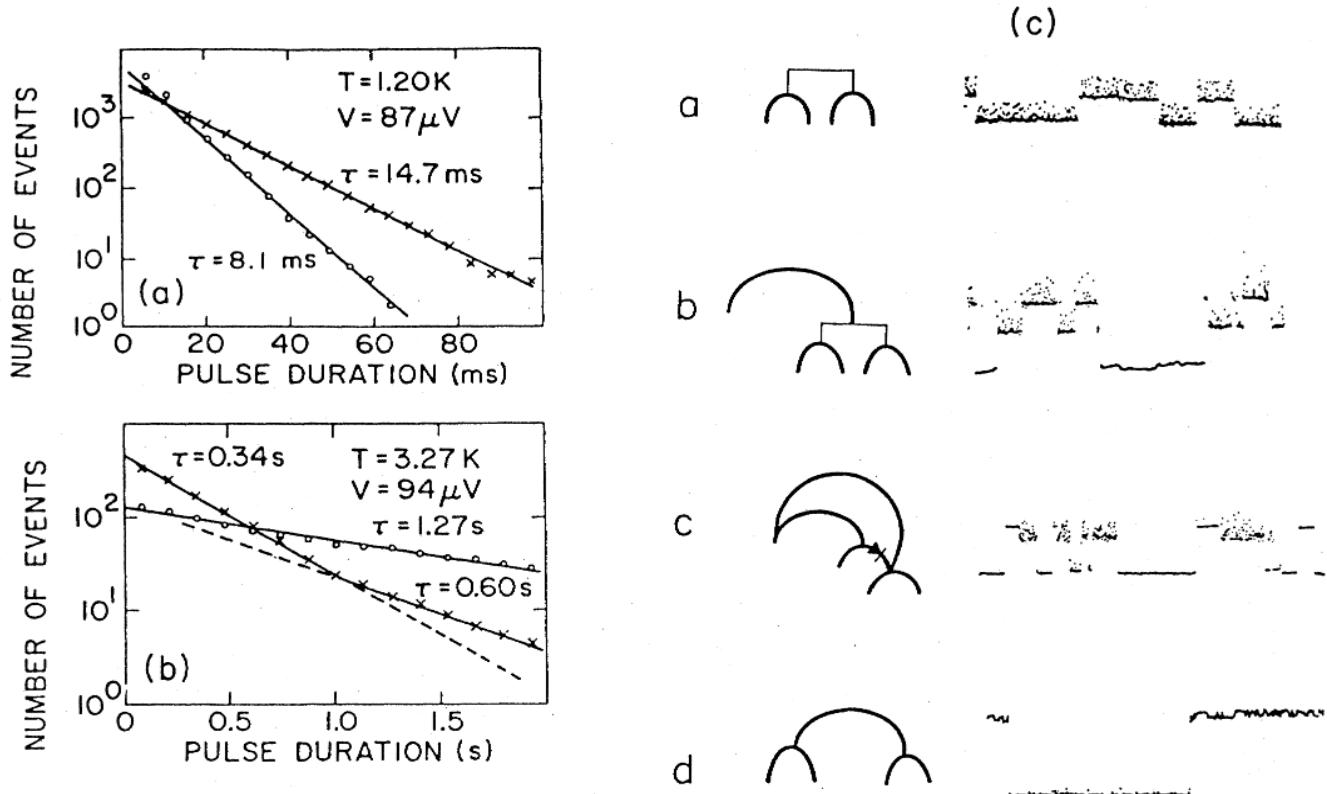


FIG. 15. Switching time histograms for two types of transitions. (a) The histogram of “on” and “off” times from a dc SQUID shows the characteristic exponential distributions of times of a Markovian two-state system, such as a simple trap. (b) This histogram of SQUID “on” and “off” times shows one simple exponential and one curve that is roughly the sum of two exponentials. It could arise from a three-state system, e.g., free electron + weak trap \leftrightarrow weakly trapped electron \leftrightarrow deeply trapped electron. Note that although three characteristic times are present in such a model, the device characteristics see only two states—trapped and untrapped, as observed (from Wakai and Van Harlingen, 1986). (c) Time-dependent voltages and apparent kinetic pathways for several junctions. Square brackets indicate transitions that do not alter those hanging off them. The curved upside-down U's indicate transitions, with different allowed transitions within each of the two states. The diode symbol indicates a transition that violated detailed balance, going only one way, which is possible due to the high current through the junction (from Wakai and Van Harlingen, 1987).

tions. Wakai and Van Harlingen (1987) have observed complicated structures of many mutually dependent switching events in Josephson junctions [see Fig. 15(c)]. Detailed accounts of multistate systems appeared almost simultaneously from a number of groups working with very small junctions and FET's (Farmer, Rogers, and

TABLE V. A covariance matrix measured from 1000 transforms in the frequency range 53 Hz to 4.8 kHz on a $\sim 1 \mu\text{m}^2$ SOS sample. The qualitative similarity to Table II and difference from Table III are evident. From Restle *et al.*, 1985.

Octave	4	5	6	7	8	9	9.5
4	1.14						
5	0.16	1.10					
6	0.09	0.10	1.14				
7	0.09	0.09	0.10	1.17			
8	0.00	0.07	-0.01	0.07	1.10		
9	0.10	0.05	-0.02	0.07	0.11	1.09	
9.5	-0.03	-0.07	-0.03	0.02	0.06	0.10	1.00

Buhrman, 1987; Restle and Bucelot, 1987; Uren, 1987a). Uren and co-workers (1987b) have shown that some non-Coulomb interaction between sites is required.

Farmer, Rogers, and Buhrman (1987) have shown that many of the transitions are not between states that differ by a single trap occupancy but rather involve many interacting trapping sites. At low temperatures, highly concerted changes can be the only ones allowed, which provides a natural explanation for the very low attempt rates (f_0) found for many of the transitions. At higher temperatures, intermediate states also have finite occupancy. In many respects the results of Farmer, Rogers, and Buhrman have more in common with complicated series kinetics than with parallel two-state systems.

The question of whether $1/f$ noise in semiconductors is primarily a bulk or surface phenomenon has much practical importance, although not necessarily much theoretical importance if the “fundamental” theories are ruled out on other grounds. Traditional lore has attributed most of the noise to the surface, not just because of the

McWhorter theory but also because the noise level is notoriously sensitive to surface treatment. Most reports that have presented evidence for bulk noise were based on the scaling between resistance and spectral density noise for point contacts with the pressure varied, but it turns out that surface noise and bulk noise show the same scaling for a good contact.

Welland and Koch (1986) have directly observed fluctuating surface states on Si-SiO₂ using a scanning tunneling microscope. The tunneling current is highly sensitive to the charge within a several-square-angstrom surface area beneath the tunneling tip. Thus a "noise map" of the surface could be made in great detail. Both isolated two-state-system traps and more complicated trap clusters were found. The scanning tunneling microscope promises to be one of the most important new tools in studying charge-trapping noise.

There are cases in which bulk noise in semiconductors has been established. HgCdTe (Bajaj *et al.*, 1985) is one of the best demonstrated cases. HgCdTe also forms unusually poor crystals, compared to silicon, GaAs, or other more conventional materials. It is likely that whenever extended defects, oxide inclusions, surfaces, or other major deviations from crystal symmetry are found $1/f$ noise can arise.

Since in Cu (Pelz and Clarke, 1985) and perhaps in Bi there is evidence that isolated defects in the bulk can give $1/f$ noise, it is possible that isolated traps could do the same. However, a silicon wafer is a much better crystal than an evaporated metal crystal. In fact, trapping noise clearly from bulk traps has been observed in JFET's (Kandiah, 1983). Single traps were observed, and these fell into several discrete classes of nearly identical traps. Noise from traps in GaAlAs in heterojunctions with GaAs also appears to be mostly from discrete trap types, and thus not of a $1/f$ form (Loreck *et al.*, 1983). Thus the association of the $1/f$ spectrum with disorder of the material seems confirmed.

It would be interesting to see if anomalous non-Gaussian effects also disappeared in these crystalline, discrete-trap-type systems. Results on several GaAs samples with discrete trap types have turned up no anomalous non-Gaussian effects, tentatively confirming that the modulation effects occur only in amorphous regions (Restle *et al.*, 1986).

These same GaAs samples, after surface treatment, showed low levels of $1/f$ noise which is full of anomalous non-Gaussian effects (Restle, 1985; Restle *et al.*, 1986). The second spectrum also has a $1/f$ form, with a slight white component (see Fig. 16). The white component led to an estimate of the number of traps contributing at any time. The $1/f$ component was large enough to indicate that each trap was unable to fluctuate about half or more of the time. From the frequency dependence of the cross correlations in the covariance matrix, it appeared that each site made a simple Lorentzian contribution to the noise. Possible connections between the GaAs noise statistics and the multistate switching discussed above

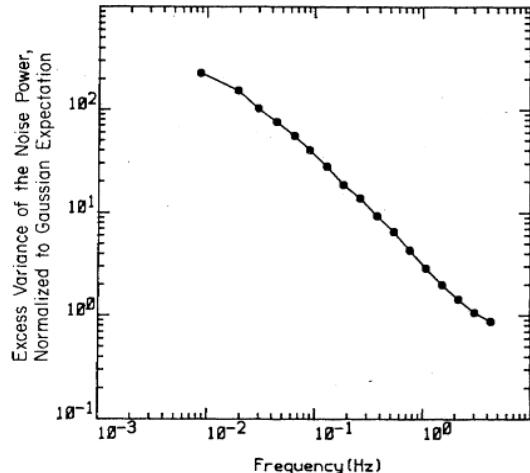


FIG. 16. A second spectrum for a GaAs resistor exhibiting $1/f$ noise. The vertical axis represents the spectral density of the fluctuations in the integral of the ordinary spectral density in an octave around 10 kHz. A constant, representing the prediction for Gaussian noise, has been subtracted. For noise from a simple superposition of independent two-state systems, second spectra are almost entirely frequency independent (Restle, 1985).

remain to be investigated.

Main and Owen (1970) measured $1/f$ noise in two chalcogenide glasses and in a copper phosphate switch glass. Large $1/f$ noise was found, with a spectral density that scaled less than quadratically with current. Contact noise was not definitively ruled out. If the noise did arise in the glass, some explanation for the unusual current-noise scaling is required. Main and Owen suggest that trapping with a distribution of trap lifetimes could be the source of the noise. That explanation would then seem to require hot-electron effects, at fields of ~ 10 V/cm, to account for the scaling. Alternatively, Main and Owen suggested that there may be a connection between the distribution of characteristic charge hopping times implied by the frequency-dependent ac conductivity and the distribution of times involved in the noise in the dc conductivity, since both distributions are logarithmically flat. Since the dc conductivity ought to depend on the locations as well as the number of trapped charges, this proposal is also reasonable. However, rather large charge displacements ($> 10 \mu\text{m}$) would seem to be required if the reduction in resistivity fluctuations at high currents were due to pinning of the fluctuating dipoles by the applied field. Clearly, more study of glassy semiconductors is required to determine their principal noise mechanisms with confidence.

Mytton and Benton (1972) measured extremely high noise levels ($\alpha_H \approx 800$) in the ferroelectric semiconductor Ba_{0.597}Sr_{0.4}La_{0.003}TiO₃. The noise was not very temperature sensitive in a range of $T_C - 30 \text{ K} < T < T_C + 70 \text{ K}$, where the nominal Curie temperature $T_C \approx 265 \text{ K}$. An-

nealing the ceramic sufficiently to remove grain boundary resistance had little effect on the noise. Mytton and Benton suggested that if all but a small fraction ($\sim 10^{-5}$) of the charge carriers were trapped, the noise would not be particularly unusual. However, it is not clear whether such a proposal would be consistent with the conductivity of the material, given reasonable limits on the mobility of the free carriers. A possible alternative explanation would lie in the orientation fluctuations of the ferroelectric domains, which presumably have anisotropic conductivities. Such domains are known to form well above T_C in solid-solution ferroelectrics (Isupov and Pronin, 1980). At temperatures sufficiently above T_C , of course, such domain noise would disappear.

The quasi-one-dimensional semiconductor TaS_3 has been investigated by Maeda, Naito, and Tanaka (1983); by Zettl and Gruner (1983); and by Bhattacharya, Stokes, Robbins, and Klemm (1985). Below the transition temperature for formation of a charge-density wave (CDW), very large $1/f$ noise appears when the applied electric field exceeds the threshold field needed to depin the CDW and initiate CDW conduction. Bhattacharya, Stokes, Robbins, and Klemm attribute the noise to fluctuations in the threshold field as the CDW switches between metastable configurations. They calculate a coherence volume of $\sim 10^{-12} \text{ cm}^3$ for these threshold field fluctuations. Although not explicitly stated, that volume appears to be a minimum value for the measured noise spectra. Obviously, in this collective CDW noise, coherence lengths are much larger than for conventional noise sources.

Although NbSe_3 is a quasi-one-dimensional metal rather than a semiconductor, its $1/f$ noise properties are so similar to TaS_3 that I shall include it here. Maeda, Naito, and Tanaka (1983) noted strong similarities in the dependence of the noise on applied field in the vicinity of the threshold fields for CDW motion. The mean-square conductance fluctuations for the two materials are similar, although the background non-CDW conductivity is much larger in NbSe_3 . There is no reason to think that the CDW noise mechanisms in the two materials are essentially different.

Davis *et al.* (1983) also found the CDW noise in NbSe_3 , as well as a component with $S(f)\alpha f^{-2}$ which appeared very close to the threshold field, apparently due to critical fluctuations. In addition, unusually large $1/f$ noise for a metal was found above the upper CDW transition temperature, in the range 175–300 K. If this noise is really from the sample, not the contacts, it would suggest that incipient CDW instabilities might provide a large noise source affecting the normal electron conductivity. Unless some CDW domains remain in the vicinity of some defects or impurities even at high temperatures, it would be hard to account for the effect.

Quasi-one-dimensional semiconducting salts of TCNQ also exhibit large $1/f$ noise, with thermally activated features (Jos, Zijlstra, and Ike, 1983; Rommelmann *et al.*, 1985). Very large noise coherence volumes

($\gtrsim 10^{-15} \text{ cm}^3$) were inferred by both experimental groups using somewhat different reasoning. In contrast to the cases of TaS_3 and NbSe_3 , in TCNQ the spectral density normalized by the squared voltage does not change drastically even when the I - V curve is well into the nonlinear regime (Rommelmann *et al.*, 1985). The doping level has little effect on the noise. In view of the apparent large coherence volume, some collective electronic effects may well be involved in these conductors, although they are not as well understood as TaS_3 and NbSe_3 .

D. Metals and semimetals

The traditional argument as to whether mobility or number fluctuations are the source of $1/f$ noise has always been irrelevant to most metals, for which no plausible carrier number fluctuation model has been proposed, since the required number of fluctuating traps would be very large. For example, the integrated noise magnitudes in Cu and Ag given by Dutta and Horn (1981), typical of most metallic samples, would require more than one trapping state per atom within a few kT of the Fermi level. Each trap can contribute a variance in the carrier number of at most 0.25. If the Dutta-Horn estimated extrapolation of the spectrum is correct, more than 3.6 active traps/atom would be required in Ag and more than 1.2 in Cu. This is clearly impossible. Scofield, Mantese, and Webb (1986) have pointed out uncertainties in the extrapolation of the spectrum outside the observed frequency range, but such uncertainties could not reduce the estimates above by more than an order of magnitude. Furthermore, there is no special reason for traps made by, say, oxide impurities to have energies close to the Fermi level. It seems most unlikely that such a prominent effect would have gone unnoticed by other techniques. Metals do not give scalar fluctuations, which would be expected for number fluctuations. Therefore one must find a source for mobility fluctuations.

Most results on most metallic films indicate that the noise originates in the bulk of the film, since the mean-square fractional fluctuations scale inversely with thickness (Hooge and Hoppenbrouwers, 1969; Voss and Clarke, 1976; Dutta and Horn, 1981; Fleetwood, Masden, and Giordano, 1983; Zimmerman, Scofield, Mantese, and Webb, 1986). One exception to this scaling is Bi, for which thin samples are relatively quiet (Hooge, Kedzia, and Vandamme, 1979; Black, Restle, and Weissman, 1983b; Bisschop and de Kuijper, 1984). This effect may be accounted for if in Bi only conduction from the large crystalline regions is noisy, with the surface conduction relatively constant. Al films show some geometry dependence associated with grain structure (Koch, Lloyd, and Cronin, 1985). There are some indications that in Ag near 100 K the surface may be noisier than the bulk (Dutta and Horn, 1981). Furthermore, in room-temperature Nb films grown by MBE it seems that the thinner films are noisier, but this effect may result from their inferior crystal quality rather than directly

from surface effects (Garfunkel, 1985). The same uncertainty applies to the differences in noise magnitude found in some metals at some temperatures as a function of substrate (Dutta and Horn, 1981; Fleetwood and Giordano, 1982).

Overall, then, some reason is needed for the mobility to fluctuate in the bulk of a typical metal film, but this fact should not be taken to mean that $1/f$ fluctuations would be present in a perfect crystal, nor that surface effects are irrelevant. The reason or reasons sought should not be too particular, since most metallic elements and alloys show similar effects, but neither should the explanation be too general, since some show unusually little noise.

In an ordinary metal at room temperature there are not many slow processes. No one has suggested any local processes (not involving macroscopic transport) with rates in the $1/f$ regime other than those involving impurities and other defects, except in special cases near phase transitions or in phases with domain structures. Most $1/f$ noise is very unlikely to be the result of equilibrium defects in a good crystal, because the concentration of such defects shows strong thermal activation, which is rarely found for the noise magnitude. Therefore it is reasonable to expect *a priori* that $1/f$ noise in metal films results from motion of impurities or nonequilibrium defects left in films as they are deposited or annealed. Such motions can change the cross sections for impurity scattering and thus show up as electrical noise.

In semimetals, such as Bi, the sensitivity of the Fermi surface to strain can give resistivity fluctuations from defect motion via an anisotropic density of states, rather than via anisotropic scattering.

Of course, thermally activated trapping and detraping of charges on impurities can provide another type of fluctuating cross section. As with traps near semiconductor surfaces, however, one cannot account for thermally activated trapping unless it is accompanied by atomic motion (Ralls *et al.*, 1984). However, unlike semiconductors, metals have inverse Fermi wave vectors comparable to the distance scale of atomic motion, and also have very effective screening of charges. Therefore those atomic motions (presumably a small fraction) which happen to be accompanied by a changing charge state would not show up disproportionately in the mobility fluctuations.

For fluctuating scattering cross sections one can calculate a maximum α_A under reasonable assumptions. [Here $\alpha_A \equiv fS_R(f)/R^2N_A$, where N_A is the number of atoms. α_a is a more convenient parameter than α_H for metals and semimetals.] So long as the variance in the cross section at some site does not exceed the square of the average cross section, one finds

$$\alpha_A \leq n_A \sigma \lambda \frac{r}{(1+r)^2} \left/ \ln \left[\frac{f_{\max}}{f_{\min}} \right] \right., \quad (5)$$

where n_A is the concentration of atoms, σ is the scattering cross section, λ is the mean free path of the pure met-

al, r is the ratio of impurity scattering to phonon scattering, and the logarithmic factor is an approximation to the frequency integral of the $1/f$ spectral shape.

If, as one normally finds, most of the impurities do not have fluctuating cross sections on the relevant time scale, α_A is reduced. Likewise, typical scatterers that do have fluctuating cross sections due to reorientation usually still have fractional variances more than an order of magnitude less than one (Pelz and Clarke, 1987). Using reasonable values [$n_A = 10^{23}$ cm $^{-3}$, $\sigma = 3 \times 10^{-16}$ cm $^{-2}$, $\lambda = 3 \times 10^{-6}$ cm, $\ln(f_{\max}/f_{\min}) = 25$], one finds $\alpha_A \leq 1$, which agrees with all the data of which I am aware, except those for whiskers (Dutta, Eberhard, and Horn, 1977; Leeman, Skove, and Stillwell, 1980). Not surprisingly, given the generous assumptions required to obtain $\alpha_A \approx 1$, all of the data on stable samples of ordinary metals at room temperature have $\alpha_A \leq 5 \times 10^{-2}$ (e.g., Fleetwood and Giordano, 1983a; Scofield, Mantese, and Webb, 1986). Larger values of α_A generally imply either that the Fermi surface itself can fluctuate (as in the strain-sensitive semimetal Bi or the spin-density-wave phase Cr) or that some very large, highly fluctuating scatterers are present, or that the fluctuations in resistivity from motions of a single scatterer are larger than its average contribution to the resistivity, as occurs only in the rather unusual universal conductance fluctuation regime (Feng, Lee, and Stone, 1986; Pelz and Clarke, 1987).

Several lines of evidence now fit nicely with this defect-motion picture, but some problems remain. The DH relation usually holds approximately, as one would expect (Fleetwood and Giordano, 1985; Scofield, Mantese, and Webb, 1986). Several types of sample modifications expected to affect defect levels do change the noise level.

Clear evidence was obtained recently that the noise spectral density in Cu can be increased by irradiation with 500-keV electrons (Pelz and Clarke, 1985). The effect is reversible upon annealing, with an initial annealing stage removing most of the irradiation-induced noise and a bit of the resistance, while a higher-temperature anneal removed the remaining noise and resistance (Pelz and Clarke, 1985) (see Fig. 17). As the resistance was increased with increasing radiation dose, the noise increase was proportional to about the 0.6 power of the resistance change, indicating that the first defects formed are more likely to generate noise than subsequent ones. Condensation could occur in the initial annealing to defects with fewer transitions available between equal-energy states, while further annealing apparently removes all the induced defects. The important point is that the original noisy defects, probably isolated sites in Cu crystals, still showed enough spread in transition rates to give $1/f$ noise.

Subsequent work (Pelz, Clarke, and King, 1987) has shown that, in fine-grained In-doped Cu films, the excess spectral density is linearly proportional to the excess resistivity induced by radiation damage, as would be expected if defect recombination and rearrangement pro-

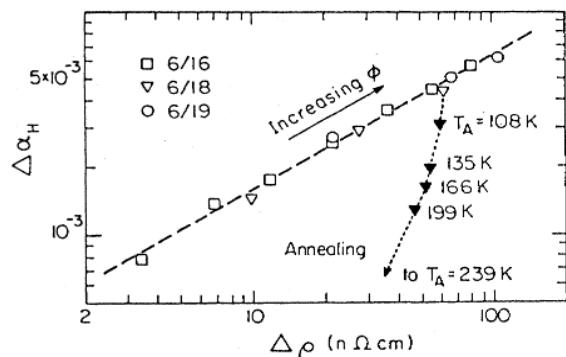


FIG. 17. The change in $1/f$ noise level is plotted vs the change in resistivity for Cu samples irradiated with electrons and subsequently annealed: dashed line [$\Delta\alpha_H \propto (\Delta\rho)^{0.6}$], irradiated samples, for which the resistance changes as the square root of the dose, ϕ ; dotted line, the recovery of both variables at increasing annealing temperatures T_A . For $T_A = 239$ K (not shown), $\Delta\rho \approx 11.6$ n Ω cm and $\Delta\alpha_H \approx 7 \times 10^{-5}$. Clearly the defects mainly responsible for the resistance change and those mainly responsible for the noise are not the same (Pelz and Clarke, 1985).

cesses were suppressed in the fine-grained film. Kr⁺ bombardment, which produces defect clusters rather than isolated defects, gave somewhat less excess noise per excess resistivity than did electron bombardment. The results all make sense in terms of known defect properties. In particular, the relatively noisy defects introduced by the initial electron bombardment of clean crystals seem to be vacancies.

Zimmerman (1987) has very recently observed a simple linear scaling of the spectral density in H-doped Pd with H concentration, strongly indicating that H diffusion is responsible for the noise. Near 20 K, the noise has a $1/f$ spectrum and is geometry independent.

Introducing mechanical strains in films can increase the noise level, which can then relax on time scales of hours to months (Fleetwood and Giordano, 1983b). Creation and annealing of defects is presumed to be responsible.

Fleetwood and Giordano (1985) have shown that annealing an amorphous AuPd film substantially reduces the noise spectral density (about an order of magnitude at $T = 300$ K) while only reducing the resistance by about a factor of 1.3, as would be expected if defects contributing much of the noise but not much of the resistance were annealed out. Below the annealing temperature, Eq. (3) applied regardless of the previous extent of annealing (see Fig. 11).

The most interesting result on AuPd was that defects were interpreted in terms of a somewhat oversimplified model, in which no distinction was made between the activation energies for noise kinetics and for annealing. Although a more complicated model is required to fit the data, they do show that it is quite likely that different sorts of defects predominate in different parts of the spectrum.

In some cases the noise magnitude depends a bit on the substrate, which presumably can affect the defect level. Zhigal'skiy, Sokov, and Tomson (1979) showed a smooth dependence of the spectral density in Al films on the stress induced by the differential thermal contraction of the Al and its substrates. A minimum spectral density was found for zero-stress films, as expected for a defect process. Since such substrate effects are temperature dependent, it is likely that the defects associated with strain near the substrate have different distributions of transition activation energies from those found in the bulk (Fleetwood, Beutler, Masden, and Giordano, 1987). The interpretation suggested for a while by Dutta and Horn (1981) and by Fleetwood and Giordano (1982) that the substrate dependence was due to varying thermal coupling affecting temperature fluctuations does not fit with any reasonable theory of temperature fluctuations nor with the random-seeming pattern of temperatures and metals in which the effect occurs.

A relatively specific defect-motion model now has rather strong evidence in Al films. Koch, Lloyd, and Cronin (1985) have shown that the characteristic activation energy of the noise changes as impurities are added in a way that parallels closely the changes in activation energies for diffusion along grain boundaries. Since the impurities segregate to the grain boundaries, they have little effect on the bulk. The characteristic times of the noise appear to be times for single hops of Al atoms, not times for large-scale diffusion. Presumably the detailed atomic positions along the boundary determine scattering rates there. This case may provide especially interesting statistical properties, since the environment of each random walker consists of other random walkers, so that highly non-Gaussian effects are likely.

In Au microstructures about half of the noise appears to be missing in samples that have no grain boundaries between the leads (Verbruggen, Koch, and Umbach, 1987). Furthermore, much of the remaining noise scales as a surface effect (as a function of thickness), unlike the noise in polycrystalline samples.

Very extensive measurements on films of ten different metals (Scofield, Mantese, and Webb, 1985) prepared under a variety of conditions show a strong relation between the mean-square resistivity fluctuations and the fraction of the scattering due to defects, estimated from the residual resistivity ratio (see Fig. 18). In the case of Cr and Ni there was some evidence that the most important source of the noise was oxide impurities. In Au the grain structure had little effect on the noise, suggesting that impurities were more important than structural defects in these samples.

Interestingly, the mean-square resistivity fluctuations seemed to scale roughly as the residual resistivity to the powers ranging from about 1 to 2, with the steeper slopes occurring for higher defect concentrations. For high defect concentrations, one does in fact expect that interference terms between scattering from different defects would enhance the noise above the level anticipated from

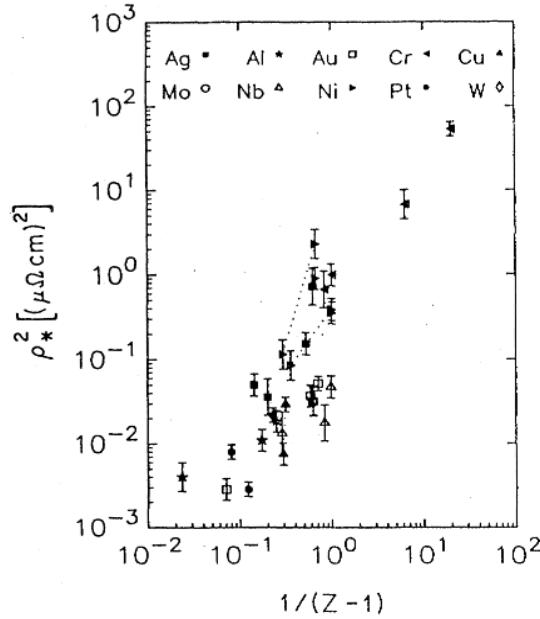


FIG. 18. The noise level $\rho_*^2 \equiv N_A [f S_\rho(f)]_{1\text{ Hz}}$ plotted vs $1/(Z-1)$, where Z is the residual resistivity ratio $\rho(300\text{ K})/\rho(4.2\text{ K})$, for a variety of metal film samples. The quantity $1/(Z-1)$ approximately represents the ratio of impurity scattering to phonon scattering (Scofield, Mantese, and Webb, 1985).

simply summing noise from individual mobile defects.

An additional line of evidence for defect motion comes from the noise symmetry measurements in several metals. As discussed above, motion of an isolated defect between states of equal energy should, barring accidental degeneracies, give only a traceless fluctuation. As mentioned, studies of the effect of thickness and of grain size on the noise in the semimetal Bi indicate that it comes from the interior of the relatively large crystalline regions. Bi also shows a dramatically negative S value, as expected (Black, Restle, and Weissman, 1983b). It is worth noting that no theories yet proposed other than defect motion and domain rotation (see below) seem to be compatible with, let alone predictive of, negative S values.

In Nb, the better-quality films show barely detectable $1/f$ noise ($\alpha_A \approx 3 \times 10^{-5}$; Scofield, Mantese, and Webb, 1985), while films with poor crystal quality (by x-ray analysis) show $S \approx 0$, again fitting a picture in which noise arising from grain boundaries and amorphous regions is neither close to scalar nor close to traceless (Black, 1984). A thin gold film also showed $S \approx 0$, but the film probably was too discontinuous to draw any conclusions about the local mechanism (Black, Snow, and Weissman, 1982). Lead films were found to give $0 < S < 0.4$ with some sample-to-sample variation, but it is not yet established whether the noise comes from crystalline regions or from extended defects (Rizk and Weissman, 1985). At any rate, it is unlikely to come from a Snoek mechanism. $S = 0$ was found in In (Rizk, 1984) al-

most certainly because the noise came from high-resistance boundaries between crystallites.

Some data are now available on noise statistics in metals (Garfunkel and Weissman, 1987). Nb samples as large as 10^{-11} cm^3 consistently show non-Gaussian effects. The only known distinguishing feature of Nb is the presence of mobile H (Scofield and Webb, 1985). Large defect clusters containing many H atoms could have enough coupled degrees of freedom to give the peculiar noise statistics.

At least some samples of Ag and AgPd with volumes of $\sim 10^{-15}\text{ cm}^3$ and of Bi with volumes of $\sim 10^{-14}\text{ cm}^3$ showed no measurable signs either of non-Gaussian statistics or of spectral features indicative of small numbers of two-state systems. In AgPd the upper limit on the size of the fluctuating cross sections was about 10^{-16} cm^2 , consistent with defect-motion models. The concentration of two-state systems in Bi exceeds $\sim 10^{16}\text{ cm}^{-3}$. Samples of Al with volumes of $\sim 10^{-15}\text{ cm}^{-3}$ showed slight non-Gaussian effects which did not fit a parallel two-state-system picture, but could be consistent with the grain-boundary diffusion model of Koch, Lloyd, and Cronin (1985).

The $1/f$ noise in antiferromagnetic Cr is a special and interesting case. Scofield (1985; also Scofield, Mantese, and Webb, 1986) has shown a sharp increase of the spectral density by about 2 orders of magnitude as the films are cooled below $\sim 350\text{ K}$ to $\sim 280\text{ K}$ followed by a more gradual decrease of about the same amount as the temperature is lowered below $\sim 100\text{ K}$ (see Fig. 19). At least the higher-temperature change in spectral density was much too abrupt to be of the kinetic, Dutta-Horn type, since the spectrum remained close to an f^{-1} form. The kinetics may still be thermally activated, but the change in spectral density requires some nonkinetic explanation. Scofield pointed out that, with allowances for the effects of having a film rather than a single crystal, the two temperatures correspond approximately to those of a paramagnetic to an antiferromagnetic transition and to a

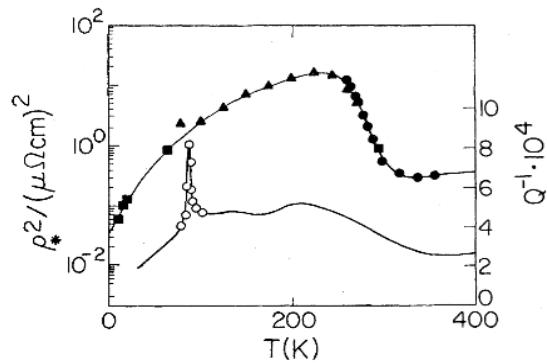


FIG. 19. The large plateau in $1/f$ noise in a relatively pure Cr film (Scofield, 1985) compared with a shallower plateau in the internal friction for a bulk Cr sample (Weller and Moser, 1981). The lower curve is for internal friction, with its scale on the right. The two effects may arise from the same mechanism.

spin-flop antiferromagnetic transition (Werner, Abbott, and Kendrick, 1967).

It happens that there is a well-known process which should cause resistivity fluctuations only in the high-temperature antiferromagnetic phase (Werner, Abbott, and Kendrick, 1967). That phase has a spin-polarization axis perpendicular to the transition vector of the antiferromagnetic order. This polarization axis is known to have thermally activated rotations around the translation direction. Below the spin-flop transition the two axes become parallel, so this process disappears. Not much anisotropy in the conductivity would be required to generate the observed noise from these thermally activated rotations of the order in the antiferromagnetic domains. The anisotropy between the translation axis and the polarization plane is known to be very large, $\sim 6\%$, but the anisotropy within the plane has not been measured (Muir and Strom-Olsen, 1971).

Recent experiments (a preliminary report is given in Israeloff, Weissman, and Scofield, 1987) have confirmed that the noise does in fact result from antiferromagnetic polarization fluctuations. The dependence of the upper onset temperature on sample purity and strain, as well as the negative symmetry parameter and its dependence on sample strain both fit this model. From experiments on small samples (10^{-12} cm^3) the size of the fluctuating domains (about 10^{-16} cm^3) and the resistivity anisotropy (about 10^{-2}) associated with the polarization can be estimated. The origin of the reduction in spectral density at low temperatures was not uniquely determined, since kinetic freezing (i.e., Dutta-Horn effects), thermodynamic freezing (inequality in the energies of the polarization states due to strains), and loss of the relevant degree of freedom via the spin-flop transition could all have been present.

The strong deviation from the Dutta-Horn relations at the upper onset temperature, however, is clearly due to the disappearance of the relevant degree of freedom when the antiferromagnetic structure melts. In the two-state systems language, it is the temperature dependence of $D(E^\pm, \Delta E)$, not its form at any one temperature, which causes the sharp temperature dependence. Within the transverse antiferromagnetic state, however, the domains behave as ordinary two-state systems.

Lebed' *et al.* (1985) showed that electrical resistivity fluctuations in several ferrites and an amorphous ferromagnet change sharply near the Curie point and depend strongly on magnetic field history. Although a detailed picture like that for antiferromagnetic Cr is not yet known, the electrical noise clearly arises from fluctuations in the magnetic domain structure.

The semimetal Bi is one of the best-studied noise sources. There is general agreement that in evaporated films almost all of the noise comes from crystalline regions. The mechanism must be the rotation of something, as is evident from the symmetry parameter. The spectral density in Bi films deviates sufficiently from $1/f$ to allow the measurement of an activation energy and a

prefactor (Black, Restle, and Weissman, 1983b). The prefactor is about $3 \times 10^5 \text{ Hz}$, which would be extraordinarily low for defect motion.

Below about 250 K, noise from Bi whiskers is roughly similar to the noise from polycrystalline evaporated films, but the spectral density increases rapidly at higher temperatures, with α_A exceeding 1 near 350 K. The temperature of the noise increase onset varied significantly between samples. The increase in spectral density was not associated with a sharp spectral slope and thus was not of the kinetic, Dutta-Horn type (Leeman, Skove, and Stillwell, 1980). Apparently some new degrees of freedom were present at high temperatures in the Bi whiskers, unlike Bi films. Sputtered Bi films also have high noise levels, but below 300 K they are substantially quieter than evaporated films (Fleetwood, Beutler, Madsen, and Giordano, 1987). The temperature dependences of the noise in the two types of films are not similar.

The reproducibility of the spectral density in Bi is hard to account for. Not only within laboratories but between them the spectral density in evaporated films, controlling for film thickness, seems reproducible to within about a factor of 2—which is about the uncertainty in the measurements. If a contaminant is required, it is not easy to believe that its concentration is so reproducible. If some defects are frozen in as the films are cooled, it is surprising that their concentration cannot be strongly affected by annealing. It would be worth considering electronic symmetry-breaking effects as an explanation for the unusual noise properties of evaporated Bi films and Bi whiskers.

Beutler, Meisenheimer, and Giordano (1987) have found discrete resistance steps in sputtered Bi films with volumes of $\sim 10^{-12}$ to $\sim 10^{-13} \text{ cm}^3$ at temperatures of 70 mK to about 1 K. The size of the resistance steps was about an order of magnitude larger than the value predicted from a universal conductance fluctuation coupling mechanism in the 100-mK range. The discrepancy appears to be larger at higher temperatures. These data were particularly interesting in that they included not only rapid, reversible steps, but also slower, continuous changes on a time scale of minutes. The origin of these latter changes is not understood, but some multatom rearrangement was suggested.

Meisenheimer, Beutler, and Giordano (1987) have found discrete resistance steps in sub-kelvin disordered Pt films about 1 nm thick and have taken further data on Bi. Qualitative agreement with the universal conductance fluctuation predictions was still found, but some surprisingly large steps were present in Pt also. It is not clear at this point whether some features of the experiment introduce complications not considered in the theory.

E. Possible problems for the defect-motion model

In addition to the extensive evidence supporting the importance of defect motion in generating $1/f$ noise

there are pieces of evidence that do not easily fit this picture. $1/f$ noise has been found in liquid metals, and has even been found not to change noticeably as a function of whether a gallium resistor was solid or liquid (Kedzia and Vandamme, 1978). The experimenters drew the conclusion that the noise resulted from some fundamental source, which was not quite justified. Nevertheless, in this case crystal defects in the gallium were obviously not important. Other nonfundamental sources were entirely possible, however, since contact to the metal was made via a nonoxidized spot on an oxidized tungsten tip. Because the field lines in such a spreading resistor bunch up at the insulator edge, the resistance is very sensitive to fluctuations of that edge, including charge fluctuations in the insulator (Black, Restle, and Weissman, 1983a). Thus $1/f$ trapping noise in the contact oxide is consistent with the experimental results. Scofield, Epworth, and Tennant (1987) have recently shown, using four-probe measurements, that the spectral density in In is reduced by at least an order of magnitude when it melts.

The defect-motion model could run into some problems, not only for a few unusual little-studied systems, but also because of some unexplained regularities in the results on conventional systems. Fleetwood and Giordano (1983) found, after measuring many metallic films, a well-defined minimum noise level, $\alpha_A \approx \rho_0/\rho$ with $\rho_0 \approx 6 \times 10^{-9} \Omega \text{ cm}$, with many samples close to the minimum, as a function of resistivity. The defect-motion picture does not easily account for the well-defined minimum, since in principle arbitrarily quiet samples should be possible. The inverse scaling of α_A with ρ is only crudely accountable for in the defect-motion picture. One of the factors giving a high ρ is large phonon scattering, which would dilute out the noisier impurity scattering (Weissman, 1981a; Scofield, Mantese, and Webb, 1985). However, there is no good explanation for why the minima for different types of films that happen to have similar ρ should be so nearly equal. More recent results suggest that this apparent pattern does not survive when more data are taken. Nb, Pt, and, to a lesser extent, Au, Al, and Mo show less noise than the Fleetwood-Giordano relation predicts (Scofield, Mantese, and Webb, 1985).

Metal whiskers of Cu (Dutta, Eberhard, and Horn, 1977) and Bi (as well as Sn and Zn; Leeman, Skove, and Stillwell, 1980) have been found to show much more $1/f$ noise than films of similar volume with higher surface-to-volume ratios. Since whiskers are believed to be relatively free of defects (or at least of extended defects) this result does not fit especially well with a defect-motion model. With whiskers one cannot remove the voltage-sensing leads from the current path, so that some contact noise from local shunts is to be expected. However, the noise increases with the distance between the contacts, which indicates a genuine whisker noise.

Metallic whiskers are not the only "clean" metallic system to be noisier than corresponding "dirty" systems. Bi films have the same property, but the detailed noise

source in them is not known, and complications arise from the semimetal properties. Clean Cr films are much noisier than dirty films, over a significant temperature range, because dirt suppresses the antiferromagnetic transition. It is just barely conceivable that delicate charge-density-wave or spin-density-wave structures, for example, might be present in some very clean metallic systems that have been hard to detect by other means (Overhauser, 1978). However, recent experiments by N. E. Israeloff in my laboratory have not found any large $1/f$ noise in Cu whiskers with soldered contacts, so there is reason to doubt whether the effect is real, at least in the normal metals.

F. Internal friction

One argument for the defect-motion model, put forward by Kogan and Nagaev (1982), links $1/f$ noise in metals to their anomalous low-frequency friction. A simple fluctuation-dissipation argument implies that frequency-independent friction requires $1/f$ fluctuations in mechanical strain. If the electrical resistivity depends on the same displacements that appear as strain, then $1/f$ noise would necessarily accompany frequency-independent friction. By assuming that simple atomic displacements are involved, Kogan and Nagaev calculated the approximate magnitude of $1/f$ noise expected for typical anomalous friction levels and found typical $1/f$ noise levels.

Kogan and Nagaev (1982) derived an expression for the resistivity power spectrum in terms of the internal damping factor Q^{-1} :

$$S_R(f)/r^2 = \frac{2}{\pi} \frac{kT(l\sigma_s)^2}{BVf} Q^{-1} = \frac{2}{\pi} \frac{kT(l\sigma_s)^2 n_A Q^{-1}}{B} \left[\frac{1}{N_A f} \right], \quad (6)$$

where l is the mean free path, σ_s is the typical change in scattering cross section along some projection caused by an anelastic relaxation transition, V is the sample volume, N_A is the number of atoms, $n_A \equiv N_A/V$, and B is related to the contribution to Young's modulus made by the transition site. The argument assumes that the noise is measured far from any mechanical resonance of the system, although Q is ordinarily measured in resonance. It is further implicitly assumed that σ_s is well defined without worrying about whether it is measured at constant stress or at constant strain or at some intermediate constraint. They proceed to calculate a typical Hooge value,

$$\alpha_A = \frac{2}{\pi} \frac{kT(l\sigma_s)^2 n_A Q^{-1}}{B}$$

using $10^{-4} - 10^{-3} \approx Q^{-1}$, $10^{22} - 10^{23} \text{ cm}^{-3} \approx n_A$, $10^{-34} - 10^{-33} \text{ ergs cm}^3 \approx B$, obtaining $\alpha_A \sim 10^{-3}$, within about an order of magnitude. The typical Q^{-1} value

used was taken from polycrystalline, not very pure, metals.

A key element needed to buttress the noise-friction connection is an empirical estimate of the resistivity changes (essentially σ_s) associated with typical anelastic defect motions. It happens that in several systems the resistivity changes associated with anelastic relaxation of tensile stress in wires were measured many years ago (Berry and Orehotsky, 1964). The results are summarized in Table VI, where M'_A is the ratio of the logarithmic change in resistance to the logarithmic change in length of the wire (due to the same anelastic process). The values for M_A , given in the original paper, consider changes in resistivity rather than resistance and thus include allowances for the change in the sample dimensions accompanying the anelastic strain.

We may rewrite Eq. (5) using quantities closer to those obtained experimentally:

$$\alpha_A \approx \left[\frac{2}{\pi} \right] \frac{kTn_A Q^{-1}}{E} M^2, \quad (7)$$

where E is Young's modulus and M^2 is the mean-square logarithmic resistance change divided by the mean-square logarithmic length change for the anelastic processes. (The mean of course involves some weighting of the different transitions if they are not all between states of equal energy.) We have assumed that the slow processes do not contribute too much to E .

Two difficulties remain before the noise-friction connection can be verified. First, the noise and friction measurements have not been made on the same systems. Since, with special precautions, internal friction may be measured in films just thin enough to allow noise measurements, this difficulty should be surmountable. Second, the piezoresistance measurements give only the mean piezoresistive coefficient of defects, while the noise magnitude depends on the mean-square value. For a homogeneous system, such as the 0 in Ta, each site

TABLE VI. The ratio of the anelastic logarithmic change in resistance (M'_A) or resistivity (M_A) to the anelastic logarithmic change in length is shown for various mechanically stressed wires. The frequency of the measurements was in the neighborhood of 3 Hz. Since the stress is mechanical, the mechanical strain resulting at every site contributes with the same sign. The resistivity change, which is sensitive to local asymmetries at each site, may vary in both magnitude and sign from site to site—these data represent an average (Berry and Orehotsky, 1964).

Alloy	M_A	M'_A
Ta-0.3% O	10	12
Ag-32.4% Zn	0.42	2.1
Ag-22.2% Zn	0.86	2.7
Ag-17.2% In	-15	-13
Cu-29% Zn	5.8	6.0
Manganin	-6.4	-4.2

should have nearly the same M ratio, so this distinction is unimportant. Since the sign of M varies from system to system it is clear that in an inhomogeneous system the mean value might result from averaging both positive and negative terms. In the alloy with small $|M_A|$, that value depends strongly on composition, again indicating that very different ratios are found at different sites. The smaller values probably are due to an approximate cancellation between opposing effects. Except for these small values, $M'_A \approx M_A$, although the differences are not negligible and could be related to the small variations found in noise magnitudes as a function of substrate adhesion (Fleetwood and Giordano, 1982).

Taking $M_A^2 \approx 100$, $n_A \approx 6 \times 10^{22} \text{ cm}^{-3}$, $k_b T \approx 4 \times 10^{-14} \text{ ergs}$, and $E \approx 10^{12} \text{ ergs/cm}^3$, we obtain a rule-of-thumb relation

$$\alpha_A \approx 0.2 Q^{-1}. \quad (8)$$

The typical film value, $\alpha_A \approx 2 \times 10^{-3}$, then corresponds to rather large internal friction $10^{-2} \approx Q^{-1}$. This estimate is somewhat higher than that of Kogan and Nagaev. Since typical films are equivalent to very poor quality bulk samples, these values are nonetheless quite plausible.

Pelz and Clarke (1987) have used the calculations of Martin (1972) to estimate fluctuating resistivities due to defect motions. The magnitudes they obtain essentially agree with those inferred from Berry and Orehotsky (1964) and are an order of magnitude lower than the estimates of Kogan and Nagaev.

One obvious question concerning the relation between $1/f$ noise and internal friction arises from the contrast between the nearly featureless $1/f$ spectrum and the identifiable peaks found in internal friction measurements. The difference partly arises from the form of presentation—when the spectral density $S(f, T)$ is plotted versus temperature rather than frequency often the features are just as prominent as for internal friction (Dutta and Horn, 1981). However, the noise in metal films never shows features as narrow as those found for internal friction peaks. Thus if the broad distributions of activation energies found in $1/f$ noise are from essentially the same processes as give a broad distribution of activation energies in internal friction, one must explain the apparent absence from the noise of the processes that have a narrow distribution.

The principal difference is very likely that the noise measurements are almost always made in films with a high surface-to-volume ratio, dense defect structures, internal strains, and other nonidealities. Even in the bulk samples used for internal friction studies, one routinely reads of peaks three times as wide as Debye peaks, i.e., about four decades wide (full width at half maximum on a power-per-octave plot). Interaction effects particularly broaden the activation energy distribution, often with the low-activation-energy tail of the distribution being particularly extended.

Finally we note that the plateau in the internal friction

in Cr between ~ 120 and ~ 300 K (Weller and Moser, 1981) has also been found in the noise magnitude (see Fig. 19). This plateau is connected with one of the anti-ferromagnetic phases, as we have discussed, but there is also a friction peak associated with the spin-flop transition, which has not yet been observed in the noise, perhaps because the spin-flop transition is greatly altered in thin films. The ratio of noise α_A to friction Q^{-1} can be approximately calculated for the SDW noise in Cr. Not surprisingly, since the effect is primarily electronic, M^2 is greater than 10^6 , rather than roughly 100 for defect motion. Some such effects might be required to account for the extraordinarily large noise in whiskers, which does not seem to be accompanied by large internal friction (Postnikov, Belikov, Belyavskii, and Yurev, 1972).

If one accepts the connection between internal friction and observable resistance fluctuations, several experimental avenues are opened. Noise measurements allow broad-band data to be collected on single samples—no resonant structure is required. Furthermore, noise measurements can be made on very tiny samples, from which non-Gaussian effects provide new information on the types of transitions allowed. The narrow overlap regime of materials suitable for noise and friction studies may provide particularly interesting experiments in the next few years.

G. SQUID's

Superconducting quantum interference devices (SQUID's) show $1/f$ noise that has much practical import, since it can be the dominant noise up to 1 MHz in these sensitive flux/current detectors. Much of the noise in devices with very small ($\lesssim 1 \mu\text{m}^2$) Josephson junctions originates in the junctions by mechanisms similar to those in semiconductors and MIM junctions. However, Koch *et al.* (1983) have found evidence that an additional noise source, not yet understood, is also often present.

In a series of SQUID's with widely varying geometries and consisting of two types of junctions, the SQUID noise was consistently larger than was expected from noise in similar junctions. Moreover, the noise did not depend on either the current bias or the magnetic flux bias in the way expected for fluctuations in the critical current of the junctions. These dependences did, however, match those expected if the noise came from fluctuations in the flux through the SQUID. Oddly enough, the magnitude of the apparent fluctuations in magnetic flux was about the same in all the SQUID's, although their loop areas varied by a factor of 10^6 and their inductances varied by nearly a factor of 10^3 . No real explanation of this behavior is known, although flux creep in the superconducting loop might be somehow involved. The apparent flux noise is drastically reduced in some SQUID's (Tesche *et al.*, 1984). The reason for the reduction is not precisely known, but may have something to do with fabrication procedures (Tesche, 1985).

Wellstood, Urbina, and Clarke (1987) have found noise

with a relatively flat spectral slope, $0.58 < \alpha < 0.80$ in a variety of SQUID's at temperatures in the 100-mK to 1-K range. The spectral density became nearly temperature independent at the lowest temperatures and also became nearly sample independent when expressed in units of magnetic flux. Examination of 12 more or less plausible explanations for the noise found that none fit the data. Two explanations—noise from the substrate or mount and noise from trapped flux in the SQUID—were not absolutely ruled out, although they were not considered very likely. Clearly, more work on this puzzle would be helpful. Savo, Wellstood, and Clarke (1987) have developed particularly low-noise junctions, which should help in isolating any sources other than the junctions.

H. Magnetic noise

Recent results on magnetization fluctuations have important implications for theories of spin-glass states. Unlike noise in resistivity, or in magnetic susceptibility, noise in magnetic moment is related to another measurable quantity, magnetic susceptibility, by a fluctuation-dissipation relation, at least for equilibrium systems (e.g., Brophy, 1965). In fact, once a magnetic material with an imaginary component of its ac susceptibility is included in an electrical pickup circuit, the resulting impedance directly gives the noise spectrum via the Nyquist relation. Using a normal copper-coil pickup system, Brophy (1965) long ago measured $1/f$ noise from ferrite samples. The noise spectrum agreed with the fluctuation-dissipation prediction from the imaginary component of the ac susceptibility.

$1/f$ noise from magnetization fluctuation in a spin-glass with a spectral density not too far from the fluctuation-dissipation relation were observed a number of years ago using a superconducting detector (Weissman, 1981c). However, since there are theoretical predictions of violations of the fluctuation-dissipation relations in spin-glasses, which are not necessarily equilibrium systems (Sompolinsky and Zippelius, 1982), it is important to check that relation carefully.

Ocio, Bouchiat, and Monod (1985, 1986) made extensive measurements of the magnetization noise of three insulating spin-glasses. They found that after sufficient equilibration time (\sim one day) spectra in the frequency range 10^{-3} – 10^{-1} Hz were roughly of a $1/f$ form and in approximate agreement with the fluctuation-dissipation relation. However, during sample "aging" a large extra noise component with a steeper frequency dependence was found, even in the absence of an applied field. This component may be connected with the complicated series kinetics believed to be present in spin-glasses. Reim *et al.* (1986) have very carefully checked the fluctuation-dissipation relation in the insulating spin-glass $\text{Eu}_{0.4}\text{Sr}_{0.6}\text{S}$ above the below the freezing temperature. They found agreement to an accuracy of about 10%. Furthermore, they found slight deviations from a simple $f^{-\alpha}$ spectral

shape. Current theories cannot account for that deviation.

Non-Gaussian statistical measurements on spin-glass fluctuations have the potential of testing spin-glass dynamics theories much more rigorously than simple noise spectral densities. Since spin-glass coherence lengths are believed to extend up to about $0.2 \mu\text{m}$ (Lévy and Ogielski, 1986), the pickup coil size would have to be reduced to less than the $17 \mu\text{m}$ of Reim *et al.* (1986) in order not to average over too many coherence volumes. The technical feasibility of using small enough coils while maintaining adequate sensitivity is uncertain. Alternately, since, as we saw previously, magnetic fluctuations in ferromagnets and antiferromagnets can be detected in resistivity measurements, it is conceivable that resistivity noise could be used to study metallic spin-glasses. Fabrication of sufficiently small samples would be trivial.

Feng, Bray, Lee, and Moore (1987) have pointed out that metallic spin-glasses with even short-range interactions should have enough coupling between their spin states and their resistivities to exhibit spin-driven electrical noise in the universal conductance fluctuation regime, e.g., at 1 K. At higher temperatures, spin-glasses with frustrated spin-density-wave interactions should show appreciable spin-driven noise analogous to the $1/f$ noise in antiferromagnetic Cr.

I. Putting the nuisance to work

Most attempts over the years to use $1/f$ noise to obtain information about something else have been premature, since not enough was known about the noise itself. As $1/f$ noise becomes better characterized, it also is starting to be useful. In particular, the well-established information that the $1/f$ noise comes from local, independent, and often uniformly distributed resistance fluctuations is proving useful in probing inhomogeneous current patterns.

Koch, Lloyd, and Cronin (1985) have demonstrated a simple practical application in predicting the failure of Al film resistors on the basis of an increase in their noise which occurs well before any significant change in resistance. The reason is essentially that the noise probes the fourth moment of the current density, while the resistance probes only the second moment, so the noise can show a thin spot or crack more easily. In fact, the variance of the spectral density, which probes the eighth moment, might turn out to be even more useful.

The same weighting property of the noise has also been useful in a more theoretical problem—investigating the distribution of current in a percolating network. Several predictions have been made for the noise scaling near the percolation threshold in various cases (Garfunkel and Weissman, 1985b; Rammal, 1985; Rammal, Tannous, and Tremblay, 1985; Tremblay, Feng, and Breton, 1986;

Wright, Bergman, and Kantor 1986).

Several groups have also made measurements of this noise scaling in three dimensions (Chen and Chou, 1985; Mantese and Webb, 1985; Rudman, Calabrese, and Garland, 1986) and in two dimensions (Garfunkel and Weissman, 1985b; Koch, Laibowitz, Allessandri, and Vigiano 1985). The two-dimensional results have demonstrated an important point—that realistic continuum percolation problems can show very different scaling properties than lattice models do. Some of the *ad hoc* scaling arguments used to interpret the 3D results (Chen and Chou, 1985) were incorrect. For example, in general, the number of sites on the percolation cluster is not proportional to $1/R$. All the experimental results to date require some caution in interpretation, either because of inhomogeneities in the concentration or because of noisy conduction paths parallel to the percolation clusters, or in some cases because of both.

Mantese, Curtin, and Webb (1986) have recently demonstrated that in some metal-insulator composites the noise arises almost exclusively from tunneling paths through the insulator, even above the metal percolation threshold. Both the conductance and the noise, as a function of metal fraction, can be nicely fit by assuming that there are two types of resistors in a lattice percolation model—one with high resistance and with noise, the other with low resistance and no noise.

In cases where frequency-dependent conductivity is found near percolation threshold, measurements of $1/f$ noise in the impedance as a function of ac probe frequency, as well as complex cross spectra of the fluctuations at different probe frequencies, should provide some stringent constraints on models of the ac conduction paths. The cross spectra in particular would allow a determination of the extent to which the high-current links are shared by the different networks carrying different frequencies of current.

In a few cases, the highly localized spontaneous perturbations giving $1/f$ noise have been useful in probing the homogeneous conduction process itself, not just effects of inhomogeneous geometries. In particular, for low-temperature semiconductors the scattering from a single site disturbs the current flow over distances of greater than a mean free path, which can exceed the spacing between leads. To measure this effect, reliably, it is necessary to switch the scattering from some site on or off—which is not easy to do. However, the spontaneous fluctuation switching serves almost as well (Howard, Jackel, Mankiewich, and Skocpol, 1986).

In very-low-temperature 1D or 2D conductors, the large quantum coherence lengths of the conduction process are directly measurable in the spatial coherence of the low-temperature remnants of the $1/f$ noise (Skocpol *et al.*, 1986). The noise continues to come from localized trapping transitions (in semiconductors), so the spatial correlation is due to the highly nonlocal properties of the scattering itself, i.e., interference terms between scattering sites dominate.

V. CONCLUSIONS

Overall, a detailed look at $1/f$ noise in a variety of materials fails to confirm the initial impression of universality. Whether one focuses on the mechanisms by which conductivity is affected or on the kinetic patterns leading to the spectral form, superficially similar systems turn out to be quite different.

One broad class of $1/f$ -noise-making systems now seems substantially understood. Semiconductors with insulating amorphous surfaces or inclusions and junctions with barriers of similar materials have noise dominated by occupation fluctuations of electron traps. These fluctuations couple to the measured variable by a variety of obvious mechanisms. The range of characteristic times results mainly from a spread of activation energies for trapping-detrappling, but also somewhat from a range of temperature-independent rate factors. The activation energies probably involve coupling of the electronic state to displacements of atoms that have enough mass to inhibit tunneling and whose arrangements are sufficiently varied to give the spread of kinetic parameters. There is almost no correlation between the trap depth and the kinetic activation energy.

Even in semiconductors, however, there is some suggestion of variety in the origins of the noise. Some of the traps on Si and in other systems are tightly coupled in ways that do not allow them to be described as two-state systems. In GaAs the individual traps have not been observed, but noise statistics show that decomposition into parallel two-state systems is impossible.

The case of metals, the other broad class of $1/f$ -noise-making materials, has not been fully solved and seems likely to fit an even less uniform picture. Current evidence indicates that almost all the noise in metals comes from defect motions, although in Cr rotating antiferromagnetic polarization is implicated. The defect motions seem to fall into two categories. One corresponds to Snoek internal friction, with transitions between rotated versions of some defect state. The other involves transitions in more complicated environments. The first type of mechanism has a distinctive almost traceless symmetry parameter, should be Gaussian in the smallest currently practical samples, and should not be increased in samples with many extended defects. Cu irradiated with electrons provides a very probable example of such $1/f$ noise, and Bi may provide another if no more exotic mechanism is involved. The second type is more likely to show nonscalar symmetry without being nearly traceless, to show interesting non-Gaussian effects, and to be sensitive to the grosser features of crystal structure. Pb, In, Nb, and Al are possible candidates for this general category.

Much work remains to be done using symmetry, non-Gaussian effects (especially single-site measurements, when possible), systematic changes in sample properties, and internal friction to pin down the nature of the defects involved in different materials. The reproducibility of the

noise in many metals is somewhat puzzling and may provide some information on how defects are formed and removed from films.

Two peculiarities remain. No satisfactory theory exists for the high levels of $1/f$ flux noise in dc SQUID's. The high noise levels in metallic whiskers seem extremely anomalous, unless they can be accounted for by contact artifacts.

The results on $1/f$ noise have several important implications for the general question of nonexponential kinetics in amorphous systems. First, since many cases of $1/f$ noise arise from simple superpositions of two-state systems, one should not assume that even the most severe nonexponentiality requires series kinetics explanations. If the lowest approximation to the distribution of rates for isolated transitions in an amorphous material is very broad (i.e., $1/f$) rather than very narrow (i.e., Lorentzian), the dynamic nature of the inhomogeneities might best be viewed as narrowing the kinetics to a stretched exponential form, rather than as broadening them. Second, the experimental techniques devised to distinguish between series and parallel kinetics in $1/f$ noise should be transferable with little modification to other broad kinetics in amorphous systems, so long as the spontaneous fluctuations may be detected in sufficiently small samples.

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