Guided Hamiltonians Are Easy: physically-motivated states enable **classical** and **quantum** algorithms

Goldilocks Zone

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Local Hamiltonian Systems

$$H = \sum_{j=1}^{m} h_j \qquad \Upsilon = \{(\lambda_s, \phi_s)\}_{s=0}^{2^n-1}$$

Each h_j acts non-trivially on at most a constant number of qubits

ϕ_0 and λ_0 denote the **ground state** and **ground state energy**

Read our paper!

arXiv:2509.25815

"Physically-Motivated Guiding States for Local Hamiltonians"

Guiding States

A guiding state is a quantum state $|\xi\rangle$ that has overlap with the **ground state** of a local Hamiltonian: $|\langle\xi|\phi_0\rangle|^2 = \delta$ and admits a **succinct classical description**.

*Are there **families of states** with interesting **physical** and/or **computational** attributes that we can use to **estimate the ground state energy**?*

Classical Result

Given a 2-local Hamiltonian, and a guiding state with **O(1) overlap**, deciding the ground state energy to **O(1) additive-error** is in **BPP**

Semi-Classical Subset States[1]

$$|\hat{C}\rangle = \frac{1}{\sqrt{|C|}} \sum_{x \in C} |x\rangle$$

Matrix Product States

$$|\Psi\rangle = \sum_{\underline{\sigma}} \mathrm{Tr}\Big[\prod_{j \in [n]} A^{\sigma_j}\Big] |\underline{\sigma}\rangle$$

Weight-k States

$$|X_{n,k}\rangle = \sum_{\substack{x \in \{0,1\}^n \\ |x|_1 = k}} \alpha_x |x\rangle$$

Gaussian States

$$M_{k,l} = -\frac{i}{2} \operatorname{tr}(\varphi[c_k, c_l])$$

Quantum Result

Given a 2-local Hamiltonian, and a guiding state with **Ω(1/poly) overlap**, deciding the ground state energy to **O(1/poly) additive-error** is **BQP-complete**

Dequantisation Algorithm[1]

Define a filter function on the Hamiltonian:

$$f(H) = \sum_{s=0}^{2^n-1} f(\lambda_s) |\phi_s\rangle\langle\phi_s|$$

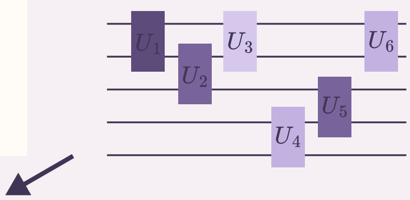
$$f(H) \succeq \sum_{s:\lambda_s \in [0,a]} |\phi_s\rangle\langle\phi_s|$$

Given query access to Hamiltonian elements: $\mathcal{Q}_H$, query and sample access to the guiding state: $\mathcal{Q}_\xi$, $\mathcal{S}_\xi$

Construct a polynomial: $p(x) \approx_\epsilon f(x)$

Evaluate: $p(H)|\xi\rangle$

Circuit-to-Hamiltonian Construction[2]



$$\hat{H}_\mu = \Delta(H_{\text{in}} + H_{\text{clock},\mu} + H_{\text{prop}}) + H_{\text{out}}$$

$$|\eta\rangle = \frac{1}{\sqrt{K+1}} \sum_{t=0}^{K} |\varphi_t\rangle |\mu(t)\rangle$$

References

[1]: S. Gharibianand F. Le Gall, *Dequantizing the Quantum Singular Value Transformation: Hardness and Applications to Quantum Chemistry and the Quantum PCP Conjecture*, 2023

[2]: A. Y. Kitaev, A. H. Shen, and M. N. Vyalyi, *Classical and Quantum Computation*, 2002