Lista 6- Geometria Analítica

Aluna: Gabriela Mazon Rabello de Souza R.A: 2025.1.08.006

Número 1

a) $\vec{u}=(1,1,1)_E$ $\|\vec{u}\|=\sqrt{3}$

$\|\vec{u}\|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

b) $\vec{u}=3\vec{i}+4\vec{k}$ $\|\vec{u}\|=5$

$\vec{u}=(3,0,4)_E$

$\|\vec{u}\|=\sqrt{3^2+0^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

c) $\vec{u}=-\vec{i}+\vec{j}$ $\|\vec{u}\|=\sqrt{2}$

$\vec{u}=(-1,1,0)_E$

$\|\vec{u}\|=\sqrt{(-1)^2+1^2+0^2}=\sqrt{1+1}=\sqrt{2}$

d) $\vec{u}=4\vec{i}+3\vec{j}-\vec{k}$ $\|\vec{u}\|=\sqrt{26}$

$\vec{u}=(4,3,-1)_E$

$\|\vec{u}\|=\sqrt{4^2+3^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}$

Número 2

a) $E=(e_1,e_2,e_3)_E$ é uma base ortonormal pois cada vetor representa uma aresta do cubo, e como a aresta do cubo é unitária, a norma de cada vetor é 1, logo, E é uma base ortonormal

b) $\vec{u}=(0,-1,-1)_E$ $\vec{w}=(1,1,-1)_E$

$\vec{v}=(0,1,-1)_E$

$\vec{u}=\vec{CD}+\vec{CB}=-\vec{e_2}-\vec{e_3}=(0,-1,-1)_E$

$\vec{v}=\vec{DC}+\vec{CB}=\vec{e_2}-\vec{e_3}=(0,1,-1)_E$

$\vec{w}=\vec{GC}=\vec{e_1}+\vec{e_2}-\vec{e_3}=(1,1,-1)_E$

c) Todas as normas são 1 após a normalização, portanto, F é uma base ortonormal.

$\vec{f_1}=\frac{\vec{u}}{\|\vec{u}\|}=\frac{(0,-1,-1)}{\sqrt{2}}=\left(0,\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$

$\vec{f_2}=\frac{\vec{v}}{\|\vec{v}\|}=\frac{(0,1,-1)}{\sqrt{2}}=\left(0,\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$

$\vec{f_3}=\vec{w}=(1,1,-1)$

$\vec{f_1}\cdot\vec{f_2}=0$ $\vec{f_1}\cdot\vec{f_3}=0$ $\vec{f_2}\cdot\vec{f_3}=0$

d) Para ser ortogonal, $M^TM=I$. Como F é ortonormal, M é ortogonal.

$$M=\begin{bmatrix} 0 & 0 & -1 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

e) $\vec{HB}=\vec{B}-\vec{H}=(-1,1,1)_E=(0,\sqrt{2},1)_F$

Em F:

$$\vec{HB}_F=M^T\cdot\vec{HB}_E=\begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}=\begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix}$$

Número 3

a) $\vec{AB} = (3,-3,-6)$ $\vec{BC} = (-5,-4,4)$ $\vec{CA} = (2,7,2)$

$\vec{AB} = B - A = (3,-3,-6)$

$\vec{BC} = C - B = (-5,-4,4)$

$\vec{CA} = A - C = (2,7,2)$

b) $\|\vec{AB}\| = 3\sqrt{6}$ $\|\vec{BC}\| = \sqrt{57}$ $\|\vec{CA}\| = \sqrt{57}$

$\|\vec{AB}\| = \sqrt{3^2+(-3)^2+(-6)^2} = \sqrt{9+9+36} = \sqrt{54} = 3\sqrt{6}$

$\|\vec{BC}\| = \sqrt{(-5)^2+(-4)^2+4^2} = \sqrt{25+16+16} = \sqrt{57}$

$\|\vec{CA}\| = \sqrt{2^2+7^2+2^2} = \sqrt{4+49+4} = \sqrt{57}$

ABC é isosceles pois $\|\vec{CA}\| = \|\vec{BC}\| = \sqrt{57}$

c) $M_{AB} = \left(\frac{7}{2}, \frac{5}{2}, 0\right)$

$M_{BC} = \left(\frac{5}{2}, -1, -1\right)$

$M_{CA} = \left(1, \frac{1}{2}, 2\right)$

Mediana relativa a AB: $\vec{CM_{AB}} = M_{AB} - C$

$= \left(\frac{7}{2}, \frac{11}{2}, -1\right)$

d) Ângulo $B\hat{C}A = \arccos\left(-\frac{10}{19}\right)$

$\cos\theta = \dfrac{\vec{CB}\cdot\vec{CA}}{\|\vec{CB}\|\,\|\vec{CA}\|} = \dfrac{(5,4,-4)\cdot(2,7,2)}{\sqrt{57}\cdot\sqrt{57}} = \dfrac{10+28-8}{57}$

$= \dfrac{30}{57} \Rightarrow \theta = \cos^{-1}\left(\frac{10}{19}\right)$

e) $\vec{AB} + \vec{BC} + \vec{CA} = (0,0,0)$

$= (3-5+2, -3-4+7, -6+4+2) = (0,0,0)$

Número 4

a) $\vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cdot\cos(\theta) \Rightarrow |\vec{u}\cdot\vec{v}| \leqslant \|\vec{u}\|\cdot\|\vec{v}\|$

Igualdade ocorre se $\cos(\theta) = \pm 1$, ou seja, vetores paralelos. //

b) $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + 2\vec{u}\cdot\vec{v} + \|v\|^2$

$2\vec{u}\cdot\vec{v} \leqslant 2\|\vec{u}\|\cdot\|\vec{v}\| \therefore \|\vec{u}+\vec{v}\| \leqslant \|\vec{u}\| + \|\vec{v}\|$ //

c) $\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2 + 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$

$\|\vec{u}-\vec{v}\|^2 = \|\vec{u}\|^2 - 2\vec{u}\cdot\vec{v} + \|\vec{v}\|^2$

$\|\vec{u}+\vec{v}\|^2 - \|\vec{u}-\vec{v}\|^2 = 4\vec{u}\cdot\vec{v}$ //

Número 5

a) $\theta = \frac{\pi}{2}$ rad

$\cos\theta = \dfrac{1\cdot(-2)+0\cdot 10+1\cdot 2}{\sqrt{2}\cdot\sqrt{108}} = 0 \to \theta = \frac{\pi}{2}$

b) $\theta = \cos^{-1}\left(\frac{1}{3}\right) = 1{,}23096\pi$ rad

$\cos\theta = \dfrac{-1+1+1}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{1}{3} \to \theta = \cos^{-1}\left(\frac{1}{3}\right) \cong 70{,}5288^\circ$

180°	π
$70{,}5288$	x

$x = \dfrac{70{,}5288}{180}\pi = 1{,}23096\pi$

c) $\theta = \frac{\pi}{4}$ rad

$\cos\theta = \dfrac{6+3+0}{\sqrt{18}\cdot\sqrt{9}} = \dfrac{9}{3\sqrt{18}} = \dfrac{\sqrt{2}}{2} \Rightarrow \theta = \dfrac{\pi}{4}$

d) $\theta = \frac{\pi}{3}$ rad

$\cos\theta = \dfrac{3+1+0}{\sqrt{4}\cdot\sqrt{16}} = \dfrac{4}{8} = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

Número 6

a) $x = \pm\sqrt{6}$ //

$$(x+1).(x-1)+1.(-1)+2.(-2)=0$$

$$x^2-1-1-4=0 \rightarrow x^2-6=0$$

$$x = \pm\sqrt{6}$$ //

b) $x = -2$ //

$$x.4+x.x+4.1=0$$

$$x^2+4x+4=0$$

$$(x+2)^2=0$$

$$x=-2$$ //

Número 7

a) $\vec{u} = (1,-1,-1)$

$$\vec{v} \times \vec{w} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 5 \\ 1 & -2 & 3 \end{bmatrix} = \vec{i}(-3+10)-\vec{j}(12-5)+\vec{k}(-8+1)$$

$$= (7,-7,-7)$$

$$\vec{u} = k(7,-7,-7)$$

$$\vec{u}.(1,1,1) = 7k-7k-7k = -7k = -1 \quad k = 1/7$$

$$\vec{u} = (1,-1,-1)$$ //

b) $\vec{u} = (3,-3,-3)$

$$\vec{v} \times \vec{w} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & -1 \\ 2 & -4 & 6 \end{bmatrix} = \vec{i}(18-4)-\vec{j}(12+2)+\vec{k}(-8-6)$$

$$= (14,-14,-14) \qquad \vec{u} = k(14,-14,-14)$$

$$\|\vec{u}\| = |k|\sqrt{14^2+(-14)^2+(-14)^2} = |k|\sqrt{588} = |k|.14\sqrt{3} = 3\sqrt{3}$$

$$|k| = \frac{3}{14} \rightarrow \vec{u} = (3,-3,-3)$$ //

c) $\theta = \arccos\left(\frac{4}{\sqrt{26}}\right)$

$$\vec{u}.\vec{v} = \|\vec{u}\|.\|\vec{v}\|.\cos\left(\frac{\pi}{4}\right) = \sqrt{5}.1.\frac{\sqrt{2}}{2} = \frac{\sqrt{10}}{2}$$

$$\|\vec{u}+\vec{v}\|^2 = \|\vec{u}\|^2+\|\vec{v}\|^2+2\vec{u}.\vec{v} = 5+1+\sqrt{10} = 6+\sqrt{10}$$

$$\|\vec{u}-\vec{v}\|^2 = \|\vec{u}\|^2+\|\vec{v}\|^2-2\vec{u}.\vec{v} = 5+1-\sqrt{10} = 6-\sqrt{10}$$

$$(\vec{u}+\vec{v}).(\vec{u}-\vec{v}) = \|\vec{u}\|^2-\|\vec{v}\|^2 = 5-1=4$$

$$\cos\theta = \frac{4}{\sqrt{6+\sqrt{10}}.\sqrt{6-\sqrt{10}}} = \frac{4}{\sqrt{36-10}} = \frac{4}{\sqrt{26}}$$ //

Número 8

a) $proj_{\vec{u}}\vec{v} = \left(\frac{18}{11}, \frac{-6}{11}, \frac{6}{11}\right)$ //

$$\vec{u}.\vec{v} = 3+1+2=6 \ ; \ \|\vec{u}\|^2 = 9+1+1=11$$

$$proj_{\vec{u}}\vec{v} = \frac{6}{11}(3,-1,1) = \left(\frac{18}{11}, \frac{-6}{11}, \frac{6}{11}\right)$$ //

c) $proj_{\vec{u}}\vec{v} = \left(-\frac{10}{9}, \frac{5}{9}, \frac{10}{9}\right)$ //

$$\vec{u}.\vec{v} = 2+1+2=5 \quad \|\vec{u}\|^2 = 4+1+4=9$$

$$proj_{\vec{u}}\vec{v} = \frac{5}{9}(-2,1,2) = \left(-\frac{10}{9}, \frac{5}{9}, \frac{10}{9}\right)$$ //

b) $proj_{\vec{u}}\vec{v} = (0,0,0)$ //

$$\vec{u}.\vec{v} = -3+3+0=0 \ ; \ \|\vec{u}\|^2 = 9+1=10$$

$$proj_{\vec{u}}\vec{v} = \frac{0}{10}(-3,1,0) = (0,0,0)$$ //

d) $proj_{\vec{u}}\vec{v} = \vec{v} = (1,2,4)$ //

$\vec{u} = -2(1,2,4)$ $\quad \vec{u}$ e $\vec{v}$ são paralelos

Numero 9

a) • $Proj_{\vec{u}}\vec{v} = (4,-4,2)$

$\vec{u}.\vec{v} = 6+12+0 = 18$; $\|\vec{u}\|^2 = 4+4+1 = 9$

$proj_{\vec{u}}\vec{v} = \frac{18}{9}(2,-2,1) = (4,-4,2)$

• $Proj_{\vec{v}}\vec{u} = \left(\frac{6}{5}, \frac{-12}{5}, 0\right)$

$\vec{u}.\vec{v} = 18$; $\|\vec{v}\|^2 = 9+36 = 45$

$proj_{\vec{v}}\vec{u} = \frac{18}{45}(3,-6,0) = \left(\frac{6}{5}, -\frac{12}{5}, 0\right)$

b) • $\vec{p} = (4,-4,2)$

$\vec{p} = proj_{\vec{u}}\vec{v} = (4,-4,2)$

• $\vec{q} = (-1,-2,-2)$

$\vec{q} = \vec{v} - \vec{p} = (3,-6,0) - (4,-4,2)$

$\vec{q} = (-1,-2,-2)$

c) $\|\vec{u}\times\vec{v}\| = 9$

$$\vec{u}\times\vec{v} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 1 \\ 3 & -6 & 0 \end{bmatrix} = \vec{i}(0+6) - \vec{j}(0-3) + \vec{k}(-12+6) = (6,3,-6)$$

$\|\vec{u}\times\vec{v}\| = \sqrt{36+9+36} = \sqrt{81} = 9$

Número 10

a) $\vec{u}\times\vec{v} = (0,0,3)$ $\|\vec{u}\times\vec{v}\| = 3$

$$\vec{u}\times\vec{v} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 3 & 0 \\ 5 & 4 & 0 \end{bmatrix} = \vec{k}(12-15) = -3\vec{k} = (0,0,-3)$$

Norma: $\|\vec{u}\times\vec{v}\| = 3$

b) $\vec{u}\times\vec{v} = (10,2,14)$ $\|\vec{u}\times\vec{v}\| = 10\sqrt{3}$

$$\vec{u}\times\vec{v} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 0 & -5 \\ 1 & 2 & -1 \end{bmatrix} = \vec{i}(0+10) - \vec{j}(-7+5) + \vec{k}(14-0) = (10,2,14)$$

Norma: $\|\vec{u}\times\vec{v}\| = \sqrt{100+4+196} = \sqrt{300} = 10\sqrt{3}$

c) $\vec{u}\times\vec{v} = (-13,-3,4)$ $\|\vec{u}\times\vec{v}\| = \sqrt{194}$

$$\vec{u}\times\vec{v} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{bmatrix} = \vec{i}(-12-1) - \vec{j}(4-1) + \vec{k}(1+3) = (-13,-3,4)$$

Norma: $\|\vec{u}\times\vec{v}\| = \sqrt{169+9+16} = \sqrt{194}$

d) $\vec{u}\times\vec{v} = (0,0,0)$ $\|\vec{u}\times\vec{v}\| = 0$

$\vec{v} = 2\vec{u}$, então são paralelos.

Número 11

a) $\vec{u}.\vec{v} = \|\vec{u}\|.\|\vec{v}\|.\cos\theta$

$\|\vec{u}\times\vec{v}\| = \|\vec{u}\|\|\vec{v}\|.\operatorname{sen}\theta$

$\|\vec{u}\times\vec{v}\|^2 = \|\vec{u}\|^2.\|\vec{v}\|^2.\operatorname{sen}^2\theta$

$\operatorname{sen}^2\theta + \cos^2\theta = 1 \therefore \|\vec{u}\times\vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2(1-\cos^2\theta)$

$= \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u}.\vec{v})^2$

b) $\|\vec{u}\times\vec{v}\| = \sqrt{16} = 4$

$\|\vec{u}\times\vec{v}\|^2 = 1^2.(5)^2 - (3)^2 = 25 - 9 = 16$

$\|\vec{u}\times\vec{v}\| = \sqrt{16} = 4$

c) $\|\vec{AB}\times\vec{AC}\| = l^2.\frac{\sqrt{3}}{2}$

$\|\vec{AB}\times\vec{AC}\| = l.l.\operatorname{sen}60°$

$= l^2.\frac{\sqrt{3}}{2}$

Número 12

a) $\vec{x} = \vec{i} + \vec{j} + \vec{k}$

$2a + 3b + 4c = 9$

$\vec{x} \times (-\vec{i} + \vec{j} - \vec{k}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ -1 & 1 & -1 \end{vmatrix} = (-b-c)\vec{i} - (-a+c)\vec{j} + (a+b)\vec{k}$

$-2\vec{i} + 2\vec{k} : \begin{cases} -b-c = -2 \to b = 1 \\ a - c = 0 \to a = c = 1 \\ a + b = 2 \to c + b = 2 \end{cases} \quad \therefore \vec{x} = \vec{i} + \vec{j} + \vec{k}$

c) $\vec{x} = (-1, -1, -1)$

$\vec{x} \cdot \vec{u} = -3x + 0y + 3z = 0 \to -x + z = 0 \to x = z$

$\vec{x} \cdot \vec{v} = 2x - 2y + 0z = 0 \to x - y = 0 \to x = y$

$\vec{x} = (x, x, x)$

Norma: $\|\vec{x}\| = \sqrt{x^2 + x^2 + x^2} = \sqrt{3x^2} = \sqrt{3}\,|x| = \sqrt{3} \to x = \pm 1$

Ângulo com $\vec{j}$:

$\cos\theta = \dfrac{\vec{x} \cdot \vec{j}}{\|\vec{x}\|\|\vec{j}\|} = \dfrac{x}{\sqrt{3} \cdot 1}$

$\cos\theta < 0 \to x < 0$

$x = -1$

$\vec{x} = (-1, -1, -1)$

b) $\vec{x} = (-1, 2, 1)$

$\vec{x} \times (1, 0, 1) = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 1 & 0 & 1 \end{bmatrix} = (y)\vec{i} - (x - z)\vec{j} + (-y)\vec{k}$
$= (y, z - x, -y)$

$2(1, 1, -1) = (2, 2, -2)$

$\begin{cases} y = 2 \\ z - x = 2 \\ -y = -2 \end{cases} \quad y = 2 \quad z = x + 2$

Norma de $\vec{x}$:

$\|\vec{x}\| = \sqrt{x^2 + y^2 + z^2} = \sqrt{x^2 + 4 + (x+2)^2} = \sqrt{6}$

$x^2 + 4 + x^2 + 4x + 4 = 6 \to 2x^2 + 4x + 2 = 0$

$x^2 + 2 + 1 = 0 \to (x+1)^2 = 0 \to x = -1$

$z = -1 + 2 = 1$

$\vec{x} = (-1, 2, 1)$

Número 13

a) área $= \sqrt{62}$

$\vec{AD} = D - A = (5 - 3, 3 - 2, 3 - (-1)) = (2, 1, 4)$

$\vec{AB} \times \vec{AD} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 2 & 1 & 4 \end{bmatrix} = (4+1)\vec{i} - (4+2)\vec{j} + (1-2)\vec{k}$
$= (5, -6, -1)$

$\|\vec{AB} \times \vec{AD}\| = \sqrt{5^2 + (-6)^2 + (-1)^2} = \sqrt{25 + 36 + 1} = \sqrt{62}$

b) área $= \dfrac{\sqrt{19}}{2}$

$\vec{AB} \times \vec{AC} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = (3-0)\vec{i} - (-3-0)\vec{j} + (-1-0)\vec{k}$
$= (3, 3, -1)$

$\|\vec{AB} \times \vec{AC}\| = \sqrt{3^2 + 3^2 + (-1)^2} = \sqrt{19}$

Área do triângulo $= \dfrac{1}{2} \times \sqrt{19} = \dfrac{\sqrt{19}}{2}$

Altura:

$h = \dfrac{\|\vec{AB} \times \vec{AC}\|}{\|\vec{BC}\|} = \dfrac{\sqrt{19}}{\sqrt{10}} = \dfrac{\sqrt{19} \cdot \sqrt{10}}{10} = \dfrac{\sqrt{190}}{10}$

$\vec{BC} = (0 - (-1), 1 - 1, 3 - 0) = (1, 0, 3)$

$\|\vec{BC}\| = \sqrt{1^2 + 0^2 + 3^2} = \sqrt{10}$

Número 14

a) $\vec{u}\cdot(\vec{v}\times\vec{w}) = (\vec{u}\times\vec{v})\cdot\vec{w}$

$\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix}$ o determinante é invariante sob permutação cíclica das linhas.

Logo, $\vec{u}\cdot(\vec{v}\times\vec{w}) = (\vec{u}\times\vec{v})\cdot\vec{w}$ //

b) • $[\vec{u}, \vec{w}, \vec{v}] = -[\vec{u}, \vec{v}, \vec{w}] = 0$

• $[\vec{v}, 2\vec{w}, \vec{u}] = 2[\vec{v}, \vec{w}, \vec{u}] = 0$

• $[\vec{v}, 3\vec{v} - 2\vec{u}, \vec{w} + 3\vec{v}] = 0 \rightarrow$ os vetores são coplanares //

$[\vec{u}, \vec{v}, \vec{w}] = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -2 \\ -1 & 2 & 0 \end{bmatrix} = 1.(1.0-(-2).2) - 3.(0.0-(-2).(-1)) + 2(0.2-1.(-1))$

$= 1.4 - 3.2 + 2.1 = 4 - 6 + 2 = 0$ //

Número 15

a) Área da base $= 3\sqrt{3}$

$\vec{AD} = \vec{AF} - \vec{BE} = (3,5,6) - (2,2,2) = (1,3,4)$

$\vec{AB}\times\vec{AD} = \begin{bmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & 3 & 4 \end{bmatrix} = i(0.4-1.3) - j(1.4-1.1) + k(1.3-0.1) = (-3,-3,3)$

$\|\vec{AB}\times\vec{AD}\| = \sqrt{(-3)^2+(-3)^2+3^2} = \sqrt{27} = 3\sqrt{3}$ //

b) Volume do paralelepípedo $= 6$ unidades cúbicas

$\vec{BE}\times\vec{AF} = \begin{bmatrix} i & j & k \\ 2 & 2 & 2 \\ 3 & 5 & 6 \end{bmatrix} = i(2.6-2.5) - j(2.6-2.3) + k(2.5-2.3) = (12-10, -(12-6), 10-6) = (2,-6,4)$

$\vec{AB}\cdot(\vec{BE}\times\vec{AF}) = 1.2 + 0.(-6) + 1.4 = 6$ //

c) Altura relativa a base $= \frac{2\sqrt{3}}{3}$

Volume x Área $\Rightarrow 6 = 3\sqrt{3}.h \Rightarrow h = \frac{6}{3\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ //

d) Volume do tetaedra $= 1$ unidade cúbica

$V = \frac{1}{6}.6 = 1$ //

e) Altura $= \frac{2\sqrt{5}}{15}$

$DB = -\vec{AB} + \vec{AD} = (-1, 3, 2)$

$DE = -\vec{AD} + AE = (0, -1, 0)$

$\|\vec{DB}\times\vec{DE}\| = \sqrt{5}$

Área da base

$A = \frac{1}{2}.\|\vec{DB}\times\vec{DE}\| = \frac{\sqrt{5}}{2}$

Volume $= \frac{1}{3}$. área da base . altura

$\text{altura} = \frac{\frac{\text{Volume}}{3}}{\text{area da base}} = \frac{\frac{1}{3}}{\text{área da base}} = \frac{\frac{1}{3}}{\frac{2}{\sqrt{5}}} = \frac{1}{3}\cdot\frac{2}{\sqrt{5}} = \frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{15}$ //