A local photon perspective on the Abraham-Minkowski controversy

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Abstract

The Abraham-Minkowski controversy debates the definition of light's momentum in dielectric media. We use a local photon approach, linking dynamical momentum to photonic wave packet translation. Introducing a locally acting mirror Hamiltonian, we analyse dynamics and show that momentum and energy are conserved during light transition air to denser medium. Our theory aligns photon rates with Fresnel coefficients in classical optics.

Motivation

Classical elastic scattering problems require two conservation laws: energy and momentum. The wave packet description offered by classical electrodynamics must be consistent with these conservation laws. The Abraham-Minkowski controversy arises from the fact that there are several different definitions for the momentum of light within a dielectric medium.

$$p_{\text{Abr.}}\,\hat{\boldsymbol{x}} = \frac{A}{c^2}\int_{-\infty}^{\infty} \mathrm{d}x\ \boldsymbol{E}(x)\times\boldsymbol{H}(x) \qquad p_{\text{Min.}}\,\hat{\boldsymbol{x}} = \frac{A}{v^2}\int_{-\infty}^{\infty} \mathrm{d}x\ \boldsymbol{E}(x)\times\boldsymbol{H}(x)$$

We use a new quantum approach that describes localised wave packets of light that travel at constant speed, without dispersion — *bosons localised in position* (blips). Can we use this theory to describe the dynamics of light in the presence of mirrors and dielectric media? Furthermore, does our theory agree with the predictions of Abraham or Minkowski?

Free space dynamics

Define dynamical Hamiltonian and momentum observables based on the Schrödinger equation. Our description includes both positive and negative frequency photons [1–5]. We start from the first-order differential equation derived from Maxwell's equations

$$\left(\frac{\partial}{\partial x}+\frac{s}{c}\frac{\partial}{\partial t}\right)\boldsymbol{\mathcal{O}}_{s\lambda}(x,t)=0 \qquad \boldsymbol{\mathcal{O}}_{s\lambda}(x,t)=\boldsymbol{\mathcal{O}}_{s\lambda}(x-sct,0)$$

In the Heisenberg picture, blip states are operators $a_{s\lambda}(x,t)$. Free space propagation dynamics in position representation

$$\left[a_{s\lambda}(x),a^{\dagger}_{s'\lambda'}(x')\right]=\delta_{ss'}\,\delta_{\lambda\lambda'}\,\delta(x-x') \qquad H_{\text{dyn}}=\mathrm{i}\hbar\sum_{s=\pm1}\sum_{\lambda=\mathsf{H},\mathsf{V}}\int_{-\infty}^{\infty}\mathrm{d}x\ sc\,a^{\dagger}_{s\lambda}(x)\,\frac{\partial}{\partial x}a_{s\lambda}(x) \qquad \frac{\partial}{\partial t}a_{s\lambda}(x,t)=-sc\frac{\partial}{\partial x}a_{s\lambda}(x,t)$$

Complex local electric and magnetic field observables constructed from photonic wave packets are Lorentz covariant under an appropriate regularisation operator. From these observables a familiar expression for the energy Hamiltonian can be constructed.

Mirror dynamics

The mirror scattering operator and transition matrix recover Stoke's relations. We have complex reflection and transmission coefficients.

$$r^*_{-1}t_1+t^*_{-1}r_1=0 \qquad |r_{\pm1}|^2+|t_{\pm1}|^2=1$$

The mirror acts locally and hence the associated Hamiltonian is comprised of two parts.

$$H_{\text{mir}}=H_{\text{dyn}}+H_{\text{int}} \qquad H_{\text{int}}=\sum_{\lambda=\mathsf{H},\mathsf{V}}\hbar\,\Omega\,a^{\dagger}_{-1\lambda}(0)\,a_{1\lambda}(0)+\text{h.c.}$$

For an incoming left moving single-blip excitation

$$H_{\text{int}}\,U_{\text{dyn}}(\tau,0)\,|1_{1\lambda}(x)\rangle=\hbar\,\Omega\,|1_{-1\lambda}(0)\rangle\,\delta(x+c\tau)$$
$$U_{\text{mir}}(\tau,0)\,|1_{1\lambda}(x)\rangle=t_1\,|1_{1\lambda}(x+c\tau)\rangle+r_1\,|1_{-1\lambda}(-x-c\tau)\rangle$$

The reflection and transmission coefficients are given by

$$\tilde{r}=-\frac{\Omega/c}{1+(|\Omega|/2c)^2} \qquad \tilde{t}=\frac{1-(|\Omega|/2c)^2}{1+(|\Omega|/2c)^2}$$

Medium dynamics

Free space and mirror dynamics can be used to characterise the behaviour of light in a medium. Given a medium with refractive index $n>1$, the propagation of light is easily characterised. It is important however to preserve photon number.

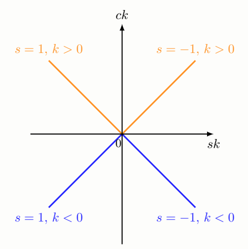
$$H_{\text{dyn}}=-\mathrm{i}\hbar\sum_{s=\pm1}\sum_{\lambda=\mathsf{H},\mathsf{V}}\int_{-\infty}^{\infty}\mathrm{d}x\ sc\,b^{\dagger}_{s\lambda}(x)\,\frac{\partial}{\partial x}b_{s\lambda}(x)$$

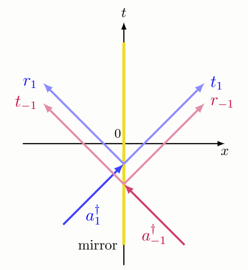
$$b_{s\lambda}(x)=\begin{cases}a_{s\lambda}(x) & \text{for}\ \ x\le 0\\ a_{s\lambda}(x/n)/\sqrt{n} & \text{for}\ \ x>0\end{cases}$$

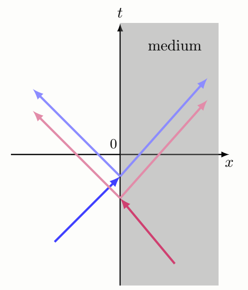
The dynamics of light incident on a medium can be described by the mirror Hamiltonian.

$$U_{\text{mir}}(\tau,0)\,|1_{1\lambda}(x)\rangle=t_1\,|1_{1\lambda}((x+c\tau)/n)\rangle+r_1\,|1_{-1\lambda}(-x-c\tau)\rangle$$

The interaction Hamiltonian is more or less the same as the one used for mirrors. The $1/\sqrt{n}$ factor in the medium dynamics is a direct result of the conservation of photon number.







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Conclusion

$$p_{\text{dyn}}=\mathrm{i}\hbar\sum_{s=\pm1}\sum_{\lambda=\mathsf{H},\mathsf{V}}\int_{-\infty}^{\infty}\mathrm{d}x\ a^{\dagger}_{s\lambda}(x)\,\frac{\partial}{\partial x}a_{s\lambda}(x)$$

$$p_{\text{dyn}}=\hbar\sum_{s=\pm1}\sum_{\lambda=\mathsf{H},\mathsf{V}}\int_{-\infty}^{\infty}\mathrm{d}k\ \ s\,k\,\tilde{a}^{\dagger}_{s\lambda}(k)\,\tilde{a}_{s\lambda}(k)$$

$$[H_{\text{eng}},H_{\text{dyn}}]=0 \qquad [p_{\text{dyn}},H_{\text{dyn}}]=0$$

- The dynamics of light in the presence of mirrors and dielectric media can be described by a local photon theory.
- The predictions of our theory align with the predictions of classical optics.
- The mirror coupling constant Ω depends on the photon speed and the reflection and transmission coefficients.
- Photons transitioning from air to a dielectric medium have their momentum *increase* by a factor n upon entering the medium and *decrease* by a factor n upon exiting the medium — Minkowski!