

Lista 5 - Geometria Analítica

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Número 1

$$a) \overrightarrow{BF} = \overrightarrow{AF} - \overrightarrow{AB}$$

$$\overrightarrow{BF} = \overrightarrow{f} - \overrightarrow{b}$$

$$\overrightarrow{BF} = \overrightarrow{f} - \overrightarrow{b}$$

$$b) \overrightarrow{AG} = \overrightarrow{AC} + \overrightarrow{CG} = \overrightarrow{AC} + \overrightarrow{BF}$$

$$\overrightarrow{AG} = \overrightarrow{AC} + \overrightarrow{AF} - \overrightarrow{AB}$$

$$\overrightarrow{AG} = \overrightarrow{c} + \overrightarrow{f} - \overrightarrow{b}$$

$$c) \overrightarrow{AE} = \overrightarrow{Af} + \overrightarrow{AB}$$

$$\overrightarrow{Ef} = \overrightarrow{AB}$$

$$\overrightarrow{AE} = \overrightarrow{f} - \overrightarrow{b}$$

$$d) \overrightarrow{BG} = \overrightarrow{c} - \overrightarrow{b} + \overrightarrow{f} - \overrightarrow{b}$$

$$\overrightarrow{c} - \overrightarrow{b} + \overrightarrow{f} - \overrightarrow{b}$$

$$e) \overrightarrow{HB} = \overrightarrow{c} + \overrightarrow{f} - \overrightarrow{b}$$

$$\overrightarrow{c} + \overrightarrow{f} - \overrightarrow{b}$$

$$f) \overrightarrow{AB} + \overrightarrow{FG} = \overrightarrow{b} + \overrightarrow{c} - \overrightarrow{b} = \overrightarrow{c}$$

$$\overrightarrow{b} + \overrightarrow{c} - \overrightarrow{b} = \overrightarrow{c}$$

$$g) \overrightarrow{AD} + \overrightarrow{HG} = \overrightarrow{c} - \overrightarrow{b} + \overrightarrow{b} = \overrightarrow{c}$$

$$\overrightarrow{b} \cancel{c}$$

$$h) \overrightarrow{HF} + \overrightarrow{AG} - \overrightarrow{EF} = \overrightarrow{f}$$

$$(2\overrightarrow{b} - \overrightarrow{c}) + (\overrightarrow{c} + \overrightarrow{f} - \overrightarrow{b}) - (\overrightarrow{b}) = \overrightarrow{f}$$

$$i) 2\overrightarrow{AD} - \overrightarrow{FG} - \overrightarrow{BH} + \overrightarrow{GH} = 2\overrightarrow{c} - 3\overrightarrow{b} + \overrightarrow{f}$$

$$2\overrightarrow{AD} = 2(\overrightarrow{c} - \overrightarrow{b})$$

$$\overrightarrow{FG} = \overrightarrow{b}$$

$$\overrightarrow{BH} = \overrightarrow{AH} - \overrightarrow{AB} = \overrightarrow{f} - \overrightarrow{b}$$

$$\overrightarrow{GH} = -\overrightarrow{HG} = -\overrightarrow{b}$$

$$2(\overrightarrow{c} - \overrightarrow{b}) - \overrightarrow{b} - (\overrightarrow{f} - \overrightarrow{b}) + (-\overrightarrow{b}) = 2\overrightarrow{c} - 2\overrightarrow{b} - \overrightarrow{b} - \overrightarrow{f} + \overrightarrow{b} - \overrightarrow{b}$$

$$= 2\overrightarrow{c} - 3\overrightarrow{b} + \overrightarrow{f}$$

Número 2

$$a) \overrightarrow{DF} = \overrightarrow{DE} + \overrightarrow{DC}$$

$$\overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{DE} + \overrightarrow{DC}$$

$$b) \overrightarrow{DA} = -2\overrightarrow{DE}$$

$$\overrightarrow{DA} = -\overrightarrow{AD}$$

$$\overrightarrow{AD} = 2\overrightarrow{DE}$$

$$c) \overrightarrow{DB} = 2\overrightarrow{DC}$$

$$\overrightarrow{DC} + \overrightarrow{CB}$$

$$\overrightarrow{CB} = \overrightarrow{DC}$$

$$d) \overrightarrow{DO} = -\overrightarrow{DE}$$

$$\overrightarrow{DO} = \frac{1}{2}\overrightarrow{DA} = -\overrightarrow{DE}$$

$$e) \overrightarrow{EC} = \overrightarrow{DC} - \overrightarrow{DE}$$

$$\overrightarrow{ED} + \overrightarrow{DC}$$

$$\overrightarrow{ED} = -\overrightarrow{DE}$$

$$f) \overrightarrow{EB} = 2\overrightarrow{DC}$$

$$\overrightarrow{EO} + \overrightarrow{OB}$$

$$\overrightarrow{EO} = \overrightarrow{OB} = \overrightarrow{DC}$$

$$g) \overrightarrow{OB} > \overrightarrow{DC}$$

$$\angle DOB = 1/2(\angle EDC)$$

$$h) \overrightarrow{AF} = -\overrightarrow{DC}$$

$$\angle DAB = \angle DBC - (\angle EDC)$$

Número 3

a) $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} = (-\vec{d}) + (-\vec{e}) + \overrightarrow{OC} + \vec{d} + \vec{e} + \overrightarrow{OF} = \overrightarrow{OC} + \overrightarrow{OF} - \overrightarrow{OC} - \overrightarrow{OC} = \overrightarrow{0}$
 $\overrightarrow{OA} = -\vec{d}$ $\overrightarrow{OB} = -\vec{e}$ $\overrightarrow{OD} = \vec{d}$
 $\overrightarrow{OE} = \vec{e}$ $\overrightarrow{OC} = -\overrightarrow{OF}$

b) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FA} = (\vec{d} - \vec{e}) + (\overrightarrow{OC} + \vec{e}) + (\vec{d} - \overrightarrow{OC}) + (\vec{e} - \vec{d}) + (\overrightarrow{OF} - \vec{e}) + (-\vec{d} - \overrightarrow{OF})$
 $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\vec{e} - (-\vec{d}) = \vec{d} - \vec{e}$ $= \vec{d} - \vec{e} + \overrightarrow{OC} + \vec{e} + \vec{d} - \overrightarrow{OC} + \vec{e} - \vec{d} + \overrightarrow{OF} - \vec{e} - \vec{d} - \overrightarrow{OF} = \overrightarrow{0}$
 $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \overrightarrow{OC} - (-\vec{e}) = \overrightarrow{OC} + \vec{e}$
 $\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC} = \vec{d} - \overrightarrow{OC}$
 $\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} = \vec{e} - \vec{d}$
 $\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE} = \overrightarrow{OF} - \vec{e}$
 $\overrightarrow{FA} = \overrightarrow{OA} - \overrightarrow{OF} = -\vec{d} - \overrightarrow{OF}$

c) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{AF} = \overrightarrow{OF} - \overrightarrow{OA} = \overrightarrow{OF} - (-\vec{d}) = \overrightarrow{OF} + \vec{d}$
 $\overrightarrow{OF} = -\overrightarrow{OC}$ $\overrightarrow{OC} = \vec{d} + \vec{e}$

d) $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OD} + \overrightarrow{OE} = (-\vec{d}) + (-\vec{e}) + \vec{d} + \vec{e} = \overrightarrow{0}$

e) $\overrightarrow{OC} + \overrightarrow{AF} + \overrightarrow{EF} = \overrightarrow{0}$

$\overrightarrow{OC} + \vec{e} + (-\vec{d}) = \vec{d} + (-\vec{e}) + \vec{e} + (-\vec{d}) = \overrightarrow{0}$
 $\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{OB} = \vec{d} + (-\vec{e})$

f) $\overrightarrow{AF} + \overrightarrow{DE} = -\vec{d} + 2\vec{e}$

$\overrightarrow{AF} = \vec{e}$ $\overrightarrow{OF} = \overrightarrow{OA} + \overrightarrow{OE} = -\vec{d} + \vec{e}$ $\overrightarrow{AF} + \overrightarrow{DE} = \vec{e} + \vec{e} + (-\vec{d}) = 2\vec{e} + (-\vec{d})$
 $\overrightarrow{DE} = \overrightarrow{OF}$

Número 4

Vetor \overrightarrow{BP}

P em relação a A e C:

$$\overrightarrow{AP} = \frac{1}{2} \overrightarrow{AC}$$

$$\overrightarrow{BP} = \overrightarrow{AP} - \overrightarrow{AB} = \frac{1}{2} \overrightarrow{AC} - \overrightarrow{AB}$$

$$\overrightarrow{BP} = \frac{1}{2} \overrightarrow{AC} - \overrightarrow{AB}$$

$$\overrightarrow{BP} = \frac{1}{2} \overrightarrow{AC} - \overrightarrow{AB}$$

Vetor \overrightarrow{AN}

$$\overrightarrow{BN} = \frac{1}{2} \overrightarrow{BC}$$

$$\overrightarrow{BN} = \frac{1}{2} (\overrightarrow{AC} - \overrightarrow{AB})$$

$$\overrightarrow{AN} = \frac{1}{2} \overrightarrow{AB} + \frac{1}{2} \overrightarrow{AC}$$

$$\overrightarrow{AN} = \frac{1}{2} \overrightarrow{AB} + \frac{1}{2} \overrightarrow{AC}$$

Vetor \overrightarrow{CM}

$$\overrightarrow{AM} = \frac{1}{2} \overrightarrow{AB}$$

$$\overrightarrow{CM} = \overrightarrow{AM} - \overrightarrow{AC} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{AC}$$

$$\overrightarrow{CM} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{AC}$$

$$\overrightarrow{CM} = \frac{1}{2} \overrightarrow{AB} - \overrightarrow{AC}$$

Número 5

a) Lado \vec{CD} : $\vec{CD} = 2\vec{v} - 2\vec{v}$
 $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DA} = \vec{0}$
 $\vec{DA} = -\vec{AD} = -5\vec{v}$
 $2\vec{v} + 3\vec{v} + \vec{CD} - 5\vec{v} = \vec{0}$
 $\vec{CD} = 2\vec{v} - 2\vec{v}$

Diagonal \vec{BD} : $\vec{BD} = 5\vec{v} - 2\vec{v}$
 $\vec{BD} = \vec{BA} + \vec{AD} = -\vec{AB} + \vec{AD} = -2\vec{v} + 5\vec{v}$
 $\vec{BD} = 5\vec{v} - 2\vec{v}$

Diagonal \vec{CA} : $\vec{CA} = -3\vec{v} - 2\vec{v}$
 $\vec{CA} = \vec{CB} + \vec{BA}$
 $= -\vec{BC} - \vec{AB}$
 $\vec{CA} = -3\vec{v} - 2\vec{v}$

b) $\vec{AB} = 2\vec{v}$, $\vec{CD} = 2\vec{v} - 2\vec{v}$
 $\vec{AD} = 5\vec{v}$, $\vec{BC} = 3\vec{v}$ $\rightarrow \vec{CD} = k\vec{AB} \rightarrow 2\vec{v} - 2\vec{v} = k \cdot 2\vec{v} \rightarrow 2\vec{v} = (2k+2)\vec{v} \rightarrow k = 1$
 $\vec{AD} = \frac{5}{3}\vec{BC} \rightarrow$ são paralelos, pois um é múltiplo escalar do outro.
 ABCD é um trapézio porque \vec{AD} e \vec{BC} são vetores paralelos e \vec{AB} e \vec{DC} não são.

Número 6

$$\vec{OD} = \vec{OA} + \vec{AD} = \vec{a} + \frac{1}{4}\vec{c}$$

$$\vec{OE} = \vec{OB} + \vec{BE} = \vec{b} + \frac{5}{6}\vec{a}$$

$$\vec{DE} = \vec{OE} - \vec{OD} = \left(\vec{b} + \frac{5}{6}\vec{a}\right) - \left(\vec{a} + \frac{1}{4}\vec{c}\right)$$

$$\vec{DE} = \frac{5}{6}\vec{a} - \vec{a} + \vec{b} - \frac{1}{4}\vec{c}$$

$$\vec{DE} = -\frac{1}{6}\vec{a} + \vec{b} - \frac{1}{4}\vec{c}$$

$$\vec{DE} = -\frac{1}{6}\vec{a} + \vec{b} - \frac{1}{4}\vec{c}$$

Número 7 $x = 2 //$

$$\vec{AC} = \vec{OC} - \vec{OA} = (5\vec{a} + x\vec{b}) - (\vec{a} + 2\vec{b}) = 4\vec{a} + (x-2)\vec{b}$$

$$\vec{BC} = \vec{OC} - \vec{OB} = (5\vec{a} + x\vec{b}) - (3\vec{a} + 2\vec{b}) = 2\vec{a} + (x-2)\vec{b}$$

$$\begin{cases} 4 = 2K \rightarrow K = 2 \\ x-2 = K(x-2) \end{cases}$$

$$\rightarrow x-2 = 2(x-2) \rightarrow x-2 = 2x-4 \rightarrow -x = -2 \rightarrow x = 2 //$$

$$\vec{AC} = K \cdot \vec{BC}$$

$$4\vec{a} + (x-2)\vec{b} = K(2\vec{a} + (x-2)\vec{b})$$

Número 8 $\vec{AB} = \vec{OB} - \vec{OA} = \frac{1}{n}\vec{n} - \vec{a}$

Para ser LD: $\vec{AC} = K \cdot \vec{AB}$

$$-\frac{n}{1+n}\vec{a} + \frac{1}{1+n}\vec{n} = K \cdot \left(-\vec{a} + \frac{1}{n}\vec{n}\right)$$

$$\begin{cases} -\frac{n}{1+n} = -K \rightarrow K = \frac{n}{1+n} \\ \frac{1}{1+n} = \frac{K}{n} \rightarrow \frac{1}{1+n} = \frac{n}{1+n} \rightarrow \frac{1}{1+n} = \frac{1}{1+n} \end{cases}$$

$$\begin{cases} \frac{1}{1+n} = \frac{K}{n} \rightarrow \frac{1}{1+n} = \frac{n}{1+n} \rightarrow \frac{1}{1+n} = \frac{1}{1+n} \\ K = \frac{1}{1+n} \end{cases}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = \frac{1}{1+n}(\vec{a} + \vec{n}) - \vec{a} = \frac{1}{1+n}\vec{a} + \frac{1}{1+n}\vec{n} - \vec{a}$$

$$\vec{AC} = \left(\frac{1}{1+n} - 1\right)\vec{a} + \frac{1}{1+n}\vec{n} = -\frac{n}{1+n}\vec{a} + \frac{1}{1+n}\vec{n}$$

\vec{AC} é múltiplo de \vec{AB} o que confirma que eles são linearmente dependentes, portanto, A, B e C são colineares.

Número 9

$2\vec{v} + \vec{r}$ e $\vec{v} - 2\vec{r}$ devem ser L.I.,
ou seja: $a(2\vec{v} + \vec{r}) + b(\vec{v} - 2\vec{r}) = 0$
 $a = 0$ $b = 0$

$$2a\vec{v} + a\vec{r} + b\vec{v} - 2b\vec{r} = 0$$

$$(2a+b)\vec{v} + (a-2b)\vec{r} = 0$$

$$\begin{cases} 2m+n=x \\ m-2n=y \end{cases} \Rightarrow \begin{cases} n=x-2m \\ m-2(x-2m)=y \end{cases} \Rightarrow m-2x+4m=y \Rightarrow 5m-2x=y \Rightarrow m=\frac{y+2x}{5}$$

*m e n sempre satisfazem o sistema.

Conclusão: $\{\vec{v} + \vec{r}, \vec{v} - 2\vec{r}\}$ é uma base do plano,

$$\begin{cases} 2a+b=0 \\ a-2b=0 \Rightarrow a=2b \end{cases}$$

$$2.(2b)+b=0 \Rightarrow 5b=0 \Rightarrow b=0,$$

$$a=2.0=0,$$

Seja $\vec{w} = x\vec{v} + y\vec{r}$. Os escalares m e n são:

$$m(2\vec{v} + \vec{r}) + n(\vec{v} - 2\vec{r}) = x\vec{v} + y\vec{r}$$

$$(2m+n)\vec{v} + (m-2n)\vec{r} = x\vec{v} + y\vec{r}$$

$$m=x-2\left(\frac{y+2x}{5}\right) = \frac{5x-2y-4x}{5} = \boxed{\frac{x-2y}{5}}$$

$$n=x-2\left(\frac{y+2x}{5}\right) = \frac{5x-2y-4x}{5} = \boxed{\frac{x-2y}{5}}$$

Número 10

a) $\alpha(\vec{v} + \vec{r}) + \beta(\vec{v} - \vec{r} + \vec{w}) + \gamma(\vec{v} + \vec{r} + \vec{w}) = \vec{0} \rightarrow (\alpha + \beta + \gamma)\vec{v} + (\alpha - \beta + \gamma)\vec{r} + (\beta + \gamma)\vec{w} = \vec{0}$

$$\begin{cases} \alpha + \beta + \gamma = 0 \Rightarrow \alpha + \beta + (-\beta) = \alpha = 0 \\ \alpha - \beta + \gamma = 0 \Rightarrow 0 - \beta - \beta \Rightarrow 2\beta = 0 \Rightarrow \beta = 0 = \gamma = 0 \\ \beta + \gamma = 0 \Rightarrow \gamma = -\beta \end{cases}$$

Conclusão: $\vec{v} + \vec{r}, \vec{v} - \vec{r} + \vec{w}, \vec{v} + \vec{r} + \vec{w}$ são L.I.

b) $\vec{v} + \vec{r} = (1+a)\vec{v} + b\vec{r} + c\vec{w}$

$$\vec{v} + \vec{r} = a\vec{v} + (1+b)\vec{r} + c\vec{w}$$

$$\vec{w} + \vec{r} = a\vec{v} + b\vec{r} + (1+c)\vec{w}$$

$$M = \begin{bmatrix} 1+a & a & a \\ b & 1+b & b \\ c & c & 1+c \end{bmatrix} \quad C_2 \leftarrow C_2 - C_1 \quad C_3 \leftarrow C_3 - C_1$$

$$M' = \begin{bmatrix} 1+a & -1 & -1 \\ b & 1 & 0 \\ c & 0 & 1 \end{bmatrix}$$

$$\det(M') = (1+a) \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - (-1) \cdot \begin{vmatrix} b & 0 \\ c & 1 \end{vmatrix} + (-1) \begin{vmatrix} b & 1 \\ c & 0 \end{vmatrix} \Rightarrow \det = (1+a) + b - (-c) = 1 + a + b + c //$$

Se $a + b + c = -1$, então $\det = 0$ o que implica que os vetores são L.D.

Número 11

a) $\vec{AB} = (1, 0, -1) - (1, 3, 2) = (0, -3, -3)$

$$\vec{BC} = (1, 1, 0) - (1, 0, -1) = (0, 1, 1)$$

$$\vec{CA} = (1, 3, 2) - (1, 1, 0) = (0, 2, 2)$$

b) $\vec{AB} + \frac{2}{3} \vec{BC} = (0, -\frac{7}{3}, -\frac{7}{3})$

$$\frac{2}{3} \vec{BC} = (0, \frac{2}{3}, \frac{2}{3})$$

$$\vec{AB} + \frac{2}{3} \vec{BC} = (0, -3, -3) + (0, \frac{2}{3}, \frac{2}{3}) \\ = (0, -\frac{7}{3}, -\frac{7}{3})$$

11-c) $C + \frac{1}{2} \vec{AB} = (1, -\frac{1}{2}, -\frac{3}{2})$

$$\frac{1}{2} \vec{AB} = (0, -\frac{3}{2}, -\frac{3}{2})$$

$$C + \frac{1}{2} \vec{AB} = (1, 1, 0) + (0, -\frac{3}{2}, -\frac{3}{2}) = (1, -\frac{1}{2}, -\frac{3}{2})$$

11-d) $A - 2\vec{BC} = (1, 1, 0)$

$$2\vec{BC} = (0, 2, 2)$$

$$(1, 3, 2) - (0, 2, 2) = (1, 1, 0)$$

Número 12

a) L.I., não há múltiplos entre os vetores.

$$(2, 3) = \lambda \begin{pmatrix} 0 \\ x_2 \end{pmatrix}, \text{ não existe } \lambda \text{ escalar}$$

pois $x_1 = 2$ e $x_2 = 0$.

c) L.I. $(0, 3, 3) \neq \lambda(2, 3, 4)$ para qualquer λ
pois não há como fazer o 1º componente ser 0.

e) L.I., $\det = 1 \neq 0$.

$$\begin{vmatrix} 1 & -1 & -1 & 1 & -1 \\ -1 & 2 & 2 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ -2 & 2 & 2 & 4 & -2 \end{vmatrix} \quad (1+0+(-2))-(2+0+(-1)) \\ \det = 3 - 2 = 1$$

b) L.D, ambos tem componente $y=0$.

São múltiplos entre si.

$$(3, 0) = -\frac{3}{2} (-2, 0)$$

d) L.I., $\det = -2 \neq 0$.

$$\begin{bmatrix} 1 & -1 & 2 & 1 & -1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ 2 & 0 & -1 & 1 & 0 \end{bmatrix} \quad (1+0+(-2))-(2+0+(-1)) \\ -1 - 1 = -2 \\ \det = -2$$

f. L.D, a segunda linha da matriz é 0 o que indica que o det é 0.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 1 & 1 & 5 \end{bmatrix}$$

b) $\vec{z} = 2\vec{a} + \vec{b} - \vec{c}$

$$\vec{z} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} \rightarrow (1, 2, 3) = \alpha(1, 1, 1) + \beta(0, 1, 1) + \gamma(1, 1, 0)$$

$$\vec{z} = (\alpha + \gamma, \alpha + \beta + \gamma, \alpha + \beta)$$

$$\begin{cases} \alpha + \gamma = 1 \Rightarrow \gamma = 1 - \alpha \\ \alpha + \beta + \gamma = 2 \Rightarrow \alpha + \beta + (1 - \alpha) = 2 \Rightarrow \beta + 1 = 2 \Rightarrow \beta = 1 \\ \alpha + \beta = 3 \Rightarrow \alpha = 3 - 1 = 2 \Rightarrow \alpha + \gamma = 1 \Rightarrow \gamma = 1 - 2 = -1 \end{cases}$$

Número 13

a) $\vec{w} = 2\vec{v} - 3\vec{u}$

$$\vec{w} = \alpha\vec{u} + \beta\vec{v} \rightarrow (1, 1) = \alpha(2, -1) + \beta(1, -1)$$

$$\begin{cases} 2\alpha + \beta = 1 \\ -\alpha - \beta = 1 \end{cases} \rightarrow 2\cdot 2 + \beta = 1 \rightarrow \beta = 1 - 4 = -3 \\ \alpha = 2$$

Número 14

a) $m = 2, n = 1$ ou $m = -2, n = 3$

$$\frac{1}{m} = \frac{m-1}{2n} = \frac{m}{4}$$

$$\frac{1}{m} = \frac{m}{4} \rightarrow m^2 = 4 \rightarrow m = \pm 2$$

$$\frac{1}{m} = \frac{m-1}{2n} \rightarrow 2n = m(m-1)$$

$$\text{Se } m = 2; 2n = 2(2-1) \Rightarrow 2n = 2 \Rightarrow n = 1$$

$$\text{Se } m = -2; 2n = -2(-2-1) \Rightarrow 2n = 6 \Rightarrow n = 3$$

b) $m = 0, n = -1$ ou $m = 2, n = 3$

$$\frac{1}{m} = \frac{m}{n+1} = \frac{n+1}{8} \quad \frac{1}{m} = \frac{m}{n+1} \rightarrow (n+1) = m^2 \rightarrow n = m^2 - 1$$

$$\frac{m}{n+1} = \frac{n+1}{8} \rightarrow 8m = (n+1)^2 \rightarrow 8m = (m^2)^2 = m^4 \rightarrow m^4 - 8m = 0$$

$$m(m^3 - 8) = 0 \rightarrow m = 0 \rightarrow m^3 = 8 \rightarrow m = 2$$

$$\text{Se } m = 0: n = m^2 - 1 = 0 - 1 = -1$$

$$\text{Se } m = 2: n = m^2 - 1 \rightarrow n = 4 - 1 = 3$$

Número 15

Para os vetores \vec{u}, \vec{v} e \vec{w} serem L.I., $\det \neq 0$.

$$\det \begin{bmatrix} m & m^2+1 & \bar{m} \\ -1 & m & 1 \\ m^2+1 & 0 & 1 \end{bmatrix} \neq 0$$

$$\det = (m^2+1) \cdot \begin{bmatrix} m & \bar{m} \\ m & 1 \end{bmatrix} - 0 \cdot \begin{bmatrix} m & \bar{m} \\ -1 & 1 \end{bmatrix} + 1 \cdot \begin{bmatrix} m & m^2+1 \\ -1 & m \end{bmatrix}$$

$$\downarrow \quad \downarrow$$

$$(m^2+1)(1-m^2) \quad m^2+m^2+1$$

$$m^2+1-m^2=1 \quad -2m^2+1$$

$$\begin{aligned} \det &= (m^2+1) \cdot (1) + 1 \cdot (2m^2+1) \\ &= m^2+1+2m^2+1 \end{aligned}$$

$$\det = 3m^2+2$$

Como $m^2 \geq 0$ para todo $m \in \mathbb{R}$, temos $3m^2+2 \geq 2 > 0$

Logo $\det \neq 0$ para todo $m \in \mathbb{R}$, portanto $\vec{u}, \vec{v}, \vec{w}$ são L.I.

Número 16

a) Cé uma base de V^3 , uma vez que $\det \neq 0$, mostrando que os vetores são L.I.

$$\det \begin{bmatrix} 1 & 1 & 1 & 1 & -1 \\ 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \rightarrow \det = (0+0+1) - (0+1-1)$$

$$b) \vec{v}_B = (12, 9, 4)$$

$$\vec{v} = 2(1, 1, 0)_B + 3(1, 0, 1)_B + 7(1, 1, -1)_B \rightarrow \vec{v} = (2+3+7, 2+0+7, 0+3-7)_B \\ \vec{v} = (12, 9, -4)$$

$$C) \vec{V_C} = (11, -1, -8)$$

$$\vec{V} = x \vec{f_1} + y \vec{f_2} + z \vec{f_3}$$

$$(2, 3, 7)_B = x(1, 1, 0)_B + y(1, 0, 1)_B + z(1, 1, -1)_B$$

$$\begin{cases} x+y+z = 2 \\ x+z = 3 \rightarrow x = 3-z \\ y-z = 7 \rightarrow y = 7+z \end{cases} \rightarrow (3-z) + (7+z) + z = 2 \quad | \quad 10 + z = 2 \quad | \quad z = 8 \quad | \quad x = 3 - (-8) \quad | \quad y = 7 + (-8)$$

$$\vec{v}_c = (11, -1, -8)$$