Manifold Sculpting: an overview
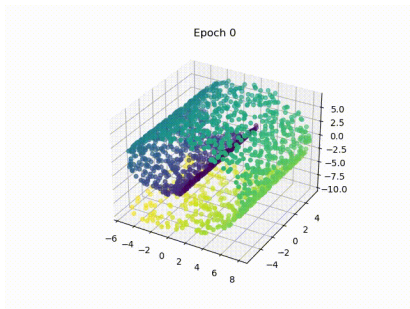
Gabriele Codega

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Idea

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Rearrange points in such a way to **preserve local relationships**
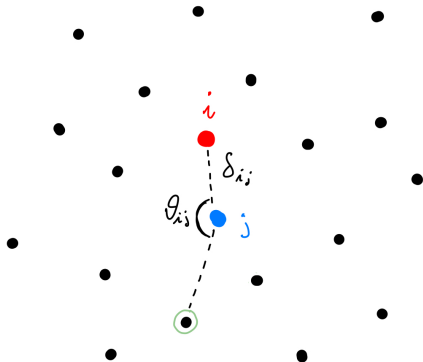
Algorithm

Algorithm

1. Find k-Nearest Neighbours
2. Compute relationships
3. Preprocess with PCA
4. Until convergence
   4a. Scale down dimensions to discard
   4b. Scale up and shift dimensions to keep
5. Embed by discarding unwanted dimensions

Relationships

- Distance δ_{ij}
- Angle to most colinear neighbour θ_{ij}

Optimisation

Adjust points in order to minimise

$$e_i = \sum_j w_{ij} \left(\left(\frac{\delta_{ij} - \delta_{ij0}}{\delta_{ave}} \right)^2 + \left(\frac{\theta_{ij} - \theta_{ij0}}{\pi} \right)^2 \right)$$

Parameters

Parameters

- Number of neighbours k
- Scale factor σ
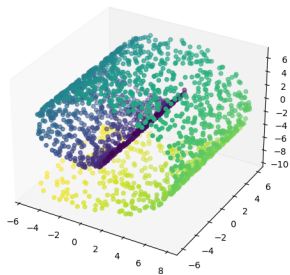- Learning rate
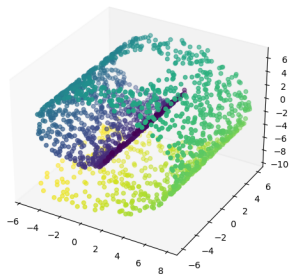- Stopping criterion
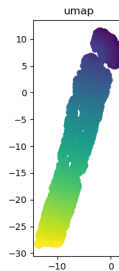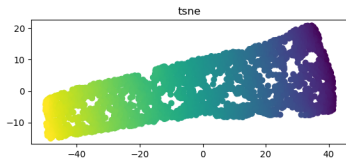
Results
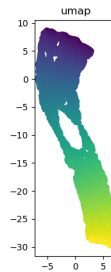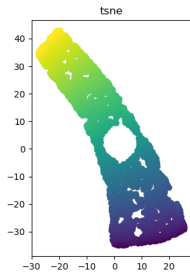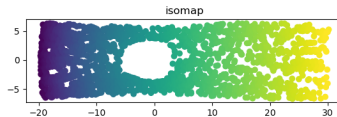
Results
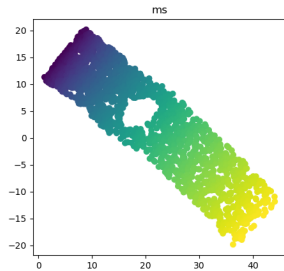

Figure 1: Swiss roll

Figure 2: Swiss hole

Comparison: roll
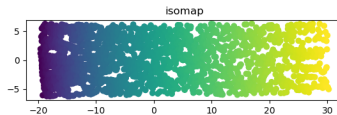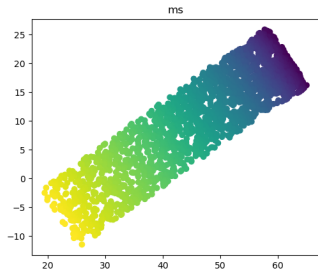
Comparison: hole

Comments

Comments

Pros
- ▶ Accurate
- ▶ Can improve other embeddings

Cons
- ▶ Hard to tune
- ▶ Possibly very slow
- ▶ Possible convergence to local minima