Lista 1 - GA

1.

a) $A + 2B$

$A = \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 5 \\ 2 & -2 \end{pmatrix}$ $\quad 2B = (2\,b_{ij})_{2\times2} =$

$\begin{pmatrix} 2.0 & 2.5 \\ 2.2 & 2.(-2) \end{pmatrix} = \begin{pmatrix} 0 & 10 \\ 4 & -4 \end{pmatrix}$

$A + 2B = \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix}_{2\times2} + \begin{pmatrix} 0 & 10 \\ 4 & -4 \end{pmatrix}_{2\times2} = \begin{pmatrix} (1+0) & (0+10) \\ (3+4) & (7+(-4)) \end{pmatrix} = \begin{pmatrix} 1 & 10 \\ 7 & 3 \end{pmatrix}_{2\times2}$

b) $AB - BA$

$AB = \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} . \begin{pmatrix} 0 & 5 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} (1.0 + 0.2) & (1.5 + 0.(-2)) \\ (3.0 + 7.2) & (3.5 + 7.(-2)) \end{pmatrix} =$

$\begin{pmatrix} 0 & 5 \\ 14 & 1 \end{pmatrix}$

$BA = \begin{pmatrix} 0 & 5 \\ 2 & -2 \end{pmatrix} . \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} (0.1 + 5.3) & (0.0 + 5.7) \\ (2.1 + (-2).3) & (2.0 + (-2).7) \end{pmatrix} = \begin{pmatrix} 15 & 35 \\ -4 & -14 \end{pmatrix}$

$AB - BA = \begin{pmatrix} 0 & 5 \\ 14 & 1 \end{pmatrix}_{2\times2} - \begin{pmatrix} 15 & 35 \\ -4 & -14 \end{pmatrix}_{2\times2} = \begin{pmatrix} (0-15) & (5-35) \\ (14-(-4)) & (1-(-14)) \end{pmatrix} = \begin{pmatrix} -15 & -30 \\ 18 & 15 \end{pmatrix}_{2\times2}$

c) $2C - D$

$C = \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix}_{2\times3}$ $\quad D = \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix}_{3\times3}$

Subtração não definida, pois C e D possuem ordens distintas.

d) $2D^T - 3E^T$

$$D = \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix}_{3\times 3} \rightarrow D^T = \begin{pmatrix} -3 & 1 & -2 \\ 2 & 1 & 0 \\ 0 & 4 & 2 \end{pmatrix}_{3\times 3}$$

$$E = \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix}_{3\times 3} \rightarrow E^T = \begin{pmatrix} 2 & -1 & -6 \\ 4 & 0 & 0 \\ -3 & -4 & -1 \end{pmatrix}_{3\times 3}$$

$$2D^T = \begin{pmatrix} 2.(-3) & 2.1 & 2.(-2) \\ 2.2 & 2.1 & 2.0 \\ 2.0 & 2.4 & 2.2 \end{pmatrix}_{3\times 3} = \begin{pmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{pmatrix}_{3\times 3}$$

$$3E^T = \begin{pmatrix} 3.2 & 3.(-1) & 3.(-6) \\ 3.4 & 3.0 & 3.0 \\ 3.(-3) & 3.(-4) & 3.(-1) \end{pmatrix}_{3\times 3} = \begin{pmatrix} 6 & -3 & -18 \\ 12 & 0 & 0 \\ -9 & -12 & -3 \end{pmatrix}_{3\times 3}$$

$$2D^T - 3E^T = \begin{pmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{pmatrix} - \begin{pmatrix} 6 & -3 & -18 \\ 12 & 0 & 0 \\ -9 & -12 & -3 \end{pmatrix} =$$

$$\begin{pmatrix} -6-6 & 2-(-3) & -4-(-18) \\ 4-12 & 2-0 & 0-0 \\ 0-(-9) & 8-(-12) & 4-(-3) \end{pmatrix} = \begin{pmatrix} -12 & 5 & 14 \\ -8 & 2 & 0 \\ 9 & 20 & 7 \end{pmatrix}_{3\times 3}$$

e) $D^2 + DE$

$$D^2 = \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} =$$

$$\begin{pmatrix} (-3.(-3) + 2.1 + 0.(-2)) & (-3.2 + 2.1 + 0.0) & (-3.0 + 2.4 + 0.2) \\ (1.(-3) + 1.1 + 4.(-2)) & (1.2 + 1.1 + 4.0) & (1.0 + 1.4 + 4.2) \\ (-2.(-3) + 0.1 + 2.(-2)) & (-2.2 + 0.1 + 2.0) & (-2.0 + 0.4 + 2.2) \end{pmatrix}$$

$$= \begin{pmatrix} 9+2+0 & -6+2+0 & 0+8+0 \\ -3+1+(-8) & 2+1+0 & 0+4+8 \\ 6+0+(-4) & -4+0+0 & 0+0+4 \end{pmatrix} = \begin{pmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{pmatrix}$$

$$DE = \begin{pmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix} =$$

$$\begin{pmatrix} (-3\cdot 2 + 2\cdot(-1) + 0\cdot(-6)) & (-3\cdot 4 + 2\cdot 0 + 0\cdot 0) & (-3\cdot(-3) + 2\cdot(-4) + 0\cdot(-1)) \\ (1\cdot 2 + 1\cdot(-1) + 4\cdot(-6)) & (1\cdot 4 + 1\cdot 0 + 4\cdot 0) & (1\cdot(-3) + 1\cdot(-4) + 4\cdot(-1)) \\ (-2\cdot 2 + 0\cdot(-1) + 2\cdot(-6)) & (-2\cdot 4 + 0\cdot 0 + 2\cdot 0) & (-2\cdot(-3) + 0\cdot(-4) + 2\cdot(-1)) \end{pmatrix}$$

$$= \begin{pmatrix} (-6-2+0) & (-12+0+0) & (9-8+0) \\ (2-1-24) & (4+0+0) & (-3-4-4) \\ (-4-0-12) & (-8+0+0) & (6-0-2) \end{pmatrix} = \begin{pmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{pmatrix}$$

$$D^2 + DE = \begin{pmatrix} 11 & -4 & 8 \\ -10 & 3 & 12 \\ 2 & -4 & 4 \end{pmatrix} + \begin{pmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{pmatrix} =$$

$$\begin{pmatrix} 11+(-8) & -4+(-12) & 8+1 \\ -10+(-23) & 3+4 & 12+(-11) \\ 2+(-16) & -4+(-8) & 4+4 \end{pmatrix} = \begin{pmatrix} 3 & -16 & 9 \\ -33 & 7 & 1 \\ -14 & -12 & 8 \end{pmatrix}_{3\times 3}$$

f) $C^T A$

$$C = \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix}_{2\times 3} \rightarrow C^T = \begin{pmatrix} -2 & 7 \\ 3 & -3 \\ -7 & -2 \end{pmatrix}_{3\times 2}$$

$$C^T A = \begin{pmatrix} -2 & 7 \\ 3 & -3 \\ -7 & -2 \end{pmatrix}_{3\times 2} \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix}_{2\times 2} = \begin{pmatrix} -2\cdot 1 + 7\cdot 3 & -2\cdot 0 + 7\cdot 7 \\ 3\cdot 1 + (-3)\cdot 3 & 3\cdot 0 + (-3)\cdot 7 \\ -7\cdot 1 + (-2)\cdot 3 & -7\cdot 0 + (-2)\cdot 7 \end{pmatrix}$$

$$= \begin{pmatrix} -2+21 & 0+49 \\ 3-9 & 0-21 \\ -7-6 & 0-14 \end{pmatrix} = \begin{pmatrix} 19 & 49 \\ -6 & -21 \\ -13 & -14 \end{pmatrix}$$

g) $E - AC$

$$AC = \begin{pmatrix} 1 & 0 \\ 3 & 7 \end{pmatrix} \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix} =$$

$$\begin{pmatrix} 1.(-2)+0.7 & 1.3+0.(-3) & 1.(-7)+0.(-2) \\ 3.(-2)+7.7 & 3.3+7.(-3) & 3.(-7)+7.(-2) \end{pmatrix} =$$

$$\begin{pmatrix} -2 & 3 & -7 \\ 43 & -12 & -35 \end{pmatrix}$$

$$E - AC = \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix}_{3\times3} - \begin{pmatrix} -2 & 3 & -7 \\ 43 & -12 & -35 \end{pmatrix}_{2\times3}$$

Subtração não definida, pois E e AC possuem ordens distintas.

h) $F^t E$

$$F = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}_{3\times1} \longrightarrow F^t = \begin{pmatrix} 1 & -2 & 0 \end{pmatrix}_{1\times3}$$

$$F^t E = \begin{pmatrix} 1 & -2 & 0 \end{pmatrix}_{1\times3} \begin{pmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{pmatrix}_{3\times3} =$$

$(1.2 + (-2)(-1) + 0(5) \quad 1.4 + (-2).0 + 0.0 \quad 1(-3) + (-2)(-1) +$

$= (4 \quad 4 \quad 5)_{1\times 3}$

i) BCF

$$BC = \begin{pmatrix} 0 & 5 \\ 2 & -2 \end{pmatrix}_{2\times 2} \begin{pmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{pmatrix}_{2\times 3} =$$

$$\begin{pmatrix} 0(-2) + 5.7 & 0.3 + 5.(-3) & 0.(-7) + 5.(-2) \\ 2(-2) + (-2).7 & 2.3 + (-2).(-3) & 2.(-7) + (-2).(-2) \end{pmatrix} =$$

$$\begin{pmatrix} 35 & -15 & -10 \\ -18 & 12 & -10 \end{pmatrix}_{2\times 3}$$

$$BCF = \begin{pmatrix} 35 & -15 & -10 \\ -18 & 12 & -10 \end{pmatrix}_{2\times 3} \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}_{3\times 1} =$$

$$\begin{pmatrix} 35.1 + (-15).(-2) + (-10).0 \\ -18.1 + 12.(-2) + (-10).0 \end{pmatrix} = \begin{pmatrix} 65 \\ -42 \end{pmatrix}_{2\times 1}$$

2.

a) $A_{\boxed{2}\times 3} B_{3\times \boxed{4}} = C_{2\times 4}$ $\quad B_{3\times \boxed{4}} A_{\boxed{2}\times 3}$ Não definido, pois $4 \neq 2$

b) $A_{\boxed{4}\times 1} B_{1\times \boxed{2}} = C_{4\times 2}$ $\quad B_{1\times \boxed{2}} A_{\boxed{4}\times 1}$ Não definido, pois $2 \neq 4$.

c) $A_{1\times 2} B_{3\times 1} = \emptyset$ Produto não existe, pois $2 \neq 3$.
$B_{3\times 1} A_{1\times 2}$ Está definido, pois $1 = 1$.

d) $A_{\boxed{5}\times 2} \cdot B_{2\times \boxed{3}} = \boxed{C_{5\times 3}}$ $B_{2\times 3} \cdot A_{5\times 2}$ Não definido, pois $3 \neq 5$.

e) $A_{4\times \boxed{4}} B_{\boxed{3}\times 3} = \emptyset$ Produto não existe, pois $4 \neq 3$.
$B_{3\times 3} A_{4\times 4}$ Não está definido, pois $3 \neq 4$

f) $A_{\boxed{4}\times 2} \cdot B_{2\times \boxed{4}} = \boxed{C_{4\times 4}}$ $B_{2\times 4} A_{4\times 2}$ Está definido, pois $4 = 4$.

g) $A_{\boxed{2}\times 1} B_{1\times \boxed{3}} = \boxed{C_{2\times 3}}$ $B_{1\times \boxed{3}} A_{\boxed{2}\times 1}$ Não definido, pois $3 \neq 2$.

h) $A_{\boxed{2}\times 2} B_{2\times \boxed{2}} = \boxed{C_{2\times 2}}$ $B_{2\times \boxed{2}} A_{\boxed{2}\times 2}$ Está definido, pois $2 = 2$

3.

a) $A = (a_{ij})_{2\times 3}$, onde $a_{ij} = 3i - 2j$

$a_{11} = 3.1 - 2.1 = 1$
$a_{12} = 3.1 - 2.2 = -1$
$a_{13} = 3.1 - 2.3 = -3$
$a_{21} = 3.2 - 2.1 = 4$
$a_{22} = 3.2 - 2.2 = 2$
$a_{23} = 3.2 - 2.3 = 0$

$$A = \begin{pmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{pmatrix}$$

b) $B = (b_{ij})_{3\times 3}$, onde $b_{ij} = \begin{cases} 2i + j, \text{ se } i = j \\ i^2 - j, \text{ se } i \neq j \end{cases}$

$b_{11} = 2.1 + 1 = 3$ $b_{12} = 1^2 - 2 = -1$ $b_{13} = 1^2 - 3 = -2$
$b_{21} = 2^2 - 1 = 3$ $b_{22} = 2.2 + 2 = 6$ $b_{23} = 2^2 - 3 = 1$
$b_{31} = 3^2 - 1 = 8$ $b_{32} = 3^2 - 2 = 7$ $b_{33} = 2.3 + 3 = 9$

$$B = \begin{pmatrix} 3 & -1 & -2 \\ 3 & 6 & 1 \\ 8 & 7 & 9 \end{pmatrix}$$

c) $C = (c_{ij})_{1\times4}$, onde $c_{ij} = j^{i}$

$c_{11} = 1^1 = 1 \quad c_{12} = 1^2 = 1 \quad c_{13} = 1^3 = 1 \quad c_{14} = 1^4 = 1$

$C = (1 \ 1 \ 1 \ 1)$

d) $D = (d_{ij})_{4\times4}$, onde $d_{ij} = \begin{cases} i^2 + j^2, \text{ se } i = j \\ 2ij, \text{ se } i \neq j \end{cases}$

$d_{11} = 1^2 + 1^2 = 2 \quad d_{12} = 2.1.2 = 4 \quad d_{13} = 2.1.3 = 6$
$d_{14} = 2.1.4 = 8 \quad d_{21} = 2.2.1 = 4 \quad d_{22} = 2^2 + 2^2 = 8$
$d_{23} = 2.2.3 = 12 \quad d_{24} = 2.2.4 = 16 \quad d_{31} = 2.3.1 = 6$
$d_{32} = 2.3.2 = 12 \quad d_{33} = 3^2 + 3^2 = 18 \quad d_{34} = 2.3.4 = 24$
$d_{41} = 2.4.1 = 8 \quad d_{42} = 2.4.2 = 16 \quad d_{43} = 2.4.3 = 24$
$d_{44} = 4^2 + 4^2 = 32$

$$D = \begin{pmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{pmatrix}$$

4.

a) $[BA]_{23} = 2.1 + (-1).2 + 4.5 = 2 - 2 + 20$
$= 20$

b) $[AB]_{23} = -2.3 + (-3).4 + 2.(-17) = -6 + (-12) + (-34)$
$= -52$

FORONI

c) $[B^2]_{31} = -3.1 + (-1).2 + (-17).(-3) = -3 - 2 + 51$
$= 46$

d) $Tr(A) = a_{11} + a_{22} + a_{33} = 1 + (-3) + 5 = 3$

e) $Tr(B^T) = [B^T]_{11} + [B^T]_{22} + [B^T]_{33} = 1 + (-1) + (-17)$
$= -17$

$$B = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & -1 & -17 \end{pmatrix} \rightarrow B^T = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -1 & -1 \\ 3 & 4 & -17 \end{pmatrix}$$

f) $Tr(A-B) = [A-B]_{11} + [A-B]_{22} + [A-B]_{33}$
$= (1-1) + (-3-(-1)) + (5-(-17)) = -2 + 22$
$= 20$

g) $Tr(AB) = [AB]_{11} + [AB]_{22} + [AB]_{33}$
$= (1.1 + 2.2 + 1.(-3)) + (-2.0 + (-3).(-1) + 2.(-1)) +$
$(1.3 + 4.4 + 5.(-17)) = 2 + 1 + (-66) = -63$

5.

a) $2X + A = 3B + C$

$$2X + \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 3.2 & 3.1 \\ 3.4 & 3.3 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} \qquad 2X = \begin{pmatrix} 6 & 3 \\ 12 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix}$$

$$2X = \begin{pmatrix} 6+0 & 3+2 \\ 12+1 & 4+0 \end{pmatrix} - \begin{pmatrix} -1 & 7 \\ 2 & 6 \end{pmatrix} \qquad 2X = \begin{pmatrix} 6-(-1) & 5-7 \\ 13-2 & 4-6 \end{pmatrix}$$

$$2X = \begin{pmatrix} 7 & -2 \\ 11 & 3 \end{pmatrix} \qquad X = \begin{pmatrix} \frac{7}{2} & -1 \\ \frac{11}{2} & \frac{3}{2} \end{pmatrix}$$

FORONI:

b) $Y + A = \frac{1}{2}(B - C)^T$

$Y + \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2 & 3\\ -1 & 3\end{pmatrix}$ $\quad Y = \begin{pmatrix}\frac{1}{2}\cdot 2 & \frac{1}{2}\cdot 3\\ \frac{1}{2}(-1) & \frac{1}{2}\cdot 3\end{pmatrix} - \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix}$

$Y = \begin{pmatrix}1 & \frac{3}{2}\\ -\frac{1}{2} & \frac{3}{2}\end{pmatrix} - \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix}$ $\quad Y = \begin{pmatrix}0 & -\frac{11}{2}\\ -\frac{5}{2} & -\frac{9}{2}\end{pmatrix}$

c) $3x + A = B - X$

$3x + \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} = \begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix} - X$ $\quad 4X = \begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix} - \begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix}$

$4x = \begin{pmatrix}3 & -6\\ 2 & -3\end{pmatrix}$ $\quad X = \begin{pmatrix}\frac{3}{4} & -\frac{3}{2}\\ \frac{1}{2} & -\frac{3}{4}\end{pmatrix}$

d) $\begin{cases} X + Y = 3A \\ X - Y = 2B + C \end{cases}$

$2X = 3A + 2B + C$ $\quad 2x = 3\begin{pmatrix}-1 & 7\\ 2 & 6\end{pmatrix} + 2\begin{pmatrix}2 & 1\\ 4 & 3\end{pmatrix} + \begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}$

$2x = \begin{pmatrix}-3 & 21\\ 6 & 18\end{pmatrix} + \begin{pmatrix}4 & 2\\ 8 & 6\end{pmatrix} + \begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}$ $\quad 2x = \begin{pmatrix}1 & 23\\ 14 & 24\end{pmatrix} + \begin{pmatrix}0 & 2\\ 1 & 0\end{pmatrix}$

$2x = \begin{pmatrix}1 & 25\\ 15 & 24\end{pmatrix}$ $\quad X = \begin{pmatrix}\frac{1}{2} & \frac{25}{2}\\ \frac{15}{2} & 12\end{pmatrix}$

6. $A^2 = \begin{pmatrix}1 & \frac{1}{x}\\ x & 1\end{pmatrix} \cdot \begin{pmatrix}1 & \frac{1}{x}\\ x & 1\end{pmatrix} = \begin{pmatrix}1\cdot 1 + \frac{1}{x}\cdot x & 1\cdot\frac{1}{x} + \frac{1}{x}\cdot 1\\ x\cdot 1 + 1\cdot x & x\cdot\frac{1}{x} + 1\cdot 1\end{pmatrix}$

$= \begin{pmatrix} 1+1 & \frac{1}{x}+\frac{1}{x} \\ x+x & 1+1 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$

$2A = 2\begin{pmatrix} 1 & \frac{1}{x} \\ x & 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 1 & 2 \cdot \frac{1}{x} \\ 2 . x & 2 . 1 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$

$A^2 = 2A \rightarrow \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{pmatrix}$

$A^2 = 2A$

$A^3 = A^2 . A = 2A . A = 2A^2 = 4A$

$A^4 = A^2 . A^2 = 2A . 2A = 4A^2 = 8A$

$\underline{A^n = 2^{n-1} A}$ //

7.

a) $A(B+C) \leftrightarrow AB + AC \leftrightarrow X + Y$

b) $B^T A^T \leftrightarrow X^T$

c) $C^T A^T \leftrightarrow Y^T$

d) $(ABA)C \leftrightarrow (XA)C \leftrightarrow CX\,CA \leftrightarrow CX\,Y$

8.

a) $A = \begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ $A^T = \begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}$

$\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix} = \begin{pmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{pmatrix}$

FORONI: $x+2 = 2x-3$ $3+2 = 2x-x$ $5 = x$

$$A = \begin{pmatrix} 4 & 7 \\ 7 & 6 \end{pmatrix}$$

b) $$B = \begin{pmatrix} 0 & -4 & 2 \\ x & 0 & 1-z \\ y & 2z & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1-z & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 & -2 \\ -x & 0 & 1+z \\ -y & -2z & 0 \end{pmatrix}$$

$x = 4$ $\quad y = -2$ $\quad 1-z = -2z$

$1 = -z$

$z = -1$

9.

$$3\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3x & 3y \\ 3z & 3t \end{pmatrix} = \begin{pmatrix} x+4 & 6+x+y \\ -1+z+t & 2t+3 \end{pmatrix}$$

$3x = x+4$ $\quad 3y = 6+2+y$ $\quad 3t = 2t+3$

$3x - x = 4$ $\quad 3y - y = 8$ $\quad 3t - 2t = 3$

$x = \frac{4}{2}$ $\quad y = \frac{8}{2}$ $\quad t = 3$

$x = 2$ $\quad y = 4$

$3z = -1 + z + 3$

$3z - z = 2$

$z = \frac{2}{2}$

$z = 1$

10.

a) $R(\theta) = \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}$ $\quad R(\theta)^T = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}$

$$R(\theta)\,R(\theta)^T = \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta.\cos\theta + \operatorname{sen}\theta.\operatorname{sen}\theta & \cos\theta.(-\operatorname{sen}\theta) + \operatorname{sen}\theta.\cos\theta \\ -\operatorname{sen}\theta.\cos\theta + \cos\theta.\operatorname{sen}\theta & -\operatorname{sen}\theta.(-\operatorname{sen}\theta) + \cos\theta.\cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta + \operatorname{sen}^2\theta & \cos\theta.(-\operatorname{sen}\theta) + \operatorname{sen}\theta.\cos\theta \\ -\operatorname{sen}\theta.\cos\theta + \cos\theta\operatorname{sen}\theta & \operatorname{sen}^2\theta + \cos^2\theta \end{pmatrix}$$

$$\operatorname{sen}^2\theta + \cos^2\theta = 1$$

$$\begin{pmatrix} 1 & \cos\theta.(-\operatorname{sen}\theta) + \operatorname{sen}\theta.\cos\theta \\ -\operatorname{sen}\theta.\cos\theta + \cos\theta\operatorname{sen}\theta & 1 \end{pmatrix}$$

$$R(\theta)^T R(\theta) = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta.\cos\theta + (-\operatorname{sen}\theta)(-\operatorname{sen}\theta) & \cos\theta\operatorname{sen}\theta + (-\operatorname{sen}\theta)\cos\theta \\ \operatorname{sen}\theta.\cos\theta + \cos\theta(-\operatorname{sen}\theta) & \operatorname{sen}\theta\operatorname{sen}\theta + \cos\theta\cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos^2\theta + \operatorname{sen}^2\theta & \cos\theta\operatorname{sen}\theta + (-\operatorname{sen}\theta)\cos\theta \\ \operatorname{sen}\theta.\cos\theta + \cos\theta(-\operatorname{sen}\theta) & \operatorname{sen}^2\theta + \cos^2\theta \end{pmatrix}$$

$$\begin{pmatrix} 1 & \cos\theta\operatorname{sen}\theta + (-\operatorname{sen}\theta)\cos\theta \\ \operatorname{sen}\theta.\cos\theta + \cos\theta(-\operatorname{sen}\theta) & 1 \end{pmatrix}$$

b)

$$A = \begin{pmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{pmatrix} \qquad A^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ x & y & z \end{pmatrix}$$

$$A.A^* = \begin{pmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ x & y & z \end{pmatrix}$$

$$\begin{pmatrix} 1.1+0.0+x.x & 1.0+0.\frac{1}{\sqrt{2}}+xy & 1.0+0.\frac{1}{\sqrt{2}}+xz \\ 0.1+\frac{1}{\sqrt{2}}.0+y.x & 0.0+\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}+y.y & 0.0+\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}+yx \\ 0.1+\frac{1}{\sqrt{2}}.0+z.x & 0.0+\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}+z.y & 0.0+\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}+z.z \end{pmatrix}$$

$$\begin{pmatrix} 1+x^2 & xy & xz \\ yx & \frac{1}{\sqrt{2}}^2+y^2 & \frac{1}{\sqrt{2}}^2+yx \\ zx & \frac{1}{\sqrt{2}}^2+zy & \frac{1}{\sqrt{2}}^2+z^2 \end{pmatrix} \to \begin{pmatrix} 1+x^2 & xy & xz \\ yx & \frac{1}{2}+y^2 & \frac{1}{2}+yx \\ zx & \frac{1}{2}+zy & \frac{1}{2}+z^2 \end{pmatrix}$$

$$A^*.A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ x & y & z \end{pmatrix}\begin{pmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{pmatrix}$$

$$\begin{pmatrix} 1.1+0.0+0.0 & 1.0+0.\frac{1}{\sqrt{2}}+0.\frac{1}{\sqrt{2}} & 1x+0y+0z \\ 0.1+\frac{1}{\sqrt{2}}.0+\frac{1}{\sqrt{2}}.0 & 0.0+\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}} & 0.x+\frac{1}{\sqrt{2}}y+\frac{1}{\sqrt{2}}z \\ x.1+y.0+z.0 & x.0+y.\frac{1}{\sqrt{2}}+z.\frac{1}{\sqrt{2}} & x.x+y.y+z.z \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y\frac{1}{\sqrt{2}}+z\frac{1}{\sqrt{2}} \\ x & y\frac{1}{\sqrt{2}}+z\frac{1}{\sqrt{2}} & x^2+y^2+z^2 \end{pmatrix}$$

$$\begin{pmatrix} 1+x^2 & xy & xz \\ yx & \frac{1}{2}+y^2 & \frac{1}{2}+yx \\ zx & \frac{1}{2}+zy & \frac{1}{2}+z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y\frac{1}{\sqrt{2}}+z\frac{1}{\sqrt{2}} \\ x & y\frac{1}{\sqrt{2}}+z\frac{1}{\sqrt{2}} & x^2+y^2+z^2 \end{pmatrix}$$

$1+x^2=1 \qquad \frac{1}{2}+y^2=1 \qquad \frac{1}{2}+z^2=1$

$x^2=0 \qquad y^2=1-\frac{1}{2} \qquad z^2=1-\frac{1}{2}$

$x=0 \qquad y^2=\frac{1}{2} \qquad z^2=\frac{1}{2}$

$y=\sqrt{\frac{1}{2}} \qquad z=\sqrt{\frac{1}{2}}$