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Lista 7 - GA

1.

(a) $A = (3,5,1)$ $B = (-2,3,2)$

$\vec{AB} = B - A = (-2-3, 3-5, 2-1) = (-5,-2,1)$

$X = A + \lambda \vec{AB}$

$r: X = (3,5,1) + \lambda(-5,-2,1) \; ; \; \lambda \in \mathbb{R}$

(i) Equação paramétrica:

$$(x,y,z) = (3,5,1) + \lambda(-5,-2,1)$$
$$= (3-5\lambda, 5-2\lambda, 1+\lambda)$$

$$r: \begin{cases} x = 3 - 5\lambda \\ y = 5 - 2\lambda \\ z = 1 + \lambda \end{cases}, \quad \lambda \in \mathbb{R}$$

(ii) Equação simétrica:

$$\rightarrow \begin{cases} \lambda = \dfrac{x-3}{-5} \\ \lambda = \dfrac{y-5}{-2} \\ \lambda = \dfrac{z-1}{1} \end{cases} \iff r: \frac{x-3}{-5} = \frac{y-5}{-2} = \frac{z-1}{1}$$

(b) $A = (0,1,0)$ $B = (1,0,0)$

$\vec{AB} = B - A = (1-0, 0-1, 0-0) = (1,-1,0)$

$X = A + \lambda \vec{AB}$

$r: X = (0,1,0) + \lambda(1,-1,0) ; \lambda \in \mathbb{R}$

(i) Equação paramétrica:

$(x,y,z) = (0,1,0) + \lambda(1,-1,0)$
$= (0+\lambda, 1-\lambda, 0+0)$

$r: \begin{cases} x = \lambda \\ y = 1-\lambda \quad , \quad \lambda \in \mathbb{R} \\ z = 0 \end{cases}$

(ii) Equação simétrica:

Vetor diretor $\vec{AB}$ possui elemento nulo, logo a forma não existe.

(c) $A = (0,1,1) \quad B = (0,0,0)$

$\vec{AB} = B - A = (0-0, 0-1, 0-1) = (0,-1,-1)$

$X = A + \lambda \vec{AB}$

$r: X = (0,1,1) + \lambda(0,-1,-1) ; \lambda \in \mathbb{R}$

(i) Equação paramétrica:

$(x,y,z) = (0,1,1) + \lambda(0,-1,-1)$
$= (0+0, 1-\lambda, 1-\lambda)$

$r: \begin{cases} x = 0 \\ y = 1-\lambda \quad , \quad \lambda \in \mathbb{R} \\ z = 1-\lambda \end{cases}$

FORON:

(ii) Equação simétrica:

Vetor diretor $\vec{AB}$ possui elemento nulo, logo a forma não existe.

(d) $A = (3, 2, 1)$ $B = (6, 1, 4)$

$\vec{AB} = B - A = (6-3, 1-2, 4-1) = (3, -1, 3)$

$X = A + \lambda \vec{AB}$

$r: X = (3, 2, 1) + \lambda(3, -1, 3)$; $\lambda \in \mathbb{R}$

(i) Equação paramétrica:

$(x, y, z) = (3, 2, 1) + \lambda(3, -1, 3)$
$= (3 + 3\lambda, 2 + (-\lambda), 1 + 3\lambda)$

$$r: \begin{cases} x = 3 + 3\lambda \\ y = 2 - \lambda \\ z = 1 + 3\lambda \end{cases}, \quad \lambda \in \mathbb{R}$$

(ii) Equação simétrica:

$$\begin{cases} \lambda = \dfrac{x-3}{3} \\ \lambda = \dfrac{y-2}{-1} \\ \lambda = \dfrac{z-1}{3} \end{cases}, \cancel{\lambda \in} \iff r: \frac{x-3}{3} = \frac{y-2}{-1} = \frac{z-1}{3}$$

2.

(a)

$$\begin{cases} x = 1 - \lambda \\ y = \lambda \\ z = 4 + 2\lambda \end{cases} ; \lambda \in \mathbb{R}$$

$$r : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} ; \lambda \in \mathbb{R}$$

$\hookrightarrow (x, y, z) = (1, 0, 4) + \lambda(-1, 1, 2)$

$A = (1, 0, 4) \quad \vec{AB} = (-1, 1, 2)$

$\vec{AB} = B - A \rightarrow (-1, 1, 2) = (x_B, y_B, z_B) - (1, 0, 4)$

$(x_B, y_B, z_B) = (-1, 1, 2) + (1, 0, 4)$

$(x_B, y_B, z_B) = (0, 1, 6)$

Pontos: $A = (1, 0, 4) \quad B = (0, 1, 6)$

Vetores diretores: $\vec{AB} = (-1, 1, 2)$

$\vec{BA} = A - B = (1-0, 0-1, 4-6) = (1, -1, -2)$

(b)

- $P \in r : P = A + \lambda \vec{AB}$

FORON:

$$\begin{cases} 1 = 1 - \lambda & \rightarrow \lambda = 0 \\ 3 = \lambda & \rightarrow \lambda = 3 \\ -3 = 4 + 2\lambda & \rightarrow \lambda = -\frac{7}{2} \end{cases}$$

$\Rightarrow \nexists \; \lambda \in \mathbb{R}$

Conclusão: $P \notin r$ $(\nexists \; \lambda \in \mathbb{R})$

- $Q \in r$: $Q = A + \lambda \vec{AB}$

$$\begin{cases} -3 = 1 - \lambda & \rightarrow -\lambda = -4 \;.(-1) \rightarrow \lambda = 4 \\ 4 = \lambda & \rightarrow \lambda = 4 \\ 12 = 4 + 2\lambda & \rightarrow \lambda = \frac{8}{2} \rightarrow \lambda = 4 \end{cases}$$

Conclusão: $Q \in r$ (e $\lambda = 4$)

c)

$r: X = (1, 4, -7) + \lambda(-1, 1, 2)$

$$\begin{cases} x = 1 - \lambda \\ y = 4 + \lambda \\ z = -7 + 2\lambda \end{cases} ; \lambda \in \mathbb{R}$$

$$r: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} ; \lambda \in \mathbb{R}$$

3. $A = (3, 6, -7)$ $B = (-5, 2, 3)$ $C = (4, -7, -6)$

a)

- Vetor diretor: $AB = B - A = (-5-3, 2-6, 3+7)$
$= (-8, -4, 10)$

FORONI

$$\begin{cases} x = 3 - 8\lambda \\ y = 6 - 4\lambda \\ z = -7 + 10\lambda \end{cases}, \lambda \in \mathbb{R}$$

- Vetor diretor: $BC = C - B = (4+5, -7-2, -6-3)$
$= (9, -9, -9)$

$$\begin{cases} x = -5 + 9\lambda \\ y = 2 - 9\lambda \\ z = 3 - 9\lambda \end{cases}, \lambda \in \mathbb{R}$$

- Vetor diretor: $AC = C - A = (4-3, -7-6, -6+7)$
$= (1, -13, 1)$

$$\begin{cases} x = 4 + \lambda \\ y = -7 - 13\lambda \\ z = -6 + \lambda \end{cases}, \lambda \in \mathbb{R}$$

Se os vetores diretores forem colineares, não formam triângulo.

$\vec{AB} = \lambda \vec{BC} \rightarrow (-8, -4, 10) = \lambda(9, -9, -9)$

$$\begin{cases} \lambda = \dfrac{-8}{9} \\ \lambda = \dfrac{-4}{-9} \\ \lambda = \dfrac{10}{-9} \end{cases} \Rightarrow \nexists \lambda \in \mathbb{R}$$

conclusão: $\vec{AB}$ e $\vec{BC}$ não são colineares

$\vec{BC} = \lambda \vec{AC} \rightarrow (9, -9, -9) = \lambda(1, -13, 1)$

FORONI:

$$\begin{cases} \lambda = 9 \\ \lambda = \frac{-9}{-13} \\ \lambda = -9 \end{cases} \Rightarrow \nexists\ \lambda \in \mathbb{R}$$

Conclusão: $\overrightarrow{AB}$ e $\overrightarrow{BC}$ não são colineares

$\overrightarrow{AC} = \lambda \overrightarrow{AB} \rightarrow (1,-13,1) = \lambda(-8,-4,10)$

$$\begin{cases} \lambda = \frac{1}{-8} \\ \lambda = \frac{-13}{-4} \\ \lambda = \frac{1}{10} \end{cases} \Longrightarrow \nexists\ \lambda \in \mathbb{R}$$

Conclusão: $\overrightarrow{AC}$ e $\overrightarrow{AB}$ não são colineares

Logo, os pontos A, B e C são vértices de um triângulo.

b)

Ponto médio de $\overrightarrow{AB}$: $\left(\frac{3-5}{2}, \frac{6+2}{2}, \frac{-7+3}{2}\right) = (-1,4,-2)$

Vetor diretor $\overrightarrow{AC} = (1,-13,1)$

$r: X = (-1,4,-2) + \lambda(1,-13,1)$

4.

a) $A = (0,1,8) \quad B = (-3,0,9)$

$r: X = (1,2,0) + \lambda(1,1,-3)$

$C = (1+\lambda, 2+\lambda, -3\lambda)$

$\overrightarrow{AB} = B - A = (-3,0,9) - (0,1,8) = (-3,-1,1)$

$\overrightarrow{AC} = C - A = (1+\lambda, 2+\lambda, -3\lambda) - (0,1,8)$
$= (1+\lambda, 1+\lambda, -3\lambda - 8)$

$\overrightarrow{BC} = C - B = (1+\lambda, 2+\lambda, -3\lambda) - (-3,0,9)$
$= (4+\lambda, 2+\lambda, -3\lambda - 9)$

Condição de ortogonalidade:

$\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$

$\overrightarrow{AB} \cdot \overrightarrow{AC} = (-3,-1,1) \cdot (1+\lambda, 1+\lambda, -3\lambda-8)$
$= -3(1+\lambda) - (1+\lambda) + 1(-3\lambda - 8)$
$= -3 - 3\lambda - 1 - \lambda - 3\lambda - 8$
$= -12 - 7\lambda$

Logo, $-12 - 7\lambda = 0 \rightarrow 7\lambda = -12 \rightarrow \lambda = -\frac{12}{7}$

Substituindo no ponto C:

$C = \left(1 - \frac{12}{7}, 2 - \frac{12}{7}, -3\left(-\frac{12}{7}\right)\right)$

$= \left(\frac{7-12}{7}, \frac{14-12}{7}, \frac{36}{7}\right) = \left(-\frac{5}{7}, \frac{2}{7}, \frac{36}{7}\right)$

b)

$X = (1,0,0) + \lambda(1,1,1)$
$C = (1+\lambda, \lambda, \lambda)$

$|CA| = |CB|$

FORON:

- $\vec{CA} = C - A = (1+\lambda, \lambda, \lambda) - (1,1,1)$
$= (\lambda, \lambda-1, \lambda-1)$

$|CA| = \sqrt{(\lambda)^2 + (\lambda-1)^2 + (\lambda-1)^2}$

$= \sqrt{\lambda^2 + (\lambda^2 - 2\lambda + 1) + (\lambda^2 - 2\lambda + 1)}$

$= \sqrt{3\lambda^2 - 4\lambda + 2}$

- $\vec{CB} = C - B = (1+\lambda, \lambda, \lambda) - (0,0,1)$
$= (1+\lambda, \lambda, \lambda-1)$

$|CB| = \sqrt{(1+\lambda)^2 + \lambda^2 + (\lambda-1)^2}$

$= \sqrt{(1+2\lambda+\lambda^2) + \lambda^2 + (\lambda^2 - 2\lambda + 1)}$

$= \sqrt{3\lambda^2 + 2}$

$|CA| = |CB| \rightarrow \sqrt{3\lambda^2 - 4\lambda + 2} = \sqrt{3\lambda^2 + 2}$

$3\lambda^2 - 4\lambda + 2 = 3\lambda^2 + 2$

$3\lambda^2 - 4\lambda + 2 - 3\lambda^2 - 2 = 0 \rightarrow -4\lambda = 0 \rightarrow \lambda = 0$

Substituindo no ponto C:

$C = (1+0, 0, 0) = (1,0,0)$

5.
a) $A = (1,2,0)$ $\vec{u} = (1,1,0)$ $\vec{v} = (2,3,-1)$

$\pi: X = (1,2,0) + \alpha(1,1,0) + \beta(2,3,-1);$
$\alpha, \beta \in \mathbb{R}$

Para obter as equações paramétricas:

$(x, y, z) = (1, 2, 0) + \alpha(1, 1, 0) + \beta(2, 3, -1)$

$$\pi: \begin{cases} x = 1 + \alpha + 2\beta \\ y = 2 + \alpha + 3\beta \quad ; \alpha, \beta \in \mathbb{R} \\ z = -\beta \end{cases}$$

b) $A = (1, 1, 0)$ $B = (1, -1, -1)$ $\vec{v} = (2, 1, 0)$

$\overrightarrow{AB} = B - A = (1, -1, -1) - (1, 1, 0) = (0, -2, -1)$

$\pi: X = (1, 1, 0) + \alpha(0, -2, -1) + \beta(2, 1, 0);$
$\alpha, \beta \in \mathbb{R}$

$(x, y, z) = (1, 1, 0) + \alpha(0, -2, -1) + \beta(2, 1, 0)$

$$\pi: \begin{cases} x = 1 + 2\beta \\ y = 1 - 2\alpha + \beta \quad ; \alpha, \beta \in \mathbb{R} \\ z = -\alpha \end{cases}$$

c) $A = (1, 0, 1)$ $B = (2, 1, -2)$ $C = (1, -1, 0)$

$\overrightarrow{AB} = B - A = (2, 1, -2) - (1, 0, 1) = (1, 1, -3)$

$\overrightarrow{AC} = C - A = (1, -1, 0) - (1, 0, 1) = (0, -1, -1)$

$\pi: X = (1, 0, 1) + \alpha(1, 1, -3) + \beta(0, -1, -1)$

$$\pi: \begin{cases} x = 1 + \alpha \\ y = \alpha - \beta \quad ; \alpha, \beta \in \mathbb{R} \\ z = 1 - 3\alpha - \beta \end{cases}$$

6.

a) $A = (9, -1, 0)$ $\vec{u} = (0,1,0)$ $\vec{v} = (1,1,1)$

Encontrar o vetor normal:

$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= i(1.1 - 0.1) - j(0.1 - 0.1) + k(0.1 - 1.1)$$

$$= i - k = (1, 0, -1)$$

$$\vec{AX} \cdot \vec{n} = 0$$

$$(x-9, y+1, z-0) \cdot (1, 0, -1) = 0$$

$$1(x-9) + 0(y+1) - 1(z-0) = 0$$

$$x - 9 - z = 0 \rightarrow x - z - 9 = 0$$

b) $A = (1,0,1)$ $B = (-1,0,1)$ $C = (2,1,2)$

Encontrar os vetores diretores do plano π:

$$\vec{AB} = B - A = (-1,0,1) - (1,0,1) = (-2,0,0)$$
$$\vec{AC} = C - A = (2,1,2) - (1,0,1) = (1,1,1)$$

Encontrar o vetor normal:

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -2 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$= i(0.1-0.1) - j(-2.1-0.1) + k(-2.1-0.1)$

$= 2j - 2k = (0, 2, -2)$

$\vec{AX} \cdot \vec{n} = 0$

$(x-1, y-0, z-1) \cdot (0, 2, -2) = 0$

$0(x-1) + 2(y-0) - 2(z-1) = 0$

$2y - 2z + 2 = 0 \quad (\div 2) \rightarrow y - z + 1 = 0$

c) $A = (1,1,0)$ $B = (1,-1,-1)$ $\vec{u} = (2,1,0)$

$\vec{AB} = B - A = (1,-1,-1) - (1,1,0) = (0,-2,-1)$

$$\vec{n} = \vec{AB} \times \vec{u} = \begin{vmatrix} i & j & k \\ 0 & -2 & -1 \\ 2 & 1 & 0 \end{vmatrix}$$

$= i(-2.0-(-1).1) - j(0.0-(-1).2) + k(0.1-(-2).2)$

$= i - 2j + 4k = (1,-2,4)$

$\vec{AX} \cdot \vec{n} = 0$

$(x-1, y-1, z-0) \cdot (1,-2,4) = 0$

$1(x-1) - 2(y-1) + 4(z) = 0$

$x - 1 - 2y + 2 + 4z = 0 \rightarrow x - 2y + 4z + 1 = 0$

FORON:

d) $P = (1,-1,1)$

$r: X = (0,2,2) + \lambda(1,1,-1)$

$Q = (0,2,2)$ $\quad \vec{d} = (1,1,-1)$; p/ $\lambda = 1$

$\vec{PQ} = Q - P = (0,2,2) - (1,-1,1) = (-1,3,1)$

$$\vec{n} = \vec{d} \times \vec{PQ} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -1 & 3 & 1 \end{vmatrix}$$

$= i(1 - (-1).3) - j(1 - (-1).(-1)) + k(1.3 - 1.(-1))$

$= 4i + 4k = (4,0,4)$

$\vec{PX} \cdot \vec{n} = 0$

$(x-1, y+1, z-1).(4,0,4) = 0$

$4(x-1) + 0(y+1) + 4(z-1) = 0$

$4x - 4 + 4z - 4 = 0 \rightarrow 4x + 4z - 8 = 0 \ (\div 4)$

$\rightarrow x + z - 2 = 0$

7.

a) $4x + 2y - z + 5 = 0$

$z = 4x + 2y + 5$

$$\pi: \begin{cases} x = \alpha \\ y = \beta \\ z = 5 + 4\alpha + 2\beta \end{cases} ; \alpha, \beta \in \mathbb{R}$$

b) $5x - y - 1 = 0$

$y = 5x - 1$

$$\pi : \begin{cases} x = \alpha \\ y = -1 + 5\alpha \\ z = z \end{cases} ; \alpha \in \mathbb{R}$$

c) $z - 3 = 0$

$z = 3$

$$\pi : \begin{cases} x = x \\ y = y \\ z = 3 \end{cases}$$

d) $y - z - 2 = 0$

$y = z + 2$

$$\pi : \begin{cases} x = x \\ y = 2 + \alpha \\ z = \alpha \end{cases} ; \alpha \in \mathbb{R}$$

8.

a) $\begin{cases} x = 1 + \lambda - \mu \\ y = 2\lambda + \mu \\ z = 3 - \mu \end{cases}$

$x = 1 + \frac{y - \mu}{2} - \mu \; (.2)$

$\lambda = \frac{y - \mu}{2}$

$2x = 2 + y - \mu - 2\mu$

FORONI:

$$x = \frac{y - 3\mu + 2}{2} \rightarrow x = \frac{y}{2} - \frac{3\mu}{2} + 1$$

$$\mu = 3 - z$$

$$x = \frac{y}{2} - \frac{3(3-z)}{2} + 1 \rightarrow x = \frac{y}{2} - \frac{9-3z}{2} + 1$$

$$x = \frac{y}{2} - \frac{9}{2} + \frac{3z}{2} + 1 \rightarrow x + \frac{1}{2}y + \frac{3}{2}z + \frac{11}{2} = 0 \quad (\cdot 2)$$

$$\rightarrow 2x + y + 3z + 11 = 0$$

b) $\begin{cases} x = 1 + \lambda \\ y = 2 \\ z = 3 - \lambda + \mu \end{cases}$

$\lambda = x - 1$

$$z = 3 - (x-1) + \mu$$

$$z = 3 - x + 1 + \mu$$

$$z = -x + \mu + 4$$

$$z + x = \mu + 4$$

$$x + z - 4 = 0$$

c) $\begin{cases} x = -2 + \lambda - \mu \\ y = 2\lambda + 2\mu \\ z = \lambda + \mu \end{cases}$

$\lambda = x + 2 - \mu \qquad \mu = \frac{y}{2} - x - 1$

$$z = x + 2 - \mu + \frac{y}{2} - x - 1$$

$$z = x + 2 - \cancel{\frac{y}{2}} + \cancel{x} + \cancel{1} + \frac{y}{2} - \cancel{x} - \cancel{1}$$

$z = x + 2 \rightarrow x - z + 2 = 0$

9.

a) $r: \begin{cases} x = 1 + 2\lambda \\ y = 1 \\ z = 1 + 3\lambda \end{cases}$

$s: \begin{cases} x = -1 + 4\mu \\ y = -1 + 2\mu \\ z = -2 + 6\mu \end{cases}$

Reescrevendo as retas:

$r: X = (1,1,1) + \lambda(2,0,3)$

$s: X = (-1,-1,-2) + \mu(4,2,6)$

Vetores diretores:

$\vec{u} = (2,0,3)$ e $\vec{v} = (4,2,6)$, $\{\vec{u},\vec{v}\}$ é L.I.

Encontrar vetor $\vec{AB}$ com os pontos:

$A = (1,1,1)$ e $B = (-1,-1,-2)$

$\vec{AB} = B - A = (-1,-1,-2) - (1,1,1) = (-2,-2,-3)$

$$\vec{AB}\cdot(\vec{u}\times\vec{v}) = \begin{vmatrix} -2 & -2 & -3 \\ 2 & 0 & 3 \\ 4 & 2 & 6 \end{vmatrix}$$

$$= 2\cdot(-1)^{2+1}\begin{vmatrix} -2 & 3 \\ 2 & 6 \end{vmatrix} + 3\cdot(-1)^{2+3}\begin{vmatrix} -2 & -2 \\ 4 & 2 \end{vmatrix}$$

$= -2(-2 \cdot 6 - (-3) \cdot 2) - 3(-2 \cdot 2 - (-2) \cdot 4)$

$= -2 \cdot (-6) - 3(4) = 12 - 12 = 0$ (L.D.)

Conclusão: r e s são concorrentes.

Interseção: $r \cap s = P$

$$\begin{cases} X = (1,1,1) + \lambda(2,0,3) \\ X = (-1,-1,-2) + \mu(4,2,6) \end{cases}$$

$(1,1,1) + \lambda(2,0,3) = (-1,-1,-2) + \mu(4,2,6)$

$(1+2\lambda, 1, 1+3\lambda) = (-1+4\mu, -1+2\mu, -2+6\mu)$

$$\begin{cases} 1 + 2\lambda = -1 + 4\mu \\ 1 = -1 + 2\mu \quad \rightarrow \quad \mu = 1 \\ 1 + 3\lambda = -2 + 6\mu \end{cases}$$

$1 + 3\lambda = -2 + 6(1) \iff 3\lambda = 3 \iff \lambda = 1$

Conclusão: $P = (1,1,1) + 1(2,0,3) = (3,1,4)$

Equação do plano:

$\overrightarrow{PX} \cdot (\vec{u} \times \vec{v}) = 0$

$$\begin{vmatrix} x-3 & y-1 & z-4 \\ 2 & 0 & 3 \\ 4 & 2 & 6 \end{vmatrix} = 0$$

$2 \cdot (-1)^{2+1} \begin{vmatrix} y-1 & z-4 \\ 2 & 6 \end{vmatrix} + 3 \cdot (-1)^{2+3} \begin{vmatrix} x-3 & y-1 \\ 4 & 2 \end{vmatrix}$

$-2((y-1)6-(z-4).2)-3((x-3).2-(y-1).1)$ –

$-2(6y-6-2z+8)-3(2x-6-4y+4)=0$

$-12y+\cancel{12}+4z-16-6x+18+12y-\cancel{12}=0$

$\boxed{-6x+4z+2=0}$

b)

$r: X=(1,1,0)+\lambda(1,2,3)$

$s: X=(2,3,3)+\mu(3,2,1)$

Vetores diretores:

$\vec{u}=(1,2,3)$ e $\vec{v}=(3,2,1)$, $\{\vec{u},\vec{v}\}$ é L.I.

$A=(1,1,0)$ e $B=(2,3,3)$

$\vec{AB}=B-A=(2,3,3)-(1,1,0)=(1,2,3)$

$\vec{AB}\cdot(\vec{u}\times\vec{v})=\begin{vmatrix}1&2&3\\1&2&3\\3&2&1\end{vmatrix}$ $l_1 \leftarrow l_1-l_2$

$\begin{vmatrix}0&0&0\\1&2&3\\3&2&1\end{vmatrix}=0$ (L.D.)

Conclusão: r e s são concorrentes.

Interseção: $r \cap s = P$

FORON:

$\begin{cases}X=(1,1,0)+\lambda(1,2,3)\\X=(2,3,3)+\mu(3,2,1)\end{cases}$

$(1,1,0) + \lambda(1,2,3) = (2,3,3) + \mu(3,2,1)$

$(1+\lambda, 1+2\lambda, 3\lambda) = (2+3\mu, 3+2\mu, 3+\mu)$

$$\begin{cases} 1+\lambda = 2+3\mu \rightarrow \lambda = 1+3\mu \\ 1+2\lambda = 3+2\mu \\ 3\lambda = 3+\mu \end{cases}$$

$1 + 2(1+3\mu) = 3+2\mu \rightarrow 1+2+6\mu = 3+2\mu$

$6\mu - 2\mu = 3-3 \rightarrow \mu = 0$

$3\lambda = 3+\mu \rightarrow \lambda = 1$

<u>Conclusão</u>: $P = (1,1,0) + 1(1,2,3)$
$= (2,3,3)$

Equação do plano: $\vec{PX} \cdot (\vec{u} \times \vec{v}) = 0$

$$\begin{vmatrix} x-2 & y-3 & z-3 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0 \quad l_2 \leftarrow l_2 - l_3$$

$$\begin{vmatrix} x-2 & y-3 & z-3 \\ -2 & 0 & 2 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$$-2\cdot(-1)^{2+1} \begin{vmatrix} y-3 & z-3 \\ 2 & 1 \end{vmatrix} + 2\cdot(-1)^{2+3} \begin{vmatrix} x-2 & y-3 \\ 3 & 2 \end{vmatrix}$$

$2\cdot(y-3-(z-3)\cdot 2) - 2\cdot((x-2)\cdot 2 - (y-3)\cdot 3) = 0$

$2\cdot(y-3-2z+6) - 2\cdot(2x-4-3y+9) = 0$

$2y - 6 - 4z + 12 - 4x + 8 + 6y - 18 = 0$

FORONI

$-4x + 8y - 4z - 4 = 0 \quad (\div -4)$

$$\boxed{x - 2y + z + 1 = 0}$$

c)

$$r: \begin{cases} x = 2 - 4\lambda \\ y = 4 + 5\lambda \\ z = 11 \end{cases}$$

$$s: \frac{x}{2} = \frac{y-1}{-2} = z$$

Reescrevendo as retas:

$r: X = (2, 4, 11) + \lambda(-4, 5, 0)$

$s:$ p/ $z = \mu$ $\quad \frac{x}{2} = \mu \rightarrow x = 2\mu$

$\frac{y-1}{-2} = \mu \rightarrow y = 1 - 2\mu$

$z = \mu$

$$s: \begin{cases} x = 2\mu \\ y = 1 - 2\mu \\ z = \mu \end{cases} \rightarrow s: X = (0, 1, 0) + \mu(2, -2, 1)$$

Vetores diretores:

$\vec{u} = (-4, 5, 0)$ e $\vec{v} = (2, -2, 1)$, $\{\vec{u}, \vec{v}\}$ é L.I.

$A = (2, 4, 11)$ e $B = (0, 1, 0)$

FORONI: $\vec{AB} = B - A = (0, 1, 0) - (2, 4, 11) = (-2, \ldots$

$$\vec{AB} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} -2 & -3 & 11 \\ -4 & 5 & 0 \\ 2 & -2 & 1 \end{vmatrix}$$

$$-4 \cdot (-1)^{2+1} \begin{vmatrix} -3 & -11 \\ -2 & 1 \end{vmatrix} + 5 \cdot (-1)^{2+2} \begin{vmatrix} -2 & 11 \\ 2 & 1 \end{vmatrix}$$

$$4(-3 \cdot 1 - (-11) \cdot (-2)) + 5(-2 \cdot 1 - (-11) \cdot 2)$$
$$4(-3 - 22) + 5(-2 + 22) = 0$$

Conclusão: r e s são concorrentes.

Interseção: $r \cap s = P$

$$\begin{cases} X = (2, 4, 11) + \lambda(-4, 5, 0) \\ X = (0, 1, 0) + \mu(2, -2, 1) \end{cases}$$

$$(2, 4, 11) + \lambda(-4, 5, 0) = (0, 1, 0) + \mu(2, -2, 1)$$

$$(2 - 4\lambda, 4 + 5\lambda, 11) = (2\mu, 1 - 2\mu, \mu)$$

$$\begin{cases} 2 - 4\lambda = 2\mu \\ 4 + 5\lambda = 1 - 2\mu \\ 11 = \mu \end{cases}$$

Conclusão: $P = (0, 1, 0) + 11(2, -2, 1)$
$= (22, -21, 11)$

Equação do plano: $\vec{PX} \cdot (\vec{u} \times \vec{v}) = 0$

$$\begin{vmatrix} x - 22 & y + 21 & z - 11 \\ -4 & 5 & 0 \\ 2 & -2 & 1 \end{vmatrix} = 0$$

$$-4\cdot(-1)^{2+1}\begin{vmatrix} y+21 & z-11 \\ -2 & 1 \end{vmatrix} + 5\cdot(-1)^{2+2}\begin{vmatrix} x-22 & z-11 \\ 2 & 1 \end{vmatrix}$$

$$4\,(y+21-(z-11)\cdot(-2)) + 5\,(x-22-(z-11)\cdot 2)$$

$$4\,(y+21+2z-22) + 5\,(x-\cancel{22}-2z+\cancel{22})$$

$$4\,(y+2z-1) + 5\,(x-2z) = 0$$

$$4y+8z-4+5x-10z=0$$

$$\boxed{5x+4y-2z-4=0}$$

d)

$$r:\ \frac{x-2}{3} = \frac{y+2}{4} = z$$

$$s:\ \frac{x}{4} = \frac{y}{2} = \frac{z-3}{2}$$

Reescrevendo as retas:

r: p/ $z = \mu$

$$\frac{x-2}{3} = \mu \rightarrow x = 2+3\mu$$

$$\frac{y+2}{4} = \mu \rightarrow y = -2+4\mu$$

$$z = \mu$$

$$r: \begin{cases} x = 2+3\mu \\ y = -2+4\mu \\ z = \mu \end{cases} \rightarrow \quad r: X = (2,-2,0) + \mu(3,4,1)$$

s: p/ $z = \lambda$

$$\frac{x}{4} = \frac{\lambda-3}{2} \rightarrow x = \frac{4\lambda-12}{2}$$

$$x = 2\lambda - 6$$

$$\frac{y}{2} = \frac{\lambda-3}{2} \rightarrow y = \lambda - 3$$

$$z = \lambda$$

$$s: \begin{cases} x = -6 + 2\lambda \\ y = -3 + \lambda \\ z = \lambda \end{cases} \rightarrow \quad s: X = (-6,-3,0) + \lambda(2,1,1)$$

Vetores diretores:

$\vec{u} = (3,4,1)$ e $\vec{v} = (2,1,1)$; $\{\vec{u},\vec{v}\}$ é L.I.

$A = (2,-2,0)$ e $B = (-6,-3,0)$

$\vec{AB} = B - A = (-6,-3,0) - (2,-2,0) = (-8,-1,0)$

$$\vec{AB}\cdot(\vec{u}\times\vec{v}) = \begin{vmatrix} -8 & -1 & 0 \\ 3 & 4 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$-8(-1)^{1+1}\begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} - 1\cdot(-1)^{1+2}\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix}$$

$$-8(4-1) + 1(3-2) = -23 \neq 0$$

Conclusão: r e s são reversas

10.

a)

$$r: \begin{cases} x + 2y + 3z - 1 = 0 \\ x - y + 2z = 0 \end{cases} \rightarrow x = y - 2z$$

$$(y - 2z) + 2y + 3z - 1 = 0$$
$$3y + z - 1 = 0 \rightarrow z = 1 - 3y$$

$$x = y - 2(1-3y) = y - 2 + 6y = 7y - 2$$

$(x, y, z) = (7y - 2, y, 1 - 3y)$

p / $y = 0$

$(x, y, z) = (7(0) - 2, 0, 1 - 3(0)) = (-2, 0, 1)$

Ponto A da reta: $A = (-2, 0, 1)$

Vetores normais:

$\vec{n}_1 = (1, 2, 3)$ e $\vec{n}_2 = (1, -1, 2)$

Vetor diretor:

$$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & -1 & 2 \end{vmatrix}$$

$\vec{d} = i(2.2 - 3.(-1)) - j(1.2 - 3.1) + k(1.(-1) - 2.1)$

$\vec{d} = i(4 + 3) - j(2 - 3) + k(-1 - 2)$

$\vec{d} = 7i + j - 3k \rightarrow \vec{d} = (7, 1, -3)$

Logo:

$r: X = (-2, 0, 1) + \alpha(7, 1, -3)$

$$r: \begin{cases} x = -2 + 7\alpha \\ y = \alpha \\ z = 1 - 3\alpha \end{cases}$$

FORON:

1-)

$$r: \begin{cases} x + y + z - 1 = 0 \\ x + y - z = 0 \rightarrow x = -y + z \end{cases}$$

$(-y + z) + y + z - 1 = 0 \rightarrow z = \frac{1}{2}$

$x = \frac{1}{2} - y$

p/ $y = 0$

$x = \frac{1}{2} - y \rightarrow x = \frac{1}{2} - 0 = \frac{1}{2}$

Ponto A da reta: $A = (\frac{1}{2}, 0, \frac{1}{2})$

$\vec{n}_1 = (1, 1, 1)$ e $\vec{n}_2 = (1, 1, -1)$

$$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$\vec{d} = i(1.(-1) - 1.1) - j(1.(-1) - 1.1) + k(1.1 - 1.1)$

$\vec{d} = i(-1-1) - j(-1-1) + k(1-1)$

$\vec{d} = -2i + 2j \rightarrow \vec{d} = (-2, 2, 0)$

Logo:

$r: X = (\frac{1}{2}, 0, \frac{1}{2}) + \alpha(-2, 2, 0)$

$$r: \begin{cases} x = \frac{1}{2} - 2\alpha \\ y = 2\alpha \\ z = \frac{1}{2} \end{cases}$$

FORONI

c)

$r: \begin{cases} x = 3 \\ 2x - z + 1 = 0 \end{cases}$

$2(3) - z + 1 = 0 \to -z = -1 - 6 \to z = 7$

p/ $y = 0$

Ponto A da reta: $A = (3, 0, 7)$

$\vec{u} = (0, 1, 0)$

Logo:

$r: X = (3, 0, 7) + \alpha(0, 1, 0)$

$r: \begin{cases} x = 3 \\ y = \alpha \\ z = 7 \end{cases}$

d)

$r: \begin{cases} y = 2 \\ z = 0 \end{cases}$

p/ $y = 0$

Ponto A da reta: $A = (2, 0, 0)$

$\vec{u} = (0, 1, 0)$

Logo:

FORONI: $r: X = (2, 0, 0) + \alpha(0, 1, 0)$

$r: \begin{cases} x = 2 \\ y = \alpha \\ z = 0 \end{cases}$

11.

a) $r: X = (1,-1,1) + \lambda(-2,1,-1)$

$s: \begin{cases} y + z = 3 \\ x + y - z = 6 \end{cases}$

$s: \ y = 3 - z \qquad x = 6 - y + z$

$x = 6 - (3 - z) + z$

$x = 3 + 2z$

p/ $z = 0 \qquad y = 3 \qquad x = 3$

Ponto na reta s: $B = (3, 3, 0)$

p/ $z = \alpha$

$\begin{cases} x = 3 + 2\alpha \\ y = 3 - \alpha \\ z = \alpha \end{cases} \rightarrow s: X = (3,3,0) + \alpha(2,-1,1)$

$\vec{u} = (-2, 1, -1)$ e $\vec{v} = (2,-1,1)$

$\{\vec{u}, \vec{v}\}$ é L.D., pois $(-2,1,-1) = -1(2,-1,1)$, sendo $\lambda = -1$

Logo, r e s são paralelas

$A = (1,-1,1) \in r$, conferir se $A \in s$:

$(1,-1,1) = (3,3,0) + \alpha(2,-1,1)$

$(1,-1,1) - (3,3,0) = \alpha(2,-1,1)$

$(-2,-4,1) = \alpha(2,-1,1)$

$\nexists \ \alpha \in \mathbb{R}$

Conclusão: $r \neq s$ (distintas)

b) $r: \frac{x+1}{2} = \frac{y}{3} = \frac{z+1}{2}$

$s: X = (0,0,0) + \lambda(1,2,0)$

r: Usando parâmetro π:

$\frac{x+1}{2} = \pi \rightarrow x = -1 + 2\pi$

$\frac{y}{3} = \pi \rightarrow y = 3\pi$

$\frac{z+1}{2} = \pi \rightarrow z = -1 + 2\pi$

Logo: $X = (-1, 0, -1) + \pi(2, 3, 2)$

$\vec{u} = (2,3,2)$ e $\vec{v} = (1,2,0)$

$\{\vec{u}, \vec{v}\}$ é L.I., pois $\nexists\ \lambda \in \mathbb{R}$ tal que
$\vec{u} = \lambda \vec{v} \iff (2,3,2) = \lambda(1,2,0)$

Então, r e s não são paralelas.

$A = (-1,0,-1)$ e $B = (0,0,0)$

$\vec{AB} = B - A = (0,0,0) - (-1,0,-1) = (1,0,1)$

$$\vec{AB} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 0 \end{vmatrix}$$

$$= 1 \cdot (-1)^{1+1} \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} + 1 \cdot (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix}$$

$1(0-4)+1(4-3) = -3 \neq 0$

Conclusão: r e s são reversas

c) $r: X = (8,1,9) + \lambda(2,-1,3)$

$s: X = (3,-4,4) + \mu(1,-2,2)$

$\vec{u} = (2,-1,3)$ e $\vec{v} = (1,-2,2)$

$\{\vec{u}, \vec{v}\}$ é LI, pois $\nexists\ \lambda \in \mathbb{R}$ tal que $\vec{u} = \lambda \vec{v} \Longleftrightarrow (2,-1,3) = \lambda(1,-2,2)$

Logo, r e s não são paralelas

$A = (8,1,9)$ e $B = (3,-4,4)$

$\vec{AB} = B - A = (3,-4,4) - (8,1,9) = (-5,-5,-5)$

$$\vec{AB} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} -5 & -5 & -5 \\ 2 & -1 & 3 \\ 1 & -2 & 2 \end{vmatrix}$$

$$1 \cdot (-1)^{3+1} \begin{vmatrix} -5 & -5 \\ -1 & 3 \end{vmatrix} - 2 \cdot (-1)^{3+2} \begin{vmatrix} -5 & -5 \\ 2 & 3 \end{vmatrix} + 2 \cdot (-1)^{3+3} \begin{vmatrix} -5 & -5 \\ 2 & -1 \end{vmatrix}$$

$1 \cdot (-5 \cdot 3 - (-5) \cdot (-1)) + 2(-5 \cdot 3 - (-5) \cdot 2) + 2(-5 - \ldots)$

$(-15 - 5) + 2(-15 + 10) + 2(-5 + 10)$

$-20 - 30 + 20 - 10 + 20 = -20 \neq 0$

Conclusão: r e s são reversas.

d) $r: \frac{x+1}{2} = y = z$

$s: \begin{cases} x + y - 3z = 1 \\ 2x - y - 2z = 0 \end{cases}$

r: usando parâmetro π:

$\frac{x+1}{2} = \pi \rightarrow x = -1 + 2\pi$

$y = \pi$

$z = \pi$

Logo: $x = (-1, 0, 0) + \pi(2, 1, 1)$

s: $y = 1 + 3z - x$

$2x - (1 + 3z - x) - 2z = 0$

$3x - 5z = 1 \rightarrow x = \frac{1 + 5z}{3}$

$y = 1 + 3z - \frac{1 + 5z}{3} \rightarrow y = 1 - \frac{1}{3} + 3z - \frac{5z}{3}$

$y = \frac{3-1}{3} + \frac{9z - 5z}{3} \rightarrow y = \frac{2}{3} + \frac{4z}{3}$

Logo: $x = \left(\frac{1}{3}, \frac{2}{3}, 0\right) + z\left(\frac{5}{3}, \frac{4}{3}, 1\right)$

$\vec{u} = (2, 1, 1)$ e $\vec{v} = \left(\frac{5}{3}, \frac{4}{3}, 1\right)$

$\{\vec{u}, \vec{v}\}$ é L I, pois $\nexists\ \lambda \in \mathbb{R}$ tal que
$\vec{u} = \lambda \vec{v} \Leftrightarrow (2, 1, 1) = \lambda\left(\frac{5}{3}, \frac{4}{3}, 1\right)$

Logo, r e s não são paralelos

$A = (-1, 0, 0)$ e $B = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$

$\overrightarrow{AB} = B - A = \left(\frac{1}{3}, \frac{2}{3}, 0\right) - (-1, 0, 0) = \left(\frac{4}{3}, \frac{2}{3}, 0\right)$

$$\overrightarrow{AB} \cdot (\vec{u} \times \vec{v}) = \begin{vmatrix} \frac{4}{3} & \frac{2}{3} & 0 \\ 2 & 1 & 1 \\ \frac{5}{3} & \frac{4}{3} & 1 \end{vmatrix}$$

$$= 2(-1)^{2+1} \begin{vmatrix} \frac{2}{3} & 0 \\ \frac{4}{3} & 1 \end{vmatrix} + 1 \cdot (-1)^{2+2} \begin{vmatrix} \frac{4}{3} & 0 \\ \frac{5}{3} & 1 \end{vmatrix} + 1 \cdot (-1)^{2+3} \begin{vmatrix} \frac{4}{3} & \frac{2}{3} \\ \frac{5}{3} & \frac{4}{3} \end{vmatrix}$$

$$= -2\left(\frac{2}{3}\right) + 1\left(\frac{4}{3}\right) - 1\left(\frac{4}{3} \cdot \frac{4}{3} - \frac{2}{3} \cdot \frac{5}{3}\right)$$

$$= \cancel{-\frac{4}{3}} + \cancel{\frac{4}{3}} - 1\left(\frac{16}{9} - \frac{10}{9}\right) = -\frac{6}{9} = -\frac{2}{3} \neq 0$$

Conclusão: r e s são reversas.

12.

a) $r: X = (1, 1, 0) + \lambda(0, 1, 1)$
$\pi: x - y - z = 2$

Vetor normal do plano π: $\vec{n} = (1, -1, -1)$
Vetor diretor da reta r: $\vec{r} = (0, 1, 1)$

$\vec{r} \cdot \vec{n} = 0 \cdot 1 + 1 \cdot (-1) + 1 \cdot (-1) = 0 - 1 - 1 = -2 \neq 0$

Logo, r e π são transversais (concorrentes)

$x = 1, \ y = 1 + \lambda, \ z = \lambda$

$1-(1+\lambda)-\lambda=2 \rightarrow 1-1-\lambda-\lambda=2$

$-2\lambda=2 \rightarrow \lambda=-1$

$P=(1,1,0)+(-1)(0,1,1)=(1,0,-1)$

b) $r: \frac{x-1}{2}=y=z$

$\pi: X=(3,0,1)+\lambda(1,0,1)+\mu(2,2,0)$

r: Usando parâmetro π:

$\frac{x-1}{2}=\pi \rightarrow x=1+2\pi$

$y=\pi$ Logo: $X=(1,0,0)+\pi(2,1,1)$

$z=\pi$

$\vec{r}=(2,1,1)$

2: Equação geral de π:

$\vec{AX}\cdot(\vec{AB}\times\vec{AC})=0$

$$\begin{vmatrix} x-3 & y-0 & z-1 \\ 1 & 0 & 1 \\ 2 & 2 & 0 \end{vmatrix}=0$$

$\Leftrightarrow 1\cdot(-1)^{2+1}\begin{vmatrix} y & z-1 \\ 2 & 0 \end{vmatrix}+1\cdot(-1)^{2+3}\begin{vmatrix} x-3 & y-0 \\ 2 & 2 \end{vmatrix}=0$

$-1(-(z-1)\cdot2)-1((x-3)\cdot2-y\cdot2)=0$

$-1(-2z+2)-1(2x-6-2y)=0$

$2z-2-2x+6+2y=0$

$-2x+2y+2z+3=0 \quad \vec{n}=(-2,2,2)$

FORON:

$\vec{u} \cdot \vec{n} = 2 \cdot (-2) + 1 \cdot 2 + 1 \cdot 2 = -4 + 2 + 2 = 0$

Logo, r e π são paralelos.

c) $r: \begin{cases} x - y = 1 \\ x - 2y = 0 \end{cases}$

$\pi: x + y = 2$

r: Usando parâmetro $t = y$:

$\begin{cases} x = 1 + t \\ y = t \\ z = z \end{cases} \rightarrow x = (1,0,0) + t(1,1,0)$

$\vec{u} = (1,1,0)$

$\vec{n} = (1,1,0)$

$\vec{u} \cdot \vec{n} = 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0 = 2 \neq 0$

Logo, r e π são transversais (concorrentes)

$x = 1 + t, \ y = t, \ z = z$

$1 + t + t = 2 \rightarrow 2t = 1 \rightarrow t = \frac{1}{2}$

$P = (1,0,0) + \frac{1}{2}(1,1,0) = \left(\frac{3}{2}, \frac{1}{2}, 0\right)$

d) $r: x - 2y = 3 - 2z + y = 2x - z$

$\pi: x = (1,4,0) + \lambda(1,1,1) + \mu(2,1,0)$

r: Usando parâmetro t:

FORON:

$$\begin{cases} x - 2y = \pi \\ 3 - 2z + y = \pi \\ 2x - z = \pi \end{cases} \qquad \begin{aligned} x &= \pi + 2y \\ y &= \pi - 3 + 2z \end{aligned}$$

$$x = \pi + 2(\pi - 3 + 2z) \rightarrow x = \pi + 2\pi - 6 + 4z$$
$$x = 3\pi - 6 + 4z$$

$$2(3\pi - 6 + 4z) - z = \pi \rightarrow 6\pi - 12 + 8z - z = \pi$$
$$6\pi - 12 + 7z = \pi \rightarrow 6\pi - \pi = 12 - 7z$$
$$5\pi = 12 - 7z \rightarrow 7z = 12 - 5\pi$$
$$z = \frac{12 - 5\pi}{7}$$

$$y = \pi - 3 + 2\left(\frac{12 - 5\pi}{7}\right) \rightarrow y = \pi - 3 + \frac{24 - 10\pi}{7}$$
$$y = \frac{7\pi - 21 + 24 - 10\pi}{7} \rightarrow y = \frac{3\pi + 3}{7}$$

$$x = 3\pi - 6 + 4\left(\frac{12 - 5\pi}{7}\right) \rightarrow x = 3\pi - 6 + \frac{48 - 20\pi}{7}$$
$$x = \frac{21\pi - 42 + 48 - 20\pi}{7} \rightarrow x = \frac{\pi + 6}{7}$$

$$\begin{cases} x = \frac{\pi + 6}{7} \\ y = \frac{3\pi + 3}{7} \\ z = \frac{12 - 5\pi}{7} \end{cases} \rightarrow X = \left(\frac{6}{7}, \frac{3}{7}, \frac{12}{7}\right) + \pi\left(\frac{1}{7}, \frac{3}{7}, -\frac{5}{7}\right)$$

$$\vec{u} = \left(\frac{1}{7}, \frac{3}{7}, -\frac{5}{7}\right)$$

2: Equação geral de π:

$$\vec{AX} \cdot (\vec{AB} \times \vec{AC}) = 0$$

FORONI

$$\begin{vmatrix} x-1 & y-4 & z \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = 0$$

$$\Leftrightarrow 2.(-1)^{3+1} \begin{vmatrix} y-4 & z \\ 1 & 0 \end{vmatrix} + 1.(-1)^{3+2} \begin{vmatrix} x-1 & z \\ 1 & 1 \end{vmatrix} = 0$$

$2.(-z) - 1(x-1-z) = 0$

$-2z - x + 1 + z = 0 \rightarrow \boxed{-x - z + 1 = 0}$

$\vec{n} = (-1, 0, -1)$

$\vec{u} . \vec{n} = \frac{1}{7}.(-1) + \frac{3}{7}.0 + (-\frac{5}{7}).(-1) = -\frac{1}{7} + \frac{5}{7} = \frac{4}{7} \neq 0$

Logo, r e π são transversais (concorrentes)

$x = \frac{\lambda+6}{7}, \; y = \frac{3\lambda+3}{7}, \; z = \frac{12-5\lambda}{7}$

$-\left(\frac{\lambda+6}{7}\right) - \left(\frac{12-5\lambda}{7}\right) + 1 = 0$

$\frac{-\lambda - 6 - 12 + 5\lambda}{7} + 1 = 0 \rightarrow \frac{4\lambda - 18 + 7}{7} = 0$

$4\lambda - 11 = 7.0 \rightarrow \lambda = \frac{11}{4}$

$P = \left(\frac{6}{7}, \frac{3}{7}, \frac{12}{7}\right) + \frac{11}{4}\left(\frac{1}{7}, \frac{3}{7}, -\frac{5}{7}\right) = \left(\frac{5}{4}, \frac{45}{28}, -\frac{1}{4}\right)$

13.

a) $r: X = (1,1,1) + \lambda(2, m, 1)$

$\pi: X = (0,0,0) + \alpha(1,2,0) + \beta(1,0,1)$

FORONI

$\vec{r} = (2, m, 1)$ $\vec{n} = (1, 2, 0) \times (1, 0, 1)$

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

$\vec{n} = i(2.1) - j(1.1) + k(-2.1)$
$\vec{n} = i(2) - j(1) + k(-2)$

$\vec{n} = (2, -1, -2)$

Para r e π serem paralelas: $\vec{r} \cdot \vec{n} = 0$

$(2, m, 1) \cdot (2, -1, -2) = 2 \cdot 2 + m \cdot (-1) + 1 \cdot (-2)$
$4 - m - 2 = 0 \rightarrow 2 - m = 0 \rightarrow m = 2$

Conclusão: Para que r e π sejam paralelas,
$\boxed{m = 2}$

e) $r: X = (n, 2, 0) + \lambda(2, m, m)$
$\pi: x - 3y + z = 1$

$\vec{r} = (2, m, m)$ $\vec{n} = (1, -3, 1)$

$\vec{r} \cdot \vec{n} = 0$

$(2, m, m) \cdot (1, -3, 1) = 2 \cdot 1 + m \cdot (-3) + m \cdot 1$
$2 - 3m + m = 0 \rightarrow 2 - 2m = 0 \rightarrow 2 = 2m$
$m = 1$

Substituindo o ponto $(n, 2, 0)$ da reta r na equação do plano π:

$n - 3(2) + 0 = 1 \rightarrow n - 6 = 1 \rightarrow n = 7$

Conclusão: Para que r esteja contida em π, $\boxed{m = 1}$ e $\boxed{n = 7}$

c)

$r: \dfrac{x-1}{m} = \dfrac{y}{2} = \dfrac{z}{m}$

$\pi: x + my + z = 0$

r: Usando parâmetro λ:

$x - 1 = m\lambda \Rightarrow x = m\lambda + 1$

$y = 2\lambda$

$z = m\lambda$

Logo: $X = (1, 0, 0) + \lambda(m, 2, m)$

$\vec{d} = (m, 2, m)$ $\vec{n} = (1, m, 1)$

$\vec{d} \cdot \vec{n} \neq 0$

$m \cdot 1 + 2 \cdot m + m \cdot 1 \neq 0$

$4m \neq 0 \rightarrow m \neq 0$

Conclusão: a reta r é transversal ao plano π para todos os valores de m diferentes de zero.

14.

a) $\pi: X = (4, 2, 4) + \lambda(1, 1, 2) + \mu(3, 3, 1)$

$\sim: X = (3, 0, 0) + \lambda(1, 1, 0) + \mu(0, 1, 4)$

Equação geral de π_1:

$\vec{AX} \cdot (\vec{AB} \times \vec{AC}) = 0$

$$\begin{vmatrix} x-4 & y-2 & z-4 \\ 1 & 1 & 2 \\ 3 & 3 & 1 \end{vmatrix} = 0$$

$$\Leftrightarrow 1 \cdot (-1)^{2+1} \begin{vmatrix} y-2 & z-4 \\ 3 & 1 \end{vmatrix} + 1 \cdot (-1)^{2+2} \begin{vmatrix} x-4 & z-4 \\ 3 & 1 \end{vmatrix}$$
$$+ 2 \cdot (-1)^{2+3} \begin{vmatrix} x-4 & y-2 \\ 3 & 3 \end{vmatrix} = 0$$

$-1(y-2-3z+12) + 1(x-4-3z+12) - 2((x-4)3$
$-(y-2).3) = 0$

$-y+2+3z-12+x-4-3z+12-6x+24+6y-6$
$= 0$

$x-6x-y+6y+3z-3z+2-12-4+12+24-6=0$
$-5x+5y+16=0 \quad \vec{n}_1 = (-5,5,0)$

Equação geral de π_2:

$\vec{AX} (\vec{AB} \times \vec{AC}) = 0$

$$\begin{vmatrix} x-3 & y & z \\ 1 & 1 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 0$$

$$\Leftrightarrow 1 \cdot (-1)^{2+1} \begin{vmatrix} y & z \\ 1 & 4 \end{vmatrix} + 1 \cdot (-1)^{2+2} \begin{vmatrix} x-3 & z \\ 0 & 4 \end{vmatrix} = 0$$

FORON:

$-1(4y - z) + 1(4x - 12) = 0$

$-4y + z + 4x - 12 = 0$

$\vec{n}_2 = (4, -4, 1)$

$\{\vec{n}_1, \vec{n}_2\}$ é L.I., logo os planos π_1 e π_2 são transversais. $(\vec{n}_1 \neq \lambda \vec{n}_2)$

$$\pi_1 : \begin{cases} x = 4 + \lambda_1 + 3\mu_1 \\ y = 2 + \lambda_1 + 3\mu_1 \\ z = 4 + 2\lambda_1 + \mu_1 \end{cases}$$

$$\pi_2 : \begin{cases} x = 3 + \lambda_2 \\ y = 0 + \lambda_2 + \mu_2 \\ z = 0 + 4\mu_2 \end{cases}$$

$$\begin{cases} 4 + \lambda_1 + 3\mu_1 = 3 + \lambda_2 \quad \ldots (1) \\ 2 + \lambda_1 + 3\mu_1 = \lambda_2 + \mu_2 \quad \ldots (2) \\ 4 + 2\lambda_1 + \mu_1 = 4\mu_2 \quad \ldots (3) \end{cases}$$

(3) $2\lambda_1 + \mu_1 = 4\mu_2 - 4$ (1) $\lambda_2 = 1 + \lambda_1 + 3\mu_1$

$(1) \to (2)$ $2 + \cancel{\lambda_1} + \cancel{3\mu_1} = 1 + \cancel{\lambda_1} + \cancel{3\mu_1} + \mu_2$

$2 = 1 + \mu_2 \Rightarrow \mu_2 = 1$

$\mu_2 = 1 \to (3)$ $2\lambda_1 + \mu_1 = 4(1) - 4$

$2\lambda_1 + \mu_1 = 0$

$\mu_1 = -2\lambda_1$

$\mu_1 = -2\lambda_1 \to (1)$ $\lambda_2 = 1 + \lambda_1 + 3(-2\lambda_1)$

$\lambda_2 = 1 + \lambda_1 - 6\lambda_1$

$\lambda_2 = 1 - 5\lambda_1$

$p / \lambda_1 = 0$

$\mu_1 = -2\lambda_1 \qquad \lambda_2 = 1 - 5\lambda_1$

$\mu_1 = 0 \qquad \lambda_2 = 1$

$P = (3,0,0) + 1(1,1,0) + 1(0,1,4)$

$= (4,2,4)$

$\vec{d} = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix} = i(0-2) - j(0-2) + k(1-1)$

$= (-2,2,0)$

Logo: $r: X = (4,2,4) + t(-2,2,0);\ t \in \mathbb{R}$

b) $\pi_1: x - y + 2z - 2 = 0$

$\pi_2: X = (0,0,1) + \lambda(1,0,3) + \mu(-1,1,1)$

Vetores normais:

$\vec{n_1} = (1,-1,2)$

$\vec{n_2} = (1,0,3) \times (-1,1,1)$

$\vec{n_2} = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ -1 & 1 & 1 \end{vmatrix}$

$\vec{n_2} = i(-3) - j(1+3) + k(1)$

$n_2 = (-3,-4,1)$

$\{\vec{n_1}, \vec{n_2}\}$ é L.I, logo π_1 e π_2 são transversais.

Equação geral de π_2:

$-3(x-0)-4.(y-0)+1(z-1)=0$

$-3x-4y+z-1=0$

$$\begin{cases} x-y+2z-2=0 & .(1) \quad .3 \\ -3x-4y+z-1=0 & .(2) \end{cases}$$

$3.(1)\quad 3x-3y+6z-6=0$

$(3x-3y+6z-6)+(-3x-4y+z-1)=0$

$-7y+7z-7=0 \quad (\div -7)$

$y-z+1=0 \rightarrow y=z-1$

$(1)\quad x-(z-1)+2z-2=0 \qquad x-z+1+2z-2=0$

$x+z-1=0 \qquad x=1-z$

Usando parâmetro λ: $z=\lambda$

$$\begin{cases} x=1-\lambda \\ y=-1+\lambda \\ z=\lambda \end{cases}$$

Logo: $r: X=(1,-1,0)+\lambda(-1,1,1)$

c) $\pi_1: 2x-y+z-1=0$

$\pi_2: 4x-2y+2z-9=0$

Vetores normais:

$\vec{n}_1=(2,-1,1) \qquad \{\vec{n}_1,\vec{n}_2\}$ é L.D., no

$\vec{n}_2=(4,-2,2) \qquad$ qual $\vec{n}_1=\lambda\vec{n}_2$, sendo

$\lambda=2$

Logo π_1 e π_2 são paralelos.

$\pi_2: 4x - 2y + 2z = 9 \quad (\div 2)$

$2x - y + z = 4,5$

$\neq$

$\pi_1: 2x - y + z = 1$

Portanto, os planos são distintos.

d) $\pi_1: A = (0,1,6)$, $B = (5,0,1)$ e $C = (4,0,0)$

$\pi_2: 4x + 40y - 4z - 16 = 0$

Vetores diretores de π_1:

- $\vec{AB} = B - A = (5,0,1) - (0,1,6) = (5,-1,-5)$
- $\vec{AC} = C - A = (4,0,0) - (0,1,6) = (4,-1,-6)$

$$\vec{n}_1 = \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 5 & -1 & -5 \\ 4 & -1 & -6 \end{vmatrix}$$

$$\vec{n}_1 = i(6-5) - j(-30+20) + k(-5-4)$$

$$= i(1) - j(-10) + k(-9)$$

$$\vec{n}_1 = (1, 10, -9)$$

Equação geral de π_1:

$$1(x-0) + 10(y-1) - 9(z-6) = 0$$

$$x + 10y - 10 - 9z + 54 = 0$$

$$x + 10y - 9z + 44 = 0$$

FORON:

$\vec{n_2} = (1, 10, -1)$

$\{\vec{n_1}, \vec{n_2}\}$ são L.I., logo π_1 e π_2 são transversais.

$$\begin{cases} x + 10y - 9z + 44 = 0 \\ x + 10y - z - 4 = 0 \end{cases}$$

$(x + 10y - 9z + 44) - (x + 10y - z - 4) = 0$

$-9z + z + 44 + 4 = 0$

$-8z + 48 = 0 \rightarrow z = 6$

Usando parametro λ : $y = \lambda$

$x + 10\lambda - 6 - 4 = 0 \rightarrow x = 10 - 10\lambda$

$$\begin{cases} x = 10 - 10\lambda \\ y = \lambda \\ z = 6 \end{cases}$$

Logo : $r : X = (10, 0, 6) + \lambda(-10, 1, 0)$

15.

a) $\pi_1 : \begin{cases} x = -\lambda_1 + 2\mu_1 \\ y = m\lambda_1 \\ z = \lambda_1 + \mu_1 \end{cases}$

$\pi_2 : \begin{cases} x = 1 + m\lambda_2 + \mu_2 \\ y = 2 + \lambda_2 \\ z = 3 + m\mu_2 \end{cases}$

$\vec{n}_1 = (-1, m, 1) \times (2, 0, 1)$

$$\begin{vmatrix} i & j & k \\ -1 & m & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$= i(m) - j(-1-2) + k(-2m)$

$\vec{n}_1 = (m, 3, -2m)$

$\vec{n}_2 = (m, 1, 0) \times (1, 0, m)$

$$\begin{vmatrix} i & j & k \\ m & 1 & 0 \\ 1 & 0 & m \end{vmatrix}$$

$= i(m) - j(m^2) + k(-1)$

$\vec{n}_2 = (m, -m^2, -1)$

$\{\vec{n}_1, \vec{n}_2\}$ é L.I, logo π_1 e π_2 são transversais independente do valor de m, pois $\nexists\ \lambda \in \mathbb{R}$ tal que $\vec{n}_1 = \lambda \vec{n}_2 \iff (m, 3, -2m) = \lambda(m, -m^2, -1)$

b) $\pi_1 : X = (1,1,0) + \lambda(m,1,1) + \mu(1,1,m)$
$\pi_2 : 2x + 3y + 2z + n = 0$

Para $(m,1,1)$ e $(2,3,2)$

$(m,1,1) \cdot (2,3,2) = 2m + 3 + 2 = 0$
$2m = -5 \rightarrow m = -\frac{5}{2}$

FORON:

Para $(1, 1, m)$ e $(2, 3, 2)$

$(1, 1, m) \cdot (2, 3, 2) = 2 + 3 + 2m = 0$

$m = -\frac{5}{2}$

Ponto de $\pi_1 = (1, 1, 0)$ em π_2:

$2x + 3y + 2z + n = 0$

$2(1) + 3(1) + 2(0) + n = 0$

$2 + 3 + n = 0$

$n = -5$

Conclusão: Para que os planos π_1 e π_2 sejam paralelos distintos, $m = -\frac{5}{2}$ e $n = -5$.

FORONI