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$$1-a) A+2B = \begin{bmatrix} 1 & 0 \\ 2 & 7 \end{bmatrix} + 2 \begin{bmatrix} 0 & 5 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 7 \end{bmatrix} + \begin{bmatrix} 0 & 10 \\ 6 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ 8 & 3 \end{bmatrix}$$

$$b) AB-BA = \begin{bmatrix} 0 & 5 \\ 21 & -4 \end{bmatrix} + \begin{bmatrix} -10 & -35 \\ +1 & +14 \end{bmatrix} = \begin{bmatrix} -10 & -30 \\ 22 & 10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 5 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 \cdot 0 + 0 \cdot 3 & 1 \cdot 5 + 0 \cdot (-2) \\ 2 \cdot 0 + 7 \cdot 3 & 2 \cdot 5 + 7 \cdot (-2) \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ 21 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 5 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 0 \cdot 1 + 5 \cdot 2 & 0 \cdot 0 + 5 \cdot 7 \\ 3 \cdot 1 + (-2) \cdot 2 & 3 \cdot 0 + (-2) \cdot 7 \end{bmatrix} = \begin{bmatrix} 10 & 35 \\ -1 & -14 \end{bmatrix}$$

$$c) 2C - D = \begin{bmatrix} -4 & 6 & -14 \\ 14 & -6 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 0 \\ -1 & -1 & -4 \\ 2 & 0 & -2 \end{bmatrix} \text{ Soma não definida, a ordem das matrizes é diferente.}$$

$$d) 2D^t - 3E^t$$

$$D^t = \begin{bmatrix} -3 & 1 & -2 \\ 2 & 1 & 0 \\ 0 & 4 & 2 \end{bmatrix} \quad 2D^t = \begin{bmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{bmatrix}$$

$$E^t = \begin{bmatrix} 2 & -1 & -6 \\ 4 & 0 & 0 \\ -3 & -4 & 1 \end{bmatrix} \quad 3E^t = \begin{bmatrix} 6 & -3 & -18 \\ 12 & 0 & 0 \\ -9 & -12 & 3 \end{bmatrix}$$

$$2D^t - 3E^t = \begin{bmatrix} -6 & 2 & -4 \\ 4 & 2 & 0 \\ 0 & 8 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 3 & 18 \\ -12 & 0 & 0 \\ 9 & 12 & -3 \end{bmatrix} = \begin{bmatrix} -12 & 5 & 14 \\ -8 & 2 & 0 \\ 9 & 20 & 1 \end{bmatrix}$$

$$e) D^2 + DE =$$

$$D^2 = \begin{bmatrix} 9 & 4 & 0 \\ 1 & 1 & 16 \\ 4 & 0 & 4 \end{bmatrix}$$

$$DE = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 1 & 4 \\ -2 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{bmatrix} = \begin{bmatrix} (-3) \cdot 2 + 2 \cdot (-1) + 0 \cdot (-6) & (-3) \cdot 4 + 2 \cdot 0 + 0 \cdot 0 & (-3) \cdot (-3) + 2 \cdot (-4) + 0 \cdot (-1) \\ 1 \cdot 2 + 1 \cdot (-1) + 4 \cdot (-6) & 1 \cdot 4 + 1 \cdot 0 + 4 \cdot 0 & 1 \cdot (-3) + 1 \cdot (-4) + 4 \cdot (-1) \\ (-2) \cdot 2 + 0 \cdot (-1) + 2 \cdot (-6) & (-2) \cdot 4 + 0 \cdot 0 + 2 \cdot 0 & (-2) \cdot (-3) + 0 \cdot (-4) + 2 \cdot (-1) \end{bmatrix}$$

$$DE = \begin{bmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{bmatrix} \quad D^2 + DE = \begin{bmatrix} 9 & 4 & 0 \\ 1 & 1 & 16 \\ 4 & 0 & 4 \end{bmatrix} + \begin{bmatrix} -8 & -12 & 1 \\ -23 & 4 & -11 \\ -16 & -8 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -8 & 1 \\ -22 & 5 & 5 \\ -12 & -8 & 8 \end{bmatrix}$$

$$f) C^t A = \begin{bmatrix} -2 & 7 \\ 3 & -3 \\ 7 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} (-2) \cdot 1 + 7 \cdot 2 & (-2) \cdot 0 + 7 \cdot 7 \\ 3 \cdot 1 + (-3) \cdot 2 & 3 \cdot 0 + (-3) \cdot 7 \\ 7 \cdot 1 + (-2) \cdot 2 & 7 \cdot 0 + (-2) \cdot 7 \end{bmatrix} = \begin{bmatrix} 12 & 49 \\ -3 & -21 \\ 3 & -14 \end{bmatrix}$$

$$g) E - AC = \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{bmatrix} - \begin{bmatrix} -2 & 3 & -7 \\ 45 & -15 & -28 \end{bmatrix} \quad \text{Subtração não definida, a ordem das matrizes é diferente.}$$

$$AC = \begin{bmatrix} 1 & 0 \\ 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-2) + 0 \cdot 7 & 1 \cdot 3 + 0 \cdot (-3) & 1 \cdot (-7) + 0 \cdot (-2) \\ 2 \cdot (-2) + 7 \cdot 7 & 2 \cdot 3 + 7 \cdot (-3) & 2 \cdot (-7) + 7 \cdot (-2) \end{bmatrix} = \begin{bmatrix} -2 & 3 & -7 \\ 45 & -15 & -28 \end{bmatrix}$$

$$h) F^t E = \begin{bmatrix} 1 & -2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & -3 \\ -1 & 0 & -4 \\ -6 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 2 + (-2) \cdot (-1) + 0 \cdot (-6) & 1 \cdot 4 + (-2) \cdot 0 + 0 \cdot 0 & 1 \cdot (-3) + (-2) \cdot (-4) + 0 \cdot (-1) \end{bmatrix}$$

$$\downarrow$$

$$F^t E = \begin{bmatrix} 4 & 4 & 5 \end{bmatrix}$$

$$F^t = \begin{bmatrix} 1 & -2 & 0 \end{bmatrix}$$

i) BCF

$$BC = \begin{bmatrix} 0 & 5 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} -2 & 3 & -7 \\ 7 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 0 \cdot (-2) + 5 \cdot 7 & 0 \cdot 3 + 5 \cdot (-3) & 0 \cdot (-7) + 5 \cdot (-2) \\ 3 \cdot (-2) + (-2) \cdot 7 & 3 \cdot 3 + (-2) \cdot (-3) & 3 \cdot (-7) + (-2) \cdot (-2) \end{bmatrix}$$

$$BC = \begin{bmatrix} 35 & -15 & -10 \\ -20 & 15 & -17 \end{bmatrix}$$

$$BCF = \begin{bmatrix} 35 & -15 & -10 \\ -20 & 15 & -17 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 35 \cdot 1 + (-15) \cdot (-2) + (-10) \cdot 0 \\ (-20) \cdot 1 + 15 \cdot (-2) + (-17) \cdot 0 \end{bmatrix}$$
$$BCF = \begin{bmatrix} 65 \\ -50 \end{bmatrix}$$

2-a)  $A_{2 \times 3} B_{3 \times 4} = C_{2 \times 4}$

BA não está definido

b)  $A_{4 \times 1} B_{1 \times 2} = C_{4 \times 2}$

BA não está definido

c)  $A_{1 \times 2} B_{3 \times 1}$

O produto AB não está definido.

BA está definido e tem ordem  $3 \times 2$ .

d)  $A_{5 \times 2} B_{2 \times 3} = C_{5 \times 3}$

BA não está definida

e)  $A_{4 \times 4} B_{3 \times 3}$

AB e BA não estão definidos.

f)  $A_{4 \times 2} B_{2 \times 4} = C_{4 \times 4}$

BA está definido.

h)  $A_{2 \times 2} B_{2 \times 2} = C_{2 \times 2}$

BA está definido e tem ordem  $2 \times 2$ .

g)  $A_{2 \times 1} B_{1 \times 3} = C_{2 \times 3}$

BA não está definido



3-a)  $A = (a_{ij})_{2 \times 3}$   $\begin{bmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \end{bmatrix}$

$a_{ij} = 3i - 2j$

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$a_{11} = 1$

$a_{13} = 3 \cdot 1 - 2 \cdot 3 = -3$

$a_{22} = 3 \cdot 2 - 2 \cdot 2 = 2$

$a_{12} = -1$

$a_{21} = 3 \cdot 2 - 2 \cdot 1 = 4$

$a_{23} = 3 \cdot 2 - 2 \cdot 3 = 0$

b)  $B = (b_{ij})_{3 \times 3}$   $\begin{bmatrix} 4 & -1 & -2 \\ 3 & 8 & 1 \\ 8 & 7 & 12 \end{bmatrix}$

$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$b_{11} = 4$

$b_{12} = -1$

$b_{13} = 1^2 - 3 = -2$

$b_{21} = 2^2 - 1 = 3$

$b_{22} = 3 \cdot 2 + 2 = 8$

$b_{23} = 1$

$b_{31} = 8$

$b_{32} = 7$

$b_{33} = 12$

c)  $C = (c_{ij})_{1 \times 4}$   $[1 \ 2 \ 3 \ 4]$

$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \end{bmatrix}$

$c_{11} = 1^1 = 1$

$c_{12} = 2^1 = 2$

$c_{13} = 3^1 = 3$

$c_{14} = 4^1 = 4$

d)  $D = (d_{ij})_{4 \times 4}$   $\begin{bmatrix} 2 & 4 & 6 & 8 \\ 4 & 8 & 12 & 16 \\ 6 & 12 & 18 & 24 \\ 8 & 16 & 24 & 32 \end{bmatrix}$

$i^2 + j^2, \text{ se } i = j$

$2ij, \text{ se } i \neq j$

$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$

$a_{11} = 1^2 + 1^2 = 2$

$a_{12} = 2 \cdot 1 \cdot 2 = 4$

$a_{13} = 2 \cdot 1 \cdot 3 = 6$

$a_{14} = 2 \cdot 1 \cdot 4 = 8$

$a_{21} = 2 \cdot 2 \cdot 1 = 4$

$a_{22} = 2^2 + 2^2 = 8$

$a_{23} = 2 \cdot 2 \cdot 3 = 12$

$a_{24} = 2 \cdot 2 \cdot 4 = 16$

$a_{31} = 2 \cdot 3 \cdot 1 = 6$

$a_{32} = 2 \cdot 3 \cdot 2 = 12$

$a_{33} = 3^2 + 3^2 = 18$

$a_{34} = 2 \cdot 3 \cdot 4 = 24$

$a_{41} = 2 \cdot 4 \cdot 1 = 8$

$a_{42} = 2 \cdot 2 \cdot 4 = 16$

$a_{43} = 2 \cdot 4 \cdot 3 = 24$

$a_{44} = 4^2 + 4^2 = 32$

4-a)  $[BA]_{23} = 20$

$[BA]_{23} = (2 \cdot 1) + (-1 \cdot 2) + (4 \cdot 5) = 2 - 2 + 20 = 20$

b)  $[AB]_{23} = -32$

$[AB]_{23} = (-2 \cdot 3) + (-3 \cdot 4) + (2 \cdot -7) = -6 - 12 - 14 = -32$

$$c) [B^2]_{31} = 16$$

$$[B^2]_{31} = (-3 \cdot 1) + (-1 \cdot 2) + (-7 \cdot -3) = -3 - 2 + 21 = 16$$

$$d) \text{tr}(A) = a_{11} + a_{22} + a_{33} = 3$$

$$\text{tr}(A) = 1 + (-3) + 5 = 3$$

$$e) \text{tr}(B^T) = [B^T]_{11} + [B^T]_{22} + [B^T]_{33} = -7$$

$$\text{tr}(B^T) = 1 + (-1) + (-7) = -7$$

$$f) \text{tr}(A-B) = [A-B]_{11} + [A-B]_{22} + [A-B]_{33} = 10$$

$$A-B = \begin{bmatrix} 1-1 & 2-0 & 1-3 \\ -2-2 & -3-(-1) & 2-4 \\ 1-(-3) & 4-(-1) & 5-(-7) \end{bmatrix} = \begin{bmatrix} 0 & 2 & -2 \\ -4 & -2 & -2 \\ 4 & 5 & 12 \end{bmatrix}$$

$$\text{tr}(A-B) = [A-B]_{11} + [A-B]_{22} + [A-B]_{33}$$

$$\text{tr}(A-B) = 0 + (-2) + 12 = 10$$

$$\text{tr}(A-B) = 10$$

$$g) \text{tr}(AB) = [AB]_{11} + [AB]_{22} + [AB]_{33} = -13$$

$$[AB]_{11} = (1 \cdot 1) + (2 \cdot 2) + (1 \cdot -3) = 1 + 4 - 3 = 2$$

$$\text{tr}(AB) = 2 + 1 + (-16) = -13$$

$$[AB]_{22} = (-2 \cdot 0) + (3 \cdot -1) + (2 \cdot -1) = 0 - 3 - 2 = -5$$

$$[AB]_{33} = (1 \cdot 3) + (4 \cdot 4) + (5 \cdot -7) = 3 + 16 - 35 = -16$$

$$5-a) 2X + A = 3B + C$$

$$X = \begin{bmatrix} \frac{7}{2} & -1 \\ \frac{11}{2} & \frac{3}{2} \end{bmatrix}$$

$$3B = 3 \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \rightarrow 3B = \begin{bmatrix} 6 & 3 \\ 12 & 9 \end{bmatrix}$$

$$3B + C = \begin{bmatrix} 6 & 3 \\ 12 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 13 & 9 \end{bmatrix}$$

$$2X = 3B + C - A \quad 2X = \begin{bmatrix} 6 & 5 \\ 13 & 9 \end{bmatrix} - \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix} \rightarrow 2X = \begin{bmatrix} 7 & -2 \\ 11 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{7}{2} & -\frac{2}{2} \\ \frac{11}{2} & \frac{3}{2} \end{bmatrix} \quad X = \begin{bmatrix} \frac{7}{2} & -1 \\ \frac{11}{2} & \frac{3}{2} \end{bmatrix}$$

$$b) Y + A = \frac{1}{2} (B - C)^T \rightarrow Y = \begin{bmatrix} 2 & -1\frac{1}{2} \\ -5\frac{1}{2} & -9\frac{1}{2} \end{bmatrix}$$

$$B - C = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 3 \end{bmatrix} \quad (B - C)^T = \begin{bmatrix} 2 & 3 \\ -1 & 3 \end{bmatrix}$$

$$\frac{1}{2} \cdot (B - C)^T = \begin{bmatrix} 1 & \frac{3}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} \quad Y = \begin{bmatrix} 1 & \frac{3}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix} - \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 2 & -1\frac{1}{2} \\ -5\frac{1}{2} & -9\frac{1}{2} \end{bmatrix}$$

$$c) 3X + A = B - X \rightarrow X = \begin{bmatrix} \frac{3}{4} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$$

$$3X + X = B - A$$

$$4X = B - A$$

$$B - A = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 7 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 2 & -3 \end{bmatrix} \quad X = \frac{1}{4} \cdot \begin{bmatrix} 3 & -6 \\ 2 & -3 \end{bmatrix} \quad X = \begin{bmatrix} \frac{3}{4} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$$

$$d) \begin{cases} X + Y = 3A \\ X - Y = 2B + C \end{cases} \rightarrow X = \begin{bmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{bmatrix}; Y = \begin{bmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{bmatrix}$$

$$3A = \begin{bmatrix} -3 & 21 \\ 6 & 18 \end{bmatrix} \quad 2B = \begin{bmatrix} 4 & 2 \\ 8 & 6 \end{bmatrix} \quad 2B + C = \begin{bmatrix} 4 & 2 \\ 8 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 9 & 6 \end{bmatrix}$$

$$\begin{cases} X + Y = 3A \\ X - Y = 2B + C \end{cases} \quad 2X = 3A + (2B + C) \rightarrow 2X = \begin{bmatrix} -3 & 21 \\ 6 & 18 \end{bmatrix} + \begin{bmatrix} 4 & 4 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 25 \\ 15 & 24 \end{bmatrix}$$

$$2X = 3A + (2B + C) \quad X = \begin{bmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{bmatrix}$$

$$Y = 3A - X \rightarrow Y = \begin{bmatrix} -3 & 21 \\ 6 & 18 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{25}{2} \\ \frac{15}{2} & 12 \end{bmatrix} \rightarrow Y = \begin{bmatrix} -\frac{7}{2} & \frac{17}{2} \\ -\frac{3}{2} & 6 \end{bmatrix}$$



$$6) A^2 = \begin{bmatrix} 1 & \frac{1}{x} \\ x & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & \frac{1}{x} \\ x & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + \frac{1}{x} \cdot x & 1 \cdot \frac{1}{x} + \frac{1}{x} \cdot 1 \\ x \cdot 1 + 1 \cdot x & x \cdot \frac{1}{x} + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{bmatrix} \therefore A^2 = 2A \rightarrow \begin{bmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{bmatrix} = \begin{bmatrix} 2 & \frac{2}{x} \\ 2x & 2 \end{bmatrix}$$

$$A^2 = 2A \therefore A^3 = A^2 \cdot A = (2A) \cdot A = 2(A \cdot A) = 2A^2$$

$$A^3 = 2 \cdot (2A) = 4A$$

$$A^4 = A^3 \cdot A = (4A) \cdot A = 4A^2$$

$$A^4 = 4(2A) = 8A$$

$$A^n = 2^{n-1}A, \text{ para } n \geq 1$$

$$7-a) A(B+C) = X+Y$$

$$A(B+C) = AB + AC = X+Y$$

$$b) (B^T A^T) = X^T$$

$$(BA)^T = A^T B^T \rightarrow B^T A^T = (AB)^T \rightarrow (AB)^T = (X)^T$$

$$c) C^T A^T = Y^T$$

$$(CA)^T = A^T C^T \Rightarrow C^T A^T = (AC)^T = Y^T$$

$$d) (ABA)C = XY$$

$$(ABA)C = (XA)C \rightarrow X(AC) \rightarrow XY$$

$$8-a) x=5 \text{ e } A = \begin{bmatrix} 4 & 7 \\ 7 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 4 & 2x-3 \\ x+2 & x+1 \end{bmatrix}$$

$$\lambda + 2 = 2x + 3$$

$$5 = 2x - \lambda$$

$$x = 5$$

$$A = \begin{bmatrix} 4 & 5+2 \\ 2 \cdot 5 - 3 & 5+1 \end{bmatrix} \rightarrow A = \begin{bmatrix} 4 & 7 \\ 7 & 6 \end{bmatrix}$$

$$b) x=4, y=-2 \text{ e } z=-\frac{1}{3}$$

$$B = \begin{bmatrix} 0 & -4 & 2 \\ x & 0 & 1-z \\ y & 2z & 0 \end{bmatrix} \quad B^T = -B \rightarrow \begin{bmatrix} 0 & x & y \\ -4 & 0 & 2z \\ 2 & 1-z & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -2 \\ -x & 0 & -(1-z) \\ -y & -2z & 0 \end{bmatrix}$$

$$-x = -4 \rightarrow x=4$$

$$2z = -(1-z)$$

$$-y = 2 \rightarrow y = -2$$

$$2z + z = -1$$

$$z = -\frac{1}{3}$$

$$z = -\frac{1}{3}$$

$$9) x=2; y=4; z=1 \text{ e } t=3.$$

$$3 \begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+t & 2t+3 \end{bmatrix}$$

$$3x = x+4$$

$$3y = 6+x+y$$

$$3z = -1+z+t$$

$$3t = 2t+3$$

$$2x = 4$$

$$3y - y = 6+2$$

$$2z + 1 = t$$

$$t = 3$$

$$x = 2$$

$$2y = 8$$

$$2z + 1 = 3$$

$$y = 4$$

$$2z = 2 \rightarrow z = 1$$

$$10 - a) R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad R(\theta)R^T(\theta) = I_2$$

$$R^T(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$R(\theta)R^T(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\begin{bmatrix} \cos \theta \cdot \cos \theta + \sin \theta \cdot \sin \theta & \cos \theta \cdot (-\sin \theta) + \sin \theta \cdot \cos \theta \\ (-\sin \theta) \cdot \cos \theta + \cos \theta \cdot \sin \theta & (-\sin \theta) \cdot (-\sin \theta) + \cos \theta \cdot \cos \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R(\theta)R^T(\theta) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2, \text{ portanto, } R(\theta) \text{ é uma matriz ortogonal.}$$



$$b) A = \begin{bmatrix} 1 & 0 & x \\ 0 & \frac{1}{\sqrt{2}} & y \\ 0 & \frac{1}{\sqrt{2}} & z \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ x & y & z \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 \cdot 1 + 0 \cdot 0 + x \cdot x & 1 \cdot 0 + 0 \cdot \frac{1}{\sqrt{2}} + x \cdot y & 1 \cdot 0 + 0 \cdot \frac{1}{\sqrt{2}} + x \cdot z \\ 0 \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 + y \cdot x & 0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + y \cdot y & 0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + y \cdot z \\ 0 \cdot 1 + \frac{1}{\sqrt{2}} \cdot 0 + z \cdot x & 0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + z \cdot y & 0 \cdot 0 + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + z \cdot z \end{bmatrix}$$

$$AA^T = \begin{bmatrix} x^2 + 1 & xy & xz \\ xy & \frac{1}{2} + y^2 & \frac{1}{2} + yz \\ xz & \frac{1}{2} + yz & \frac{1}{2} + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{lllll} 1 + x^2 = 1 & xy = 0 & \frac{1}{2} + y^2 = 1 & \frac{1}{2} + yz = 0 & \frac{1}{2} + z^2 = 1 \\ x^2 = 0 & xz = 0 & y^2 = \frac{1}{2} & yz = -\frac{1}{2} & z^2 = \frac{1}{2} \\ x = 0 & & y = \pm \frac{1}{\sqrt{2}} & & z = \pm \frac{1}{\sqrt{2}} \end{array}$$

Os possíveis valores são:  $x=0$ ;  $y = \frac{1}{\sqrt{2}}$ ,  $z = -\frac{1}{\sqrt{2}}$  ou  $y = -\frac{1}{\sqrt{2}}$ ,  $z = \frac{1}{\sqrt{2}}$ .