Lista 8- Geometria Analítica

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Número 1

a) vetores diretores:

$\vec{v_r} = (\frac{1}{2}, 1, 1)$

$\vec{v_s} = (1, \frac{3}{2}, 3)$

$$sen\,\alpha = \frac{|\vec{v_r} \times \vec{v_s}|}{|\vec{v_r}|.|\vec{v_s}|} = \frac{\frac{\sqrt{37}}{4}}{\frac{3}{2}\cdot\frac{7}{2}} = \frac{\sqrt{37}}{21}$$

$$\vec{v_r} \times \vec{v_s} = \left(1.3 - 1.\frac{3}{2}, -\left(\frac{1}{2}.3 - 1.1\right), \frac{1}{2}\cdot\frac{3}{2} - 1.1\right)$$

$$\vec{v_r} \times \vec{v_s} = \left(\frac{3}{2}, \frac{1}{2}, -\frac{1}{4}\right)$$

$$|\vec{v_r} \times \vec{v_s}| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{4}\right)^2} = \sqrt{\frac{9}{4} + \frac{1}{4} + \frac{1}{16}} = \sqrt{\frac{37}{16}} = \frac{\sqrt{37}}{4}$$

$$|\vec{v_r}| = \sqrt{\left(\frac{1}{2}\right)^2 + 1^2 + 1^2} = \sqrt{\frac{1}{4} + 1 + 1} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$|\vec{v_s}| = \sqrt{1^2 + \left(\frac{3}{2}\right)^2 + 3^2} = \sqrt{1 + \frac{9}{4} + 9} = \sqrt{\frac{49}{4}} = \frac{7}{2}$$

b) vetores diretores:

$\vec{v_r} = (0, -1, 1)$

$\vec{v_s} = (1, 1, 0)$

$$\vec{v_r} \times \vec{v_s} = (-1.0 - 1.1, -(0.0 - 1.1), 0.1 - (-1).1)$$

$$\vec{v_r} \times \vec{v_s} = (-1, 1, 1)$$

$$|\vec{v_r} \times \vec{v_s}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\vec{v_r}| = \sqrt{0^2 + (-1)^2 + 1^2} = \sqrt{2}$$

$$|\vec{v_s}| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$$

$$sen\,\alpha = \frac{\sqrt{3}}{\sqrt{2}.\sqrt{2}} = \frac{\sqrt{3}}{2}$$

c) vetores diretores:

$\vec{v_r} = (-3, 0, 1)$

$\vec{v_s} = (2, 0, 1)$

$$\cos\alpha = \frac{|\vec{v_r} \times \vec{v_s}|}{|\vec{v_r}|.|\vec{v_s}|} = \frac{|(-3).2 + 0.0 + 1.1|}{\sqrt{(-3)^2 + 0^2 + 1^2}.\sqrt{2^2 + 0^2 + 1^2}}$$

$$\cos\alpha = \frac{5}{\sqrt{10}.\sqrt{5}} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

d) vetores diretores:

$\vec{v_r} = (1, -2, 3)$

$\vec{v_s} = (1, 2, 1)$

$$\cos\alpha = \frac{|1.1 + (-2).2 + 3.1|}{\sqrt{1^2 + (-2)^2 + 3^2}.\sqrt{1^2 + 2^2 + 1^2}}$$

$$\cos\alpha = \frac{|0|}{\sqrt{14}.\sqrt{6}} = 0$$ (perpendiculares)

Número 2

Retas:

$r: X = (0, 2, 0) + \lambda(0, 1, 0)$

$s: X = (1, 2, 0) + \mu(0, 0, 1)$

Pontos genéricos:

$P = (0, 2+\lambda, 0) \rightarrow$ em r

$Q = (1, 2, \mu) \rightarrow$ em s

$\vec{PQ} = (1, -\lambda, \mu)$

Condições:

• 45° com r:

$$\cos 45° = \frac{|\vec{PQ}.\vec{v_r}|}{|\vec{PQ}|.|\vec{v_r}|} \Rightarrow \frac{\sqrt{2}}{2} = \frac{|-\lambda|}{\sqrt{1+\lambda^2+\mu^2}.1} \rightarrow \sqrt{2}\sqrt{1+\lambda^2+\mu^2} = 2|\lambda|$$

$$\sqrt{2}.2|\mu| = 2|\lambda| \Rightarrow \lambda = \sqrt{2}\mu$$

• 60° com s:

$$\cos 60° = \frac{|\vec{PQ}.\vec{v_s}|}{|\vec{PQ}|.|\vec{v_s}|} \Rightarrow \frac{1}{2} = \frac{|\mu|}{\sqrt{1+\lambda^2+\mu^2}.1} \rightarrow \sqrt{1+\lambda^2+\mu^2} = 2|\mu|$$

$$\sqrt{1+3\mu^2} = 2|\mu|$$

$$1 + 3\mu^2 = 4\mu^2$$

$$\mu^2 = 1$$

$$\mu = \pm 1$$

se $\mu = 1$; $\lambda = \sqrt{2}$

se $\mu = -1$; $\lambda = -\sqrt{2}$

Logo:

$P = (0, 2 \pm \sqrt{2}, 0)$ e $Q = (1, 2, \pm 1)$

Número 3

a) vetor diretor de r: $\vec{v_r} = (0, 1, 1)$

vetor normal de π: $\vec{n_\pi} = (0, 0, 1)$

$$\text{sen } \theta = \frac{|\vec{v_r} \cdot \vec{n_\pi}|}{|\vec{v_r}| \cdot |\vec{n_\pi}|} = \frac{1}{\sqrt{2} \cdot 1} = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4} \text{ radianos}$$

b) $\vec{v_r} = (-1, 1, 2)$; $\vec{n_\pi} = (2, -1, 0)$

$$\text{sen } \theta = \frac{|(-1) \cdot 2 + 1 \cdot (-1) + 2 \cdot 0|}{\sqrt{6} \cdot \sqrt{5}} = \frac{3}{\sqrt{30}} = \frac{\sqrt{30}}{10}$$

$$\theta = \text{arcsen}\left(\frac{\sqrt{30}}{10}\right) \text{ radianos}$$

c) $\vec{v_r} = (1, 1, -2)$; $\vec{n_\pi} = (1, 1, -1)$

$$\text{sen } \theta = \frac{|1 + 1 + 2|}{\sqrt{6} \cdot \sqrt{3}} = \frac{4}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}$$

$$\theta = \text{arcsen}\left(\frac{2\sqrt{2}}{3}\right) \text{ radianos}$$

Número 4

$\vec{n_1} = (1, 1, 1)$; $\vec{n_2} = (1, -1, 0)$

vetor diretor $\vec{v}$: $\vec{v} = (a, b, c)$

- $\vec{v} \cdot \vec{n_1} = 0$ (paralelo)

$a + b + c = 0$ (-1)

- $$\text{sen } 45° = \frac{|\vec{v} \cdot \vec{n_2}|}{|\vec{v}| \cdot |\vec{n_2}|} = \frac{|a - b|}{\sqrt{2} \cdot \sqrt{a^2 + b^2 + c^2}}$$

$$\frac{\sqrt{2}}{2} = \frac{|a - b|}{\sqrt{2} \cdot \sqrt{a^2 + b^2 + c^2}} \rightarrow |a - b| = \sqrt{a^2 + b^2 + c^2}$$

$$(a - b)^2 = a^2 + b^2 + c^2 \rightarrow -2ab = c^2$$

$$-2ab = (a + b)^2 \rightarrow -2ab = a^2 + 2ab + b^2$$

$$a^2 + 4ab + b^2 = 0$$

$$b = \frac{-4a \pm \sqrt{16a^2 - 4a^2}}{2} = \frac{-4a \pm 2\sqrt{3}a}{2} = (-2 \pm \sqrt{3})a$$

$$a = 1, \; b = -2 + \sqrt{3}, \; c = -1 - (-2 + \sqrt{3}) = 1 - \sqrt{3}$$

Vetor unitário:

$\vec{v} = (1, -2 + \sqrt{3}, 1 - \sqrt{3})$:

$$|\vec{v}| = \sqrt{1^2 + (-2 + \sqrt{3})^2 + (1 - \sqrt{3})^2} = \sqrt{1 + 4 - 4\sqrt{3} + 3 + 1 - 2\sqrt{3} + 3} = \sqrt{12 - 6\sqrt{3}}$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{(1, -2 + \sqrt{3}, 1 - \sqrt{3})}{\sqrt{12 - 6\sqrt{3}}}$$

Número 5

a) $\vec{n_1} = (2, 1, -1)$; $\vec{n_2} = (1, -1, 3)$ $\rightarrow$ $\theta = \text{arccos}\left(\frac{\sqrt{66}}{33}\right)$ radianos

$$\cos \theta = \frac{|\vec{n_1} \times \vec{n_2}|}{|\vec{n_1}| \times |\vec{n_2}|} = \frac{|2 \cdot 1 + 1 \cdot (-1) + (-1) \cdot 3|}{\sqrt{6} \cdot \sqrt{11}} = \frac{|2 - 1 - 3|}{\sqrt{66}} = \frac{2}{\sqrt{66}} = \frac{\sqrt{66}}{33}$$

b) $\vec{n_1} = (1, 0, 1) \times (-1, 0, 0) = (0, -1, 0)$; $\vec{n_2} = (1, 1, 1)$

$$\cos \theta = \frac{|0 \cdot 1 + (-1) \cdot 1 + 0 \cdot 1|}{\sqrt{1} \cdot \sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \rightarrow \theta = \text{arccos}\left(\frac{\sqrt{3}}{3}\right) \text{ radianos}$$

c) $\vec{n_1} = (0, -1, 1)$; $\vec{n_2} = (0, 0, -1)$

$$\cos \theta = \frac{|0 \cdot 0 + (-1) \cdot 0 + 1 \cdot (-1)|}{\sqrt{2} \cdot \sqrt{1}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \rightarrow \theta = \frac{\pi}{4} \text{ radianos}$$

Número 6

$\vec{n_1}=(2,-1,1)$; $\vec{n_2}=(1,-2,1)$

$\vec{n_1}\cdot\vec{n_2}=2\cdot1+(-1)\cdot(-2)+1\cdot1=2+2+1=5$

$|\vec{n_1}|=\sqrt{6}$

$|\vec{n_2}|=\sqrt{6}$

Ângulo: $\cos\theta=\frac{|5|}{\sqrt{6}\cdot\sqrt{6}}=\frac{5}{6}\rightarrow\theta=\arccos\left(\frac{5}{6}\right)$

Número 7

a) Ponto genérico em r: $X=(1+t, t/2, t)$

distâncias ao quadrado:

$d(X,A)^2=t^2+(\frac{t}{2}-1)^2+t^2=2t^2+\frac{t^2}{4}-t+1$

$d(X,B)^2=(1+t)^2+(\frac{t}{2}-1)^2+(t-1)^2=(1+2t+t^2)+(\frac{t^2}{4}-t+1)+(t^2-2t+1)=2t^2+\frac{t^2}{4}-t+3$

Equidistância:

$2t^2+\frac{t^2}{4}-t+1=2t^2+\frac{t^2}{4}-t+3\rightarrow 1\neq3$

Não existem pontos na reta r que equidistam de A e B.

b) $X=(4\lambda, 2\lambda, 4-3\lambda)$

$d(X,A)^2=(4\lambda-2)^2+(2\lambda-2)^2+(4-3\lambda-5)^2=29\lambda^2-18\lambda+9$

$d(X,B)^2=(4\lambda)^2+(2\lambda)^2+(4-3\lambda-1)^2=29\lambda^2-18\lambda+9$

Equidistância:

$29\lambda^2-18\lambda+9=29\lambda^2-18\lambda+9$

Conclusão: Todos os pontos da reta r equidistam de A e B.

c) $X=(2+\lambda, 3+\lambda, -3+\lambda)$

$d(X,A)^2=(1+\lambda)^2+(2+\lambda)^2+(-3+\lambda)^2=3\lambda^2+14$

$d(X,B)^2=(\lambda)^2+(1+\lambda)^2+(-7+\lambda)^2=3\lambda^2-12\lambda+50$

Equidistância:

$3\lambda^2+14=3\lambda^2-12\lambda+50\rightarrow12\lambda=36$

$\lambda=3$

Conclusão: para $\lambda=3$, $X=(5,6,0)$

Número 8

$d(P,r)=\frac{\|\vec{P_0P}\times\vec{v}\|}{\|\vec{v}\|}$

a) $\vec{P_0}=(1,-2,0)$; $\vec{v}=(3,2,1)$; $\vec{P_0P}=(-3,2,1)$

$\vec{P_0P}\times\vec{v}=(0,6,-12)$

$\|\vec{P_0P}\times\vec{v}\|=\sqrt{0^2+6^2+(-12)^2}=\sqrt{180}=6\sqrt{5}$

$\|\vec{v}\|=\sqrt{3^2+2^2+1^2}=\sqrt{14}$

$d(P,r)=\frac{6\sqrt{5}}{\sqrt{14}}=\frac{6\sqrt{70}}{14}=\frac{3\sqrt{70}}{7}$

b) $\vec{P_0}=(2,0,1)$; $\vec{v}=(4,-3,-2)$; $\vec{P_0P}=(-1,-1,3)$

$\vec{P_0P}\times\vec{v}=(11,10,7)$

$\|\vec{P_0P}\times\vec{v}\|=\sqrt{11^2+10^2+7^2}=\sqrt{270}=3\sqrt{30}$

$\|\vec{v}\|=\sqrt{4^2+(-3)^2+(-2)^2}=\sqrt{29}$

$d(P,r)=\frac{3\sqrt{30}}{\sqrt{29}}=\frac{3\sqrt{870}}{29}$

c) $\vec{P_0}=(0,3/2,1/2)$, $\vec{v}=(1,1/2,1/2)$, $\vec{P_0P}=(0,-5/2,-1/2)$

$\vec{P_0P}\times\vec{v}=(-2,-1,5)$ $\vec{v}=(2,1,1)$

$\|\vec{P_0P}\times\vec{v}\|=\sqrt{(-2)^2+(-1)^2+5^2}=\sqrt{30}$

$\|\vec{v}\|=\sqrt{2^2+1^2+1^2}=\sqrt{6}$

$d(P,r)=\frac{\sqrt{30}}{\sqrt{6}}=\sqrt{5}$

Número 9

• Reta de intersecção: $x+y=2$ e $x=y+z$; $r:(t, 2-t, 2t-2)$

• Reta s: $s:(k, k, k-1)$

• Ponto genérico em r: $P_r=(t, 2-t, 2t-2)$

• Distância de P_r a s: $P_s=(0,0,-1)$; $\vec{v_s}=(1,1,1)$ $\overrightarrow{P_sP_r}=(t, 2-t, 2t-1)$

$\overrightarrow{P_sP_r}\times\vec{v_s}=(3-3t, t-1, 2t-2)$ $\quad \|\overrightarrow{P_sP_r}\times\vec{v_s}\|^2=(3-3t)^2+(t-1)^2+(2t-2)^2=9(t-1)^2+(t-1)^2+4(t-1)^2$

$\|\vec{v_s}\|^2=1^2+1^2+1^2=3$ $\quad \|\overrightarrow{P_sP_r}\times\vec{v_s}\|^2=14(t-1)^2$

$d(P_r,s)^2=\frac{14(t-1)^2}{3}$ $\longrightarrow$ $\frac{14(t-1)^2}{3}=\frac{14}{3}\to(t-1)^2=1\to t-1=\pm1.$

Pontos: Para $t=2$, $P=(2,0,2)$. Para $t=0$, $P=(0,2,-2)$.

Número 10

a) Equação geral de π: $\vec{n_\pi}=(1,0,0)\times(-1,0,3)=(0,-3,0)$ ou $(0,1,0)$. $P_0=(1,0,0)$

$0(x-1)+1(y-0)+0(z-0)=0\to y=0$ $\quad d(P,\pi)=\frac{|0.1+1.3+0.4+0|}{\sqrt{0^2+1^2+0^2}}=\frac{|3|}{1}=3$

b) $d(P,\pi)=\frac{|1.0-2.0-2.(-6)-6|}{\sqrt{1^2+(-2)^2+(-2)^2}}=\frac{|12-6|}{\sqrt{9}}=\frac{6}{3}$

$d(P,\pi)=2$

c) $d(P,\pi)=\frac{|2.1-1.1+2.1-3|}{\sqrt{2^2+(-1)^2+2^2}}=\frac{|2-1+2-3|}{\sqrt{9}}=\frac{|0|}{3}$

$d(P,\pi)=0$ (P está no plano)

Número 11

• Ponto genérico em r: $x=2-y\to x=2-t$; $y+z=y\to z=0$. Logo $P_r=(2-t, t, 0)$

• $x-2y-z-1=0$ $\quad$ • $d(P_r,\pi)=\frac{|1(2-t)-2(t)-1.(0)-1|}{\sqrt{1^2+(-2)^2+(-1)^2}}=\frac{|2-t-2t-1|}{\sqrt{6}}=\frac{|1-3t|}{\sqrt{6}}$

• $\frac{|1-3t|}{\sqrt{6}}=\sqrt{6}\to|1-3t|=6$ $\quad$ • $t\Rightarrow 1-3t=6\to t=\frac{-5}{3}$ ou $t\Rightarrow 1-3t=-6\to t=\frac{7}{3}$

• Para $t=-\frac{5}{3}$, $P_1=(\frac{11}{3},-\frac{5}{3},0)$. Para $t=\frac{7}{3}$, $P_2=(-\frac{1}{3},\frac{7}{3},0)$

Número 12

a) r: $P_1=(2,1,0)$, $\vec{v_1}=(1,-1,1)$ s: $P_2=(0,-1,1)$; $\vec{v_2}=(1,2,-3)$

$\overrightarrow{P_1P_2}=(-2,-2,1)$. $\vec{v_1}\times\vec{v_2}=(1,4,3)$; $(\overrightarrow{P_1P_2})\cdot(\vec{v_1}\times\vec{v_2})=-2.1+(-2).4+1.3=-2-8+3=-7$

$\|\vec{v_1}\times\vec{v_2}\|=\sqrt{1^2+4^2+3^2}=\sqrt{26}$ $\quad$ • $d(r,s)=\frac{|-7|}{\sqrt{26}}=\frac{7\sqrt{26}}{26}$

b) $P_1=(-4,0,-5)$, $\vec{v_1}=(3,4,-2)$ • $P_2=(21,-5,2)$, $\vec{v_2}=(6,-4,-1)$

$\overrightarrow{P_1P_2}=(25,-5,7)$. $\vec{v_1}\times\vec{v_2}=(-12,-9,-36)\rightsquigarrow\vec{n}=(4,3,12)$

$(\overrightarrow{P_1P_2})\times\vec{n}=25.4+(-5).3+7.12=100-15+84=169$

$\|\vec{n}\|=\sqrt{4^2+3^2+12^2}=\sqrt{169}=13$ $\quad$ • $d(r,s)=\frac{|169|}{13}=13$

c) $P_1 = (1,0,0)$, $\vec{v_1} = (-4,1,2)$. $P_2 = (0,0,2)$, $\vec{v_2} = (-4,1,2)$ $\vec{v_1} = \vec{v_2} \rightarrow$ paralelas

$\overrightarrow{P_2P_1} = (1,0,-2)$ $\overrightarrow{P_2P_1} \times \vec{v_2} = (2,6,1)$ $\|\overrightarrow{P_2P_1} \times \vec{v_2}\| = \sqrt{2^2+6^2+1^2} = \sqrt{41}$

$\|\vec{v_2}\| = \sqrt{(-4)^2+1^2+2^2} = \sqrt{21}$

• $d(r,s) = \frac{\sqrt{41}}{\sqrt{21}} = \frac{\sqrt{861}}{21}$

Número 13

a) $\vec{v_r} = (3,3,3)$ ou $(1,1,1)$ $\vec{n_\pi} = (1,0,0) \times (0,1,0) = (0,0,1)$

$\vec{v_r} \times \vec{n_\pi} = 1.0 + 1.0 + 1.1 = 1 \neq 0 \rightsquigarrow$ Reta e plano não são paralelos, logo a distância é 0.

b) $\vec{v_r} = (1,-1,1) \times (2,1,-1) = (0,3,3)$ ou $(0,1,1)$ $\vec{n_\pi} = (1,1,-1)$

$\vec{v_r} \times \vec{n_\pi} = 0.1 + 1.1 + 1.(-1) = 0$. São paralelos Ponto em r: $P_r = (1,1,0)$

$d(P_r,\pi) = \frac{|1.1 + 1.1 - 1.0 - 4|}{\sqrt{1^2+1^2+(-1)^2}} = \frac{|-2|}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

c) $\vec{v_r} = (1,1,1)$, $\vec{n_\pi} = (2,1,-3)$ $\vec{v_r} \times \vec{n_\pi} = 1.2 + 1.1 + 1.(-3) = 0$. São paralelos.

Ponto em r: $P_r = (0,1,-3)$ $d(P_r,\pi) = \frac{|2.0 + 1.1 - 3.(-3) - 10|}{\sqrt{2^2+1^2+(-3)^2}} = \frac{|0|}{\sqrt{14}} = 0$. A reta está contida no plano.

Número 14

a) $\vec{n_1} = (2,-1,2)$, $\vec{n_2} = (4,-2,4) \rightarrow \vec{n_2} = 2\vec{n_1}$ $\pi_1: 4x - 2y + 4z + 0 = 0$

$d(\pi_1,\pi_2) = \frac{|0-(-21)|}{\sqrt{4^2+(-2)^2+4^2}} = \frac{|21|}{\sqrt{36}} = \frac{21}{6} = \frac{7}{2}$

b) $\vec{n_1} = (2,2,2)$ ou $(1,1,1)$. Para π_2, $\vec{n_2} = (-1,0,3) \times (1,1,0) = (-3,3,1)$.

Não são paralelos, distância = 0

c) $\vec{n_1} = (1,1,1)$; $\vec{n_2} = (2,1,1)$.

Não são paralelos, distância = 0

Número 15

Reta r: $y = 1, z = 5 - x$. Ponto $P_r = (0,1,5)$; $\vec{v_r} = (1,0,-1)$

Reta s: $P_s = (4,1,1)$, $\vec{v_s} = (4,2,-3)$

As retas interceptam em $(4,1,1)$ (para r, $t=4$)

• $\vec{n_\pi} = \vec{v_r} \times \vec{v_s} = (1,0,-1) \times (4,2,-3) = (2,-1,2)$

• Equação de π ($P_s = (4,1,1)$ e $\vec{n_\pi} = (2,-1,2)$):

$2(x-4) - 1(y-1) + 2(z-1) = 0 \rightarrow 2x - y + 2z - 9 = 0$

$2x - y + 2z + d' = 0$

A distância entre $2x - y + 2z - 9 = 0$ e $2x - y + 2z + d' = 0$ é $\frac{|-9-d'|}{\sqrt{2^2+(-1)^2+2^2}} = \frac{|-9-d'|}{3}$

$\frac{|-9-d'|}{3} = 2 \rightarrow |-9-d'| = 6$

Valores de d':

1. $-9 - d' = 6 \rightarrow d' = -15$, Plano: $2x - y + 2z - 15 = 0$
2. $-9 - d' = -6 \rightarrow d' = -3$, Plano: $2x - y + 2z - 3 = 0$