Solutions for Introduction to Linear Algebra 5th - Gilbert Strang
Chapter1

Gabriel Romão

Contents

Chapter 1

Problem Set 1.1

Question 1: Describe geometrically (line, plane or all of $\mathbb{R}^3$) all linear combinations of

a.
$\begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$ b. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ c. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

Solution:

a.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} * 3 = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$$

, line

Question 2: 5

Solution:

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$$

$$\mathbf{u} + \mathbf{v} + \mathbf{w} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2\mathbf{u} + 2\mathbf{v} + \mathbf{w} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

c and d:

$$\begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = c \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + d \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$$

$$c = -1, d = -1$$

Chapter 2

Problem Set 1.2

Question 1: 1

$$\mathbf{u} \cdot \mathbf{v} = \begin{bmatrix} -2.4 \\ 2.4 \end{bmatrix}$$

$$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) =$$
$$= \begin{bmatrix} -0.6 \\ 0.8 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$
$$= \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

Question 2: 2

$$\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$$
$$= \sqrt{(-0.6 * -0.6) + (0.8 * 0.8)}$$
$$= \sqrt{0.36 + 0.64}$$
$$= 1$$

$$\|\mathbf{v}\| = \sqrt{(4 * 4) + (3 * 3)} = 5$$

Question 3: 4

if the lenght of **v** is 1, **v** is a unit vector.
a.

$$\|\mathbf{v}\| = 1$$
$$\mathbf{v} \cdot \mathbf{v} = 1$$
$$\rightsquigarrow \mathbf{v} \cdot -\mathbf{v} = -1$$

b.

$$
\begin{aligned}
\begin{bmatrix} v_0 + w_0 \\ \vdots \\ v_n + w_n \end{bmatrix} \cdot \begin{bmatrix} v_0 - w_0 \\ \vdots \\ v_n - w_n \end{bmatrix} &= \\
&= \sum_{i=0}^{n} v_0^2 - w_0^2 \\
&= \sum_{i=0}^{n} v_0^2 - \sum_{i=0}^{n} w_0^2 \\
&= \|\mathbf{v}\| - \|\mathbf{w}\| = 0
\end{aligned}
$$