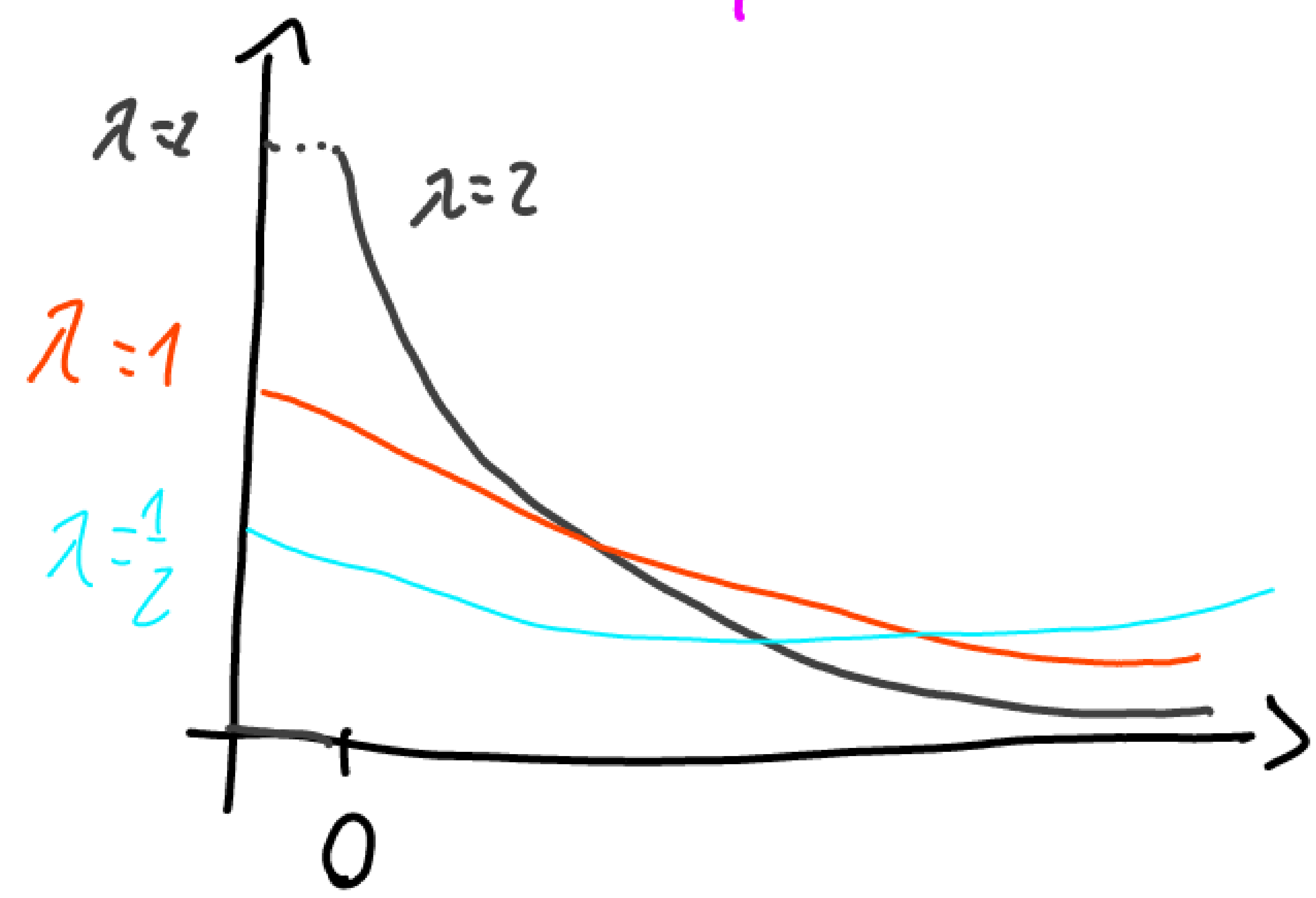
(22) exponential distribution $x \sim Exp(\lambda)$, $\lambda > 0$

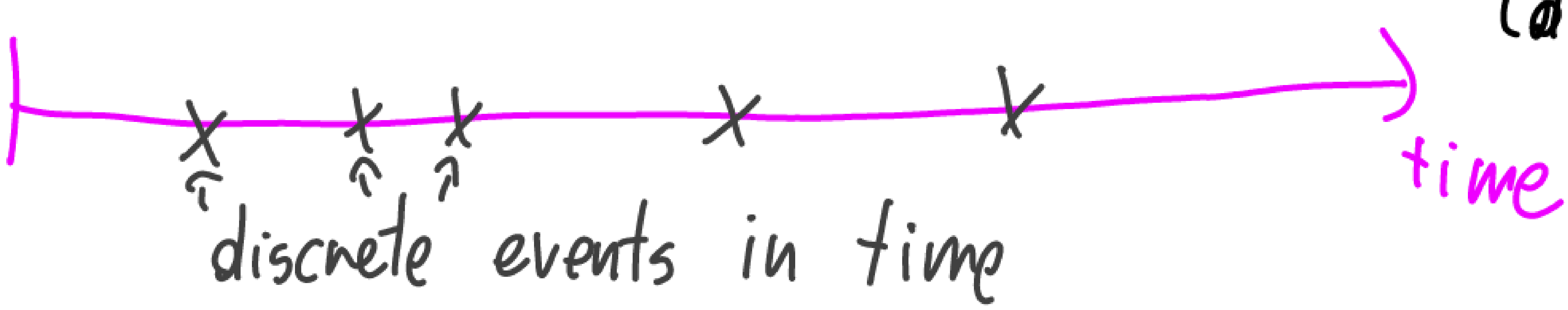
PDF: $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

CDF: $F(X) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$



usage: It is used to model times between events.

So let's imagine we have time (continues) and events (discrete)



the number of events happening in interval is given by Poisson distribution.

Exponential distribution is used for ansering questions like what is the probability that interval will have some length before the event.

So it's used to describe "distance" (continues) between events (discrete).

Lack of memory, so Pr between time 0 and X.

a) $\cdot\ E[X] = \int_0^\infty x\lambda e^{-\lambda x}dx = \frac{1}{\lambda}\int_0^\infty \lambda \cdot x\lambda e^{-\lambda x}dx = \frac{1}{\lambda}\int_0^\infty u\, e^{-u}du$

$u = x\lambda$
$u' = \lambda$
$u(0) = 0$
$u(\infty) = \infty$

$$= \frac{1}{\lambda}\left[-e^{-u} - ue^{-u}\right]_0^\infty = \frac{1}{\lambda}\left(0-(-1)\right) = \frac{1}{\lambda}$$

$$\int xe^x dx = \int u\,dv = uv - \int v\,du = xe^x - \int e^x dx = xe^x - e^x + C$$

$u = x \qquad v = e^x$
$dv = e^x dx \qquad du = dx$

$\cdot\ Var(X) = E[X^2] - E[X]^2 =$

$$E[X^2] = \int_0^\infty x^2\lambda e^{-\lambda x}dx = \frac{1}{\lambda^2}\int_0^\infty u^2 e^{-u}du = \frac{1}{\lambda^2}\left[-2e^{-u} - 2ue^{-u} - u^2e^{-u}\right]_0^\infty$$

$$= \frac{2}{\lambda^2}$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda} = \frac{1}{\lambda^2}$$

6) $\underset{\text{Uni}}{\sim} \text{Gen}_u \rightarrow u \in [0,1)$

TheInversion Method (Gen_u, F^{-1}):
| for x in $F^{-1}(\text{Gen}_u)$:
| | yield x

in this case F^{-1}:

Generating values from the exponential distribution

- $S \sim \text{Exp}(\lambda)$, $E[S] = \frac{1}{\lambda}$, $f(x) = \lambda e^{-\lambda x}$
- CDF $F(x) = 1 - e^{-\lambda x}$, $x \geq 0$, $\lambda > 0$

we look for inverse of F.

- $u = 1 - e^{-\lambda x} \rightarrow x = \frac{\ln(1-u)}{\lambda} \rightarrow F^{-1}(u) = \frac{\ln(1-u)}{\lambda}$

$-(u-1) = e^{-\lambda x}$ $/\ln()$

$\ln(1-u) = -\lambda x \rightarrow x = \frac{\ln(1-u)}{-\lambda} \approx \frac{\ln(u)}{-\lambda}$

Inverse Transform Sampling

Let F be CDF on R. We define inverse F^{-1} by equation

$$F^{-1}(u) = \inf\{x : F(x) = u, 0 < u < 1\}$$

1) Then for $U \sim \text{Unf}$, $F^{-1}(U) \sim F$

2) if $X \sim F$, then $F(X) \sim \text{Unif}[0,1]$