Solutions for Introduction to Linear Algebra 5th - Gilbert Strang
Chapter1

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Contents

Chapter 1

Problem Set 1.1

Question 1: Describe geometrically (line, plane or all of $\mathbb{R}^3$) all linear combinations of

a.
$\begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ b. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ c. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

Solution:

a.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} * 3 = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

, line

Question 2: 5

Solution:

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$$

$$\mathbf{u} + \mathbf{v} + \mathbf{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2\mathbf{u} + 2\mathbf{v} + \mathbf{w} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

c and d:

$$\begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = c \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + d \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$$

$$c = -1, d = -1$$

Chapter 2

Problem Set 1.2

Question 1: 1

$$\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} -2.4 \\ 2.4 \end{bmatrix}$$

$$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) =$$
$$= \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$
$$= \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

Question 2: 2

$$||\mathbf{u}|| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$$
$$= \sqrt{(-0.6 * -0.6) + (0.8 * 0.8)}$$
$$= \sqrt{0.36 + 0.64}$$
$$= 1$$

$$||\mathbf{v}|| = \sqrt{(4 * 4) + (3 * 3)} = 5$$

Question 3: 4

if the lenght of $\mathbf{v}$ is 1, $\mathbf{v}$ is a unit vector.
a.

$$||\mathbf{v}|| = 1$$
$$\mathbf{v} \cdot \mathbf{v} = 1$$
$$\rightsquigarrow \mathbf{v} \cdot -\mathbf{v} = -1$$

b.

$$\begin{bmatrix} v_0 + w_0 \\ \vdots \\ v_n + w_n \end{bmatrix} \cdot \begin{bmatrix} v_0 - w_0 \\ \vdots \\ v_n - w_n \end{bmatrix} =$$

$$= \sum_{i=0}^{n} v_0^2 - w_0^2$$

$$= \sum_{i=0}^{n} v_0^2 - \sum_{i=0}^{n} w_0^2$$

$$= ||\mathbf{v}|| - ||\mathbf{w}|| = 0$$

Question 4: 6

a.

$$\mathbf{w} \cdot \mathbf{v} = 0$$

$$(w_1 * 2) + (w_2 * -1) = 0$$

$$2w_1 - w_2 = 0$$

$$2w_1 = w_2$$

Question 5: 7

$$\cos\theta = \frac{\mathbf{w} \cdot \mathbf{v}}{||\mathbf{w}|| \cdot ||\mathbf{v}||}$$

c.

$$\mathbf{w} \cdot \mathbf{v} = \frac{1}{\sqrt{3}} \cdot \frac{-1}{\sqrt{3}}$$

$$= -1 + 3 = 2$$

$$||\mathbf{w}|| = \sqrt{(-1 * -1) + (\sqrt{3} * \sqrt{3})}$$

$$= \sqrt{1 + 3} = 2$$

$$||\mathbf{v}|| = \sqrt{(1 * 1) + (\sqrt{3} * \sqrt{3})}$$

$$= \sqrt{1 + 3} = 2$$

$$\cos\theta = \frac{2}{2 * 2} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Question 6: 12

$$
\begin{aligned}
(\mathbf{w} - c\mathbf{v}) \cdot \mathbf{v} &= 0 \\
(\begin{bmatrix} 1 \\ 5 \end{bmatrix} - c \begin{bmatrix} 1 \\ 1 \end{bmatrix}) \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} &= 0 \\
\begin{bmatrix} 1 - c \\ 5 - c \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} &= 0 \\
(1 - c) + (5 - c) &= 0 \\
6 - 2c &= 0 \\
c &= 3
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{w} - c\mathbf{v}) \cdot \mathbf{v} &= 0 \\
\mathbf{w} \cdot \mathbf{v} - c\mathbf{v} \cdot \mathbf{v} &= 0 \\
-c\mathbf{v} \cdot \mathbf{v} &= \mathbf{w} \cdot \mathbf{v} \\
c &= -\frac{\mathbf{w} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}}
\end{aligned}
$$

Question 7: 13

$$
\begin{aligned}
\mathbf{w} \cdot (1, 0, 1) &= 0 \\
w_1 + w_2 &= 0 \\
w_1 &= -w_2 \\
\mathbf{w} &= (1, 0, -1)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v} \cdot \mathbf{w} &= 0 \qquad (2.1) \\
\mathbf{v} \cdot (1, 0, 1) &= 0 \qquad (2.2)
\end{aligned}
$$

2.1

$$
\begin{aligned}
(v_1, v_2, v_3) \cdot (1, 0, -1) &= 0 \\
v_1 - v_3 &= 0 \\
v_1 &= v_3
\end{aligned}
$$

2.2

$$
\begin{aligned}
(v_1, v_2, v_3) \cdot (1, 0, 1) &= 0 \\
v_1 + v_3 &= 0 \\
v_1 &= -v_3 \\
v_1 &= 0 \\
v_3 &= 0
\end{aligned}
$$

$$
\mathbf{v} = (0, v_2, 0), v_2 \in \mathbb{R}
$$

Question 8: 19

$$\|\mathbf{v} + \mathbf{w}\|^2$$
$$\|\mathbf{u}\|$$
$$\mathbf{u} \cdot \mathbf{u}$$
$$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$$
$$\mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$$
$$(\mathbf{v} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{w}) + (\mathbf{w} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w})$$
$$\rightsquigarrow \mathbf{v} \cdot \mathbf{v} + 2\mathbf{v} \cdot \mathbf{w} + \mathbf{w} \cdot \mathbf{w}$$