# HM(X)

Gabriela Moreira

gabrielamoreira05@gmail.com

Departamento de Ciência da Computação
Centro de Ciências e Tecnológicas
Universidade do Estado de Santa Catarina

4 de Dezembro de 2017

# Bibliografia

## *The Essence of ML Type Inference*


François Pottier e Didier Rémy

`bit.ly/tipos-HMX`

Capítulo 10 do livro *Advanced Topics in Types and Programming Languages* - Coletânea editada por Benjamin C. Pierce

$$
\begin{array}{llll}
\text{Tipo} & \sigma ::= & \forall \bar{X}[C].T & \\
 & & & \\
\text{Restrição} & C, D ::= & true & \text{(Verdade)} \\
 & & \mid false & \text{(Falsidade)} \\
 & & \mid P T_1 \ldots T_n & \text{(Aplicação de Predicado)} \\
 & & \mid C \wedge C & \text{(Conjunção)} \\
 & & \mid \exists \bar{X}.C & \text{(Quantificação Existencial)} \\
 & & \mid def \; x : \sigma \; in \; C & \text{(Introdução de Tipo)} \\
 & & \mid x \preceq T & \text{(Instanciação de Tipo)}
\end{array}
$$

# Açúcar Sintático

Seja $\sigma$ um tipo da forma $\forall\bar{X}[D].T$ onde $X \notin \mathit{ftv}(T')$.

Escrevemos $\sigma \preceq T'$ (leia $T'$ é uma instância de $\sigma$) para $\exists\bar{X}.(D \wedge T \leqslant T')$.

Escrevemos $\exists\sigma$ (leia $\sigma$ tem uma instância) para $\exists\bar{X}.D$.

Escrevemos $\mathit{let}\ x : \sigma\ \mathit{in}\ C$ para $\exists\sigma \wedge \mathit{def}\ \ x : \sigma\ \mathit{in}\ C$.

Os tipos do Sistema Damas-Milner (DM) podem ser vistos como subtipos de tipos restritivos, já que para qualquer tipo $\forall \bar{X}.T$ do sistema Damas-Milner, existe um tipo restritivo $\forall \bar{X}[true].T$.

A definição de instanciação também é mais geral do que em DM. O tipo $Y \rightarrow Z$ não é uma instância de $\forall X.X \rightarrow X$ em DM, porque $Y$ e $Z$ são variáveis de tipo diferentes.

Em HM(X), $\forall X.X \rightarrow X \preceq Y \rightarrow Z$ é uma restrição.

$$\exists X.(true \wedge X \rightarrow X \leqslant Y \rightarrow Z)$$
$$\exists X.(Y \leqslant X \wedge X \leqslant Z)$$
$$Y \leqslant Z$$

Um contexto restritivo $\mathbb{C}$ pode ser definido por:

$\mathbb{C}$ ::= $[\,]$ | C | $\mathbb{C} \wedge \mathbb{C}$ | $\exists \bar{X} \mathbb{C}$ | def x : $\sigma$ in $\mathbb{C}$ | def x : $\forall \bar{X}[\mathbb{C}].T$ in C

# Interpretação de restrições

$$\phi, \psi \vDash \textit{true} (\textit{CM-TRUE})$$

$$\frac{P(\phi(T_1), \ldots, \phi(T_n))}{\phi, \psi \vDash P T_1 \ldots T_n} \; (\textit{CM}-\textit{PREDICATE})$$

$$\frac{\phi, \psi \vDash C_1 \quad \phi, \psi \vDash C_2}{\phi, \psi \vDash C_1 \wedge C_2} \; (\textit{CM}-\textit{AND})$$

$$\frac{\phi[\vec{X} \mapsto \vec{t}], \psi \vDash C}{\phi, \psi \vDash \exists \bar{X}.C} \; (\textit{CM}-\textit{EXISTS})$$

$$\frac{\phi, \psi[x \mapsto (\phi, \psi)\sigma] \vDash C}{\phi, \psi \vDash \textit{def} \;\; x : \sigma \;\; \textit{in} \;\; C} \; (\textit{CM}-\textit{DEF})$$

$$\frac{\phi(T) \in \psi(x)}{\phi, \psi \vDash x \preceq T} \; (\textit{CM}-\textit{INSTANCE})$$

$C_1 \Vdash C_2$ se e somente se, para todo $\phi$ e $\psi$, $\phi, \psi \vDash C_1$ implica em $\phi, \psi \vDash C_2$

$C_1 \equiv C_2$ se e somente se $C_1 \Vdash C_2$ e $C_2 \Vdash C_1$

| | | | | |
|---|---|---|---|---|
| $C_1 \wedge C_2$ | $\equiv$ | $C_2 \wedge C_1$ | | (C-And) |
| $(C_1 \wedge C_2) \wedge C_3$ | $\equiv$ | $C_1 \wedge (C_2 \wedge C_3)$ | | (C-AndAnd) |
| $C_1 \wedge C_2$ | $\equiv$ | $C_1$ | if $C_1 \Vdash C_2$ | (C-Dup) |
| $\exists \bar{\mathsf{X}}.\exists \bar{\mathsf{Y}}.C$ | $\equiv$ | $\exists \bar{\mathsf{X}}\bar{\mathsf{Y}}.C$ | | (C-ExEx) |
| $\exists \bar{\mathsf{X}}.C$ | $\equiv$ | $C$ | if $\bar{\mathsf{X}} \# ftv(C)$ | (C-Ex*) |
| $(\exists \bar{\mathsf{X}}.C_1) \wedge C_2$ | $\equiv$ | $\exists \bar{\mathsf{X}}.(C_1 \wedge C_2)$ | if $\bar{\mathsf{X}} \# ftv(C_2)$ | (C-ExAnd) |
| $\exists \mathsf{Z}.(\sigma \leq \mathsf{Z} \wedge \mathsf{Z} \leq \mathsf{T})$ | $\equiv$ | $\sigma \leq \mathsf{T}$ | if $\mathsf{Z} \notin ftv(\sigma, \mathsf{T})$ | (C-ExTrans) |
| $\mathsf{let}\ \mathsf{x} : \sigma\ \mathsf{in}\ \mathbb{C}[\mathsf{x} \leq \mathsf{T}]$ | $\equiv$ | $\mathsf{let}\ \mathsf{x} : \sigma\ \mathsf{in}\ \mathbb{C}[\sigma \leq \mathsf{T}]$ if $\mathsf{x} \notin dpi(\mathbb{C})$ and $dtv(\mathbb{C}) \# ftv(\sigma)$ and $\{\mathsf{x}\} \cup dpi(\mathbb{C}) \# fpi(\sigma)$ | | (C-InId) |
| $\mathsf{let}\ \Gamma\ \mathsf{in}\ C$ | $\equiv$ | $\exists \Gamma \wedge C$ | if $dpi(\Gamma) \# fpi(C)$ | (C-In*) |
| $\mathsf{let}\ \Gamma\ \mathsf{in}\ (C_1 \wedge C_2)$ | $\equiv$ | $(\mathsf{let}\ \Gamma\ \mathsf{in}\ C_1) \wedge (\mathsf{let}\ \Gamma\ \mathsf{in}\ C_2)$ | | (C-InAnd) |
| $\mathsf{let}\ \Gamma\ \mathsf{in}\ (C_1 \wedge C_2)$ | $\equiv$ | $(\mathsf{let}\ \Gamma\ \mathsf{in}\ C_1) \wedge C_2$ | if $dpi(\Gamma) \# fpi(C_2)$ | (C-InAnd*) |
| $\mathsf{let}\ \Gamma\ \mathsf{in}\ \exists \bar{\mathsf{X}}.C$ | $\equiv$ | $\exists \bar{\mathsf{X}}.\mathsf{let}\ \Gamma\ \mathsf{in}\ C$ | if $\bar{\mathsf{X}} \# ftv(\Gamma)$ | (C-InEx) |
| $\mathsf{let}\ \Gamma_1; \Gamma_2\ \mathsf{in}\ C$ | $\equiv$ | $\mathsf{let}\ \Gamma_2; \Gamma_1\ \mathsf{in}\ C$ if $dpi(\Gamma_1) \# dpi(\Gamma_2)$ and $dpi(\Gamma_2) \# fpi(\Gamma_1)$ and $dpi(\Gamma_1) \# fpi(\Gamma_2)$ | | (C-LetLet) |
| $\mathsf{let}\ \mathsf{x} : \forall \bar{\mathsf{X}}[C_1 \wedge C_2].\mathsf{T}\ \mathsf{in}\ C_3$ | $\equiv$ | $C_1 \wedge \mathsf{let}\ \mathsf{x} : \forall \bar{\mathsf{X}}[C_2].\mathsf{T}\ \mathsf{in}\ C_3$ | if $\bar{\mathsf{X}} \# ftv(C_1)$ | (C-LetAnd) |
| $\mathsf{let}\ \Gamma; \mathsf{x} : \forall \bar{\mathsf{X}}[C_1].\mathsf{T}\ \mathsf{in}\ C_2$ | $\equiv$ | $\mathsf{let}\ \Gamma; \mathsf{x} : \forall \bar{\mathsf{X}}[\mathsf{let}\ \Gamma\ \mathsf{in}\ C_1].\mathsf{T}\ \mathsf{in}\ C_2$ if $\bar{\mathsf{X}} \# ftv(\Gamma)$ and $dpi(\Gamma) \# fpi(\Gamma)$ | | (C-LetDup) |
| $\mathsf{let}\ \mathsf{x} : \forall \bar{\mathsf{X}}[\exists \bar{\mathsf{Y}}.C_1].\mathsf{T}\ \mathsf{in}\ C_2$ | $\equiv$ | $\mathsf{let}\ \mathsf{x} : \forall \bar{\mathsf{X}}\bar{\mathsf{Y}}[C_1].\mathsf{T}\ \mathsf{in}\ C_2$ | if $\bar{\mathsf{Y}} \# ftv(\mathsf{T})$ | (C-LetEx) |
| $\mathsf{let}\ \mathsf{x} : \forall \bar{\mathsf{X}}\bar{\mathsf{Y}}[C_1].\mathsf{T}\ \mathsf{in}\ C_2$ | $\equiv$ | $\exists \bar{\mathsf{Y}}.\mathsf{let}\ \mathsf{x} : \forall \bar{\mathsf{X}}[C_1].\mathsf{T}\ \mathsf{in}\ C_2$ if $\bar{\mathsf{Y}} \# ftv(C_2)$ and $\exists \bar{\mathsf{X}}.C_1$ determines $\bar{\mathsf{Y}}$ | | (C-LetAll) |
| $\exists \mathsf{X}.(\mathsf{T} \leq \mathsf{X} \wedge \mathsf{let}\ \mathsf{x} : \mathsf{X}\ \mathsf{in}\ C)$ | $\equiv$ | $\mathsf{let}\ \mathsf{x} : \mathsf{T}\ \mathsf{in}\ C$ | if $\mathsf{X} \notin ftv(\mathsf{T}, C)$ | (C-LetSub) |
| $\vec{\mathsf{X}} = \vec{\mathsf{T}} \wedge [\vec{\mathsf{X}} \mapsto \vec{\mathsf{T}}]C$ | $\equiv$ | $\vec{\mathsf{X}} = \vec{\mathsf{T}} \wedge C$ | | (C-Eq) |
| $\mathsf{true}$ | $\equiv$ | $\exists \bar{\mathsf{X}}.(\vec{\mathsf{X}} = \vec{\mathsf{T}})$ | if $\bar{\mathsf{X}} \# ftv(\vec{\mathsf{T}})$ | (C-Name) |
| $[\vec{\mathsf{X}} \mapsto \vec{\mathsf{T}}]C$ | $\equiv$ | $\exists \bar{\mathsf{X}}.(\vec{\mathsf{X}} = \vec{\mathsf{T}} \wedge C)$ | if $\bar{\mathsf{X}} \# ftv(\vec{\mathsf{T}})$ | (C-NameEq) |

**Figure 10-6: Constraint equivalence laws**

Fim =)