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Lista 2 - GA

1.

a) $\begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix}$

$\det A = 2 \cdot (-1)^{1+1} \cdot 3 + 1 \cdot (-1)^{1+2} \cdot (-4) \Leftrightarrow \det A = 6 + 4$

$\det A = 10$

b) $\begin{vmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{vmatrix}$

$\det B = 2 \cdot (-1)^{2+1} \cdot 3\sqrt{6} + \sqrt{3} \cdot (-1)^{2+2} \cdot \sqrt{2}$

$\det B = -6\sqrt{6} + \sqrt{6} \quad \det B = -5\sqrt{6}$

c) $\begin{vmatrix} \pi & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{vmatrix}$

$\det C = 1 \cdot (-1)^{3+1} \cdot \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} + \cancel{0 \cdot (-1)^{3+2} \cdot \begin{vmatrix} \pi & 2 \\ 5 & 1 \end{vmatrix}} + \cancel{0 \cdot (-1)^{3+3} \begin{vmatrix} \pi & 0 \\ 5 & -1 \end{vmatrix}}$

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$\det C = 1 \cdot (0 \cdot 1 - (2 \cdot (-1))) \quad \det C = 2$

d) $\begin{vmatrix} -2 & 1 & -1 \\ 1 & 5 & 4 \\ -3 & 4 & 2 \end{vmatrix}$

$\det D = -2 \cdot (-1)^{1+1} \cdot \begin{vmatrix} 5 & 4 \\ 4 & 2 \end{vmatrix} + 1 \cdot (-1)^{1+2} \cdot \begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix} + (-1) \cdot (-1)^{1+3+1} \begin{vmatrix} 1 & 5 \\ -3 & 4 \end{vmatrix}$

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$\det D \Rightarrow -2(5 \cdot 2 - 4 \cdot 4) - (1 \cdot 2 - 4 \cdot (-3)) - (1 \cdot 4 - 5(-3))$

$12 - 14 - 19 \Rightarrow \det D = -21$

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e) $\begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ 2 & -1 & 2 \end{vmatrix}$

$$\det E = \cancel{0.(-1)^{1+1}} \cdot \begin{vmatrix} 3 & 5 \\ -1 & 2 \end{vmatrix} + 2.(-1)^{1+2} \cdot \begin{vmatrix} 1 & 5 \\ 2 & 2 \end{vmatrix} + \cancel{0\,(-1)^{1+3}} \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix}$$

$$\det E = -2.(2-10) \qquad \det E = 16$$

f) $\begin{vmatrix} 3 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix}$

$$\det F = \cancel{0.(-1)^{2+1} \cdot \begin{vmatrix} -1 & 1 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}} + 1.(-1)^{2+2} \cdot \begin{vmatrix} 3 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix} + \cancel{0.(-1)^{2+3} \cdot \begin{vmatrix} 3 & -1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}} + 1.(-1)^{2+4} \begin{vmatrix} 3 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\det F = \cancel{0.(-1)^{2+1}} \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} + 1.(-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 0 & -1 \end{vmatrix} + \cancel{0.(-1)^{2+3}} \begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix}$$
$$+ \cancel{0.(-1)^{3+1}} \begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix} + \cancel{0.(-1)^{3+2}} \begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} + 1.(-1)^{3+3} \begin{vmatrix} 3 & -1 \\ 0 & -1 \end{vmatrix}$$

$$\det F = 1.(-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 0 & -1 \end{vmatrix} + 1.(-1)^{3+3} \begin{vmatrix} 3 & -1 \\ 0 & -1 \end{vmatrix}$$

$$\det F = (3\,(-1) - 1.0) + (3.(-1) - (-1).0)$$

$$\det F = -6$$

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g) $\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 3 \\ 7 & 2 & \sqrt{3} & 0 & 0 \\ 10 & -3 & 6 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 \end{vmatrix}$

$\det G = 1\cdot(-1)^{1+1} \cdot \begin{vmatrix} 1 & 2 & 5 & 3 \\ 2 & \sqrt{3} & 0 & 0 \\ -3 & 6 & 1 & 0 \\ -3 & 0 & 0 & 0 \end{vmatrix}$

$\det G = -3\cdot(-1)^{4+1} \cdot \begin{vmatrix} 2 & 5 & 3 \\ \sqrt{3} & 0 & 0 \\ 6 & 1 & 0 \end{vmatrix}$

$\det G = 3\cdot\left(6\cdot(-1)^{3+1} \begin{vmatrix} 5 & 3 \\ 0 & 0 \end{vmatrix} + 1\cdot(-1)^{3+2} \begin{vmatrix} 2 & 3 \\ \sqrt{3} & 0 \end{vmatrix}\right)$

$\det G = 3\cdot(6\cdot(5\cdot0 - 3\cdot0) - 1\cdot(2\cdot0 - 3\cdot\sqrt{3}))$

$\det G = 3\cdot 3\sqrt{3}$ $\qquad$ $\det G = 9\sqrt{3}$

h) $\begin{vmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{vmatrix}$

$\det M = 1\cdot(-1)^{4+4} \cdot \begin{vmatrix} 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & -2 & 0 & 0 \end{vmatrix}$ $\qquad$ $\det M = 2\cdot(-1)^{3+3} \begin{vmatrix} 3 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & -2 & 0 \end{vmatrix}$

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$\det M = 2\left(3\,(-1)^{1+1} \begin{vmatrix} 0 & -2 \\ -2 & 0 \end{vmatrix}\right)$ $\det M = 2(3\,(0.0-(-2).(-2)))$

$\det M = 2\,(-12)$ $\det M = -24$

2.

a) $\det(A+B)$

$$A+B = \begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} + \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix} = \begin{pmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 4 & -8 & 2 \end{pmatrix}$$

$$\det A+B = 3.(-1)^{2+1} \begin{vmatrix} -2 & 14 \\ -8 & 2 \end{vmatrix} + 2.(-1)^{2+2} \begin{vmatrix} 7 & 14 \\ 4 & 2 \end{vmatrix} + 10.(-1)^{2+3} \begin{vmatrix} 7 & -2 \\ 4 & -8 \end{vmatrix}$$

$$\det A+B = -3\,(-2.2 - 14.(-8)) + 2\,(7.2 - 14.4) + 10.(-1)\,(7.(-8) - (-2).4)$$

$\det A+B = 72$

b) $\det(AB)$

$$AB = \begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix} = \begin{pmatrix} 3.4 + (-5)(-1) + 7.3 \\ 4.4 + (2)(-1) + 8.3 \\ 1.4 + (-9)(-1) + 6.3 \end{pmatrix}$$

$$\begin{matrix} 3.3 + (-5).0 + 7.1 & 3.7 + (-5)2 + 7.(-4) \\ 4.3 + 2.0 + 8.1 & 4.7 + 2.2 + 8.(-4) \\ 1.3 + (-9).0 + 6.1 & 1.7 + (-9).2 + 6.(-4) \end{matrix} = \begin{pmatrix} 38 & 16 & -17 \\ 38 & 20 & 0 \\ 31 & 9 & -35 \end{pmatrix}$$

$$\det AB = 38.(-1)^{2+1} \begin{vmatrix} 16 & -17 \\ 9 & -35 \end{vmatrix} + 20.(-1)^{2+2} \begin{vmatrix} 38 & -17 \\ 31 & -35 \end{vmatrix}$$

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$$\det AB = -38(16.(-35) - (-17).9) + 20(38.(-35) - (-17).31)$$

$$\det AB = -594$$

c) $\det(B^T A^T)$

$$A^T = \begin{pmatrix} 3 & 4 & 1 \\ -5 & 2 & -9 \\ 7 & 8 & 6 \end{pmatrix} \qquad B^T = \begin{pmatrix} 4 & -1 & 3 \\ 3 & 0 & 1 \\ 7 & 2 & -4 \end{pmatrix}$$

$$B^T A^T = \begin{pmatrix} 4 & -1 & 3 \\ 3 & 0 & 1 \\ 7 & 2 & -4 \end{pmatrix} \begin{pmatrix} 3 & 4 & 1 \\ -5 & 2 & -9 \\ 7 & 8 & 6 \end{pmatrix} = \begin{pmatrix} 4.3 + (-1)(-5) + 3.7 & 4.4 + (-1).2 + 3.8 & 4.1 + (-1).(-9) + 3.6 \\ 3.3 + 0.(-5) + 1.7 & 3.4 + 0.2 + 1.8 & 3.1 + 0.(-9) + 1.6 \\ 7.3 + 2.(-5) + (-4).7 & 7.4 + 2.2 + (-4).8 & 7.1 + 2.(-9) + (-4).6 \end{pmatrix} = \begin{pmatrix} 38 & 38 & 31 \\ 16 & 20 & 9 \\ \boxed{-17 \quad 0 \quad -35} \end{pmatrix}$$

$$\det B^T A^T = -17.(-1)^{3+1} \begin{vmatrix} 38 & 31 \\ 20 & 9 \end{vmatrix} - 35.(-1)^{3+3} \begin{vmatrix} 38 & 38 \\ 16 & 20 \end{vmatrix}$$

$$\det B^T A^T = -17(38.9 - 31.20) - 35(38.20 - 38.16)$$

$$\det B^T A^T = -594$$

d) $\det(2A - 3C + B)$

$$2A = 2.\begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} = \begin{pmatrix} 6 & -10 & 14 \\ 8 & 4 & 16 \\ 2 & -18 & 12 \end{pmatrix}$$

$$3C = 3\begin{pmatrix} 2 & 3 & -1 \\ 6 & 9 & -2 \\ 8 & 12 & -3 \end{pmatrix} = \begin{pmatrix} 6 & 9 & -3 \\ 18 & 27 & -6 \\ 24 & 36 & -9 \end{pmatrix}$$

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$$2A - 3C = \begin{pmatrix} 6 & -10 & 14 \\ 8 & 4 & 16 \\ 2 & -18 & 12 \end{pmatrix} - \begin{pmatrix} 6 & 9 & -3 \\ 18 & 27 & -6 \\ 24 & 36 & -9 \end{pmatrix} = \begin{pmatrix} 0 & -19 & 17 \\ -10 & -23 & 22 \\ -22 & -54 & 21 \end{pmatrix}$$

$$2A - 3C + B = \begin{pmatrix} 0 & -19 & 17 \\ -10 & -23 & 22 \\ -22 & -54 & 21 \end{pmatrix} + \begin{pmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{pmatrix} = \begin{pmatrix} 4 & -16 & 24 \\ -11 & -23 & 24 \\ -19 & -53 & 17 \end{pmatrix}$$

$$\det 2A - 3C + B = 4\,(-1)^{1+1} \begin{vmatrix} -23 & 24 \\ -53 & 17 \end{vmatrix} - 16\,.(-1)^{1+2} \begin{vmatrix} -11 & 24 \\ -19 & 17 \end{vmatrix}$$
$$+ 24\,.(-1)^{1+3} \begin{vmatrix} -11 & -23 \\ -19 & -53 \end{vmatrix}$$

$$\det 2A - 3C + B = 4\,(-23.17 - 24.(-53)) + 16.(-11.17 + 24(-19))$$
$$+ 24\,(-11.(-53) - (-23).(-19))$$

$$\det 2A - 3C + B = -3260$$

e) $\det(AC^T)$

$$AC^T = \begin{pmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{pmatrix} \begin{pmatrix} 2 & 6 & 8 \\ 3 & 9 & 12 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 3.2 + (-5).3 + 7\,(-1) & 3.6 + (-5).9 + 7(-2) & 3.8 + (-5).12 + 7.(-3) \\ 4.2 + 2.3 + 8\,(-1) & 4.6 + 2.9 + 8(-2) & 4.8 + 2.12 + 8.(-3) \\ 1.2 + (-9).3 + 6\,(-1) & 1.6 + (-9).9 + 6\,(-2) & 1.8 + (-9).12 + 6.(-3) \end{pmatrix} =$$

$$\begin{pmatrix} -16 & -41 & -57 \\ 6 & 26 & 32 \\ -31 & -87 & -118 \end{pmatrix}$$

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$\det AC^T = 6.(-1)^{2+1} \begin{vmatrix} -41 & -57 \\ -87 & -118 \end{vmatrix} + 26(-1)^{2+2} \begin{vmatrix} -16 & -57 \\ -31 & -118 \end{vmatrix}$
$+ 32 . (-1)^{2+3} \begin{vmatrix} -16 & -41 \\ -31 & -87 \end{vmatrix}$

$\det AC^T = -6(-41.(-118) - (-57)(-87)) + 26(-16.(-118) - (-57)(-31)) - 32(-16.(-87) - (-41)(-31))$

$\det AC^T = 0$

3.

a) $\det(A^T) = \det(A) = -2$

b) $\det(5A) = 5.\det(A) = 5.(-2) = -10$

c) $\det(A^6) = (-2)^6 = 64$

d) $\det(A^{-1}) = \frac{1}{(-2)} = -\frac{1}{2}$

4.

a) $\begin{vmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{vmatrix} = 5.(-3) = -15$

b) $\begin{vmatrix} a & b & -2c \\ 3d & 3e & -6f \\ g & h & -2i \end{vmatrix} = 0$ (proporcionalidade nas linhas 1 e 2)

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c) $\begin{vmatrix} -a & -b & -c \\ g & h & i \\ -d & -e & -f \end{vmatrix} = -(-3) = 3$ (troca de posições das linhas)

d) $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix} = -(-3) = 3$ (troca de posições das linhas)

e) $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix} = -3$ (soma entre linhas multiplicada por um número)

f) $\begin{vmatrix} Ka+a & Kb+b & Kc+c \\ d & e & f \\ g & h & i \end{vmatrix} = -3$ (soma entre linhas multiplicada por um número)

5.

$$A = \begin{bmatrix} 5 & 4 & 20 & 1 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 9 \\ 3 & -6 & -30 & 12 \end{bmatrix}$$ linha 1 $\leftarrow$ linha 1 + 1 . linha 3

$$A = \begin{bmatrix} 0 & -3 & -10 & 10 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 9 \\ 3 & -6 & -30 & 12 \end{bmatrix}$$ linha 2 $\leftarrow$ linha 2 + 2 . linha 1

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$$A = \begin{bmatrix} 0 & -3 & -10 & 10 \\ 4 & 0 & 0 & 16 \\ -5 & -7 & -30 & 9 \\ 3 & -6 & -30 & 12 \end{bmatrix}$$

$$A = \begin{vmatrix} 0 & -3 & -10 & 10 \\ \boxed{4} & \boxed{0} & \boxed{0} & \boxed{16} \\ -5 & -7 & -30 & 9 \\ 3 & -6 & -30 & 12 \end{vmatrix}$$

linha 2 ← linha 2 + (-1). linha 3

$$\det A = 4.(-1)^{2+1} \begin{vmatrix} -3 & -10 & 10 \\ -7 & -30 & 9 \\ -6 & -30 & 12 \end{vmatrix} + 16.(-1)^{2+4} \begin{vmatrix} 0 & -3 & -10 \\ -5 & -7 & -30 \\ 3 & -6 & -30 \end{vmatrix}$$

linha 2 ← linha 2 + (-1). linha 3

$$\det A = 4.(-1)^{2+1} \begin{vmatrix} -3 & -10 & 10 \\ -1 & 0 & -3 \\ -6 & -30 & 12 \end{vmatrix} + 16.(-1)^{2+4} \begin{vmatrix} 0 & -3 & -10 \\ -8 & -1 & 0 \\ 3 & -6 & -30 \end{vmatrix}$$

$$\det A = 4.(-1)^{2+1} \left(-1.(-1)^{2+1} \begin{vmatrix} -10 & 10 \\ -30 & 12 \end{vmatrix} - 3.(-1)^{2+3} \begin{vmatrix} -3 & -10 \\ -6 & -30 \end{vmatrix}\right) +$$

$$16.(-1)^{2+4} \left(-8.(-1)^{2+1} \begin{vmatrix} -3 & -10 \\ -6 & -30 \end{vmatrix} - 1.(-1)^{2+2} \begin{vmatrix} 0 & -10 \\ 3 & -30 \end{vmatrix}\right)$$

$$\det A = -4\,(1\,(-10.12 - 10(-30))) + 3\,(-3(-30) - (-10)(-6))) +$$
$$16\,(8\,(-3(-30) - (-10)(-6)) - 1\,(0(-30) - (-10)\,3))$$

$$\det A = 3180$$

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6.

a) linha 2 ← linha 2 + (-2) linha 3

$$\begin{vmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{vmatrix} = -128 \qquad \begin{vmatrix} 4 & 6 & x \\ -3 & 0 & 4x \\ 5 & 2 & -x \end{vmatrix} = -128$$

$$-3 \cdot (-1)^{2+1} \begin{vmatrix} 6 & x \\ 2 & -x \end{vmatrix} + 4x \cdot (-1)^{2+3} \begin{vmatrix} 4 & 6 \\ 5 & 2 \end{vmatrix} = -128$$

$$3(6 \cdot (-x) - x \cdot 2) - 4x(4 \cdot 2 - 6 \cdot 5) = -128$$

$$-24x + 88x = -128 \qquad 64x = -128 \qquad x = -2$$

b)

$$\begin{vmatrix} 3 & 5 & 7 \\ \boxed{2x \quad x \quad 3x} \\ 4 & 6 & 7 \end{vmatrix} = 39$$

$$2x \cdot (-1)^{2+1} \begin{vmatrix} 5 & 7 \\ 6 & 7 \end{vmatrix} + x \cdot (-1)^{2+2} \begin{vmatrix} 3 & 7 \\ 4 & 7 \end{vmatrix} + 3x \cdot (-1)^{2+3} \begin{vmatrix} 3 & 5 \\ 4 & 6 \end{vmatrix} = 39$$

$$-2x(5 \cdot 7 - 7 \cdot 6) + x(3 \cdot 7 - 7 \cdot 4) + 3x \cdot (-1)(3 \cdot 6 - 5 \cdot 4) = 39$$

$$14x - 7x + 6x = 39 \qquad 13x = 39 \qquad x = 3$$

c)

$$\begin{vmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{vmatrix} = -7 \qquad \begin{vmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 1 & 0 & 1 \end{vmatrix} = -7$$

↪ linha 3 + (-2) linha 2

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$(X+3)(-1)^{1+1}\begin{vmatrix}5 & 3\\ 0 & 1\end{vmatrix} + (X+1)(-1)^{1+2}\begin{vmatrix}4 & 3\\ 1 & 1\end{vmatrix} + (X+4)(-1)^{1+3}$
$\begin{vmatrix}4 & 5\\ 1 & 0\end{vmatrix} = -7$

$(X+3)(5.1-3.0)-(X+1)(4.1-3.1)+(X+4)(4.0-5.1)$
$=-7$

$(X+3)5 - (X+1) + (X+4)(-5) = -7$
$5X+15-(X+1)-5X-20=-7$
$-X-1-5=-7 \qquad -X=-1 \qquad X=1$

d)
$\begin{pmatrix}X & X+2\\ 1 & X\end{pmatrix}$ é singular, logo det $=0$

$\begin{vmatrix}X & X+2\\ 1 & X\end{vmatrix} = 0 \qquad (X.X-(X+2)1) = 0$

$X^2 - X - 2 = 0$ | $a=1$, $b=-1$, $c=-2$ | $\Delta = b^2-4.a.c$ $\quad \Delta=(-1)^2-4.1.(-2)$
$\Delta = 1+8 \quad \Delta = 9$

$X = \dfrac{-(-1)\pm\sqrt{9}}{2.1} \qquad X = \dfrac{1\pm 3}{2} \qquad X_1 = 2$
$X_2 = -1$

e)
$\begin{pmatrix}x-4 & 0 & 3\\ 2 & 0 & x-9\\ 0 & 3 & 0\end{pmatrix}$ é inversível, logo $A.A^{-1} = A^{-1}A = I$

$I = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \qquad X-4=1 \quad X=5$
$X-9=0 \quad X=9$

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7.

a)

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Condição de existência: $\det(A) \neq 0$

b)

$$A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} \qquad \det(A) \neq 0$$

$$A^{-1} = \frac{1}{3.2 - 1.5} \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} -9\frac{2}{3} & 4\frac{5}{6} \\ 24\frac{1}{6} & -14\frac{1}{2} \end{bmatrix}$$

$$B = \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix} \qquad \det(B) \neq 0$$

$$B^{-1} = \frac{1}{4.2 - 7.1} \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -13\,3/4 & 48\,1/8 \\ 6\,7/8 & -27\,1/2 \end{bmatrix}$$

$$AB = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 4 & 7 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 3.4 + 1.1 & 3.7 + 1.2 \\ 5.4 + 2.1 & 5.7 + 2.2 \end{pmatrix}$$

$$= \begin{pmatrix} 13 & 23 \\ 22 & 39 \end{pmatrix} \qquad \det(AB) \neq 0$$

$$(AB)^{-1} = \frac{1}{13.39 - 23\,22} \begin{bmatrix} 39 & -23 \\ -22 & 13 \end{bmatrix} = \begin{bmatrix} -19733\,12/13 & 11637\,484/507 \\ 11131\,485/507 & -6577\,31/39 \end{bmatrix}$$

8.

a)

$$A = \begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix} \longrightarrow Cof(A) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$a_{11} = (-1)^{1+1} \begin{vmatrix} 1 \end{vmatrix} = 1$$

$$= \begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$$

$$a_{12} = (-1)^{1+2} \begin{vmatrix} 3 \end{vmatrix} = -3$$

$$a_{21} = (-1)^{2+1} \begin{vmatrix} -2 \end{vmatrix} = 2$$

$$a_{22} = (-1)^{2+2} \begin{vmatrix} 2 \end{vmatrix} = 2$$

b)

$$B = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \longrightarrow Cof(B) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = (2.(-1)) - 1.1 = -3$$

$$= \begin{pmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{pmatrix}$$

$$a_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = -(1.(-1)) - 1.0 = 1$$

$$a_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = (1.1) - 2.0 = 1$$

$$a_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} = -(-2.(-1)) - 0.1 = -2$$

$$a_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = (2.(-1)) - 0.0 = -2$$

$$a_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} = -(2.1) - (-2).0 = -2$$

$$a_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix} = (-2.1) - (0.2) = -2$$

$$a_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = -(2.1) - (0.1) = -2$$

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$$a_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = (2.2) - (-2.1) = 6$$

9.

a)

$A = \begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix}$ $\quad \det(A) = \begin{vmatrix} 2 & -2 \\ 3 & 1 \end{vmatrix} = 2 \cdot 1 - (-2) \cdot 3 = 8$

$cof(A) = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{bmatrix}$

$\tilde{a}_{11} = (-1)^{1+1} \cdot 1 = 1$

$\tilde{a}_{12} = (-1)^{1+2} \cdot 3 = -3$

$\tilde{a}_{21} = (-1)^{2+1} \cdot (-2) = 2$

$\tilde{a}_{22} = (-1)^{2+2} \cdot 2 = 2$

$\therefore cof(A) = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}$

$adj(A) = [cof(A)]^T = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{\det A} (adj(A)) = \frac{1}{8} \begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 1/8 & 1/4 \\ -3/8 & 1/4 \end{bmatrix}$

b)

$B = \begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ $\quad \det(B) = \left| \begin{matrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{matrix} \right| \begin{matrix} 2 & -2 \\ 1 & 2 \\ 0 & 1 \end{matrix}$

$0 \quad 2 \quad 2$

$-4 \quad 0 \quad 0$

$\det(B) = -4 - 4 = -8$

$cof(B) = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix}$ $\quad \tilde{a}_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = (2 \cdot (-1)) - 1$

$= -3$

$\tilde{a}_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 0 & -1 \end{vmatrix} = -(1 \cdot (-1)) - 1 \cdot 0 = 1$

$\tilde{a}_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \cdot 1 - 2 \cdot 0 = 1$

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$$\tilde{a}_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} = -(-2 \cdot (-1)) - 0 \cdot 1 = -2$$

$$\tilde{a}_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = 2 \cdot (-1) = -2$$

$$\tilde{a}_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} = -(2 \cdot 1) - 2 \cdot 0 = -2$$

$$\tilde{a}_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix} = -2 \cdot 1 - 0 \cdot 2 = -2$$

$$\tilde{a}_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = -(2 \cdot 1) = -2$$

$$\tilde{a}_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -2 \\ 1 & 2 \end{vmatrix} = 2 \cdot 2 - (-2) \cdot 1 = 6$$

$$\therefore \ cof(B) = \begin{bmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{bmatrix}$$

$$adj(B) = [cof(B)]^T = \begin{bmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{bmatrix}$$

$$B^{-1} = \frac{1}{\det B} adj(B) = \frac{1}{-8} \begin{bmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 3/8 & 1/4 & 1/4 \\ -1/8 & 1/4 & 1/4 \\ -1/8 & 1/4 & -3/4 \end{bmatrix}$$

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c)

$$C = \begin{pmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

$\det(C) = 1.(-1)^{1+1} \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + 1(-1)^{3+2} \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} = (-1(-1)) - 1.0 - 0(-1) - 1.2 = -1$

$$cof(C) = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix}$$

$\tilde{a}_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} = 0.0 - (-1).1 = 1$

$\tilde{a}_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} = -(2.0) - (-1).1 = 1$

$\tilde{a}_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2.1 - 0.1 = 2$

$\tilde{a}_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} = -(-1.0) - 1.1 = -1$

$\tilde{a}_{22} = (-1)^{2+2} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0.0 - 1.1 = -1$

$\tilde{a}_{23} = (-1)^{2+3} \begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix} = -(0.1) - (-1).1 = 1$

$\tilde{a}_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} = -1.(-1) - 1.0 = 1$

$\tilde{a}_{32} = (-1)^{3+2} \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} = -(0.(-1)) - 1.2 = -2$

$\tilde{a}_{33} = (-1)^{3+3} \begin{vmatrix} 0 & -1 \\ 2 & 0 \end{vmatrix} = 0.0 - (-1).2 = 2$

$$\therefore cof(C) = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -1 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

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$$\mathrm{adj}(C) = [\mathrm{cof}(C)]^T = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -1 & 1 \\ 1 & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

$$C^{-1} = \frac{1}{\det C}\,\mathrm{adj}(C) = \frac{1}{-1}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & -2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1 \\ -1 & 1 & 2 \\ -2 & -1 & -2 \end{bmatrix}$$

d)

$$D = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 \end{pmatrix} \qquad \det(D) = 1 . (-1)^{1+1}$$

$$\det(D) = 1\left(-1 . (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}\right) = (1.3 - 1.2) = 1$$

$$\mathrm{cof}(D) = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} & \tilde{a}_{14} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} & \tilde{a}_{24} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} & \tilde{a}_{34} \\ \tilde{a}_{41} & \tilde{a}_{42} & \tilde{a}_{43} & \tilde{a}_{44} \end{bmatrix}$$

$$\tilde{a}_{11} = (-1)^{1+1}\begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = 1 . (-1)^{1+1}\begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} = 2.0 - 3.(-1) = 3$$

$$\tilde{a}_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 0 & 0 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = -1 . 0 = 0$$

$$\tilde{a}_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 1 . 0 = 0$$

$$\tilde{a}_{14} = (-1)^{1+4}\begin{vmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} = -1 \left(-1 . (-1)^{3+3}\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix}\right) =$$

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$-(-1 \quad 1.0.0 - 1.2) = 2$

$\tilde{a}_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = -1.0 = 0$

$\tilde{a}_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = -1.(-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1.3 - 1.2 = 1$

$\tilde{a}_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{vmatrix} = -1.0 = 0$

$\tilde{a}_{24} = (-1)^{2+4} \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 2 \\ 0 & 0 & -1 \end{vmatrix} = 1.(-1)^{1+1} \begin{vmatrix} 0 & 2 \\ 0 & -1 \end{vmatrix} = 0$

$\tilde{a}_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = 1.(-1)^{2+1} \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = -1.0.0 - 1.(-1) = 1$

$\tilde{a}_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = -1.0 = 0$

$\tilde{a}_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 1.0 = 0$

$\tilde{a}_{34} = (-1)^{3+4} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -1\left(1.(-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}\right) = 1$

$\tilde{a}_{41} = (-1)^{4+1} \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 2 & 3 \end{vmatrix} = -1\left(1.(-1)^{2+1} \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix}\right) = -2$

$\tilde{a}_{42} = (-1)^{4+2} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 2 & 3 \end{vmatrix} = 1.0 = 0$

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$$\bar{a}_{43} = (-1)^{4+3} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = -1 \cdot \left(1 \cdot (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}\right) = -1$$

$$\bar{a}_{44} = (-1)^{4+4} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 2 \end{vmatrix} = 1 \cdot \left(1 \cdot (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}\right) = +2$$

$$\therefore \operatorname{cof}(D) = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{bmatrix}$$

$$\operatorname{adj}(D) = [\operatorname{cof}(D)]^T = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & 0 & 1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 2 & 0 & 1 & 2 \end{bmatrix}$$

$$D^{-1} = \frac{1}{\det D} \operatorname{adj}(D) = \frac{1}{1} \begin{bmatrix} 3 & 0 & 1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 2 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 2 & 0 & 1 & 2 \end{bmatrix}$$

10.

a)

$$A^T - BX = 2C \qquad X = \frac{2C - A^T}{-B}$$

$$X = 2C - A^T \cdot (-B)^{-1}$$

Na matriz B seu determinante deve ser diferente de zero.

Foroni

b)

$$X = 2C - A^T \cdot (-B)^{-1}$$

$$2C = 2 \cdot \begin{pmatrix} 0 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 6 \\ 2 & 0 & 4 \\ 6 & 4 & 0 \end{pmatrix}$$

$$A^T = \begin{pmatrix} -1 & 2 & 6 \\ 2 & -1 & 4 \\ 6 & 4 & -1 \end{pmatrix} \qquad -B = -1 \cdot \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix} = \begin{pmatrix} -1 & 0 & -2 \\ -2 & 1 & -3 \\ -4 & -1 & -8 \end{pmatrix}$$

$$(-B)^{-1} = \begin{pmatrix} 11 & -2 & -2 \\ 4 & 0 & -1 \\ -6 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & -2 \\ -2 & 1 & -3 \\ -4 & -1 & -8 \end{pmatrix} \qquad \det(-B) = -1 \cdot (-1)^{1+1} \begin{vmatrix} 1 & -3 \\ -1 & -8 \end{vmatrix} - 2 \cdot (-1)^{1+3} \begin{vmatrix} -2 & 1 \\ -4 & -1 \end{vmatrix}$$

$$\det(-B) = -1$$

$$\operatorname{cof}(-B) = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix}$$

$$\tilde{a}_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -3 \\ -1 & -8 \end{vmatrix} = 1.(1.(-8) - (-3).(-1)) = -11$$

$$\tilde{a}_{12} = (-1)^{1+2} \begin{vmatrix} -2 & -3 \\ -4 & -8 \end{vmatrix} = -1.(-2.(-8) - (-3).(-4)) = -4$$

$$\tilde{a}_{13} = (-1)^{1+3} \begin{vmatrix} -2 & 1 \\ -4 & -1 \end{vmatrix} = 1.(-2.(-1) - 1.(-4)) = 6$$

$$\tilde{a}_{21} = (-1)^{2+1} \begin{vmatrix} 0 & -2 \\ -1 & -8 \end{vmatrix} = -1.(0.(-8) - (-2).(-1)) = 2$$

$$\tilde{a}_{22} = (-1)^{2+2} \begin{vmatrix} -1 & -2 \\ -4 & -8 \end{vmatrix} = 1.(-1.(-8) - (-2).(-4)) = 0$$

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$$\bar{a}_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 0 \\ -4 & 1 \end{vmatrix} = -1 \cdot (-1 \, (1) - 0 \, (-4)) = -1$$

$$\bar{a}_{31} = (-1)^{3+1} \begin{vmatrix} 0 & -2 \\ 1 & -3 \end{vmatrix} = 1 \cdot (0 \cdot (-3) - (-2) \cdot 1) = 2$$

$$\bar{a}_{32} = (-1)^{3+2} \begin{vmatrix} -1 & -2 \\ -2 & -3 \end{vmatrix} = -1 \cdot (-1 \, (-3) - (-2) \cdot (-2)) = 1$$

$$\bar{a}_{33} = (-1)^{3+3} \begin{vmatrix} -1 & 0 \\ -2 & 1 \end{vmatrix} = 1 \cdot (-1 \cdot 1 - 0 \, (-2)) = -1$$

$$\therefore \operatorname{cof}(-B) = \begin{bmatrix} -11 & -4 & 6 \\ 2 & 0 & -1 \\ 2 & 1 & -1 \end{bmatrix}$$

$$\operatorname{adj}(-B) = [\operatorname{cof}(-B)]^T = \begin{bmatrix} -11 & -4 & 6 \\ 2 & 0 & -1 \\ 2 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$$

$$(-B)^{-1} = \frac{1}{\det -B} \operatorname{adj}(-B) = \frac{1}{-1} \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 11 & -2 & -2 \\ 4 & 0 & -1 \\ -6 & 1 & 1 \end{bmatrix}$$

$$X = \begin{pmatrix} 0 & 2 & 6 \\ 2 & 0 & 4 \\ 6 & 4 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 6 \\ 2 & -1 & 4 \\ 6 & 4 & -1 \end{pmatrix} \begin{pmatrix} 11 & -2 & -2 \\ 4 & 0 & -1 \\ -6 & 1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2 & 6 \\ 2 & -1 & 4 \\ 6 & 4 & -1 \end{pmatrix} \begin{pmatrix} 11 & -2 & -2 \\ 4 & 0 & -1 \\ -6 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 \cdot 11 + 2 \cdot 4 + 6 \, (-6) & -1 \, (-2) + 2 \cdot 0 + 6 \cdot 1 & -1 \cdot (-2) + 2 \cdot (-1) + 6 \cdot 1 \\ 2 \cdot 11 + (-1) \cdot 4 + 4 \cdot (-6) & 2 \cdot (-2) + (-1) \cdot 0 + 4 \cdot 1 & 2 \cdot (-2) + (-1) \cdot (-1) + 4 \cdot 1 \\ 6 \cdot 11 + 4 \cdot 4 + (-1) \cdot (-6) & 6 \cdot (-2) + 4 \cdot 0 + (-1) \cdot 1 & 6 \cdot (-2) + 4 \cdot (-1) + (-1) \cdot 1 \end{pmatrix} =$$

FORONI:

$$\begin{pmatrix} -39 & 8 & 6 \\ -6 & 0 & 1 \\ 88 & -13 & -17 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 2 & 6 \\ 2 & 0 & 4 \\ 6 & 4 & 0 \end{pmatrix} - \begin{pmatrix} -39 & 8 & 6 \\ -6 & 0 & 1 \\ 88 & -13 & -17 \end{pmatrix}$$

$$X = \begin{pmatrix} -39 & -6 & 0 \\ 8 & 0 & 3 \\ -82 & 17 & 17 \end{pmatrix}$$

Foroni