

Capítulo 14

1. Índice 1.
2. Índice = 2
4. 25 m 5. 10 m
6. a) $\frac{3}{5}$ c) $\frac{3}{4}$ e) $\frac{3}{5}$
b) $\frac{4}{5}$ d) $\frac{4}{5}$ f) $\frac{4}{3}$
7. Sim
8. a) 90°
b) $\sin \hat{B} = \frac{z}{x}$; $\cos \hat{B} = \frac{y}{x}$; $\tan \hat{B} = \frac{z}{y}$;
 $\sin \hat{C} = \frac{y}{x}$; $\cos \hat{C} = \frac{z}{x}$; $\tan \hat{C} = \frac{y}{z}$
9. a) $\sin \hat{G} = \frac{\sqrt{11}}{6}$; $\cos \hat{G} = \frac{5}{6}$;
 $\tan \hat{G} = \frac{\sqrt{11}}{5}$
b) 25 cm e $5\sqrt{11}$ cm
c) 1; $\frac{5\sqrt{11}}{11}$; 1; $\frac{\sqrt{11}}{5}$
11. a) $\sin \alpha = \frac{5}{13}$; $\tan \alpha = \frac{5}{12}$
b) 41,6 m
12. 0,173 13. 0,3098
14. Poste: 18 m; fios: 21,2 m e 26,9 m aproximadamente
15. AC = 8 cm; $m(\hat{B}) \approx 53^\circ$; $m(\hat{C}) \approx 37^\circ$
16. a) $\sin \hat{B} = 0,6$; $\cos \hat{B} = 0,8$;
 $\tan \hat{B} = 0,75$; $\hat{B} \approx 37^\circ$
b) $\sin \hat{B} = \frac{1}{2}$; $\cos \hat{B} = \frac{\sqrt{3}}{2}$;
 $\tan \hat{B} = \frac{\sqrt{3}}{3}$; $\hat{B} = 30^\circ$
17. a) $\sin 45^\circ = \frac{\sqrt{2}}{2}$; $\cos 45^\circ = \frac{\sqrt{2}}{2}$;
 $\tan 45^\circ = 1$
b) $\sin 30^\circ = \frac{1}{2}$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$;
 $\tan 30^\circ = \frac{\sqrt{3}}{3}$
c) $\sin 60^\circ = \frac{\sqrt{3}}{2}$; $\cos 60^\circ = \frac{1}{2}$;
 $\tan 60^\circ = \sqrt{3}$
18. a) $x = 6$ c) $x = 6$ e) $x = 10$
b) $x = 9,4$ d) $x = 6$ f) $x = 5,12$
19. $x = 50\sqrt{3}$ 20. 5,1 m 21. 22 m
22. $(20 + 6\sqrt{3})$ m 23. $50\sqrt{3}$ m
24. 540 m; aproximadamente 2 062 m.
25. R\$ 10,00 26. 59,7 m 27. 5,04 m
28. 120 m

29. b) $600(3 - \sqrt{3})$ m ≈ 780 m
30. 4,08 m 31. $5\sqrt{3}$ e 5
32. 188 unidades de comprimento
33. 391,40 m 34. 6 633 km
35. 30° 36. Aproximadamente 41° .
37. a) 5 m/s^2 b) $35\sqrt{3}$ N
38. 160 J 39. $\frac{\sqrt{6}}{2}$

Desafio em dupla

$$h = p \cdot \frac{\tan \beta \cdot \tan \alpha}{\tan \alpha - \tan \beta}$$

Para refletir

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- Para que o índice seja 1, a altura e o afastamento devem ser iguais.
Para que o índice seja maior que 1, a altura deve ser maior que o afastamento.
- A de índice de subida maior.

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- $\alpha < \beta$; $\frac{h_1}{a_1} < \frac{h_2}{a_2}$.
- Ângulo α ; $\sin \alpha$; ângulo de subida α (de seno maior).
- A que tem ângulo de subida β ; $\cos \beta$; ângulo de subida α (de cosseno menor).

Capítulo 15

1. a) $\frac{\sqrt{2}}{2}$ c) $\frac{1}{2}$
b) $-\frac{\sqrt{2}}{2}$ d) $-\frac{\sqrt{3}}{2}$
2. a) 0 b) 0
3. $100\sqrt{2}$ 4. $x = 2$; $y = 0,73$
5. $\sqrt{3}$; $\frac{\sqrt{6} + \sqrt{2}}{2}$ 6. $\sqrt{2}$
7. a) Aproximadamente 9,151.
b) Aproximadamente 5,959.
c) Aproximadamente 0,705.
8. $\frac{m \cdot \sin \alpha}{\sin \beta}$ 9. $\frac{50(\sin \alpha + \sin \beta)}{\sin (\alpha + \beta)}$
10. $\frac{m \cdot \sin (45^\circ - \alpha)}{\sin \alpha}$ 11. $\sqrt{7}$
12. $\sqrt{10}$ 13. $\frac{1}{9}$
14. 14 cm 15. $2\sqrt{17}$
16. Diagonal BD = $2\sqrt{39}$ cm;
diagonal AC = $2\sqrt{109}$ cm
17. 45° 18. $4\sqrt{19}$ N 19. $\frac{3\sqrt{7}}{8}$

20. $r\sqrt{2(1 - \cos 36^\circ)}$
21. $\alpha = 62^\circ$; $x = 4,13$; $y = 4,76$
22. Aproximadamente 5 459 m.
23. Aproximadamente 496,7 m.
24. $2\sqrt{39}$ m 25. 111,6 km

Desafio em equipe

Aproximadamente $56,6^\circ$.

Para refletir

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No losango dado, cada ângulo mede 20° . Como a soma das medidas dos ângulos internos de qualquer quadrilátero é 360° , temos:
 $x + x + 20^\circ + 20^\circ = 360^\circ \Rightarrow$
 $\Rightarrow 2x = 360^\circ - 40^\circ = 320^\circ \Rightarrow x = 160^\circ$

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1. a) $\frac{\pi}{3}$ rad d) $\frac{5\pi}{3}$ rad
b) $\frac{\pi}{4}$ rad e) $\frac{3\pi}{8}$ rad
c) $\frac{7\pi}{6}$ rad f) $\frac{11\pi}{48}$ rad
2. a) 30° d) 150°
b) 36° e) $67^\circ 30'$
c) 45° f) $11^\circ 15'$
3. 5 rad 4. Aproximadamente 1,57 cm.
5. 31,4 cm
6. Aproximadamente 15,7 cm.
7. a) $\alpha = 60^\circ + k \cdot 360^\circ$, com $k \in \mathbb{Z}$
b) $\alpha = 120^\circ + k \cdot 360^\circ$, com $k \in \mathbb{Z}$
c) $\alpha = 240^\circ + k \cdot 360^\circ$, com $k \in \mathbb{Z}$
d) $\alpha = 300^\circ + k \cdot 360^\circ$, com $k \in \mathbb{Z}$
e) $\alpha = \frac{\pi}{3} + 2k\pi$, com $k \in \mathbb{Z}$
f) $\alpha = \frac{2\pi}{3} + 2k\pi$, com $k \in \mathbb{Z}$
g) $\alpha = \frac{5\pi}{4} + 2k\pi$, com $k \in \mathbb{Z}$
h) $\alpha = \frac{11\pi}{6} + 2k\pi$, com $k \in \mathbb{Z}$
8. a) 325° g) $\frac{3\pi}{2}$ rad
b) 60° h) $\frac{4\pi}{3}$ rad
c) 60° i) $\frac{11\pi}{6}$ rad
d) 130° j) $\frac{\pi}{5}$ rad
e) 320° l) $\frac{\pi}{2}$ rad
f) 130° m) $\frac{\pi}{4}$ rad
9. a) $\alpha = 80^\circ + k \cdot 360^\circ$, $k \in \mathbb{Z}$
b) $\alpha = 60^\circ + k \cdot 360^\circ$, $k \in \mathbb{Z}$
c) $\alpha = 200^\circ + k \cdot 360^\circ$, $k \in \mathbb{Z}$

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- d) $\alpha = \frac{\pi}{4} + 2k\pi$, com $k \in \mathbb{Z}$
 e) $\alpha = \frac{\pi}{3} + 2k\pi$, com $k \in \mathbb{Z}$
 f) $\alpha = \frac{3\pi}{5} + 2k\pi$, com $k \in \mathbb{Z}$
10. a) $30^\circ + k \cdot 360^\circ$, com $k \in \mathbb{Z}$ ou $\frac{\pi}{6} + 2k\pi$, com $k \in \mathbb{Z}$
 b) $45^\circ + k \cdot 180^\circ$, com $k \in \mathbb{Z}$ ou $\frac{\pi}{4} + k\pi$, com $k \in \mathbb{Z}$
 c) $180^\circ + k \cdot 360^\circ$, com $k \in \mathbb{Z}$ ou $\pi + 2k\pi$, com $k \in \mathbb{Z}$ ou $(2k+1)180^\circ$, com $k \in \mathbb{Z}$ ou $(2k+1)\pi$, com $k \in \mathbb{Z}$
 d) $120^\circ + k \cdot 360^\circ$, com $k \in \mathbb{Z}$ ou $\frac{2\pi}{3} + 2k\pi$, com $k \in \mathbb{Z}$
 e) $-60^\circ + k \cdot 360^\circ$, com $k \in \mathbb{Z}$ ou $-\frac{\pi}{3} + 2k\pi$, com $k \in \mathbb{Z}$
 f) $150^\circ + k \cdot 180^\circ$, com $k \in \mathbb{Z}$ ou $\frac{5\pi}{6} + k\pi$, com $k \in \mathbb{Z}$
11. a) Fakie 360: giro de uma volta completa; 540 McTwist: giro de uma volta completa e mais meia-volta; 720 McHawk: giro de duas voltas completas; 900: giro de duas voltas completas e mais meia-volta.
 b) 540 McTwist e 900.
12. a) 315°
 b) $\frac{\pi}{2}$ cm
 c) 130°
 d) $\frac{2\pi}{3} + 2k\pi$, com $k \in \mathbb{Z}$
13. b 14. e 15. d

Desafio em equipe

- a) 200 gr; 400 gr c) 63,66 gr
 b) 3º quadrante d) $0,9^\circ = 54'$

Para refletir

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- Sim; não.
- Aproximadamente 58° .

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- Porque os arcos são considerados com medidas positivas, negativas ou nulas.
- $B(0, 1)$; $A'(-1, 0)$ e $B'(0, -1)$

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- Quando k é negativo, estamos percorrendo a circunferência no sentido horário.
- Do 2º exemplo, podemos afirmar que são congruos 45° e 765° ou $\frac{\pi}{4}$ e $-\frac{17\pi}{4}$.

- Do 3º exemplo, podemos afirmar que são congruos 60° e -1020° ou $\frac{\pi}{3}$ e $-\frac{17\pi}{3}$.

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Significa um número positivo ou zero.

1. a) 3º quadrante
 b) 1º quadrante
 c) 4º quadrante
2. a) 3º ou 4º quadrante
 b) 2º ou 3º quadrante
 c) 1º ou 4º quadrante
 d) 1º ou 2º quadrante
3. $-\frac{4}{5}$
4. a) $\frac{1}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $-\frac{1}{2}$
5. a) 0,985 e) 0,956
 b) -0,423 f) -0,087
 c) -0,766 g) 0,998
 d) -0,927 h) -0,574
6. a) 270° c) $\frac{\pi}{3}$ e) $\nexists x$
 b) $\frac{\pi}{6}$ ou $\frac{5\pi}{6}$ d) 0
7. a) $-\frac{\sqrt{3}}{2}$ d) $\frac{\sqrt{3}}{2}$
 b) $\frac{\sqrt{2}}{2}$ e) $-\frac{\sqrt{2}}{2}$
 c) $-\frac{1}{2}$ f) $-\frac{1}{2}$
8. a) 0,883 c) -0,996
 b) -0,643 d) 0,643
9. a) 60° ou 330° b) $\frac{3\pi}{4}$ ou $\frac{5\pi}{4}$
10. a) $\frac{1}{2}$ d) $\frac{\sqrt{2}}{2}$ g) 1
 b) $\frac{\sqrt{2}}{2}$ e) -1 h) $-\frac{1}{2}$
 c) 0 f) $-\frac{\sqrt{3}}{2}$
11. a) -0,643 c) -0,643
 b) -0,242 d) 0,588
12. a) $\frac{3\pi}{2} + 2k\pi$, com $k \in \mathbb{Z}$
 b) $\frac{\pi}{4} + 2k\pi$ ou $\frac{3\pi}{4} + 2k\pi$, com $k \in \mathbb{Z}$
 c) $\frac{7\pi}{6} + 2k\pi$ ou $\frac{11\pi}{6} + 2k\pi$, com $k \in \mathbb{Z}$
 d) $k\pi$, com $k \in \mathbb{Z}$
13. a) $\frac{\sqrt{2}}{2}$ d) $\frac{1}{2}$ g) -1
 b) $\frac{\sqrt{3}}{2}$ e) $\frac{\sqrt{3}}{2}$ h) $-\frac{\sqrt{3}}{2}$
 c) 0 f) $\frac{\sqrt{2}}{2}$
14. a) -0,766 c) 0,122
 b) 0,985 d) 0,707

15. a) $\frac{\sqrt{2}}{2}$ g) $\frac{1+\sqrt{3}}{2}$
 b) $\frac{7\sqrt{3}}{2}$ h) 0
 c) $\sqrt{3}$ i) $\frac{\sqrt{3}}{2}$
 d) 1 j) $\sqrt{3}$
 e) 1 l) $\frac{\sqrt{3}}{2}$
 f) $\frac{1+\sqrt{3}}{2}$
16. a) 45° ou 315°
 b) $\frac{\pi}{2}$ ou $\frac{3\pi}{2}$
 c) $\frac{5\pi}{6} + 2k\pi$ ou $\frac{7\pi}{6} + 2k\pi$, com $k \in \mathbb{Z}$
 d) $\pi + 2k\pi$, com $k \in \mathbb{Z}$ ou $(2k+1)\pi$, com $k \in \mathbb{Z}$
17. $-\frac{\sqrt{6}}{3}$
18. $\cos x = \pm \frac{\sqrt{3}}{2}$; $x = \frac{\pi}{6}$ e $x = \frac{5\pi}{6}$
19. a) 0 g) $\frac{\sqrt{3}}{3}$
 b) 0 h) $-\sqrt{3}$
 c) $\frac{\sqrt{3}}{3}$ i) -1
 d) Não é definida. j) $\sqrt{3}$
 e) 1 l) $\frac{\sqrt{3}}{3}$
 f) $\sqrt{3}$ m) $-\frac{\sqrt{3}}{3}$
20. a) -5,671 d) 0,176
 b) 0,364 e) 2,050
 c) -1,428 f) 1,192
21. $x \in \mathbb{R} \mid x = \frac{\pi}{4} + k\pi$, com $k \in \mathbb{Z}$
22. a) $\{x \in \mathbb{R} \mid x = \frac{\pi}{3} + k\pi\}$
 b) $\{x \in \mathbb{R} \mid x = \frac{3\pi}{4} + k\pi\}$
23. a) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$
 b) $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$
24. $-\frac{3}{4}$ 25. $\frac{\sqrt{7}}{5}$ 26. -1

Desafios em dupla

- 40° , 50° e 14°
- 0

Para refletir

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Porque são os mais usados.

Capítulo 18

1. a) $\cos x = \frac{\sqrt{3}}{2}$, $\lg x = -\frac{\sqrt{3}}{3}$,
 $\cotg x = -\sqrt{3}$, $\sec x = \frac{2\sqrt{3}}{3}$,
 $\operatorname{cosec} x = -2$

$$b) \operatorname{sen} x = \frac{2\sqrt{2}}{3}, \operatorname{tg} x = 2\sqrt{2},$$

$$\operatorname{cotg} x = \frac{\sqrt{2}}{4}, \sec x = 3,$$

$$\operatorname{cosec} x = \frac{3\sqrt{2}}{4}$$

$$c) \operatorname{sen} x = -\frac{\sqrt{2}}{2}, \cos x = -\frac{\sqrt{2}}{2},$$

$$\operatorname{tg} x = 1, \operatorname{cotg} x = 1, \sec x = -\sqrt{2}$$

$$d) \operatorname{sen} x = \frac{\sqrt{3}}{2}, \cos x = \frac{1}{2},$$

$$\operatorname{cotg} x = \frac{\sqrt{3}}{3}, \sec x = 2;$$

$$\operatorname{cosec} x = \frac{2\sqrt{3}}{3}$$

$$2. -\frac{36}{25} \quad 3. \frac{1}{5} \quad 4. 2\sqrt{2} \quad 5. \frac{\sqrt{3}}{9}$$

$$6. a) \sec x \quad b) 1 \quad 7. \frac{1}{2} \quad 8. -2\sqrt{2}$$

$$9. \frac{9}{4} \quad 10. \frac{1}{2} \quad 11. \frac{1 + \cos^2 x}{\cos x}$$

$$13. \frac{t}{t-1}$$

$$19. a) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{4} + 2k\pi \text{ ou } x = \frac{7\pi}{4} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$b) \left\{ x \in \mathbb{R} \mid x = \frac{2\pi}{3} + k\pi, k \in \mathbb{Z} \right\}$$

$$c) \left\{ x \in \mathbb{R} \mid x = \frac{7\pi}{6} + 2k\pi \text{ ou } x = \frac{11\pi}{6} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$d) \{ x \in \mathbb{R} \mid x = \pi + 2k\pi, k \in \mathbb{Z} \}$$

$$e) \emptyset$$

$$f) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{4} + 2k\pi \text{ ou } x = \frac{7\pi}{4} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$20. a) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{6} + k \cdot \frac{2\pi}{3}, k \in \mathbb{Z} \right\}$$

$$b) \left\{ x \in \mathbb{R} \mid x = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$c) \left\{ x \in \mathbb{R} \mid x = k \cdot \frac{\pi}{5}, k \in \mathbb{Z} \right\}$$

$$d) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{6} + k \cdot \frac{2\pi}{3} \text{ ou } x = \frac{\pi}{3} + k \cdot \frac{2\pi}{3}, k \in \mathbb{Z} \right\}$$

$$e) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{4} + k \cdot \frac{\pi}{2}, k \in \mathbb{Z} \right\}$$

$$f) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{12} + k \cdot \frac{\pi}{2}, k \in \mathbb{Z} \right\}$$

$$g) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{8} + k\pi \text{ ou } x = \frac{7\pi}{8} + k\pi, k \in \mathbb{Z} \right\}$$

$$h) \emptyset$$

$$21. a) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + k\pi \text{ ou } x = \frac{\pi}{6} + 2k\pi \text{ ou } x = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$k \in \mathbb{Z}$$

$$b) \left\{ x \in \mathbb{R} \mid x = k\pi \text{ ou } x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$c) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{3} + k\pi \text{ ou } x = \frac{2\pi}{3} + k\pi, k \in \mathbb{Z} \right\}$$

$$d) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{6} + 2k\pi \text{ ou } x = \frac{5\pi}{6} + 2k\pi \text{ ou } x = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$x = \frac{5\pi}{6} + 2k\pi \text{ ou } x = \frac{3\pi}{2} + 2k\pi, k \in \mathbb{Z}$$

$$e) \left\{ x \in \mathbb{R} \mid x = \frac{11\pi}{36} + k\pi \text{ ou } x = \frac{61\pi}{36} + k\pi, k \in \mathbb{Z} \right\}$$

$$f) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{3} + 2k\pi \text{ ou } x = \frac{5\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$22. a) \left\{ x \in \mathbb{R} \mid x = \frac{5\pi}{4} + 2k\pi \text{ ou } x = \frac{7\pi}{4} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$b) \left\{ x \in \mathbb{R} \mid x = \frac{7\pi}{4} + k\pi, k \in \mathbb{Z} \right\}$$

$$c) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + k\pi \text{ ou } x = \frac{\pi}{6} + 2k\pi \text{ ou } x = \frac{11\pi}{6} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$d) \left\{ x \in \mathbb{R} \mid x = \frac{2\pi}{3} + 2k\pi \text{ ou } x = \frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$e) \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{6} + k\pi, k \in \mathbb{Z} \right\}$$

$$f) \left\{ x \in \mathbb{R} \mid x = \frac{k\pi}{4}, k \in \mathbb{Z} \right\}$$

23. \emptyset

Desafios em dupla

$$\bullet \left\{ x \in \mathbb{R} \mid \frac{\pi}{4} < x < \frac{3\pi}{4} \right\}$$

Para refletir

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$$x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

Capítulo 19

$$1. a) \sqrt{3} - 2$$

$$d) 2 + \sqrt{3}$$

$$b) \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$e) \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$c) \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$f) -\frac{\sqrt{6} + \sqrt{2}}{4}$$

$$2. \frac{6}{7}$$

$$3. \operatorname{sen}(a+b) = \frac{56}{65}; \cos(a-b) = \frac{63}{65};$$

$$\operatorname{tg}(a+b) = -\frac{56}{33}$$

$$4. \frac{t}{2+t^2} \quad 5. \operatorname{tg} x \quad 6. \frac{3}{7} \quad 7. c$$

$$8. A'(4\sqrt{3} - 3, 3\sqrt{3} + 4)$$

$$9. B'\left(\frac{4+7\sqrt{2}}{2}, \frac{19\sqrt{2}}{2}\right)$$

$$10. \operatorname{sen} 2x = 2mn; \cos 2x = n^2 - m^2; \operatorname{tg} 2x = \frac{2mn}{n^2 - m^2} \quad 11. \frac{8}{15}$$

$$14. \operatorname{sen} 2a = \frac{4\sqrt{5}}{9}; \cos 2a = \frac{1}{9};$$

$$\operatorname{tg} 2a = 4\sqrt{5}$$

$$15. -\frac{31}{17} \quad 16. 1 \quad 18. \sec x \quad 19. \frac{2}{3}$$

$$20. \frac{1}{8} \quad 23. \text{Aproximadamente } 3,6.$$

$$24. a) \frac{\sqrt{3}}{2}$$

$$b) \frac{\sqrt{6}}{6}$$

$$c) \operatorname{sen} \frac{x}{2} = \frac{\sqrt{10}}{10};$$

$$\cos \frac{x}{2} = \frac{3\sqrt{10}}{10}; \operatorname{tg} \frac{x}{2} = \frac{1}{3}$$

$$d) \operatorname{sen} 67^\circ 30' = \frac{\sqrt{2} + \sqrt{2}}{2};$$

$$\cos 67^\circ 30' = \frac{\sqrt{2} - \sqrt{2}}{2}$$

$$26. a) 2 \cdot \operatorname{sen} 10^\circ \cdot \cos 50^\circ$$

$$b) 2 \cdot \operatorname{sen} 2x \cdot \cos y$$

$$c) -2 \cdot \operatorname{sen} 3x \cdot \operatorname{sen} 2x$$

$$d) 2 \cdot \cos(a+b) \cdot \cos c$$

$$e) 4 \cdot \operatorname{sen} 4x \cdot \cos x \cdot \cos 2x$$

$$f) -\operatorname{cotg} \frac{x+y}{2} \text{ ou } -\cos \frac{x+y}{2} \cdot \operatorname{cosec} \frac{x+y}{2}$$

$$27. a) \operatorname{tg} \frac{a+b}{2} \quad b) \operatorname{cotg} a$$

$$28. a) 2 \cdot \operatorname{sen}^2 \frac{x}{2}$$

$$b) 2 \cdot \cos^2 x$$

$$c) \sqrt{2} \cdot \operatorname{sen}\left(x - \frac{\pi}{4}\right)$$

$$d) 2 \cdot \operatorname{sen} 55^\circ \cdot \cos 5^\circ$$

$$29. a) -4 \cdot \operatorname{sen} x \cdot \operatorname{sen}^2 \frac{x}{2}$$

$$b) 2 \cdot \operatorname{sen} x \cdot \cos^2 2x$$

Capítulo 20

$$1. a) \{m \in \mathbb{R} \mid 3 \leq m \leq 4\}$$

$$b) \{m \in \mathbb{R} \mid 4 \leq m \leq 6\}$$

$$c) \left\{ m \in \mathbb{R} \mid \frac{1}{3} \leq m \leq 1 \right\}$$

$$d) \{m \in \mathbb{R} \mid -2 \leq m \leq -1 \text{ ou } 0 \leq m \leq 1\}$$

$$e) \{m \in \mathbb{R} \mid -\sqrt{2} \leq m \leq \sqrt{2}\}$$

$$f) \left\{ m \in \mathbb{R} \mid 0 \leq m \leq \frac{1}{2} \right\}$$

$$2. a) \{m \in \mathbb{R} \mid -3 \leq m \leq -2\}$$

$$b) \{m \in \mathbb{R} \mid 2 \leq m \leq 4\}$$

- c) $\{m \in \mathbb{R} \mid -\frac{5}{3} \leq m \leq -1\}$
 d) $\{m \in \mathbb{R} \mid -\frac{2}{3} \leq m \leq 0 \text{ ou } \frac{1}{3} \leq m \leq 1\}$
 e) $\{m \in \mathbb{R} \mid -\sqrt{2} \leq m \leq \sqrt{2}\}$
 f) $\{m \in \mathbb{R} \mid 1 \leq m \leq \frac{7}{5}\}$
3. a) $f(\pi) = 0; g(\pi) = -1;$
 $f\left(\frac{\pi}{3}\right) - g\left(\frac{\pi}{4}\right) = \frac{\sqrt{3} - \sqrt{2}}{2};$
 $\frac{f\left(\frac{\pi}{6}\right)}{g\left(\frac{\pi}{6}\right)} = \frac{\sqrt{3}}{3};$
 $f\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2};$
 $g\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}.$
- b) $\frac{\pi}{4}$ ou $\frac{5\pi}{4}$
 c) Não existe.
 d) $\frac{3\pi}{2} \leq x < 2\pi$
4. a) -9 e -11 c) 4 e 1
 b) 16 e 4 d) 2 e -2
5. a) 0
 b) 2
 c) $\frac{\sqrt{3}}{2}$
 d) \mathbb{R}
 e) $[0, 2]$
 f) $\frac{\pi}{8}$ ou $\frac{5\pi}{8}$ ou $\frac{9\pi}{8}$ ou $\frac{13\pi}{8}$
6. a) $D(f) = \mathbb{R}; \text{Im}(f) = [-1, 1]; p = \frac{2\pi}{3}$
 b) $D = \mathbb{R}; \text{Im} = [0, 1]; p = \pi$
7. $\text{Im}(f) = [2, 4];$
 $\text{Im}(g) = [-3, 3]$
8. $D(g) = \mathbb{R}; \text{Im}(g) = [-2, 2]; p = 4\pi$
9. d) $\frac{2\pi}{7}$ b) π c) π d) 1 e) 2
10. a) $\pm \frac{2}{3}$ b) $\pm \frac{5}{2}$ c) ± 2
11. d 12. e
13. a) $\{t \mid t = 12k - \frac{15}{2}, t \in \mathbb{Z}\}$
 b) 4,5 horas
14. a) $\{12\}$ b) $20^\circ; 15h$
15. $\forall x, y = 2 \cdot \sin\left(\frac{\pi}{4}x\right)$
16. $h(x) = 0,3 \cdot \sin(\pi x)$ 17. b
18. $\frac{1}{2}$ e 2 19. b
20. $A = 2, \omega = \frac{\pi}{2}$ e $\phi = -\frac{3\pi}{2}$

Para refletir

Página 231

Como $k \in \mathbb{Z}$, temos:

$$x = 0 + 2k\pi \Rightarrow \sin x = 0$$

$$x = \frac{\pi}{2} + 2k\pi \Rightarrow \sin x = 1$$

$$x = \pi + 2k\pi \Rightarrow \sin x = 0$$

$$x = \frac{3\pi}{2} + 2k\pi \Rightarrow \sin x = -1$$

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$$\cos(0 + 2k\pi) = 1; \cos\left(\frac{\pi}{2} + 2k\pi\right) = 0;$$

$$\cos(\pi + 2k\pi) = -1; \cos\left(\frac{3\pi}{2} + 2k\pi\right) = 0$$

Página 234

Para $f(x) = 3 \cdot \sin x$, o intervalo da imagem triplicou.Para $f(x) = 1 + \cos x$, os extremos do intervalo que representa a imagem ficaram somados de uma unidade.

Capítulo 21

1.
$$\begin{bmatrix} 6 & 4 & 5 & 8 \\ 5 & 7 & 5 & 5 \\ 5 & 6 & 7 & 4 \end{bmatrix}$$

2. a) 2×2

c) 5×3

b) Matriz-coluna 3×1

3. a) $a_{11} = 2; a_{22} = -5; a_{13} = 10$

b) $a_{31} = 6; a_{23} = 2; a_{33} = \sqrt{2}$

4. a)
$$\begin{bmatrix} 2 & 5 & 10 \\ 5 & 8 & 13 \end{bmatrix}$$
 d)
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

b)
$$\begin{bmatrix} 0 & 2 & 4 \\ 3 & 5 & 7 \\ 6 & 8 & 10 \end{bmatrix}$$
 e)
$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

c)
$$\begin{bmatrix} 1 & 0 \\ 7 & 6 \\ 17 & 16 \\ 31 & 30 \end{bmatrix}$$
 f)
$$\begin{bmatrix} 2 & -2 \\ -4 & 4 \end{bmatrix}$$

5. -8 6. 18

7. a)
$$\begin{bmatrix} 5 & 3 \\ 9 & 7 \end{bmatrix}$$
 b)
$$\begin{bmatrix} -1 & -3 & -5 \\ 6 & 4 & 2 \\ 25 & 23 & 21 \end{bmatrix}$$

8. a)
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$
 b)
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

9. a)
$$\begin{bmatrix} 4 & -2 & -2 & -2 \\ 0 & 16 & -2 & -2 \\ 0 & 0 & 36 & -2 \\ 0 & 0 & 0 & 64 \end{bmatrix}$$

b)
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 8 & 8 \\ 0 & 0 & 27 \end{bmatrix}$$

10. $x = 6$ e $y = 2$

11. $a = 6; b = 3; c = -4; d = -2$

12. $x = 2; y = 0; z = 1; t = 3$

13. $m = 0$ e $n = 1$

14. $a = 1, b = 0$ e $c = \frac{1}{3}$

15. $x = 3; y = 10; z = 3$

16. a)
$$\begin{pmatrix} 2 & 6 \\ -6 & -1 \end{pmatrix}$$
 c)
$$\begin{pmatrix} 7 & 2 \\ -11 & 2 \end{pmatrix}$$

b)
$$\begin{pmatrix} 1 & 4 \\ -5 & 1 \end{pmatrix}$$
 d)
$$\begin{pmatrix} 5 & 6 \\ -11 & 1 \end{pmatrix}$$

17. a) $x = 7, y = -3$ e $z = 0$

b) $x = 5, y = -4, z = 6$ e $t = 1$

18. a)
$$\begin{bmatrix} 8 & 2 \\ 6 & -8 \end{bmatrix}$$
 c)
$$\begin{bmatrix} 24 & 6 \\ 18 & -24 \end{bmatrix}$$

b)
$$\begin{bmatrix} 12 & 3 \\ 9 & -12 \end{bmatrix}$$

19. a)
$$\begin{bmatrix} 4 & 2 \\ 10 & 8 \\ 16 & 14 \end{bmatrix}$$
 b)
$$\begin{bmatrix} 2 & 1 \\ 5 & 4 \\ 8 & 7 \end{bmatrix}$$

20.
$$\begin{bmatrix} 4 & 0 \\ 12 & 14 \end{bmatrix}$$
 e
$$\begin{bmatrix} 4 & 0 \\ 12 & 14 \end{bmatrix}$$

21.
$$\begin{pmatrix} 4 & 11 \\ 6 & 13 \end{pmatrix}$$

22.
$$\begin{bmatrix} 0 & -3 \\ -2 & -5 \end{bmatrix}$$
 23.
$$\begin{bmatrix} 3 & -2 & 1 \\ -6 & 10 & -7 \end{bmatrix}$$

24. a)
$$\begin{pmatrix} 3 \\ 8 \\ 7 \end{pmatrix}$$
 b)
$$\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$$
 c)
$$\begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$$

25. a) $x = 13, y = -9$ e $z = 2$

b) $x = 6, y = 2$ e $z = -1$

26.
$$\begin{bmatrix} 0 & -2 \\ -12 & -2 \end{bmatrix}$$

27. a)
$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$
 d)
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

b)
$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$
 e)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

c)
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$
 f)
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

28. a)
$$\begin{pmatrix} 10 & 0 & -5 \\ -20 & 5 & 15 \end{pmatrix}$$

b)
$$\begin{pmatrix} 0 & 2 & -4 \\ -10 & 0 & -12 \end{pmatrix}$$

c)
$$\begin{pmatrix} 1 & 0 & -\frac{1}{2} \\ -2 & \frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

d)
$$\begin{pmatrix} 4 & -3 & 4 \\ 7 & 2 & 24 \end{pmatrix}$$