# Lista 4 - Geometria Analítica

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R. A.: 2025.1.08.006

## Número 1

a. $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix}$ $\begin{cases} x_1 = 2 \\ x_2 = -1 \end{cases}$ $(2, -1)$

b. $\begin{pmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$ $\begin{cases} x_1 = 4 \\ x_2 = 3 \\ x_3 = 2 \\ x_4 = 1 \end{cases}$ $(4, 3, 2, 1)$

c. $\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & \phantom{0} \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} \end{array}$ $\begin{cases} x_1 = 6 \\ x_2 = 3 \\ x_3 + x_4 = 2 \rightarrow x_3 = 2 - x_4 \end{cases}$ $(6, 3, 2 - x_4, x_4)$, onde $x_4$ é livre.

d. $\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 2 \end{pmatrix}$ $\begin{cases} x_1 + 3x_3 = 1 \rightarrow 1 - 3x_3 \\ x_2 - x_3 = 2 \rightarrow 2 + x_3 \end{cases}$ $(1 - 3x_3, 2 + x_3, x_3)$, onde $x_3$ é livre.

e. $\begin{pmatrix} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & -5 \end{pmatrix}$ $\begin{cases} x_1 - 7x_4 = 8 \rightarrow x_1 = 8 + 7x_4 \\ x_2 + 3x_4 = 2 \rightarrow x_2 = 2 - 3x_4 \\ x_3 + x_4 = -5 \rightarrow x_3 = -5 - x_4 \end{cases}$ $(8 + 7x_4, 2 - 3x_4, -5 - x_4, x_4)$, onde $x_4$ é livre.

f. $\begin{pmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $\begin{cases} x_1 - 6x_2 + 3x_5 = -2 \rightarrow x_1 = -2 + 6x_2 - 3x_5 \\ x_3 + 4x_5 = 7 \rightarrow x_3 = 7 - 4x_5 \\ x_4 + 5x_5 = 8 \rightarrow x_4 = 8 - 5x_5 \end{cases}$

$(-2 + 6x_2 - 3x_5, 7 - 4x_5, 8 - 5x_5, x_5)$, onde $x_2$ e $x_5$ são livres.

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Número 2

a. $\begin{cases} 3x - 4y = 1 \\ x + 3y = 9 \end{cases}$ $(3, 2)$

$\left(\begin{array}{cc|c} 3 & -4 & 1 \\ 1 & 3 & 9 \end{array}\right) L_1 \leftrightarrow L_2 \begin{pmatrix} 1 & 3 & 9 \\ 3 & -4 & 1 \end{pmatrix} L_2 \leftarrow L_2 - 3L_1 \begin{pmatrix} 1 & 3 & 9 \\ 0 & -13 & -26 \end{pmatrix} L_2 \leftarrow L_2/-13$

$\begin{pmatrix} 1 & 3 & 9 \\ 0 & 1 & 2 \end{pmatrix} L_1 \leftarrow L_1 - 3L_2 \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{pmatrix} \begin{cases} x = 3 \\ y = 2 \end{cases}$

b. $\begin{cases} 5x + 8y = 34 \\ 10x + 16y = 50 \end{cases}$ Sistema impossível, não há solução.

$\begin{pmatrix} 5 & 8 & 34 \\ 10 & 16 & 50 \end{pmatrix} L_2 \leftarrow L_2/2 \begin{pmatrix} 5 & 8 & 34 \\ 5 & 8 & 25 \end{pmatrix}$ $34 \neq 25$

c. $\begin{cases} x + 2y = 5 \\ 2x - 3y = -4 \end{cases}$ $(1, 2)$

$\begin{pmatrix} 1 & 2 & 5 \\ 2 & -3 & -4 \end{pmatrix} L_2 \leftarrow L_2 - 2L_1 \begin{pmatrix} 1 & 2 & 5 \\ 0 & -7 & -14 \end{pmatrix} L_2 \leftarrow L_2/-7 \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \end{pmatrix} L_1 \leftarrow L_1 - 2L_2$

$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{cases} x = 1 \\ y = 2 \end{cases}$

d. $\begin{cases} 3x + 2y - 5z = 8 \\ 2x - 4y - 2z = -4 \\ x - 2y - 3z = -4 \end{cases}$ $(3, 2, 1)$

$\begin{pmatrix} 3 & 2 & -5 & 8 \\ 2 & -4 & -2 & -4 \\ 1 & -2 & -3 & -4 \end{pmatrix} L_1 \leftrightarrow L_3 \begin{pmatrix} 1 & -2 & -3 & -4 \\ 2 & -4 & -2 & -4 \\ 3 & 2 & -5 & 8 \end{pmatrix} \begin{matrix} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \end{matrix} \begin{pmatrix} 1 & -2 & -3 & -4 \\ 0 & 0 & 4 & 4 \\ 0 & 8 & 4 & 20 \end{pmatrix} L_2 \leftrightarrow L_3$

$\begin{pmatrix} 1 & -2 & -3 & -4 \\ 0 & 8 & 4 & 20 \\ 0 & 0 & 4 & 4 \end{pmatrix} \begin{matrix} L_2 \leftarrow L_2/8 \\ L_3 \leftarrow L_3/4 \end{matrix} \begin{pmatrix} 1 & -2 & -3 & -4 \\ 0 & 1 & 0,5 & 2,5 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} L_1 \leftarrow L_1 + 3L_3 \\ L_2 \leftarrow L_2 - 0,5L_3 \end{matrix} \begin{pmatrix} -1 & -2 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix} L_1 \leftarrow L_1 + 2L_2$

$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{cases} x = 3 \\ y = 2 \\ z = 1 \end{cases}$

e. $\begin{cases} 2x - 6y = -4 \\ x + 3y = 1 \\ 4x + 12y = 2 \end{cases}$ Sistema impossível, não há solução.

$$\begin{pmatrix} 2 & -6 & -4 \\ 1 & 3 & 1 \\ 4 & 12 & 2 \end{pmatrix} L_1 \leftarrow \frac{L_1}{2} \begin{pmatrix} 1 & -3 & -2 \\ 1 & 3 & 1 \\ 4 & 12 & 2 \end{pmatrix} \begin{matrix} L_2 \leftarrow L_2 - L_1 \\ L_3 \leftarrow L_3 - 4L_1 \end{matrix} \begin{pmatrix} 1 & -3 & -2 \\ 0 & 6 & 3 \\ 0 & 24 & 10 \end{pmatrix} L_2 \leftarrow \frac{L_2}{6}$$

$$\begin{pmatrix} 1 & -3 & -2 \\ 0 & 1 & 0,5 \\ 0 & 0 & -2 \end{pmatrix} \Rightarrow 0 \neq -2.$$

f. $\begin{cases} x + 2y - z = 2 \\ 2x - y + 3z = 9 \\ 3x + 3y - 2z = 3 \end{cases}$ $(1, 2, 3)$

$$\begin{pmatrix} 1 & 2 & -1 & 2 \\ 2 & -1 & 3 & 9 \\ 3 & 3 & -2 & 3 \end{pmatrix} \begin{matrix} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \end{matrix} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & -5 & 5 & 5 \\ 0 & -3 & 1 & -3 \end{pmatrix} \Rightarrow L_2 \leftarrow \frac{L_2}{-5} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & -3 & 1 & -3 \end{pmatrix} L_3 \leftarrow L_3 + 3L_2$$

$$\begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -2 & -6 \end{pmatrix} L_3 \leftarrow \frac{L_3}{-2} \begin{pmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{pmatrix} \begin{matrix} L_1 \leftarrow L_1 + L_3 \\ L_2 \leftarrow L_2 + L_3 \end{matrix} \begin{pmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} L_1 \leftarrow L_1 - 2L_2$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} \begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$$

g. $\begin{cases} x + 3z = -8 \\ 2x - 4y = -4 \\ 3x - 2y - 5z = 26 \end{cases}$ $(4, 3, -4)$

$$\begin{pmatrix} 1 & 0 & 3 & -8 \\ 2 & -4 & 0 & -4 \\ 3 & -2 & -5 & 26 \end{pmatrix} \begin{matrix} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \end{matrix} \begin{pmatrix} 1 & 0 & 3 & -8 \\ 0 & -4 & -6 & 12 \\ 0 & -2 & -14 & 50 \end{pmatrix} L_2 \leftarrow \frac{L_2}{-2} \begin{pmatrix} 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -6 \\ 0 & -2 & -14 & 50 \end{pmatrix} L_3 \leftarrow L_3 + L_2$$

$$\begin{pmatrix} 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -6 \\ 0 & 0 & -11 & 44 \end{pmatrix} L_3 \leftarrow \frac{L_3}{-11} \begin{pmatrix} 1 & 0 & 3 & -8 \\ 0 & 2 & 3 & -6 \\ 0 & 0 & 1 & -4 \end{pmatrix} \begin{matrix} L_1 \leftarrow L_1 - 3L_3 \\ L_2 \leftarrow L_2 - 3L_3 \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{pmatrix} L_2 \leftarrow \frac{L_2}{2}$$

$$\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -4 \end{pmatrix} \begin{cases} x = 4 \\ y = 3 \\ z = -4 \end{cases}$$

h. $\begin{cases} x + 2y + 3z = 10 \\ 3x + 4y + 6z = 23 \\ 2x + 2y + 3z = 13 \end{cases}$ $(3; 3,5 - 1,5z; z)$, onde $z$ é livre.

$$\begin{pmatrix} 1 & 2 & 3 & 10 \\ 3 & 4 & 6 & 23 \\ 2 & 2 & 3 & 13 \end{pmatrix} \begin{matrix} \\ L_2 \leftarrow L_2 - 3L_1 \\ L_3 \leftarrow L_3 - 2L_1 \end{matrix} \begin{pmatrix} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 0 & -2 & -3 & -7 \end{pmatrix} L_3 \leftarrow L_3 - L_2$$

$$\begin{pmatrix} 1 & 2 & 3 & 10 \\ 0 & -2 & -3 & -7 \\ 0 & 0 & 0 & 0 \end{pmatrix} L_2 \leftarrow L_2/_{-2} \begin{pmatrix} 1 & 2 & 3 & 10 \\ 0 & 1 & 1,5 & 3,5 \\ 0 & 0 & 0 & 0 \end{pmatrix} L_1 \leftarrow L_1 - 2L_2 \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 1,5 & 3,5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$0 = 0 \rightarrow z$ é livre. $x = 3$; $y = 3,5 - 1,5z$; $z$ livre

i. $\begin{cases} x - 3y + 4z - w = 2 \\ 2x - y + 3z - 2w = 19 \end{cases}$ $(11 - z + w, 3 + z, z, w)$, onde $z$ e $w$ são livres.

$$\begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 2 & -1 & 3 & -2 & 19 \end{pmatrix} L_2 \leftarrow L_2 - 2L_1 \begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 5 & -5 & 0 & 15 \end{pmatrix} L_2 \leftarrow L_2/5$$

$$\begin{pmatrix} 1 & -3 & 4 & -1 & 2 \\ 0 & 1 & -1 & 0 & 3 \end{pmatrix} L_1 \leftarrow L_1 + 3L_2 \begin{pmatrix} 1 & 0 & 1 & -1 & 11 \\ 0 & 1 & -1 & 0 & 3 \end{pmatrix}$$

$x = 11 - z + w$; $y = 3 + z$; $z =$ livre; $w =$ livre

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# Número 3

a)

$$\left(\begin{array}{ccccc} 1 & 2 & 3 & 1 & 8 \\ 1 & 3 & 0 & 1 & 7 \\ 1 & 0 & 2 & 1 & 3 \end{array}\right) \begin{array}{l} \\ l_2 \leftarrow l_2 - l_1 \\ l_3 \leftarrow l_3 - l_1 \end{array} \left|\begin{array}{ccccc} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & -2 & -1 & 0 & -5 \end{array}\right| l_3 \leftarrow l_3 + 2l_2$$

$$\begin{cases} x = 1 - w \\ y = 2 \\ z = 1 \\ w \in \mathbb{R} \end{cases}$$

$$\left[\begin{array}{ccccc} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & -7 & 0 & -7 \end{array}\right] l_3 \leftarrow l_3 / -7 \rightarrow \left[\begin{array}{ccccc} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right] l_2 \leftarrow l_2 + 3l_3$$

$$\left[\begin{array}{ccccc} 1 & 2 & 3 & 1 & 8 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right] l_1 \leftarrow l_1 - 3l_3 \left[\begin{array}{ccccc} 1 & 2 & 0 & 1 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right] l_1 \leftarrow l_1 - 2l_2 \left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \end{array}\right] \rightarrow \begin{cases} x + w = 1 \\ y = 2 \\ z = 1 \end{cases}$$

b)

$$\left|\begin{array}{ccccc} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 1 & 0 & 2 & -1 & -1 \end{array}\right| l_3 \leftarrow l_3 - l_1 \left|\begin{array}{ccccc} 1 & 1 & 3 & -3 & 0 \\ 0 & 2 & 1 & -3 & 3 \\ 0 & -1 & -1 & 2 & -1 \end{array}\right| \begin{array}{l} \\ l_2 \leftrightarrow l_3 \\ l_3 \leftarrow l_3 \cdot -1 \end{array} \left|\begin{array}{ccccc} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 2 & 1 & -3 & 3 \end{array}\right| l_3 \leftarrow l_3 - 2l_1$$

$$\left|\begin{array}{ccccc} 1 & 1 & 3 & -3 & 0 \\ 0 & 1 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 & 1 \end{array}\right| \begin{array}{l} l_1 \leftarrow l_1 - l_2 \\ l_2 \leftarrow l_2 + l_3 \\ \\ \end{array} \left|\begin{array}{ccccc} 1 & 0 & 2 & -1 & -1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & -1 & 1 & 1 \end{array}\right| l_1 \leftarrow l_1 + l_3 \left|\begin{array}{ccccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & -1 & 1 & 1 \end{array}\right| l_1 \leftarrow l_1 + l_3$$

$$\left|\begin{array}{ccccc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & -1 & 1 & 1 \end{array}\right| l_3 \leftarrow l_3 . (-1) \left|\begin{array}{ccccc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right| \rightarrow \begin{cases} x + w = 1 \\ y - w = 2 \\ z - w = -1 \end{cases}$$

$$\begin{cases} x = 1 - w \\ y = 2 + w \\ z = -1 + w \\ w \in \mathbb{R} \end{cases}$$

c)

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & 3 & 0 \end{array}\right] \rightarrow \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases} \qquad \begin{cases} x = 0 \\ y = 0 \\ z = 0 \end{cases}$$

Número 4

a. $(9-3z,\ 4-z,\ z)$; (infinitas soluções, $z$ livre).

b. Não há solução (sistema impossível).

$$\left(\begin{array}{ccc|cc} 1 & -2 & 1 & 1 & 2 \\ 2 & -5 & 1 & -2 & -1 \\ 3 & -7 & 2 & -1 & 2 \end{array}\right) \begin{array}{l} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 3L_1 \end{array} \left(\begin{array}{ccc|cc} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & -1 & -1 & -4 & -4 \end{array}\right) L_3 \leftarrow L_3 - L_2$$

$$\left(\begin{array}{ccc|cc} 1 & -2 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 & -5 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

Para (a) $\rightarrow 0 = 0$ o que determina um SPI.

Para (b) $\rightarrow 0 = 1$ o que é impossível, SI.

Resolução do sistema (a):

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} x = 9 - 3z \\ y = 4 - z \\ z = \text{livre} \end{array}$$

Resolução do sistema (b):

$$\left(\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & -1 & -1 & -5 \\ 0 & 0 & 0 & 1 \end{array}\right) \quad 0 \neq 1$$

Não há solução.

Número 5

a. $(A+4I_3)X=0 \rightarrow X = x_3\begin{pmatrix}-1\\0\\1\end{pmatrix}$, onde $x_3$ é uma constante arbitrária.

$A=\begin{pmatrix}1&0&5\\1&1&1\\0&1&-4\end{pmatrix}$ $\quad X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$ $\quad 0=\begin{pmatrix}0\\0\\0\end{pmatrix}$

$A+4I_3=\begin{pmatrix}1&0&5\\1&1&1\\0&1&-4\end{pmatrix}+\begin{pmatrix}4&0&0\\0&4&0\\0&0&4\end{pmatrix}=\begin{pmatrix}5&0&5\\1&5&1\\0&1&0\end{pmatrix}$

$(A+4I_3)X=0 \rightarrow \begin{cases}5x_1+0x_2+5x_3=0\\1x_1+5x_2+1x_3=0\\0x_1+1x_2+0x_3=0\end{cases}$

$x_2=0$

$5x_1+5x_3=0$

$x_1=-x_3$

$x_1+x_3=0$

$x_1=-x_3$

$X=\begin{pmatrix}-x_3\\0\\x_3\end{pmatrix}=x_3\begin{pmatrix}-1\\0\\1\end{pmatrix}$, $x_3 \in \mathbb{R}$.

b. $AX=2X \rightarrow X=x_3\begin{pmatrix}5\\6\\1\end{pmatrix}$, onde $x_3$ é uma constante arbitrária.

$AX=2X$

$(A-2I_3)X=0$

$A-2I_3=\begin{pmatrix}1&0&5\\1&1&1\\0&1&-4\end{pmatrix}-\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}=\begin{pmatrix}-1&0&5\\1&-1&1\\0&1&-6\end{pmatrix}$

$(A-2I_3)X=0 \rightarrow \begin{cases}-x_1+0x_2+5x_3=0 \rightarrow x_1=5x_3\\x_1-x_2+x_3=0 \rightarrow 5x_3-6x_3+x_3=0 \rightarrow 0=0\\0x_1+x_2-6x_3=0 \rightarrow x_2=6x_3\end{cases}$

$X=\begin{pmatrix}5x_3\\6x_3\\x_3\end{pmatrix}=x_3\begin{pmatrix}5\\6\\1\end{pmatrix}$, $x_3 \in \mathbb{R}$.

## Número 6

a. $\begin{cases} x+y+z=2 \\ 2x+3y+2z=5 \\ 2x+3y+(a^2-1)z=a+1 \end{cases}$

Nenhuma solução: $a=\pm\sqrt{3}$
Solução única: $a \neq \pm\sqrt{3}$
Infinitas soluções: não existe a que satisfaça.

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & a^2-1 & a+1 \end{array}\right) \begin{array}{l} L_2 \leftarrow L_2 - 2L_1 \\ L_3 \leftarrow L_3 - 2L_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & a^2-3 & a-3 \end{array}\right) L_3 \leftarrow L_3 - L_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & a^2-3 & a-4 \end{array}\right)$$

Solução única: $a^2-3 \neq 0$, $a \neq \pm\sqrt{3}$
Infinitas soluções: $a^2-3=0$ e $a-4=0$, não há valor
Nenhuma solução: $a^2-3=0$ e $a-4\neq 0$, $a=\pm\sqrt{3}$.

b. $\begin{cases} x+2y-3z=4 \\ 3x-y+5z=2 \\ 4x+y+(a^2-14)z=a+2 \end{cases}$

Nenhuma solução: $a=-4$.
Solução única: $a \neq \pm 4$.
Infinitas soluções: $a=4$.

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{array}\right) \begin{array}{l} L_2 \leftarrow L_2 - 3L_1 \\ L_3 \leftarrow L_3 - 4L_1 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right) L_3 \leftarrow L_3 - L_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right)$$

Solução única: $a^2-16 \neq 0 \rightarrow a \neq \pm 4$.
Infinitas soluções: $a^2-16=0$ e $a-4=0 \rightarrow a=4$.
Nenhuma solução: $a^2-16=0$ e $a-4\neq 0 \rightarrow a=-4$.

## Número 7

a. $A=\begin{pmatrix} 2 & -2 \\ 3 & 1 \end{pmatrix}$  $A^{-1}=\begin{pmatrix} 1/8 & 1/4 \\ -3/8 & 1/4 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) L_1 \leftarrow \tfrac{1}{2}L_1 \left(\begin{array}{cc|cc} 1 & -1 & 1/2 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) L_2 \leftarrow L_2 - 3L_1 \left(\begin{array}{cc|cc} 1 & -1 & 1/2 & 0 \\ 0 & 4 & -3/2 & 1 \end{array}\right) L_2 \leftarrow \tfrac{1}{4}L_2$$

$$\left(\begin{array}{cc|cc} 1 & -1 & 1/2 & 0 \\ 0 & 1 & -3/8 & 1/4 \end{array}\right) L_1 \leftarrow L_1 + L_2 \left(\begin{array}{cc|cc} 1 & 0 & 1/8 & 1/4 \\ 0 & 1 & -3/8 & 1/4 \end{array}\right)$$

b. $B=\begin{pmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ $\quad B^{-1}=\begin{bmatrix} 3/8 & 1/4 & 1/4 \\ -1/8 & 1/4 & 1/4 \\ -1/8 & 1/4 & -3/4 \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 2 & -2 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) L_1 \leftrightarrow L_2 \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) L_2 \leftarrow L_2 - 2L_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -6 & -2 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right) L_2 \leftrightarrow L_3 \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & -6 & -2 & 1 & -2 & 0 \end{array}\right) L_3 \leftarrow L_3 + 6L_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & -8 & 1 & -2 & 6 \end{array}\right) L_3 \leftarrow -\tfrac{1}{8} L_3 \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{array}\right) \begin{array}{l} L_1 \leftarrow L_1 - L_3 \\ L_2 \leftarrow L_2 + L_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{1}{8} & \frac{3}{4} & \frac{3}{4} \\ 0 & 1 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{array}\right) L_1 \leftarrow L_1 - 2L_2 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{array}\right) //$$

c. $C=\begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ $\quad C^{-1}=\begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) L_1 \leftrightarrow L_2 \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 3 & 5 & 1 & 0 \end{array}\right) L_2 \leftarrow L_2 - 3L_1 \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -3 \end{array}\right) L_2 \leftarrow -L_2$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & 1 & -1 & 3 \end{array}\right) L_1 \leftarrow L_1 - 2L_2 \left(\begin{array}{cc|cc} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array}\right) \quad \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} //$$

d. $D=\begin{pmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ $\quad D^{-1}=\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 0 & -1 & 1 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) L_1 \leftrightarrow L_3 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \end{array}\right) L_2 \leftarrow L_2 - 2L_1 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -2 & -1 & 0 & 1 & -2 \\ 0 & -1 & 1 & 1 & 0 & 0 \end{array}\right) L_2 \leftrightarrow L_3$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 0 & 1 & -2 \end{array}\right) L_3 \leftarrow L_3 - 2L_2 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -3 & -2 & 1 & -2 \end{array}\right) L_3 \leftarrow \tfrac{1}{3} L_3 \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{array}\right) \begin{array}{l} L_1 \leftarrow L_1 - L_2 \\ L_2 \leftarrow L_2 - L_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -1 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{array}\right) L_2 \leftarrow -L_2 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{array}\right) \quad D^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

e. $E=\begin{pmatrix} 2 & -1 & 0 & 1 \\ 0 & 2 & 0 & -1 \\ 1 & 0 & 2 & 3 \\ 0 & 0 & -2 & 0 \end{pmatrix}$ $\quad E^{-1}=\begin{bmatrix} -\frac{2}{3} & -\frac{1}{3} & -\frac{7}{3} & -\frac{7}{3} \\ \frac{1}{9} & \frac{5}{9} & -\frac{2}{9} & \frac{2}{9} \\ 0 & 0 & 0 & \frac{1}{2} \\ \frac{2}{9} & \frac{1}{9} & -\frac{4}{9} & \frac{4}{9} \end{bmatrix}$

$$\left[\begin{array}{cccc|cccc} 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} l_3 \leftrightarrow l_1 \\ l_3 \leftrightarrow l_4 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\ 2 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \end{array}\right] l_4 \leftarrow l_4 - 2l_1$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & -4 & -5 & 1 & 0 & -2 & 0 \end{array}\right] l_4 \leftarrow l_4 + l_2/2 \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -4 & \frac{9}{2} & 1 & \frac{1}{2} & -2 & 0 \end{array}\right] l_4 \leftarrow l_4 + 2l_3$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{9}{2} & 1 & \frac{1}{2} & -2 & 2 \end{array}\right] L_4 \leftarrow L_4 \cdot \frac{2}{9} \left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & \frac{2}{9} & \frac{1}{9} & -\frac{4}{9} & \frac{4}{9} \end{array}\right] \begin{array}{l} L_2 \leftarrow l_2 + L_4 \\ L_3 \leftarrow l_3 / 2 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & \frac{2}{9} & \frac{10}{9} & -\frac{4}{9} & \frac{4}{9} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{2}{9} & \frac{1}{9} & -\frac{4}{9} & \frac{4}{9} \end{array}\right] \begin{array}{l} L_1 \leftarrow l_1 - 2l_3 \\ L_2 \leftarrow L_2/2 \end{array} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 3 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & \frac{1}{9} & \frac{5}{9} & -\frac{2}{9} & \frac{2}{9} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{2}{9} & \frac{1}{9} & -\frac{4}{9} & \frac{4}{9} \end{array}\right] L_1 \leftarrow l_1 - 3l_3$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{2}{3} & -\frac{1}{3} & -\frac{7}{3} & -\frac{7}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{9} & \frac{5}{9} & -\frac{2}{9} & \frac{2}{9} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{2}{9} & \frac{1}{9} & -\frac{4}{9} & \frac{4}{9} \end{array}\right]$$

**Número 8** Resposta: Calça R$ 4,00; Short R$ 8,00; Blusa R$ 2,00.

calça → $x = 4$
short → $y = 8$
blusa → $z = 2$

$$\begin{cases} x + 2y + 3z = 26 & (1) \\ 2x + 5y + 6z = 60 & (2) \\ 2x + 3y + 4z = 40 & (3) \end{cases}$$

$(3) - (1) = (4) \rightarrow (2x + 3y + 4z) - (x + 2y + 3z) = 40 - 26$

$x + y + z = 14 \; (4)$

$(2) - 2 \cdot (1) \rightarrow (2x + 5y + 6z) - 2(x + 2y + 3z) = 60 - 52 \rightarrow \boxed{y = 8}$

$x + y + z = 14$ $\qquad x + 16 + 3z = 26$

$x + z = 6$ $\qquad x + 3z = 10$

$(x + 3z) - (x + z) = 10 - 6 \rightarrow 2z = 4 \rightarrow \boxed{z = 2}$

$x + 2 = 6 \rightarrow \boxed{x = 4}$

**Número 9** Resposta: 200 sundaes, 300 casquinhas e 100 bananas split.

x: quantidade de sundaes 200
y: quantidade de casquinhas 300
z: quantidade de bananas split 100

$$\begin{cases} 1.\; 5x + 2y + 6z = 2200 \\ 2.\; y = 3z \\ 3.\; y = x + z \end{cases}$$

$3z = x + z \rightarrow x = 2z$

$y = 3z \Rightarrow y = 3 \cdot 100 = \boxed{300} \rightarrow y$

$x = 2z \rightarrow z = 2 \cdot 100 = \boxed{200} \rightarrow x$

(1.) $5 \cdot (2z) + 2 \cdot (3z) + 6z = 2200$

$10z + 6z + 6z = 2200$

$22z = 2200$

$\boxed{z = 100}$

**Número 10** Resposta: Torta de carne R$ 140; Salada R$ 20; Pizza R$ 80.

t: torta de carne: 140
s: salada: 20
p: pizza: 80

$$\begin{cases} 1.\; 40t + 30s + 10p = 7000 & (\div 10) \\ 2.\; 20t + 40s + 30p = 6000 & (\div 10) \\ 3.\; 10t + 20s + 40p = 5000 & (\div 10) \end{cases}$$

$$\begin{cases} 4t + 3s + p = 700 \rightarrow p = 700 - 4t - 3s \\ 2t + 4s + 3p = 600 \rightarrow 2t + 4s + 3(700 - 4t - 3s) = 600 \rightarrow 2t + 4s + 2100 - 12t - 9s = 600 \\ t + 2s + 4p = 500 \end{cases}$$

$-10t - 5s = -1500$

$t + 2s + 4(700 - 4t - 3s = 500 \rightarrow -15 - 10s = -2300$

4. $2t + s = 300$

5. $3t + 2s = 460$

5. $3t + 2(300 - 2t) = 460$

$3t + 600 - 4t = 460$

$-t = -140$

$\boxed{t = 140}$

$s = 300 - 2 \cdot 140$

$s = 300 - 280 \Rightarrow \boxed{s = 20}$

$p = 700 - 4 \cdot 140 - 3 \cdot 20$

$p = 700 - 560 - 60 \rightarrow \boxed{p = 80}$

Número 11

A: comprimento da pista A

B: comprimento da pista B

C: comprimento da pista C

$$\begin{cases} (1)\ 2A + 3B + C = 8420 \\ (2)\ A + 2B + 2C = 7940 \\ (3)\ 4A + 3B = 8110 \end{cases}$$

(3) $B = \dfrac{8110 - 4A}{3}$

(1) $2A + 3\left(\dfrac{8110 - 4A}{3}\right) + C = 8420$

$2A + 8110 - 4A + C = 8420$

$-2A + C = 310 \rightarrow C = 310 + 2A$

(2) $A + 2\left(\dfrac{8110 - 4A}{3}\right) + 2(2A + 310) = 7940$

$A + 16220 - 8A + 12A + 1860 = 23820$

$7A = 5740$

$\boxed{A = 820}$

$B = \dfrac{8110 - 4 \cdot 820}{3} = \dfrac{8110 - 3280}{3} = \dfrac{4830}{3}$

$\boxed{B = 1610}$

$C = 2 \cdot 820 + 310 \rightarrow C = \boxed{1950}$

C → maior
A → menor
$\rightarrow C - A = 1950 - 820 = \boxed{1130 \text{ metros}}$

Resposta: A maior pista é a C (1950 m) e a menor é a A (820 m). A diferença entre elas é de 1130 metros.

tilibra