# FYS-STK4155 Project 1

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**Abstract**

## 1 Introduction

yvycjyukioyjfchdxcgfjhbk

## 2 Data

## 3 Method

## 4 Results

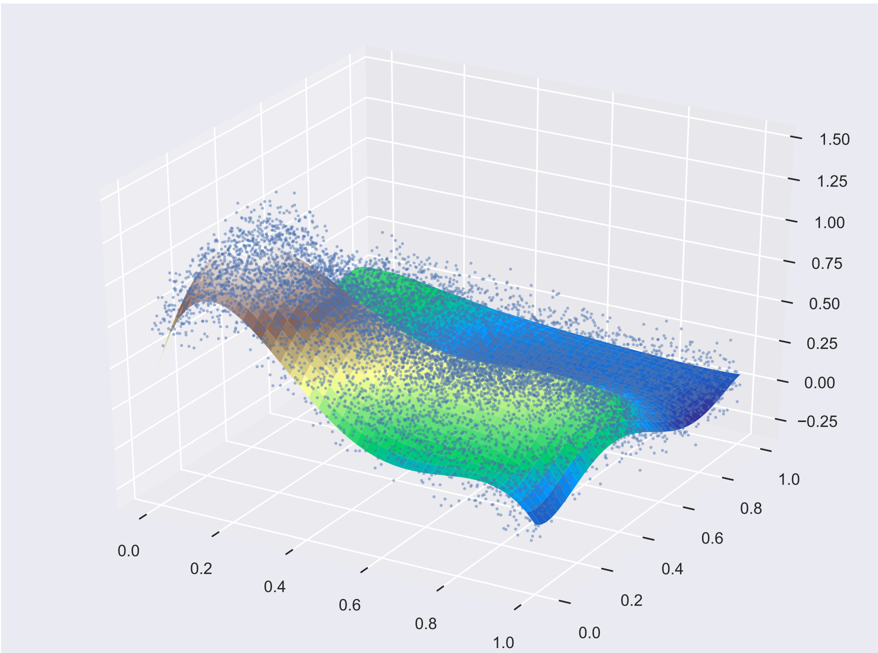


Figure 1: The resulting function after performing a standard least square regression analysis using polynomials in x and y up to fifth order on the Franke function

| | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\beta_5$ | $\beta_6$ | $\beta_7$ | $\beta_8$ | $\beta_9$ | $\beta_{10}$ | $\beta_{11}$ | $\beta_{12}$ | $\beta_{13}$ | $\beta_{14}$ | $\beta_{15}$ | $\beta_{16}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SLS | 0.00789 | 0.322 | 2.46 | 4.81 | 5.61 | 1.76 | 0.322 | 1.85 | 4.3 | 5.97 | 2.83 | 2.46 | 4.3 | 5.97 | 4.23 | 4.81 | 5.97 |
| $k$-fold | 0.0321 | 4.03 | 94 | 474 | 517 | 79.5 | 4.03 | 57.3 | 260 | 301 | 60.1 | 93.9 | 260 | 261 | 57.1 | 474 | 301 |

| | $\beta_{17}$ | $\beta_{18}$ | $\beta_{19}$ | $\beta_{20}$ |
|---|---|---|---|---|
| SLS | 4.23 | 5.61 | 2.83 | 1.76 |
| $k$-fold | 57.1 | 517 | 60.1 | 79.5 |

Table 1: The variance of $\beta$ for the standard least square regression and for $k$-fold validation

| | MSE | $R^2$ |
|---|---|---|
| SLS | 0.015 | 0.84 |
| $k$-fold | 0.012 | 0.87 |

Table 2: MSE and R2 for a and b

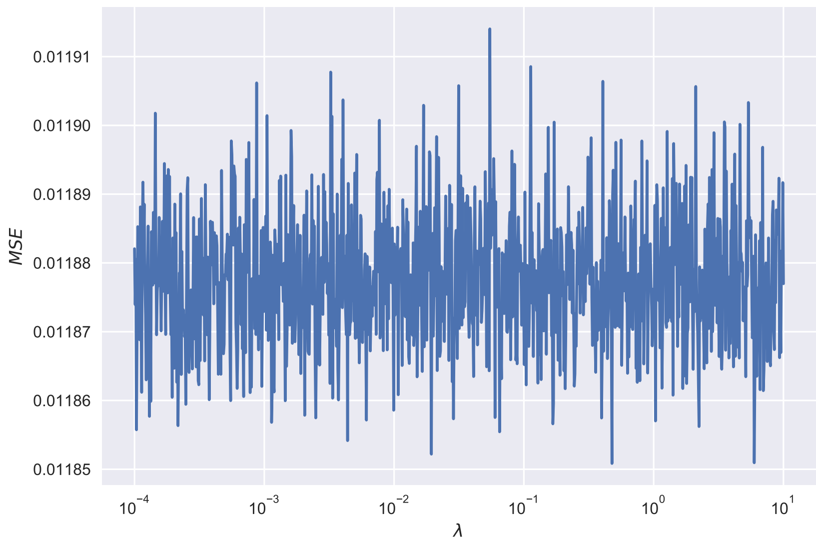


Figure 2: The Mean Squared Error for the Ridge method for different values of $\lambda$

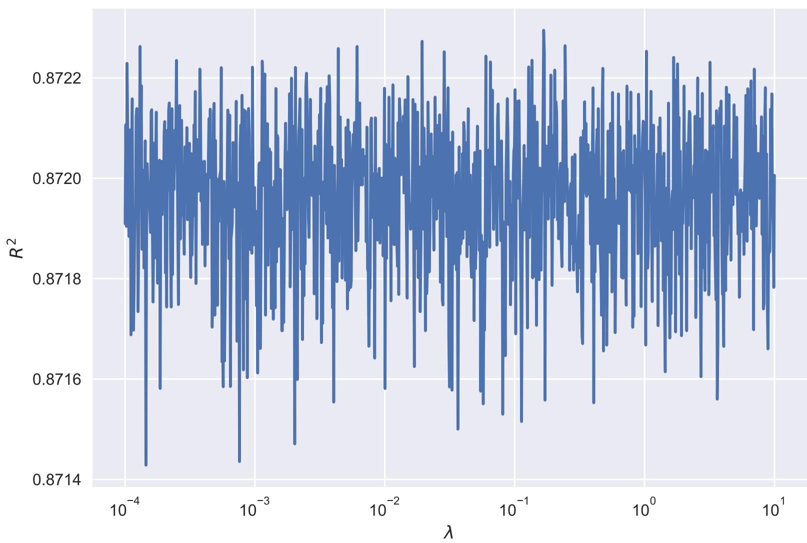


Figure 3: $R^2$-score for the Ridge method for different values of $\lambda$

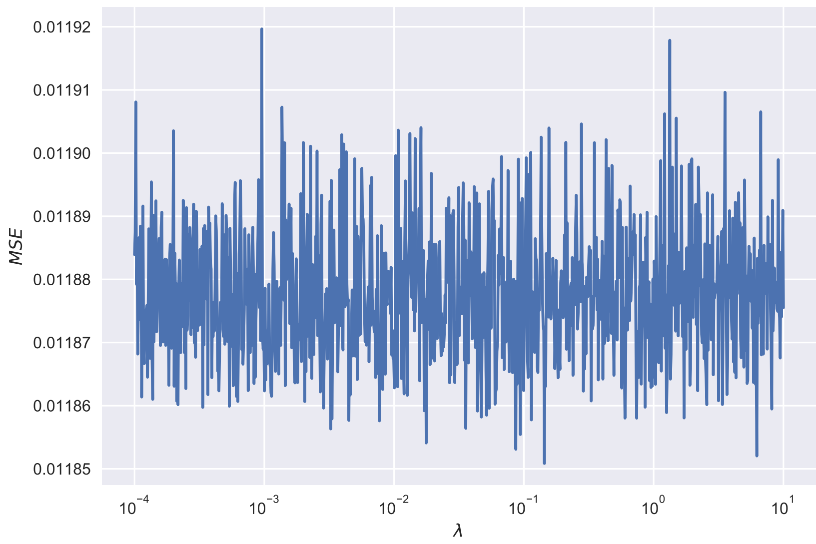


Figure 4: The Mean Squared Error for the Lasso method for different values of $\lambda$

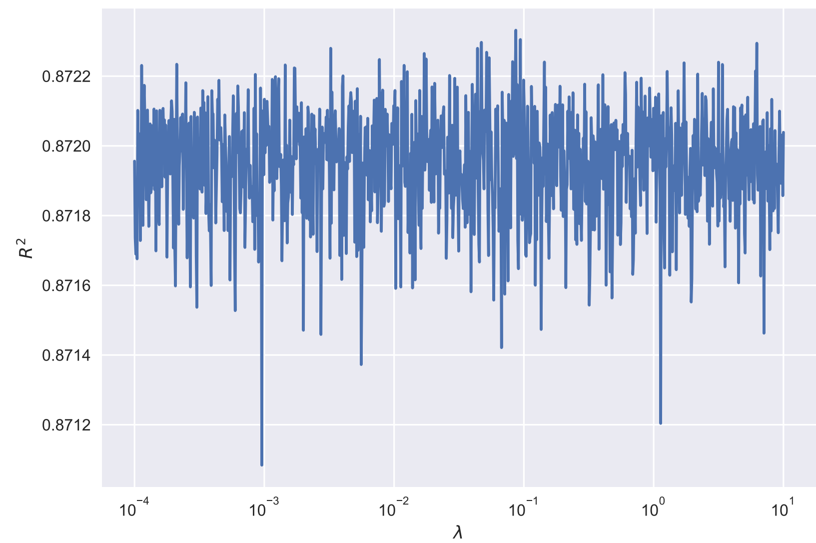


Figure 5: $R^2$-score for the Lasso method for different values of $\lambda$

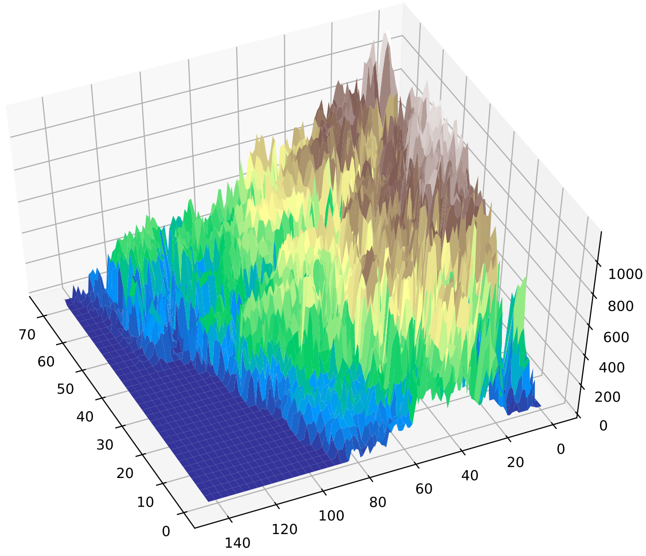


Figure 6: The terrain-data we are studying, from Møsvatn Austfjell in Norway

# 5 Discussion

# 6 Appendix

## 6.1 Part c math

We are too show that

$$C(\boldsymbol{X}, \boldsymbol{\beta}) = \frac{1}{n}\sum_{i=0}^{n-1}(y_i - \tilde{y}_i)^2 = \mathbf{E}\left[(\boldsymbol{y} - \tilde{\boldsymbol{y}})^2\right] = \frac{1}{n}\sum_i (f_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 + \frac{1}{n}\sum_i (\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 + \sigma^2 \tag{1}$$

$$\begin{aligned}
\mathbf{E}\left[(\boldsymbol{y} - \tilde{\boldsymbol{y}})^2\right] =& \frac{1}{n}\sum_i (y_i - \tilde{y}_i)^2 = \frac{1}{n}\sum_i (f_i + \varepsilon - \tilde{y}_i)^2 && (2)\\
=& \frac{1}{n}\sum_i (f_i + \varepsilon - \tilde{y}_i + \mathbf{E}[\tilde{\boldsymbol{y}}] - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 \qquad \mid \text{introduce } a = f_i - \mathbf{E}[\tilde{\boldsymbol{y}}] \text{ and } b = \tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}] && (3)\\
=& \frac{1}{n}\sum_i (a - b + \varepsilon)^2 = \frac{1}{n}\sum_i \left(a^2 - 2ab + b^2 - 2b\varepsilon + \varepsilon^2 + 2a\varepsilon\right) && (4)\\
=& \frac{1}{n}\sum_i (f_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 + \frac{1}{n}\sum_i \left(\varepsilon^2\right) + \frac{1}{n}\sum_i (\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 - 2\frac{1}{n}\sum_i \varepsilon(\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}]) + 2\frac{1}{n}\sum_i \varepsilon(f_i - \mathbf{E}[\tilde{\boldsymbol{y}}]) && (5)\\
&- 2\frac{1}{n}\sum_i (f_i - \mathbf{E}[\tilde{\boldsymbol{y}}])(\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}]) && (6)\\
=& \frac{1}{n}\sum_i (f_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 + \frac{1}{n}\sum_i (\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 + \sigma^2 - 2\mathbf{E}[\varepsilon]\frac{1}{n}\sum_i (\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}]) + 2\mathbf{E}[\varepsilon]\frac{1}{n}\sum_i (f_i - \mathbf{E}[\tilde{\boldsymbol{y}}]) && (7)\\
&- 2\frac{1}{n}\sum_i (f_i - \mathbf{E}[\tilde{\boldsymbol{y}}])(\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}]) && (8)\\
=& \frac{1}{n}\sum_i (f_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 + \frac{1}{n}\sum_i (\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2 + \sigma^2 \quad \blacksquare && (9)
\end{aligned}$$

Where $\frac{1}{n}\sum_i (f_i - \mathbf{E}[\tilde{\boldsymbol{y}}])$ is the bias and $\frac{1}{n}\sum_i (\tilde{y}_i - \mathbf{E}[\tilde{\boldsymbol{y}}])^2$ is the variance.
(skal vi gjøre noe annet og?)