Gabriel Ribeiro

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Lista 3 - GA

1.

$A = (a_{ij})_{3\times 3}$, tal que $a_{ij} = \begin{cases} i+j, & i<j \\ 2i-j, & i=j \\ j-i, & i>j \end{cases}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 2.1-1 & 1+2 & 1+3 \\ 1-2 & 2.2-2 & 2+3 \\ 1-3 & 2-3 & 2.3-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 4 \\ -1 & 2 & 5 \\ -2 & -1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$$

$$AX = B \longrightarrow X = A^{-1}B$$

$$A^{-1} = \frac{1}{\det A} \cdot [\operatorname{cof}(A)]^T$$

$$\det A = \begin{vmatrix} 1 & 3 & 4 \\ -1 & 2 & 5 \\ -2 & -1 & 3 \end{vmatrix} \quad \begin{matrix} l_1 \leftarrow l_1 + l_2 \\ \\ l_3 \leftarrow l_3 + 2.l_1 \end{matrix}$$

$$\begin{vmatrix} \boxed{0} & 5 & 9 \\ \boxed{-1} & 2 & 5 \\ \boxed{0} & 5 & 11 \end{vmatrix} = -1.(-1)^{2+1} \begin{vmatrix} 5 & 9 \\ 5 & 11 \end{vmatrix} = 5.11 - 9.5 = 10$$

$$[\operatorname{cof}(A)]^T = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix}^T$$

$$\tilde{a}_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 5 \\ -1 & 3 \end{vmatrix} = 1(2.3 - 5(-1)) = 11$$

$$\tilde{a}_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 5 \\ -2 & 3 \end{vmatrix} = -1.(-1.3 - 5(-2)) = -7$$

$$\tilde{a}_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 2 \\ -2 & -1 \end{vmatrix} = 1.(-1.(-1) - 2(-2)) = 5$$

$$\tilde{a}_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 4 \\ -1 & 3 \end{vmatrix} = -1(3.3 - 4(-1)) = -10$$

$$\tilde{a}_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 4 \\ -2 & 3 \end{vmatrix} = 1(1.3 - 4(-2)) = 11$$

$$\tilde{a}_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 3 \\ -2 & -1 \end{vmatrix} = -1(1.(-1) - 3(-2)) = -5$$

$$\tilde{a}_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix} = 1(3.5 - 4.2) = 7$$

$$\tilde{a}_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 4 \\ -1 & 5 \end{vmatrix} = -1(1.5 - 4(-1)) = -9$$

$$\tilde{a}_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} = 1(1.2 - 3(-1)) = 5$$

$$\therefore \; [\operatorname{cof}(A)]^T = \begin{bmatrix} 11 & -7 & 5 \\ -10 & 11 & -5 \\ 7 & -9 & 5 \end{bmatrix}^T = \begin{bmatrix} 11 & -10 & 7 \\ -7 & 11 & -9 \\ 5 & -5 & 5 \end{bmatrix}$$

FORON:

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 11 & -10 & 7 \\ -7 & 11 & -9 \\ 5 & -5 & 5 \end{bmatrix} = \begin{bmatrix} 11/10 & -1 & 7/10 \\ -7/10 & 11/10 & -9/10 \\ 1/2 & -1/2 & 1/2 \end{bmatrix}$$

$$X = A^{-1}B \qquad X = \begin{bmatrix} 11/10 & -1 & 7/10 \\ -7/10 & 11/10 & -9/10 \\ 1/2 & -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$$

$$X = \begin{bmatrix} 11/10 \cdot (-1) + (-1) \cdot 2 + 7/10 \cdot (-3) \\ -7/10 \cdot (-1) + 11/10 \cdot 2 + -9/10 \cdot (-3) \\ 1/2 \cdot (-1) + (-1/2) \cdot 2 + 1/2 \cdot (-3) \end{bmatrix} = \begin{bmatrix} -5\,(1/5) \\ 5\,(3/5) \\ -3 \end{bmatrix}$$

2.

a)

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} X = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \rightarrow X = \begin{pmatrix} +3 & -4 \\ -2 & +1 \end{pmatrix}\begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$X = \begin{pmatrix} 3 \cdot (-1) + (-4) \cdot (-1) \\ -2 \cdot (-1) + 1 \cdot (-1) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

b)

$$\begin{pmatrix} 2 & 3 \\ 5 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} Y = \begin{pmatrix} 1 & 7 \\ 2 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} Y = \begin{pmatrix} 1 & 7 \\ 2 & 7 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ 5 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} Y = \begin{pmatrix} -1 & 4 \\ -3 & 2 \end{pmatrix} \rightarrow Y = \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ -3 & 2 \end{pmatrix}$$

$$Y = \begin{pmatrix} 5(-1) + (-2)(-3) & 5 \cdot 4 + (-2) \cdot 2 \\ -3(-1) + 1 \cdot (-3) & -3 \cdot 4 + 1 \cdot 2 \end{pmatrix} = \begin{pmatrix} 1 & 16 \\ 0 & -10 \end{pmatrix}$$

c)

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix} W = \begin{pmatrix} 5 \\ 7 \\ 2 \end{pmatrix} \rightarrow W = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -3 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 7 \\ 2 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 0 & \boxed{0} \\ 2 & 1 & \boxed{0} \\ 2 & 3 & \boxed{1} \end{vmatrix} = 1 \cdot (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1 \cdot 1 - 0 \cdot 2 = 1$$

$$\tilde{a}_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} = 1 \cdot 1 - 0 \cdot 3 = 1$$

$$\tilde{a}_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = -1(2 \cdot 1 - 0 \cdot 2) = -2$$

$$\tilde{a}_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 2 \cdot 3 - 1 \cdot 2 = 4$$

$$\tilde{a}_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ 3 & 1 \end{vmatrix} = 0$$

$$\tilde{a}_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1 \cdot 1 - 0 \cdot 2 = 1$$

$$\tilde{a}_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = -1(1 \cdot 3 - 0 \cdot 2) = -3$$

$$\tilde{a}_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$$

$$\tilde{a}_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = 0$$

$$\tilde{a}_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1 \cdot 1 - 0 \cdot 2 = 1$$

$$\begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}^T \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -3 & 1 \end{pmatrix}$$

FORON:

$$W = \begin{pmatrix} 1\cdot 5 + 0\cdot 7 + 0\cdot 2 \\ -2\cdot 5 + 1\cdot 7 + 0\cdot 2 \\ 4\cdot 5 + (-3)\cdot 7 + 1\cdot 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \\ 1 \end{pmatrix}$$

3.

a) $AXB = C \rightarrow X = A^{-1}(B^{-1}C)$

b) $A(B+X) = A \rightarrow X = I - B$

c) $ACXB = C \rightarrow X = (AC)^{-1}(B^{-1}C)$

d) $(AB)^{-1}(AX) = CC^{-1} \rightarrow X = A^{-1}(ABI)$

e) $AB^{T}XB^{-1} = A^{T} \rightarrow X = (AB^{T})^{-1}(BA^{T})$

4.

a) $\begin{cases} 3x - 4y = 1 \\ x + 3y = 9 \end{cases} \longleftrightarrow \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$

$A = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix}$ e $B = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$

i) Determinantes:

- $\det(A) = \begin{vmatrix} 3 & -4 \\ 1 & 3 \end{vmatrix} = 3\cdot 3 - (-4)\cdot 1 = 13$

- $\det(A_x) = \begin{vmatrix} 1 & -4 \\ 9 & 3 \end{vmatrix} = 1\cdot 3 - (-4)\cdot 9 = 39$

- $\det(A_y) = \begin{vmatrix} 3 & 1 \\ 1 & 9 \end{vmatrix} = 3\cdot 9 - 1\cdot 1 = 26$

ii) Regra de Cramer:

- $X = \dfrac{\det(A_x)}{\det(A)} = \dfrac{39}{13} = 3$

- $Y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{26}{13} = 2$

Solução: $\begin{cases} X = 3 \\ Y = 2 \end{cases}$

b) $\begin{cases} 5X + 8Y = 34 \\ 10X + 16Y = 50 \end{cases} \longleftrightarrow \begin{bmatrix} 5 & 8 \\ 10 & 16 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 34 \\ 50 \end{bmatrix}$

$A = \begin{bmatrix} 5 & 8 \\ 10 & 16 \end{bmatrix}$ e $B = \begin{bmatrix} 34 \\ 50 \end{bmatrix}$

i) Determinantes:

- $\det(A) = \begin{vmatrix} 5 & 8 \\ 10 & 16 \end{vmatrix} = 5.16 - 8.10 = 0$

- $\det(A_x) = \begin{vmatrix} 34 & 8 \\ 50 & 16 \end{vmatrix} = 34.16 - 8.50 = 144$

- $\det(A_y) = \begin{vmatrix} 5 & 34 \\ 10 & 50 \end{vmatrix} = 5.50 - 34.10 = -90$

Solução: $\begin{cases} \det(A) = 0 \\ \det(A_x) \neq 0 \\ \det(A_y) \neq 0 \end{cases}$, logo: SI

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c) $\begin{cases} x + 2y = 5 \\ 2x - 3y = -4 \end{cases} \longleftrightarrow \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}$ e $B = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

i) Determinantes:

- $\det(A) = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 1 \cdot (-3) - 2 \cdot 2 = -7$

- $\det(A_x) = \begin{vmatrix} 5 & 2 \\ -4 & -3 \end{vmatrix} = 5 \cdot (-3) - 2(-4) = -7$

- $\det(A_y) = \begin{vmatrix} 1 & 5 \\ 2 & -4 \end{vmatrix} = 1 \cdot (-4) - 5 \cdot 2 = -14$

ii) Regra de Cramer:

- $x = \frac{\det(A_x)}{\det(A)} = \frac{-7}{-7} = 1$

- $y = \frac{\det(A_y)}{\det(A)} = \frac{-14}{-7} = 2$

Solução: $\begin{cases} x = 1 \\ y = 2 \end{cases}$

d) $\begin{cases} 3x + 2y - 5z = 8 \\ 2x - 4y - 2z = -4 \\ x - 2y - 3z = -4 \end{cases}$ $A = \begin{bmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{bmatrix}$ e $B = \begin{bmatrix} 8 \\ -4 \\ -4 \end{bmatrix}$

i) Determinantes:

• $\det(A) = \begin{vmatrix} 3 & 2 & -5 \\ 2 & -4 & -2 \\ 1 & -2 & -3 \end{vmatrix}$ $\quad l_2 \leftarrow l_2 - 2l_3$ $\quad l_1 \leftarrow l_1 + l_3$

$\begin{vmatrix} 4 & 0 & -8 \\ \boxed{0 \quad 0 \quad 4} \\ 1 & -2 & -3 \end{vmatrix} = 4.(-1)^{2+3} \begin{vmatrix} 4 & 0 \\ 1 & -2 \end{vmatrix} = -4(4.(-2) - 0\,1)$
$= 32$

• $\det(A_x) = \begin{vmatrix} 8 & 2 & -5 \\ -4 & -4 & -2 \\ -4 & -2 & -3 \end{vmatrix}$ $\quad l_1 \leftarrow l_1 + 2l_3$ $\quad l_2 \leftarrow l_2 - l_3$

$\begin{vmatrix} 0 & -2 & -11 \\ 0 & -2 & 1 \\ -4 & -2 & -3 \end{vmatrix} = -4.(-1)^{3+1} \begin{vmatrix} -2 & -11 \\ -2 & 1 \end{vmatrix} = -4(-2.1 - (-11)(-2))$
$= 96$

• $\det(A_y) = \begin{vmatrix} 3 & 8 & -5 \\ 2 & -4 & -2 \\ 1 & -4 & -3 \end{vmatrix}$ $\quad l_1 \leftarrow l_1 + 2l_3$ $\quad l_2 \leftarrow l_2 - 2l_3$

$\begin{vmatrix} 5 & 0 & -11 \\ 0 & 4 & 4 \\ 1 & -4 & -3 \end{vmatrix} = 5.(-1)^{1+1} \begin{vmatrix} 4 & 4 \\ -4 & -3 \end{vmatrix} + 1.(-1)^{3+1} \begin{vmatrix} 0 & -11 \\ 4 & 4 \end{vmatrix}$

$= 5(4.(-3) - 4(-4)) + (0.4 - (-11).4) = 64$

• $\det(A_z) = \begin{vmatrix} 3 & 2 & 8 \\ 2 & -4 & -4 \\ 1 & -2 & -4 \end{vmatrix}$ $\quad l_2 \leftarrow l_2 - 2l_3$

$\begin{vmatrix} 3 & 2 & 8 \\ \boxed{0 \quad 0 \quad 4} \\ 1 & -2 & -4 \end{vmatrix} = 4.(-1)^{2+3} \begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} = -4(3(-2) - 2.1)$
$= 32$

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ii) Regra de Cramer:

- $x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{96}{32} = 3$

- $y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{64}{32} = 2$

- $z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{32}{32} = 1$

Solução: $\begin{cases} x = 3 \\ y = 2 \\ z = 1 \end{cases}$

e) $\begin{cases} x + 2y - z = 2 \\ 2x - y + 3z = 9 \\ 3x + 3y - 2z = 3 \end{cases}$ $\quad A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 3 & -2 \end{bmatrix}$ e $B = \begin{bmatrix} 2 \\ 9 \\ 3 \end{bmatrix}$

i) Determinantes:

- $\det(A) = \begin{vmatrix} \boxed{1 \quad 2 \quad -1} \\ 2 \quad -1 \quad 3 \\ 3 \quad 3 \quad -2 \end{vmatrix} = 1.(-1)^{1+1} \begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix} + 2.(-1)^{1+2} \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$

$+ (-1).(-1)^{1+3} \begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} = (-1.(-2) - 3.3) - 2(2.(-2) - 3.3) + (2.3 - (-1).3) = 28$

- $\det(A_x) = \begin{vmatrix} \boxed{2 \quad 2 \quad -1} \\ 9 \quad -1 \quad 3 \\ 3 \quad 3 \quad -2 \end{vmatrix} = 2.(-1)^{1+1} \begin{vmatrix} -1 & 3 \\ 3 & -2 \end{vmatrix} + 2.(-1)^{1+2} \begin{vmatrix} 9 & 3 \\ 3 & -2 \end{vmatrix}$
$-1.(-1)^{1+3} \begin{vmatrix} 9 & -1 \\ 3 & 3 \end{vmatrix} =$

$2(-1.(-2) - 3.3) - 2(9.(-2) - 3.3) + (9.3 - (-1).3) = 70$

FORON:

- $\det(A_y) = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 3 \\ 3 & 3 & -2 \end{vmatrix} \begin{matrix} \\ l_2 \leftarrow l_2 + 3l_1 \\ l_3 \leftarrow l_3 - 2l_1 \end{matrix}$

$\begin{vmatrix} 1 & 2 & \boxed{-1} \\ 5 & 15 & \boxed{0} \\ 1 & -1 & \boxed{0} \end{vmatrix} = -1 \cdot (-1)^{1+3} \begin{vmatrix} 5 & 15 \\ 1 & -1 \end{vmatrix} = -1(5 \cdot (-1) - 15 \cdot 1) = 20$

- $\det(A_z) = \begin{vmatrix} 1 & 2 & 2 \\ 2 & -1 & 9 \\ 3 & 3 & 3 \end{vmatrix} \begin{matrix} \\ l_2 \leftarrow l_2 - 2l_1 \\ l_3 \leftarrow l_3 - 3l_1 \end{matrix}$

$\begin{vmatrix} \boxed{1} & 2 & 2 \\ \boxed{0} & -5 & 5 \\ \boxed{0} & -3 & -3 \end{vmatrix} = 1 \cdot (-1)^{1+1} \begin{vmatrix} -5 & 5 \\ -3 & -3 \end{vmatrix} = -5 \cdot (-3) - 5(-3) = 30$

ii) Regra de Cramer:

- $X = \dfrac{\det(A_x)}{\det(A)} = \dfrac{70^{\div 7}}{28^{\div 7}} = \dfrac{10^{\div 2}}{4^{\div 2}} = \dfrac{5}{2}$

- $Y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{20^{\div 4}}{28^{\div 4}} = \dfrac{5}{7}$

- $Z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{30^{\div 2}}{28^{\div 2}} = \dfrac{15}{14}$

Solução: $\begin{cases} x = \dfrac{5}{2} \\ y = \dfrac{5}{7} \\ z = \dfrac{15}{14} \end{cases}$

f) $\begin{cases} x + 3z = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases}$ $\quad A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -4 & 0 \\ 3 & -2 & -5 \end{bmatrix}$ e $B = \begin{bmatrix} -8 \\ -4 \\ 26 \end{bmatrix}$

i) Determinantes:

• $\det(A) = \begin{vmatrix} 1 & 0 & 3 \\ 2 & -4 & 0 \\ 3 & -2 & -5 \end{vmatrix} \quad l_2 \leftarrow l_2 - 2l_3$

$\begin{vmatrix} 1 & 0 & 3 \\ -4 & 0 & 10 \\ 3 & -2 & -5 \end{vmatrix} = -2(-1)^{3+2} \begin{vmatrix} 1 & 3 \\ -4 & 10 \end{vmatrix} = 2(1.10 - 3(-4)) = 44$

• $\det(A_x) = \begin{vmatrix} -8 & 0 & 3 \\ -4 & -4 & 0 \\ 26 & -2 & -5 \end{vmatrix} = -4.(-1)^{2+1} \begin{vmatrix} 0 & 3 \\ -2 & -5 \end{vmatrix} - 4.(-1)^{2+2} \begin{vmatrix} -8 & 3 \\ 26 & -5 \end{vmatrix}$

$= 4(0.(-5) - 3(-2)) - 4(-8.(-5) - 3.26) = 176$

• $\det(A_y) = \begin{vmatrix} 1 & -8 & 3 \\ 2 & -4 & 0 \\ 3 & 26 & -5 \end{vmatrix} = 2.(-1)^{2+1} \begin{vmatrix} -8 & 3 \\ 26 & -5 \end{vmatrix} - 4.(-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 3 & -5 \end{vmatrix}$

$= -2(-8.(-5) - 3.26) + 4(1.(-5) - 3.3) = 20$

• $\det(A_z) = \begin{vmatrix} 1 & 0 & -8 \\ 2 & -4 & -4 \\ 3 & -2 & 26 \end{vmatrix} = 1.(-1)^{1+1} \begin{vmatrix} -4 & -4 \\ -2 & 26 \end{vmatrix} - 8.(-1)^{1+3} \begin{vmatrix} 2 & -4 \\ 3 & -2 \end{vmatrix}$

$= (-4.26 - (-4)(-2)) + 8(2(-2) - (-4).3) = -48$

ii) Regra de Cramer:

• $x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{176}{44} = 4$

$$\cdot\ y = \frac{\det(A_y)}{\det(A)} = \frac{20^{:4}}{44_{:4}} = \frac{5}{11}$$

$$\cdot\ z = \frac{\det(A_z)}{\det(A)} = \frac{-48^{:4}}{44_{:4}} = -\frac{12}{11}$$

Solução: $\begin{cases} x = 4 \\ y = 5/11 \\ z = -12/11 \end{cases}$

g) $\begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 3x + 2y + 3z = 10 \end{cases}$ $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 3 & 2 & 3 \end{bmatrix}$ e $B = \begin{bmatrix} 10 \\ 23 \\ 10 \end{bmatrix}$

i) Determinantes:

$$\cdot\ \det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 6 \\ 3 & 2 & 3 \end{vmatrix} \quad l_2 \leftarrow l_2 - 2l_1$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 3 & 2 & 3 \end{vmatrix} = 1.(-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = -1(2.3 - 3.2) = 0$$

$$\cdot\ \det(A_x) = \begin{vmatrix} 10 & 2 & 3 \\ 23 & 4 & 6 \\ 10 & 2 & 3 \end{vmatrix} \quad l_3 \leftarrow l_3 - l_1$$

$$\begin{vmatrix} 10 & 2 & 3 \\ 23 & 4 & 6 \\ 0 & 0 & 0 \end{vmatrix} = 0.$$

FORON:

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- $\det(A_y) = \begin{vmatrix} 1 & 10 & 3 \\ 3 & 23 & 6 \\ 3 & 10 & 3 \end{vmatrix}$ $l_1 \leftarrow l_1 - l_3$

$\begin{vmatrix} -2 & 0 & 0 \\ 3 & 23 & 6 \\ 3 & 10 & 3 \end{vmatrix} = -2 \cdot (-1)^{1+1} \begin{vmatrix} 23 & 6 \\ 10 & 3 \end{vmatrix} = -2(23 \cdot 3 - 6 \cdot 10) = -18$

- $\det(A_z) = \begin{vmatrix} 1 & 2 & 10 \\ 3 & 4 & 23 \\ 3 & 2 & 10 \end{vmatrix}$ $l_3 \leftarrow l_3 - l_1$

$\begin{vmatrix} 1 & 2 & 10 \\ 3 & 4 & 23 \\ 2 & 0 & 0 \end{vmatrix} = 2 \cdot (-1)^{3+1} \begin{vmatrix} 2 & 10 \\ 4 & 23 \end{vmatrix} = 2(2 \cdot 23 - 10 \cdot 4) = 12$

Solução: $\begin{cases} \det(A) = 0 \\ \det(A_x) = 0 \\ \det(A_y) \neq 0 \\ \det(A_z) \neq 0 \end{cases}$, logo: SI

5.

a) $\begin{cases} 3x_1 - 4x_2 = 0 \\ -6x_1 + 8x_2 = 0 \end{cases}$ $A = \begin{bmatrix} 3 & -4 \\ -6 & 8 \end{bmatrix}$ e $B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

i) Determinantes:

- $\det(A) = \begin{vmatrix} 3 & -4 \\ -6 & 8 \end{vmatrix} = 3 \cdot 8 - (-4)(-6) = 0$

- $\det(A_{x_1}) = \begin{vmatrix} 0 & -4 \\ 0 & 8 \end{vmatrix} = 0 \cdot 8 - (-4) \cdot 0 = 0$

$\cdot \det(A_{x_2}) = \begin{vmatrix} 3 & 0 \\ -6 & 0 \end{vmatrix} = 0.(3) - 0.(-6) = 0$

Solução : $\begin{cases} \det(A) = 0 \\ \det(A_{x_1}) = 0 \\ \det(A_{x_2}) = 0 \end{cases}$, logo : SPI

b) $\begin{cases} x + y + z = 0 \\ 2x + 2y + 4z = 0 \\ x + y + 3z = 0 \end{cases}$ $\quad A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

i) Determinantes :

$\cdot \det(A) = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ 1 & 1 & 3 \end{vmatrix} \quad l_3 \leftarrow l_3 - l_1$

$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ \boxed{0 \quad 0 \quad 2} \end{vmatrix} = 2.(-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2(1.2 - 1.2) = 0$

$\cdot \det(A_x) = \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 1 & 3 \end{vmatrix} = 0$

$\cdot \det(A_y) = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 4 \\ 1 & 0 & 3 \end{vmatrix} = 0$

$\cdot \det(A_z) = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$

Solução: $\begin{cases} det(A) = 0 \\ det(A_x) = 0 \\ det(A_y) = 0 \\ det(A_z) = 0 \end{cases}$, logo: SPI

c) $\begin{cases} X + Y + 2Z = 0 \\ X - Y - 3Z = 0 \\ X + 4Y = 0 \end{cases}$ $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & -3 \\ 1 & 4 & 0 \end{bmatrix}$ e $B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

i) Determinantes:

• $det(A) = \begin{vmatrix} 1 & 1 & 2 \\ 1 & -1 & -3 \\ 1 & 4 & 0 \end{vmatrix} = 1 . (-1)^{3+1} \begin{vmatrix} 1 & 2 \\ -1 & -3 \end{vmatrix} + 4 . (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix}$

$= 1 . (-3) - 2 . (-1) - 4 . (1 . (-3) - 2 . 1)$

$= 19$

• $det(A_x) = \begin{vmatrix} 0 & 1 & 2 \\ 0 & -1 & -3 \\ 0 & 4 & 0 \end{vmatrix} = 0$

• $det(A_y) = \begin{vmatrix} 1 & 0 & 2 \\ 1 & 0 & -3 \\ 1 & 0 & 0 \end{vmatrix} = 0$

• $det(A_z) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 4 & 0 \end{vmatrix} = 0$

Solução: $\begin{cases} det(A) \neq 0 \\ det(A_x) = 0 \\ det(A_y) = 0 \\ det(A_z) = 0 \end{cases}$, logo: SI

6.

a) $\begin{cases} 3x + my = 2 \\ x - y = 1 \end{cases}$ $A = \begin{bmatrix} 3 & m \\ 1 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

i) Determinantes:

• $\det(A) = \begin{vmatrix} 3 & m \\ 1 & -1 \end{vmatrix} = 3.(-1) - m = -3 - m$

• $\det(A_x) = \begin{vmatrix} 2 & m \\ 1 & -1 \end{vmatrix} = 2.(-1) - m = -2 - m$

• $\det(A_y) = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3.1 - 2.1 = 1$

ii) Estudo de solução:

• SPD: $\det(A) \neq 0$

$\begin{cases} -3 - m \neq 0 & (1) \\ -2 - m \neq 0 & (2) \\ 1 \neq 0 & (3) \end{cases}$ $-m \neq 3 \Leftrightarrow m \neq -3$
$-2 \neq m \Leftrightarrow 2 \neq -m$

Conclusão: $(m) \neq -3$ ou $(m) \neq -2$ tornam o sistema dado em SPD.

b) $\begin{cases} 3x + 2(m-1)y = 1 \\ mx - 4y = 0 \end{cases}$ $A = \begin{bmatrix} 3 & 2(m-1) \\ m & -4 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

i) Determinantes:

$\cdot \det(A) = \begin{vmatrix} 3 & 2(m-1) \\ m & -4 \end{vmatrix} = 3.(-4) - 2(m-1).m = -12-2m^2+2m$

$\cdot \det(A_x) = \begin{vmatrix} 1 & 2(m-1) \\ 0 & -4 \end{vmatrix} = 1.(-4) - 2(m-1).0 = -4$

$\cdot \det(A_y) = \begin{vmatrix} 3 & 1 \\ m & 0 \end{vmatrix} = 3.0 - m = -m$

ii) Estudo de solução:

$\cdot$ SPD: $\det(A) \neq 0$

$$\begin{cases} -12-2m^2+2m \neq 0 \\ -4 \neq 0 \\ -m \neq 0 \end{cases}$$

$-2m^2+2m-12 \neq 0 \qquad \Delta = 2^2 - 4.(-2).(-12)$

$\Delta = -92$

Conclusão: $\Delta < 0$, logo sem solução.
$(m) \neq 0$ torna o sistema dado em SPD.

e) $\begin{cases} x - y = 2 \\ x + my = -z \\ -x + y - z = 4 \end{cases} \qquad A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & m & 1 \\ -1 & 1 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$

$\downarrow$

$$\begin{cases} x - y + 0 = 2 \\ x + my + z = 0 \\ -x + y - z = 4 \end{cases}$$

i) Determinantes:

• $\det(A) = \begin{vmatrix} 1 & -1 & 0 \\ 1 & m & 1 \\ -1 & 1 & -1 \end{vmatrix} = 1.(-1)^{2+1} \begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} + m.(-1)^{2+2} \begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} + 1.(-1)^{2+3} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = -1(-1.(-1) - 0.1)$

$+ m(1.(-1) - 0.(-1)) - 1(1.1 - (-1).(-1)) = -1 - m$

• $\det(A_x) = \begin{vmatrix} 2 & -1 & 0 \\ 0 & m & 1 \\ 4 & 1 & -1 \end{vmatrix} = m.(-1)^{2+2} \begin{vmatrix} 2 & 0 \\ 4 & -1 \end{vmatrix} + 1.(-1)^{2+3} \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix}$

$= m.(2.(-1) - 0.4) - (2.1 - (-1).4)$

$= -2m - 6$

• $\det(A_y) = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ -1 & 4 & -1 \end{vmatrix} = 1.(-1)^{2+1} \begin{vmatrix} 2 & 0 \\ 4 & -1 \end{vmatrix} + 1.(-1)^{2+3} \begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix}$

$= -1(2.(-1) - 0.4) - 1(1.4 - 2.(-1)) = -4$

• $\det(A_z) = \begin{vmatrix} 1 & -1 & 2 \\ 1 & m & 0 \\ -1 & 1 & 4 \end{vmatrix} = 1.(-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} + m.(-1)^{2+2} \begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix}$

$= -1(-1.4 - 2.1) + m(1.4 - 2(-1))$

$= 6 + 6m$

ii) Estudo de solução

• SPD: $\det(A) \neq 0$.

$$\begin{cases} -1 - m \neq 0 \rightarrow -m \neq 1 \leftrightarrow m \neq -1 \\ -2m - 6 \neq 0 \rightarrow -m \neq 6/2 \leftrightarrow m \neq -3 \\ -4 \neq 0 \\ 6 + 6m \neq 0 \rightarrow 6m \neq -6 \leftrightarrow m \neq -1 \end{cases}$$

Conclusão: $(m) \neq -1$ ou $(m) \neq -3$ tornam o sistema dado em SPD.

d) $\begin{cases} mx + y - z = 4 \\ x + my + z = 0 \\ x - y + 0 = 2 \end{cases}$ $A = \begin{bmatrix} m & 1 & -1 \\ 1 & m & 1 \\ 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

i) Determinantes:

- $\det(A) = \begin{vmatrix} m & 1 & -1 \\ 1 & m & 1 \\ \boxed{1 \quad -1 \quad 0} \end{vmatrix} = 1.(-1)^{3+1} \begin{vmatrix} 1 & -1 \\ m & 1 \end{vmatrix} - 1.(-1)^{3+2} \begin{vmatrix} m & -1 \\ 1 & 1 \end{vmatrix}$

$= 1.1 - (-1).m - 1(m.1 - (-1).1) = 2m - m - 1$
$= m - 1$

- $\det(A_x) = \begin{vmatrix} 4 & 1 & -1 \\ \boxed{0 \quad m \quad 1} \\ 2 & -1 & 0 \end{vmatrix} = m.(-1)^{2+2} \begin{vmatrix} 4 & -1 \\ 2 & 0 \end{vmatrix} + 1.(-1)^{2+3} \begin{vmatrix} 4 & 1 \\ 2 & -1 \end{vmatrix}$

$= m(4.0 - (-1).2) - 1(4.(-1) - 1.2) = 2m + 6$

- $\det(A_y) = \begin{vmatrix} m & 4 & -1 \\ \boxed{1 \quad 0 \quad 1} \\ 1 & 2 & 0 \end{vmatrix} = 1.(-1)^{2+1} \begin{vmatrix} 4 & -1 \\ 2 & 0 \end{vmatrix} + 1.(-1)^{2+3} \begin{vmatrix} m & 4 \\ 1 & 2 \end{vmatrix}$

$= -1(4.0 - (-1).2) - 1(m.2 - 4.1) = -2 - 2m + 4$
$= -2m + 2$

- $\det(A_z) = \begin{vmatrix} m & 1 & 4 \\ \boxed{1 \quad m \quad 0} \\ 1 & -1 & 2 \end{vmatrix} = 1.(-1)^{2+1} \begin{vmatrix} 1 & 4 \\ -1 & 2 \end{vmatrix} + m.(-1)^{2+2} \begin{vmatrix} m & 4 \\ 1 & 2 \end{vmatrix}$

$= -1(1.2 - 4.(-1)) + m(m.2 - 4.1) = -6 + 2m^2 - 4m$

ii) Estudo de solução

· SPD = det(A) ≠ 0

$$\begin{cases} m-1 \neq 0 \rightarrow m \neq 1 \\ 2m+6 \neq 0 \rightarrow 2m \neq -6 \leftrightarrow m \neq -3 \\ -2m+2 \neq 0 \rightarrow -2m \neq -2 \leftrightarrow m \neq 1 \\ -6+2m^2-4m \neq 0 \rightarrow \end{cases}$$

$2m^2-4m-6 \neq 0$ $\Delta = (-4)^2 - 4 \cdot 2 \cdot (-6)$

$\Delta = 64$

$m \neq \frac{-(-4) \pm \sqrt{64}}{2 \cdot 2} \rightarrow m \neq \frac{4 \pm 8}{4}$ $m_1 \neq \frac{12}{4} \leftrightarrow 3$

$m_2 \neq \frac{-4}{4} \leftrightarrow -1$

Solução: $(m) \neq -3$ ou $(m) \neq -1$ ou $(m) \neq 1$ ou $(m) \neq 3$ tornam o sistema dado em SPD.

7.

a)

· Lucro obtido: $6 \cdot X - 2 \cdot Y = 750$

· Peças produzidas: $X + Y = 225$

$$\begin{cases} 6X - 2Y = 750 \\ X + Y = 225 \end{cases} \quad A = \begin{bmatrix} 6 & -2 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 750 \\ 225 \end{bmatrix}$$

i) Determinantes:

$$\det(A) = \begin{vmatrix} 6 & -2 \\ 1 & 1 \end{vmatrix} = 6.1 - (-2).1 = 8$$

$$\det(A_x) = \begin{vmatrix} 750 & -2 \\ 225 & 1 \end{vmatrix} = 750.1 - (-2).225 = 1200$$

$$\det(A_y) = \begin{vmatrix} 6 & 750 \\ 1 & 225 \end{vmatrix} = -750.1 + 6.225 = 600$$

ii) Regra de Cramer:

- $X = \dfrac{\det(Ax)}{\det(A)} = \dfrac{1200}{8} = 150$

- $Y = \dfrac{\det(Ay)}{\det(A)} = \dfrac{600}{8} = 75$

Conclusão: a assistente da oficina produziu corretamente 150 peças.

8.

- Quilômetros rodados: $X + Y = 540$

- Custo mensal: $0,6X + 0,2Y = 300$

$$\begin{cases} X + Y = 540 \\ 0,6X + 0,2Y = 300 \end{cases} \quad A = \begin{bmatrix} 1 & 1 \\ 0,6 & 0,2 \end{bmatrix} \quad B = \begin{bmatrix} 540 \\ 300 \end{bmatrix}$$

i) Determinantes:

$$\det(A) = \begin{vmatrix} 1 & 1 \\ 0,6 & 0,2 \end{vmatrix} = -0,6 + 0,2 = -0,4$$

$\det(A_x) = \begin{vmatrix} 540 & 1 \\ 300 & 0{,}2 \end{vmatrix} = 540 \cdot 0{,}2 - 1 \cdot 300 = -192$

$\det(A_y) = \begin{vmatrix} 1 & 540 \\ 0{,}6 & 300 \end{vmatrix} = 1 \cdot 300 - 540 \cdot 0{,}6 = -24$

ii) Regra de Cramer:

- $X = \dfrac{\det(A_x)}{\det(A)} = \dfrac{-192}{-0{,}4} = 480$

- $Y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{-24}{-0{,}4} = 60$

Conclusão: Takumi deve percorrer no mês, com o Toyota AE86 e a motocicleta Suzuki respectivamente, 480 Km e 60 Km para o custo ser de 300 reais.

9.

- Quantia Total: $2X + 5Y + 10Z = 500$

- Cédulas: $X + Y + Z = 92$

- Igualdade: $X - Z = 0$

$\begin{cases} 2X + 5Y + 10Z = 500 \\ X + Y + Z = 92 \\ X + 0 - Z = 0 \end{cases}$ $\quad A = \begin{bmatrix} 2 & 5 & 10 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$

$B = \begin{bmatrix} 500 \\ 92 \\ 0 \end{bmatrix}$

i) Determinantes:

$$\det(A) = \begin{vmatrix} 2 & 5 & 10 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix} = 1.(-1)^{3+1} \begin{vmatrix} 5 & 10 \\ 1 & 1 \end{vmatrix} - 1.(-1)^{3+3} \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix}$$

$$= 5.1 - 10.1 - 1(2.1 - 5.1) = -2$$

$$\det(A_x) = \begin{vmatrix} 500 & 5 & 10 \\ 92 & 1 & 1 \\ 0 & 0 & -1 \end{vmatrix} = -1.(-1)^{3+3} \begin{vmatrix} 500 & 5 \\ 92 & 1 \end{vmatrix}$$

$$= -1(500.1 - 5.92) = -40$$

$$\det(A_y) = \begin{vmatrix} 2 & 500 & 10 \\ 1 & 92 & 1 \\ 1 & 0 & -1 \end{vmatrix} = 1.(-1)^{3+1} \begin{vmatrix} 500 & 10 \\ 92 & 1 \end{vmatrix} - 1.(-1)^{3+3} \begin{vmatrix} 2 & 500 \\ 1 & 92 \end{vmatrix}$$

$$= 500.1 - 92.10 - 1(2.92 - 500.1) = -104$$

$$\det(A_z) = \begin{vmatrix} 2 & 5 & 500 \\ 1 & 1 & 92 \\ 1 & 0 & 0 \end{vmatrix} = 1.(-1)^{3+1} \begin{vmatrix} 5 & 500 \\ 1 & 92 \end{vmatrix} = 5.92 - 500.1$$

$$= -40$$

ii) Regra de Cramer:

- $x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{-40}{-2} = 20$

- $y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{-104}{-2} = 52$

[/ /]

- $z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{-40}{-2} = 20$

Conclusão: Nikaido vai precisar de 52 cédulas de cinco reais.

10.

- Kika e Akamaru: $x + y = 109$
- Kika e Tamaki: $x + z = 142$
- Tamaki e Akamaru: $z + y = 97$

$$\begin{cases} x + y + 0 = 109 \\ x + 0 + z = 142 \\ 0 + y + z = 97 \end{cases} \qquad A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 109 \\ 142 \\ 97 \end{bmatrix}$$

i) Determinantes:

$$\det(A) = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1.(-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1.(-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 0.1 - 1.1 - 1(1.1 - 1.0) = -2$$

$$\det(A_x) = \begin{vmatrix} 109 & 1 & 0 \\ 142 & 0 & 1 \\ 97 & 1 & 1 \end{vmatrix} = 109.(-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1.(-1)^{1+2} \begin{vmatrix} 142 & 1 \\ 97 & 1 \end{vmatrix}$$

$$= 109.(0.1 - 1.1) - 1(142.1 - 1.97) = -154$$

$$\det(A_y) = \begin{vmatrix} 1 & 109 & 0 \\ 1 & 142 & 1 \\ 0 & 97 & 1 \end{vmatrix} = 1.(-1)^{1+1} \begin{vmatrix} 142 & 1 \\ 97 & 1 \end{vmatrix} + 109.(-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 142.1 - 1.97 - 109.(1.1 - 1.0) = -64$$

$$\det(A_z) = \begin{vmatrix} 1 & 1 & 109 \\ 1 & 0 & 142 \\ 0 & 1 & 97 \end{vmatrix} = 1.(-1)^{3+2} \begin{vmatrix} 1 & 109 \\ 1 & 142 \end{vmatrix} + 97.(-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -1(1.142 - 109.1) + 97(1.0 - 1.1) = -130$$

ii) Regra de Cramer:

- $x = \dfrac{\det(A_x)}{\det(A)} = \dfrac{-154}{-2} = 77$

- $y = \dfrac{\det(A_y)}{\det(A)} = \dfrac{-64}{-2} = 32$

- $z = \dfrac{\det(A_z)}{\det(A)} = \dfrac{-130}{-2} = 65$

Conclusão: Kila tem um peso de 77 Kg, Akamaru tem um peso de 32 Kg e a Tamaki tem um peso de 65 Kg.