Lista 2

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1- a) $Det(A) = 11$

$A = \begin{bmatrix} 1 & 2 \\ -4 & 3 \end{bmatrix} \rightarrow det(A) = a_{11} . a_{22} - a_{12} . a_{21}$

$det(A) = (1 . 3) - (2 . -4) \rightarrow det(A) = 3 - (-8) \rightarrow det(A) = 11$

b) $Det(B) = -5\sqrt{6}$

$B = \begin{bmatrix} \sqrt{2} & 3\sqrt{6} \\ 2 & \sqrt{3} \end{bmatrix} \rightarrow det(B) = b_{11} . b_{22} - b_{12} . b_{21}$

$det(B) = (\sqrt{2} . \sqrt{3}) - (3\sqrt{6} . 2) \rightarrow det(B) = \sqrt{6} - 6\sqrt{6} \rightarrow det(B) = -5\sqrt{6}$

c) $Det(C) = 2$

$C = \begin{bmatrix} \pi & 0 & 2 \\ 5 & -1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $det(C) = \pi . \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} - 0 . \begin{bmatrix} 5 & 1 \\ 1 & 0 \end{bmatrix} + 2 . \begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix}$

$det(C) = \pi . 0 - 0 . (-1) + 2 . 1 \therefore det(C) = 2$

$det = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} = (-1) . 0 - (1) . 0 = 0$

$det \begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix} = 5 . 0 - (-1) . 1 \rightarrow 0 - (-1) = +1$

$det \rightarrow \begin{bmatrix} 5 & 1 \\ 1 & 0 \end{bmatrix} = 5 . 0 - 1 . 1 = 0 - 1 = -1$

d) $Det(D) = -21$

$D = \begin{bmatrix} -2 & 1 & -1 \\ 1 & 5 & 4 \\ -3 & 4 & 2 \end{bmatrix}$ $det(D) = (-2) . (-6) + 1 . (-14) + (-1) . 19$

$det(D) = 12 - 14 - 19$

$det(D) = -21$

Cofator $C_{11} = (-1)^{1+1} . \begin{bmatrix} 5 & 4 \\ 4 & 2 \end{bmatrix} = 1 . (5 . 2 - 4 . 4) = 1 . (10 - 16) = -6$

Cofator $C_{12} = (-1)^{1+2} . \begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix} = -1 (1 . 2 - (4 . (-3)) = -1 . (2 + 12) = -14$

Cofator $C_{13} = (-1)^{1+3} . \begin{bmatrix} 1 & 5 \\ -3 & 4 \end{bmatrix} = 1 . (1 . 4 - 5 . (-3)) = 1 . (4 + 15) = 19$

e) Det (E) = 16

$$E=\begin{bmatrix} 0 & 2 & 0 \\ 1 & 3 & 5 \\ 2 & -1 & 2 \end{bmatrix}$$

det (E) = 0 . 11 + 2 . 8 + 0 . -7

det (E) = 0 + 16 + 0

det (E) = 16 //

cofator $C_{11}=(-1)^{1+1}.\begin{bmatrix} 3 & 5 \\ -1 & 2 \end{bmatrix}=1.(3.2-5.(-1))=1.(6+5)=11$

$C_{12}=(-1)^{1+2}.\begin{bmatrix} 1 & 5 \\ 2 & 2 \end{bmatrix}=-1.(1.2-5.2)=-1(2-10)=8$

$C_{13}=(-1)^{1+3}.\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}=1.(1.(-1)-3.2)=1.(-1-6)=-7$

f) Det (F) = -6

$$\begin{bmatrix} 3 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

det (F) = $a_{11} . C_{11} + a_{12} . C_{12} + a_{13} . C_{13} + a_{14} . C_{14}$

det (F) = 3 . (-2) + (-1) . 0 + 1 . 0 + 1 . 0

det (F) = -6 + 0 + 0 + 0 → Det (F) = -6 //

Cofator $C_{11}=(-1)^{1+1}.\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ det $=1.\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}+0\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}+1.\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$

$C_{11}=1.(-2)=-2$ //

det = 1.(-1) + 0. 1 + 1 (-1) = -1 - 1 = -2

$C_{12}=(-1)^{1+2}.\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ det $=0.\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}+0.\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}+1\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$C_{12}=-1.0=0$ //

det = 0 + 0 + 1.0 = 0 //

$C_{13}=(-1)^{1+3}.\begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ det $=0.\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}-1.\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}+1.\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$

$C_{13}=1.0=0$ //

det = 0.1 - 1.0 + 1.0 = 0

$C_{14}=(-1)^{1+4}\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ det $=0.\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}-1.\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}+0\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$

$C_{14}=0$ //

det = 0 - 1.0 + 0 = 0 //

g) Det $(G) = -9\sqrt{5}$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 3 \\ 7 & 2 & \sqrt{5} & 0 & 0 \\ 10 & -3 & 6 & 1 & 0 \\ -2 & -3 & 0 & 0 & 0 \end{bmatrix}$$

$\det(G) = 1 \cdot (-1)^2 \cdot |\tilde{G}_{11}|$

$\det(G) = 1 \cdot (-9\sqrt{5}) = -9\sqrt{5}$

$$\tilde{G}_{11} = \begin{bmatrix} 1 & 2 & 5 & 3 \\ 2 & \sqrt{5} & 0 & 0 \\ -3 & 6 & 1 & 0 \\ -3 & 0 & 0 & 0 \end{bmatrix} = 3 \cdot (-1)^5 \cdot |\tilde{g}_{14}| = -9\sqrt{5}$$

$$\tilde{g}_{14} = \begin{bmatrix} 2 & \sqrt{5} & 0 \\ -3 & 6 & 1 \\ -3 & 0 & 0 \end{bmatrix} = 1 \cdot (-1)^5 \cdot |\tilde{G}_{31}| = -3\sqrt{5}$$

$$\tilde{G}_{31} = \begin{bmatrix} 2 & \sqrt{5} \\ -3 & 0 \end{bmatrix} = 2 \cdot (-1)^2 \cdot 0 + (-3) \cdot (-1)^3 \cdot \sqrt{5} = 3\sqrt{5}$$

h) Det $(H) = -24$

$$H = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -2 & 0 & 0 & 0 \end{bmatrix}$$

$\det(H) = \sum_{j=1}^{5} (-1)^{1+j} \cdot h_{1j} \cdot \det(H_{1j})$

$\det(H) = (-1)^{1+1} \cdot 3 \cdot \det(H_{11})$

$\det(H) = (-1)^2 \cdot 3 \cdot (-8) = -24$

$$H_{11} = \begin{bmatrix} 0 & 0 & 0 & -2 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \end{bmatrix}$$

$\det(H_{11}) = (-1)^{1+4} \cdot (-2) \cdot \det(H_{11,14})$

$$H_{11,14} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ -2 & 0 & 0 \end{bmatrix}$$

$\det(H_{11,14}) = (-1)^{1+1} \cdot 0 \cdot \det(H_{11,14,11}) + (-1)^{1+2} \cdot 2 \cdot \det(H_{11,14,12}) + (-1)^{1+3} \cdot 0 \cdot \det(H_{11,14,13}) \rightarrow \det(H_{11,14}) = (-1)^{1+2} \cdot 2 \cdot \det(H_{11,14,12})$

$H_{(11,14,12)} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$ $\det(H_{11,14,12}) = (0.0) - (1.(-2)) = 2$

$\det(H_{11,14}) = (-1)^{1+2}.2.2 = -4$

$\det(H_{11}) = (-1)^{1+4}.(-2).(-4) = (-1)^5.(-8) = -8$

2-a) $\det(A+B) = 72$

$A+B = \begin{bmatrix} 3 & -5 & 7 \\ 4 & 2 & 8 \\ 1 & -9 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 7 \\ -1 & 0 & 2 \\ 3 & 1 & -4 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 14 \\ 3 & 2 & 10 \\ 4 & -8 & 2 \end{bmatrix}$

$\det(A+B) = 7.(2.2 - 10.(-8)) - (-2).(3.2 - 10.4) + 14.(3.(-8) - 2.4)$

$\det(A+B) = 7.(4+80) + 2.(6-40) + 14.(-24-8)$

$\det(A+B) = 7.84 + 2.(-34) + 14.(-32)$

$\det(A+B) = 588 - 68 - 448 = 72$

b) $\det(AB) = -594$

$A.B = \begin{bmatrix} 3.4+(-5).(-1)+7.3 & 3.3+(-5).0+7.1 & 3.7+(-5).2+7.(-4) \\ 4.4+2.(-1)+8.3 & 4.3+2.0+8.1 & 4.7+2.2+8.(-4) \\ 1.4+(-9).(-1)+6.3 & 1.3+(-9).0+6.1 & 1.7+(-9).2+6.(-4) \end{bmatrix}$

$AB = \begin{bmatrix} 12+5+21 & 9+0+7 & 21-10-28 \\ 16-2+24 & 12+0+8 & 28+4-32 \\ 4+9+18 & 3+0+6 & 7-18-24 \end{bmatrix} = \begin{bmatrix} 38 & 16 & -17 \\ 38 & 20 & 0 \\ 31 & 9 & -35 \end{bmatrix}$

$\det(AB) = 38.(20.(-35) - 0.9) - 16.(38.(-35) - 0.31) + (-17).(38.9 - 20.31)$

$\det(AB) = 38.(-700) - 16.(-1330) + (-17).(342-620)$

$\det(AB) = -26600 + 21280 + (-17).(-278)$

$\det(AB) = -26600 + 21280 + 4726$

$\det(AB) = -26600 + 26006$

$\det(AB) = -594$

c) $\det(B^t A^t) = -594$

$\det(B^t A^t) = \det(A^t B^t) = \det(AB)$

$\det(B^T A^T) = \det(AB) = -594$

d) $\det(3A - 2C + B) = 14280$

$3A = \begin{bmatrix} 9 & -15 & 21 \\ 12 & 6 & 24 \\ 3 & -27 & 18 \end{bmatrix}$, $-2C = \begin{bmatrix} -4 & -6 & 2 \\ -12 & -18 & 4 \\ -16 & -24 & 6 \end{bmatrix}$

$3A - 2C + B = \begin{bmatrix} 9-4+4 & -15-6+3 & 21+2+7 \\ 12-12-1 & 6-18+0 & 24+4+2 \\ 3-16+3 & -27-24+1 & 18+6-4 \end{bmatrix} = \begin{bmatrix} 9 & -18 & 30 \\ -1 & -12 & 30 \\ -10 & -50 & 20 \end{bmatrix}$

$\det(3A-2C+B) = 9.(-12.20-30.(-50)) - (-18).(-1.20-30.(-10)) + 30.(-1.(-50)-(-12).(-10))$
$\det(3A-2C+B) = 9.(-240+1500) + 18.(-20+300) + 30.(50-120)$
$\det(3A-2C+B) = 9.1260 + 18.280 + 30.(-70)$
$\det(3A-2C+B) = 11340 + 5040 - 2100$
$\det(3A-2C+B) = 14280$

e) $\det(AC^t) = 0$

$C^t = \begin{bmatrix} 2 & 6 & 8 \\ 3 & 9 & 12 \\ -1 & -2 & -3 \end{bmatrix}$ $AC^T = \begin{bmatrix} 3.2+(-5).3+7.(-1) & 3.6+(-5).9+7.(-2) & 3.8+(-5).12+7.(-3) \\ 4.2+2.3+8.(-1) & 4.6+2.9+8.(-2) & 4.8+2.12+8.(-3) \\ 1.2+(-9).3+6.(-1) & 1.6+(-9).9+6.(-2) & 1.8+(-9).12+6.(-3) \end{bmatrix}$

$AC^T = \begin{bmatrix} 6-15-7 & 18-45-14 & 24-60-21 \\ 8+6-8 & 24+18-16 & 32+24-24 \\ 2-27-6 & 6-81-12 & 8-108-18 \end{bmatrix} = \begin{bmatrix} -16 & -41 & -57 \\ 6 & 26 & 32 \\ -31 & -87 & -118 \end{bmatrix}$

$\det(AC^T) = -16.(26.(-118)-32.(-87)) - (-41).(6.(-118)-32.(-31)) + (-57).(6.(-87)-26.(-31))$
$\det(AC^T) = -16.(-3068+2784) + 41.(-708+992) - 57(-522+806)$
$\det(AC^T) = -16.(-284) + 41.284 - 57.284$
$\det AC^T = 4544 + 11644 - 16188 = 0$

3- a) $\det(A^T) = -2$

$\det(A^T) = \det(A) = -2$

b) $\det(6A) = -2592$

Por escalar: $\det = K^n$

$K = 6$ e $n = 4$

$\det(6A) = 6^4 . \det(A) = 1296 . (-2) = -2592$

c) $\det(A^7) = -128$

$\det(A^7) = (\det(A))^7 = (-2)^7 = -128$

d) $\det(A^{-1}) = -\frac{1}{2}$

$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{-2} = -\frac{1}{2}$

4-a) $\begin{bmatrix} a & b & c \\ d & e & f \\ 4g & 4h & 4i \end{bmatrix} = 4 . \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 4 . -3 = -12$

b) $\begin{bmatrix} a & b & -2c \\ 3d & 3e & -6f \\ g & h & -2i \end{bmatrix} = -6 . \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = (-6).(-3) = 18$ $(-2 . 3 = -6)$

c) $\begin{bmatrix} -a & -b & -c \\ d & e & f \\ -g & -h & -i \end{bmatrix} = (-1).(-1). \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 1 . -3 = -3$

d) $\begin{bmatrix} g & h & i \\ a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -3 = -3$

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e) $\begin{bmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{bmatrix} = 2.\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -6$

A soma de múltiplos de uma linha a outra não altera o determinante.

f) $\begin{bmatrix} ka+a & kb+b & ck+c \\ d & e & f \\ g & h & i \end{bmatrix} = (k+1).\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = (k+1).(-3) = -3(k+1)$

$Det(A) = -3.1120 = -3360$

5) $A = \begin{bmatrix} 10 & 8 & 40 & -2 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 1 \\ 3 & -6 & -30 & 12 \end{bmatrix} \rightarrow det(A) = 3.det \begin{bmatrix} 10 & 8 & 40 & -2 \\ 4 & 6 & 20 & -4 \\ -5 & -7 & -30 & 1 \\ 1 & -2 & -10 & 4 \end{bmatrix} \begin{matrix} \rightarrow L_1 \leftarrow L_1 - 10.L_4 \\ \rightarrow L_2 \leftarrow L_2 - 4.L_4 \\ \rightarrow L_3 \leftarrow L_3 + 5.L_4 \\ \end{matrix}$

$\begin{bmatrix} 0 & 28 & 140 & -42 \\ 0 & 14 & 60 & -20 \\ 0 & -17 & -80 & 21 \\ 1 & -2 & -10 & 4 \end{bmatrix} det(A) = 3.1.(-1)^{4+1}.det \begin{bmatrix} 28 & 140 & -42 \\ 14 & 60 & -20 \\ -17 & -80 & 21 \end{bmatrix} L_1 \leftarrow L_1 - 2.L_2 = \begin{bmatrix} 0 & 20 & -2 \\ 14 & 60 & -20 \\ -17 & -80 & 21 \end{bmatrix}$

$det = 0.(cofator) - 20.det \begin{bmatrix} 14 & -20 \\ -17 & 21 \end{bmatrix} + (-2).det \begin{bmatrix} 14 & 60 \\ -17 & 80 \end{bmatrix}$

6-a) $\begin{bmatrix} 4 & 6 & x \\ 7 & 4 & 2x \\ 5 & 2 & -x \end{bmatrix} = -128 \qquad x = -2$

$det = 4.(4.(-x) - 2x.2) - 6.(7.(-x) - 2x.5) + x(7.2 - 4.5)$

$det = 4.(-4x - 4x) - 6.(-7x - 10x) + x(14 - 20)$

$det = 4.(-8x) - 6(-17x) + x(-6)$

$det = -32x + 102x - 6x = 64x$

$64x = -128$

$x = \frac{-128}{64} \rightarrow x = -2.$

$det \begin{bmatrix} 14 & -20 \\ -17 & 21 \end{bmatrix} =$

$294 - 340 = -46$

$det \begin{bmatrix} 14 & 60 \\ -17 & 80 \end{bmatrix} =$

$-1120 + 1020 = -100$

$det = -20.(-46) + (-2).100$

$det = 920 + 200 = 1120$

$det(A) = -3.1120 = -3360$

b) $\begin{bmatrix} 3 & 5 & 7 \\ 2x & x & 3x \\ 4 & 6 & 7 \end{bmatrix} = 39$ e $x = 3$ //

$det = 3.(x.7 - 3x.6) - 5.(2x.7 - 3x.4) + 7.(2x.6 - x.4)$
$det = 3.(7x - 18x) - 5(14x - 12x) + 7(12x - 4x)$
$det = 3.(-11x) - 5.(2x) + 7.(8x)$
$det = -33x - 10x + 56x = 13x$
$13x = 39 \rightarrow x = 3$ //

c) $\begin{bmatrix} x+3 & x+1 & x+4 \\ 4 & 5 & 3 \\ 9 & 10 & 7 \end{bmatrix} = -7$ e $x = 1$ //

$det = (x+3).(5.7 - 3.10) - (x+1).(4.7 - 3.9) + (x+4).(4.10 - 5.9)$
$det = (x+3).(35 - 30) - (x+1).(28 - 27) + (x+4).(40 - 45)$
$det = (x+3).(5) - (x+1).(1) + (x+4).(-5)$
$det = 5x + 15 - x - 1 - 5x - 20$ $\qquad -x - 6 = -7$
$det = -x - 6$ $\qquad -x = -1$
$x = 1$ //

d) $\begin{bmatrix} x & x+2 \\ 1 & x \end{bmatrix}$ é singular. $\qquad x = 2$ ou $x = -1$ //
$\hookrightarrow det = 0$

$det = x.x - (x+2).1 \rightarrow det = x^2 - x - 2$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0 \rightarrow x = 2$ ou $x = -1$ //

e) $\begin{bmatrix} x-4 & 0 & 3 \\ 2 & 0 & x-9 \\ 0 & 3 & 0 \end{bmatrix}$ é inversível. $\qquad x \neq 3$ e $x \neq 10$
$\hookrightarrow det \neq 0$

$det = (x-4).(0.0 - (x-9).3) - 0.(2.0 - (x-9).0) + 3.(2.3 - 0.0) = (x-4).(-3(x-9)) + 3.6 =$
$-3(x-4)(x-9) + 18 = 0 \rightarrow (x-4)(x-9) = 6$
$x^2 - 13x + 36 = 6 \rightarrow x^2 - 13x + 30 = 0 \rightarrow (x-10)(x-3) = 0$
$x \neq 10$ // $\quad x \neq 3$ //

7 - a) $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, a condição é que $ad-bc \neq 0$.

$A^{-1} = \frac{1}{det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $det(A) = ad - bc$

A inversa A^{-1} só existe se $det(A) \neq 0$, ou seja, $ad-bc \neq 0$//

b) $A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$, $(AB)^{-1} = \begin{bmatrix} 39 & -23 \\ -22 & 13 \end{bmatrix}$.

$det(A) = 3.2 - 1.5 = 6 - 5 = 1$//
$det(B) = 4.2 - 7.1 = 8 - 7 = 1$//

$A^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$B^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3.4+1.1 & 3.7+1.2 \\ 5.4+2.1 & 5.7+2.2 \end{bmatrix} = \begin{bmatrix} 12+1 & 21+2 \\ 20+2 & 35+4 \end{bmatrix} = \begin{bmatrix} 13 & 23 \\ 22 & 39 \end{bmatrix}$

$det(AB) = 13.39 - 23.22 = 507 - 506 = 1$//

$AB^{-1} = \frac{1}{1}\begin{bmatrix} 39 & -23 \\ -22 & 13 \end{bmatrix} = \begin{bmatrix} 39 & -23 \\ -22 & 13 \end{bmatrix}$

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8- a) $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$. A matriz dos cofatores é $C = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}$

$a_{11} = 2 \therefore C_{11} = (-1)^{1+1} \cdot \det(M_{11}) = 1 \cdot \det(1) = 1_{//}$

$a_{12} = -2 \therefore C_{12} = (-1)^{1+2} \cdot \det(M_{12}) = -1 \cdot \det(3) = -3_{//}$

$a_{21} = 3 \therefore C_{21} = (-1)^{2+1} \cdot \det(M_{21}) = -1 \cdot \det(-2) = 2_{//}$

$a_{22} = 1 \therefore C_{22} = (-1)^{2+2} \cdot \det(M_{22}) = 1 \cdot \det(2) = 2_{//}$

$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}$

b) $B = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$. A matriz dos cofatores é $C = \begin{bmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{bmatrix}$.

$b_{11} = 2 \therefore C_{11} = (-1)^{1+1} \cdot \det(M_{11}) = 1 \cdot \det \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} = 1 \cdot (2 \cdot (-1) - 1 \cdot 1) = -3_{//}$

$b_{12} = -2 \therefore C_{12} = (-1)^{1+2} \cdot \det(M_{12}) = -1 \cdot \det \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = -1 \cdot (1 \cdot (-1) - 1 \cdot 0) = 1_{//}$

$b_{13} = 0 \therefore C_{13} = (-1)^{1+3} \cdot \det(M_{13}) = 1 \cdot \det \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = 1 \cdot (1 \cdot 1 - 2 \cdot 0) = 1_{//}$

$b_{21} = 1 \therefore C_{21} = (-1)^{2+1} \cdot \det(M_{21}) = -1 \cdot \det \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} = -1((-2) \cdot (-1) - 0 \cdot 1) = -2_{//}$

$b_{22} = 2 \therefore C_{22} = (-1)^{2+2} \cdot \det(M_{22}) = 1 \cdot \det \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = 1 \cdot (2 \cdot (-1) - 0 \cdot 0) = -2_{//}$

$b_{23} = 1 \therefore C_{23} = (-1)^{2+3} \cdot \det(M_{23}) = -1 \cdot \det \begin{bmatrix} 2 & -2 \\ 0 & 1 \end{bmatrix} = -1 \cdot (2 \cdot 1 - (-2) \cdot 0) = -2_{//}$

$b_{31} = 0 \therefore C_{31} = (-1)^{3+1} \cdot \det(M_{31}) = 1 \cdot \det \begin{bmatrix} -2 & 0 \\ 2 & 1 \end{bmatrix} = 1 \cdot ((-2) \cdot 1 - 0 \cdot 2) = -2_{//}$

$b_{32} = 1 \therefore C_{32} = (-1)^{3+2} \cdot \det(M_{32}) = -1 \cdot \det \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = -1 \cdot (2 \cdot 1 - 0 \cdot 1) = -2_{//}$

$b_{33} = -1 \therefore C_{33} = (-1)^{3+3} \cdot \det(M_{33}) = 1 \cdot \det \begin{bmatrix} 2 & -2 \\ 1 & 2 \end{bmatrix} = 1 \cdot (2 \cdot 2 - (-2) \cdot 1) = 6_{//}$

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \rightarrow C = \begin{bmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & 6 \end{bmatrix}_{//}$$

9-a) $A = \begin{bmatrix} 2 & -2 \\ 3 & 1 \end{bmatrix}$. $A^{-1} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{4} \end{bmatrix}$

$A^{-1} = \frac{1}{\det(A)} \cdot adj(A)$ $\det(A) = 2.1 - (-2).3 = 2 + 6 = 8$

$C = \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix} \therefore adj(A) = C^T = \begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{8} \cdot \begin{bmatrix} 1 & 2 \\ -3 & 2 \end{bmatrix} \rightarrow A^{-1} = \begin{bmatrix} \frac{1}{8} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{4} \end{bmatrix}$

b) $B = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ $B^{-1} = \begin{bmatrix} \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$

$\det(B) = 2.(2.(-1) - 1.1) - (-2).(1.(-1) - 1.0) + 0.(1.1 - 2.0)$
$\det(B) = 2.(-2-1) + 2.(-1-0) + 0.(1-0)$
$\det(B) = 2.(-3) + 2.(-1) + 0.(1) = -6 - 2 + 0 = -8$

$C = \begin{bmatrix} -3 & 1 & 1 \\ -2 & -2 & -2 \\ -2 & -2 & -6 \end{bmatrix}$ $adj(B) = C^T = \begin{bmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{bmatrix}$; $B^{-1} = \frac{1}{-8} \cdot \begin{bmatrix} -3 & -2 & -2 \\ 1 & -2 & -2 \\ 1 & -2 & 6 \end{bmatrix} \rightarrow B^{-1} = \begin{bmatrix} \frac{3}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{8} & \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$

c) $C = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$ $C^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$\det(C) = 0.(0.0 - (-1).1) - (-1).(2.0 - (-1).1) + 1.(2.1 - 0.1) = 0.(0+1) + 1.(0+1) + 1.(2-0)$
$\det(C) = 0 + 1 + 2 = 3$ $C = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$; $adj(C) = C^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

matriz dos cofatores

$C^{-1} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$C_{ij} = (-1)^{i+j} \cdot \det(M_{ij})$

d) $D = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 3 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ $\det(D) = 1 \cdot \begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 3 \\ 0 & -1 & 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 3 \\ -1 & 0 \end{vmatrix} - 0 \cdot \begin{vmatrix} 2 & 3 \\ 0 & 0 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 2 \\ 0 & -1 \end{vmatrix}$

$\det(D) = 1$

$= 1 \cdot (2 \cdot 0 - 3 \cdot (-1)) - 0 + 1 \cdot (2 \cdot (-1) - 2 \cdot 0) = 1 \cdot (0+3) + 1 \cdot (-2-0) = 3 - 2 = 1$

$C_{11} = 1 \cdot (1 \cdot (2 \cdot 0 - 3 \cdot (-1))) = 1 \cdot (0+3) = 3$

$C_{12} = -1 \cdot 0 = 0$

$C_{13} = 1 \cdot 0 = 0$

$C_{14} = -1 \cdot (0 - 1 \cdot (2 \cdot (-1) - 2 \cdot 0)) = -1 \cdot (-1 \cdot (-2)) = -2$

$C_{21} = -1 \cdot 0 = 0$

$C_{22} = 1 \cdot (1 \cdot (2 \cdot 0 - 3 \cdot (-1)) - 0 + 1 \cdot (2 \cdot (-1) - 2 \cdot 0)) = 1 \cdot 1 = 1$

$C_{23} = -1 \cdot 0 = 0$

$C_{24} = 1 \cdot (1 \cdot (2 \cdot (-1) - 2 \cdot (-1))) = 1 \cdot (-2+2) = 0$

$C_{31} = 1 \cdot (0 - 0 + 1 \cdot (1 \cdot (-1) - 0 \cdot 0)) = 1 \cdot (-1) = -1$

$C_{32} = -1 \cdot 0 = 0$

$C_{33} = 1 \cdot 0 = 0$

$C_{34} = -1 \cdot (1 \cdot (1 \cdot (-1) - 0 \cdot 0)) = -1 \cdot (-1) = 1$

$C_{41} = -1 \cdot (0 - 0 + 1 \cdot (1 \cdot 2 - 0 \cdot 0)) = -1 \cdot 2 = -2$

$C_{42} = 1 \cdot 0 = 0$ $C_{44} = 1 \cdot (1 \cdot (1 \cdot 2 - 0 \cdot 0)) = 1 \cdot 2 = 2$

$C_{43} = -1 \cdot (1 \cdot (1 \cdot 3 - 0 \cdot 0) - 0 + 1 \cdot (0 \cdot 0 - 1 \cdot 2)) = -1 \cdot (3-2) = -1$

$C = \begin{vmatrix} 3 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \\ -2 & 0 & -1 & 2 \end{vmatrix}$

$adj(D) = C^T = \begin{vmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{vmatrix}$

$D^{-1} = \frac{1}{\det(D)} \cdot adj(D) = adj(D)$

$D^{-1} = adj(D) = \begin{vmatrix} 3 & 0 & -1 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -2 & 0 & 1 & 2 \end{vmatrix}$

10 - a) $2A^T = C - XB$ $X = (C - 2A^T)B^{-1}$

$XB = C - 2A^T$

$X = (C - 2A^T)B^{-1}$

A matriz B deve ser inversível, ou seja, $\det(B) \neq 0$. Caso contrário, a inversa B^{-1} não existe, assim a solução para X não é possível.

10-b) $X = (C - 2A^T)B^{-1}$

$$X = \begin{bmatrix} 6 & -12 & 8 \\ 2 & -6 & 6 \\ -2 & 4 & -2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ -1 & 0 & 2 \end{bmatrix} ; \quad 2A^T \begin{bmatrix} 2 & 4 & 0 \\ 0 & 6 & -2 \\ -2 & 0 & 4 \end{bmatrix}$$

$$C - 2A^T = \begin{bmatrix} 4 & 4 & 0 \\ 0 & 8 & -2 \\ -2 & 0 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 0 \\ 0 & 6 & -2 \\ -2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$\det(B) = 3.(-1.3-5.0) - (-2).(2.3-5.1) + 6.(2.0-(-1).1)$

$\det(B) = 3.(-3) + 2.(6-5) + 6.(0+1)$

$\det(B) = -9 + 2 + 6 = -1$ //

Matriz dos cofatores de B: $C_B = \begin{bmatrix} -3 & -1 & 1 \\ 6 & 3 & -2 \\ -4 & -3 & 1 \end{bmatrix}$

$$adj(B) = C_B^T = \begin{bmatrix} -3 & 6 & -4 \\ -1 & 3 & -3 \\ 1 & -2 & 1 \end{bmatrix} ; \quad B^{-1} = \frac{1}{-1} \cdot \begin{bmatrix} -3 & 6 & -4 \\ -1 & 3 & -3 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 & 4 \\ 1 & -3 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$X = (C - 2A^T)B^{-1} \rightarrow X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & -6 & 4 \\ 1 & -3 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} 2.3+0.1+0.(-1) & 2.(-6)+0.(-3)+0.2 & 2.4+0.3+0.(-1) \\ 0.3+2.1+0.(-1) & 0.(-6)+2.(-3)+0.2 & 0.4+2.3+0.(-1) \\ 0.3+0.1+2.(-1) & 0.(-6)+0.(-3)+2.2 & 0.4+0.3+2.(-1) \end{bmatrix}$$

$$X = \begin{bmatrix} 6 & -12 & 8 \\ 2 & -6 & 6 \\ -2 & 4 & -2 \end{bmatrix} //$$

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