Problem 7.5

Consider the design of a system for changing the sampling rate of a signal from 10 kHz to 16 kHz:

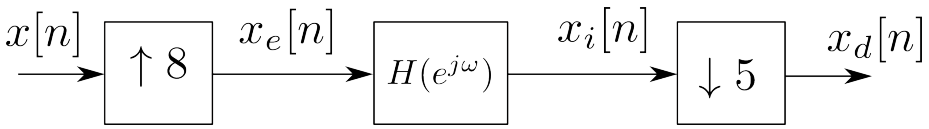

Figure 7.5a

The ideal lowpass filter $H(e^{j\omega})$ in the system above would have a cutoff frequency of 5 kHz. We approximate the ideal filter with IIR and FIR filter designs that meet the following specifications:

1. Average of passband and stopband edges at the ideal filter cutoff frequency.
2. Width of transition band: 4 kHz.
3. Maximum gain in the passband: 0 dB.
4. Minimum gain in the passband: -1 dB.
5. Maximum gain in the stopband: -40 dB.

The pole-zero diagrams of the resulting 4th-order elliptic IIR filter and 30th-order Parks-McClellan FIR filter satisfying these requirements are shown below.

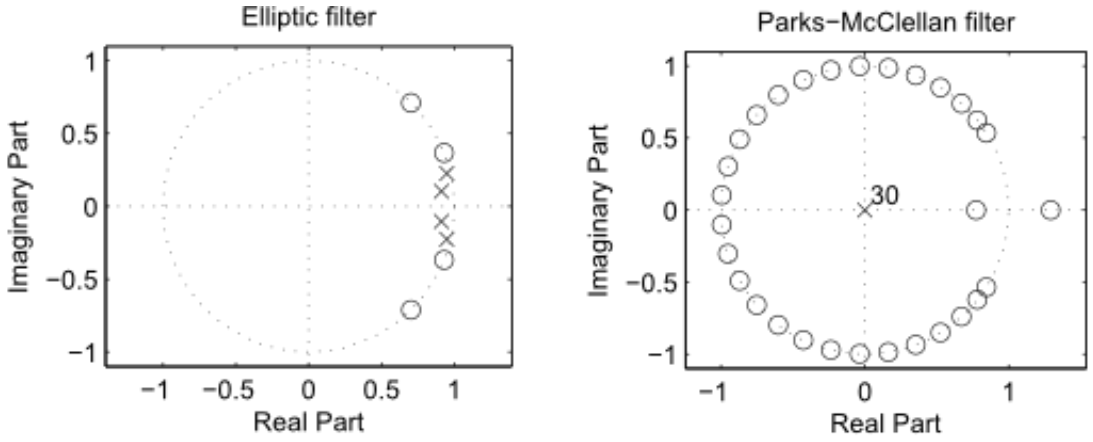

Figure 7.5b

Problem 7.5 Part a

0.0/1.0 point (ungraded)

Fill in the table describing how many multiplies per input sample and multiplies per output sample the direct form implementation of each of these filters requires.

Filter Type	Per Input Sample	Per Output Sample
Elliptic	**Answer:** 72	**Answer:** 45
Parks-McClellan	**Answer:** 248	**Answer:** 155

Which filter requires fewer multiplies per output sample?

- ◉ Elliptic ✔
- ○ Parks-McClellan

Answer: (see below)

Reasoning: The direct form implementation of the elliptic IIR filter requires 5 multiplies for the nonrecursive part and 4 multiplies for the recursive part, so it has

9 multiplies in total. The direct form implementation of the Parks-McClellan FIR filter has 31 multiplies, since the FIR filter has 31 coefficients. The number of multiplies per input sample is eight times this, and the number of multiplies per output sample is five times this. The table of multiplies for these two filters is shown below:

Filter Type	Per Input Sample	Per Output Sample
Elliptic	72	45
Parks-McClellan	248	155

Implemented in direct form, the elliptic filter is the more efficient filter in terms of the number of multiplies.

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Answers are displayed within the problem

Problem 7.5 Part b

0.0/1.0 point (ungraded)

Consider the most multiplication-efficient polyphase decompositions of the direct form implementations of the nonrecursive parts of each filter. How many multiplies per input sample and multiplies per output sample are required for each filter using polyphase decomposition?

Filter Type	Per Input Sample	Per Output Sample
Elliptic		
	Answer: 37	**Answer:** 23.125
Parks-McClellan		
	Answer: 31	**Answer:** 19.375

Which filter requires fewer multiplies per output sample?

Elliptic

Parks-McClellan ✔

Answer: (see below)

Reasoning: The polyphase decomposition can only be applied to the nonrecursive parts, leaving the poles of the elliptic filter unaffected. The most efficient polyphase decomposition is achieved by splitting the nonrecursive part and commuting across the expander, since the expansion rate is higher than the decimation rate. Thus, the number of multiplies per input sample and per output sample for the nonrecursive parts is reduced by a factor of 8. In particular,

- for the elliptic filter:
  the number of multiplies per input sample: $\mathbf{5 + 4 \times 8 = 37}$
  the number of multiplies per output sample: $\mathbf{37 \times \frac{5}{8} = 23.125}$

- for the Parks-McClellan filter:
  the number of multiplies per input sample: $\mathbf{31}$
  the number of multiplies per output sample: $\mathbf{31 \times \frac{5}{8} = 19.375}$

The results are summarized in the following table.

Filter Type	Per Input Sample	Per Output Sample
Elliptic	37	23.125
Parks-McClellan	31	19.375

Implemented using polyphase decomposition, the Parks-McClellan filter now requires fewer multiplies.

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Answers are displayed within the problem