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Lista 4 - GA

1.

a) Matriz ampliada:

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} = [\tilde{A} | \tilde{B}] \to \tilde{A} X = \tilde{B}$$

Sistema equivalente:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$\begin{cases} X = 2 \\ Y = -1 \end{cases}$$

b) Matriz ampliada:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} = [\tilde{A} | \tilde{B}] \to \tilde{A} X = \tilde{B}$$

Sistema equivalente:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix} \quad \begin{cases} X = 4 \\ Y = 3 \\ Z = 2 \\ W = 1 \end{cases}$$

c) Matriz ampliada:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} = [\tilde{A} | \tilde{B}] \to \tilde{A} Y = \tilde{B}$$

Sistema equivalente:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \quad \begin{cases} x = 6 \\ y = 3 \\ z + w = 2 \end{cases} \longleftrightarrow$$

$$\begin{cases} x = 6 \\ y = 3 \\ z = 2 - w \end{cases}$$

Parâmetros:

Sejam $w = \alpha$; $\alpha \in \mathbb{R}$

$$\begin{cases} x = 6 \\ y = 3 \\ z = 2 - \alpha \end{cases} ; \quad \alpha \in \mathbb{R}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ -1 \\ +1 \end{bmatrix}, \quad \alpha \in \mathbb{R}$$

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d) Matriz ampliada:

$$\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{pmatrix} = [\tilde{A} | \tilde{B}] \to \tilde{A}X = \tilde{B}$$

Sistema equivalente:

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad \begin{cases} X + 3Z = 1 \\ Y - Z = 2 \end{cases} \leftrightarrow$$

$$\begin{cases} X = 1 - 3Z \\ Y = 2 + Z \end{cases}$$

Parâmetro:

Seja $Z = \alpha$; $\alpha \in \mathbb{R}$

$$\begin{cases} X = 1 - 3\alpha \\ Y = 2 + \alpha \end{cases} ; \alpha \in \mathbb{R}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}, \alpha \in \mathbb{R}$$

e) Matriz ampliada:

$$\begin{pmatrix} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & -5 \end{pmatrix} = [\tilde{A} | \tilde{B}] \to \tilde{A}X = \tilde{B}$$

Sistema equivalente:

$$\begin{pmatrix} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ -5 \end{pmatrix}$$

$$\begin{cases} X - 7W = 8 \\ Y + 3W = 2 \\ Z + W = -5 \end{cases} \longleftrightarrow \begin{cases} X = 8 + 7W \\ Y = 2 - 3W \\ Z = -5 - W \end{cases}$$

Parâmetro:

Seja $W = \alpha$; $\alpha \in \mathbb{R}$

$$\begin{cases} X = 8 + 7\alpha \\ Y = 2 - 3\alpha \\ Z = -5 - \alpha \\ W = \alpha \end{cases} ; \alpha \in \mathbb{R}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ -5 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 7 \\ -3 \\ -1 \\ 1 \end{bmatrix}, \alpha \in \mathbb{R}$$

f) Matriz ampliada:

$$\begin{pmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = [\tilde{A} | \tilde{B}] \rightarrow \tilde{A}X = \tilde{B}$$

Sistema equivalente:

$$\begin{pmatrix} 1 & -6 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \\ T \end{pmatrix} = \begin{pmatrix} -2 \\ 7 \\ 8 \\ 0 \end{pmatrix}$$

$$\begin{cases} X - 6Y + 3T = -2 \\ Z + 4T = 7 \\ W + 5T = 8 \\ 0 = 0 \end{cases} \longleftrightarrow \begin{cases} X = -2 + 6Y - 3T \\ Z = 7 - 4T \\ W = 8 - 5T \end{cases}$$

Parâmetros

Sejam $Y = \alpha$ e $T = \beta$; $\alpha, \beta \in \mathbb{R}$

$$\begin{cases} X = -2 + 6\alpha - 3\beta \\ Y = \alpha \\ Z = 7 - 4\beta \\ W = 8 - 5\beta \\ T = \beta \end{cases} ; \alpha, \beta \in \mathbb{R}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ W \\ T \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 7 \\ 8 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -3 \\ 0 \\ -4 \\ -5 \\ 1 \end{bmatrix}, \alpha, \beta \in \mathbb{R}$$

2.

a) $\begin{cases} 3X - 4Y = 1 \\ X + 3Y = 9 \end{cases}$ $A = \begin{pmatrix} 3 & -4 \\ 1 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$

$$M = [A|B] = \left(\begin{array}{cc|c} 3 & -4 & 1 \\ 1 & 3 & 9 \end{array}\right) \; l_2 \leftarrow 3l_2 - l_1$$

$$\begin{pmatrix} 3 & -4 & 1 \\ 0 & 5 & 26 \end{pmatrix} \; l_1 \leftarrow 5l_1 + 4l_2$$

$$\begin{pmatrix} 15 & 0 & 109 \\ 0 & 5 & 26 \end{pmatrix} \; \begin{array}{l} l_1 \leftarrow l_1/15 \\ l_2 \leftarrow l_2/5 \end{array} \quad \begin{pmatrix} 1 & 0 & 109/15 \\ 0 & 1 & 26/5 \end{pmatrix}$$

$$\tilde{M} = [\tilde{A}|\tilde{B}] = \begin{pmatrix} 1 & 0 & 109/15 \\ 0 & 1 & 26/5 \end{pmatrix}$$

$$\tilde{A}X = \tilde{B}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 109/15 \\ 26/5 \end{bmatrix} \Leftrightarrow \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 109/15 \\ 26/5 \end{bmatrix}$$

$$\begin{cases} X = 109/15 \\ Y = 26/5 \end{cases} \quad SPD$$

b) $\begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$ $A = \begin{pmatrix} 5 & 8 \\ 10 & 16 \end{pmatrix}$ $B = \begin{pmatrix} 34 \\ 50 \end{pmatrix}$

$$M = [A|B] = \begin{pmatrix} 5 & 8 & 34 \\ 10 & 16 & 50 \end{pmatrix} \; l_2 \leftarrow l_2 - 2l_1$$

$$\begin{pmatrix} 5 & 8 & 34 \\ 0 & 0 & -18 \end{pmatrix} = \tilde{M} = [\tilde{A}|\tilde{B}]$$

$$\tilde{A}X = \tilde{B}$$

$$\begin{bmatrix} 5 & 8 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 34 \\ -18 \end{bmatrix} \Leftrightarrow \begin{cases} 5x + 8y = 34 \\ 0 = -18 \end{cases}$$

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c) $\begin{cases} x + 2y = 5 \\ 2x - 3y = -4 \end{cases}$ $A = \begin{pmatrix} 1 & 2 \\ 2 & -3 \end{pmatrix}$ $B = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

$M = [A|B] = \begin{pmatrix} 1 & 2 & 5 \\ 2 & -3 & -4 \end{pmatrix}$ $l_2 \leftarrow l_2 - 2l_1$

$\begin{pmatrix} 1 & 2 & 5 \\ 0 & -7 & -14 \end{pmatrix}$ $l_1 \leftarrow 7l_1 + 2l_2$

$\begin{pmatrix} 7 & 0 & 7 \\ 0 & -7 & -14 \end{pmatrix}$ $l_1 \leftarrow l_1/7$, $l_2 \leftarrow l_2/(-7)$ $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

$\tilde{M} = [\tilde{A} | \tilde{B}] = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

$\tilde{A} x = \tilde{B}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \Longleftrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\begin{cases} x = 1 \\ y = 2 \end{cases}$ SPD

d) $\begin{cases} 3x + 2y - 5z = 8 \\ 2x - 4y - 2z = -4 \\ x - 2y - 3z = -4 \end{cases}$ $A = \begin{pmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{pmatrix}$

$B = \begin{pmatrix} 8 \\ -4 \\ -4 \end{pmatrix}$ $M = [A|B] = \begin{pmatrix} 3 & 2 & -5 & 8 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{pmatrix}$

$l_2 \leftarrow l_2 - 2l_3$ $\begin{pmatrix} 3 & 2 & -5 & 8 \\ 0 & 0 & 4 & 4 \\ 1 & -2 & -3 & -4 \end{pmatrix}$

$$l_3 \leftarrow 3l_3 \sim \begin{pmatrix} 3 & 2 & -5 & 8 \\ 0 & 0 & 4 & 4 \\ 3 & -6 & -9 & -12 \end{pmatrix} \; l_3 \leftarrow l_3 - l_1 \sim$$

$$\begin{pmatrix} 3 & 2 & -5 & 8 \\ 0 & 0 & 4 & 4 \\ 0 & -8 & -4 & -20 \end{pmatrix} \; l_2 \leftarrow l_2 - l_3 \sim$$

$$\begin{pmatrix} 3 & 2 & -5 & 8 \\ 0 & 8 & 8 & 24 \\ 0 & -8 & -4 & -20 \end{pmatrix} \; l_1 \leftarrow l_1 + l_3/4 \sim$$

$$\begin{pmatrix} 3 & 0 & -6 & 3 \\ 0 & 8 & 8 & 24 \\ 0 & -8 & -4 & -20 \end{pmatrix} \; l_3 \leftarrow l_3 + l_2 \sim$$

$$\begin{pmatrix} 3 & 0 & -6 & 3 \\ 0 & 8 & 8 & 24 \\ 0 & 0 & 4 & 4 \end{pmatrix} \; l_2 \leftarrow l_2 - 2l_3$$

$$\begin{pmatrix} 3 & 0 & -6 & 3 \\ 0 & 8 & 0 & 16 \\ 0 & 0 & 4 & 4 \end{pmatrix} \; \begin{matrix} l_1 \leftarrow l_1/3 \\ l_2 \leftarrow l_2/8 \\ l_3 \leftarrow l_3/4 \end{matrix} \sim$$

$$\begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \; l_1 \leftarrow l_1 + 2l_3 \sim$$

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix} = \bar{M} = [\tilde{A} \mid \tilde{B}]$$

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$$\tilde{A}x = \tilde{B}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \longleftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$\begin{cases} x = 3 \\ y = 2 \\ z = 1 \end{cases} \quad \text{SPD}$$

l) $\begin{cases} 2x - 6y = -4 \\ x + 3y = 1 \\ 4x + 12y = 2 \end{cases}$ $A = \begin{pmatrix} 2 & -6 \\ 1 & 3 \\ 4 & 12 \end{pmatrix}$ $B = \begin{pmatrix} -4 \\ 1 \\ 2 \end{pmatrix}$

$$M = [A|B] = \begin{pmatrix} 2 & -6 & -4 \\ 1 & 3 & 1 \\ 4 & 12 & 2 \end{pmatrix} \quad \begin{matrix} l_1 \leftarrow l_1/2 \\ \sim \\ l_3 \leftarrow l_3/2 \end{matrix}$$

$$\begin{pmatrix} 1 & -3 & -2 \\ 1 & 3 & 1 \\ 2 & 6 & 1 \end{pmatrix} \quad \underset{\sim}{l_2 \leftarrow l_2 - l_1}$$

$$\begin{pmatrix} 1 & -3 & -2 \\ 0 & 6 & 3 \\ 2 & 6 & 1 \end{pmatrix} \quad \underset{\sim}{l_3 \leftarrow l_3 - 2l_1} \quad \begin{pmatrix} 1 & -3 & -2 \\ 0 & 6 & 3 \\ 0 & 12 & 5 \end{pmatrix}$$

$$\underset{\sim}{l_2 \leftarrow l_2/3} \begin{pmatrix} 1 & -3 & -2 \\ 0 & 2 & 1 \\ 0 & 12 & 5 \end{pmatrix} \quad l_1 \leftarrow l_1 + l_3/4$$

$$\begin{pmatrix} 1 & 0 & -3/4 \\ 0 & 2 & 1 \\ 0 & 12 & 5 \end{pmatrix} \quad \underset{\sim}{l_3 \leftarrow l_3 - 6l_2} \quad \begin{pmatrix} 1 & 0 & -3/4 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

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$$l_2 \leftarrow l_2/2 \quad \sim \quad \begin{pmatrix} 1 & 0 & -3/4 \\ 0 & 1 & 1/2 \\ 0 & 0 & -1 \end{pmatrix} = \tilde{M} = [\tilde{A} \mid \tilde{B}]$$

$$\tilde{A}X = \tilde{B}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3/4 \\ 1/2 \\ -1 \end{pmatrix} \Leftrightarrow \begin{cases} x = -3/4 \\ y = 1/2 \\ 0 = -1 \end{cases} \quad SI$$

f) $\begin{cases} x + 2y - z = 2 \\ 2x - y + 3z = 9 \\ 3x + 3y - 2z = 3 \end{cases}$ $\quad A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{pmatrix}$ $\quad B = \begin{pmatrix} 2 \\ 9 \\ 3 \end{pmatrix}$

$$M = [A \mid B] = \begin{pmatrix} 1 & 2 & -1 & 2 \\ 2 & -1 & 3 & 9 \\ 3 & 3 & -2 & 3 \end{pmatrix} \begin{matrix} l_2 \leftarrow l_2 - 2l_1 \\ \sim \\ l_3 \leftarrow l_3 - 3l_1 \end{matrix}$$

$$\begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & -5 & 5 & 5 \\ 0 & -3 & 1 & -3 \end{pmatrix} \begin{matrix} l_1 \leftarrow l_1 . 3 \\ \sim \\ l_3 \leftarrow l_3 . 2 \end{matrix} \begin{pmatrix} 3 & 6 & -3 & 6 \\ 0 & -5 & 5 & 5 \\ 0 & -6 & 2 & -6 \end{pmatrix}$$

$$l_1 \leftarrow l_1 + l_3 \quad \sim \quad \begin{pmatrix} 3 & 0 & -1 & 0 \\ 0 & -5 & 5 & 5 \\ 0 & -6 & 2 & -6 \end{pmatrix} \quad l_3 \leftarrow l_3 - l_2 \quad \sim$$

$$\begin{pmatrix} 3 & 0 & -1 & 0 \\ 0 & -5 & 5 & 5 \\ 0 & -1 & -3 & -11 \end{pmatrix} \quad l_2 \leftarrow l_2/(-5) \quad \sim \quad \begin{pmatrix} 3 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & -3 & -11 \end{pmatrix}$$

$$l_3 \leftarrow l_3 + l_2$$

$$\begin{pmatrix} 3 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -10 \end{pmatrix} \quad \sim \quad l_3 \leftarrow l_3/(-2)$$

$$\begin{pmatrix} 3 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & +1 & +5 \end{pmatrix} \quad l_2 \leftarrow l_2 + l_3$$

$$\begin{pmatrix} 3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{pmatrix} \quad l_1 \leftarrow l_1 + l_3$$

$$\begin{pmatrix} 3 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{pmatrix} = \tilde{M} = [\tilde{A} \mid \tilde{B}]$$

$$\tilde{A} X = \tilde{B}$$

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 5 \end{pmatrix} \Longleftrightarrow \begin{cases} 3x = 5 \\ y = 4 \\ z = 5 \end{cases}$$

$$\begin{cases} x = 5/3 \\ y = 4 \\ z = 5 \end{cases}$$

g) $\begin{cases} x + 3z = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases}$ $\quad A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -4 & 0 \\ 3 & -2 & -5 \end{pmatrix} \quad B = \begin{pmatrix} -8 \\ -4 \\ 26 \end{pmatrix}$

$$M = [A \mid B] = \begin{pmatrix} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ 3 & -2 & -5 & 26 \end{pmatrix} \begin{array}{l} \\ l_2 \leftarrow l_2 - 2l_1 \\ l_3 \leftarrow l_3 - 3l_1 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 3 & -8 \\ 0 & -4 & -6 & 12 \\ 0 & -2 & -14 & 50 \end{pmatrix} \quad l_2 \leftarrow l_2/(-2) \quad \begin{pmatrix} 1 & 0 & 3 & -8 \\ 0 & +2 & 3 & -6 \\ 0 & -2 & -14 & 50 \end{pmatrix}$$

$\ell_3 \leftarrow \ell_3 + \ell_2$ $\begin{pmatrix} 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -6 \\ 0 & 0 & -11 & 44 \end{pmatrix}$ $\ell_1 \leftarrow \ell_1 - \ell_3$, $\ell_2 \leftarrow \ell_2 + \ell_3$

$\begin{pmatrix} 1 & 0 & 14 & -52 \\ 0 & 2 & -8 & 38 \\ 0 & 0 & -11 & 44 \end{pmatrix}$ $\ell_2 \leftarrow \ell_2/2$, $\ell_3 \leftarrow \ell_3/(11)$

$\begin{pmatrix} 1 & 0 & 14 & -52 \\ 0 & 1 & -4 & 19 \\ 0 & 0 & 1 & -4 \end{pmatrix}$ $\ell_2 \leftarrow \ell_2 + 4\ell_3$

$\begin{pmatrix} 1 & 0 & 14 & -52 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{pmatrix}$ $\ell_1 \leftarrow \ell_1 - 14\ell_3$

$\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{pmatrix} = \tilde{M} = [\tilde{A} | \tilde{B}]$

$\tilde{A}X = \tilde{B}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ -4 \end{pmatrix} \Leftrightarrow \begin{cases} x = 4 \\ y = 3 \\ z = -4 \end{cases}$ SPD

b) $\begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 2x + 2y + 3z = 13 \end{cases}$ $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 2 & 2 & 3 \end{pmatrix}$

$B = \begin{pmatrix} 10 \\ 23 \\ 13 \end{pmatrix}$ $M = [A | B] = \begin{pmatrix} 1 & 2 & 3 & 10 \\ 3 & 4 & 6 & 23 \\ 2 & 2 & 3 & 13 \end{pmatrix}$

$$\begin{matrix} l_3 \leftarrow l_3 - 2l_1 \\ l_2 \leftarrow l_2 - 3l_1 \end{matrix} \sim \begin{pmatrix} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 0 & -2 & -3 & -16 \end{pmatrix} \; l_1 \leftarrow l_1 + l_3 \sim$$

$$\begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & -2 & -3 & -7 \\ 0 & -2 & -3 & -16 \end{pmatrix} \; l_2 \leftarrow l_2 - l_3 \sim$$

$$\begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & 0 & 0 & 9 \\ 0 & -2 & -3 & -16 \end{pmatrix} = \tilde{M} = [\tilde{A} \mid \tilde{B}]$$

$$\tilde{A} x = \tilde{B}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -6 \\ 9 \\ -16 \end{pmatrix} \Leftrightarrow \begin{cases} x = -6 \\ 0 = 9 \\ -2y - 3z = -16 \end{cases}$$

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i) $\begin{cases} x - 3y + 4z - w = 2 \\ 2x - y + 3z - 2w = 19 \end{cases}$

$$A = \begin{pmatrix} 1 & -3 & 4 & -1 \\ 2 & -1 & 3 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 2 \\ 19 \end{pmatrix}$$

$$M = [A \mid B] = \begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 2 & -1 & 3 & -2 & 19 \end{pmatrix} \; l_2 \leftarrow l_2 - 2l_1$$

$$\begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 5 & -5 & 0 & 15 \end{pmatrix} \; l_2 \leftarrow l_2 / 5$$

$$\begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 1 & -1 & 0 & 3 \end{pmatrix} \; l_1 \leftarrow l_1 + 3l_2$$

$$\begin{pmatrix} 1 & 0 & 1 & -1 & 11 \\ 0 & 1 & -1 & 0 & 3 \end{pmatrix} \quad l_1 \leftarrow l_1 + l_2$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 14 \\ 0 & 1 & -1 & 0 & 3 \end{pmatrix} = \tilde{M} = [\tilde{A} \mid \tilde{B}]$$

$$\tilde{A} X = \tilde{B}$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 14 \\ 3 \end{pmatrix}$$

$$\begin{cases} x - w = 14 \\ y - z = 3 \end{cases} \Leftrightarrow \begin{cases} x = 14 + w \\ y = 3 + z \end{cases} \quad SPI$$

3.

a) $$\begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{pmatrix} = [\tilde{A} \mid \tilde{B}] \rightarrow \tilde{A} X = \tilde{B}$$

$$l_2 \leftarrow l_2 - l_1$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 1 & 0 & 2 & 1 & 3 \end{pmatrix} \quad l_3 \leftarrow l_3 - l_1$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{pmatrix} \quad l_1 \leftarrow l_1 - 2 l_2$$

$$\sim \begin{pmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{pmatrix} \quad l_3 \leftarrow l_3 + 2 l_2$$

$$\begin{pmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{pmatrix} \quad l_1 \leftarrow l_1 + 3 l_2$$

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$$\begin{pmatrix} 1 & 3 & 0 & 1 & 7 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{pmatrix} \quad l_1 \leftarrow l_1 - 3 l_2$$

$$\sim$$

$$\begin{pmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{pmatrix} \quad \sim \quad l_3 \leftarrow l_3 / (-7)$$

$$\begin{pmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \quad l_2 \leftarrow l_2 + 3 l_3$$

$$\begin{pmatrix} 1 & 0 & 9 & 1 & 10 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} \quad l_1 \leftarrow l_1 - 9 l_3$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix} = \tilde{M} = [\tilde{A} \mid \tilde{B}]$$

$$\tilde{A} X = \tilde{B} \qquad \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix}$$

$$\begin{cases} x + w = 1 \\ y = -4 \\ z = 1 \end{cases} \iff \begin{cases} x = 1 - w \\ y = -4 \\ z = 1 \end{cases}$$

Seja $w = \alpha$; $\alpha \in \mathbb{R}$

$$\begin{cases} x = 1 - \alpha \\ y = -4 \\ z = 1 \end{cases} ; \alpha \in \mathbb{R}$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 1 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \alpha \in \mathbb{R}$$

b) $$\left(\begin{array}{ccccc} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array}\right) \sim \; l_3 \leftarrow l_3 - l_1$$

$$\left(\begin{array}{ccccc} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{array}\right) l_1 \to l_1 + l_3 \sim$$

$$\left(\begin{array}{ccccc} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -2 \end{array}\right) \sim \; l_3 \leftarrow l_3 + l_2$$

$$\left(\begin{array}{ccccc} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & 1 & 0 & -1 & 2 \end{array}\right) = [\tilde{A} | \tilde{B}]$$

$$\tilde{A}X = \tilde{B} \quad \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & -3 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

$$\begin{cases} x + 2z + w = -1 \\ 2y + z + (-3w) = 3 \\ y - w = 2 \end{cases} \Leftrightarrow \begin{cases} x = -1 - 2z - w \\ 2y = 3 + 3w - z \\ y = 2 + w \end{cases}$$

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c) $\begin{pmatrix} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{pmatrix} = [\tilde{A} \mid \tilde{B}] \rightarrow \tilde{A}x = \tilde{B}$

$\cancel{A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}$

$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & +1 & -1 & 0 \\ 0 & +1 & -2 & 0 \\ 0 & 3 & 3 & 0 \end{pmatrix} \begin{matrix} \\ l_2 \leftarrow l_2 - l_1 \\ l_3 \leftarrow l_3 - l_1 \\ l_4 \leftarrow l_4 - l_1 \end{matrix} \sim \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad l_1 \leftarrow l_1 + 2 l_2 \quad \sim$

$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \sim \quad l_3 \leftarrow l_3 + l_4$

$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad l_1 \leftarrow l_1 + l_3 \quad \sim$

$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad l_2 \leftarrow l_2 - 2 l_3$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{matrix} l_2 \leftarrow l_2/(-1) \\ l_3 \leftarrow l_3/(-1) \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = [\tilde{A} | \tilde{B}]$$

$$\tilde{A}X = \tilde{B} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} X = 0 \\ Y = 0 \\ Z = 0 \\ Y = 0 \end{cases}$$

4.

a) $\begin{cases} X - 2Y + Z = 1 \\ 2X - 5Y + Z = -2 \\ 3X - 7Y + 2Z = -1 \end{cases}$; b) $\begin{cases} X - 2Y + Z = 2 \\ 2X - 5Y + Z = -1 \\ 3X - 7Y + 2Z = 2 \end{cases}$

Matriz ampliada: $[A | B_1 | B_2]$

$$M = \left[\begin{array}{ccc|c|c} 1 & -2 & 1 & 1 & 2 \\ 2 & -5 & 1 & -2 & -1 \\ 3 & -7 & 2 & -1 & 2 \end{array}\right] \begin{matrix} l_2 \leftarrow l_2 - 2l_1 \\ \\ l_3 \leftarrow l_3 - 3l_1 \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & -1 & -1 & -4 & -4 \end{bmatrix} \begin{array}{l} l_1 \leftarrow l_1 - 2l_2 \\ \sim \\ l_3 \leftarrow l_3 - l_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 3 & 9 & 12 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \sim \\ l_2 \leftarrow -l_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 3 & 9 & 12 \\ 0 & 1 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = [\tilde{A} \mid \tilde{B}_1 \mid \tilde{B}_2]$$

a) $$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x + 3z = 9 \\ y + z = 4 \\ 0 = 0 \end{cases} \iff \begin{cases} x = 9 - 3z \\ y = 4 - z \end{cases}$$

Seja $z = \lambda$, $\lambda \in \mathbb{R}$:

$$\begin{cases} x = 9 - 3\lambda \\ y = 4 - \lambda \\ z = \lambda \end{cases} ; \lambda \in \mathbb{R} \quad \text{ou} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -1 \\ 1 \end{pmatrix}$$

b) $$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 5 \\ 1 \end{bmatrix}$$

$$\begin{cases} x + 3z = 12 \\ y + z = 5 \\ 0 = 1 \end{cases}$$

(Sistema inconsistente)

SI

5.

$$A = \begin{pmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{pmatrix}$$

a) $(A + 4I_3)X = 0$, com $0 = (0_j)_{3\times 1}$ e $X = (x_j)_{3\times 1}$

$(A + 4I_3)X = \bar{0}$ (sistema homogênio)

$$\left(\begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 1+4 & 0 & 5 \\ 1 & 1+4 & 1 \\ 0 & 1 & -4+4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 5 \\ 1 & 5 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Matriz ampliada:

$$M = [A + 4I_3 \mid \bar{0}] = \begin{bmatrix} 5 & 0 & 5 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad l_1 \leftarrow l_1/5$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad l_2 \leftarrow l_2 - l_1$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \ell_2 \leftarrow \ell_2/5$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x + z = 0 \\ y = 0 \\ y = 0 \end{cases} \rightarrow \begin{cases} x = 0 - z \\ y = 0 \end{cases}$$

Seja $z = \alpha$, $\alpha \in \mathbb{R}$

$$\begin{cases} x = -\alpha \\ y = 0 \end{cases} \longleftrightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \alpha \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

2) $AX = 2X$, com $X = (x_j)_{3 \times 1}$

$$AX - 2X = 2X - 2X$$
$$AX - 2X = \bar{0}$$

$(A - 2I_3)X = \bar{0}$ (Sistema homogênio)

$$\left(\begin{bmatrix} 1 & 0 & 5 \\ 1 & 1 & 1 \\ 0 & 1 & -4 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 1-2 & 0 & 5 \\ 1 & 1-2 & 1 \\ 0 & 1 & -4-2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 5 \\ 1 & -1 & 1 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$M = [A - \lambda I_3 \mid \bar{0}] = \begin{bmatrix} -1 & 0 & 5 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -6 & 0 \end{bmatrix}$$

$$\underset{\sim}{l_2 \leftarrow l_2 + l_1} \begin{bmatrix} -1 & 0 & 5 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & 1 & -6 & 0 \end{bmatrix} \underset{\sim}{l_3 \leftarrow l_3 + l_2}$$

$$\begin{bmatrix} -1 & 0 & 5 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underset{l_2 \leftarrow -l_2}{\overset{l_1 \leftarrow -l_1}{\sim}}$$

$$\begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x - 5z = 0 \\ y - 6z = 0 \\ 0 = 0 \end{cases} \rightarrow \begin{cases} x = 5z \\ y = 6z \end{cases}$$

Seja $z = \alpha$, $\alpha \in \mathbb{R}$

$$\begin{cases} x = 5\alpha \\ y = 6\alpha \\ z = \alpha \end{cases} \leftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \alpha \begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix}$$

(/ /)

6.

a) $\begin{cases} x + y + z = 2 \\ 2x + 3y + 2z = 5 \\ 2x + 3y + (a^2-1)z = a+1 \end{cases}$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 5 \\ a+1 \end{bmatrix}$

Matriz ampliada:

$M = [A|B] = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2-1 & a+1 \end{bmatrix} \begin{matrix} l_2 \leftarrow l_2 - 2l_1 \\ \sim \\ l_3 \leftarrow l_3 - 2l_1 \end{matrix}$

$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & a^2-3 & a-3 \end{bmatrix} \begin{matrix} l_1 \leftarrow l_1 - l_2 \\ \sim \\ l_3 \leftarrow l_3 - l_2 \end{matrix}$

$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-3 & a-4 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & a^2-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ a-4 \end{bmatrix}$

$\begin{cases} x + z = 1 \\ y = 1 \\ (a^2-3)z = a-4 \end{cases} \rightarrow \begin{cases} x = 1 - z \\ y = 1 \\ (a^2-3)z = a-4 \end{cases}$

$(a^2-3)z = a-4$

1) $(a^2-3) \neq 0 \rightarrow z = \dfrac{a-4}{a^2-3}$ (SPD)

$a^2 \neq 3 \iff a \neq \pm\sqrt{3}$

ii) $(a^2-3) = 0 \implies 0z = a-4$
$= \pm\sqrt{3} - 4 \neq 0$
(SI)

$a = \pm\sqrt{3}$

Resposta: $\begin{cases} SPD: & a \neq \pm\sqrt{3} \\ SI: & a = \pm\sqrt{3} \end{cases}$

b) $\begin{cases} x + 2y - 3z = 4 \\ 3x - y + 5z = 2 \\ 4x + y + (a^2-14)z = a+2 \end{cases}$

$A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 5 \\ 4 & 1 & (a^2-14) \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 2 \\ a+2 \end{bmatrix}$

matriz ampliada:

$M = [A|B] = \left[\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2-14) & (a+2) \end{array}\right] \begin{array}{l} l_2 \leftarrow l_2 - 3l_1 \\ \sim \\ l_3 \leftarrow l_3 - 4l_1 \end{array}$

$\left[\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right] \begin{array}{l} l_1 \leftarrow 7l_1 + 2l_2 \\ \sim \\ l_3 \leftarrow l_3 + l_2 \end{array}$

$\left[\begin{array}{cccc} 7 & 0 & 7 & 8 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2+12 & a-24 \end{array}\right] \begin{array}{l} l_1 \leftarrow l_1/7 \\ l_2 \leftarrow l_2/(-7) \\ \sim \end{array}$

$\left[\begin{array}{cccc} 1 & 0 & 1 & \frac{8}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & a^2+12 & a-24 \end{array}\right] \iff \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & a^2+12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$= \begin{bmatrix} \frac{8}{7} \\ \frac{10}{7} \\ a-24 \end{bmatrix} \qquad \begin{cases} x + z = \frac{8}{7} \\ y - 2z = \frac{10}{7} \\ (a^2+12)\, z = a - 24 \end{cases} \longleftrightarrow \begin{cases} x = \frac{8}{7} - z \\ y = \frac{10}{7} + 2z \\ (a^2+12)\, z = a - 24 \end{cases}$$

$(a^2+12)\, z = a - 24$

i) $(a^2+12) \neq 0 \rightarrow z = \dfrac{a-24}{a^2+12}$ (SPD)

$a^2 \neq 12 \longleftrightarrow a \neq \pm\sqrt{12}$

ii) $(a^2+12) = 0 \implies 0 . z = a - 24$
$= \pm\sqrt{12} - 24 \neq 0$
(SI)

$a = \pm\sqrt{12}$

Conclusão: $\begin{cases} SPD : a \neq \pm\sqrt{12} \\ SI : a = \pm\sqrt{12} \end{cases}$

7.

a)
$A = \begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix} \qquad M = [A \mid I_2] \longrightarrow [I_2 \mid A^{-1}]$

$M = \left(\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) \quad l_1 \leftarrow l_1/2$

$\left(\begin{array}{cccc} 1 & -1 & \frac{1}{2} & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) \quad l_2 \leftarrow l_2 - 3l_1$

$\left(\begin{array}{cccc} 1 & -1 & \frac{1}{2} & 0 \\ 0 & 4 & -\frac{3}{2} & 1 \end{array}\right) \quad l_1 \leftarrow l_1 . 4 + l_2$

$$\begin{pmatrix} 4 & 0 & \frac{1}{2} & 1 \\ 0 & 4 & -\frac{3}{2} & 1 \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1/4 \\ l_2 \leftarrow l_1/4 \end{array}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{8} & \frac{1}{4} \\ 0 & 1 & -\frac{3}{8} & \frac{1}{4} \end{array}\right) = [I_2 \mid A^{-1}]$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{4} \end{bmatrix}$$

b)

$$B = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \qquad M = [B \mid I_3] \longrightarrow [I_3 \mid B^{-1}]$$

$$M = \left(\begin{array}{ccc|ccc} 2 & -2 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \; l_1 \leftarrow l_1/2$$

$$\begin{pmatrix} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \; l_2 \leftarrow l_2 - l_1$$

$$\begin{pmatrix} 1 & -1 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 3 & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \; l_1 \leftarrow l_1 + l_3$$

$$\begin{pmatrix} 1 & 0 & -1 & \frac{1}{2} & 0 & 1 \\ 0 & 3 & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \; l_2 \leftarrow l_2/3$$

$$\begin{pmatrix} 1 & 0 & -1 & \frac{1}{2} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \; l_3 \leftarrow l_3 - l_2$$

$$\begin{pmatrix} 1 & 0 & -1 & \frac{1}{2} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{4}{3} & \frac{1}{6} & -\frac{1}{3} & 1 \end{pmatrix} \quad l_1 \leftarrow l_1/3$$

$$\sim$$

$$\begin{pmatrix} \frac{1}{3} & 0 & -\frac{1}{3} & \frac{1}{6} & 0 & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{4}{3} & \frac{1}{6} & -\frac{1}{3} & 1 \end{pmatrix} \quad \begin{matrix} l_1 \leftarrow l_1 \cdot 3 \\ l_2 \leftarrow l_2 \cdot 3 \\ l_3 \leftarrow l_3 \cdot 3 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & -1 & \frac{1}{2} & 0 & 1 \\ 0 & 3 & 1 & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & -4 & \frac{1}{2} & -1 & 3 \end{pmatrix} \quad \begin{matrix} l_3 \leftarrow l_3/(-4) \\ l_2 \leftarrow l_2/3 \end{matrix}$$

$$\sim$$

$$\begin{pmatrix} 1 & 0 & -1 & \frac{1}{2} & 0 & 1 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix} \quad l_1 \leftarrow l_1 + l_3$$

$$\sim$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{6} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{pmatrix} \quad l_2 \leftarrow l_2 - l_3/3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{array}\right) = [I_3 \mid B^{-1}]$$

$$\Rightarrow B^{-1} = \begin{bmatrix} \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$$

c)

$$C = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \qquad M = [C \mid I_2] \to [I_2 \mid C^{-1}]$$

$$M = \left(\begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \quad l_2 \leftarrow l_2 - l_1/3$$

$$\sim$$

FORON:

$2 - \frac{5}{3}$

$$\left(\begin{array}{cccc} 3 & 5 & 1 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 1 \end{array}\right) l_1 \leftarrow l_1/3$$

$$\sim$$

$$\left(\begin{array}{cccc} 1 & \frac{5}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 1 \end{array}\right) l_2 \leftarrow l_1 - l_2 \cdot 5$$

$$\sim$$

$$\left(\begin{array}{cccc} 1 & 0 & 2 & -5 \\ 0 & \frac{1}{3} & \frac{1}{3} & 1 \end{array}\right) l_2 \leftarrow l_2 \cdot 3$$

$$\sim$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array}\right) = [I_2 \mid C^{-1}]$$

$$\Rightarrow C^{-1} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

d)

$$D = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \qquad M = [D \mid I_3] \longrightarrow [I_3 \mid D^{-1}]$$

$$M = \left[\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] l_1 \leftrightarrow l_3$$

$$\sim$$

$$\left[\begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \end{array}\right] l_2 \leftarrow l_2 - 2 l_1$$

$$\sim$$

$$\left[\begin{array}{cccccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -2 & -1 & 0 & 1 & -2 \\ 0 & -1 & 1 & 1 & 1 & 0 \end{array}\right] l_2 \leftarrow l_3$$

$$\sim$$

FORON:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 0 & 1 & -2 \end{bmatrix} \begin{matrix} l_1 \leftarrow l_1 + l_2 \\ \sim \\ l_3 \leftarrow l_3 - 2l_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & -3 & -2 & -1 & -2 \end{bmatrix} \begin{matrix} l_1 \leftarrow 3l_1 \\ \sim \\ l_2 \leftarrow 3l_2 \end{matrix}$$

$$\begin{bmatrix} 3 & 0 & 3 & 3 & 3 & 3 \\ 0 & -3 & 3 & 3 & 3 & 0 \\ 0 & 0 & -3 & -2 & -1 & -2 \end{bmatrix} \begin{matrix} l_1 \leftarrow l_1 + l_3 \\ \sim \\ l_2 \leftarrow l_2 + l_3 \end{matrix}$$

$$\begin{bmatrix} 3 & 0 & 0 & 1 & 2 & 1 \\ 0 & -3 & 0 & 1 & 2 & -2 \\ 0 & 0 & -3 & -2 & -1 & -2 \end{bmatrix} \begin{matrix} l_1 \leftarrow l_1/3 \\ l_2 \leftarrow l_2/(-3) \sim \\ l_3 \leftarrow l_3/(-3) \end{matrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & 0 & -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{array}\right] = [I_3 | D^{-1}] \Rightarrow D^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

l)

$$E = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 2 & 0 & -1 \\ 1 & 0 & 2 & 3 \\ 0 & 0 & -2 & 0 \end{pmatrix} \qquad M = [E | I_4] \to [I_4 | E^{-1}]$$

$$M = \left(\begin{array}{cccc|cccc} 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{array}\right) l_4 \leftarrow l_4 + l_1$$

$$\begin{pmatrix} 2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 2 & 1 & 2 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1/2 \\ l_2 \leftarrow l_2/2 \\ l_3 \leftarrow l_3/2 \\ \end{array}$$

$$\begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 1 & \frac{3}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 2 & 1 & 2 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} \\ \\ l_3 \leftarrow l_3 - l_1/2 \\ l_4 \leftarrow l_4 - 2l_1 \end{array}$$

$$\begin{pmatrix} 1 & -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 1 & \frac{5}{4} & -\frac{1}{4} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1 + l_2/2 \\ \\ l_3 \leftarrow l_3 - l_2/4 \\ \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{8} & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{2} & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{array}{l} \\ \\ \\ l_4 \leftarrow l_4 + 2l_3 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & \frac{11}{8} & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{11}{4} & -\frac{1}{2} & -\frac{1}{4} & 1 & 1 \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1 . 4 \\ l_2 \leftarrow l_2 . 2 \\ l_3 \leftarrow l_3 . 8 \\ l_4 \leftarrow l_4 . 4 \end{array}$$

$$\begin{pmatrix} 4 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 8 & 11 & -2 & -1 & 4 & 0 \\ 0 & 0 & 0 & 11 & -2 & -1 & 4 & 4 \end{pmatrix} \begin{array}{l} \\ \\ l_3 \leftarrow l_3/11 \\ l_4 \leftarrow l_4/11 \end{array}$$

FORON:

$$\begin{pmatrix} 4 & 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{11} & 1 & \frac{2}{11} & \frac{1}{11} & \frac{4}{11} & 0 \\ 0 & 0 & 0 & 1 & \frac{2}{11} & \frac{1}{11} & \frac{4}{11} & \frac{4}{11} \end{pmatrix} \begin{array}{l} l_1 \leftarrow l_1 - l_4 \\ l_2 \leftarrow l_2 + l_4 \\ l_3 \leftarrow l_3 - l_4 \\ \end{array}$$

$$\left(\begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & \frac{24}{11} & \frac{12}{11} & -\frac{4}{11} & -\frac{4}{11} \\ 0 & 2 & 0 & 0 & -\frac{2}{11} & \frac{10}{11} & \frac{4}{11} & \frac{4}{11} \\ 0 & 0 & \frac{8}{11} & 0 & 0 & 0 & 0 & -\frac{4}{11} \\ 0 & 0 & 0 & 1 & -\frac{2}{11} & -\frac{1}{11} & \frac{4}{11} & \frac{4}{11} \end{array}\right) \quad l_3 \leftarrow l_3 \cdot 11$$

$$\left(\begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & \frac{24}{11} & \frac{12}{11} & -\frac{4}{11} & -\frac{4}{11} \\ 0 & 2 & 0 & 0 & -\frac{2}{11} & \frac{10}{11} & \frac{4}{11} & \frac{4}{11} \\ 0 & 0 & 8 & 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 & -\frac{2}{11} & -\frac{1}{11} & \frac{4}{11} & \frac{4}{11} \end{array}\right) \quad \begin{array}{l} l_1 \leftarrow l_1/4 \\ l_2 \leftarrow l_2/2 \\ l_3 \leftarrow l_3/8 \end{array}$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{6}{11} & \frac{3}{11} & -\frac{1}{11} & -\frac{1}{11} \\ 0 & 1 & 0 & 0 & -\frac{1}{11} & \frac{5}{11} & \frac{2}{11} & \frac{2}{11} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & -\frac{2}{11} & -\frac{1}{11} & \frac{4}{11} & \frac{4}{11} \end{array}\right) = [\, I_4 \mid E^{-1} \,]$$

$$\Rightarrow E^{-1} = \begin{bmatrix} \frac{6}{11} & \frac{3}{11} & -\frac{1}{11} & -\frac{1}{11} \\ -\frac{1}{11} & \frac{5}{11} & \frac{2}{11} & \frac{2}{11} \\ 0 & 0 & 0 & -\frac{1}{2} \\ -\frac{2}{11} & -\frac{1}{11} & \frac{4}{11} & \frac{4}{11} \end{bmatrix}$$

8.

$$\begin{cases} 1x + 2y + 3z = \cancel{8}\ 26 \\ 2x + 5y + 6z = 60 \\ 2x + 3y + 4z = 40 \end{cases}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 2 & 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 26 \\ 60 \\ 40 \end{bmatrix}$$

i) Determinantes:

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 6 \\ 2 & 3 & 4 \end{vmatrix} \quad \begin{array}{l} l_2 \leftarrow l_2 - 2l_1 \\ l_3 \leftarrow l_3 - 2l_1 \end{array}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ \boxed{0 \quad 1 \quad 0} \\ 0 & -1 & -2 \end{vmatrix} = 1 \cdot (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 0 & -2 \end{vmatrix} = 1 \cdot (-2) - 3 \cdot 0 = -2$$

$$\det(A_x) = \begin{vmatrix} 26 & 2 & 3 \\ 60 & 5 & 6 \\ 40 & 3 & 4 \end{vmatrix}$$

$$= 26 \cdot (-1)^{1+1} \begin{vmatrix} 5 & 6 \\ 3 & 4 \end{vmatrix} + 2 \cdot (-1)^{1+2} \begin{vmatrix} 60 & 6 \\ 40 & 4 \end{vmatrix} + 3 \cdot (-1)^{1+3} \begin{vmatrix} 60 & 5 \\ 40 & 3 \end{vmatrix}$$

$$= 26(5 \cdot 4 - 6 \cdot 3) - 2(60 \cdot 4 - 6 \cdot 40) + 3(60 \cdot 3 - 5 \cdot 40)$$

$$= -8$$

$$\det(A_y) = \begin{vmatrix} 1 & 26 & 3 \\ 2 & 60 & 6 \\ 2 & 40 & 4 \end{vmatrix}$$

$$= 1 \cdot (-1)^{1+1} \begin{vmatrix} 60 & 6 \\ 40 & 4 \end{vmatrix} + 2 \cdot (-1)^{2+1} \begin{vmatrix} 26 & 3 \\ 40 & 4 \end{vmatrix} + 2 \cdot (-1)^{3+1} \begin{vmatrix} 26 & 3 \\ 60 & 6 \end{vmatrix}$$

$$= (60 \cdot 4 - 6 \cdot 40) - 2(26 \cdot 4 - 3 \cdot 40) + 2(26 \cdot 6 - 3 \cdot 60)$$

$$= -16$$

$$\det(A_z) = \begin{vmatrix} 1 & 2 & 26 \\ 2 & 5 & 60 \\ 2 & 3 & 40 \end{vmatrix}$$

$$= 1 \cdot (-1)^{1+1} \begin{vmatrix} 5 & 60 \\ 3 & 40 \end{vmatrix} + 2 \cdot (-1)^{2+1} \begin{vmatrix} 2 & 26 \\ 3 & 40 \end{vmatrix} + 2 \cdot (-1)^{3+1} \begin{vmatrix} 2 & 26 \\ 5 & 60 \end{vmatrix}$$

FORON:

$= (5 \cdot 40 - 60 \cdot 3) - 2(2 \cdot 40 - 26 \cdot 3) + 2(2 \cdot 60 - 26 \cdot 5)$

$= -4$

ii) Regra de Cramer:

- $X = \dfrac{\det(A_x)}{\det(A)} = \dfrac{-8}{-2} = 4$

- $Y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{-16}{-2} = 8$

- $Z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{-4}{-2} = 2$

Solução: $\begin{cases} X = 4 \\ Y = 8 \\ Z = 2 \end{cases}$

9.

$$\begin{cases} 5X + 2Y + 6Z = 2200 \\ 3Z - Y = 0 \\ Y - X - Z = 0 \end{cases}$$

$$A = \begin{bmatrix} 5 & 2 & 6 \\ 0 & -1 & 3 \\ -1 & 1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 2200 \\ 0 \\ 0 \end{bmatrix}$$

i) Determinantes:

$$\det(A) = \begin{vmatrix} 5 & 2 & 6 \\ \boxed{0 \quad -1 \quad 3} \\ -1 & 1 & -1 \end{vmatrix}$$

$$= -1\cdot(-1)^{2+2}\begin{vmatrix}5 & 6\\ -1 & -1\end{vmatrix} + 3\cdot(-1)^{2+3}\begin{vmatrix}5 & 2\\ -1 & 1\end{vmatrix}$$

$$= -\left(5\cdot(-1) - 6\cdot(-1)\right) - 3\left(5\cdot 1 - 2\cdot(-1)\right)$$

$$= -22$$

$$\det(A_x) = \begin{vmatrix}2200 & 2 & 6\\ 0 & -1 & 3\\ 0 & 1 & -1\end{vmatrix}$$

$$= 1\cdot(-1)^{3+2}\begin{vmatrix}2200 & 6\\ 0 & 3\end{vmatrix} - 1\cdot(-1)^{3+3}\begin{vmatrix}2200 & 2\\ 0 & -1\end{vmatrix}$$

$$= -\left(2200\cdot 3 - 6\cdot 0\right) - \left(2200\cdot(-1) - 2\cdot 0\right)$$

$$= -4400$$

$$\det(A_y) = \begin{vmatrix}5 & 2200 & 6\\ 0 & 0 & 3\\ -1 & 0 & -1\end{vmatrix}$$

$$= 3(-1)^{2+3}\begin{vmatrix}5 & 2200\\ -1 & 0\end{vmatrix} = -3\left(5\cdot 0 - 2200(-1)\right)$$

$$= -6600$$

$$\det(A_z) = \begin{vmatrix}5 & 2 & 2200\\ 0 & -1 & 0\\ -1 & 1 & 0\end{vmatrix}$$

$$= -1\cdot(-1)^{2+2}\begin{vmatrix}5 & 2200\\ -1 & 0\end{vmatrix} = -\left(5\cdot 0 - 2200(-1)\right)$$
$$= -2200$$

ii) Regra de Cramer:

- $x = \frac{\det(A_x)}{\det(A)} = \frac{-4400}{-22} = 200$

- $y = \frac{\det(A_y)}{\det(A)} = \frac{-6600}{-22} = 300$

- $z = \frac{\det(A_z)}{\det(A)} = \frac{-2200}{-22} = 100$

Solução: $\begin{cases} x = 200 \\ y = 300 \\ z = 100 \end{cases}$

10.

$$\begin{cases} 40x + 30y + 10z = 7000 \\ 20x + 40y + 30z = 6000 \\ 10x + 20y + 40z = 5000 \end{cases}$$

$$A = \begin{bmatrix} 40 & 30 & 10 \\ 20 & 40 & 30 \\ 10 & 20 & 40 \end{bmatrix} \quad B = \begin{bmatrix} 7000 \\ 6000 \\ 5000 \end{bmatrix}$$

i) Determinantes:

$$\det(A) = \begin{vmatrix} 40 & 30 & 10 \\ 20 & 40 & 30 \\ 10 & 20 & 40 \end{vmatrix} \begin{matrix} l_1 \leftarrow l_1/10 \\ l_2 \leftarrow l_2/10 \\ l_3 \leftarrow l_3/10 \end{matrix}$$

$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 3 \\ 1 & 2 & 4 \end{vmatrix} \begin{matrix} l_1 \leftarrow l_1 - 4l_3 \\ l_2 \leftarrow l_2 - 2l_3 \end{matrix}$$

$$\begin{vmatrix} 0 & -5 & -15 \\ \boxed{0 \quad 0 \quad -5} \\ 1 & 2 & 4 \end{vmatrix} = -5 \cdot (-1)^{2+3} \begin{vmatrix} 0 & -5 \\ 1 & 2 \end{vmatrix}$$

$$= 5\,(0 \cdot 2 - (-5) \cdot 1) = 25$$

$$\det(A_x) = \begin{vmatrix} 7000 & 30 & 10 \\ 6000 & 40 & 30 \\ 5000 & 20 & 40 \end{vmatrix} \begin{matrix} l_1 \leftarrow l_1/10 \\ l_2 \leftarrow l_2/10 \\ l_3 \leftarrow l_3/10 \end{matrix} \sim$$

$$\begin{vmatrix} \boxed{700 \quad 3 \quad 1} \\ 600 & 4 & 3 \\ 500 & 2 & 4 \end{vmatrix} = 700 \cdot (-1)^{1+1} \begin{vmatrix} 4 & 3 \\ 2 & 4 \end{vmatrix} + 3 \cdot (-1)^{1+2} \begin{vmatrix} 600 & 3 \\ 500 & 4 \end{vmatrix}$$

$$+ 1 \cdot (-1)^{1+3} \begin{vmatrix} 600 & 4 \\ 500 & 2 \end{vmatrix} = 700\,(4 \cdot 4 - 3 \cdot 2) - 3\,(600 \cdot 4 - 3 \cdot 500)$$

$$+ 1\,(600 \cdot 2 - 4 \cdot 500) = 3500$$

$$\det(A_y) = \begin{vmatrix} 40 & 7000 & 10 \\ 20 & 6000 & 30 \\ 10 & 5000 & 40 \end{vmatrix} \begin{matrix} l_1 \leftarrow l_1/10 \\ l_2 \leftarrow l_2/10 \\ l_3 \leftarrow l_3/10 \end{matrix} \sim$$

$$\begin{vmatrix} 4 & 700 & 1 \\ 2 & 600 & 3 \\ \boxed{1 \quad 500 \quad 4} \end{vmatrix} = 1 \cdot (-1)^{3+1} \begin{vmatrix} 700 & 1 \\ 600 & 3 \end{vmatrix} + 500 \cdot (-1)^{3+2} \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix}$$

$$+ 4 \cdot (-1)^{3+3} \begin{vmatrix} 4 & 700 \\ 2 & 600 \end{vmatrix} = (700 \cdot 3 - 1 \cdot 600) - 500\,(4 \cdot 3 - 1 \cdot 2)$$

$$+ 4\,(4 \cdot 600 - 700 \cdot 2) = 500$$

$$\det(A_z) = \begin{vmatrix} 40 & 30 & 7000 \\ 20 & 40 & 6000 \\ 10 & 20 & 5000 \end{vmatrix} \begin{matrix} l_1 \leftarrow l_1/10 \\ l_2 \leftarrow l_2/10 \\ l_3 \leftarrow l_3/10 \end{matrix} \sim$$

$$\begin{vmatrix} 4 & 3 & 700 \\ 2 & 4 & 600 \\ 1 & 2 & 500 \end{vmatrix} = 1.(-1)^{3+1} \begin{vmatrix} 3 & 700 \\ 4 & 600 \end{vmatrix} + 2.(-1)^{3+2} \begin{vmatrix} 4 & 700 \\ 2 & 600 \end{vmatrix}$$

$$+ 500.(-1)^{3+3} \begin{vmatrix} 4 & 3 \\ 2 & 4 \end{vmatrix} = +1(3.600 - 700.4) - 2(4.600 - 700.2)$$

$$+ 500(4.4 - 3.2) = 2000$$

ii) Regra de Cramer:

- $x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{3500}{25} = 140$

- $y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{500}{25} = 20$

- $z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{2000}{25} = 80$

Solução: $\begin{cases} x = 140 \\ y = 20 \\ z = 80 \end{cases}$

11.

$$\begin{cases} 2A + 3B + 1C = 8420 \\ 1A + 2B + 2C = 7440 \\ 4A + 3B + 0 = 8110 \end{cases}$$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ 4 & 3 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 8420 \\ 7940 \\ 8110 \end{bmatrix}$$

i) Determinantes:

$$\det(A) = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ 4 & 3 & 0 \end{vmatrix}$$

$$= 1\cdot(-1)^{2+1}\begin{vmatrix} 3 & 1 \\ 3 & 0 \end{vmatrix} + 2\cdot(-1)^{2+2}\begin{vmatrix} 2 & 1 \\ 4 & 0 \end{vmatrix} + 2\cdot(-1)^{2+3}\begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix}$$

$$= -1(3\cdot 0 - 1\cdot 3) + 2(2\cdot 0 - 1\cdot 4) - 2(2\cdot 3 - 3\cdot 4)$$

$$= 7$$

$$\det(A_x) = \begin{vmatrix} 8420 & 3 & 1 \\ 7940 & 2 & 2 \\ 8110 & 3 & 0 \end{vmatrix}$$

$$= 8110\cdot(-1)^{3+1}\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + 3\cdot(-1)^{3+2}\begin{vmatrix} 8420 & 1 \\ 7940 & 2 \end{vmatrix}$$

$$= 8110(3\cdot 2 - 1\cdot 2) - 3(8420\cdot 2 - 1\cdot 7940)$$

$$= 5740$$

$$\det(A_y) = \begin{vmatrix} 2 & 8420 & 1 \\ 1 & 7940 & 2 \\ 4 & 8110 & 0 \end{vmatrix}$$

$= 4 \cdot (-1)^{3+1} \begin{vmatrix} 8420 & 1 \\ 7940 & 2 \end{vmatrix} + 8110 \cdot (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$

$= 4\,(8420 \cdot 2 - 1 \cdot 7940) - 8110\,(2 \cdot 2 - 1 \cdot 1)$

$= 11270$

$\det(A_z) = \begin{vmatrix} 2 & 3 & 8420 \\ 1 & 2 & 7940 \\ 4 & 3 & 8110 \end{vmatrix}$

$= 1 \cdot (-1)^{2+1} \begin{vmatrix} 3 & 8420 \\ 3 & 8110 \end{vmatrix} + 2 \cdot (-1)^{2+2} \begin{vmatrix} 2 & 8420 \\ 4 & 8110 \end{vmatrix} + 7940 \cdot (-1)^{2+3} \cdot \begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} = -1\,(3 \cdot 8110 - 8420 \cdot 3) + 2\,(2 \cdot 8110 - 8420 \cdot 4)$

$- 7940\,(2 \cdot 3 - 3 \cdot 4) = 13650$

ii) Regra de Cramer:

- $x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{5740}{7} = 820$

- $y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{11270}{7} = 1610$

- $z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{13650}{7} = 1950$

Solução: $\begin{cases} x = 820 \\ y = 1610 \\ z = 1950 \end{cases}$

$\begin{array}{r} 1950 \\ 820 \\ \hline \boxed{1130} \end{array}$