Statistical Inference and Modelling: Assignment 2

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Contents

```
# Clear plots
if(!is.null(dev.list())) dev.off()
```

```
## null device
##           1
```

```
# Clean workspace
rm(list=ls())

library(car)
```

```
## Loading required package: carData
```

```
library(MASS)
library(missMDA)
library(visdat)
library(FactoMineR)
library(chemometrics)
```

```
## Loading required package: rpart
```

```
library(corrplot)
```

```
## corrplot 0.92 loaded
```

```
library(naniar)
library(nortest)
library(visdat)
library(tidyverse)
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.3     v readr     2.1.4
## v forcats   1.0.0     v stringr   1.5.0
## v ggplot2   3.4.3     v tibble    3.2.1
## v lubridate 1.9.3     v tidyr     1.3.0
## v purrr     1.0.2
```

```
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## x dplyr::recode() masks car::recode()
## x dplyr::select() masks MASS::select()
## x purrr::some()   masks car::some()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```r
library(lsr)
library(effects)
```

```
## lattice theme set by effectsTheme()
## See ?effectsTheme for details.
```

First thing will be to load the data into R, and redeclaring the variables to properly comply with the needs of the analysis. Notice we relable the levels of the variable SeniorCitizen from (0,1) to (No, Yes) for practical reasons.

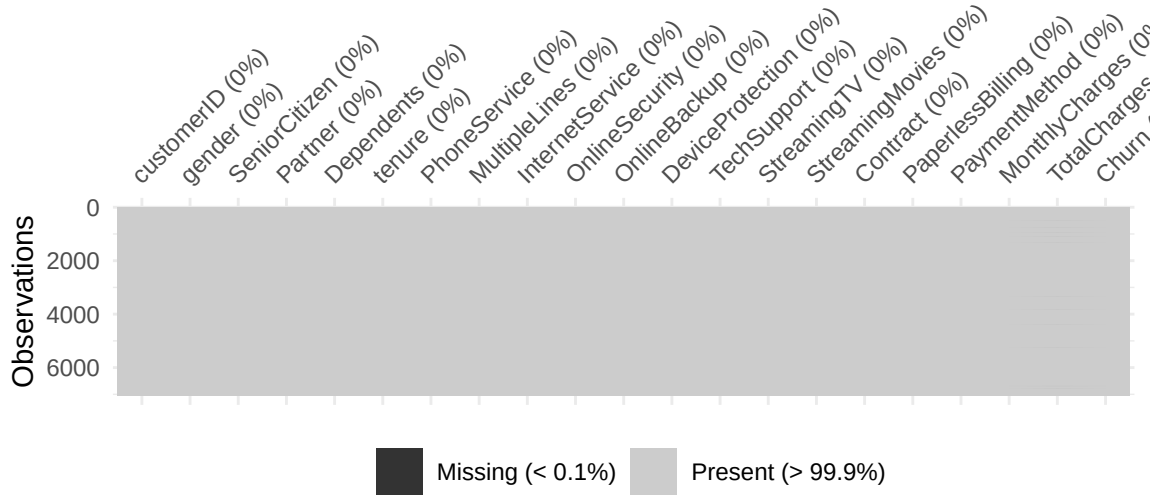
```r
#setwd("C:\\Users\\darry\\Documents\\MDS\\Statistical_Inference_And_Modelling\\SIM_Assignment2")
setwd("/Users/gabrielvayaabad/Documents/GitHub/SIM_Assignment2")
#setwd("C:/Users/Admin/Desktop/MÀSTER DATA SCIENCE/SIM/Assigment 2")
#setwd("D:/MDS/ADSDB/SIM_Assignment2")

df <- read.csv("WA_Fn-UseC_-Telco-Customer-Churn.csv")
#df <- WA_Fn_UseC_Telco_Customer_Churn

df$MonthlyCharges <- as.numeric(df$MonthlyCharges)
df$TotalCharges <- as.numeric(df$TotalCharges)
df[sapply(df, is.character)] <- lapply(df[sapply(df, is.character)], as.factor)
df$SeniorCitizen <- as.factor(df$SeniorCitizen)
df$customerID <- as.character(df$customerID)
levels(df$SeniorCitizen) <- c("No","Yes")
```

1 DataPreparation

2 Missing Values

For the missing data, we see that there are only 11 missing values. If we look even closer, we can see they are all in the same variable, TotalCharges. Analysing even further, we can see those missing values are corresponding to those clients with 0 month of tenure. Hence are imputing them with 0.

```r
vis_miss(df)
```




```
#df[is.na(df$TotalCharges),] #NAs only found in TotalCharges variable

mcar_test(df) #p-value is 0 -> not random
```

```
## # A tibble: 1 x 4
##   statistic    df p.value missing.patterns
##       <dbl> <dbl>   <dbl>            <int>
## 1      184.    20       0                2
```

```
df$tenure[is.na(df$TotalCharges)] #TotalCharges are NA when tenure is 0
```

```
##  [1] 0 0 0 0 0 0 0 0 0 0 0
```

```
df$TotalCharges[is.na(df$TotalCharges)] <- 0 #impute with 0
```

3 Deduplication

Once imputed, we look into the duplicate rows.

```
dup <- which(duplicated(df)) #No duplicate rows
```

4 Looking for errors

In the errors, we want to see, first of all we look for huge discrepancies between the total charges, and the product between MonthlyCharges and tenure months. We see some, but small, probably due to discounts or opening fees. Next, we want to see if the number of clients without phone service is the same throughout the variables. It is, 682. Finally, we do the same with internet service, and we see that it is the same, 1526.

```r
df.aux <- df
df.aux$TheoreticalTotalCharges <- df.aux$tenure*df.aux$MonthlyCharges
#df.aux[df.aux$TheoreticalTotalCharges > df.aux$TotalCharges,] #uncomment to see full table

#Look at phone service
table(df.aux$PhoneService)
```

```
##
##   No  Yes
##  682 6361
```

```r
table(df.aux$MultipleLines)
```

```
##
##               No No phone service              Yes
##             3390              682             2971
```

```r
#Look at internet service
table(df.aux$OnlineSecurity)
```

```
##
##                  No No internet service                 Yes
##                3498                1526                2019
```

```r
table(df.aux$OnlineBackup)
```

```
##
##                  No No internet service                 Yes
##                3088                1526                2429
```

```r
table(df.aux$InternetService)
```

```
##
##         DSL Fiber optic          No
##        2421        3096        1526
```

```r
table(df.aux$TechSupport)
```

```
##
##                  No No internet service                 Yes
##                3473                1526                2044
```

```r
table(df.aux$StreamingTV)
```

```
##
##                  No No internet service                 Yes
##                2810                1526                2707
```

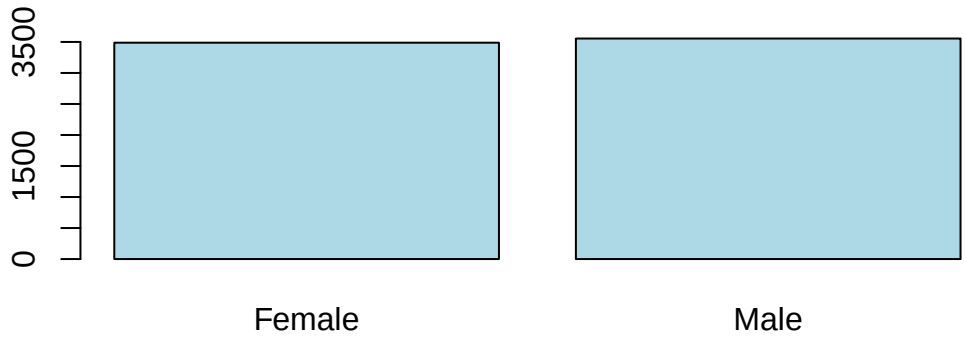
```
table(df.aux$StreamingMovies)
```

```
##
##                   No No internet service                  Yes
##                 2785                 1526                 2732
```

5 EDA (+ Univariate Outliers)

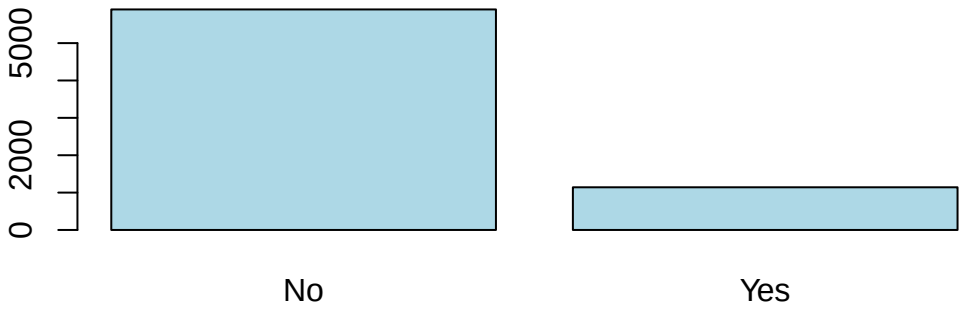
#Gender
This is the gender of the customer, which as we can see is balanced throughout the dataset.

```
na.gender <- sum(is.na(df$gender)) #No NAs
barplot(table(df$gender),col='lightblue') #Balanced
```



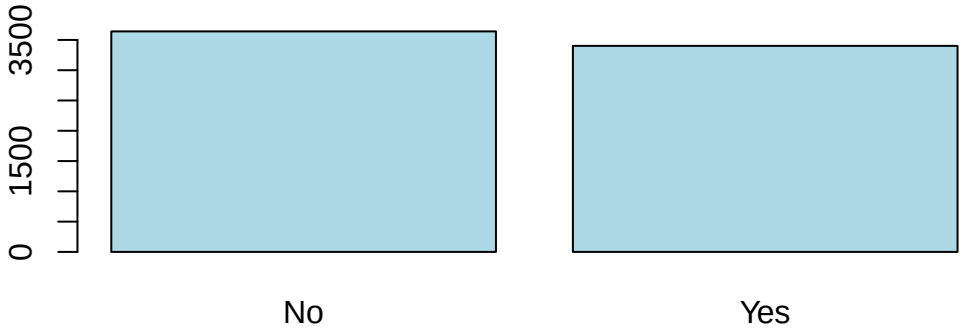
#SeniorCitizen
As far as SeniorCitizen, there is much less SeniorCitizens as you can imagine.

```
na.seniorcitizen <- sum(is.na(df$SeniorCitizen)) #No NAs
barplot(table(df$SeniorCitizen),col='lightblue') #Unbalanced
```



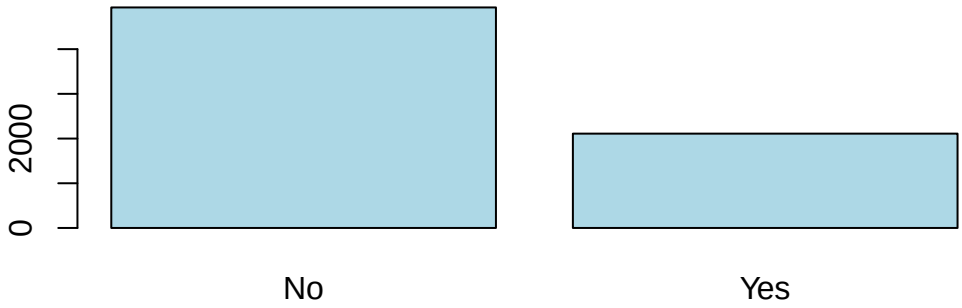
#Partner
Those are customers with partner, and we see it is pretty balanced.

```
na.partner <- sum(is.na(df$Partner)) #No NAs
barplot(table(df$Partner),col='lightblue') #Balanced
```



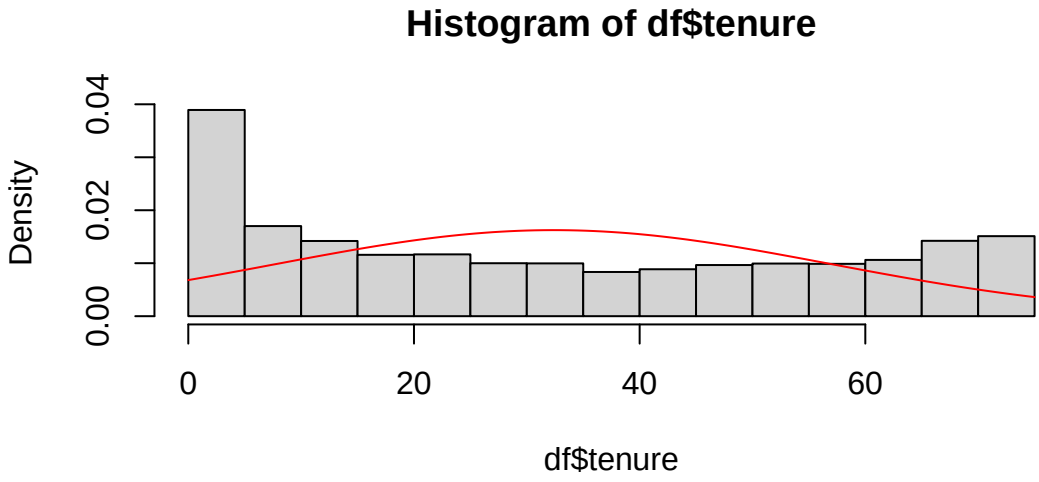
#Dependents
The dependent customers follow a similar distribution than the SeniorCitizen.

```
na.dependents <- sum(is.na(df$Dependents)) #No NAs
barplot(table(df$Dependents),col='lightblue') #Unbalanced
```



#Tenure
This is the months that a customers have been in the company. We can see that the majority of the customer base are recent customers. We can see that, using an Anderson-Darling test the variable is not normally distributied, and has no univariate outliers.

```
na.tenure <- sum(is.na(df$tenure)) #No NAs
hist(df$tenure,freq=F,15) #Young customers overrepresented
mm <- mean(df$tenure,na.rm=T);ss <- sd(df$tenure,na.rm=T);
curve(dnorm(x,mm,ss),col="red",add=T)
```

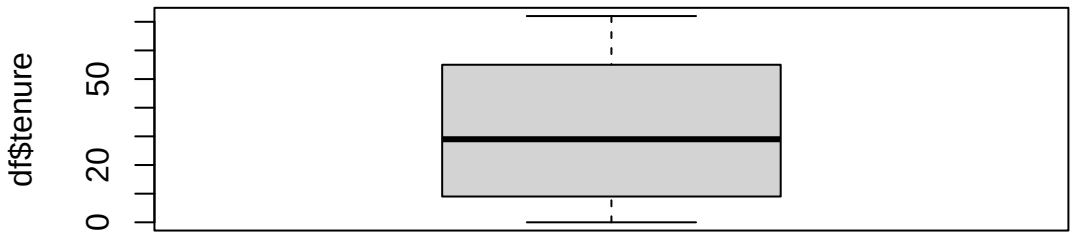



```
#shapiro.test(df$tenure) #Error: too many samples for shapiro test
ad.test(df$tenure) #Anderson-Darling test: Not normally distributed
```

```
##
##  Anderson-Darling normality test
```

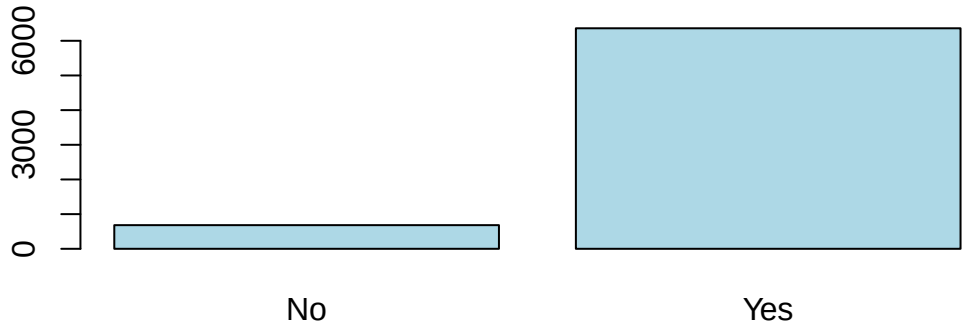
```
##
## data:  df$tenure
## A = 203.24, p-value < 2.2e-16
```

```
Boxplot(df$tenure,range=1.5,id=list(n=Inf,labels=rownames(df))) #No mild univariate outliers
Boxplot(df$tenure,range=3,id=list(n=Inf,labels=rownames(df))) #No extreme univariate outliers
```



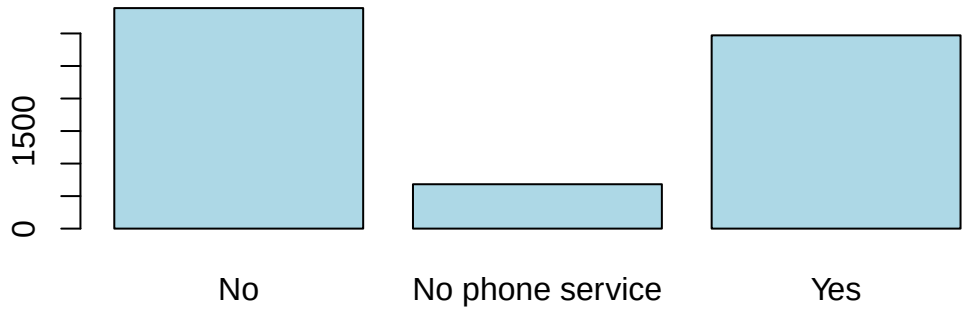
#PhoneService
Now we can see that the majority of the clients have PhoneService in the contract.

```
na.phoneservice <- sum(is.na(df$PhoneService)) #No NAs
barplot(table(df$PhoneService),col='lightblue') #Unbalanced
```



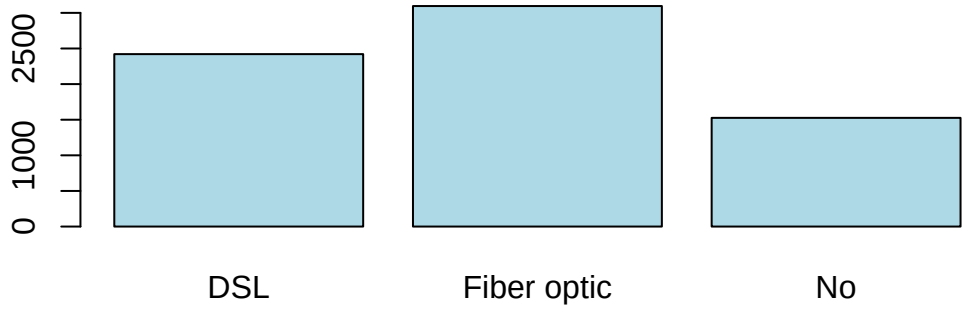
#MultipleLines
As we saw before, the majority of the clients have PhoneService, so they are underrepresented in this variable, but regarding those who have, MultipleLines is pretty balanced.

```r
na.multiplelines <- sum(is.na(df$MultipleLines)) #No NAs
barplot(table(df$MultipleLines),col='lightblue') #Unbalanced in "No phone service"
```



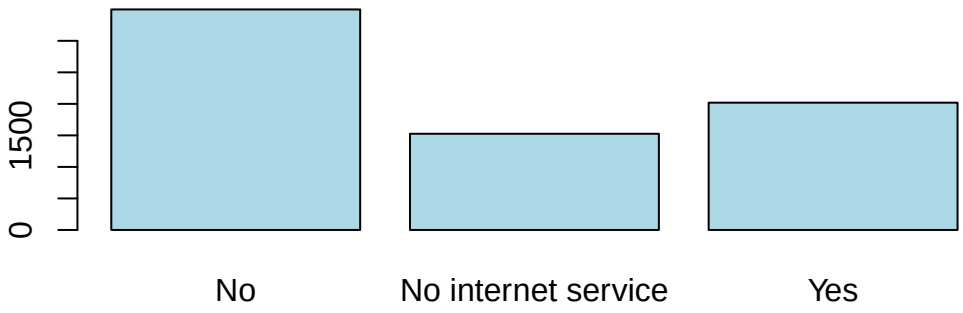
#InternetService
Most of the clients have fiber optic, and there are a few that don't have internet service.

```r
na.internetservice <- sum(is.na(df$InternetService)) #No NAs
barplot(table(df$InternetService),col='lightblue') #Unbalanced
```



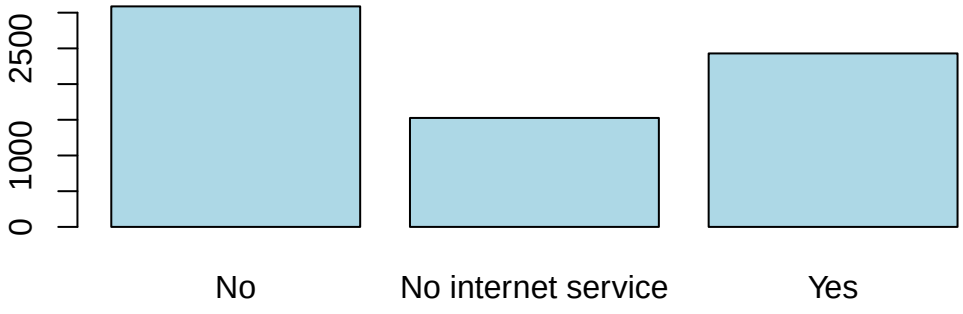
#OnlineSecurity
Those clients without internet service are a minority as they were in the previous variable, and most of those with internet service don't have OnlineSecurity.

```r
no.onlinesecurity <- sum(is.na(df$OnlineSecurity)) #No NAs
barplot(table(df$OnlineSecurity),col='lightblue') #Unbalanced in "No"
```



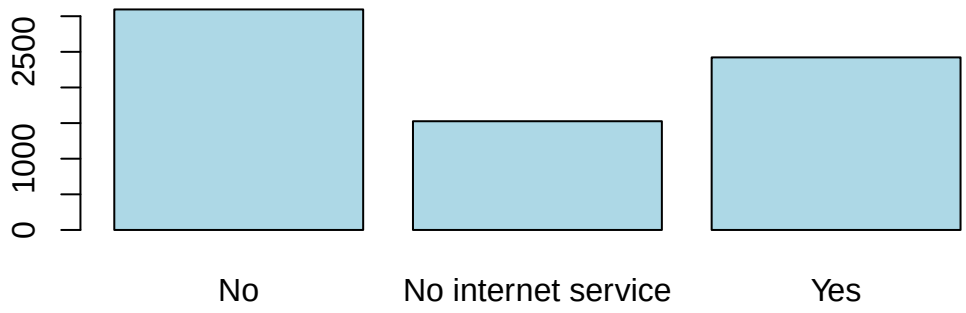
#OnlineBackup
Similar to previous variable

```
na.onlinebackup <- sum(is.na(df$OnlineBackup)) #No NAs
barplot(table(df$OnlineBackup),col='lightblue') #Unbalanced
```



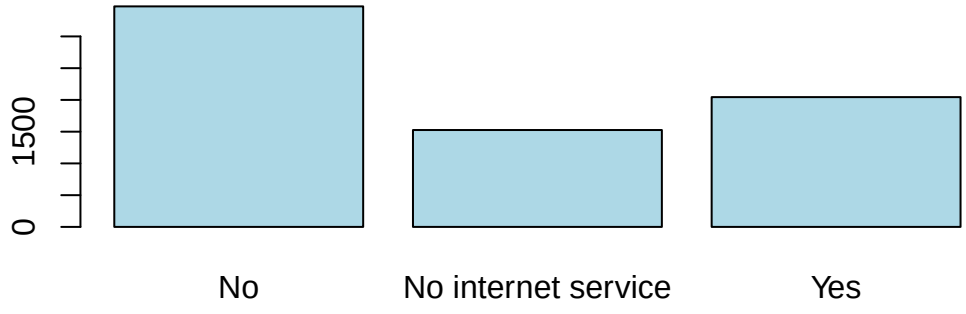
#DeviceProtection
Similar to previous variable

```
na.deviceprotection <- sum(is.na(df$DeviceProtection)) #No NAs
barplot(table(df$DeviceProtection),col='lightblue') #Unbalanced
```



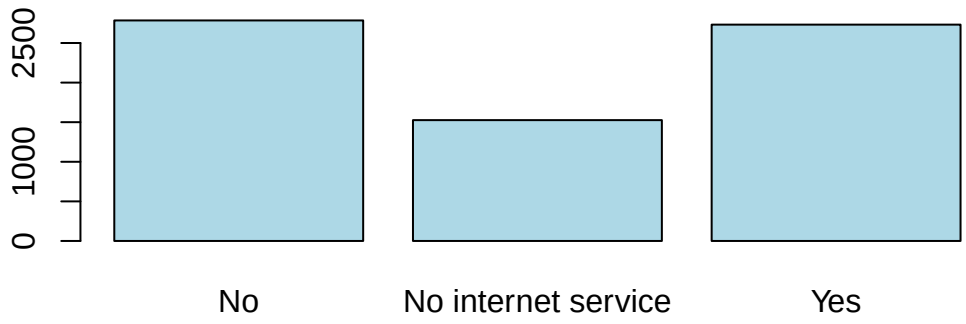
#TechSupport
Similar to previous variable

```
na.techsupport <- sum(is.na(df$TechSupport)) #No NAs
barplot(table(df$TechSupport),col='lightblue') #Unbalanced in "No"
```



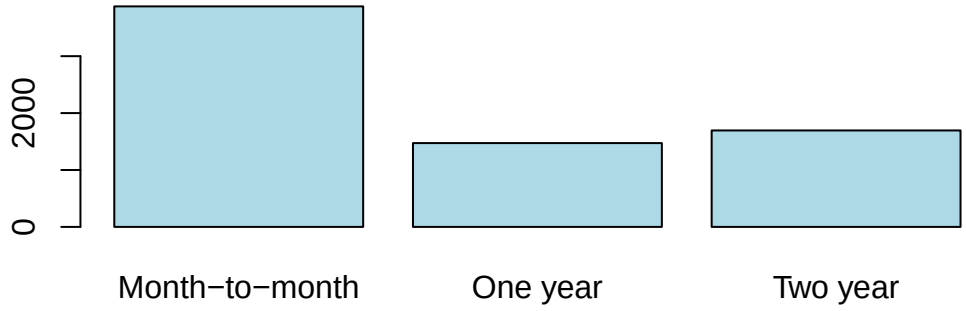
#StreamingMovies
In this case, disregarding the users without internet service, the categories are balanced.

```
na.streamingmovies <- sum(is.na(df$StreamingMovies)) #No NAs
barplot(table(df$StreamingMovies),col='lightblue') #Unbalanced in "No internet service"
```



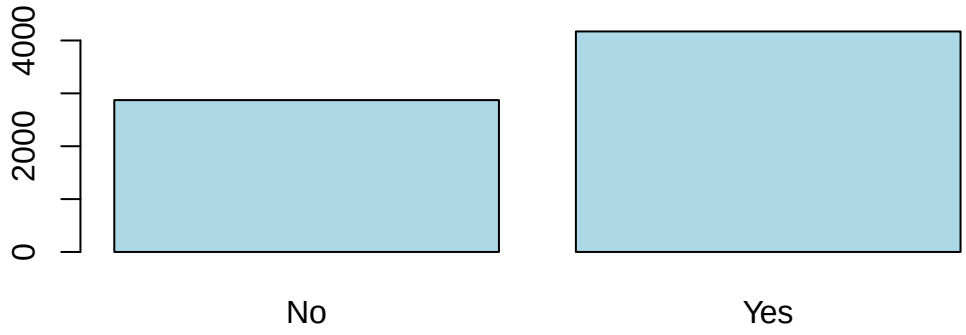
#Contract
Most of the contracts are month to month.

```
na.contract <- sum(is.na(df$Contract)) #No NAs
barplot(table(df$Contract),col='lightblue') #Unbalanced in "Month-to-month"
```



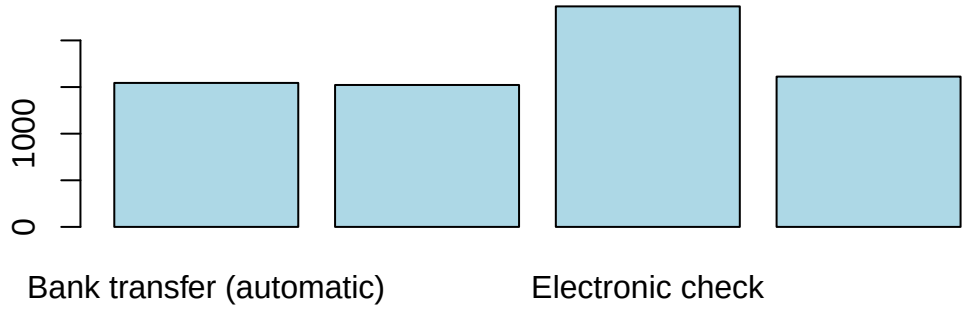
#PaperlessBilling
This variable is pretty much balanced, with more population towards having PaperlessBilling.

```
na.paperlessbilling <- sum(is.na(df$PaperlessBilling)) #No NAs
barplot(table(df$PaperlessBilling),col='lightblue') #Relatively balanced
```



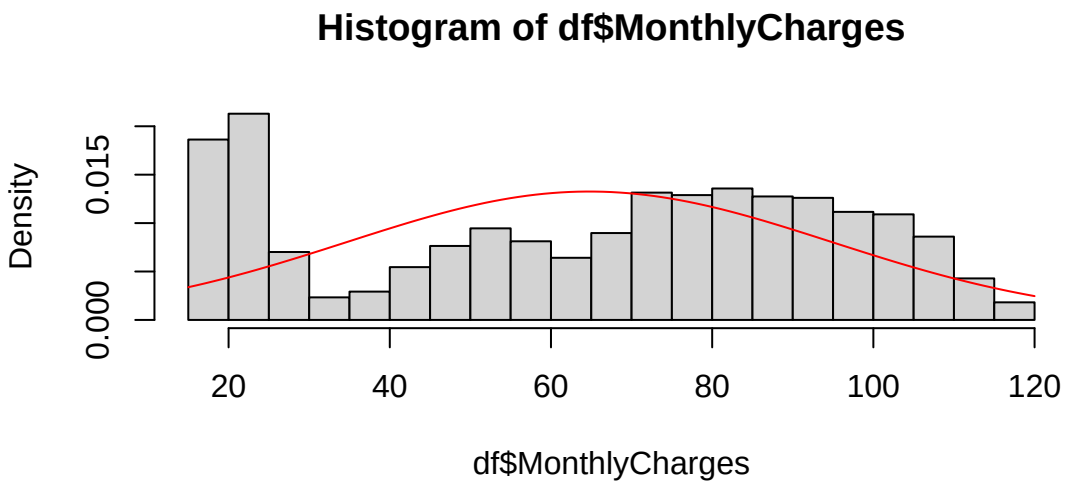
#PaymentMethod
Most of the customers use electronic check.

```
na.paymentmethod <- sum(is.na(df$PaymentMethod)) #No NAs
barplot(table(df$PaymentMethod),col='lightblue') #Unbalanced in "Electronic check"
```



#MonthlyCharges
We can see that most of population concentrates around 20 units of MothlyCharges. Not normally distributed. No univariate outliers.

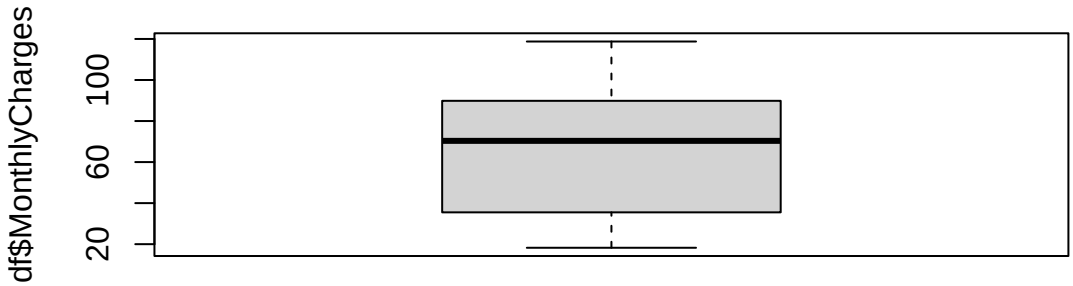
```
na.monthlycharges <- sum(is.na(df$MonthlyCharges)) #No NAs
hist(df$MonthlyCharges,freq=F,15)
mm <- mean(df$MonthlyCharges,na.rm=T)
ss <- sd(df$MonthlyCharges,na.rm=T)
curve(dnorm(x,mm,ss),col="red",add=T)
```




```
#shapiro.test(df$MonthlyCharges) #Error: too many samples for shapiro test
ad.test(df$MonthlyCharges) #Anderson-Darling test: Not normally distributed
```

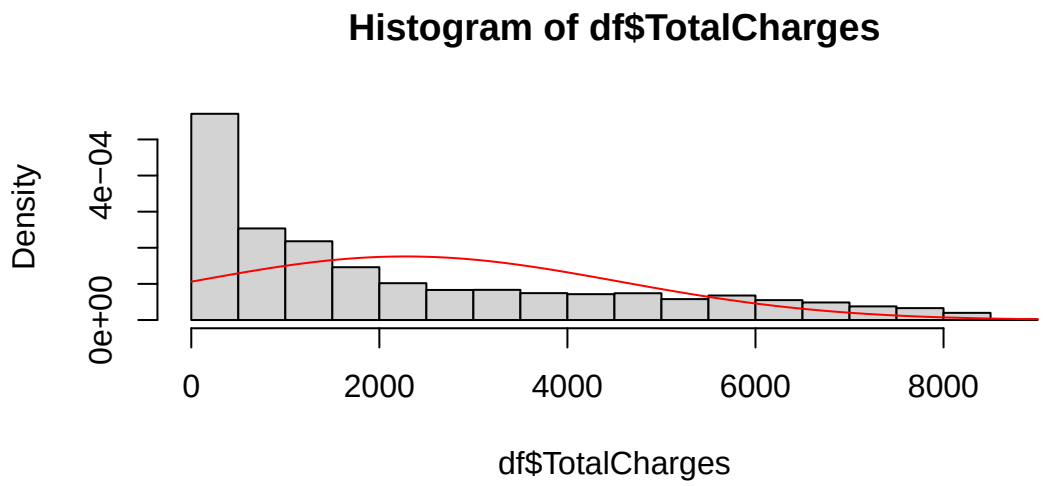
```
##
##  Anderson-Darling normality test
##
## data:  df$MonthlyCharges
## A = 170.56, p-value < 2.2e-16
```

```
Boxplot(df$MonthlyCharges,range=1.5,id=list(n=Inf,labels=rownames(df))) #No mild univariate outliers
Boxplot(df$MonthlyCharges,range=3,id=list(n=Inf,labels=rownames(df))) #No severe univariate outliers
```



#TotalCharges
Again, most of the population groups around small values. Not normally distributed. No univariate outliers.

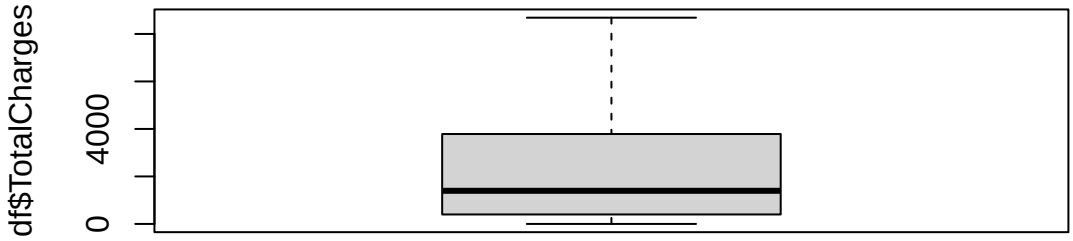
```
na.totalcharges <- sum(is.na(df$TotalCharges)) #11 NAs imputed earlier
hist(df$TotalCharges,freq=F,15)
mm <- mean(df$TotalCharges,na.rm=T)
ss <- sd(df$TotalCharges,na.rm=T)
curve(dnorm(x,mm,ss),col="red",add=T)
```




```
#shapiro.test(df$TotalCharges) #Error: too many samples for shapiro test
ad.test(df$TotalCharges) #Anderson-Darling test: Not normally distributed
```

```
##
##  Anderson-Darling normality test
##
## data:  df$TotalCharges
## A = 346.64, p-value < 2.2e-16
```

```
Boxplot(df$TotalCharges,range=1.5,id=list(n=Inf,labels=rownames(df))) #No mild univariate outliers
Boxplot(df$TotalCharges,range=3,id=list(n=Inf,labels=rownames(df))) #No severe univariate outliers
```



6 Profiling Target Variable: Churn

We can see from the summary of the target variable Churn that it is highly unbalanced. 73% of all instances do not Churn. This may be a problem when building a model that we must keep in mind.

Using catdes we can see that the most highly related categorical variables are Contract, OnlineSecurity, TechSupport, InternetService, PaymentMethod, ONlineBackup, DeviceProtection, StreamingMovies, StreamingTV, PaperlessBilling, Dependents, SeniorCitizen, Partner, and MultipleLines. All of these variables also have an extremely low p-value (far below the 5% significance level) which indicates a strong link to the target. For the quantitative variables, we can see that the target Churn is highly linked to all the numeric variables, tenure, TotalCharges, and MonthlyCharges.

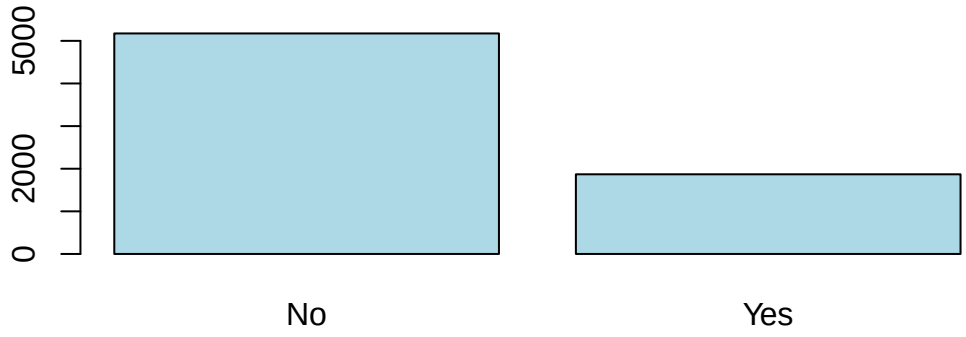
```
na.churn <- sum(is.na(df$Churn)) #No NAs
summary(df$Churn)
```

```
##   No  Yes
## 5174 1869
```

```
ptt<-prop.table(table(df$Churn));ptt
```

```
##
##        No       Yes
## 0.7346301 0.2653699
```

```
barplot(table(df$Churn),col='lightblue') #Unbalanced
```




```
catdes(df,21)
```

```
##
## Link between the cluster variable and the categorical variables (chi-square test)
## =================================================================================
##                         p.value df
## Contract          5.863038e-258  2
## OnlineSecurity    2.661150e-185  2
## TechSupport       1.443084e-180  2
## InternetService   9.571788e-160  2
## PaymentMethod     3.682355e-140  3
## OnlineBackup      2.079759e-131  2
## DeviceProtection 5.505219e-122  2
## StreamingMovies    2.667757e-82  2
## StreamingTV        5.528994e-82  2
## PaperlessBilling   2.614597e-58  1
## Dependents         3.276083e-43  1
## SeniorCitizen      9.477904e-37  1
## Partner            1.519037e-36  1
## MultipleLines      3.464383e-03  2
##
## Description of each cluster by the categories
## =============================================
## $No
##                                         Cla/Mod  Mod/Cla   Global
## Contract=Two year                      97.16814 31.83224 24.06645
## StreamingMovies=No internet service    92.59502 27.30963 21.66690
## StreamingTV=No internet service        92.59502 27.30963 21.66690
## TechSupport=No internet service        92.59502 27.30963 21.66690
## DeviceProtection=No internet service   92.59502 27.30963 21.66690
## OnlineBackup=No internet service       92.59502 27.30963 21.66690
## OnlineSecurity=No internet service     92.59502 27.30963 21.66690
## InternetService=No                     92.59502 27.30963 21.66690
## PaperlessBilling=No                    83.66992 46.44376 40.77808
## Contract=One year                      88.73048 25.26092 20.91438
```

```
## OnlineSecurity=Yes                        85.38881 33.32045 28.66676
## TechSupport=Yes                           84.83366 33.51372 29.02172
## Dependents=Yes                            84.54976 34.48009 29.95882
## Partner=Yes                               80.33510 52.82180 48.30328
## SeniorCitizen=No                          76.39383 87.12795 83.78532
## PaymentMethod=Credit card (automatic)     84.75690 24.93235 21.61011
## InternetService=DSL                       81.04089 37.92037 34.37456
## PaymentMethod=Bank transfer (automatic)   83.29016 24.85504 21.92248
## PaymentMethod=Mailed check                80.89330 25.20294 22.88797
## OnlineBackup=Yes                          78.46851 36.83804 34.48814
## DeviceProtection=Yes                      77.49794 36.27754 34.38875
## MultipleLines=No                          74.95575 49.11094 48.13290
## MultipleLines=Yes                         71.39010 40.99343 42.18373
## StreamingMovies=Yes                       70.05857 36.99266 38.79029
## StreamingTV=Yes                           69.92981 36.58678 38.43533
## StreamingTV=No                            66.47687 36.10359 39.89777
## StreamingMovies=No                        66.31957 35.69772 39.54281
## SeniorCitizen=Yes                         58.31874 12.87205 16.21468
## Partner=No                                67.04202 47.17820 51.69672
## Dependents=No                             68.72086 65.51991 70.04118
## PaperlessBilling=Yes                      66.43491 53.55624 59.22192
## DeviceProtection=No                       60.87237 36.41283 43.94434
## OnlineBackup=No                           60.07124 35.85234 43.84495
## PaymentMethod=Electronic check            54.71459 25.00966 33.57944
## InternetService=Fiber optic               58.10724 34.77000 43.95854
## TechSupport=No                            58.36453 39.17665 49.31137
## OnlineSecurity=No                         58.23328 39.36993 49.66634
## Contract=Month-to-month                   57.29032 42.90684 55.01917
##                                                p.value     v.test
## Contract=Two year                        3.588830e-187  29.178937
## StreamingMovies=No internet service       6.584621e-98  20.999812
## StreamingTV=No internet service           6.584621e-98  20.999812
## TechSupport=No internet service           6.584621e-98  20.999812
## DeviceProtection=No internet service      6.584621e-98  20.999812
## OnlineBackup=No internet service          6.584621e-98  20.999812
## OnlineSecurity=No internet service        6.584621e-98  20.999812
## InternetService=No                        6.584621e-98  20.999812
## PaperlessBilling=No                       1.072745e-60  16.435085
## Contract=One year                         3.593041e-57  15.935502
## OnlineSecurity=Yes                        1.606459e-50  14.947938
## TechSupport=Yes                           1.323174e-46  14.334963
## Dependents=Yes                            3.572324e-46  14.265846
## Partner=Yes                               6.170871e-37  12.696658
## SeniorCitizen=No                          3.024931e-34  12.202212
## PaymentMethod=Credit card (automatic)     6.408166e-32  11.758206
## InternetService=DSL                       2.545367e-26  10.614727
## PaymentMethod=Bank transfer (automatic)   1.180908e-24  10.250207
## PaymentMethod=Mailed check                3.226893e-15   7.881803
## OnlineBackup=Yes                          3.021982e-12   6.976698
## DeviceProtection=Yes                      2.173366e-08   5.597602
## MultipleLines=No                          6.262488e-03   2.733712
## MultipleLines=Yes                         7.843169e-04  -3.358271
## StreamingMovies=Yes                       2.922571e-07  -5.128373
## StreamingTV=Yes                           1.283457e-07  -5.281193
```

```
## StreamingTV=No                              6.049871e-27 -10.748094
## StreamingMovies=No                          1.092934e-27 -10.904833
## SeniorCitizen=Yes                           3.024931e-34 -12.202212
## Partner=No                                  6.170871e-37 -12.696658
## Dependents=No                               3.572324e-46 -14.265846
## PaperlessBilling=Yes                        1.072745e-60 -16.435085
## DeviceProtection=No                         1.116896e-99 -21.192627
## OnlineBackup=No                            3.366400e-112 -22.509287
## PaymentMethod=Electronic check             1.790860e-136 -24.864755
## InternetService=Fiber optic                2.289126e-148 -25.941138
## TechSupport=No                             1.899538e-183 -28.883947
## OnlineSecurity=No                          6.171504e-190 -29.396034
## Contract=Month-to-month                    3.620915e-283 -35.959308
##
## $Yes
##                                               Cla/Mod   Mod/Cla   Global
## Contract=Month-to-month                     42.709677 88.550027 55.01917
## OnlineSecurity=No                           41.766724 78.170144 49.66634
## TechSupport=No                              41.635474 77.367576 49.31137
## InternetService=Fiber optic                 41.892765 69.395399 43.95854
## PaymentMethod=Electronic check              45.285412 57.303371 33.57944
## OnlineBackup=No                             39.928756 65.971108 43.84495
## DeviceProtection=No                         39.127625 64.794007 43.94434
## PaperlessBilling=Yes                        33.565092 74.906367 59.22192
## Dependents=No                               31.279140 82.557517 70.04118
## Partner=No                                  32.957979 64.205457 51.69672
## SeniorCitizen=Yes                           41.681261 25.468165 16.21468
## StreamingMovies=No                          33.680431 50.187266 39.54281
## StreamingTV=No                              33.523132 50.401284 39.89777
## StreamingTV=Yes                             30.070188 43.552702 38.43533
## StreamingMovies=Yes                         29.941435 43.766720 38.79029
## MultipleLines=Yes                           28.609896 45.478866 42.18373
## MultipleLines=No                            25.044248 45.425361 48.13290
## DeviceProtection=Yes                        22.502064 29.159979 34.38875
## OnlineBackup=Yes                            21.531494 27.982879 34.48814
## PaymentMethod=Mailed check                  19.106700 16.479401 22.88797
## PaymentMethod=Bank transfer (automatic)     16.709845 13.804173 21.92248
## InternetService=DSL                         18.959108 24.558587 34.37456
## PaymentMethod=Credit card (automatic)       15.243101 12.413055 21.61011
## SeniorCitizen=No                            23.606168 74.531835 83.78532
## Partner=Yes                                 19.664903 35.794543 48.30328
## Dependents=Yes                              15.450237 17.442483 29.95882
## TechSupport=Yes                             15.166341 16.586410 29.02172
## OnlineSecurity=Yes                          14.611194 15.783842 28.66676
## Contract=One year                           11.269518  8.881755 20.91438
## PaperlessBilling=No                         16.330084 25.093633 40.77808
## StreamingMovies=No internet service          7.404980  6.046014 21.66690
## StreamingTV=No internet service              7.404980  6.046014 21.66690
## TechSupport=No internet service              7.404980  6.046014 21.66690
## DeviceProtection=No internet service         7.404980  6.046014 21.66690
## OnlineBackup=No internet service             7.404980  6.046014 21.66690
## OnlineSecurity=No internet service           7.404980  6.046014 21.66690
## InternetService=No                           7.404980  6.046014 21.66690
## Contract=Two year                            2.831858  2.568218 24.06645
```

```
##                                              p.value     v.test
## Contract=Month-to-month                 3.620915e-283  35.959308
## OnlineSecurity=No                       6.171504e-190  29.396034
## TechSupport=No                          1.899538e-183  28.883947
## InternetService=Fiber optic             2.289126e-148  25.941138
## PaymentMethod=Electronic check          1.790860e-136  24.864755
## OnlineBackup=No                         3.366400e-112  22.509287
## DeviceProtection=No                      1.116896e-99  21.192627
## PaperlessBilling=Yes                     1.072745e-60  16.435085
## Dependents=No                            3.572324e-46  14.265846
## Partner=No                               6.170871e-37  12.696658
## SeniorCitizen=Yes                        3.024931e-34  12.202212
## StreamingMovies=No                       1.092934e-27  10.904833
## StreamingTV=No                           6.049871e-27  10.748094
## StreamingTV=Yes                          1.283457e-07   5.281193
## StreamingMovies=Yes                      2.922571e-07   5.128373
## MultipleLines=Yes                        7.843169e-04   3.358271
## MultipleLines=No                         6.262488e-03  -2.733712
## DeviceProtection=Yes                     2.173366e-08  -5.597602
## OnlineBackup=Yes                         3.021982e-12  -6.976698
## PaymentMethod=Mailed check               3.226893e-15  -7.881803
## PaymentMethod=Bank transfer (automatic)  1.180908e-24 -10.250207
## InternetService=DSL                      2.545367e-26 -10.614727
## PaymentMethod=Credit card (automatic)    6.408166e-32 -11.758206
## SeniorCitizen=No                         3.024931e-34 -12.202212
## Partner=Yes                              6.170871e-37 -12.696658
## Dependents=Yes                           3.572324e-46 -14.265846
## TechSupport=Yes                          1.323174e-46 -14.334963
## OnlineSecurity=Yes                       1.606459e-50 -14.947938
## Contract=One year                        3.593041e-57 -15.935502
## PaperlessBilling=No                      1.072745e-60 -16.435085
## StreamingMovies=No internet service      6.584621e-98 -20.999812
## StreamingTV=No internet service          6.584621e-98 -20.999812
## TechSupport=No internet service          6.584621e-98 -20.999812
## DeviceProtection=No internet service     6.584621e-98 -20.999812
## OnlineBackup=No internet service         6.584621e-98 -20.999812
## OnlineSecurity=No internet service       6.584621e-98 -20.999812
## InternetService=No                       6.584621e-98 -20.999812
## Contract=Two year                       3.588830e-187 -29.178937
##
##
## Link between the cluster variable and the quantitative variables
## ================================================================
##                      Eta2       P-value
## tenure          0.12406504 7.999058e-205
## TotalCharges    0.03933251  2.127212e-63
## MonthlyCharges 0.03738671  2.706646e-60
##
## Description of each cluster by quantitative variables
## ======================================================
## $No
##                   v.test Mean in category Overall mean sd in category
## tenure           29.55784         37.56997     32.37115       24.11145
## TotalCharges     16.64270       2549.91144   2279.73430     2329.72904
```

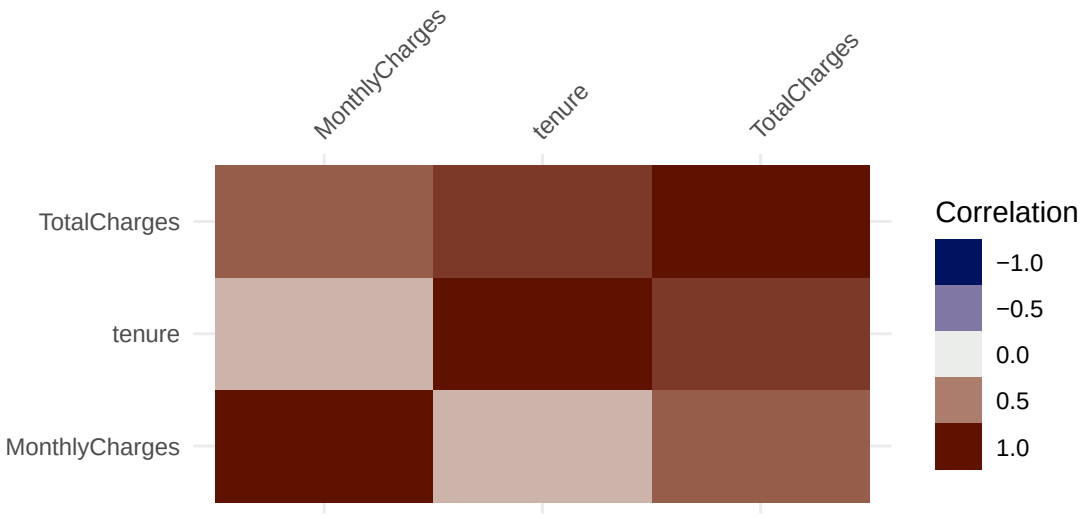
```
## MonthlyCharges -16.22582         61.26512     64.76169       31.08964
##                Overall sd       p.value
## tenure           24.55774 5.207314e-192
## TotalCharges   2266.63354  3.418341e-62
## MonthlyCharges   30.08791  3.312724e-59
##
## $Yes
##                   v.test Mean in category Overall mean sd in category
## MonthlyCharges  16.22582         74.44133     64.76169       24.65945
## TotalCharges   -16.64270       1531.79609   2279.73430     1890.31709
## tenure         -29.55784         17.97913     32.37115       19.52590
##                Overall sd       p.value
## MonthlyCharges   30.08791  3.312724e-59
## TotalCharges   2266.63354  3.418341e-62
## tenure           24.55774 5.207314e-192
```

7 Correlations and Associations

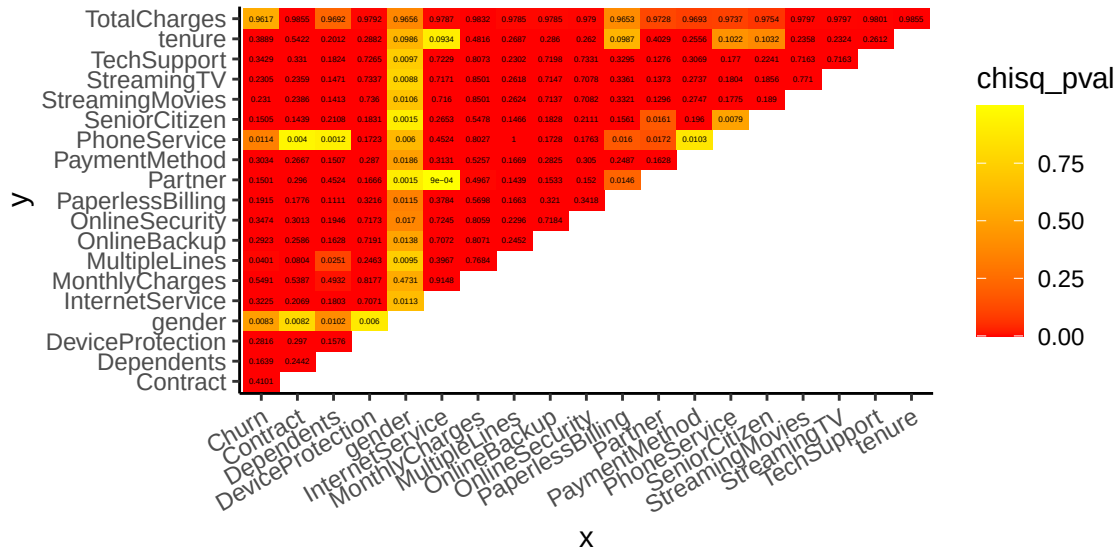
If we plot the correlations between the numeric variables, the heatmap shows that MonthlyCharges and tenure are not correlated. However, intuitively TotalCharges and MonthlyCharges are positively correlated at approximately 0.5. This makes sense as the higher your monthly charges are the higher your total charges will be. Similarly, tenure has a relatively high positive correlation with TotalCharges. Once again this makes sense because the longer you are subscribed for the higher your total overall charges will be.

To further our analysis on the relationship between variables we make use of a function that plots mixed associations using the Chisquare p-values and CramersV, for numeric and categorical variables (this function was found at https://stackoverflow.com/questions/52554336/plot-the-equivalent-of-correlation-matrix-for-factors-categorical-data-and-mi by AntoniosK on StackOverflow). Note that to interpret the graph, the color of the cells indicate the Chisquared p-value (red meaning highly related), and the label found in the cells indicated the CramersV (1 indicated a perfect association). Interestingly it seems that most variables are related according to the Chisquared test, as most cells are deep red, except for gender which according to the Chisquared independence test does not show any significant relationship with the other variables. When looking at the CramersV (labels in the cells), we can see that TotalCharges has an almost perfect association with Churn the target, and all the other variables. Aside from TotalCharges, no other variable has a near perfect association with another variables.

```
num <- which(sapply(df,is.numeric))

# Correlations
vis_cor(df[,num])
```




```
# Mixed Associations (using Chisquared pvalue and CramersV)
cat <- which(sapply(df, function(x) is.factor(x) || is.character(x)))

df_corr <- df[,-1]

# function to get chi square p value and Cramers V
f = function(x,y) {
    tbl = df %>% select(x,y) %>% table()
    chisq_pval = round(chisq.test(tbl)$p.value, 4)
    cramV = round(cramersV(tbl), 4)
    data.frame(x, y, chisq_pval, cramV) }

# create unique combinations of column names
# sorting will help getting a better plot (upper triangular)
df_comb = data.frame(t(combn(sort(names(df_corr)), 2)), stringsAsFactors = F)

# apply function to each variable combination
df_res = map2_df(df_comb$X1, df_comb$X2, f)

# plot results
df_res %>%
  ggplot(aes(x,y,fill=chisq_pval))+
  geom_tile()+
  geom_text(aes(x,y,label=cramV), size=1)+
  scale_fill_gradient(low="red", high="yellow")+
  theme_classic()+
  theme(axis.text.x = element_text(angle = 30, hjust = 1))
```




```
# Function to find mixed associations found at:
# https://stackoverflow.com/questions/52554336/plot-the-equivalent-of-correlation-matrix-for-factors-ca
# By AntoniosK on StackOverflow
```

8 Multivariate Outliers

Using the Moutlier function at a 1% significance level we find that there are 62 outliers. Checking the summary of the outliers we see that they have abnormally high tenure, and very high TotalCharges. However, when we plot the robust distance versus the mahalanobis distance we see that there is a lot of continuity in the points. We believe that all the points so closely together, springing out in three continuous spikes means that these outliers are expected, as many other observations come close to this cutoff. Thus, we decide to keep all of the multivariate outliers. This is becuase of this strong continuity in the graph and the relatedness of all the points close to the cutoff. Losing these points mean we lose valuable information as there is a clear pattern, namely the higher the tenure the higher the overall total charges, even if they are very high numbers that are shown to be outliers by the Moutlier function. We want this relationship to be captured in the model.

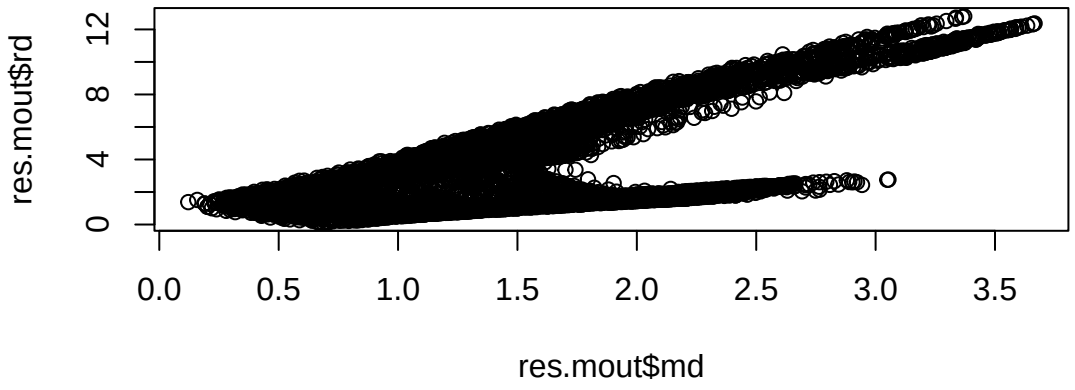
```
res.mout <- Moutlier(df[,num],quantile=0.99,plot=F)
length(which(res.mout$md > res.mout$cutoff))
```

```
## [1] 62
```

```
mout <- which(res.mout$md > res.mout$cutoff)
summary(df[mout, num]) #Summary of outliers
```

```
##      tenure      MonthlyCharges    TotalCharges
##  Min.   :68.00   Min.   : 19.10   Min.   :1194
##  1st Qu.:71.00   1st Qu.: 19.70   1st Qu.:1361
##  Median :71.50   Median : 19.88   Median :1406
##  Mean   :71.18   Mean   : 21.80   Mean   :1540
##  3rd Qu.:72.00   3rd Qu.: 20.34   3rd Qu.:1490
##  Max.   :72.00   Max.   :117.80   Max.   :8685
```

```r
plot(res.mout$md, res.mout$rd)
```




```r
#df <- df[-m.out,] #remove multivariate outliers
```

9 Modelling

Now we enter into the modelling stage of the analysis. We first want to construct a robust numerical model, in order to add transformations in the numerical variables. Afterwards we will add the main categorical variables into this best numerical model, and finally we will look at interactions between variables to make the final model.

10 Model with numerical variables

We only have 3 numerical variables, so, in the first place, we construct a model with all 3 numerical variables: tenure, MonthlyCharges and TotalCharges. We are suspicious of multicolinearity regarding TotalCharges, since the number calculated by multiplying the tenure months by MonthlyCharges gives a similar result than the TotalCharges value (with small a deviation probably coming from opening fees or discounts in the different contracts).

```r
attach(df)
nm1 <- glm(Churn ~ tenure + MonthlyCharges + TotalCharges, family="binomial", data = df)
vif.nm1 <- vif(nm1);vif.nm1
```

```
##          tenure MonthlyCharges   TotalCharges
##       13.650324       2.293852      17.715584
```

```r
step(nm1,k= log(nrow(df)))
```

```
## Start:  AIC=6424.6
```

```
## Churn ~ tenure + MonthlyCharges + TotalCharges
##
##                  Df Deviance    AIC
## - TotalCharges    1   6394.4 6420.9
## <none>                6389.2 6424.6
## - tenure          1   6576.5 6603.1
## - MonthlyCharges  1   6737.3 6763.8
##
## Step:  AIC=6420.93
## Churn ~ tenure + MonthlyCharges
##
##                  Df Deviance    AIC
## <none>                6394.4 6420.9
## - MonthlyCharges  1   7191.9 7209.6
## - tenure          1   7878.2 7895.9

##
## Call:  glm(formula = Churn ~ tenure + MonthlyCharges, family = "binomial",
##     data = df)
##
## Coefficients:
##    (Intercept)          tenure  MonthlyCharges
##       -1.80244        -0.05485         0.03295
##
## Degrees of Freedom: 7042 Total (i.e. Null);  7040 Residual
## Null Deviance:       8150
## Residual Deviance: 6394  AIC: 6400
```

We perform an anova test to see if the variance explained by the two models is the same

```
nm2 <- glm(Churn ~ tenure + MonthlyCharges, family="binomial", data = df)
anova(nm2,nm1,test="Chisq") #p-value: 0.02277
```

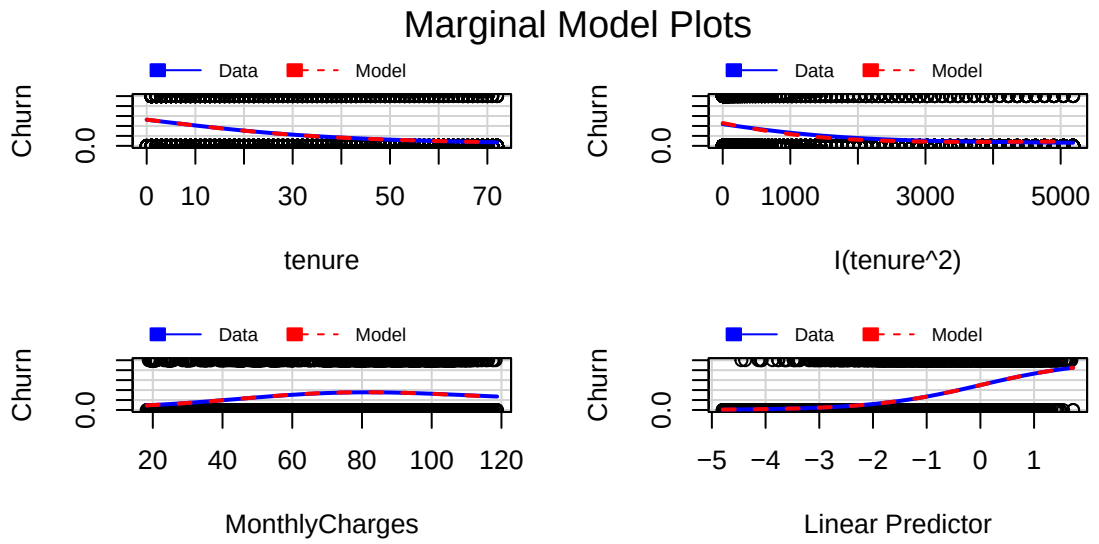
```
## Analysis of Deviance Table
##
## Model 1: Churn ~ tenure + MonthlyCharges
## Model 2: Churn ~ tenure + MonthlyCharges + TotalCharges
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      7040     6394.4
## 2      7039     6389.2  1   5.1862  0.02277 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Since the p-value is 0.02277, at 99% confidence we do not reject and accept the simple model as the best one.

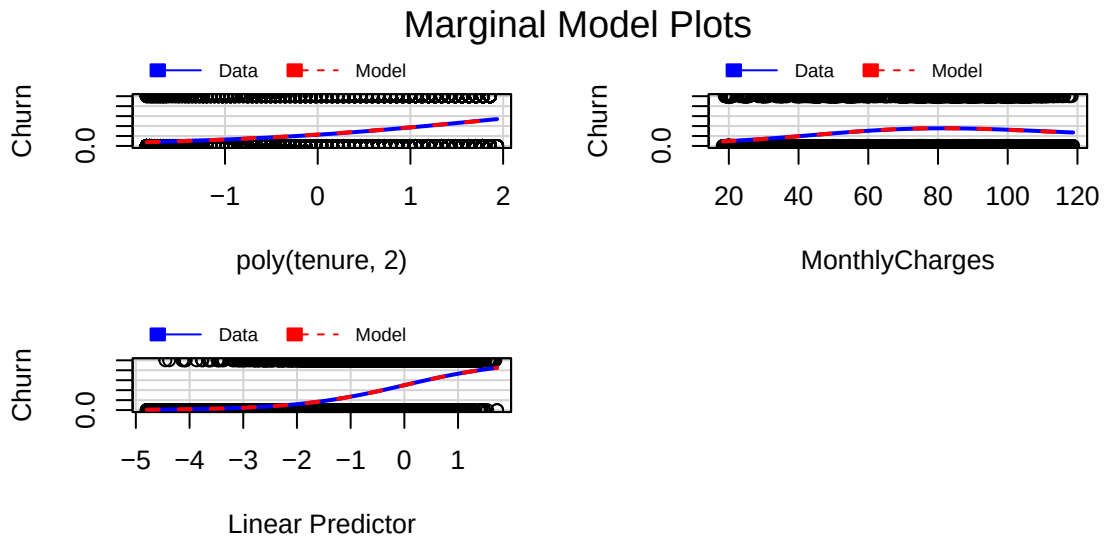
11 Transformations of numerical variables

Firstly, we examine marginalModelPlots to gain insights into which variables effectively fit the model. Subsequently, we observed the necessity for a transformation in the tenure variable. A reduction of one unit in tenure is associated with a log-odds increase of -0.054850 for Churn. Consequently, implementing a Square Root Transformation becomes imperative.

```
nm3 <- glm(Churn ~ tenure+ I(tenure^2) + MonthlyCharges, family="binomial", data = df)
marginalModelPlots(nm3)
```




```
nm4 <- glm(Churn ~ poly(tenure,2) + MonthlyCharges, family="binomial", data = df)
marginalModelPlots(nm4)
```



12 NM4: Residual analysis and Influential Data

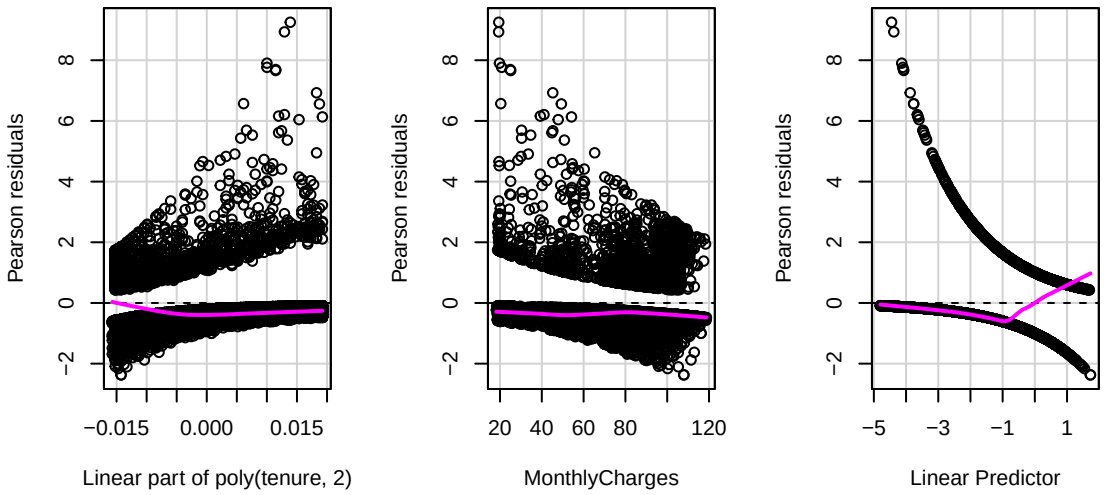
(INTERPRETATION OF RESIDUAL PLOTS)

When looking at the hat values, using the influenceIndexPlot, we see that there are many no points that stand out relative to the rest. We have a similar case when looking at the influencePlot. We can confirm that there are no outstanding hat values by drawing a boxplot and a typical cutoff line at 4*p/n. This plot

shows us there are no points with hat values above this cutoff and therefore no further action needs to be taken.

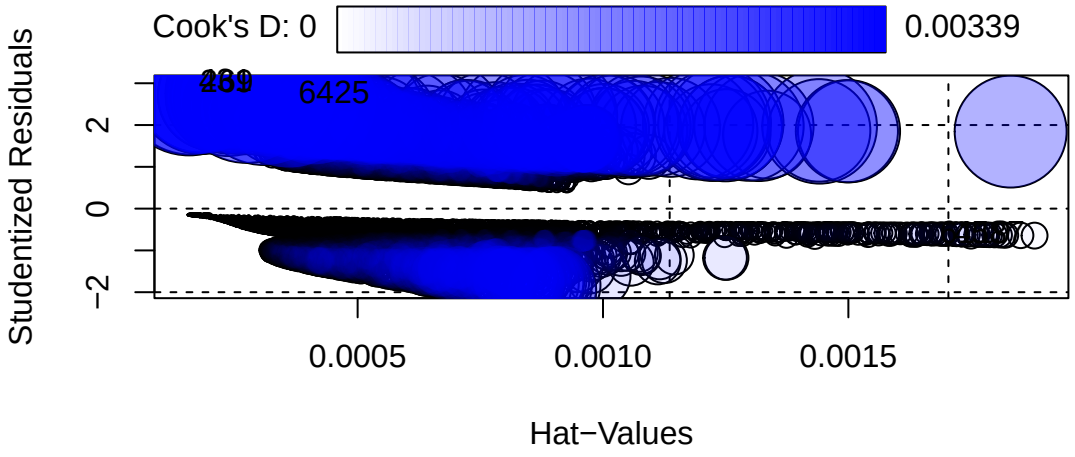
In a similar way, when we plot the influenceIndexPlot for Cooks' Distance, we find a few (approximately 5) points that stand out above the rest. To further investigate we draw a boxplot of Cooks' Distance and find that indeed there are four points that are relatively further out than others. We can draw a clear cutoff line at 0.003 to separate these points. When we further inspect these points by comparing their summary statistics against overall summaries of Tenure and Monthly Charges we find that tenure is significantly higher for these outlier observations while the average MonthlyCharges are almost half of the overall average. We decide to remove these points.

```
# Residuals
residualPlots(nm4, layout=c(1, 3))
```



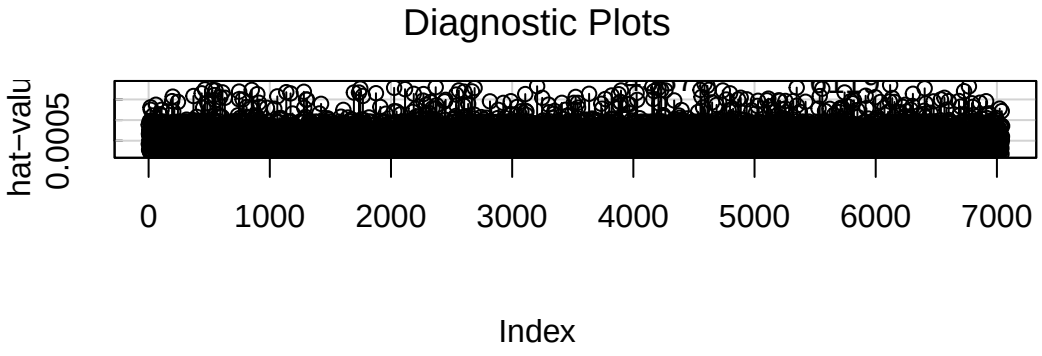

```
##                 Test stat Pr(>|Test stat|)
## poly(tenure, 2)
## MonthlyCharges     0.0255           0.8732
```

```
influencePlot(nm4)
```




```
##          StudRes          Hat        CookD
## 269   2.9660363 0.0001596978 0.0031880647
## 431   2.9893903 0.0001567313 0.0033558372
## 4587 -0.6462575 0.0018793961 0.0001093968
## 6119 -0.6408873 0.0018510422 0.0001057701
## 6425  2.7056019 0.0003598658 0.0033855901
```

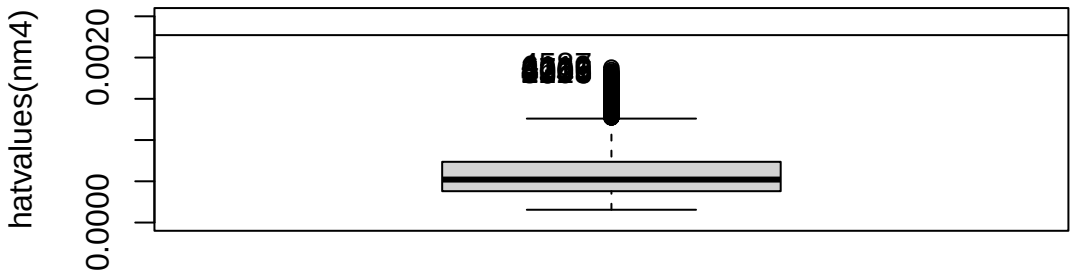
```
# Hat values
influenceIndexPlot(nm4, id.n=10, vars=c('hat'))
```



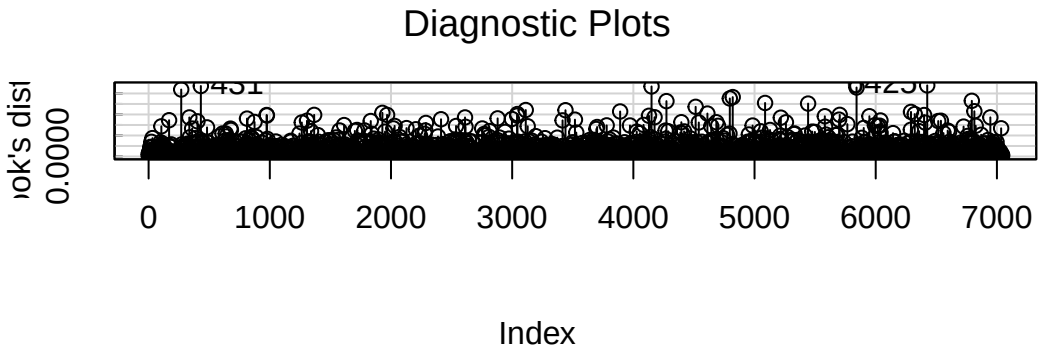

```
Boxplot(hatvalues(nm4), ylim=c(0,0.0025))
```

```
##  [1] 4587 6119 4611 3206 6769 4156 2369 5348 2026 4207
```

```
abline(h=4*length(coef(nm4))/nrow(df))
```



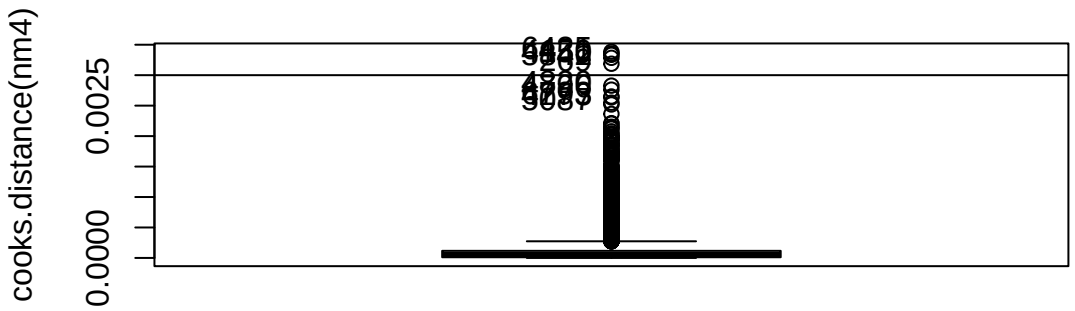

```
# Cooks distance
influenceIndexPlot(nm4, id.n=10, vars=c('Cook'))
```




```
Boxplot(cooks.distance(nm4))
```

```
##  [1] 6425  431 4150 5842  269 4820 4796 6793 4273 5087
```

```
abline(h=0.003)
```




```
llcoo <- which(cooks.distance(nm4)> 0.003);
summary(df[,c('tenure', 'MonthlyCharges')])
```

```
##      tenure      MonthlyCharges
##  Min.   : 0.00   Min.   : 18.25
##  1st Qu.: 9.00   1st Qu.: 35.50
##  Median :29.00   Median : 70.35
##  Mean   :32.37   Mean   : 64.76
##  3rd Qu.:55.00   3rd Qu.: 89.85
##  Max.   :72.00   Max.   :118.75
```

```
summary(df[llcoo,c('tenure', 'MonthlyCharges')])
```

```
##      tenure     MonthlyCharges
##  Min.   :59.0   Min.   :19.35
##  1st Qu.:61.0   1st Qu.:19.40
##  Median :70.0   Median :45.25
##  Mean   :66.6   Mean   :37.51
##  3rd Qu.:71.0   3rd Qu.:49.35
##  Max.   :72.0   Max.   :54.20
```

```
df <- df[-llcoo,]
rownames(df) <- NULL
```

13 Adding main categorical effects

Now we attempt to add our factor variables to the model. First we are building an inicial model with the main numerical variables and all the categorical ones, which will obviously result in a too complex model to be analysed or properly interpreted. After that, we are conducting an Anova Chisq test to assess the significance of each categorical variable.

```r
cm1 <- glm(Churn ~ poly(tenure,2) + MonthlyCharges + gender + SeniorCitizen + Partner + Dependents + Pho
Anova(cm1,test="LR")
```

```
## Analysis of Deviance Table (Type II tests)
##
## Response: Churn
##                   LR Chisq Df Pr(>Chisq)
## poly(tenure, 2)    295.599  2  < 2.2e-16 ***
## MonthlyCharges       0.910  1   0.339996
## gender               0.100  1   0.751910
## SeniorCitizen        7.125  1   0.007602 **
## Partner              0.004  1   0.950749
## Dependents           2.063  1   0.150926
## PhoneService                0
## MultipleLines        7.968  1   0.004762 **
## InternetService      4.411  1   0.035699 *
## OnlineSecurity       1.006  1   0.315820
## OnlineBackup         0.012  1   0.913028
## DeviceProtection     0.977  1   0.322887
## TechSupport          0.562  1   0.453597
## StreamingTV          3.263  1   0.070872 .
## StreamingMovies      3.347  1   0.067312 .
## Contract           120.396  2  < 2.2e-16 ***
## PaperlessBilling    20.170  1  7.087e-06 ***
## PaymentMethod       22.268  3  5.738e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This Anova tests suggests, at 95% confidence, to take out of the modeling the following variables: gender, Partner, Dependets, PhoneService, OnlineSecurity, OnlineBackup, DeviceProtection, TechSupport, StreamingTV and StreamingMovies. Now we construct a model only with the significant variables.

```r
cm2 <- glm(Churn ~ poly(tenure,2) + MonthlyCharges + SeniorCitizen + MultipleLines + InternetService + C
```

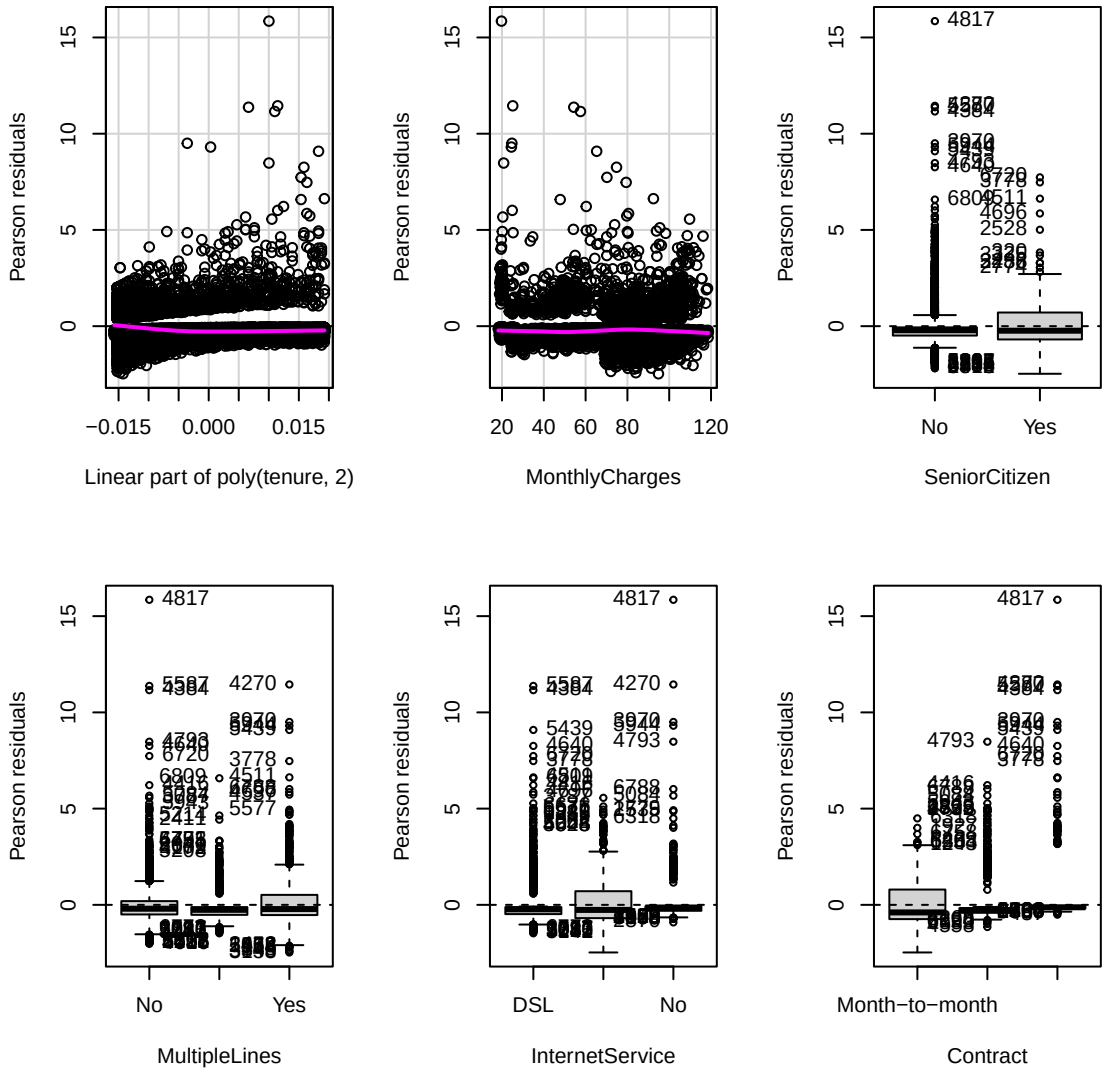
14 CM2: Residual Analysis and Influential Data

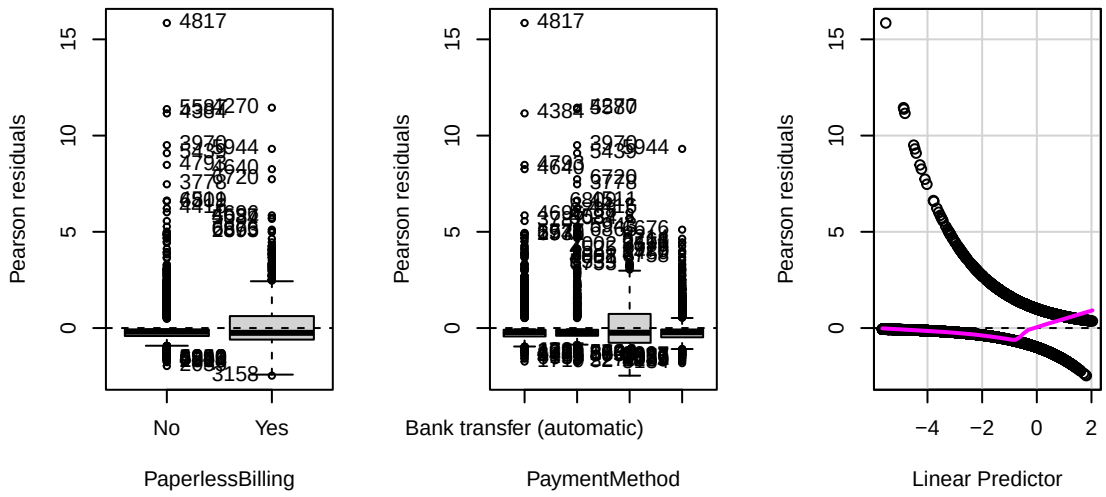
(INTERPRETATION OF RESIDUAL PLOTS)

From the general influencePlot we can see that there are a couple possible outliers that need further investivation such as observations 3821 and 489, which seem to have abnormally high hat values. These same points stand out in the influenceIndexPlot. When we now draw a boxplot of the hat values and draw a cutoff line at the typical cutoff 4*p/n, we see that point 489 is the only one above this cutoff, with point 2819 just below. Thus, we remove observation 489.

When we further investigate Cooks' Distance using influenceIndexPlot we see more observations stand out with observations 3972 and 5948 being fat above the rest. Now when we draw the boxplot, we again see these two points stand out with several other points. Here there is no clear cutoff as the observations splinter into groups, where one group has a large distance to the rest, and then the two observations mentioned even further than this. Thus, to not lose too much valuable information we decide to remove only the two largest outliers, thus moving the cutoff to 0.0033. When comparing the Cooks outliers summaries to the all the data it is difficult to see where they are different. They seem to generally have lower charges (monthly and total), with longer tenure but nothing noteworthy that would suggest why they seem to be so influential from the summary statistics alone.

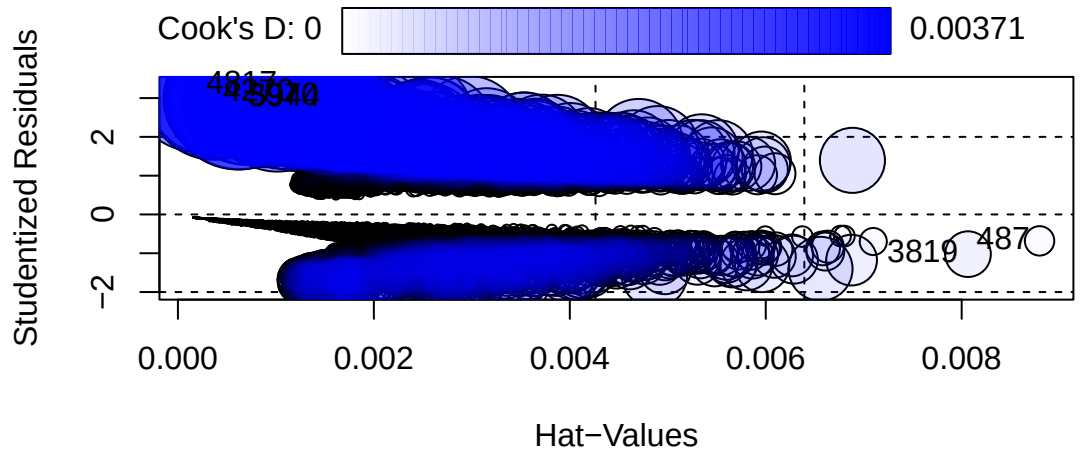
```
# Residuals
residualPlots(cm2, layout=c(1, 3))
```






```
##                   Test stat Pr(>|Test stat|)
## poly(tenure, 2)
## MonthlyCharges       6.7304         0.009479 **
## SeniorCitizen
## MultipleLines
## InternetService
## Contract
## PaperlessBilling
## PaymentMethod
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

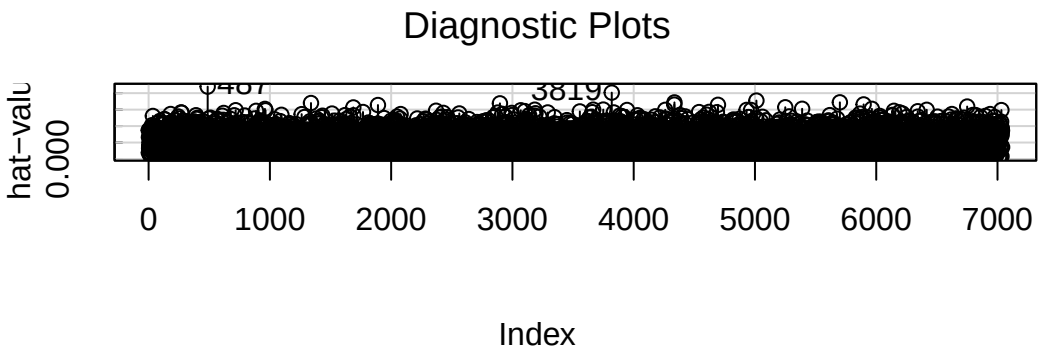
```
influencePlot(cm2)
```




```
##          StudRes          Hat         CookD
```

```
## 487  -0.6808226 0.0087949451 0.0001547851
## 3819 -1.0230782 0.0080644712 0.0003731883
## 3970  3.0140048 0.0006158875 0.0037127146
## 4270  3.1328757 0.0003653224 0.0031950636
## 4817  3.3328830 0.0001889194 0.0031652752
## 5944  3.0000940 0.0006272134 0.0036268210
```

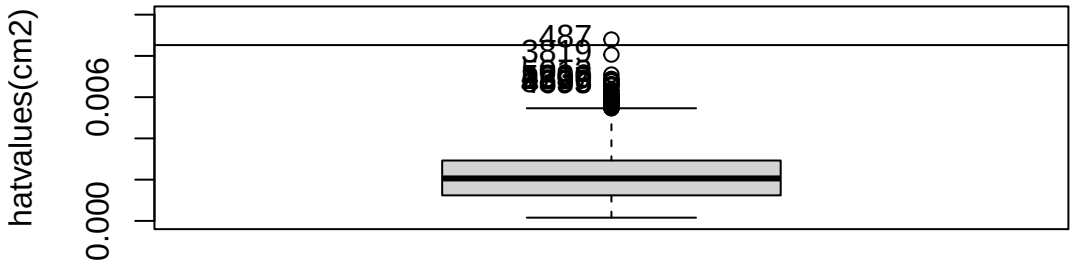
```
# Hat values
influenceIndexPlot(cm2, id.n=10, vars=c('hat'))
```



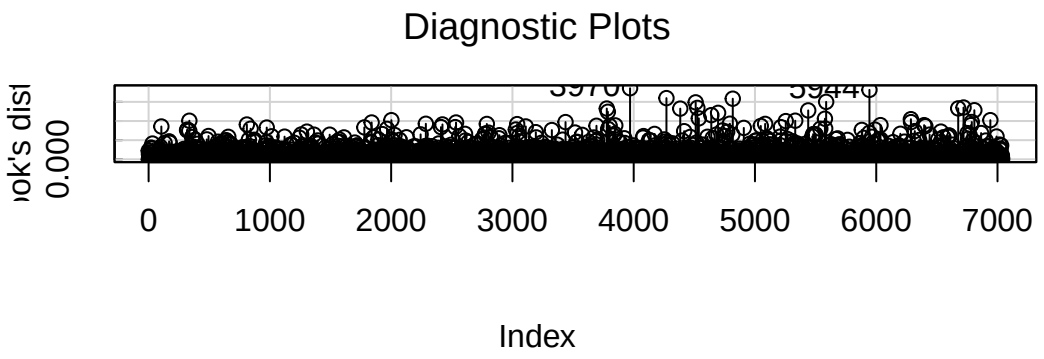

```
Boxplot(hatvalues(cm2), ylim=c(0,0.01))
```

```
##  [1]  487 3819 5012 5700 4336 1339 2897 5896 4335 4695
```

```
abline(h=4*length(coef(cm2))/nrow(df))
```



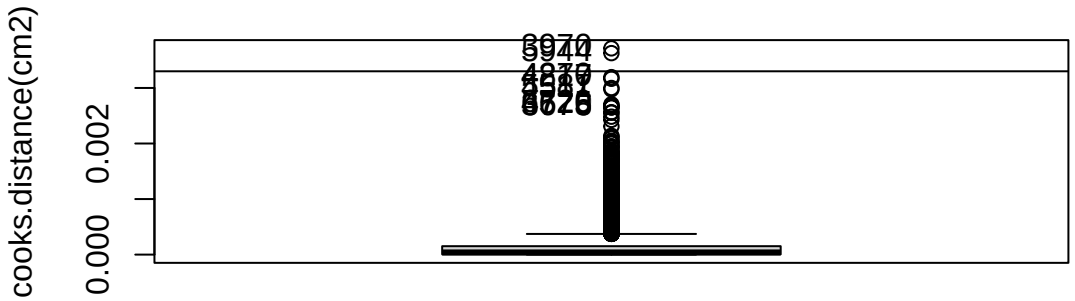
```r
# hatvalues(cm2)[487] --> referring to observation 489
df <- df[-489,]
rownames(df) <- NULL

# Cooks distance
influenceIndexPlot(cm2, id.n=10, vars=c('Cook'))
```




```r
Boxplot(cooks.distance(cm2))
```

```
##  [1] 3970 5944 4270 4817 5587 4511 6720 4525 6676 3778
```

```r
abline(h=0.0033)
```



```r
llcoo <- which(cooks.distance(cm2)> 0.0033);
#summary(df)          --> uncomment to compare
#summary(df[llcoo,])  --> uncomment to compare

df <- df[-llcoo,]
rownames(df) <- NULL
```

```r
cm2 <- glm(Churn ~ poly(tenure,2) + MonthlyCharges + SeniorCitizen + MultipleLines + InternetService + C
```

15 Interactions

Significant interactions (5% level):
poly(tenure, 2):Contract
MonthlyCharges:MultipleLines
MonthlyCharges:InternetService
MonthlyCharges:Contract
SeniorCitizen:PaymentMethod
InternetService:Contract
InternetService:PaymentMethod

```r
Anova(glm(Churn ~ (poly(tenure,2) + MonthlyCharges + SeniorCitizen + MultipleLines + InternetService + C
```

```
## Analysis of Deviance Table (Type II tests)
##
## Response: Churn
##                                  LR Chisq Df Pr(>Chisq)
## poly(tenure, 2)                    353.01  2  < 2.2e-16 ***
## MonthlyCharges                       8.47  1  0.0036015 **
## SeniorCitizen                        9.38  1  0.0021979 **
## MultipleLines                       18.01  2  0.0001228 ***
## InternetService                     40.35  2  1.727e-09 ***
## Contract                           114.48  2  < 2.2e-16 ***
## PaperlessBilling                    24.49  1  7.487e-07 ***
## PaymentMethod                       31.75  3  5.896e-07 ***
## poly(tenure, 2):MonthlyCharges       1.16  2  0.5588909
## poly(tenure, 2):SeniorCitizen        2.39  2  0.3032295
## poly(tenure, 2):MultipleLines        0.53  4  0.9701025
## poly(tenure, 2):InternetService      2.35  4  0.6721362
## poly(tenure, 2):Contract            22.56  4  0.0001547 ***
## poly(tenure, 2):PaperlessBilling     5.34  2  0.0693936 .
## poly(tenure, 2):PaymentMethod       14.00  6  0.0296045 *
## MonthlyCharges:SeniorCitizen         0.06  1  0.8083933
## MonthlyCharges:MultipleLines         7.67  2  0.0216087 *
## MonthlyCharges:InternetService      16.11  2  0.0003172 ***
## MonthlyCharges:Contract             12.63  2  0.0018079 **
## MonthlyCharges:PaperlessBilling      0.08  1  0.7748776
## MonthlyCharges:PaymentMethod         4.96  3  0.1746787
## SeniorCitizen:MultipleLines          1.89  2  0.3885162
## SeniorCitizen:InternetService        0.29  2  0.8646871
## SeniorCitizen:Contract               2.00  2  0.3686318
## SeniorCitizen:PaperlessBilling       0.01  1  0.9284968
```

```
## SeniorCitizen:PaymentMethod         12.34  3  0.0062953 **
## MultipleLines:InternetService        1.07  2  0.5865189
## MultipleLines:Contract               8.58  4  0.0723792 .
## MultipleLines:PaperlessBilling       2.05  2  0.3580582
## MultipleLines:PaymentMethod          6.48  6  0.3715010
## InternetService:Contract            18.44  4  0.0010127 **
## InternetService:PaperlessBilling     1.02  2  0.6010553
## InternetService:PaymentMethod       18.44  6  0.0052278 **
## Contract:PaperlessBilling            3.46  2  0.1770413
## Contract:PaymentMethod               4.94  6  0.5512889
## PaperlessBilling:PaymentMethod       2.76  3  0.4300596
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We will be adding interactions one by one to our model, and we will see if they have significance over the last model, and, if they do, we keep them in the model and move on. Note that to do that we will be using Fisher tests, with the null hypothesis being that the variance explained by both models are the same. First we look into the interaction between Contract and the transformed tenure. We can see that the role it performs on the model is to ... . Then, we perform an Fisher test to see if the variances explained by the two models are the same or not.

```
gm1 <- glm(Churn ~ poly(tenure,2)*Contract + MonthlyCharges + SeniorCitizen + MultipleLines + InternetS
anova(cm2,gm1,test="Chisq") #p-value 2.982e-06
```

```
## Analysis of Deviance Table
##
## Model 1: Churn ~ poly(tenure, 2) + MonthlyCharges + SeniorCitizen + MultipleLines +
##     InternetService + Contract + PaperlessBilling + PaymentMethod
## Model 2: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges + SeniorCitizen +
##     MultipleLines + InternetService + PaperlessBilling + PaymentMethod
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1      7020     5819.4
## 2      7016     5788.4  4   31.056 2.982e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We can see that the p-value in this case is 2.982e-06, so we reject the null hypothesis and we decide to keep the interaction forward. Now let's look at the interactions regarding MonthlyCharges, starting by its interaction with MultipleLines.

```
gm2 <- glm(Churn ~ poly(tenure,2)*Contract + MonthlyCharges*MultipleLines + SeniorCitizen + MultipleLin
anova(gm1,gm2,test="Chisq") #p-value 0.01424
```

```
## Analysis of Deviance Table
##
## Model 1: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges + SeniorCitizen +
##     MultipleLines + InternetService + PaperlessBilling + PaymentMethod
## Model 2: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges * MultipleLines +
##     SeniorCitizen + MultipleLines + InternetService + PaperlessBilling +
##     PaymentMethod
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      7016     5788.4
```

```
## 2      7014     5779.9  2   8.5034  0.01424 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In this case, the p-value is 0.01424, which would make us reject the hypothesis at 99% confidence, but accept it at 95%. We decide to take it out of the model.

```
gm3 <- glm(Churn ~ poly(tenure,2)*Contract + MonthlyCharges*InternetService + SeniorCitizen + MultipleLi
anova(gm1,gm3,test="Chisq") #p-value 0.005535
```

```
## Analysis of Deviance Table
##
## Model 1: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges + SeniorCitizen +
##     MultipleLines + InternetService + PaperlessBilling + PaymentMethod
## Model 2: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges * InternetService +
##     SeniorCitizen + MultipleLines + InternetService + PaperlessBilling +
##     PaymentMethod
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      7016     5788.4
## 2      7014     5778.0  2   10.393 0.005535 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This time, the p-value is 0.005535, so we will keep the interaction in the model.

```
gm4 <- glm(Churn ~ poly(tenure,2)*Contract + MonthlyCharges*(InternetService + Contract) + SeniorCitizer
anova(gm3,gm4,test="Chisq") #p-value 0.8
```

```
## Analysis of Deviance Table
##
## Model 1: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges * InternetService +
##     SeniorCitizen + MultipleLines + InternetService + PaperlessBilling +
##     PaymentMethod
## Model 2: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges * (InternetService +
##     Contract) + SeniorCitizen + MultipleLines + InternetService +
##     PaperlessBilling + PaymentMethod
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      7014     5778.0
## 2      7012     5777.5  2   0.4462      0.8
```

Now the p-value is 0.8, so we accept the null hypothesis and not keep this interaction.

```
gm5 <- glm(Churn ~ poly(tenure,2)*Contract + MonthlyCharges*InternetService + SeniorCitizen*PaymentMeth
anova(gm3,gm5,test="Chisq") #p-value 0.035
```

```
## Analysis of Deviance Table
##
## Model 1: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges * InternetService +
##     SeniorCitizen + MultipleLines + InternetService + PaperlessBilling +
##     PaymentMethod
## Model 2: Churn ~ poly(tenure, 2) * Contract + MonthlyCharges * InternetService +
```

```
##     SeniorCitizen * PaymentMethod + MultipleLines + InternetService +
##     PaperlessBilling
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      7014     5778.0
## 2      7011     5769.4  3   8.6069    0.035 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In this final case, we could reject the null hypothesis at 95% confidence, but we decide to take this interaction out of the model. The best model is gm3.

16 Train-test validation & Final interpretation

This is the final model as it is. First of all we are going to make a residual analysis and, if there is any, take out the influential data to get the model performing at its best, before intepreting it and testing it.

```
fm <- glm(Churn ~ poly(tenure,2)*Contract + MonthlyCharges*InternetService + SeniorCitizen + MultipleLi
```

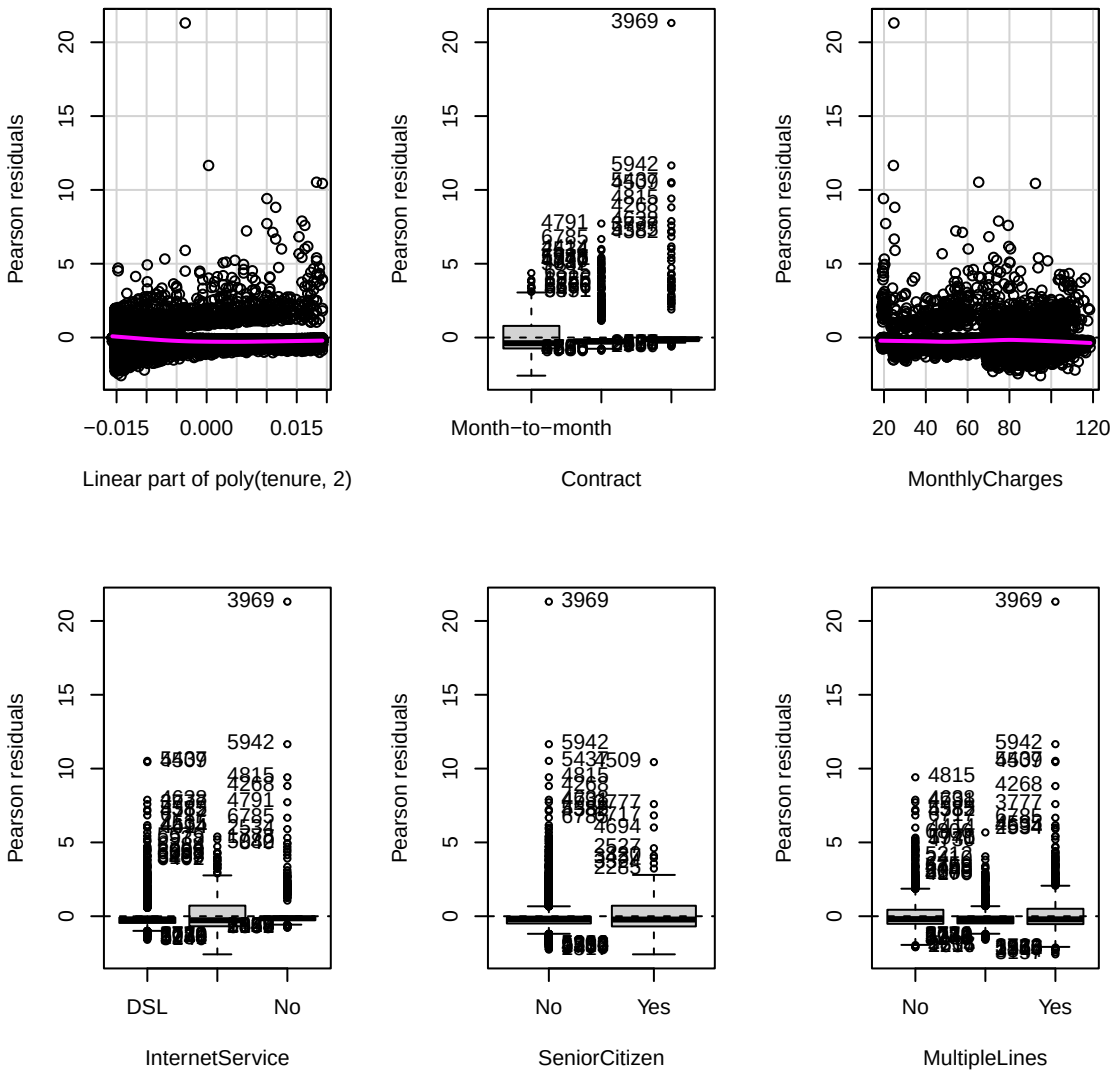
17 FM: Residual Analysis and Influential Data

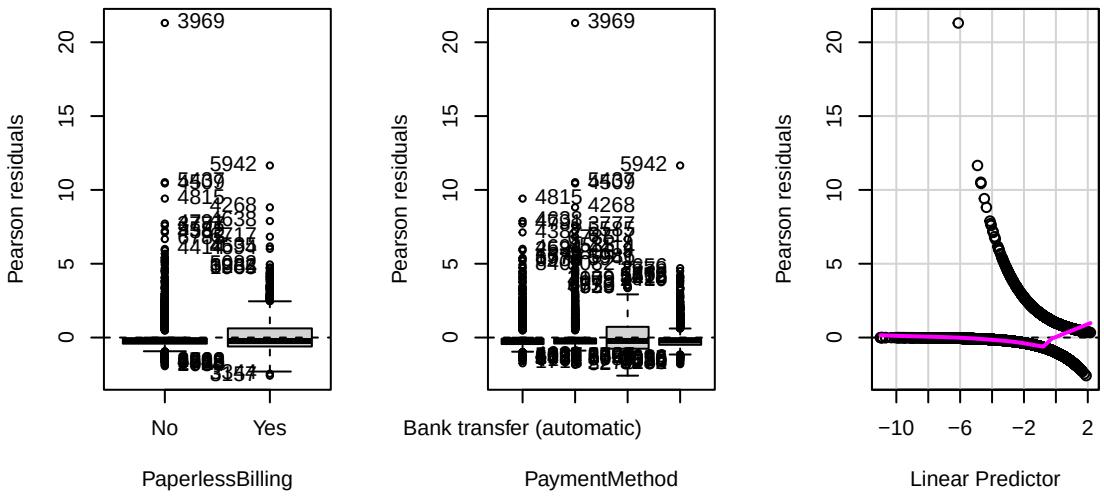
(INTERPRETATION OF RESIDUAL PLOTS)

From the influencePlot of the final model we see that there may be a lot of observations with abnormally high hat values that must be investigated, as well as some observations with extreme Cooks' Distances. When checking the influenceIndexPlot we can see many observations that stand out above the rest, for example observation 2367 and 3663. When the boxplot is plotted with the cutoff at 4*p/n we find that 56 observations are above this cutoff. We decide to remove all of these observations.

When analyzing the Cooks' Distance using influenceIndexPlot we see only two observations that stand out far above the rest, 3969 and 5942. When we continue by drawing the boxplot we get the same result, and thus set the cutoff at 0.01 to remove these observations. Once again the analysis of the summary statistics is not very revealing, the outliers seem to have much lower charges (monthly and total), however the comparison between other factors is not adequate as there are only two outliers.

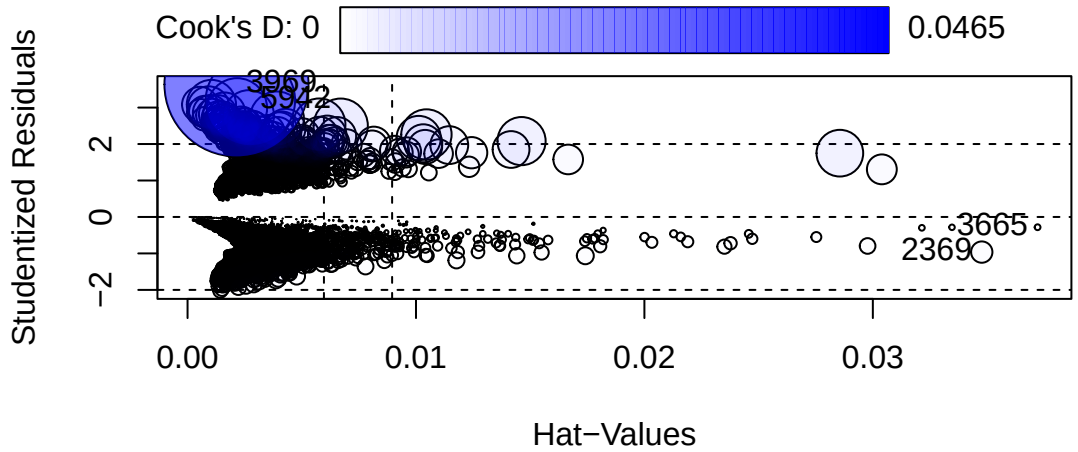
```
# Residuals
residualPlots(fm, layout=c(1, 3))
```


Pearson residuals
Linear part of poly(tenure, 2)
Contract
Month−to−month
MonthlyCharges
InternetService
DSL
No
SeniorCitizen
No
Yes
MultipleLines
No
Yes




```
##                   Test stat Pr(>|Test stat|)
## poly(tenure, 2)
## Contract
## MonthlyCharges       0.5814           0.4458
## InternetService
## SeniorCitizen
## MultipleLines
## PaperlessBilling
## PaymentMethod
```

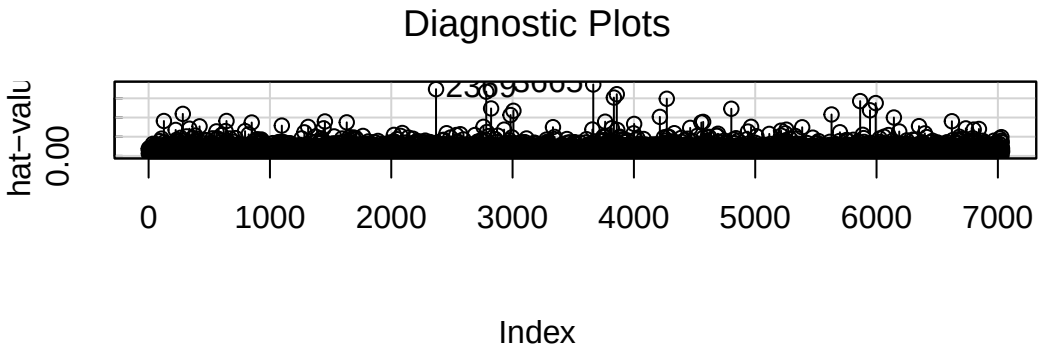
```
influencePlot(fm)
```




```
##         StudRes         Hat        CookD
## 2369 -0.9631290 0.034768311 0.0010196241
## 3665 -0.2815715 0.037210783 0.0000757755
```

```
## 3969  3.6352511 0.002142920 0.0465189488
## 5942  3.1956020 0.002751547 0.0178927286
```

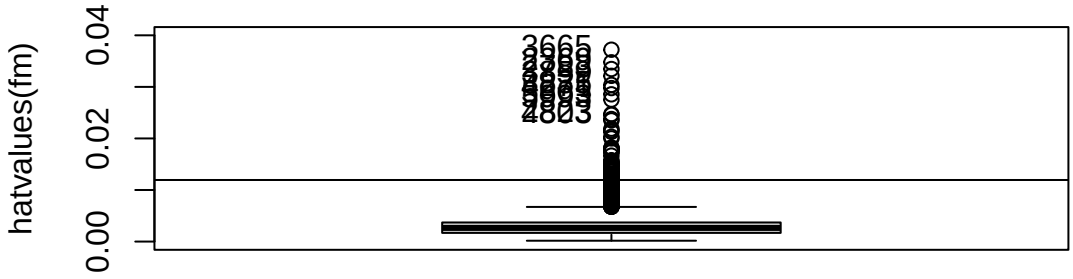
```
# Hat values
influenceIndexPlot(fm, id.n=10, vars=c('hat'))
```



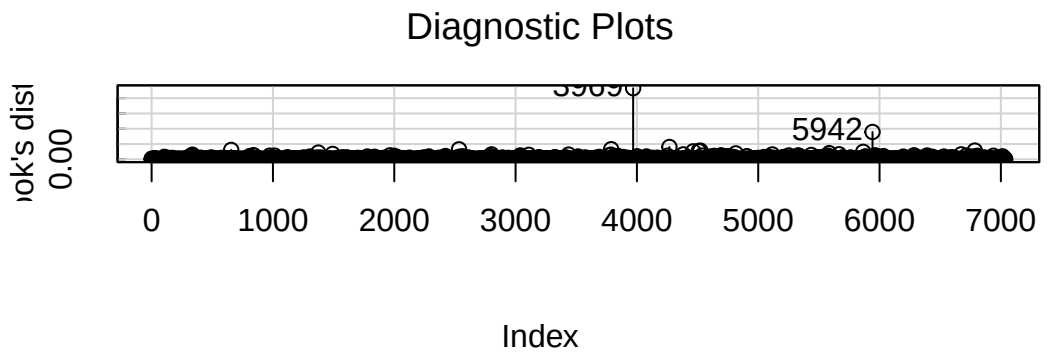

```
Boxplot(hatvalues(fm), ylim=c(0,0.04))
```

```
##  [1] 3665 2369 2783 3859 3835 4271 5865 5993 2823 4803
```

```
abline(h=4*length(coef(fm))/nrow(df))
```



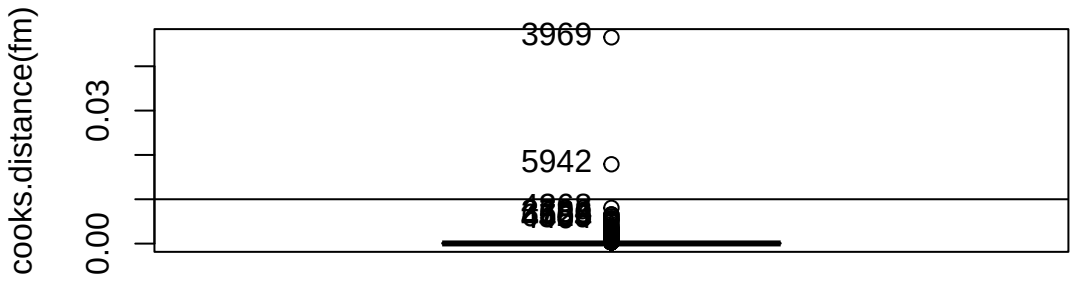
```
llhat <- which(hatvalues(fm)>4*length(coef(fm))/nrow(df))
df <- df[-llhat,]
rownames(df) <- NULL

# Cooks distance
influenceIndexPlot(fm, id.n=10, vars=c('Cook'))
```




```
Boxplot(cooks.distance(fm))
```

```
##  [1] 3969 5942 4268 3788 2534  656 6785 4523 4509 4464
```

```
abline(h=0.01)
```



```r
llcoo <- which(cooks.distance(fm)> 0.01);
summary(df[,])
```

```
##   customerID           gender     SeniorCitizen Partner    Dependents
##  Length:6979        Female:3459   No :5843      No :3597   No :4890
##  Class :character   Male  :3520   Yes:1136      Yes:3382   Yes:2089
##  Mode  :character
##
##
##
##      tenure      PhoneService          MultipleLines     InternetService
##  Min.   : 0.00   No : 675     No              :3373   DSL        :2411
##  1st Qu.: 9.00   Yes:6304     No phone service: 675   Fiber optic:3070
##  Median :29.00                Yes             :2931   No         :1498
##  Mean   :32.45
##  3rd Qu.:56.00
##  Max.   :72.00
##              OnlineSecurity              OnlineBackup
##  No                 :3475   No                 :3071
##  No internet service:1498   No internet service:1498
##  Yes                :2006   Yes                :2410
##
##
##
##            DeviceProtection              TechSupport
##  No                 :3084    No                 :3457
##  No internet service:1498    No internet service:1498
##  Yes                :2397    Yes                :2024
##
##
##
##              StreamingTV              StreamingMovies           Contract
##  No                 :2804   No                 :2777   Month-to-month:3846
##  No internet service:1498   No internet service:1498   One year      :1460
##  Yes                :2677   Yes                :2704   Two year      :1673
##
##
##
##  PaperlessBilling                   PaymentMethod  MonthlyCharges
##  No :2856         Bank transfer (automatic):1535   Min.   : 18.25
##  Yes:4123         Credit card (automatic)  :1513   1st Qu.: 35.80
##                   Electronic check         :2340   Median : 70.35
##                   Mailed check             :1591   Mean   : 64.80
##                                                    3rd Qu.: 89.85
##                                                    Max.   :118.75
##   TotalCharges  Churn
##  Min.   :   0   No :5122
##  1st Qu.: 403   Yes:1857
##  Median :1398
##  Mean   :2284
##  3rd Qu.:3790
##  Max.   :8685
```

```r
summary(df[llcoo,])
```

```
##   customerID           gender  SeniorCitizen Partner Dependents     tenure
##  Length:2           Female:0   No :2         No :1   No :2      Min.   :62.0
##  Class :character   Male  :2   Yes:0         Yes:1   Yes:0      1st Qu.:64.5
##  Mode  :character                                               Median :67.0
##                                                                 Mean   :67.0
##                                                                 3rd Qu.:69.5
##                                                                 Max.   :72.0
##  PhoneService          MultipleLines    InternetService
##  No :0        No              :1     DSL        :1
##  Yes:2        No phone service:0     Fiber optic:0
##               Yes             :1     No         :1
##
##
##
##              OnlineSecurity              OnlineBackup
##  No                 :0      No                 :0
##  No internet service:1      No internet service:1
##  Yes                :1      Yes                :1
##
##
##
##             DeviceProtection              TechSupport              StreamingTV
##  No                 :0       No                 :0    No                 :1
##  No internet service:1       No internet service:1    No internet service:1
##  Yes                :1       Yes                :1    Yes                :0
##
##
##
##             StreamingMovies           Contract PaperlessBilling
##  No                 :1      Month-to-month:0   No :1
##  No internet service:1      One year      :0   Yes:1
##  Yes                :0      Two year      :2
##
##
##
##                   PaymentMethod MonthlyCharges   TotalCharges  Churn
##  Bank transfer (automatic):2     Min.   :19.85   Min.   :1254   No :2
##  Credit card (automatic)  :0     1st Qu.:31.84   1st Qu.:2141   Yes:0
##  Electronic check         :0     Median :43.83   Median :3029
##  Mailed check             :0     Mean   :43.83   Mean   :3029
##                                  3rd Qu.:55.81   3rd Qu.:3917
##                                  Max.   :67.80   Max.   :4805
```

```r
df <- df[-llcoo,]
rownames(df) <- NULL
```

18 Interpretation

Now that we have our influential data removed, let's move on to the interpretation of the model. Let's keep in mind that the base class for this model is Churn:No. To start with the numerical variables, MonthlyCharges

parameter is negative, which means that, as the amount charged by the month increases, it decreases the probability in the logodds scale that a customer will leave by 1.514e-03 units, which is not very much so we could consider it neutral. In the case of tenure, we see that as a client stays more months in the company, the probability of it to leave is 6.892e+0 units less in the logodds scale. This, a priori, speaks well of the company, since it is clear that clients tend to leave less as they experiment its services over time. However, the transformation of tenure^2 has the opposite effect, meaning that with clients that have been a lot of months in the company are more probable to leave.

Now for the categorical variables. Looking at contract we can see that, over the base class which is month to month, customers with contracts of one year are a little less probable to leave, and even less for contracts of two years, that are 3.798e+00 units less probable to leave in the logodds scale. In the case of InternetService, No internet service and Fiber are less probable to leave than DSL. Then, we see that SeniorCitizens are more probable to leave, which maybe can be explained because they are more probable to die. Futhermore, those clients with No phone service or multiple lines are more likely to leave than those with only one phone line. The paperless bill also plays a part, since those using a paperless (online) bill are 3.838e-01 units more probable to leave in the logodds scale than those with paper bill. That can be maybe attributed to the fact that many paper bill users don't even see the bill, since paper mail is given less attention these days. In terms of payment method, MailedCheck makes it less likely for the clients to leave than Bank transfer (the base class), which has to do with what was mentioned earlier in the Paperless bill. The same phenomena is observed with CreditCard payment although it is not as pronounced. The opposite is observed with Wlectronic check, since users of this payment method are more likely to leave the company.

Finally, let's look at the interactions. To start with, we have the interaction between the transformed tenure and Contracts. We can see that, regarding one year contracts, the probability of a customer to leave increases 2.276e+01 units in the logodds scale as tenure months increase compared to those customers with month to month contracts; and the probability of a customer to leave decreases 2.297e+01 units in the logodds scale as tenure^2 months increase. Similar with two-year contracts but not as pronounced. This means that, customers with one year or two year contracts are more likely to leave than those with month to month contracts over the months. However, the opposite happens with tenure^2, which means that if you look over large periods of time moving faster, customers with closed contracts are less likely to leave than those with month to month contracts. To end with, let's look at the interaction between MonthlyCharges and InternetService. We can see that those customers with Fiber optic are more likely to leave as their monthly charges go up than those with DSL, and the opposite with those with no internet service. However, the coeficients are relatively small, so no further conclusions must be dragged from this.

```
fm <- glm(Churn ~ poly(tenure,2)*Contract + MonthlyCharges*InternetService + SeniorCitizen + MultipleLi
summary(fm)
```

```
##
## Call:
## glm(formula = Churn ~ poly(tenure, 2) * Contract + MonthlyCharges *
##     InternetService + SeniorCitizen + MultipleLines + InternetService +
##     PaperlessBilling + PaymentMethod, family = "binomial", data = df)
##
## Coefficients:
##                                          Estimate Std. Error z value
## (Intercept)                            -1.928e+00  3.711e-01  -5.195
## poly(tenure, 2)1                       -6.892e+01  6.167e+00 -11.176
## poly(tenure, 2)2                        2.803e+01  4.386e+00   6.391
## ContractOne year                       -9.184e-01  1.501e-01  -6.117
## ContractTwo year                       -3.798e+00  1.061e+00  -3.578
## MonthlyCharges                         -1.514e-03  6.054e-03  -0.250
## InternetServiceFiber optic             -5.081e-01  4.658e-01  -1.091
## InternetServiceNo                      -5.015e-01  1.670e+00  -0.300
## SeniorCitizenYes                        2.917e-01  8.291e-02   3.519
```

```
## MultipleLinesNo phone service              6.009e-01  1.757e-01   3.419
## MultipleLinesYes                           3.515e-01  8.547e-02   4.113
## PaperlessBillingYes                        3.838e-01  7.506e-02   5.113
## PaymentMethodCredit card (automatic)      -8.903e-02  1.139e-01  -0.782
## PaymentMethodElectronic check              3.295e-01  9.447e-02   3.488
## PaymentMethodMailed check                 -1.110e-01  1.160e-01  -0.957
## poly(tenure, 2)1:ContractOne year          2.276e+01  1.297e+01   1.755
## poly(tenure, 2)2:ContractOne year         -2.297e+01  1.082e+01  -2.122
## poly(tenure, 2)1:ContractTwo year          1.877e+02  8.666e+01   2.166
## poly(tenure, 2)2:ContractTwo year         -1.128e+02  3.783e+01  -2.982
## MonthlyCharges:InternetServiceFiber optic  1.920e-02  6.667e-03   2.880
## MonthlyCharges:InternetServiceNo          -9.200e-03  8.114e-02  -0.113
##                                           Pr(>|z|)
## (Intercept)                               2.05e-07 ***
## poly(tenure, 2)1                           < 2e-16 ***
## poly(tenure, 2)2                          1.65e-10 ***
## ContractOne year                          9.51e-10 ***
## ContractTwo year                          0.000346 ***
## MonthlyCharges                            0.802584
## InternetServiceFiber optic                0.275304
## InternetServiceNo                         0.763921
## SeniorCitizenYes                          0.000433 ***
## MultipleLinesNo phone service             0.000627 ***
## MultipleLinesYes                          3.91e-05 ***
## PaperlessBillingYes                       3.16e-07 ***
## PaymentMethodCredit card (automatic)      0.434462
## PaymentMethodElectronic check             0.000487 ***
## PaymentMethodMailed check                 0.338514
## poly(tenure, 2)1:ContractOne year         0.079278 .
## poly(tenure, 2)2:ContractOne year         0.033796 *
## poly(tenure, 2)1:ContractTwo year         0.030309 *
## poly(tenure, 2)2:ContractTwo year         0.002868 **
## MonthlyCharges:InternetServiceFiber optic 0.003980 **
## MonthlyCharges:InternetServiceNo          0.909721
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 8085.0  on 6976  degrees of freedom
## Residual deviance: 5736.9  on 6956  degrees of freedom
## AIC: 5778.9
##
## Number of Fisher Scoring iterations: 8
```

From the summary of the model we can see that the Residual deviance is 5814.5 and the AIC is 5856.5. These are relatively high numbers, however we understand that AIC is more punishing to models with more parameters which is the case for our final model.

19 Train-test

Initially, the dataset was partitioned into two subsets, namely the training set, comprising 80% of the data, and the test set, consisting of 20%. Subsequently, the model was trained using the training data to predict

churn. The next step involved predicting dropout in the test dataset. The model achieved an accuracy of 80%.

Finally, an additional assessment was conducted using the ROC curve. The Area Under the Curve (AUC) on the Receiver Operating Characteristic (ROC) curve serves as a metric for gauging the classifier's efficacy in distinguishing between positive and negative classes. An AUC value of 0.87 suggests very robust performance. Typically values above 0.90 are not achievable in practice and therefore our model shows near excellent behaviour in this regard.

```
# Install and load the pROC package
#install.packages("pROC")
library(pROC)
```

```
## Type 'citation("pROC")' for a citation.

##
## Attaching package: 'pROC'

## The following objects are masked from 'package:stats':
##
##     cov, smooth, var
```

```
set.seed(1234)
llwork <- sample(1:nrow(df), round(0.8 * nrow(df), dig = 0))
train_data <- df[llwork, ]
test_data <- df[-llwork, ]

# Train model to predict churn
fm <- glm(Churn ~ poly(tenure, 2) * Contract + MonthlyCharges * InternetService + SeniorCitizen + Multi

# Predict churn using the model
fm_pred <- predict(fm, test_data, type = "response")

# Print first few results
head(fm_pred, 10)
```

```
##           2           8          12          18          20          22
## 0.041164279 0.322210388 0.001292906 0.041389916 0.468325456 0.044721183
##          25          28          31          37
## 0.050557470 0.586235786 0.066176091 0.633474936
```

```
# Check model performance
test_data$predict.Churn <- ifelse(fm_pred < 0.5, "No", "Yes")

# Accuracy
accuracy <- mean(test_data$predict.Churn == test_data$Churn)
cat("Accuracy:", accuracy, "\n")
```

```
## Accuracy: 0.818638
```

```r
# Create ROC curve
roc_curve <- roc(test_data$Churn, fm_pred)
```

```
## Setting levels: control = No, case = Yes
```

```
## Setting direction: controls < cases
```

```r
# Plot ROC curve
plot(roc_curve, main = "ROC Curve", col = "blue", lwd = 2)

# Add AUC to the plot
auc_value <- auc(roc_curve)
legend("bottomright", legend = paste("AUC =", round(auc_value, 2)), col = "blue", lwd = 2)
```