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Lista 8 - GA

1.

a)

$r: X = (-5, \frac{2}{3}, 0) + \lambda(\frac{1}{2}, 1, 1)$

$s: z = 3x = 2y - 16$

Vetor diretor de r:

$\vec{d}_r = (\frac{1}{2}, 1, 1)$

Usando parâmetro π: $x = \pi$

$z = 3\pi$

$3\pi = 2y - 16 \rightarrow y = \frac{3\pi + 16}{2}$

Logo: $s: (x, y, z) = (\pi, \frac{3\pi + 16}{2}, 3\pi)$

Vetor diretor de s:

$\vec{d}_s = (1, \frac{3}{2}, 3)$

$\operatorname{sen} \alpha = \frac{|d_r \times d_s|}{|d_r|\,|d_s|}$

$d_r \times d_s = \begin{vmatrix} i & j & k \\ \frac{1}{2} & 1 & 1 \\ 1 & \frac{3}{2} & 3 \end{vmatrix}$

$= i(1.3 - 1.\frac{3}{2}) - j(\frac{1}{2}.3 - 1.1) + k(\frac{1}{2}.\frac{3}{2} - 1.1)$

$= i\left(3-\frac{3}{2}\right) - j\left(\frac{3}{2}-1\right) + k\left(\frac{3}{4}-1\right)$

$= i\left(\frac{3}{2}\right) - j\left(\frac{1}{2}\right) + k\left(-\frac{1}{4}\right) = \left(\frac{3}{2}, -\frac{1}{2}, -\frac{1}{4}\right)$

$|d_1 \times d_2| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{4}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{1}{4} + \frac{1}{16}} = \sqrt{\frac{36}{16} + \frac{4}{16} + \frac{1}{16}} = \sqrt{\frac{41}{16}} = \frac{\sqrt{41}}{4}$

$|d_1| = \sqrt{\left(\frac{1}{2}\right)^2 + 1^2 + 1^2} = \sqrt{\frac{1}{4} + 1 + 1} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$|d_2| = \sqrt{1^2 + \left(\frac{3}{2}\right)^2 + 3^2} = \sqrt{1 + \frac{9}{4} + 9} = \sqrt{\frac{49}{4}} = \frac{7}{2}$

$\operatorname{sen} \alpha = \frac{|d_1 \times d_2|}{|d_1||d_2|} = \frac{\frac{\sqrt{41}}{4}}{\frac{3}{2} \cdot \frac{7}{2}} = \frac{\frac{\sqrt{41}}{4}}{\frac{21}{4}} = \frac{\sqrt{41}}{21}$

Conclusão: $\operatorname{sen} \alpha = \frac{\sqrt{41}}{21}$

b)

$r: X = (1, 1, 0) + \lambda(0, -1, 1)$

$s: x - y + 3 = z = 4$

Vetor diretor de r:

$\vec{d_1} = (0, -1, 1)$

Equações implícitas em s:

$x - y + 3 = 4 \rightarrow x - y - 1 = 0 \rightarrow x = y + 1$

$z = 4$

Usando parâmetros $y = \lambda$ e $z = 4$

$$\begin{cases} x = 1 + t \\ y = t \\ z = 4 \end{cases} \qquad r: (x, y, z) = (1 + t, t, 4)$$

Vetor diretor de r:

$\vec{d}_r = (1, 1, 0)$

$$\text{sen}\, \alpha = \frac{|d_1 \times d_2|}{|d_1|\,|d_2|}$$

$$d_1 \times d_2 = \begin{vmatrix} i & j & k \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= i(-1.0 - 1.1) - j(0.0 - 1.1) + k(0.1 - (-1.1))$$

$$i(-1) - j(-1) + k(1) = (-1, 1, 1)$$

$$|d_1 \times d_2| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|d_1| = \sqrt{0^2 + (-1)^2 + 1^2} = \sqrt{2}$$

$$|d_2| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$$

$$\text{sen}\, \alpha = \frac{|d_1 \times d_2|}{|d_1|\,|d_2|} = \frac{\sqrt{3}}{\sqrt{2} . \sqrt{2}} = \frac{\sqrt{3}}{2}$$

Conclusão: $\text{sen}\, \alpha = \frac{\sqrt{3}}{2}$

c)

$$r: \begin{cases} x + 3z = 7 \\ y = 0 \end{cases}$$

$$s: \begin{cases} x - 4y - 2z = 5 \\ y = 0 \end{cases}$$

Reta r:

$x = 7 - 3z \qquad r: (x, y, z) = (7 - 3z, 0, z)$

$\vec{v}_r = (-3, 0, 1)$

Reta s:

$x = 5 + 2z \qquad s: (x, y, z) = (5 + 2z, 0, z)$

$\vec{v}_s = (2, 0, 1)$

$$\cos \alpha = \frac{\vec{v}_r \cdot \vec{v}_s}{|\vec{v}_r|\,|\vec{v}_s|}$$

$v_r \cdot v_s = (-3) \cdot 2 + 0 \cdot 0 + 1 \cdot 1 = -6 + 0 + 1 = -5$

$|v_r| = \sqrt{(-3)^2 + 0^2 + 1^2} = \sqrt{9 + 0 + 1} = \sqrt{10}$

$|v_s| = \sqrt{2^2 + 0^2 + 1^2} = \sqrt{4 + 0 + 1} = \sqrt{5}$

$$\cos \alpha = \frac{v_r \cdot v_s}{|v_r|\,|v_s|} = \frac{-5}{\sqrt{10} \cdot \sqrt{5}} = \frac{-5}{\sqrt{50}} = \frac{-5}{5\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$= -\frac{\sqrt{2}}{2}$$

Conclusão: $\cos \alpha = -\frac{\sqrt{2}}{2}$

d)

$r: x = \frac{1 - y}{2} = \frac{z}{3}$

FORON: $s: \begin{cases} 3x + y - 5z = 0 \\ x - 2y + 3z + 1 = 0 \end{cases}$

Reta r: Parâmetro t : $y = t$

$x = \frac{1-t}{2}$ $\quad z = 3\left(\frac{1-t}{2}\right) = \frac{3(1-t)}{2}$

$r: (x, y, z) = \left(\frac{1-t}{2}, t, \frac{3(1-t)}{2}\right)$

$\vec{v}_r = \left(-\frac{1}{2}, 1, -\frac{3}{2}\right)$

Reta s: Parâmetro t : $y = t$

$3x + t - 5z = 0 \ldots (1)$
$x - 2t + 3z + 1 = 0 \ldots (2)$

$3.(1)$ $\quad 9x + 3t - 15z = 0$
$5.(2)$ $\quad 5x - 10t + 15z + 5 = 0$

$9x + 3t - 15z + 5x - 10t + 15z + 5 = 0$
$14x - 7t + 5 = 0$
$x = \frac{7t - 5}{14} \rightarrow x = \frac{t}{2} - \frac{5}{14}$

$3\left(\frac{t}{2} - \frac{5}{14}\right) + t - 5z = 0$

$\frac{3t}{2} - \frac{15}{14} + t - 5z = 0$

$\frac{5t}{2} - \frac{15}{14} - 5z = 0 \rightarrow 5z = \frac{5t}{2} - \frac{15}{14}$

$z = \frac{t}{2} - \frac{3}{14}$ $\quad s: (x, y, z) = \left(\frac{t}{2} - \frac{5}{14}, t, \frac{t}{2} - \frac{3}{14}\right)$

$\vec{v}_s = \left(\frac{1}{2}, 1, \frac{1}{2}\right)$

FORONI

$$\cos \alpha = \frac{\vec{d_r} \cdot \vec{d_s}}{|d_r|\,|d_s|}$$

$$d_r \cdot d_s = \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) + 1 \cdot 1 + \left(-\frac{3}{2}\right)\left(\frac{1}{2}\right) =$$

$$-\frac{1}{4} + 1 + -\frac{3}{4} = 1 - 1 = 0$$

Conclusão: $\cos \alpha = 0$, logo as retas são perpendiculares.

2)

Reta r:

$$X = (0,2,0) + \lambda(0,1,0)$$

Portanto: $P = (0, 2+\lambda, 0)$

Reta s:

$$X = (1,2,0) + \mu(0,0,1)$$

Portanto: $Q = (1, 2, \mu)$

Vetor PQ:

$$PQ = Q - P = (1,2,\mu) - (0, 2+\lambda, 0) = (1, -\lambda, \mu)$$

Ângulo de 45° com r:

FORON: $\vec{d_r} = (0,1,0)$

$$\cos 45° = \frac{\vec{PQ} \cdot \vec{d_1}}{|PQ|\,|d_1|} = \frac{\sqrt{2}}{2}$$

$$\frac{(1,-\lambda,\mu)\cdot(0,1,0)}{|PQ| \cdot 1} = \frac{\sqrt{2}}{2}$$

$$\frac{-\lambda}{|PQ|} = \frac{\sqrt{2}}{2} \rightarrow -\lambda = \frac{\sqrt{2}}{2}|PQ|$$

Ângulo de 60° com r:

$$\vec{d_2} = (0,0,1)$$

$$\cos 60° = \frac{\vec{PQ} \cdot \vec{d_2}}{|PQ| \cdot |d_2|} = \frac{1}{2}$$

$$\frac{(1,-\lambda,\mu)\cdot(0,0,1)}{|PQ| \cdot 1} = \frac{1}{2}$$

$$\frac{\mu}{|PQ|} = \frac{1}{2} \rightarrow \mu = \frac{1}{2}|PQ|$$

$$|PQ| = \sqrt{1^2 + (-\lambda)^2 + \mu^2} = \sqrt{1 + \lambda^2 + \mu^2}$$

$$\lambda = -\frac{\sqrt{2}}{2}|PQ| \qquad \mu = \frac{1}{2}|PQ|$$

$$|PQ| = \sqrt{1 + \left(-\frac{\sqrt{2}}{2}|PQ|\right)^2 + \left(\frac{1}{2}|PQ|\right)^2}$$

$$|PQ| = \sqrt{1 + \frac{2}{4}|PQ|^2 + \frac{1}{4}|PQ|^2}$$

$$|PQ| = \sqrt{1 + \frac{3}{4}|PQ|^2} \rightarrow |PQ|^2 = 1 + \frac{3}{4}|PQ|^2$$

$|PQ|^2 - \frac{3}{4}|PQ|^2 = 1 \rightarrow \frac{1}{4}|PQ|^2 = 1$

$|PQ|^2 = 4 \rightarrow |PQ| = 2$

Logo: $\lambda = -\frac{\sqrt{2}}{2} \cdot 2 = -\sqrt{2}$

$\mu = \frac{1}{2} \cdot 2 = 1$

Pontos P e Q:

$P = (0, 2+\lambda, 0) = (0, 2-\sqrt{2}, 0)$

$Q = (1, 2, \mu) = (1, 2, 1)$

3)

a) $r: X = Y - Z = 0$

$\pi: Z = 0$

Reta r:

$X = 0$

$Y - Z = 0 \rightarrow Y = Z$

Parametro $\lambda = Z$

$r: (X, Y, Z) = (0, \lambda, \lambda)$

$v_r = (0, 1, 1)$

Plano π:

$n_\pi = (0, 0, 1)$

FORON:

Ângulo entre reta e plano:

$\theta = \frac{\pi}{2} - ang(\vec{d_1}, \vec{n_\pi})$

$ang(\vec{d_1}, \vec{n_\pi}) = \cos^{-1}(\cos\alpha)$

$\cos\alpha = \frac{\vec{d_1} \cdot \vec{n_\pi}}{|d_1| \, |n_\pi|}$

$d_1 \cdot n_\pi = 0.0 + 1.0 + 1.1 = 1$

$|d_1| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$

$|n_\pi| = \sqrt{0^2 + 0^2 + 1^2} = 1$

$\cos\alpha = \frac{1}{\sqrt{2} \cdot 1} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

$ang(\vec{d_1}, \vec{n_\pi}) = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$\theta = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

Conclusão: O ângulo entre a reta r e o plano π é $\theta = \frac{\pi}{4}$.

b) $r: -x = y = \frac{z-1}{2}$

$\pi: 2x - y = 0$

Reta r: Parâmetro t: $t = y$

FORON

$-x = \lambda \rightarrow x = -\lambda$

$\lambda = \frac{z-1}{2} \rightarrow z = 2\lambda + 1$

Portanto: $\quad r: (x, y, z) = (-\lambda, \lambda, 2\lambda + 1)$

$$\vec{d_r} = (-1, 1, 2)$$

Plano π:

$$\vec{n_\pi} = (2, -1, 0)$$

Ângulo entre reta e plano:

$$\theta = \frac{\pi}{2} - \alpha$$

$$\cos \alpha = \frac{d_r \cdot n_\pi}{|d_r||n_\pi|}$$

$d_r \cdot n_\pi = (-1) \cdot 2 + 1 \cdot (-1) + 2 \cdot 0 = -2 - 1 + 0 = -3$

$|d_r| = \sqrt{(-1)^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$

$|n_\pi| = \sqrt{2^2 + (-1)^2 + 0^2} = \sqrt{4 + 1 + 0} = \sqrt{5}$

$$\cos \alpha = \frac{-3}{\sqrt{6} \cdot \sqrt{5}} = -\frac{3}{\sqrt{30}} = -\frac{3}{\sqrt{30}} = -\frac{3\sqrt{30}}{30} = -\frac{\sqrt{30}}{10}$$

$$\alpha = \cos^{-1}\left(-\frac{\sqrt{30}}{10}\right)$$

Conclusão:

$$\theta = \frac{\pi}{2} - \cos^{-1}\left(-\frac{\sqrt{30}}{10}\right)$$

c) $r: X = (1,0,0) + \lambda(1,1,-2)$
$\pi: x + y - z - 1 = 0$

Vetor diretor de r: $\vec{d}_r = (1,1,-2)$

Vetor normal de π: $\vec{n}_\pi = (1,1,-1)$

Ângulo entre reta e plano:

$$\theta = \frac{\pi}{2} - \alpha$$

$$\cos\alpha = \frac{d_r \cdot n_\pi}{|d_r||n_\pi|}$$

$$d_r \cdot n_\pi = 1.1 + 1.1 + (-2).(-1) = 1 + 1 + 2 = 4$$
$$|d_r| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{1+1+4} = \sqrt{6}$$
$$|n_\pi| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{1+1+1} = \sqrt{3}$$

$$\cos\alpha = \frac{4}{\sqrt{6}.\sqrt{3}} = \frac{4}{\sqrt{18}} = \frac{4}{3\sqrt{2}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

$$\alpha = \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

Conclusão:

$$\theta = \frac{\pi}{2} - \cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

4) $\pi_1: x + y + z = 0$
$\pi_2: x - y = 0$

Vetor normal de π_1:
$\vec{n}_1 = (1,1,1)$

FORON:

Vetor normal de π_2:

$$\vec{n}_2 = (1, -1, 0)$$

Vetor diretor paralelo ao plano π_1:

$\vec{d} = \vec{n}_1 = (1, 1, 1)$

Vetor diretor unitário u:

$|d| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$u = \dfrac{d}{|d|} = \dfrac{(1,1,1)}{\sqrt{3}} = \left(\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$

5)

a) $\pi_1: 2x + y - z - 1 = 0$

$\pi_2: x - y + 3z - 10 = 0$

$\vec{n}_1 = (2, 1, -1)$ e $\vec{n}_2 = (1, -1, 3)$

$\cos\alpha = \dfrac{\vec{n}_1 \cdot \vec{n}_2}{|n_1||n_2|}$

$\vec{n}_1 \cdot \vec{n}_2 = 2.1 + 1\cdot(-1) + (-1).3 = 2 - 1 - 3 = -2$

$|n_1| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{4+1+1} = \sqrt{6}$

$|n_2| = \sqrt{1^2 + (-1)^2 + 3^2} = \sqrt{1+1+9} = \sqrt{11}$

$\cos\alpha = \dfrac{-2}{\sqrt{6}.\sqrt{11}} = \dfrac{-2}{\sqrt{66}}$

$\alpha = \cos^{-1}\left(\dfrac{-2}{\sqrt{66}}\right)$ Logo: $\theta = \cos^{-1}\left(\dfrac{-2}{\sqrt{66}}\right)$

b) $\pi_1 : X = (1,0,0) + \lambda(1,0,1) + \mu(-1,0,1)$
$\pi_2 : x + y + z = 0$

Vetor diretor para π_1:

$\vec{d}_{\pi_1} = (1,0,1) + (-1,0,0) = (0,0,1)$

Vetor normal para π_2:

$\vec{n}_{\pi_2} = (1,1,1)$

$$\cos\alpha = \frac{\vec{d}_{\pi_1} \cdot \vec{n}_{\pi_2}}{|d_{\pi_1}||n_{\pi_2}|}$$

$d_{\pi_1} \cdot n_{\pi_2} = 0.1 + 0.1 + 1.1 = 0 + 0 + 1 = 1$
$|d_{\pi_1}| = \sqrt{0^2 + 0^2 + 1^2} = \sqrt{1} = 1$
$|n_{\pi_2}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$$\cos\alpha = \frac{1}{1 \cdot \sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ Logo: $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

c) $\pi_1 : X = (0,0,0) + \lambda(1,0,0) + \mu(1,1,1)$
$\pi_2 : X = (1,0,0) + \lambda(-1,2,0) + \mu(0,1,0)$

Vetor normal de π_1:

$\vec{d}_1 = (1,0,0)$, $\vec{d}_2 = (1,1,1)$

$$\vec{n}_1 = \vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix} = (0,-1,1)$$

FORONI

Vetor normal de π_2:

$\vec{d_1} = (-1, 2, 0)$, $\vec{d_2} = (0, 1, 0)$

$\vec{n_2} = d_1 \times d_2 = \begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0, 0, -2)$

$\cos \alpha = \dfrac{\vec{n_1} \cdot \vec{n_2}}{|n_1| \cdot |n_2|}$

$\vec{n_1} \cdot \vec{n_2} = 0 \cdot 0 + (-1) \cdot 0 + 1 \cdot (-2) = -2$

$|n_1| = \sqrt{0^2 + (-1)^2 + 1^2} = \sqrt{2}$

$|n_2| = \sqrt{0^2 + 0^2 + (-2)^2} = 2$

$\cos \alpha = \dfrac{-2}{\sqrt{2} \cdot 2} = -\dfrac{1}{\sqrt{2}}$

Portanto: $\theta = \cos^{-1}\left(-\dfrac{1}{\sqrt{2}}\right)$

6)

Vetor normal do primeiro plano:

$2x - y + z = 0 \rightarrow \vec{n_1} = (2, -1, 1)$

Vetor normal do segundo plano:

$v = i - 2j + k \rightarrow \vec{n_2} = (1, -2, 1)$

$\cos \alpha = \dfrac{n_1 \cdot n_2}{|n_1| |n_2|}$

FORON:

$n_1 \cdot n_2 = 2 \cdot 1 + (-1) \cdot (-2) + 1 \cdot 1 = 2 + 2 + 1 = 5$

$|n_1| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

$|n_2| = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$

$\cos \alpha = \dfrac{5}{\sqrt{6} \cdot \sqrt{6}} = \dfrac{5}{6}$

Portanto : $\theta = \cos^{-1}\left(\dfrac{5}{6}\right)$

7)

a) $r : x - 1 = 2y = z$, $A = (1,1,0)$ e $B = (0,1,1)$

$\begin{cases} x = 1 + \lambda \\ y = \dfrac{\lambda}{2} \\ z = \lambda \end{cases}$ $\qquad P = (\lambda + 1, \dfrac{\lambda}{2}, \lambda)$

$d(P, A) = \sqrt{\lambda^2 + (\dfrac{\lambda}{2} - 1)^2 + \lambda^2}$

$= \sqrt{\lambda^2 + \dfrac{\lambda^2}{4} - \lambda + 1 + \lambda^2}$

$= \sqrt{2\lambda^2 + \dfrac{\lambda^2}{4} - \lambda + 1} = \sqrt{\dfrac{9\lambda^2}{4} - \lambda + 1}$

$d(P, B) = \sqrt{(\lambda + 1)^2 + (\dfrac{\lambda}{2} - 1)^2 + (\lambda - 1)^2}$

$= \sqrt{\lambda^2 + 2\lambda + 1 + \dfrac{\lambda^2}{4} - \lambda + 1 + \lambda^2 - 2\lambda + 1}$

$= \sqrt{2\lambda^2 + \dfrac{\lambda^2}{4} + 3} = \sqrt{\dfrac{9\lambda^2}{4} + 3}$

$\sqrt{\dfrac{9\lambda^2}{4} - \lambda + 1} = \sqrt{\dfrac{9\lambda^2}{4} + 3}$

$\frac{9\lambda^2}{4} - \lambda + 1 = \frac{9\lambda^2}{4} + 3$

$-\lambda = 3 - 1 \rightarrow \lambda = -2$

$P = (1, 0, 0) + -2(1, \frac{1}{2}, 1) = (-1, -1, -2)$

b) $r : X = (0,0,4) + \lambda(4, 2, -3)$
$A = (2, 2, 5)$ e $B = (0, 0, 1)$

$P = (0 + 4\lambda, 0 + 2\lambda, 4 - 3\lambda)$
$P = (4\lambda, 2\lambda, 4 - 3\lambda)$

$d(P, A) = \sqrt{(4\lambda - 2)^2 + (2\lambda - 2)^2 + (4 - 3\lambda - 5)^2}$

$d(P, A) = \sqrt{(4\lambda - 2)^2 + (2\lambda - 2)^2 + (3 - 3\lambda)^2}$

$d(P, B) = \sqrt{(4\lambda - 0)^2 + (2\lambda - 0)^2 + (4 - 3\lambda - 1)^2}$

$d(P, B) = \sqrt{(4\lambda)^2 + (2\lambda)^2 + (3 - 3\lambda)^2}$

$\cancel{\sqrt{}}(4\lambda - 2)^2 + (2\lambda - 2)^2 + (3 - 3\lambda)^2 = \cancel{\sqrt{}}(4\lambda)^2 + (2\lambda)^2 + (3 - 3\lambda)^2$

$(4\lambda - 2)^2 + (2\lambda - 2)^2 + \cancel{(3 - 3\lambda)^2} = 16\lambda^2 + 4\lambda + \cancel{(3 - 3\lambda)^2}$

$(4\lambda - 2)^2 + (2\lambda - 2)^2 = 20\lambda^2$

$16\lambda^2 - 16\lambda + 4 + 4\lambda^2 - 8\lambda + 4 = 20\lambda^2$

$\cancel{20\lambda^2} - 24\lambda + 8 = \cancel{20\lambda^2} \rightarrow \lambda = \frac{-8}{-24} = \frac{1}{3}$

FORON:

$$\begin{array}{r} 49 \\ -13 \\ \hline 36 \end{array}$$

$$\begin{cases} x = 4\left(\frac{1}{3}\right) = \frac{4}{3} \\ y = 2\left(\frac{1}{3}\right) = \frac{2}{3} \\ z = 4 - 3\left(\frac{1}{3}\right) = 4 - 1 = 3 \end{cases}$$

$$P = \left(\frac{4}{3}, \frac{2}{3}, 3\right)$$

c) $r: X = (2, 3, -3) + \lambda(1, 1, 1)$

$A = (1, 1, 0)$ e $B = (2, 2, 4)$

$$P = (2 + \lambda, 3 + \lambda, -3 + \lambda)$$

$$d(P, A) = \sqrt{(2 + \lambda - 1)^2 + (3 + \lambda - 1)^2 + (-3 + \lambda - 0)^2}$$

$$= \sqrt{(1 + \lambda)^2 + (2 + \lambda)^2 + (\lambda - 3)^2}$$

$$d(P, B) = \sqrt{(2 + \lambda - 2)^2 + (3 + \lambda - 2)^2 + (-3 + \lambda - 4)^2}$$

$$= \sqrt{\lambda^2 + (1 + \lambda)^2 + (\lambda - 7)^2}$$

$$\cancel{\sqrt{}}\,\cancel{(1+\lambda)^2} + (2 + \lambda)^2 + (\lambda - 3)^2 = \cancel{\sqrt{}}\,\lambda^2 + \cancel{(1+\lambda)^2} + (\lambda - 7)^2$$

$$(2 + \lambda)^2 + (\lambda - 3)^2 = \lambda^2 + (\lambda - 7)^2$$

$$4 + 4\lambda + \lambda^2 + 9 - 6\lambda + \lambda^2 = \lambda^2 + 49 - 14\lambda + \lambda^2$$

$$\cancel{2\lambda^2} - 2\lambda + 13 = \cancel{2\lambda^2} - 14\lambda + 49$$

$$12\lambda = 36$$

$$\lambda = 3$$

$x = 2 + 3 = 5$ $\quad P = (5, 6, 0)$

$y = 3 + 3 = 6$

$z = -3 + 3 = 0$

FORONI

8)

a) $P = (-2, 0, 1)$, $r : X = (1, -2, 0) + \lambda(3, 2, 1)$

$A = (1, -2, 0)$ e $\vec{v} = (3, 2, 1)$

$\vec{AP} = P - A = (-2, 0, 1) - (1, -2, 0) = (-3, 2, 1)$

$d = \dfrac{\|\vec{AP} \times \vec{v}\|}{\|\vec{v}\|}$

$\vec{AP} \times \vec{v} = \begin{vmatrix} i & j & k \\ -3 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix}$

$i(2-2) - j(-3-3) + k(-6-6) \rightarrow i(0) - j(-6) + k(-12)$

$= (0, 6, -12)$

$|v| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

$d = \dfrac{\sqrt{0^2 + 6^2 + (-12)^2}}{\sqrt{14}} = \dfrac{\sqrt{180}}{\sqrt{14}} = \sqrt{\dfrac{180^{\div 2}}{14_{\div 2}}} = \sqrt{\dfrac{90}{7}}$

b) $P = (1, -1, 4)$, $r : \dfrac{x-2}{4} = \dfrac{y}{-3} = \dfrac{1-z}{2}$

$r : \begin{cases} x = 2 + 4\lambda \\ y = -3\lambda \\ z = 1 - 2\lambda \end{cases} \rightarrow r : X = (2, 0, 1) + \lambda(4, -3, -2)$

$d = \dfrac{\|\vec{AP} \times \vec{v}\|}{\|\vec{v}\|}$ $\quad A = (2, 0, 1)$ e $\vec{v} = (4, -3, -2)$

FORONI:

$AP = P - A = (1,-1,4) - (2,0,1) = (-1,-1,3)$

$$\vec{AP} \times \vec{v} = \begin{vmatrix} i & j & k \\ -1 & -1 & 3 \\ 4 & -3 & -2 \end{vmatrix}$$

$i(2+9) - j(2-12) + k(3+4)$

$\rightarrow i(11) - j(-10) + k(7) = (11, 10, 7)$

$\|\vec{AB} \times \vec{v}\| = \sqrt{(11)^2 + (10)^2 + (7)^2} = \sqrt{121 + 100 + 49} = \sqrt{270}$

$\|\vec{v}\| = \sqrt{4^2 + (-3)^2 + (-2)^2} = \sqrt{16 + 9 + 4} = \sqrt{29}$

$$d = \frac{\sqrt{270}}{\sqrt{29}} = \sqrt{\frac{270}{29}}$$

c) $P = (0,-1,0)$, $r: x = 2y - 3 = 2z - 1$

Parâmetro λ: $x = \lambda$

$2y = \lambda + 3 \rightarrow y = \frac{\lambda + 3}{2}$

$2z = \lambda + 1 \rightarrow z = \frac{\lambda + 1}{2}$

$x = \left(\lambda, \frac{\lambda+3}{2}, \frac{\lambda+1}{2}\right)$

$$d = \frac{|\vec{AP} \times \vec{v}|}{|\vec{v}|}$$

p/ $\lambda = 0$: $A = \left(0, \frac{3}{2}, \frac{1}{2}\right)$

$\vec{v} = \left(1, \frac{1}{2}, \frac{1}{2}\right)$

$AP = P - A = (0,-1,0) - \left(0, \frac{3}{2}, \frac{1}{2}\right) = \left(0, -\frac{5}{2}, -\frac{1}{2}\right)$

$$\vec{AP} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & -\frac{5}{2} & -\frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{2} \end{vmatrix}$$

$i\left(-\frac{5}{2}\cdot\frac{1}{2}-\left(-\frac{1}{2}\right)\cdot\frac{1}{2}\right)-j\left(\frac{1}{2}\right)+k\left(\frac{5}{2}\right)$

$i(-1)-j\left(\frac{1}{2}\right)+k\left(\frac{5}{2}\right) = \left(-1,-\frac{1}{2},\frac{5}{2}\right)$

$|\vec{AB}\times\vec{v}| = \sqrt{(-1)^2+\left(-\frac{1}{2}\right)^2+\left(\frac{5}{2}\right)^2} = \sqrt{7}$

$|\vec{v}| = \sqrt{1^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{2}}$

$d = \frac{|\vec{AB}\times\vec{v}|}{|\vec{v}|} = \frac{\sqrt{7}}{\sqrt{\frac{3}{2}}} = \sqrt{\frac{7}{\frac{3}{2}}} = \sqrt{7\cdot\frac{2}{3}} = \sqrt{\frac{14}{3}}$

9) $\pi_1: x+y=2 \qquad r: x=y=z+1$

$\pi_2: x=y+z$

$\begin{cases} x+y=2 \to y=2-x \\ x=y+z \to x=(2-x)+z \to z=-2+2x \end{cases}$

Parâmetro t: $x=t$

$r: \begin{cases} x=t \\ y=2-t \\ z=-2+2t \end{cases} \qquad r: X=(t,2-t,-2+2t)$

$s: \begin{cases} x=\gamma \\ y=\gamma \\ z=-1+\gamma \end{cases} \qquad s: X=(0,0,-1)+\gamma(1,1,1)$

$d(P,r) = \frac{\|\vec{AP}\times\vec{v}\|}{\|\vec{v}\|}$

$\vec{AP} = P-A = (t,2-t,-2+2t)-(0,0,-1)$
$= (t,2-t,-1+2t)$

FORONI:

$$\vec{AP} \times \vec{v} = \begin{vmatrix} i & j & k \\ \lambda & 2-\lambda & -1+2\lambda \\ 1 & 1 & 1 \end{vmatrix}$$

$$= i\,(2-\lambda-(-1+2\lambda)) - j\,(\lambda-(-1+2\lambda)) + k\,(\lambda-(2-\lambda))$$

$$i\,(2-\lambda+1-2\lambda) - j\,(\lambda+1-2\lambda) + k\,(\lambda-2+\lambda)$$

$$i\,(3-3\lambda) - j\,(-\lambda+1) + k\,(-2+2\lambda) = 0$$

$$\sqrt{\frac{14}{3}} = \frac{\sqrt{(3-3\lambda)^2+(\lambda-1)^2+(-2+2\lambda)^2}}{\sqrt{3}}$$

$$\frac{\sqrt{14}}{\cancel{\sqrt{3}}} = \frac{\sqrt{(3-3\lambda)^2+(\lambda-1)^2+(-2+2\lambda)^2}}{\cancel{\sqrt{3}}}$$

$$14 = (3-3\lambda)^2+(\lambda-1)^2+(-2+2\lambda)^2$$

$$14 = 9 + 18\lambda + 9\lambda^2 + \lambda^2 + 2\lambda + 1 + 4 - 8\lambda + 4\lambda^2$$

$$14 = 14 + 12\lambda + 14\lambda^2$$

$$14\lambda^2 + 12\lambda = 0 \quad (\div 2)$$

$$7\lambda^2 + 6\lambda = 0$$

$$\Delta = 6^2 - 4\cdot 7\cdot 0 \qquad \lambda = \frac{-6 \pm \sqrt{36}}{2\cdot 7} \rightarrow \lambda_1 = \frac{0}{14} = 0$$

$$\Delta = 36 \qquad\qquad \lambda_2 = -\frac{12}{14} = -\frac{6}{7}$$

Logo, os dois pontos são:

p/ $\lambda = 0$ $\qquad$ p/ $\lambda = -\frac{6}{7}$

$P_1 = (0, 2, -2)$ $\qquad$ $P_2 = \left(-\frac{6}{7}, \frac{20}{7}, -\frac{26}{7}\right)$

10) $\pi: X = (1,0,0) + \lambda(1,0,0) + \mu(-1,0,3)$

a) $P = (1,3,4)$

Equação geral do plano:

$$\begin{vmatrix} x-1 & y & z \\ 1 & 0 & 0 \\ -1 & 0 & 3 \end{vmatrix} = 0$$

$$x-1 \cdot (-1)^{1+1} \begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} + y(-1)^{1+2} \begin{vmatrix} 1 & 0 \\ -1 & 3 \end{vmatrix} + z(-1)^{1+3} \begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix}$$

$$x-1(0) - y(3) + z(0) = 0$$

$$-3y = 0 \rightarrow y = 0$$

$$d = \frac{|0.1 + (-3).3 + 0.4 + 0|}{\sqrt{0^2 + (-3)^2 + 0^2}} = \frac{|-9|}{3} = \frac{9}{3} = 3$$

Conclusão: a distância do ponto P ao plano π é de 3 unidades.

b) $\pi: x - 2y - 2z - 6 = 0$

$P = (0, 0, -6)$

$$d = \frac{|1.0 - 2.0 - 2.(-6) - 6|}{\sqrt{1^2 + (-2)^2 + (-2)^2}} = \frac{|6|}{\sqrt{9}} = \frac{6}{3} = 2$$

Conclusão: a distância do ponto P ao plano π é de 2 unidades.

FORON:

c) $\pi: 2x - y + 2z - 3 = 0$
$P = (1, 1, 1)$

$$d = \frac{|2 \cdot 1 - 1 \cdot 1 + 2 \cdot 1 - 3|}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{|0|}{\sqrt{9}} = \frac{0}{3} = 0$$

Conclusão: a distância do ponto P ao plano π é de 0 unidades.

11) $r: x = 2 - y = y + z$
$\pi: x - 2y - z = 1$

Reta r:

- $x = 2 - y$
- $x = y + z$

Parametro λ: $y = \lambda$

- $x = 2 - \lambda$
- $2 - \lambda = \lambda + z \rightarrow z = 2 - 2\lambda$

$$(x, y, z) = (2 - \lambda, \lambda, 2 - 2\lambda)$$

$$d = \frac{|(2 - \lambda) - 2(\lambda) - (2 - 2\lambda) - 1|}{\sqrt{1^2 + (-2)^2 + (-1)^2}}$$

$$d = \frac{|1 - \lambda|}{\sqrt{6}} = \sqrt{6} \quad (\cdot\sqrt{6}) \rightarrow |1 - \lambda| = 6$$

$1 - \lambda = 6$ ou $1 - \lambda = -6$

• $1 - \lambda = 6 \Rightarrow \lambda = -5$
• $1 - \lambda = -6 \Rightarrow \lambda = 7$

Logo:

p/ $\lambda = -5$

$(x, y, z) = (2-(-5), -5, 2-2(-5)) = (7, -5, 12)$

p/ $\lambda = 7$

$(x, y, z) = (2-7, 7, 2-2(7)) = (-5, 7, -12)$

Conclusão: os pontos da reta r que distam $\sqrt{6}$ do plano π são $(7, -5, 12)$ e $(-5, 7, -12)$.

12)

a) $r: X = (2, 1, 0) + \lambda(1, -1, 1)$
$s: x + y + z = 2x - y - 1 = 0$

Reta s:

• $x + y + z = 0$
• $2x - y - 1 = 0 \rightarrow y = 2x - 1$

$x + (2x - 1) + z = 0 \Rightarrow 3x - 1 + z = 0 \Rightarrow z = -3x + 1$

$s: (x, y, z) = (\lambda, 2\lambda - 1, -3\lambda + 1)$

$P = (2, 1, 0)$ e $Q = (0, -1, 1)$
$d_r = (1, -1, 1)$ e $d_s = (1, 2, -3)$

$\vec{PQ} = Q - P = (0,-1,1) - (2,1,0) = (-2,-2,1)$

$$d = \frac{|\vec{PQ} \cdot (d_1 \times d_2)|}{\|d_1 \times d_2\|}$$

$$d_1 \times d_2 = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 1 & 2 & -3 \end{vmatrix}$$

$= i((-1)(-3) - 1 \cdot 2) - j(1(-3) - 1 \cdot 1) + k(1 \cdot 2 - 1 \cdot 1)$
$= i(1) - j(-4) + k(1) = (1,4,1)$

$\vec{PQ} \cdot (1,4,1) = (-2,-2,1) \cdot (1,4,1) = -2 \cdot 1 + (-2) \cdot 4 + 1 \cdot 1 =$
$-2 - 8 + 1 = -9$

$\|d_1 \times d_2\| = \sqrt{1^2 + 4^2 + 1^2} = \sqrt{18} = 3\sqrt{2}$

$$d = \frac{|-9|}{3\sqrt{2}} = \frac{\cancel{9}}{\cancel{3}\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

b) $r: \dfrac{x+4}{3} = \dfrac{y}{4} = \dfrac{z+5}{-2}$

$s: X = (21,-5,2) + \lambda(6,-4,-1)$

Reta r:

- $\dfrac{x+4}{3} = \lambda \Rightarrow x = 3\lambda - 4$
- $\dfrac{y}{4} = \lambda \Rightarrow y = 4\lambda$
- $\dfrac{z+5}{-2} = \lambda \Rightarrow z = -2\lambda - 5$

$r: (x, y, z) = (3t - 4, 4t, -2t - 5)$

$\vec{d}_r = (3, 4, -2)$

Reta s:

$\vec{d}_s = (6, -4, -1)$

Ponto P na reta r: $t = 0$

$P = (3.0 - 4, 4.0, -2.0 - 5) = (-4, 0, -5)$

Ponto Q na reta s: $\lambda = 0$

$Q = (21, -5, 2)$

$\vec{PQ} = Q - P = (21 - (-4), -5 - 0, 2 - (-5)) = (25, -5, 7)$

$$\vec{d}_r \times \vec{d}_s = \begin{vmatrix} i & j & k \\ 3 & 4 & -2 \\ 6 & -4 & -1 \end{vmatrix}$$

$= i(4.(-1) - (-2).(-4)) - j(3.(-1) - (-2).6) + k(3.(-4) - 4.6)$

$= i(-4 - 8) - j(-3 + 12) + k(-12 - 24)$
$= i(-12) - j(9) + k(-36) = (-12, -9, -36)$

$\vec{PQ} . (-12, -9, -36) = (25, -5, 7) . (-12, -9, -36)$
$= 25.(-12) + (-5).(-9) + 7.(-36)$
$= -300 + 45 - 252 = -507$

FORONI:

$\|d_r \times d_s\| = \sqrt{(-12)^2 + (-9)^2 + (-36)^2} = \sqrt{144 + 81 + 1296}$
$= \sqrt{1521} = 39$

$d = \frac{507}{39} = 13$

c) $r: \frac{x-1}{-2} = 2y = z$

$s: X = (0,0,2) + \lambda(-4,1,2)$

Reta r:

- $\frac{x-1}{-2} = \pi \rightarrow x = 1 - 2\pi$
- $2y = \pi \rightarrow y = \frac{\pi}{2}$
- $z = \pi$

$r: (x,y,z) = (1-2\pi, \frac{\pi}{2}, \pi)$

$d_r = (-2, \frac{1}{2}, 1)$

Reta s:

$d_s = (-4, 1, 2)$

Ponto P na reta r: $\pi = 0$

$P = (1 - 2.0, \frac{0}{2}, 0) = (1, 0, 0)$

Ponto Q na reta s:

$Q = (0, 0, 2)$

$\vec{PQ} = Q - P = (0,0,2) - (1,0,0) = (-1,0,2)$

$$d_1 \times d_2 = \begin{vmatrix} i & j & k \\ -2 & \frac{1}{2} & 1 \\ -4 & 1 & 2 \end{vmatrix}$$

$= i(1-1) - j(-4+4) + k(-2+2) = (0,0,0)$

$$d = \frac{|(-1,0,2)\cdot(0,0,0)|}{\sqrt{0^2+0^2+0^2}} = 0$$

13)

a) $r: X = (1,9,4) + \lambda(3,3,3)$
$\pi: X = (5,7,9) + \lambda(1,0,0) + \mu(0,1,0)$

$d_r = (3,3,3)$

$d_\lambda = (1,0,0)$ e $d_\mu = (0,1,0)$

$n = d_\lambda \times d_\mu = (1,0,0) \times (0,1,0) = (0,0,1)$

$d_r \cdot n = (3,3,3)\cdot(0,0,1) = 3 \neq 0$ (Reta não paralela ao plano.

$P = (1,9,4) \qquad A = (5,7,9)$

$\vec{AP} = P - A = (1,9,4) - (5,7,9) = (-4,2,-5)$

$\|n\| = \sqrt{0^2+0^2+1^2} = 1$

$\vec{AP}\cdot n = (-4,2,-5)\cdot(0,0,1) = -5$

FORONI:

$d = \dfrac{|-5|}{1} = 5$

b) $r: x - y + z = 0 = 2x + y - z - 3$
$\pi: y - z = 4$

Reta r:

- $x - y + z = 0$
- $2x + y - z = 3$

$(x - y + z) + (2x + y - z) = 3$
$3x = 3 \Rightarrow x = 1$

$1 - y + z = 0 \Rightarrow z = y - 1$

$r: (x, y, z) = (1, \lambda, \lambda - 1)$

Ponto P na reta r: $\lambda = 0$

$P = (1, 0, 0-1) = (1, 0, -1)$

Vetor normal ao plano π:

$n = (0, 1, -1)$

$$d = \frac{|0 \cdot 1 + 1 \cdot 0 + (-1) \cdot (-1) - 4|}{\sqrt{0^2 + 1^2 + (-1)^2}}$$

$$d = \frac{|0 + 0 + 1 - 4|}{\sqrt{1+1}} = \frac{|-3|}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

c) $r: x = y - 1 = z + 3$
$\pi: 2x + y - 3z - 10 = 0$

Reta r:

- $x = t$
- $y = 1 + t$
- $z = -3 + t$

$r: (x, y, z) = (t, 1 + t, -3 + t)$

Ponto P na reta r: $t = 0$

$P = (0, 1 + 0, -3 + 0) = (0, 1, -3)$

Vetor normal ao plano π:

$n = (2, 1, -3)$

$$d = \frac{|2 \cdot 0 + 1 \cdot 1 + (-3)(-3) - 10|}{\sqrt{2^2 + 1^2 + (-3)^2}}$$

$$d = \frac{|1 - 9 - 10|}{\sqrt{14}} = \frac{18}{\sqrt{14}} = \frac{18\sqrt{14}}{14}$$

14)

a) $\pi_1: 2x - y + 2z + 0 = 0$
$\pi_2: 4x - 2y + 4z - 21 = 0$

Plano π_1: $x = 0$

$2(0) - y + 2z = 0 \Rightarrow -y + 2z = 0 \Rightarrow y = 2z$

p/ $z = 1$ e $x = 0$

$y = 2(1) = 2$

Logo: Ponto A no plano π_1:

$$A = (0, 2, 1)$$

Plano π_2: $x = 0$

$4(0) - 2y + 4z - 21 = 0 \Rightarrow -2y + 4z = 21 \Rightarrow y = \frac{21 + 4z}{-2}$

p/ $z = 1$ e $x = 0$

$y = \frac{21 + 4(1)}{-2} = -\frac{25}{2}$

Logo: Ponto B no plano π_2:

$$B = \left(0, -\frac{25}{2}, 1\right)$$

$\vec{AB} = B - A = \left(0, -\frac{25}{2}, 1\right) - (0, 2, 1) = \left(0, -\frac{29}{2}, 0\right)$

$\left(0, -\frac{29}{2}, 0\right) \cdot (2, -1, 2) = \frac{29}{2}$

$\sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{9} = 3$

$d = \frac{\frac{29}{2}}{3} = \frac{29}{2} \cdot \frac{1}{3} = \frac{29}{6}$

b) $\pi_1 : 2x + 2y + 2z = 5$

$\pi_2 : X = (2,1,2) + \lambda(-1,0,3) + \mu(1,1,0)$

Plano π_1 : $x = 0$

$2(0) + 2y + 2z = 5 \rightarrow y = \frac{5-2z}{2}$

p/ $z = 1$ e $x = 0$

$y = \frac{5-2}{2} = \frac{3}{2}$

Logo, ponto A no plano π_1 :

$A = (0, \frac{3}{2}, 1)$

$B = (2,1,2)$

Vetor normal no plano π_2 :

$$\vec{n} = \begin{vmatrix} i & j & k \\ -1 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix}$$

$i(-3) - j(-3) + k(-1) = (-3, 3, -1)$

$\vec{BA} = A - B = (0, \frac{3}{2}, 1) - (2,1,2) = (-2, \frac{1}{2}, -1)$

$(-2, \frac{1}{2}, -1) \cdot (-3, 3, -1) =$

$= (-2)\cdot(-3) + \frac{1}{2} \cdot 3 + (-1)(-1) = 6 + \frac{3}{2} + 1 = \frac{12 + [illegible] +}{2}$

FORONI: $= \frac{17}{2}$

$$\sqrt{(-3)^2 + 3^2 + (-1)^2} = \sqrt{19}$$

$$d = \frac{\frac{17}{2}}{\sqrt{19}} = \frac{17}{2} \cdot \frac{1}{\sqrt{19}} = \frac{17}{2\sqrt{19}} = \frac{34\sqrt{19}}{19}$$

c) $\pi_1 : x + y + z = 0$

$\pi_2 : 2x + y + z + 2 = 0$

Plano π_1: $x = 0$

$y = -z$

p/ $z = 1$ e $x = 0$

$y = -1$

Logo, ponto A no plano π_1:

$A = (0, -1, 1)$

Plano π_2: $x = 0$

$y = -2 - z$

p/ $z = 1$ e $x = 0$

$y = -3$

Logo, ponto B no plano π_2:

$B = (0, -3, 1)$

$\vec{AB} = B - A = (0,-3,1) - (0,-1,1) = (0,-2,0)$

$(0,-2,0) \cdot (1,1,1) = 0 \cdot 1 + (-2) \cdot 1 + 0 \cdot 1 = -2$

$\sqrt{1^2+1^2+1^2} = \sqrt{3}$

$d = \dfrac{|-2|}{\sqrt{3}} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

15) $r: x + z = 5 = y + 4$

$s: X = (4,1,1) + \lambda(4,2,-3)$

$r: \begin{cases} x + z = 5 \\ y + 4 = 5 \end{cases} \quad \begin{matrix} z = 5 - x \\ y = 1 \end{matrix}$

$r: \begin{cases} x = \alpha \\ y = 1 \\ z = 5 - \alpha \end{cases}, \quad \alpha \in \mathbb{R}$

$r: X = (0,1,5) + \alpha(1,0,-1)$

Equação geral de π determinado por r e s:

$$\begin{vmatrix} x & y-1 & z-5 \\ 4 & 2 & -3 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

$-2x - 3(y-1) - 2(z-5) + 4(y-1) = 0$

$-2x + y - 1 - 2z + 10 = 0$

$-2x + y - 2z + 9 = 0$

$\dfrac{|-2x_0 + y_0 - 2z_0 + 9|}{\sqrt{(-2)^2 + 1^2 + (-2)^2}} = 2$

FORON:

$$|-2x_0+y_0-2z_0+9|=6$$
$$-2x_0+y_0-2z_0=\pm 6-9$$

$$\pi_1: -2x+y-2z=-3$$
$$\pi_2: -2x+y-2z=-15$$

FORON