Quantifying Ionospheric Delay Variations from Satellite Ephemeris Errors in Distinct Solar Periods

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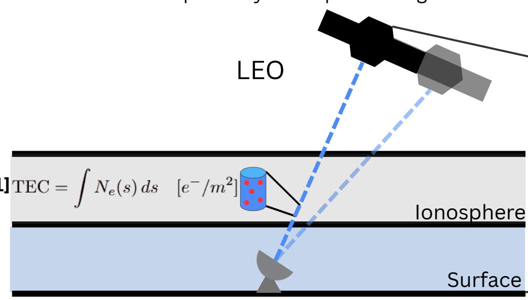
Motivation

With the rapid expansion of services and products relying on Low-Earth Orbit (LEO) satellites, it has become essential to understand how satellite positioning errors impact signal delay.

Common orbit propagation methods, such as Two-Line Element (TLE) sets, can exhibit initial errors on the order of kilometers. These errors tend to grow over time due to unmodeled forces—primarily atmospheric drag in LEO.



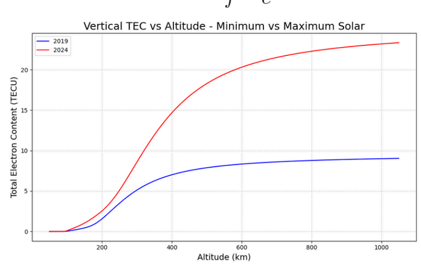
Therefore, how much can satellite ephemeris errors delay signals?

Background

The Sun's magnetic activity follows a 11-year cycle, that plays a key role in shaping the Total Electron Content (TEC) of Earth's ionosphere. Increased solar activity leads to more intense radiation and solar wind, which in turn increases ionization in the upper atmosphere.

Since ionospheric delay depends on TEC and signal frequency, the delay experienced by satellite signals also varies with the solar cycle phase. This makes understanding the solar period essential for accurate modeling of ionospheric effects.

$$\textbf{[2]}\quad \Delta t_{iono} = \frac{40.3 \cdot TEC}{f^2 \cdot c} \quad [s]$$



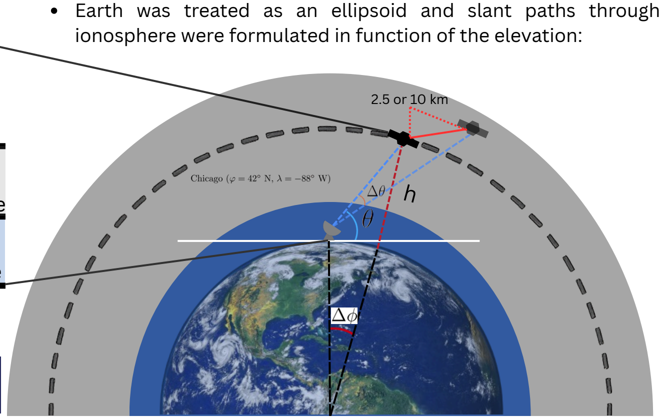
Methodology

1.Vertical TEC Profiles: Generated by the International Reference Ionosphere (IRI) model at 1 km resolution for:

- Dates: December 1, 2019 (solar minimum) and August 1, 2024 (solar maximum).
- Boundaries: 50 km and 1049 km.
- Location: Near Chicago (latitude = 42° N, longitude = -88° W); solar local time 14:00.

2. Geometry:

- Earth was treated as an ellipsoid and slant paths through ionosphere were formulated in function of the elevation:



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$$\textbf{[3]}\quad R(\phi) = \sqrt{\frac{(a^2\cos\phi)^2 + (b^2\sin\phi)^2}{(a\cos\phi)^2 + (b\sin\phi)^2}}$$

$$s_{total} = \sqrt{(R(\phi \pm \Delta\phi) + h)^2 + [R(\phi)\cdot\sin(\theta)]^2 - R(\phi \pm \Delta\phi)^2} - R(\phi)\cdot\sin(\theta)$$

[4]

$$s_{ionosphere} = s_{total} - s_{under-ionosphere}$$

- Iteration was used to find the radius at a point along the signal path by solving for the latitude angle increment ($\Delta\phi$).

$$\textbf{[5]}\quad \cos(\phi + \theta) = \frac{R(\phi \pm \Delta\phi)\cdot\sin\left(\frac{\pi}{2} + \theta\right)}{R(\phi) + h}$$

- The slant TEC was computed by integrating the vertical electron density profile along the satellite signal path. The path was discretized, and the TEC was estimated using the trapezoidal rule:

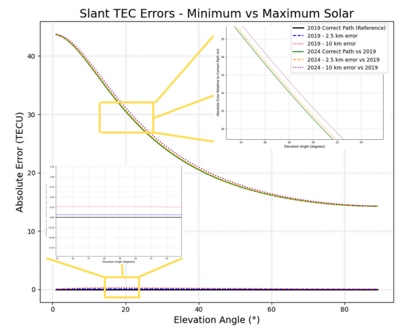
$$\textbf{[6]}\quad TEC_{slant} = \sum_{i=1}^{n-1}\left(\frac{N_e(h_i) + N_e(h_{i+1})}{2}\right)\cdot \Delta s_{h_i}$$

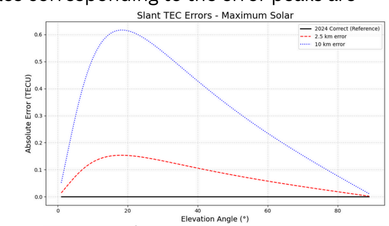
3. The impact of orbital position errors of 2.5 km and 10 km on the computed ionospheric delay was assessed by calculating both the absolute error (in TECU) and the percentage error (%) relative to the delay obtained with precise orbit data.

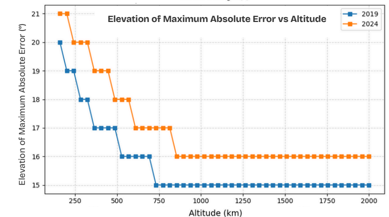
$$1\ \text{TECU} = 1 \times 10^{16}\ \text{electrons/m}^2$$

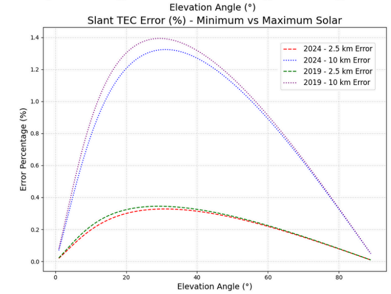
Results

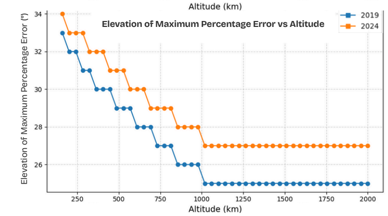
At an altitude of 500 km, slant TEC was computed for elevation angles ranging from 0° to 89°. The absolute and percentage errors peak near 20° and 30° of elevation, respectively. Additionally, the elevation angles corresponding to the error peaks are analyzed across the full LEO altitude range.











Conclusion

For a 500 km altitude satellite operating in the Ku-band (10.7 GHz), such as Starlink, a 2.5 km orbital error can introduce delays of up to 1.8 picoseconds during periods of maximum solar activity. For lower-frequency meteorological satellites in the VHF band (137 MHz), the delay can reach up to 11 nanoseconds. While small, these delays are not negligible in high-precision applications and should be considered in mission planning and system design.

Future Work

Expand the model to signal paths with azimuth angles ≠ 0° or 180°, moving beyond local meridian-only geometry using Vertical TEC profiles from IRI in other times of the day.

The orbital error was modeled to maximize the elevation angle difference along the signal path. So, future work should investigate how real satellite ephemeris errors behave, as they may exhibit different spatial patterns and directional components.

Acknowledgment

- Armour R&D Research grant.

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