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René Aïd

Electricity Derivatives

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A mon père, M. Mahand Aïd
et à ma mère, Mme Ferroudja Mohellebi.

Foreword I

The electricity market is currently entering a period of significant changes with the development of intermittent renewable energies and demand-response mechanisms.

Already quite complex to manage because electricity cannot be stored at reasonable cost and because electricity follows all available paths (according to the Kirchhoff's principles), the laws of the market and the pricing system are entering a new era of development.

The questions dealt with are quite complex on a mathematical standpoint with high level optimization problems. Do not be afraid: several mathematical formulae appear in the text.

Nevertheless, these questions are practicable and really usable by traders and the utilities market. It is not an academic book, it is a book for the industry.

René Aïd succeeds to build that bridge between two worlds.

This book is the result of a brilliant collaboration between the academic world (Ecole Polytechnique, ENSAE, Dauphine) through the FIME lab, and industrial world through EDF R&D teams.

It is the result of more than two years of research and the overall understanding of the evolution of the electricity market.

It is also a landmark of the ambition of the EDF R&D organization, to bring the best available academic knowledge into the industry.

Thanks and congratulations to René and his team for this performance.

Paris-Saclay, October 2014

Bernard Salha
Senior Vice President
Head of EDF R&D

Foreword II

This monograph is an excellent introduction to the world of electricity markets. The content is unique within the available literature by the wide spectrum which covers the subject starting from the managerial aspects of electricity generation, and arriving at the corresponding financial derivatives.

A special emphasis is put on the various specific aspects of the electricity financial market as opposed to the stock market, thus justifying the need to develop related relevant models for the derivatives of hedging and pricing, and the corresponding numerical approximation.

Through his unique positioning as one of the best international experts in the electricity market R&D, and a researcher strongly connected to the academic community, René succeeds in delivering the essential messages from electricity market practitioners. The present valuable presentation of the field will undoubtedly attract more economists and applied mathematicians, and help them to identify interesting academic questions with relevant application to the practical electricity market. The content will also serve in the opposite direction as a reference for the relevant models that have been developed in the academic literature, and are currently used by electricity markets R&D practitioners.

Paris-Saclay, October 2014

Nizar Touzi
Professor, Ecole Polytechnique

Preface

The project that led to this book started in August 2011 when Matheus Grasselli proposed the writing of a monograph on the quantitative financial aspects of energy markets in a new collection launched by Springer: Springer Briefs. We quickly defined the scope of the book and the table of contents. But, this process would certainly have taken much longer without the opportunity given to me by Fred Espen Benth. Fred invited me to give a short series of lectures at the University of Oslo in September 2013 on electricity markets and derivatives. This commitment compelled me to create a large part of the material included in this book.

To fit the requirements of the SpringerBrief series, I chose the field of electricity derivatives. Electricity markets and prices have drawn the attention of academics from many different fields: economy, regulations, statistics, finance and mathematical finance. I skipped all of the regulatory aspects which nevertheless involved first-order economic questions as well as interesting mathematical modelling problems. I also overlooked the questions of price forecasting because exhaustive monographs on this subject already exist.

The book ranges from models which allow the tractable computation of futures prices to the valuation of storage and swing options, which are the most complex options to be evaluated in this market. My purpose is to give the reader a strong foundation in this field. Thus, I first provide an explanation of the main properties of electricity as a commodity and the main characteristics of the electricity market's microstructure. With these concepts, the reader is able to go through the whole zoology of stochastic models that propose to capture the dynamic of the electricity spot and futures prices. Then, I focus on the most important derivatives: spread options, tolling contracts, power plants, and storage and swing options. I also provide the reader with a description of the problems involved with the pricing of retail contracts and weather derivatives.

This book is intended on the one hand for applied mathematicians, statisticians and economists looking for a new interesting field of research. And on the other hand, for practitioners working in energy utilities or on the commodity desks of financial institutions.

I want to take this opportunity to thank the different institutions and persons that made this book possible. The first is the EDF group. As an employee of EDF, I was given the time and the resources to write this book. My successive managers trusted me, and without this trust I would not have had the chance to finish this book. Thus, I want to personally thank my manager, Marc Ringeisen, head of the EDF's Lab Osiris department. I also want to send special thanks to Bernard Salha, Head of EDF R&D, who agreed to write a foreword for this book.

Several academic institutions also contributed greatly to the writing of this book: the University Paris-Dauphine, the Ecole Polytechnique, and the CREST (Centre for Economic and Statistic Research of the ENSAE). Together with EDF R&D, they created the Finance for Energy Market Research Centre (the FiME Lab), which I had the honour to manage from its birth in 2006 to 2012. I want to particularly thank those three institutions for providing me with document resources and the University of Paris-Dauphine for providing me with an office (with a priceless view of the Boulogne wood).

I also want to thank the people who created the FiME Lab in 2006: Nizar Touzi, professor at the Ecole Polytechnique; Elyès Jouini, Vice-President of the University Paris-Dauphine; Jean-Michel Lasry, former Scientific Advisor of Credit Agricole CIB and professor at the University Paris-Dauphine; Pierre-Louis Lions, professor at the Collège de France; and Patrick Pruvot, former head and founder of the EDF's R&D Osiris department. These five people had a significant impact on my life, particularly Nizar who had a special role. He and I have been collaborating since 2004, and these past ten years literally changed my life. I want to thank him not only for writing a foreword to this book, but for all I learned from him during these ten years.

I also want to thank my colleagues at EDF R&D and the FiME members who helped me substantively improve the manuscript of this book: Nadia Oudjane; Xavier Warin; Olivier Féron; Clémence Alasseur; Audrey Mahuet, director of EpexSpot Product Design, who helped me understand some aspects of intra-day trading and provided me with data; and Matt Davison who as a reviewer gave me simple and concrete advice on how to improve the book. Further, I want to thank Nicolas Langrené, my former Ph.D. student, and Corentin Guttierrez and Elias Daboussi for providing some nice pictures in this book. I also want to thank Jonathan Moore for transforming this text written in French–English into a book written in English. I also thank Ute McCrory, my editor at Springer, who helped me in the design of this book.

Paris-Saclay, October 2014 René Aïd

Contents

Chapter 1
Introduction

More than 30 years have passed since the first country (Chile 1981) deregulated its electricity market and quoted an electricity spot price. Nearly 20 years have passed since the first quotation of a futures contract on electricity (Scandinavian market 1993). Since then, the quantitative finance literature has followed the worldwide liberalisation of the electricity industry. Amongst the many drivers that can explain the interest in this field, three of them are worth noting in a monograph dedicated to electricity derivatives.

First, electricity fails to satisfy the basic hypothesis that sustains modern pricing theory. Electricity cannot be stored. This fact leads to the extremely spiky behaviour of spot prices. This phenomenon is now well known and well documented. More recently, electricity spot prices in Europe have frequently exhibited negative values due to the joint effects of the operational constraints of power plants and the fact that electricity cannot be stored. Second, these effects together with the lack of flexibility in the generation assets lead to complex microstructures in the electricity markets. They are designed to offer the best possible trading capacities to market operators while maintaining the security of the electric system. Third, if not specifically, electricity trading involves the pricing of path-dependent options. They involve not a unique but a sequence of exercises. They are designed to reflect the constraints in the generation assets. The pricing of theses derivatives leads to optimal control problems. They rarely admit an analytical solution. Numerical methods to efficiently provide a value and its sensitivities are needed.

The purpose of this monograph is twofold. It aims at providing a comprehensive state of the art investigation into the pricing and hedging problems raised by electricity derivatives. And it proposes methods to tackle them. Monographs already exist that provide detailed descriptions of each aspect of the electricity derivatives treated in this book. Most of them provide a global view on the microstructures in the energy markets, the main spot and futures price models, some descriptions of standard derivatives, and some methods to price them. For the reader who is interested in the field of energy derivatives, I recommend the lectures by Clewlow and Strickland [65], Eydeland and Wolyniec [85], Geman [93], and Burger et al. [47]. They provide a far more detailed description of all of the energy derivatives and provide an introduction

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to the main spot and futures price models. Pilipovic's monograph [143] is more focused on pricing and hedging in the context of a variety of spot price models based on the variations in Schwartz's models of 1997. In this context, the monograph provides detailed computations of prices and Greeks. For models on spikes, the reader should consult the reference monograph by Benth et al. [19]. This book covers the problems in electricity price modelling (estimation, calibration) and the pricing of standard derivatives (spreads) in the context of general Lévy processes. Just recently, the collected works edited by Carmona et al. [55] address the question of efficient numerical methods for specific electricity and gas derivatives (storage and swings). And even more recently, Swindle's [155] detailed monograph on the valuation of energy derivatives offers a practitioners view on all of the aspects of energy derivatives as well as the methods used by trading desks to price them.

The purpose of this brief book is not to offer an in-depth analysis like those works. Its objective is to propose a broad view of the features of electricity markets through the price models and derivatives which are specific to the electricity business, such as power plants, tolling, swings, and storage. This text could be useful to readers who are involved in electricity trading or the optimisation of generation assets. The reader will find here a concise landscape of the academic knowledge on the pricing and hedging of electricity derivatives. It could also be useful to academics who are looking for new interesting fields to investigate. The book describes the main valuation problems faced by operators of electric utilities. Crude mathematical formulations are provided to help the reader understand the mathematical difficulties involved with these methods. Moreover, I hope this book will encourage new researchers to devote part of their time to this fascinating field. Indeed, despite two decades of research and significant progress in the modelling of electricity spot prices, no model has emerged as a reference—as is the case for equity derivatives with the Black and Scholes model and the Heath-Jarrow-Morton (HJM) models for interest rates—and a lot is left to be done to reduce the gap between the operational needs for efficient and robust pricing models and the available solutions in the literature.

This book is structured in three chapters which deal successively with markets, price models, and derivatives.

Chapter 2 provides an introduction to electricity markets. This chapter explains the main features of the electricity commodity: that it cannot be stored and that it is a local commodity. I provide the reader with a crash course on electrical engineering and focus on its consequences on the market's microstructure. I do not address the reactive power or frequency regulation, but I describe the consequences of Kirchhoff's laws on trading. By using the example of the French electricity market, I explain how the three main markets of the intraday, the day-ahead, and the forward market are interrelated. Moreover, I explain the main problems faced by the utilities and the derivatives developed to respond to these needs. In particular, there is a class which can be called real derivatives by analogy with real options, which consists of physical generation assets. Indeed, power plants and hydroelectric or gas storage are assets with embedded options. I leave aside the issues surrounding regulations, even though the European electricity markets are (still) under construction as the development of capacity markets shows it. Moreover, the trading activities of electric utilities have been affected by European financial regulation after the 2008 financial crisis, such as EMIR (European Market Infrastructure Regulation) and REMIT (Regulation on Energy Market Integrity and Transparency). But, I do not deal with this topic because it is a subject all on its own.

Chapter 3 covers the models for electricity prices. Because this monograph is dedicated to derivative pricing and because futures are the basic tools used to hedge more complex derivatives, it limits itself to models on the joint dynamic of spot and futures prices. Thus, the forecast models for the spot prices are not covered here. I refer the reader to Weron's (2006) book [165] on this subject. The models presented here are sorted out in four sections. First, I deal in Sect. 3.2 with the HJM approach. This approach consists of directly modelling the dynamic of the futures prices. The spot price is then deduced as a futures price with zero time to maturity. No arbitrage conditions are encoded in the dynamic of the futures price written directly under the risk-neutral measure. This is the preferred approach of trading desks in order to stay away from the spot. The constraints to ensure no arbitrage conditions between the futures prices and spot prices, which are already known in the case of the yield curve, still apply for electricity. But, those constraints are considerably increased in the case of electricity because a futures contract on electricity involves delivery during a period that makes it closer to a swap contract. Although this approach is preferred by market operators, there is less literature on it than on the strategy that consists of modelling the spot price and deducing the futures prices through an argument of no-arbitrage (the Royal road). The literature which uses this strategy is important. Therefore, I sort the models into three categories of increasing complexity. The first class that is presented in Sect. 3.3 consists of a one-dimensional model. This class offers the simplest models and allows assessing the difficulty involved in modelling the dynamic of the spot and futures prices. This class includes a Gaussian mean-reversion model, a jump mean-reversion model, and a one-factor Lévy process, which is the most popular example of a non-Gaussian process. This class of models succeeds in capturing the spiky behaviour of electricity spot prices, but generally fails in getting all the richness of the dynamic of the forward curve. The second class that is presented in Sect. 3.4 consists of multi-factor models. The typical models in this class are the hidden Markov chain models. This class of models gives up the Markovian property of the spot price to be able to capture a realistic dynamic of the futures prices. They succeed in capturing both the behaviour of the spot and that of the futures prices but requires filtering procedures for the estimation of the parameters, because they involve the dynamic of non-observed variables. The third class that is presented in Sect. 3.5 is the structural models. The principle of these models is to deduce the spot price—and not necessarily its dynamic—as the result of an over-simplified relation between generation and consumption. The main advantages of these models is to propose a direct and consistent answer to the dependence of the electricity spot price on some other observed variable, which is often of some interest for utilities (available capacities, fuel prices, consumption). Moreover, the estimation and calibration processes are simplified by the fact that the models are set on observable variables. But, this is done at the expense of an increase in the dimension that penalises the efficiency in the pricing of complex derivatives.

The last Chap. 4 is devoted to derivatives. I concentrate on the products which are, if not specific, at least of a particular importance to electricity trading. Section 4.1 covers the spread options. These options are not unique to electricity markets, but as Chap. 2 will make clear, electricity utilities are not interested in the spread options for themselves but because they are the basic tool to evaluate many other derivatives such as power plants. These derivatives are seen as fuel spread options or, with storage facilities, as calendar spread options. Then, Sect. 4.2 deals with the valuation of power plants and their financial representation, which are tolling contracts. Taking into account their main constraints leads to the path-dependent options and optimal switching problems. This section presents an example extracted from the literature on the different ways the problem can be formulated. In particular, the example takes explicitly into account the fact that the market is incomplete and the hedge is not perfect. Section 4.3 deals with one of the most complex derivatives involved in the electricity industry, namely storage facilities. This chapter also covers swing contracts. They can be seen as a financial contract modelling a physical storage facility such as a dam or a gas storage facility. In the case of a physical storage facility, random inflows, injection or withdrawal costs from the facility make the optimal control problem more difficult to solve. Section 4.4 is devoted to the contracts which do not attract much attention from academics but which are essential to the utilities, namely retail contracts. Depending on the nature of the customer (household, small business, or industrial sites), the retail contracts can have very different settings and very different embedded options. Finally, Sect. 4.5 presents weather derivatives. Although they do not represent an important part of the derivatives traded in electricity markets, they enjoy a large degree of attention from the professionals that sell them. Indeed, the activity of electric utilities is climate sensitive: consumption depends on weather, hydroelectric generation depends on precipitation and snow falls, and wind generation depends on the wind (!). Thus, it is natural to think of developing hedging instruments against these risk factors.

The Chap. 5 concludes this monograph with some research perspectives.

Chapter 2
Electricity Markets

This chapter presents the main properties of electricity, the microstructures of the electricity market and introduces the derivatives which are specific to this market. Regarding electricity's properties, I focus only on those that have a direct consequence on pricing and trading. The fact that electricity cannot be stored cannot be understated. In the same line, the constraints on its transport make electricity a local commodity. There is no such thing as an international homogeneous electricity market or even a regional electricity market on the scales of Europe or the United States as is the case for gas markets. There are at least as many electricity markets as there are countries in the world. It is thus not possible to enter into the details of each country's specific electricity market. Here, I deal only with the most common features of their structure. Further, I introduce some of the most specific derivatives of electricity markets by presenting the context, objective, and constraints of the electricity utilities whether they are large or small, or whether they hold generation assets or are purely retailers.

2.1 Electricity Features

All commodities exhibit technical features that make them unique. For example, trading live cattle on the Chicago Board of Trade (CBOT) might be as technical as trading electricity. But, electricity presents two main characteristics that raise both theoretical and practical problems:

1. electricity cannot be stored,
2. the transport of electricity satisfies specific laws.

2.1.1 Storage

The fact that electricity cannot be stored is sometimes tempered by using the expression that it cannot be stored at reasonable cost. In fact, because electricity

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consumption is a continuous phenomenon, they exist both an energy problem and a capacity problem, that is, the pace at which energy can be released.

Regarding energy, the most economical way to store a large amount of energy for electricity generation is still hydroelectric reservoirs. Nevertheless, this is not a universal solution because it relies on the hydroelectric potential of a country. For instance, in a country like France where the hydroelectric potential was developed over the 1950s to the 1970s, it now represents 25 GW of installed capacity of a total installed capacity of 110 GW. Hydroelectric generation represents 15 % of the total generation, and its energy storage capacity is around 10 TWh as compared to an annual consumption of 500 TWh. Moreover, its availability depends on inflows coming from precipitation. Contrary to a thermal power plant whose fuel can be bought, a hydroelectric plant might be unavailable because its reservoir is empty—and rain cannot be bought (even now). This point can be illustrated by the Norwegian electricity system. In this country, more than 95 % of the generation is based on hydroelectric. The total consumption is around 130 TWh, and the hydroelectric generation is about 124 TWh. But, due to the dependence on hydrology, the energy capacity can vary from 160 TWh during wet years to 100 TWh during dry years which in this last case leads to a lack of energy. Further, the cost of building a hydroelectric power plant HIGH varies a lot according to the place where it is built. Nevertheless, the investment cost varies from 1.5 to 2.7 billion euros per GW (source: Report on the development of hydroelectric generation, French Ministry of Industry, 2006). To make a comparison, the investment cost for a combined cycle gas turbine is around 450 million euros per GW.

Regarding capacity, there has to be enough capacity to continuously satisfy the power demand. However, the energy required to satisfy the power demand over a certain period of time might exist, but not at the right capacity. To give an order of magnitude, the total installed capacity of France is around 110 GW while the annual maximum power demand has reached 100 GW in the last several decades. In the case of Norway, the installed capacity is around 30 GW with 29 GW of it hydroelectric. But, the annual maximum power demand has reached 30 GW in the last several decades too. To overcome the capacity problem, installing enough capacity to satisfy demand in any situation might be a solution. But, this method requires too many power plants that would be used for only a few hours in their lifetime. The cost of this system would be prohibitively expensive.

The fact that power cannot be stored has drastic effects on the way electricity systems have to be managed. A too long excess of demand compared to generation might first be resolved by a decrease in frequency and if not properly corrected, in dramatic blackouts. Thus, this risk has two major implications in terms of generation management. First, it implies a minute by minute real-time assessment of the equilibrium between consumption and generation because 15 min of disequilibrium can result in a major blackout. Second, the transport system operator (TSO) who is responsible for the electricity system's security and reliability needs to have at its disposal *operating reserves* to be able to cope with the uncertainties which affect generation and consumption.

Operating reserves are generation capacities that can be mobilised within a given notification time. They are sorted according to their response time. Because frequency is immediately affected by any discrepancy between consumption and generation, a first reserve consists in the capacities that can be automatically increased or decreased without any human intervention. This reserve is composed by the primary and the secondary reserves. They can be mobilized within less than 15 min. Beyond 15 min, the reserve is manually mobilised through a direct order by the system operator to the generation plant's management. This is the tertiary reserve. It consists in two parts. First, the rapid tertiary reserve provides the generation capacities that can be mobilised in 15 min for a guaranteed period of at least one hour. It offers a complement to the secondary reserve. Second, the additional tertiary reserve provides capacities that can be mobilised in 15–30 min for a guaranteed generation of 6 h. A deferred reserve is made of power that can be brought in line in more than half an hour. The volume of each reserve can vary depending on the nature of the uncertainties on a particular electricity system. To give orders of magnitude, the following minimum values hold in France where consumption on a given hour varies from 50 GW in off-peak hours of the summer to 100 GW in the peak hours of winter:

primary reserve $\approx$700 MW
secondary reserve $\approx$500 MW
tertiary reserve $\approx$1.500 MW.

Thus, at any given time t, the system operator can compute his or her *operating margin* for the maturity $t + h$ that represents the difference between the demand forecast at time $t + h$ and the sum of the secondary, tertiary, and deferred reserves. The margin required for a maturity of 15 min is set by the event of losing the group with the highest generation capacity. For a longer maturity, it is set by the probability of using exceptional measures (e.g., load shedding). For instance, in France, this risk is defined to be lower than a 1 % chance during peak hours. This risk level requires a margin of 2.3 GW to be available for the next two hours.

The reader with a greater interest in the fields of power system reliability can consult the Power System Reliability Memento [150] of the French system operator, which is available on the RTE website.

2.1.2 Transport

Electricity is transported according to Kirchhoff's laws. Basically, these laws state that the intensity at each node should be zero and the tension in each loop should also be zero. An important consequence of these laws is that, in a meshed electricity network, power goes from one point to another through *all* available paths. Figure 2.1 presents a three-node electricity network where each line is supposed to have the same technical properties and the same lengths. Line A–C can handle 180 MW while line A–B and B–C can handle 90 MW. These limitations hold in both directions. Suppose that a power operator G1 has a customer in node C whose consumption is 180 MW,

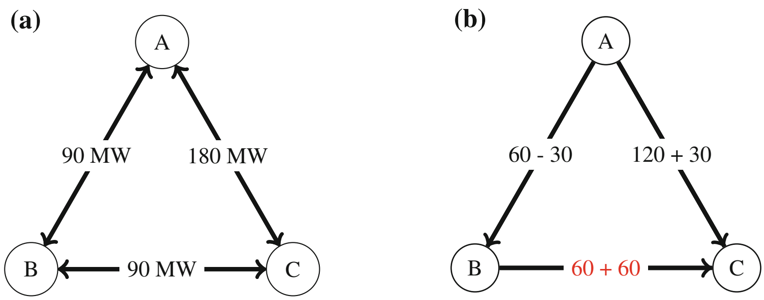

Fig. 2.1 The effect of Kirchhoff's law on electricity exchange. **a** Constraints on capacities, **b** actual flows

while a power operator G2 also has a customer in node C whose consumption is 90 MW. Suppose each operator holds enough generation capacity and that there is no generation cost advantage. If the power flows in lines A–C and B–C like trucks, there is no conflict. Operator G1 generates 180 MW and transmits it through line A–C while operator G2 generates 90 MW and transmits it transit through line B–C. But, because electricity flows through all of the available paths between points, 2/3 of the 180 MW transmitted by G1 will spread from A to C and 1/3 from A to B, and then from B to C. The same holds for the 90 MW transmitted by operator G2. These transmissions then lead to the violation of the transit capacity of line B–C by carrying 120 MW while the line is limited to 90 MW.

It is the main reason why the transfer capacities which are available for trading between countries require some electricity generation hypothesis before they can be computed. In Europe, the available net transfer capacities (NTC) are managed and published by the ENTSOE (European Network System Operator for Electricity) and are made publicly available on its website (www.entsoe.net). To handle these constraints, TSOs and the electricity markets use different methods. In the first years of the European electricity markets, continental Europe chose an explicit auction mechanism. Those who wanted to sell power from one country and have it delivered to another country had to buy the transfer capacity between the countries at an auction that was organised by the TSOs. This method has the benefit of being simple. But, it induces inefficiencies and often adverse flows (flows going from a high price country to a low price country) because of the coordination problems between the market timing of both the power and the transmission capacities.

At the same time, since its early beginning in the 1990s, the NordPool has implemented an implicit auction mechanism between its member countries. In this mechanism, the buyers and sellers post their bids without worrying about transport constraints. Their bids are assigned to their areas. Then, the market operator performs a clearing which takes into account the possibilities of exchange between the zones. While more complex than the former, this method has the benefit of avoiding adverse flows and simplifying the business of traders. This method was chosen by the different actors of the electricity market in continental Europe under the name of *market coupling*. From 2006 to now, the coupling of markets has been achieved between (in alphabetic order) Belgium, Denmark, Estonia, Finland, France, Germany and Austria, Great Britain, Latvia, Lithuania, Luxembourg, the Netherlands, Norway, Poland, Portugal, Spain, and Sweden.

For an introduction on the auction mechanism for electricity transport, I refer the reader to Stoft's book on power system economics [154, part 5].

Remark 2.1 The topic of this book is mostly about what happens at the transport level of electricity because this is the level at which the wholesale markets operate. Nevertheless, the change that occurs at the distribution level of electricity because of the introduction of renewable energies cannot be ignored. The distribution network was historically designed to transfer power from the transport level to the consumers. Electricity would flow downward from the transport level to the distribution level. With the connection of renewable sources such as solar panels and wind farms at the distribution level, now power sometimes flows upward from the distribution network to the transport network.

This phenomenon requires an adaptation of the way distribution networks are monitored and operated. The concept of *smart grids* applies to this adaptation. Smart Grids correspond to the development of the information technology infrastructure which allows the control of the distribution network to adapt to the massive introduction of intermittent sources of energy. In this regard, the smart counter is one key element of the infrastructure. It is the ability to have a fine measure of household consumption but also the capacity to communicate with the customer to send him or her price signals. But it is not the only key element. Flexibilities in consumption (the capacity to defer consumption in time even for a few hours) as well as short-term storage capacities which are able to cope with important variations in wind generation become an issue. I will come back to this point in Sect. 2.3.

2.2 Market Microstructure

The design of electricity wholesale markets differs from one country to the next. However, the properties of electricity have led to common features in the markets' microstructures. I focus here only on those common factors. In each country, the electricity market is composed of a series of markets with different time scales and different scopes. Generally, three different gross markets can be distinguished:

1. the intraday market and/or the balancing mechanism: the balancing mechanism consists of exchanges between the TSO and the market players to ensure the real-time assessment of the grid while the intraday market allows the exchanges between the market players themselves to ensure the equilibrium between their generation and the consumption of their customers.
2. the day-ahead market: quantities are exchanged one day before delivery for the next 24 h or 48 half-hours of the next day.
3. the forward market: market players can buy or sell electricity for future delivery.

To illustrate these three markets, I use France and Germany as examples.

2.2.1 Intraday Market and Balancing Mechanism

In a very short time frame (say the next 12 hours), the actors in the electricity market are most concerned with the precise equilibrium between generation and consumption of their portfolio. Each actor is made financially accountable for all of the discrepancies between the consumption of its customers and its generation. The TSO is also most concerned about a precise equilibrium between the overall generation and consumption in the system. For this reason, two systems coexist. The first one is designed by the TSO and is called the balancing mechanism which adjusts the generation and consumption. The other one is a market where the actors can exchange generation to reduce their imbalances.

The first purpose of the balancing mechanism is to ensure the security of the system. In particular, it aims at maintaining constant the frequency and the voltage to all points in the networks. To achieve this purpose, France requires some obligations from all of the market players who own generation assets which can contribute to the reserves. For instance, coal-fired plants, nuclear power plants, and hydroelectric power plants can contribute while wind farms cannot because of the way they are presently connected to the grid. Market players have the obligation to offer all of their available generation to the TSO so that he or she is able to increase or decrease generation if needed. These bids can involve a non-negligible level of complexity because they tend to reproduce precisely the dynamical constraints of the groups. At this time frame and for the purpose followed by the TSO, it is not possible to neglect the notification time, ramp constraints, and so on. But, basically, the power plant owners submit a quantity together with a price and a maturity, and the system operator selects them according to their economic efficiency.

A second objective of this mechanism is to provide a transparent market price for the cost of the imbalances in the system. If the TSO has to adjust generation, then it is because at least one actor is not generating as much power as its customers are consuming or is generating too much power. Thus, this actor has to incur the costs of the adjustments made by the TSO. It is done at a price referred to as the *imbalanced price*.

To give a concrete example of this mechanism, Table 2.1 shows the way the imbalance price is settled by the RTE, the French TSO, for a given hour h of the day. In this table, S represents the day-ahead price settled the day before for the hour h, P^d is the weighted average price of the offers used by the TSO to decrease the generation (or increase the consumption), P^u is the average price of the offers used by the TSO to increase the generation (or decrease the consumption). The table reads in the following way.

When the network needs to be adjusted upward (lack of generation, first column) and the actor is itself generating too much (first row), then the actor is paid at the spot price established the day before for each MWh. In the opposite, if the actor is itself not producing enough (second row), it pays each MWh of its imbalance at a price that is always higher than the spot price and the average cost incurred by the TSO to compensate for its generation deficit.

Table 2.1 French TSO imbalance price settlement mechanism as of April 2013

	Network adjustment trend positive	Network adjustment trend negative
Actor imbalance positive: actor is paid	S	$\min\left(S, \frac{P^d}{1+k}\right)$
Actor imbalance negative: actor pays	$\max\left(S, P^u \times (1+k)\right)$	S

When the network needs to be adjusted downward (excess of generation, second column), and the actor is producing too much (first row), the actor is paid, but at a price that is less than the average cost incurred by the TSO to decrease the generation (P^d), and less than the day-ahead spot price.

Although apparently complex, those rules are basic. It is natural to pay those who produce energy even if the system has too much of it and to make pay those who do not produce enough even though the system has too much electricity. Moreover, the system is designed to avoid any arbitrage opportunity between the spot market and the balancing mechanism. If the payment for having a negative imbalance in the case where the system is lacking power (first column, second row) is not floored by the spot price, then the temptation exists to sell power on the spot market at price S and to not deliver it. This action leads to a negative imbalance position that would be penalised in some cases at a lower price, leading to a profit.

Moreover, the factor k ensures that the TSO does not have a negative balance sheet at the end of the year because the TSO always pays those who contribute to correct the imbalance less than what the TSO pays to correct them.

Besides this mechanism where the TSO is the only buyer or seller, a market exists where players can adjust their own positions between their generation and the consumption of their customers. In this market, players are constantly exchanging power to adjust their own perimeter over the next few hours. There is growing interest by market players in this market because of the increase in short-term uncertainties. This increase in uncertainty is mainly due to renewable energies. For instance, in France, these exchanges can be done over-the-counter on the EPEX platform. Electricity can be bought or sold as much as 32 hours in advance before delivery. The exchanges performed there are not marginal. For certain hours, the exchanged volumes can be as large as 5 GW. Moreover, this market is not limited to France. It includes Germany, Austria, and Switzerland. This market shares some features with forward markets. The actors can take a commitment for delivery on a future day but reverse it in the next minute. Figure 2.2 shows the index price of the intraday market on EpexSpot (weighted average of the intraday transactions) as a function of the day-ahead price in 2012. The figure shows that there was a spike on the day-ahead market while nothing happened on the intraday, and, more interesting maybe, there was no spike on the day-ahead market but one occurred on the intraday. Price caps might differ on the day-ahead and on the intraday markets. For instance, on EPEX, the prices on the

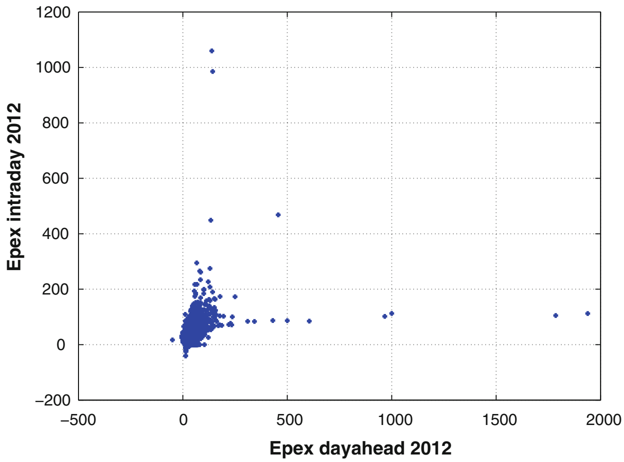

Fig. 2.2 Relation between the average intraday price for delivery at a given hour and the day-ahead spot price for the same hour in 2012. *Source* EpexSpot

day-ahead market range from −500 to +3.000 €/MWh while on the intraday market they move between −10.000 and +10.000 €/MWh.

Figure 2.3 shows an example of the evolution of the transaction price for the delivery of a given hour on the German intraday market in December 2010. The price moves from 70 €/MWh (close to the reference provided by the spot) to 120 €/MWh before going back to 80.

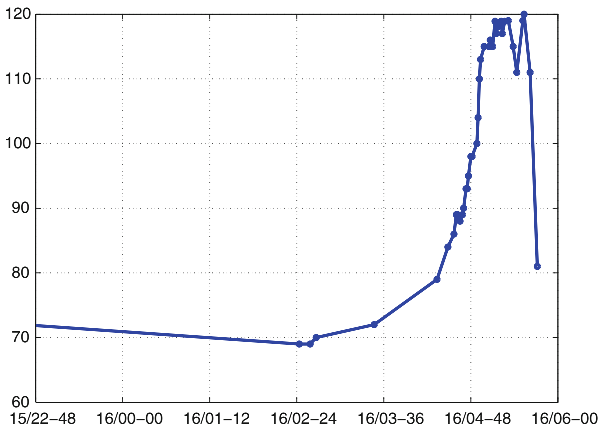

Fig. 2.3 Intraday transaction price for delivery at 7 a.m. on 16 December, 2010, on the German intraday market. X-axis reads: day/hour-minutes. *Source* EpexSpot

2.2.2 Day-Ahead Market

The day-ahead market is based on a fixed trading auction. Each day the market participants submit bids before a certain time (around 10:00 a.m.). They can bid (sales or purchases) for a particular hour of the next day or for a set of hours (order block). The bids of the market participants for a particular hour form two curves as Fig. 2.4 shows. One represents the sales curve and one the purchases. Then, around 12:00 p.m., the market organiser clears the market: the organiser fixes a price for each hour of delivery and determines the sellers and the buyers. The market players then have enough time to send the generation orders to their power plants and send their schedule to the TSO.

Moreover, Fig. 2.4 illustrates the fact that it is possible to submit negative prices for buying and selling. A negative sale price indicates that the seller is ready to pay to sell, and a negative purchase price indicates that the buyer is ready to be paid to buy. This phenomenon is the consequence of the lack of flexibility in some thermal power plants. It can be less expensive to let a coal-fired plant run during hours of the day when the spot price is below its fuel cost than shutting it down and starting it up again later. On the demand side, some customers also have the flexibility to increase their consumption if they are compensated for the cost of changing the schedule of their production process. The figure shows that Sunday the 16th of June, 2010, had particularly low demand during the night, the price settled at −154 €/MWh, but some actors were ready to pay as much as 500 €/MWh with a volume of 4 GWh.

Moreover, because the market participants are allowed to submit block orders, the clearing process results in a non-convex optimisation problem for which defining a market price requires caution. For details, see the documentation regarding the market coupling algorithm named Cosmos available on EpexSpot's website [78].

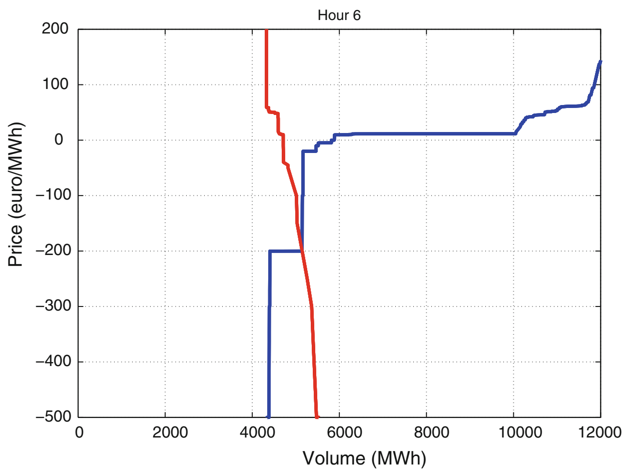

Fig. 2.4 Day-ahead market volume of sales and purchases on 16 June, 2013. *Source* EpexSpot

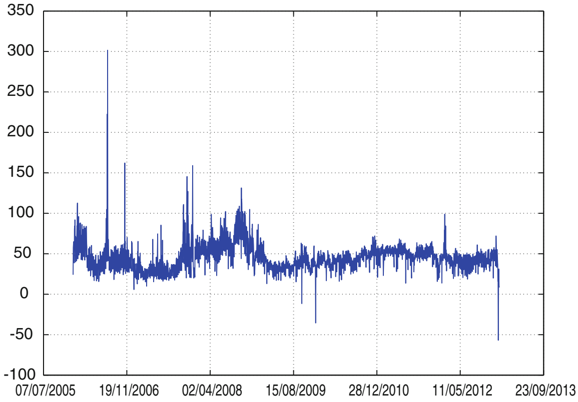

Fig. 2.5 German daily day-ahead electricity price from 2006 to June 2012. *Source* EpexSpot

A direct consequence of the fact that electricity cannot be stored is the occurrence of extreme *spikes* in the day-ahead market. This phenomenon is illustrated in Fig. 2.5 with the daily average of the German spot price during the years 2006 to 2012. The figure shows that the daily average price reaches values as high as 300 €/MWh, but it reaches values as high as 2,400 €/MWh during one hour. The figure also shows occurrences of negative price spikes and their disappearance since 2009 with the development of solar energy.

As Fig. 2.6 shows, electricity day-ahead prices exhibit all of the seasonal patterns of the economic activity of a country: daily, weekly, and annual seasonality such as the moment people wake up to go to work, the moment they get back home and switch on their home appliances, the moment they go to sleep, and the moment when offices and some factories close for the weekend. Electricity day-ahead prices also show fat tails and long memory. They also exhibit correlation with the temperatures in countries with electric heating or air conditioning.

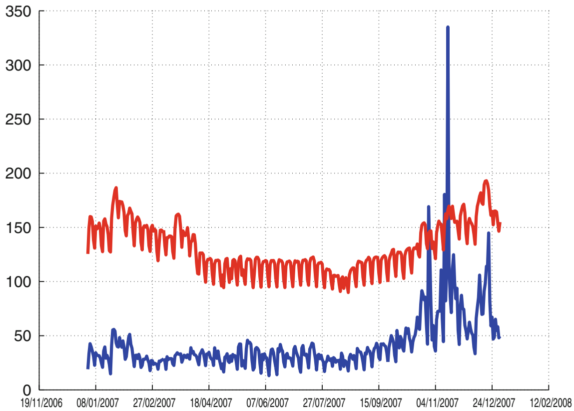

Fig. 2.6 French daily electricity consumption in relation to the daily spot prices during the year 2007. Consumption is scaled to fit the picture. *Source* EpexSpot and RTE

In the European Union, each country has its own electricity day-ahead market cleared by its own market operator. Without coordination, the resulting quoted prices might provide the wrong signals when compared to the transit flow between countries. Indeed, an inquiry performed in October 2004 by the French Ministry of Finance on electricity prices showed that there was barely no relation between the French-German price spread and the transit on the French-German interconnection [147, Sect. I.4.1.1].

Because the quoted day-ahead prices by the market operator have a transparency function, a mechanism has been developed to ensure a consistent relation between cross-border transactions and local day-ahead prices. This mechanism consists in a *market coupling* process which implements a *decentralised implicit auction* mechanism. It is the same mechanism as in the NordPool market. In each country, the market participants do not have to care about finding a counterparty in neighbouring countries. The participant just has to submit its bid in its country (sell or buy). Then, the market organisers perform a clearing process *with* transport constraints implied by the available transfer capacity. If there are no binding transit capacity constraints, then there is a single price for the clearing area. If there is at least one binding transit capacity constraint, then two prices emerge. The debate in the literature on the congestion management mechanisms for cross-border trading is important. For an introduction to this debate, one can begin with Ehrenmann and Neuhoff's [82].

Due to the importance of the relation between the spot and futures prices in the pricing theory, it is important to know what the spot price for power is. Sometimes, the intraday market is referred to as the spot market. Because the intraday price is the shortest time to maturity, it might appear as the *real* spot market, in a sense where the spot refers to a price for instantaneous delivery. But, the reference price for delivery in the futures market is the day-ahead market price. For this reason, I refer to the day-ahead market as the electricity spot market.

2.2.3 Forward Market

Electricity forward markets share many aspects with those of storable commodities such as oil, coal, or metals. In all of the countries where an electricity market exists, the organised markets have developed and proposed standardised contracts for future delivery as well as a margin call mechanism to reduce the counterparty's risk. As in any other commodity contract, the standardised contracts for future delivery specify the currency, the underlying volume, the location of delivery, the trading period, and the tick size. Because electricity is the same in all of the parts of the network grid, the question of quality is not relevant. Physical delivery is often preferred. However, a financial settlement is getting more frequent as markets become more mature. Moreover, because they are doing it for other commodities or financial products, organised markets report the prices and quantities of the OTC contracts.

The most important difference in the electricity forward contracts from the storable commodities comes from their term structure. Because electricity cannot be stored and because the spot market is a day-ahead market with an hourly time-step, a power operator who wants to hedge its generation for the next year needs to have at its disposal the forward contracts for all of the hours in the year. For a non-leap year, the quantity corresponds to 8,760 hourly forward contracts. Such a forward structure would result in a huge dispersion of liquidity because the market participants would spread their needs to all of those contracts. For this reason, electricity forward contracts aggregate hours during a *delivery period*. The delivery period specifies all of the hours during which the electricity should be delivered. The choice is not left to the seller to pick the time for delivery during the delivery period as, for instance, is the case for crude oil futures contracts on the New York Mercantile Exchange (Nymex). The delivery period refers to a month, a quarter, a year, or even a week or a day. The contract also is precise on the schedule of the delivery: the base-load if the electricity is to be delivered during all of the hours of the period and the peak hours or off-peak hours if the electricity is to be delivered only during a special period which reflects large or low demand.

This aggregation mechanism results in a sparse structure of the electricity forward curve. The further the maturity, the longer the delivery period. For instance, the European Energy Market (EEX) offers forward contracts with the following term structure for delivery in the German market: 6 Calendars, 11 Quarters, 9 Months, 4 Weeks, 2 Weekends and 8 Days.

These contracts can come in three different delivery forms: base-load (every hour of the delivery period), peak-load (7:00–20:00 Monday to Friday), and off-peak (base-load minus peak-load). The calendars, quarters, and months come in those three forms while weeks, weekends, and days come only in base and peak forms. So, each day, 106 contracts are available, which represents only a very small fraction of the 525,584 h of the next six years. Nevertheless, despite this concentration of trades on a small number of contracts, liquidity remains an issue in the electricity forward market. It is not uncommon to report no more than 20 MW available per day for two-month-ahead contracts—to be compared with the generation capacity of a standard coal power plant of 400 MW.

Figure 2.7 gives the evolution of the German month-ahead and year-ahead base-load contracts from 2006 to mid-2012. The behaviour of their prices is similar to standard financial products. They do not exhibit strong seasonal patterns with spikes. The jumps in the month-ahead prices come from the seasonality effect of changing contracts. Further, the month-ahead contracts present a much higher volatility than the year-ahead ones. The figure also shows that the effect of the boom on commodities during the years 2005 to 2008 and the financial crisis of August 2008 are reflected in the year-ahead price. It rose from 50 €/MWh to 100 in 2 years and went back to 45 in less than 6 months.

Figure 2.8 shows the different possible shapes of the German forward curve. The figure shows the relative position of the daily spot price in normal backwardation (left: the spot is slightly above the futures price) as well as in contango (right: the spot is clearly lower than the futures prices). If the month-ahead and year-ahead contracts tend to be aligned to form a nice configuration, then the spot price does not fit the plan and tends to exist independently of the remaining forward curve. The

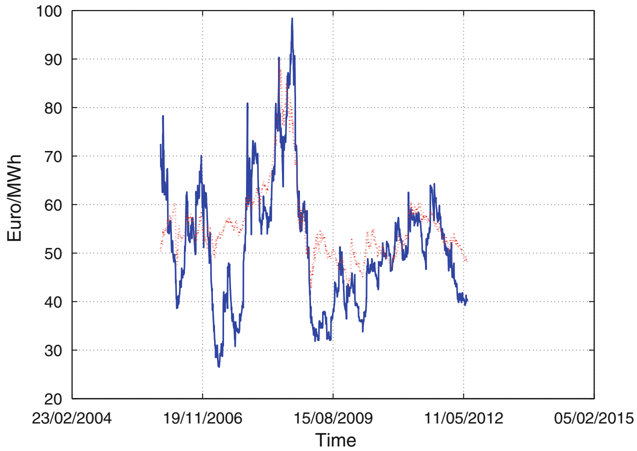

Fig. 2.7 Daily quotation of German month-ahead (*blue solid line*) and year-ahead (*red dotted line*) futures contracts from 2006 to June 2012. Time in French format dd/mm/yyyy *Source* EpexSpot

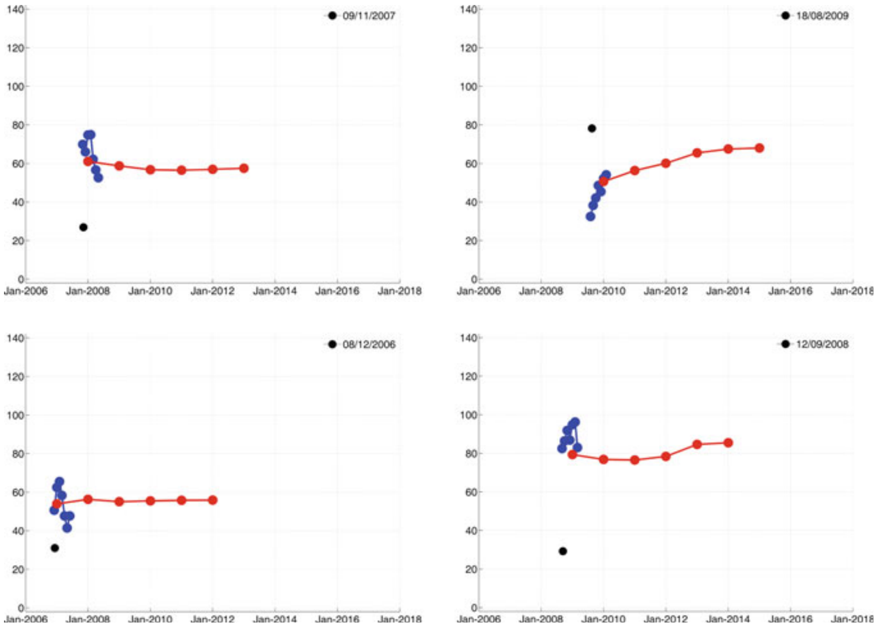

Fig. 2.8 Illustrations of four different dates of the German electricity forward curve with only year-ahead (*red dots*) and month-ahead (*blue dots*) base-load contracts. The *black dot* corresponds to the average spot price on the dates. The X-axes are the delivery date, and the Y-axes are the price in €/MWh

figure at the bottom also shows that the shapes of the term structure can be much more complex. It gives an illustration of the strong seasonal pattern that can affect the month-ahead contracts while the year-ahead part of the curve remains perfectly flat.

Convergence: Another important issue regarding the electricity forward contracts concerns the settlement mechanism of the delivery period. The electricity futures contracts are settled against the average day-ahead price of the delivery period. This mechanism ensures that the electricity futures contracts provide the hedge required by the market participants.

For example, on 1 March, a power operator owns a power plant of 1 MW. The April base-load futures price is 40 €/MWh. That price suits the operator's objectives and so, he or she immediately sells one contract. On the last business day of March, the April contract settlement price is 45 €/MWh. Thus, the operator has lost 5 €/MWh on the futures value but still holds the futures contract during the delivery period. During the month of April, independently of what is happening in the futures market, the operator operates the plant on a base-load schedule each hour with full power. Then, the operator receives exactly the average April day-ahead market price. In this example, the average April day-ahead price is 30 €/MWh. From the April futures contract the operator holds, he or she receives for each hour of April, $(45 - 30) =$ 15 €/MWh. In the end, the operator receives $-5 + 45 - 30 + 30 = 40$ €/MWh, which was the selling price objective.

This example shows that, contrary to other commodity markets, the operator cannot rely on the convergence of the futures to the spot price to reverse the operator's position in the financial market on the last days of the contract quotation.

In this example, if the operator buys back the April contract on the last business day of March, believing that the average day-ahead price will settle exactly at 45 €/MWh, then the result will be completely different. First, the operator still incurs the loss of 5 €/MWh from the margin call. But, now the operator receives 30 €/MWh on the spot market with no hedge. With the 5 €/MWh loss on the futures market, the operator's selling price is 25 €/MWh. The 15 € difference comes from the basis risk between the last quotation of the April base-load and the resulting average day-ahead price.

However, the actors on the market do not hold a year-ahead contract during its whole delivery period. Instead, as soon as the contracts with shorter maturities appear, such as quarters-ahead and months-ahead, they sell their longer maturities and buy the shorter ones. This procedure is automatically implemented in the EEX calendars and quarters contracts in a mechanism called *cascading*. A few days before the delivery period, every open position on a year-ahead futures contract is replaced by an equal position in the three months from January to March and the three quarters of April, July, and October.

Options market: There are quotations available for vanilla options on the futures contracts on the organised markets like the EEX or the NordPool. European calls and puts are open on the futures contracts with different strike prices and three different maturities. For example, on the last trading day of 2012, there were three calls and three puts with the expiration dates of 13 April, 13 July, 13 October and four calls

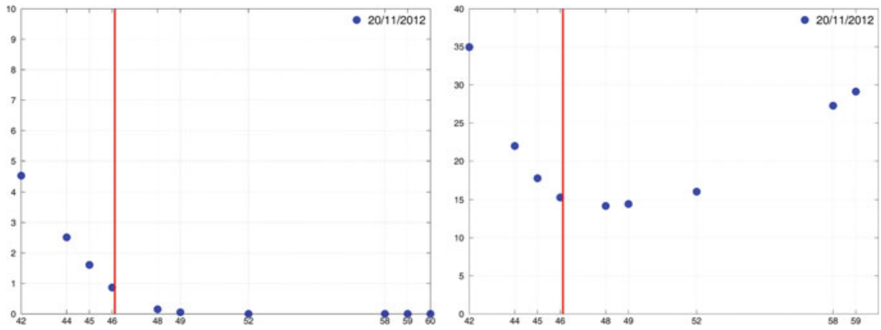

Fig. 2.9 Prices of call options on year-ahead futures base-load contract with expiry on January 2013 as of 20 November, 2012, (*left*) and their implied volatility (*right*) as a function of the strike price. The *solid red line* indicates the options in the money

and puts with the expiration date 14 January for the year-ahead contracts in 2014. The same year-ahead contracts were available for 2015 and 2016 with the slight difference that more calls and puts were available for the last expiration dates. Also, only the base-load contract was available. Thus, these contracts resulted in only 82 options available for the trade of the year-ahead contracts. The calls and puts were also available for base-load months and quarters. The prices for those contracts were provided even though there were no transactions and no open interests. Some contracts were barely traded while the options with an expiry on N January with the underlying year N grabbed all of the liquidity. Thus, in 2012, the options on year-2013 with expiry on 13 January represented all of the open interest for options. They represented an average of 58 contracts with a maximum of 300, and calls and puts were equally traded.

Figure 2.9 provides an example of the prices quoted on 20 November, 2012, for call options on the year-ahead base-load futures contracts for 2013 with expiry in January 2013. I also provide an estimation of their implied volatility. This figure depicts a situation where there is some open interest in most of the options with an available price. It shows that the implied volatility can produce a nice smile with a volatility of around 30 %.

2.2.4 The Diversity of Electricity Markets

The general presentation above hides a great deal of heterogeneity in the development of the electricity markets around the globe. Sioshansi and Pfaffenberger's book [153] on the reform implementations in the worldwide electricity market shows that these markets have followed very different paths. The markets in some countries have important growth both in terms of volume and in the complexity of their products.

An example of such developments is the Pennsylvania, New Jersey, and Maryland market (PJM) in the United States. The PJM market has existed since 1997 and now serves more than 60 million customers in the Mid-Atlantic and the District of Columbia. Its total annual consumption is approximately 800 TWh with more than 180 GW of installed capacity of which 60 % are gas-fired plants. The spot market is a bid-based mechanism with security constraints which deliver prices for each node of its operating network (more than 10,000). In addition to offering prices for energy, PJM also provides its operators financial transmission rights to hedge the locational risk spread. Since 2007, it has launched a capacity market to ensure long-term reliability of the system and to provide its investors with a market signal on the value of the capacity. Its futures and options are also available on the Nymex. The contracts come in different delivery time schedules (off-peak, peak, day-ahead, month-ahead) but also in different locations of the PJM network.

In Europe, a similar development has occurred in Scandinavia with the NordPool, in continental Europe with the EEX, and the market coupling mechanism progressively developed across Western countries.

The NordPool was launched in 1996 and covered only Sweden and Norway. It now connects Norway, Sweden, Finland, Danemark, Lithuania, Latvia, and Estonia for an annual generation of approximately 480 TWh generated by 370 companies. Its hydroelectric generation accounts for 100 TWh and its nuclear for another 100. The remaining generation comes from coal- and oil-fired plants. The NordPool's day-ahead spot price is operated by ElSpot and consists of an implicit auction mechanism between all of the countries covered by the market. An intraday market also exists which allows the NordPool to secure the imbalances. The intraday market has seen important growth thanks to the introduction of wind energy and the increasing need for intraday rescheduling. The financial market for Scandinavian electricity formerly known as Eltermin is now operated by Nasdaq OMX Commodities. It offers futures and options that cover the daily, weekly, monthly, quarterly, and the annual horizons. Each underlying is given by the hourly day-ahead spot price fixed by ElSpot.

These two markets (PJM and NordPool) disclose not only prices but a large amount of hour data, such as many time series on generation per technology, transit, planned outages, inflows, and the states of the reservoirs, which is all of the information needed by the market participants to precisely assess the equilibrium between offer and demand.

However, some electricity markets have not developed as much as might have been expected in terms of financial derivatives. This is the case for Chile. The Chilean electricity system is basically a one-dimensional network due to the geography of the country. The market is decomposed into three areas, but the main part consists of the central interconnected system which covers around 93 % of the Chilean population and 70 % of the total installed capacity. The generation is a mix of hydroelectric generation (50 %), natural gas (27 %), coal (10 %), and oil (7 %). The main feature of the spot market is that it relies on a cost-based economical dispatch performed by the Economic Load Dispatch Centre. The dispatch minimises the expected discounted value of the generation cost of serving demand in the next 48 half-hours. The system price is the running cost of the most expensive unit of generation used to satisfy the demand. The dispatch is mandatory. A price cap is set by the regulator every six months according to the value of the load lost. Moreover, each power plant receives a monthly capacity payment based on its availability. Despite the fact that this country was the first to establish a pool market for electricity in the beginning of the 1980s, there is not a developed market of standardised futures and options contracts.

This lack does not mean that the development of a financial market is not possible. New Zealand is an example of the late development of a financial market for electricity. New Zealand is a country composed of two islands with a population of 4 million people. They have an annual electricity consumption of 36 TWh and an installed capacity of 9 GW. Deregulation began in 1987, and the spot market began in 1996. It is similar to the PJM design: bid-based security constraints with nodal prices providing 48 half-hourly prices for each of the 285 nodes in its network. The forward contracts are traded on ASX, New Zealand's electricity futures and options exchange. The exchange launched in December 2013 trades in base-load monthly futures, peak-load quarterly futures, and the average rate options on base-load quarterly futures.

There is no explanation for the diversity of the financial markets' development. Possibly it comes from the generation mix, the network configuration and most of all, the amount of consumption. For instance, Cyprus is a European state where an electricity market should be designed. It has 5 TWh of annual consumption that relies mainly on three fuel power plants for 95 % of its generation. Nevertheless, the evolution of the wholesale market of England and Wales may indicate that the design of the spot market can be a key driver in the development of the financial market. Indeed, the first phase of the English and Wales market in 1992 relied on a mandatory economical dispatch very similar to the Chilean model. During that period, there was not to my knowledge a financial market for futures similar to the one in NordPool. But, now after several changes in the design of the English and Wales power market, the futures and options can be traded on the Nasdaq OMX Commodities board with the standard variety of products available (day, week, month, quarter, year, and base-load and peak-load).

2.3 Real Derivatives

Like any other commodity market, the electricity market has its options markets on quoted futures. But, the most challenging problems do not come from the valuation of these standard products but rather from their pricing, hedging, and structuring into exotic tradable products which are called *real derivatives*. The electricity operators as well as the retailers have these options embedded in their portfolios. A power plant can be seen as a strip of call options. A retail household contract with a curtailment clause which gives the retailer the right to charge the customer an exceptionally high price if the customer does not reduce his or her consumption on a sent signal, is a contract with an embedded put option. Thus, as an analogy with the idea of the real option for investment decision, these rights can be thought of as real derivatives.

Moreover, because electricity cannot be stored, the TSO always needs some flexibilities to be able to cope with uncertainties but also to cope with the dynamic constraints of the generation system. It is sometimes more economical to be able to reduce consumption or to defer it by a few hours than to satisfy it right away. On the contrary, it might be better sometimes that the electricity consumption is higher to avoid shutting down a power plant which is not flexible. An example of these flexibilities can be illustrated by household water heaters. In France, these home appliances are automatically started up with an electric signal when the consumption is at its lowest during the night. As Remark 2.1 points out, the development of intermittent sources of renewable energies has led to a growing interest from all of the market actors in all of these flexibilities. The flexibilities can be seen as real options bought by the TSO from the users or by any involved actors from the customers.

The following are the three most important real derivatives that are the daily concerns of the operators and their financial representation:

1. Power plant and tolling agreement
2. Energy storage and swing contract
3. Retail contract.

Power plant and tolling agreement: The owner of a power plant creates value by selling power and buying fuel. The owner needs to estimate the value of the generation of a power plant for investment decisions, long-term contract negotiations, and risk management applications. A power plant is an industrial facility and thus, is subject to many technical, environmental, and legal constraints. But, in a first crude approximation, a power plant can be seen as a strip of calls on the spread between its fuel price and the electricity price. For a gas-fired plant, one speaks of a spark spread; while for a coal-fired plant, one speaks of a dark spread. When the carbon emission price is taken into account, then it is called a clean dark spread and a clean spark spread. The payoff per MW of a power plant in a period of time $[0, T]$ is then given by:

$$\int_0^T (S_t - hS_t^f - gS_t^c)^+ dt, \tag{2.1}$$

where S is the spot price of power, S^f is the spot price of its fuel, and h is its heat rate. The variable S^c is the spot price of the emission permit, and g is the emission factor of the plant. The notation x^+ refers to $\max(0, x)$. The payoff (2.1) makes it clear that it is possible to identify the value of a power plant with a strip of call options. Nevertheless, the fact that the instantaneous payoff depends on three assets makes the problem technically difficult. Moreover, the existence of the operational constraints drastically reduces the possibility of capturing the successive positive spreads. The start-up costs, minimum running time, ramp-up and ramp-down constraints, and the limited number of cycles per day, all limit the flexibility of the power plants. Thus, the quantity (2.1) overestimates the profit of the power plant. Valuating power plants with operational constraints gives rise to *optimal control problems*. One way to formulate it is:

$$\sup_{q_t \in \mathscr{A}} \mathbb{E}\left[\int_0^T q_t(S_t - hS_t^f - gS_t^c - \kappa)^+ dt\right], \tag{2.2}$$

where q_t is the generation belonging to an admissible set $\mathscr{A}$, and κ is a start-up cost.

However, the problem above is not new in generation management. It is related to unit commitment problems, that is, scheduling a group of power plants for the next few hours or days. And it is also related to management problems in mid-term generation where the assessment of the generation level of a group of power plants is needed for the next few months. Over the last 40 years, a lot of efforts has been devoted to the development of numerical optimisation algorithms and software to solve those problems. These models are based on a context where monopolies try to satisfy a random demand at the lowest cost. They use mix-integer programming methods or Lagrangian relaxation methods. The introduction of spot markets has changed the problem by introducing new phases like the bidding phase where the power operators have to submit their bids to the market operator. However, the underlying optimisation methods still apply. For a more in depth description of the constraints in power plant scheduling and numerical optimisation methods, the reader can consult Wood et al.'s monograph [169] and the recent survey of Prekopa et al. [146].

The major novelty raised by the development of financial market is the question of the hedge of the payoff (2.2). Typically, the natural hedging instruments are the futures on the fuels, carbon emissions, and the futures on power. But, as we seen in Sect. 2.2.3, the futures contracts available on the electricity market do not have the fine granularity of the spot price. If T is one year, the above payoff is exposed to 8,760 risk factors whereas only a handful of futures are available. In this regard, the electricity market is incomplete. As a consequence, even the notion of the value of the first basic payoff (2.1) is ambiguous. At the present time, it has no consensual answer. This lack of consensus translates immediately into the difficulty of finding exchange prices for tolling contracts. These contracts are financial counterparts of the power plants where the owner concedes an exploitation right in exchange for a fixed premium. Depending on the price models, the constraints taken into account and the hedging capacities, there might be a large discrepancy between the prices.

Energy storage and swing contract: Hydroelectric plants are very flexible. They can provide electricity at very short notice. The valuation problems described for the thermal power plants are still valid for hydroelectric power plants. But, the existence of a limited resource of fuel leads to the problem of storage management. The simplest problem in hydroelectric storage management deals with a single reservoir. The valuation problem is:

$$\sup_{q_t \in [0,\overline{q}], \delta_t} \mathbb{E}\left[\int_0^T q_t S_t dt + g(S_T, X_T)\right], \tag{2.3}$$

where S_t is the electricity spot price, and X_t is the current level of the water in the reservoir. This level satisfies the following dynamic:

$$dX_s^{t,x} = (a_s^{t,a} - q_s - \delta_t)ds, \tag{2.4}$$

where a_s are random inflows. Moreover, X is subject to level constraints and should stay within $[\underline{x}, \overline{x}]$. The control δ_t is the spilling variable. If X_t is at its maximum level and the inflows exceed the generation capacity $\overline{q}$, the operator has no other solution than to spill the excess water. The function g represents a final value for having a certain level of water at a final time.

In a more general form, a whole *network* of hydroelectric plants and reservoirs influence one another. The constraints on the hydroelectric plants become more complex and the dimension of the system drastically increases. Hydroelectric management problems are not new. They were at the origin of the development of dynamic programming [132]. There is important literature on power systems that is devoted to these problems. The main difficulty in these problems is the dimension of the state. The higher the number of reservoirs, the higher the dimension. They are mainly tackled by using stochastic dynamic programming methods, such as the dual dynamic programming method [139], or decomposition methods.

The development of the electricity markets launched a revival of this problem because of the availability of spot prices. Other methods have then been proposed based on the *optimal switching problem* or the *stochastic control problem*.

As was the case for power plants with tolling contracts, the financial representation to storage facilities, named swing options, were developed to offer the ability to store power but without all of the technical constraints involved with a real hydroelectric system. An important class of swing contracts consists of the demand-side management contracts. The customers holding this form of contract enjoy a lower tariff during the year except for a few days when any kWh consumed costs the customer a dissuasive price. What is important here is that the days with the higher price are not known in advance by the customer. The utility warns the customer the day before. Thus, the utility holds an option on the customer's consumption. When the customer is an industrial firm, the result of the signal is deterministic. The firm cancels its consumption. But, when the contracts concern a large amount of households, the resulting total avoided consumption is random.

Retail contract: The retail pricing policy consists in determining the form of the contracts for customers as well as the price to be charged. As in the insurance business, retailers first have to decide on the forms of their contracts and their prices; then, the customers select the contract and consume. When setting the retail pricing policy, a retailer does not know what is going to be its sourcing cost and precisely how much the customers are going to consume.

The consumption of the customer drastically differs from a chemical plant to a household in a building in a big city. The customers are generally sorted into three main classes: industrial, professional, and household. They differ by their number, their consumption volumes and patterns, their economic behaviour, their needs, and by the information the retailers have on their consumption. In a country like France or Germany, industrial customers are much less numerous than households but they represent a significant part of the country's consumption. The industrial customers can benefit from very precise metering of their consumption whereas households are still only measured twice a year in many countries. The industrial customers might not be sensitive to weather whereas households are. The industrial customer can switch from one retailer to another for 1 cEuro/MWh because each cent can represent thousands of euros on the customer's electricity bill. Household switching is more viscous.

For these reasons, the contracts offered to different classes of customers are very different. The industrial contracts might be tailored to their operational constraints whereas the households are viewed as a mass market that only needs basic standardised contracts. The industrial retail contract often includes embedded options (right to resell to the market the power delivered at a certain price). The household's contracts involve fixed premiums proportional to the subscribed capacity plus a price per kWh. Sometimes they benefit from a two-level tariff depending on the hour of consumption (peak and off-peak). For a case where a detailed description of a retail market is given, the reader is referred to the analysis of the Norwegian household market in von der Fehr and Hansen [161].

This problem has received much less attention in the quantitative energy finance literature than in the economics literature. Indeed, electricity pricing policy was addressed as a problem of long-term pricing during the monopoly period of electricity generation. In that economy, the main concept which dealt with this problem was the marginal cost pricing theory, deeply illustrated in theory and in practice by Boiteux [38]. In a nutshell, the marginal cost pricing of electricity insures a maximisation of the social welfare through a regulated monopoly. With market competition and with the real gross market prices, nothing insures that selling at the marginal generation cost is the right policy to maximise the value of the firm. Indeed, the marginal cost pricing rule requires certain long-term equilibrium conditions on the generation assets to be fulfilled which are unlikely to hold in actual market conditions.

2.4 Conclusion

The structure of electricity markets can be seen as a multi-layered market with successive time horizons going from the forward market of up to six years to the real-time balancing market for the next hour. The electricity forward curve is coarse and presents a cascading structure. The spot prices are defined by the day-ahead market. They present strong seasonality patterns, spiky behavior, and positive as well as negative prices. Further, power plants, tolling contracts, storage and swing contracts and retail contracts are some of the most specific derivatives to electricity markets.

Chapter 3
Price Models

This chapter is devoted to the modelling of the electricity spot and futures prices. Since the electricity market's deregulation, an important and increasing number of publications are dedicated to the problem of modelling the electricity spot price. I exclude from the scope of this chapter the research activity on the forecasting of the spot prices. The chapter is limited to the models that provide the dynamic of the futures prices. Indeed, modelling the dynamic of the futures prices is the basic brick for hedging and pricing more complex products. A model should be able to provide them at a low computational cost.

But, even with this restriction, an important diversity of models exists for the spot and futures electricity prices. For pricing and risk management applications, an electricity price model should have the following properties: realism, consistency, efficiency, robustness, and generality. Realism concerns the ability of the models to reproduce the statistical properties of the time series of the spot and forward prices. It can be assessed by different methods with basic to sophisticated statistical tests. The consistency concerns the no-arbitrage property between the spot and the forward prices. Although electricity is non-storable, no-arbitrage conditions appear in the different models in this chapter. Efficiency is a key element. The models should lead to fast computation of the forward prices, where fast means milliseconds. Since forward prices are the basis for any pricing or risk management computation, a single computation has to be performed in sufficiently efficient way to allow the valuation of more complex products within a reasonable time (a few minutes). Robustness deals with the continuity of the estimated parameters with respect to the data and the continuity of the modelled spot and futures prices. A small modification of either the entry data or the parameters should not lead to important modifications of the modelled prices. Further, the model should be general enough to allow its estimation and calibration for the different electricity markets available. Because there are at least as many electricity markets as there are countries, the models should be generic enough to avoid the need for a specific development for each.

The comparison of electricity models within the framework above is a research topic in of itself. Here, this chapter confines itself to the presentation of the models. But even with this limited ambition, the large spectrum of models that already exist

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raises the difficulty of finding a proper way to sort them out. No taxonomy is perfect and neither is the classification used here. The prevailing idea is in a sense to sort them by increasing complexity.

So, the first class, which is presented in Sect. 3.2, consists of models that follow the Heath-Jarrow-Morton (HJM) approach. Those models start with the observed forward prices, model them, and then derive the spot price (if needed) as a future with a zero maturity limit. This is the preferred method used on trading desks and in risk management departments because it ensures that the model fits the market prices every day. This approach is inherited from the modeling of the yield curve for interest rates. Thus, many observations, results and problems are not particular to electricity forward curve modeling and can be found in classical text books on interest rates curve modeling like the book by Musiela and Rutkowski [136] and the book by Brigo and Mercurio [44]. In particular, the constraints on the volatility functions used in these models apply to the electricity forward curve. But, in the case of electricity, the existence of a delivery period for futures adds more constraints if the setting is to remain no-arbitrage.

The other classes of models take the opposite approach to the HJM method. The electricity spot price is modelled first, and the futures prices are then deduced as an expectation through a convenient probability measure from an arbitrage argument. A first category of electricity price models which follow this procedure is presented in Sect. 3.3 and consists of one-factor spot models. These models illustrate the main difficulties when preserving the Markov property of the spot, capturing spikes, and getting the dynamic of the forward curve. Although rich, this class of models does not offer enough degrees of freedom to capture the main properties of the electricity price and therefore lead to a need for more complex classes of models.

The third class of models, which is described in Sect. 3.4, consists of multi-factor models of the spot. This increase in dimensionality can be done either by adding factors that are observed or not, or mixing them both. These models show a much more realistic behaviour in the joint dynamic of the spot and the forward prices. The cost for this result is the loss of the Markov property of the spot and the need for the filtering techniques used in the estimation of the parameters of the unobserved state variables.

The last class, which is presented in Sect. 3.5, is structural models. In these models, the spot is deduced from an over-simplified version of an equilibrium model between the production cost curve and the electricity demand. The structural models naturally implement the dependencies of the spot price with the observed factors such as fuel prices, consumption, and power plant outages. Relying only on observed factors, they avoid the use of filtering methods for the estimation. But they are challenging for the computation of futures prices because the spot is generally a non-linear function of the observed factors.

But, before I proceed on this journey into price models and because of their fundamental importance to the pricing theory, I devote Sect. 3.1 to the no-arbitrage conditions and the convergence of the futures price to the spot price in the case of electricity.

Remark 3.1 In this chapter, the notations used for the same variable (e.g., the spot price and the mean-reversion value) might change from one model to another. This is my choice between being homogeneous or keeping true to the original papers where the models were presented. I thought it would be better to keep the notations from the original papers for the simplicity of understanding if one was to consult their papers for a deeper understanding.

Moreover, we will use from now on indifferently the terms futures and forwards because their difference is not significant in the context of the price models of this section.

3.1 Preliminary Remarks

For freely storable assets, the spot price S_t at time t is linked to the forward price $F(t, T)$ for delivery at time T by the relation

$$F(t, T) = e^{r(T-t)} S_t, \tag{3.1}$$

with r the interest rate that is constant. This relation is established with no-arbitrage arguments, whose details can be found in Musiela and Rutkowski [136, Sects. 1.5 and 1.6]. These arguments rely on the storable property of the asset. It has been acknowledged that the relation (3.1) does not hold for commodities such as cereals, copper, or oil. The cost of storage for a commodity is not negligible compared to a stock. This cost should significantly reduce the expected return from buying the spot. This remark leads to the thinking that the right relation for the storable commodities should be:

$$F(t, T) = e^{r(T-t)+c(T-t)} S_t, \tag{3.2}$$

with $c(T-t)$ as the cost of storage that is supposed to be only a function of the time to maturity. But, even with this correction, the relation still does not hold. In particular, the forward price could be lower than the spot. In an early paper, Kaldor [108] introduces the idea of a *convenience yield* to account for that effect. The convenience yield represents the extra return that the actors gain from holding the commodity instead of a forward contract. The relation then writes as:

$$F(t, T) = e^{r(T-t)+c(T-t)-y(t,T)} S_t, \tag{3.3}$$

with $y(t, T)$ as the convenience yield.

Remark 3.2 The arbitrage and relations above between forward price and spot price are precisely described in Eydeland and Wolyniec [85, Chap. 4, pp. 140–143].

However, this construction raises questions even for storable commodities because the convenience yield is not observable. But, in the case of electricity, it barely holds.

Further, because electricity cannot be stored, the storage argument cannot be used to justify the relation between the current spot price and the forward price. Thus, even the concept of the convenience yield cannot be applied to this market, at least without reconsideration of its underlying.

Moreover, a second hypothesis which is important for the modelling of forward and spot prices is the fact that as the time to delivery gets closer the forward price should converge to the spot price:

$$\lim_{t \to T} F(t, T) = S_T. \tag{3.4}$$

This relation ensures that the basis risk between the forward contract and the spot market is negligible. When this relation holds, an actor can reverse his or her position on the forward market and buy or sell on the spot market, while still being sure that the loss from one market will be offset by the gain from the other. In the case of electricity, Sect. 2.2 shows that the forward contracts involve a delivery period that leads the operators to implement a cascading mechanism of switching to contracts with shorter delivery periods each time they appear.

It is not obvious in this situation that the relation (3.4) occurs here. But, it is possible to test if this convergence holds or not for the electricity futures, for example, the day-ahead futures contracts. These contracts are currently traded on the European Energy Market, EEX, for instance. The underlying of those contracts is the average price of the spot price on the delivery day of the contract. For a contract with a delivery on Tuesday to Friday, the forecast of the day-ahead spot price is good enough to ensure a small discrepancy between the last quotation of the day-ahead futures price and the average spot price. But, because these contracts are financial, they are only traded on business days. Thus, the last quotation of the day-ahead futures contract of a Monday is the Friday before. Thus, the difference between the last quotation of the day-ahead futures contract and the average realised spot price gives an idea of the quality of the convergence.

The study of this point is undertaken by Viehmann [160]. The author shows that even for weekend days the discrepancy between the last quotation of a day-ahead futures quotation and the realised spot is not statistically different from zero on average for all of the hours of the day. For some particular hours like peak hours (6 p.m.), the discrepancy can be close to 5 %. Thus, if the reader is not too demanding, he or she can consider that there is convergence up to a small premium due to the fact that even from Friday to Monday events might occur during the weekend that could change the spot price.

A third point in the spot-forward relation for storable assets is that there is a risk-neutral measure which ensures that the forward prices can be written as the expected spot price, namely:

$$F(t, T) = \mathbb{E}_t^{\mathbb{Q}} [S_T]. \tag{3.5}$$

For valuation purposes, it is better that the risk-neutral measure used here, $\mathbb{Q}$, is unique. In the case of electricity, the models presented in this chapter generally lead to incomplete market models. Thus, the uniqueness of a risk-neutral measure is not guaranteed, and one has the choice amongst an infinity of them. The next sections will show the different methods used to pick up one risk-neutral measure. In this regard, the important point is less the theoretical question of the existence and uniqueness of this risk-neutral measure but the practical capacity of the model to fit all the observed futures prices.

Another important part of the literature on electricity forward prices is devoted to the analysis of the sign of the risk premium, namely:

$$R(t,T) = F(t,T) - \mathbb{E}_t^{\mathbb{P}}\left[S_T\right], \tag{3.6}$$

where the expectation of the spot price is taken under the historical probability. This premium can have an important economical meaning on the rationality of the economic agents in the markets. This premium is an indicator of the relative market power or the relative risk aversions of the producers and the consumers. The risk premium might also have an important impact on the generation management process of the utilities. Some decision-makers might judge that the futures are too expensive when compared to the expected spot price under the historical probability. But, the futures prices are not actually expensive. They only imply that the decision-makers forget that their decision is driven by their own risk aversion. Thus, despite its important economic interest, the question of the risk premium is not necessary to develop within the scope of a text dedicated to the pricing of derivatives. However, for an in-depth analysis of this quantity, the reader can consult amongst other works Bessembinder and Lemon [33], Longstaff and Wang [126], Diko et al. [77], Benth et al. [21] and Aïd et al. [4].

3.2 HJM-Style Forward Curve Models

The models described here are inspired by the methodology developed by Heath, Jarrow, and Morton for the debt market [99]. The intention of the authors was to find an alternative way to the factor model based on the idea of an instantaneous interest rate similar to the Vasicek style models [159]. The Vasicek approach suffers from the difficulty of its fit with the observed yield curve. By proposing to instead directly model the dynamic of the forward rate, Heath, Jarrow, and Morton (HJM) are able to overcome this difficulty. The same idea has prevailed in the literature on commodity prices where the first applications can be traced to Jamshidian [106] for crude oil and Cortazar [66] for copper. I first present the general principle of this method and then turn to its implementation in the case of electricity.

3.2.1 Principle

Consider the price $F(t,T)$ given at time t for delivery at the future instant $T > t$ of a commodity. I suppose from now on that there exists a risk-neutral measure and I write the dynamic of $F(t,T)$ under this probability. The dynamic for F is thus given by

$$\frac{dF}{F}(t,T) = \sum_{1 \le i \le n} \sigma_i(t,T) dW_t^i, \tag{3.7}$$

where the $\sigma_i(t,T)$ are n volatility functions, and the W^i are n Brownian motions (potentially correlated). With a storable commodity, the delivery times T are discrete and basically given by the month steps. Moreover, the initial condition required for $F(0,T)$ is given by the observed forward prices for each delivery T, which makes the fit of the forward price at the initial time a basic procedure.

The number of Brownian motions N can potentially be as many as there are delivery periods. But, the number of factors needed to reproduce the dynamic of the term structure sufficiently well is much less. The number of factors needed has been the subject of many studies based on the principal component analysis (PCA). The first studies were performed on the money market by Litterman and Scheinkman [124] and Knez et al. [120] who showed that three factors explained more than 86 % of the total variance while four grabbed 90 %. The PCA performed on the different commodity markets shows that three factors are enough to capture more than 90 % of the variance in the futures returns: for copper more than 97 % [66] and for NYMEX crude oil and gas more than 98 % [65, Sect. 8.4]. Thus, this approach not only remains in a complete market setting but it only requires a small number of factors.

Furthermore, if needed, the spot S_t can be defined as a futures contract with immediate delivery where $S_t = F(t,t)$. Its dynamic can then be derived from (3.7) by first writing the integral form of $F(t,T)$ as

$$F(t,T) = F(0,T) \exp\left(-\frac{1}{2} \int_0^t \sum_{1 \le i \le n} \sigma_i^2(s,T) ds + \int_0^t \sum_{1 \le i \le n} \sigma_i(s,T) dW_s^i \right). \tag{3.8}$$

And, because $S_t = F(t,t)$, the use of Itô's lemma shows that the spot follows the dynamic:

$$\begin{aligned} \frac{dS_t}{S_t} &= \left(\partial_2 \ln F(0,t) - \sum_{1 \le i \le n} \left[\int_0^t \sigma_i(s,t) \partial_2 \sigma_i(s,t) ds + \int_0^t \partial_2 \sigma_i(s,t) dW_s^i \right] \right) dt \\ &\quad + \sum_{1 \le i \le n} \sigma_i(t,t) dW_t^i, \end{aligned} \tag{3.9}$$

where ∂_2 denotes the derivatives of a function with respect to the second variable.

Remark 3.3 The different steps that lead to the relation (3.9) are the following. Let $X_t = \int_0^t \sigma(s,t)dW_s$ and $S_t = g(t, X_t) = \exp\left[\ln f(0,t) - \frac{1}{2}\int_0^t \sigma^2(s,t)ds + X_t\right]$. Thus

$$dS = \partial_1 g \cdot dt + \partial_2 g \cdot dX + \frac{1}{2}\partial_2^2 g \cdot \langle dX_t, dX_t \rangle.$$

Moreover,

$$\partial_1 g = g \times \left[\partial_2 \ln f(0,t) - \int_0^t \sigma(s,t)\partial_2\sigma(s,t)dt - \frac{1}{2}\sigma^2(t,t)\right]$$

It holds that $\partial_2 g = \partial_2^2 g = g$. And, one has

$$dX_t = \left[\int_0^t \partial_2\sigma(s,t)dW_s\right]dt + \sigma(t,t)dW_t$$

and

$$\langle dX_t, dX_t \rangle = \sigma^2(t,t)dt.$$

Thus, the term $-\frac{1}{2}g\sigma^2(t,t)$ from $\partial_1 g$ cancels out with the term $+\frac{1}{2}g\sigma^2(t,t)$ coming from $\frac{1}{2}\partial_2^2 g \cdot \langle dX_t, dX_t \rangle$.

An example and an important case for application is a single factor with an exponentially decreasing volatility with respect to maturity $\sigma(t,T) = \sigma_0 e^{-a(T-t)}$. By remarking that

$$\int_0^t \sigma(s,t)dW_s = \ln F(t,t) - \ln F(t,0) + \frac{1}{2}\int_0^t \sigma^2(s,t)ds, \tag{3.10}$$

this specification leads to a mean-reverting spot price dynamic given by:

$$\frac{dS_t}{S_t} = (\mu(t) - a\ln(S_t))\,dt + \sigma_0 dW_t, \tag{3.11}$$

with

$$\mu(t) = \partial_2 \ln F(0,t) + a\ln f(0,t) + \frac{\sigma_0^2}{4}(1 - e^{-2at}). \tag{3.12}$$

This example shows the relation between a term structure with decreasing volatility and a mean-reverting spot price. But, it should not hide that not all of the volatility functions are admissible. Because, if they are not suitably chosen, the Markov property of the spot price might be lost. Further, the relation which shows the dynamic of the spot (3.9) also shows that the drift term depends on the whole trajectory of the Brownian motions because of the terms:

$$\int_0^t \partial_2 \sigma_i(s,t)dW_s^i.$$

Conditions on the volatility functions have to be set to ensure the Markov property of the spot (see Musiela and Rutkowski [136], Proposition 11.2.2). Without the Markov property, any simulation of the prices or any computation of the price derivatives becomes cumbersome because the whole past of the Brownian motions has to be remembered. In the case where volatility is required to depend only on the time to delivery $T-t$, then the only possible volatility functions that preserve the Markov property of the futures prices are of the form $\sigma(t,T) = \sigma_0 e^{-a(T-t)}$ with σ_0 and a constants, which are the most commonly used volatility functions in the literature and in practice.

3.2.2 The Case of Electricity

A first technical difficulty is the existence of a strong seasonal pattern in the electricity forward curve. This seasonality requires the introduction of a time-dependent drift term and a procedure to estimate it. The situation is quite different from the case of the Black and Scholes model for stock markets where it is known that the estimation of the drift term under the historical probability requires unreasonable historical time-series. Indeed, the seasonality of the futures prices are connected to the seasonal patterns of consumption which are well-known. Several methods have been proposed to estimate the seasonality of futures prices. The first one consists of using a forecast model as in Fleten et al. [87]. Because the seasonal pattern of the electricity forward curve is driven by the seasonality of the demand and of the marginal cost of production, a spot price forecast model can be used to provide a seasonal trend in the observed futures prices. In their work, Fleten et al. [87] fit the seasonal pattern obtained by a fundamental model to the forward seasonal pattern by using a constrained least-square. They also use smoothness constraints for the variations in the futures prices with respect to maturity to avoid jumps, for example, between the last daily summer contract and the first daily autumn contract. They compare this procedure with a truncated two-term Fourier series. They find a slight improvement with the seasonal trend given by the forecast model.

A second and more important difficulty lies in the number of factors suitable to model the dynamic of the electricity forward curve. Many studies undertake this subject, particularly for the NordPool market by the Norwegian School of Financial Economics. The analyses performed by Frestad et al. [89, 90, 91] and Koekebakker et al. [121] indicate that the electricity forward curve exhibits a much more volatile behaviour than any other market. In Frestad [89], the same PCA is performed on the market price data for the bond, copper, crude oil, and electricity markets. The analysis of the dynamic of the forward products of each market is performed with a model based on the asset pricing theory. The author shows that three common factors account for 67 % of the variance, and the unique factor explains 33 % of the variance in electricity forwards while three factors succeed in explaining 99 % of the variance in copper, 100 % for crude oil, and 97 % for bonds. This phenomenon is confirmed to some extent by Koekebakker et al. [121] who use a modelling approach closer to the standard HJM approach. Applied to NordPool data from September 1995 to March 2001, they find that two factors have much less explanatory power than in other commodities markets. One factor accounts for 68 %, two factors for 75 %, three factors for 80 %, and four factors 83 %. They need to have more than seven factors to get more than 90 % of the variance.

This need for an increasing number of factors for electricity forward prices comes from the unrelated dynamic of the weekly contracts with the longer maturity contracts. The behaviour of the next week's prices has little to do with the electricity futures price for the next season or the next year. Moreover, the returns of the forward prices even with not so short maturities might not be Gaussian. Indeed, Frestad et al. [91] show that the hypothesis of the normally distributed return for the forward prices is outperformed by the normal inverse Gaussian distribution even for long-term maturity contracts. These two remarks are the main justifications for the development of the models based on the infinite dimensional Lévy processes, or ambit fields (see Barndorff-Nielsen et al. [12]).

More difficulties come from the fact that the electricity spot price is hourly while the forward prices involve a delivery period. In the relation (3.7) which gives the dynamic of the forward prices, it is not clear what is represented by T. In the case of electricity, it should be clear if it is a given observed maturity (a month, a year) or if it is an hour in the future. In fact, for the valuation purposes of the real derivatives presented in Sect. 2.3, the hourly prices are needed which quickly leads to completing the observed electricity forward curve with a forward curve of hourly contracts.

The construction of an hourly forward curve for the next three or six years when there are only a hundred available quoted contracts might be difficult. An important effort has been devoted to the construction of such a forward curve of instantaneous delivery contracts in Fleten et al. [87], Koekebakker et al. [121], Benth et al. [26], Frestad [90]. A detailed description of a method can be found in Benth et al.'s book [19, Chap. 7]. In Benth et al. [23], for instance, their model starts with

$$F(0, T_s, T_e) = \frac{1}{N} \sum_{K=1}^{N} f(0, t_k) \tag{3.13}$$

where $f(t_k)$ is unobserved futures prices for delivery at time t_k, T_s is the time of the beginning of delivery and T_e is the time of the end of delivery. The function f is decomposed into two continuous terms $f = s + \varepsilon$ with s the seasonal term and ε an adjustment term. The constraints are set on the adjustment term to get a smooth hourly forward curve. Meanwhile, the seasonal term is estimated as the seasonal component of the historical spot price. Alternative methods might be developed.

But, in any case, clearly there is no unique solution to this initial hourly forward curve.

However, I assume the function $f(0, T)$ of the unobserved price at time 0 for delivery of 1 MW for the hour T can be constructed. I also assume null interest rate. If I denote by $f(t, T)$ the unobserved price at t for delivery of 1 MW for the hour T, then the observed forward contract $F(t, T_1, T_2)$ quoted at time t and involving the delivery of 1 MW each instantly between T_1 and T_2 is:

$$F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t, s) ds. \tag{3.14}$$

The idea then is to set a dynamic on the non-observed instantaneous delivery forward price $f(t, T)$, just like for the instantaneous forward rate (see Heath et al. [99]). Further, $f(0, T)$ is used as the initial condition of this dynamic.

But, not all volatility functions are admissible if the Markov property of the spot price is to be preserved. The fact that electricity forward contracts involve a delivery period somewhat constrains the admissible volatility functions. This remark is raised by Benth and Koekebakker [25]. Indeed, they show that under mild conditions, the relation (3.14) implies that the only dynamic for the return of f that preserves the Markov property of the spot is to have a volatility function that is independent of the maturity T. This does not leave many alternatives for the dynamic of f except the volatility functions which depend only on time t, and amongst them the most popular choice is the Black model: constant volatility (see Black [37]).

Moreover, Benth and Koekebakker [25] obtain similar results with alternative ways to specify the dynamic of the forward curve. They take one-step backward and assume that it is possible to specify the dynamic of the forward prices $F(t, T_1, T_2)$ with the relation of the form:

$$dF(t, T_1, T_2) = \Sigma(t, T_1, T_2) F(t, T_1, T_2) dW_t, \tag{3.15}$$

which holds for any pair of T_1, T_2. One may think that in this setting one would avoid the integration of the volatility function in relation (3.9) which is responsible for the loss of the Markov property of the spot. But, the authors show that the only suitable volatility functions depend only on time t. As a consequence, the only log-normal volatility structure for the quoted forward contracts that complies with the no-arbitrage condition that all of the overlapping contracts must satisfy is the Black model [37].

In both cases—setting a dynamic on an hourly forward price or on all forwards with a delivery period—one has very little choice for the volatility functions. The choice is between the realism of the volatility term structure and the relation with the spot price. For this reason, to satisfy the absence of the arbitrage opportunity and still be able to take into account the dependence of the volatility function with the maturity requires limiting the modelling to only the observed contracts and not being able to deduce an instantaneous forward price and a spot price. This is precisely what is done in Benth and Koekebakker [25] and Kiesel et al. [118].

In Kiesel et al. [118], the authors develop an HJM two-factor model which is one often used in practice. The authors consider $F(t, T)$ the price for a monthly futures contract quoted at date t with a starting delivery time of T, where T corresponds to the first day of the month. They consider only monthly contracts and consider that the other contracts are a portfolio of monthly contracts. The observable forward price is modelled directly under the risk-neutral measure. The initial condition is given by the observed forward curve. The authors then use Brigo and Mercurio's [44] method to approximate the density of the yearly contract by a log-normal distribution that has the same first two moments. They perform the calibration of the following two-factor model:

$$dF(t, T) = e^{-\kappa(T-t)}\sigma_1 F(t, T)dW_t^1 + \sigma_2 F(t, T)dW_t^2,$$

with uncorrelated Brownian motions. The volatility for a given month is then:

$$Var(\ln F(t, T)) = \frac{\sigma_1^2}{2\kappa}(e^{-2\kappa(T-t)} - e^{-2\kappa T}) + \sigma_2^2 t.$$

The parameters are estimated by minimising the quadratic difference between the market variances and the model-implied variances by using at-the-money options data. This model has the benefit of simplicity even though it does not provide all of the features required by a price model for derivative valuation.

Remark 3.4 An alternative to the problem of the limited choice of a volatility function in the HJM-style model for the electricity forward price was recently formulated in De Franco et al. [71]. Their approach consists in directly writing the dynamic of $F(t, T_1, T_2)$ under the historical probability and making the valuation with a quadratic hedging criterion. In this case, no stochastic differential equation are required for the forward prices.

I will conclude this section with a word on the joint modelling of electricity forward prices and fuel prices. This point is not often addressed in the context of forward curve models for electricity. Indeed, in practice, the most basic approach is used which consists of adding the same style of factor models for fuels and putting correlations between the factors. Typically, a joint coal-power model takes the form of:

$$\frac{df^e}{f^e}(t, T) = \sigma^e(t, T)dW_t^e, \tag{3.16}$$

$$\frac{df^c}{f^c}(t, T) = \sigma^c(t, T)dW_t^c, \tag{3.17}$$

where $f^e(t, T)$ (resp. $f^c(t, T)$) stands for the forward price of power (resp. coal). The dependence between the two prices is embedded in the correlation between the factors dW_t^e and dW_t^c. Although basic, this method assigns a symmetric role to both variables whereas it is highly unexpected that a shock to the electricity futures prices will have an impact on the futures prices of coal. This point has an important impact on risk management because the utilities are exposed to fuel spreads. The correlation between the returns tends to provide a pessimistic look at the evolution of the spreads because it fails to capture the fact that if the price of coal increases, the price of electricity is likely to increase too. Indeed, the phenomenon to be captured is that the levels of the prices should move together.

A possible way to deal with this form of dependency is to use co-integrated variables as initiated in Engel and Granger [83]. Co-integrated variables present the advantage of connecting their levels rather than their returns. This approach is implemented for the relation between power and gas in Benmenzer et al. [17] and between gas and oil in Ohana [137]. Benmenzer et al. [17] develop a continuous-time co-integrated model consistent with no-arbitrage conditions. Their model writes under the historical probability:

$$\frac{df^e}{f^e}(t,T) = \sigma^e(T-t)dX_t, \tag{3.18}$$

$$\frac{df^g}{f^g}(t,T) = \sigma^g(T-t)dX_t, \tag{3.19}$$

$$dX_t = \eta_t dt + \Pi X_t dt + \Sigma dW_t. \tag{3.20}$$

The volatility functions σ^e and σ^g are vectors of length N_e and N_g. Thus, dW_t is a vector of N_e+N_g independent Brownian motions. The function η_t is a parameter used to centre the dynamic around a desired target. Further, the dependencies between the price levels are obtained because of the matrix Π. The estimation of the parameters performs well and involves the PCA and linear regressions. The dynamic of the prices under the risk-neutral measure is obtained because of a suitable market price of risk λ_t given by $dB_t = dW_t + \lambda_t dt$, and λ_t is $\lambda_t = \Sigma^{-1}(\Pi X_t + \eta_t)$. The simulation results they report tend to provide more realistic joint trajectories for gas and power and show considerable difference in the perception of risk exposure for a utility. This point is also particularly stressed in Döttling and Heider [79].

3.3 One-Factor Spot Model

The purpose of the one-factor spot models is to design an electricity model which preserves the Markov property of the spot and which allows for the computation of the whole forward curve by using an expectation of the spot price under a suitable measure. Although it might appear restrictive, many alternatives were developed in the last decade. It goes from Gaussian mean-reversion processes where the change in the measure is given by a constant market price of risk as in Lucia and Schwartz [127] to Lévy jump process with a time-dependent market price of risk as in Benth et al.

Table 3.1 Summary of one-factor spot models discussed in this section

Model	Seasonality	Process	Jumps	Change of measure
Lucia and Schwartz [127]	Deterministic	Gaussian MR		Constant
Cartea and Figueroa [61]	Deterministic	Gaussian MR	Poisson	Constant
Benth et al. [22]	Deterministic	Gaussian MR	Lévy	Time-dependant
Benth and Benth [28]	Deterministic	Non-Gaussian MR	–	Constant

[22] and Deng and Jiang [75] (Table 3.1). This section illustrates the advantages and the limits of this approach with four models with increasing complexity (Table 3.1):

- Lucia and Schwartz's model [127]: deterministic seasonal part with a Gaussian mean-reversion process and a change of measure given by a constant market price of risk
- Cartea and Figueroa's model [61]: deterministic seasonal part with a Gaussian mean-reversion term, a Poisson jump term, and a change of measure given by a constant market price of risk
- Benth et al.'s model [22]: deterministic seasonal part with a Gaussian mean-reversion term plus a Lévy jump term and a time-dependent market price of risk
- Benth and Benth's model [28]: deterministic seasonal part with a non-Gaussian mean-reversion process and a constant-time change of measure

3.3.1 Mean-Reversion Process

Because of the characteristics of the electricity spot and futures prices illustrated in Chap. 2, a Gaussian mean-reversion process with a seasonal part seems very optimistic for modelling their behaviour. Nevertheless, Lucia and Schwartz's model [127] is a benchmark for measuring the effect of standard models when being calibrated to data on the electricity spot price. The authors try different variants but one is enough to understand the problems involved in this modelling.

I concentrate on the model using the mean-reversion on the log-price of the spot P_t. Here, P_t is the daily average spot price, and not the hourly spot price, satisfies

$$\ln P_t = f(t) + Y_t \tag{3.21}$$

with $f(t)$, a deterministic known function of the time that represents the seasonal part, and

$$dY_t = -\kappa Y_t dt + \sigma dW_t \tag{3.22}$$

which is a mean-reverting process with a null long-run mean, constant reverting speed κ, and an initial condition y_0. This model is identical to Schwartz's 1997 one-factor model [152]. The law of P_t is well known and satisfies:

$$\mathbb{E}_0\left[P_t\right] = \exp\left(f(t) + (\ln P_0 - f(0))e^{-\kappa t} + \frac{\sigma^2}{4\kappa}(1 - e^{-2\kappa t})\right)$$

$$\mathrm{Var}_0\left[P_t\right] = \mathbb{E}_0(P_t)^2\left[\exp\left(\frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t})\right) - 1\right]$$

Although the underlying cannot be stored, the authors suppose that there is a unique risk-neutral measure under which it is possible to write that the futures prices are equal to the expected spot price:

$$F(t,T) = \mathbb{E}_t\left[S_T\right].$$

They suppose also that this measure is given by a constant market price of risk λ to carry the futures valuation. And, finally, as in Schwartz [152], under this risk-neutral measure, the dynamic of Y remains mean-reverting and is given by $dY_t = \kappa\left(\alpha^* - Y_t\right)dt + \sigma dW_t^*$ with $\alpha^* = -\lambda\sigma/\kappa$. As raised by Carmona and Durrleman [57, Sect. 8, p. 671], the common practice in the literature is to suppose that the risk-neutral measure preserves the mean-reversion property of the spot, although the drift might be expected equal to the risk-free rate.

To make this hypothesis convenient, the price $F(0,T)$ of the future quoted at time 0 for delivery at the instant T is given by the expectation of the spot under this measure and can be computed analytically—which is all in the interest of having a constant market price of risk:

$$\begin{aligned}\ln F(t,T) = {} & f(T) + e^{-\kappa(T-t)}\left(\ln P_t - f(t)\right) \\ & + \alpha^*\left(1 - e^{-\kappa(T-t)}\right) + \frac{\sigma^2}{4\kappa}\left(1 - e^{-2\kappa(T-t)}\right).\end{aligned} \quad (3.23)$$

These results allow for an estimation and calibration of the model. The deterministic component of the spot f is considered to be given by a sinusoidal function $f(t) = a + bD_t + c\cos\left((t+\tau)2\pi/365\right)$ where D_t equals one if t is a holiday or a weekend. An estimation of the model is thus performed on the NordPool data covering January 1993 to December 1999, which represents seven years of daily data (see Fig. 3.1).

The estimation of the parameters κ, σ, a, b, c, τ are performed in a single non-linear least-square procedure. The estimation leads to a significant non-zero mean-reversion coefficient of $\kappa = 0.016 \times \Delta t = 5.84$ per year with a time step equal to one day and a volatility of 164 %. The goodness-of-fit of the model can be measured by a mean absolute percentage error and is lower than 1 %. But, despite the fact that the model can be made close to the observations, the general aspect of the simulated

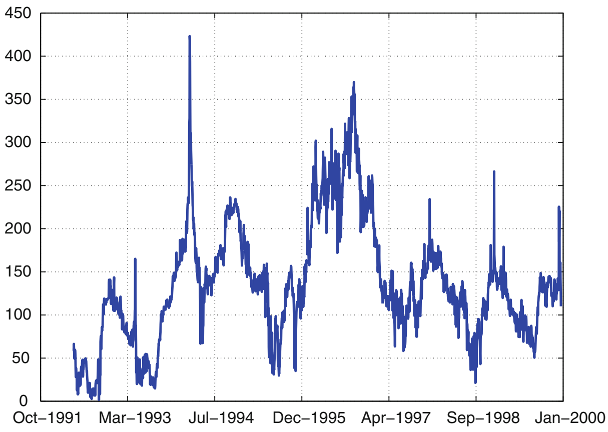


Fig. 3.1 NordPool historical daily spot prices. *source* NordPool

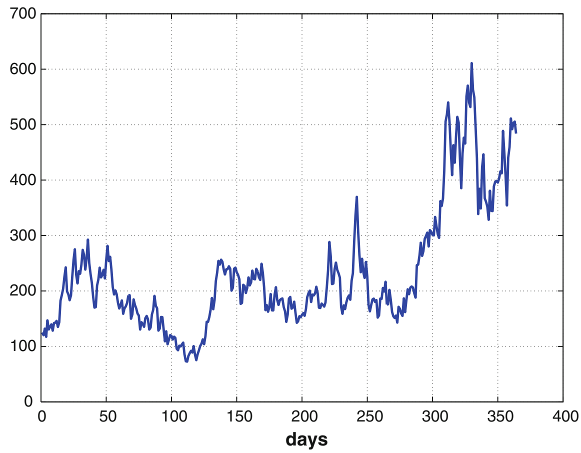


Fig. 3.2 Simulated daily spot price trajectory obtained with Lucia and Schwartz's [127] one-factor log spot model 4 with parameter values of $\kappa = 5.84$, $\sigma = 164\,\%$, $a = 4.86$, $b = -0.09$, $c = 0.306$, $\tau = 0.836$

prices is quite different from the observed NordPoool daily spot price as Fig. 3.2 shows.

The calibration of the model is thus performed by using the observed forward prices quoted during one year (end 1998 to end 1999). Denoting the number of days before the beginning of delivery by T_1 and the number of days before the end of delivery by T_2, the theoretical forward price $\mathscr{F}(0, T_1, T_2)$ is computed as

$$\mathscr{F}(0, T_1, T_2) = \frac{1}{T_2 - T_1} \sum_{T_1 \leq T \leq T_2} F(0, T).$$

By using the model above with an out-of-sample parameter estimation and a null market price of risk, the authors get a root mean square error (RMSE) over all of the

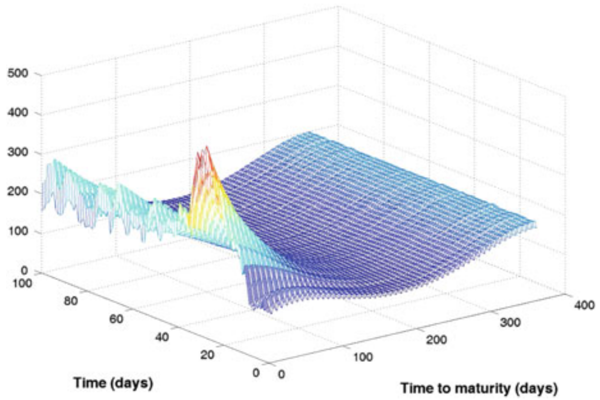


Fig. 3.3 Dynamic of the forward curve for Lucia and Schwartz's [127] one-factor log spot model 4 with parameters identical to Fig. 3.2

sample contracts of ≈9 %, or a monetary unit of 11 NOK. Thus, there are not enough parameters in the model to allow for a perfect fit of the forward curve.

Moreover, the dynamic of the forward price given by the relation (3.23) does not depend on the current time for a sufficiently long enough time to maturity because of the damping effect of the mean-reversion coefficient. Thus, the estimated value of the long time to maturity forward is quickly the seasonal component of the forward price. This point is illustrated on Fig. 3.3.

Thus, as a conclusion, it is possible to have a good fit of the historical daily spot prices with this model, but the simulated prices do not exhibit any spikes. Moreover, as expected, a constant market price of risk is not enough to succeed in getting the required perfect match between the predicted futures prices and the currently quoted ones. A one factor model is not enough to get the dynamic of the forward curve because Sect. 3.2 shows that at least two are required to get more than 80 % of the variance in the storable commodities and even more are required for electricity. A statistical analysis of this model is also performed in Wilkens and Wimschulte [168] who report poor performance. However, surprisingly despite these aspects, this model is often used in some utilities with a seasonal market price of risk as the only modification.

3.3.2 Mean-Reverting Jump-Diffusion Models

The first idea to improve the behaviour of the spot price in the model (3.21–3.22) is to add a jump component. This is the approach implemented in Cartea and Figueroa [61] and in Geman and Roncoroni [94]. I am going to focus on Cartea and Figueora's (2005) work because they provide formulas for the futures prices. The spot price S_t is now assumed to follow:

$$\ln S_t = g(t) + Y_t$$

with g as the deterministic log-seasonality that is supposed to be differentiable, and Y_t as the stochastic process that satisfies

$$dY_t = -\alpha Y_t dt + \sigma(t)dW_t + J \cdot dq_t$$

with α as the speed of the mean-reversion, $\sigma(t)$ as the time-dependent volatility, J as the proportional random jump-size, and dq_t as the Poisson process with intensity l. The jump-size J is supposed to be log-normally distributed with $\ln J \sim N(\mu_J, \sigma_J^2)$.

Given this framework, the spot price dynamic satisfies

$$dS_t = \alpha\left(\rho(t) - \ln S_t\right) S_t dt + \sigma(t) S_t dW_t + S_t(J-1)dq_t$$

with

$$\rho(t) = \frac{1}{\alpha}\left(g'(t) + \frac{1}{2}\sigma^2(t)\right) + g(t).$$

The term $S_t(J-1)dq_t$ comes from the fact that after a shock, S_{t-} moves to JS_{t-}, which makes $\Delta S_t = (J-1)S_{t-}$. Moreover, the authors assume that J satisfies $\mathbb{E}[J] = 1$ based on the argument that jumps should not lead to an extra return. This constraint limits the parameters of the model because $\mu_J = -\sigma_J^2/2$.

Figure 3.4 illustrates the behaviour of the daily average spot price for the realistic parameters estimated on the UK day-ahead market extracted from Cartea and Figueora's (2005) paper. Because the parameter values for the seasonal part of the spot price is not given in the paper, I use the same seasonal function as in Sect. 3.3.1 for Lucia and Schwartz's [127] model. I observe that the spiky behaviour is now clearly obtained. This phenomenon is obtained thanks to the strong per annum mean-reversion value $\alpha = 102$. It corresponds to a half-life of two days. Thus, although

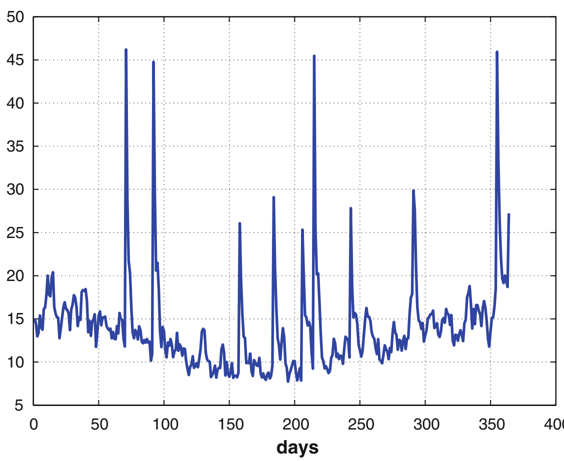


Fig. 3.4 Simulated daily spot price trajectory obtained with Cartea and Figueroa [61] with the parameter values of $\alpha = 102$, $\sigma_J = 0.67$, $l = 8.5$, $\sigma = 1.64$. The seasonal part is identical to Lucia and Schwartz's [127] model of Fig. 3.2

quick, this return to a normal level price is still too slow to represent the change in a price within an hour.

For the valuation of futures prices, the authors follow Lucia and Schwartz's [127] approach by using a constant market price of risk λ. Under this measure, $x = \ln S$ satisfies

$$dx_t = \alpha\left(\mu^*(t) - x_t\right)dt + \sigma_t dW_t^* + \ln J dq_t$$

with $\mu^*(t) = \frac{1}{\alpha}g'(t) + g(t) - \lambda\frac{\sigma(t)}{\alpha}$.

The first interesting result in this setting is that it allows an analytical computation of the futures prices. Thus, $F(t, T)$, the price quoted at time t for delivery at the instant T, is given by:

$$F(t, T) = G(T)\left(\frac{S_t}{G(t)}\right)^{e^{-\alpha(T-t)}} \mathscr{D}(t, T)\mathscr{J}(t, T),$$

where $G(t) = \exp(g(t))$ represents the seasonal part; $\mathscr{D}(t, T)$ is a term which comes from the diffusion,

$$\mathscr{D}(t, T) = \exp\left(\int_t^T \left[\frac{1}{2}\sigma^2(s)e^{-2\alpha(T-s)} - \lambda\sigma(s)e^{-\alpha(T-s)}\right]ds\right),$$

and $\mathscr{J}(t, T)$ is the term which comes from the jumps

$$\mathscr{J}(t, T) = \exp\left(l\int_t^T (\xi(T, s) - 1)ds\right)$$

with

$$\xi(T, s) = \exp\left(-\frac{\sigma_J^2}{2}\left(e^{-\ alpha(T-s)} - e^{-2\alpha(T-s)}\right)\right).$$

At this point, some remarks can be made on that formula. First, because the market price of risk is constant, there is little chance to perfectly fit the forward prices at the initial time. This point could be somehow addressed by using a seasonal market price. Second, the jump intensity l and the jump-size σ_J both have a negative impact on the forward price. It is difficult to assess a change in one parameter only because a modification of the intensity of jump l might have an impact on the estimation of the volatility σ. However, a blind comparative static leads to a decrease in the $F(t, T)$ that causes an increase in l because the term $\int_t^T (\xi(T, s) - 1)ds$ is negative. Moreover, a jump of S at time t has an impact on the whole term structure. But, because of the strong mean-reversion value of α, the effect is strongly dampened. However, this advantage is also a drawback because the mean-reversion property that is useful to quickly erase the effect of a jump on the spot market leads to a flat dynamic for the long-term maturities of the forward prices. As mentioned by the authors themselves in a footnote, it is necessary to use far lower mean-reversion values to get a realistic dynamic for the forward with long-term maturity and to avoid a forward dynamic similar to Fig. 3.3.

3.3.3 Non-Gaussian Mean-Reversion Models

An alternative to the strategy with a jump term added to a diffusion process is to directly represent the spot price with a non-Gaussian process. This alternative is quickly recognised as a possible strategy in the seminal paper of Benth et al. [22]. More works have followed this track, such as Deng and Jiang [75]. Benth et al.'s monograph [19] is the main reference in this matter. Even if in Benth et al. [22], the authors develop a richer version with more than one Lévy factor, in this section, I present the Benth and Saltyte-Benth's [28] model, which is the most basic non-Gaussian model.

In this approach, the electricity spot price S_t at time t is modelled as

$$S_t = \Lambda_t e^{X_t}, \tag{3.24}$$

where Λ_t represents a deterministic seasonal component while the random part X_t follows the dynamic

$$X_t = a(m - X_t)dt + dL_t, \tag{3.25}$$

with a as the mean-reversion parameter, m as the long-run mean of X, and L_t as the Lévy process. This method comprises the increments of the process L_t which are independent and stationary in the sense that the distribution of $L_t - L_s$ depends only on $t - s$. Here, I will spare the reader the rather technical conditions that are raised when using Lévy processes, and I will consider dL_t as a random increment following a given density. Amongst all of the possible densities, the Normal Inverse Gaussian (NIG) receives the most attention. The density function of the NIG class is explicitly known and depends on four parameters: the location of the density μ, the heaviness of the tail α, the skewness β, and the scale δ. This class of Lévy processes offers a good representation of the heavy tails of the density of the observed returns. And because the density is known explicitly, it is possible to perform an efficient maximum likelihood estimation.

Figure 3.5 illustrates the spot price modelled according to a NIG process. The parameters as well as the seasonality are chosen according to Benth and Saltyte-Benth's [28, Sect. 3.2] for the daily average NordPool spot price.

In this incomplete market setting, there are infinitely many equivalent martingale measures $\mathbb{Q}$ such that $F(t,T) = \mathbb{E}_t^{\mathbb{Q}}[S_T]$. But it is possible here to design a change of measure that will satisfy the former relation and to allow a perfect fit of the initial forward curve in certain conditions. This change of measure—based on Esscher's transform—is parameterised by a deterministic function of the time θ. In the NIG

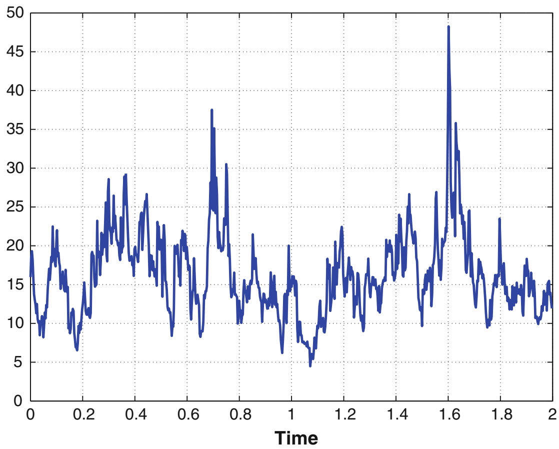


Fig. 3.5 Simulated daily spot price trajectory obtained with Benth and Saltyte-Benth [28] with the parameter values of $m = -0.3$, $a = 0.108$, $\mu = -0.01284$, $\alpha = 7.91$, $\beta = 0.73$, $\delta = 0.139$, and $\Lambda(t) = c_1 + c_2 \cos\left(\frac{2\pi}{3}(t - c_3)\right)$ with $c_1 = 19.14$, $c_2 = 3.23$, $c_3 = 10.47$

model (3.24–3.25), the forward prices are given by:

$$F^\theta(t,T) = \Lambda(T)\exp\left(\frac{\mu}{a}(1 - e^{-a(T-t)})\right)\left(\frac{S_t}{\Lambda(t)}\right)^{e^{-\alpha(T-t)}} \times \exp\left(\delta\int_t^T \left[\sqrt{\alpha^2 - (\theta(s)+\beta)^2} - \sqrt{\alpha^2 - (\theta(s)+\beta+e^{-a(T-s)})^2}\right]ds\right) \tag{3.26}$$

Thus, if at time zero, there are n forward prices $(f_i)_{i=1\dots n}$ for delivery at the non-overlapping maturities $0 < T_1 < \cdots < T_n$ and if one chooses a piece-wise constant function $\theta(t) = \theta_i$ on the interval (T_{i-1}, T_i), then $(\theta_i)_{i=1\dots n}$ satisfies:

$$f_i = q_i \times \Theta_i \tag{3.27}$$

with

$$q_i = \Lambda(T_i)\exp\left(\frac{\mu}{a}(1 - e^{-a(T_i)})\right)\left(\frac{S_0}{\Lambda(0)}\right)^{e^{-\alpha T_i}}$$

and

$$\Theta_i = \exp\left(\delta\int_0^{T_i} \underbrace{\left[\sqrt{\alpha^2 - (\theta(s)+\beta)^2} - \sqrt{\alpha^2 - (\theta(s)+\beta+e^{-a(T_i-s)})^2}\right]}_{h(s,T_i,\theta(s))} ds\right).$$

It is possible to recursively solve the system (3.27) starting with θ_1. Once θ_1 is obtained, θ_2 comes from:

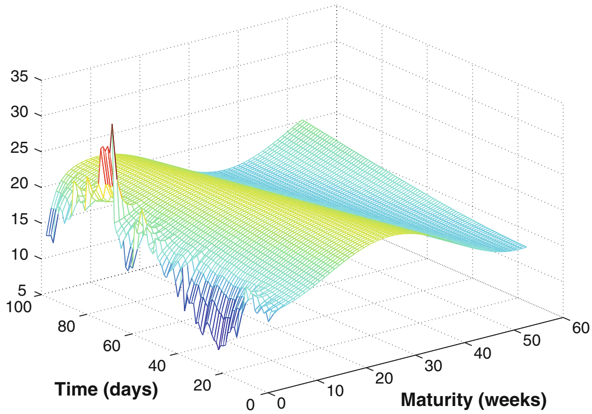


Fig. 3.6 Dynamic of the forward curve for Benth and Saltyte-Benth [28] with parameter values identical to Fig. 3.5

$$f_2 = q_2 \times \exp\left[\int_0^{T_1} h(s, T_2, \theta_1)ds\right] \times \exp\left[\int_{T_1}^{T_2} h(s, T_2, \theta_2)ds\right].$$

And so on for the next values of θ_i, $i \geq 3$. This procedure is not describe in Benth and Slatyte [28] but is in Benth et al. [22]. It is still in a Lévy model setting but with instances that make the system (3.27) linear.

However, the use of a non-Gaussian process does not circumvent the problem of the dampening effect of the mean-reversion on the forward curve dynamic. This phenomenon is illustrated on Fig. 3.6 where I use the forward prices given in the NIG case by the relation (3.26) and the parameter values given in [28, Sect. 3.2]. In particular, the mean-reversion value is rather high and equals 0.108. However, the behaviour is the same as the mean-reversion jump-diffusion process in Sect. 3.3.2. The short-term maturities present some volatility while the longer maturities are reduced to the seasonal component of the spot price.

3.3.4 Conclusion

This section shows that it is possible to obtain a realistic model of the spot price for electricity by using one-factor models. They succeed in reproducing the main stylised facts of the spot: seasonality, mean-reversion, and spikes. But, the class of one-factor spot models studied so far fail to reproduce the proper volatility in the forward price. A one-factor model with volatility functions depending on the spot could lead to better results. But, possibly not, because the forward prices are being impacted by information that has no effect on the spot, like the planned outages of power plants or movements of the forward curve of fuel prices. This point is not particular to electricity. The dynamic of the forward curve of other commodities also requires more than one factor to get a realistic modelling of its behaviour. But, the dynamic is accentuated in the case of electricity because the amount of energy that can be transported over the maturities through storage is very limited compared to consumption. This point is clearly illustrated in the study of the information premium in Benth et al. [20].

3.4 Multi-factor Spot Models

It seems that if the Markov property of the spot is given up, then the possibilities for modelling become limitless. Thus, before going through a review of the alternatives already developed in the literature (Sect. 3.4.2), I will start with an illustrative example to show how the addition of just one new factor avoids the collateral damage of the mean-reversion on the forward dynamic.

3.4.1 An Illustrative Example

In this example, I consider that the spot price S_t at time t is given by

$$S_t = \Lambda_t e^{X_t + Y_t} \tag{3.28}$$

where Λ_t stands for the deterministic seasonal component of the price, X_t follows the mean-reversion process

$$dX_t = \lambda_X(\mu_X - X_t)dt + \sigma_X dW_t,$$

Y_t is the mean-reverting jump process

$$dY_t = -\lambda_Y Y_t dt + h\, dN_t$$

with $\ln h \sim N(\mu_J, \sigma_J)$, and N_t is a Poisson process with intensity l. The forward price $F(t, T)$ is taken equal to the expectation of the spot price at maturity without any specification on the probability. The point here is to only illustrate the effect of this modelling on the dynamic of the forward curve.

In this setting, the fact that there are two different mean-reversion speeds allows me to separate the need for a quick dampening of the spikes occurring because of Y from the trend given by X. I set the parameters to standard realistic values. For the diffusion factor I use $\lambda_X = 0.03$, $\mu_X = 0$, and $\sigma_X = 1.64$ for the jump factors; and $\lambda_J = 200$, $\sigma_J = 0.3$, and $\mu_J = -\sigma_J^2/2$ for the intensity of the jumps $l = 10$. For the seasonal component, I use the function Λ_t from Benth and Saltyte [28] presented in Sect. 3.3.3.

Figure 3.7 provides an illustration of the behaviour of the electricity spot price while Fig. 3.8 provides the forward curve dynamic. It appears that the spot price

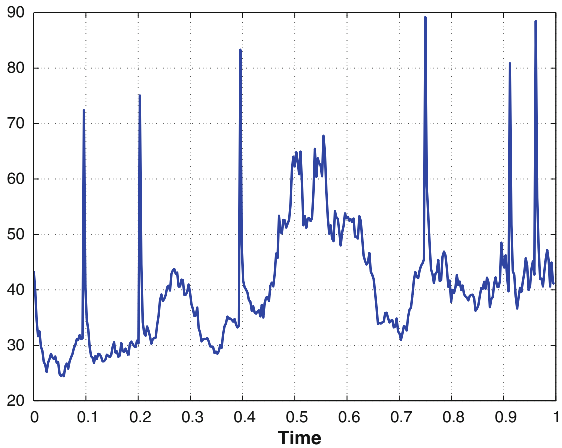


Fig. 3.7 Simulated daily spot price trajectory obtained with the model (3.28) with the parameter values of $\lambda_X = 0.036$, $\mu_X = 1.025$, $\sigma_X = 3.8$, $\lambda_J = 200$, $\mu_J = 1$, $\sigma_J = 1$, $l = 10$ and $\Lambda_t = c_1 + c_2 \cos\left(\frac{2\pi}{3}(t - c_3)\right)$ with $c_1 = 19.14$, $c_2 = 3.23$, $c_3 = 10.47$

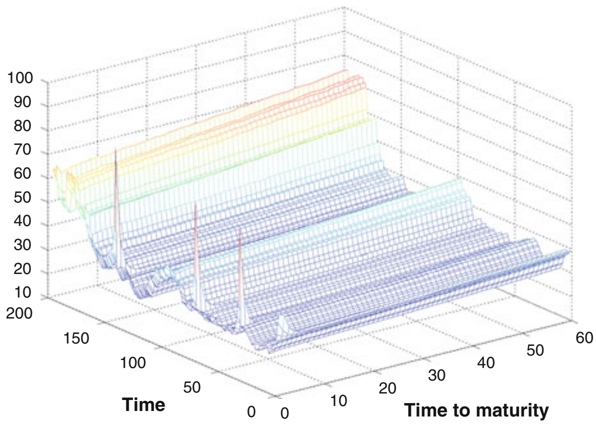


Fig. 3.8 Dynamic of the forward curve for the two-factor model (3.28) with the same parameters as in Fig. 3.7

exhibits the required spikes while the whole forward curve is no longer fixed to a constant value. The forward curve now floats according to the random variations of the diffusion factor.

This toy example does not say anything about the possibilities of estimating the different component of the price. Here, only S_t is being observed while the estimation procedure should reconstruct two different processes. Generally, using some kind of filtering methods cannot be escaped when performing the parameter estimation of a multi-factor model.

3.4.2 A Review of the Multi-factor Models

As I said in the introduction of this section, due to the important number of possibilities, it is not easy to find a logical way of sorting the different models proposed in the literature. A not perfect but feasible way is to sort them according to the dynamic of the hidden state, whether it is a discrete Markov chain or a continuous process, and according also to the Gaussian or non-Gaussian character of this dynamic.

I begin with the continuous hidden state model with Gaussian noise. Kaaresen and Husby [107] and Barlow et al. [11] develop this modelling framework. I focus on Barlow et al. [11] where the authors investigate different models in the spirit of those proposed by Schwartz [152] for storable commodities. They perform the parameter estimations for three models of the spot:

Model I: $S_t = e^{X_t}$ $dX_t = \lambda_X(Y_t - X_t)dt + \sigma_X dB_t^{(1)}$
$dY_t = \mu dt + \sigma_Y dB_t^{(2)}$.

Model II: $S_t = e^{X_t}$ $dX_t = \lambda_X(Y_t - X_t)dt + \sigma_X dB_t^{(1)}$
$dY_t = \lambda_Y(L - Y_y)dt + \sigma_Y dB_t^{(2)}$

Model III: $dS_t = \lambda_X(Y_t - S_t)dt + \sigma_S S_t dB_t^{(1)}$
$dY_t = \mu Y_t dt + \sigma_Y Y_t dB_t^{(2)}$

In these three models, $B^{(1)}$ and $B^{(2)}$ are independent Brownian motions.

In their work, Barlow et al. [11] focus on a recurrent difficulty in multi-factor models: the estimation of the parameters of the dynamic of the hidden state. By using a Kalman filter algorithm, they perform the parameter estimations of these three models both with simulated data and with real market data from North America. They conclude that due to the large variation in the spot price and the short length of the available time series compared to the length needed for the simulated data to perform a good estimation of the parameters, it is not possible to properly estimate the mean-reversion values of the models. Nevertheless, these models succeed in producing more volatile time series than the one-factor mean-reversion models considered in Sect. 3.3. However, they fail to represent the isolated spikes.

A natural alternative while maintaining the idea of a continuous hidden state is to look at multi-factor non-Gaussian models. This is the approach developed in Benth et al. [23]. They define the spot price S_t as $S(t) = \mu(t) + X(t)$ with μ as the deterministic seasonal component and X as an arithmetic average of the non-Gaussian mean-reverting process

$$X(t) = \sum_i w_i Y_i(t) \tag{3.29}$$

where

$$Y_i(t) = -\lambda_i Y_i(t)dt + \sigma_i(t)dL_i(t). \tag{3.30}$$

with L_i as the pure jump process. The main interest of the arithmetic model (3.29) is that it provides analytical formulas for the forward prices with variable delivery periods. In this model, $S(t)$ tends to go back to the seasonal component μ with a speed controlled by the parameters w_i and λ_i. The processes L_i control the variation in the spot price (both daily volatility and spikes). The functions σ_i allow for seasonal trends in the volatility. The intensity of the jump is also made seasonal. The forward prices can be computed with no more complexity than with the one-factor non-Gaussian model in Sect. 3.3.3. The simulations of the spot price are given for some instances with three Ornstein-Uhlenbeck processes that capture well the texture of the NordPool spot prices. The estimation of the parameters in this model is somehow not obvious. Because it is neither Markovian nor Gaussian, a classical Kalman filter cannot be relied on. But, there are possible methods that could overcome this difficulty.

The final alternative consists in using the hidden Markov chains. In these models, the spikes occur because a hidden variable switches from one state to another. In the economic literature on modelling a pure electricity spot price, there are a lot of articles on regime switching models. The works that compute futures prices and perform the calibration of the model with the forward curve are rarer. The first example of this kind of approach is by Kholodnyi [116]. In this work and in the references therein, the author develops a non-Markovian approach with a two-state regime switching model (one state for the spiky regime and one for the regular regime). The authors do a computation of the forward prices but have yet to publish the calibration and the implementation to the markets.

The only known full implementation of a regime switching model with the computation of the forward prices and the calibration in a electricity market was conducted in Monfort and Féron [135]. Their approach is based on a discrete time affine model and a stochastic discount factor. Their work presents more a class of models than a single one. The computation of the futures prices requires the use of a Laplace transform and produces semi-explicit expressions. The estimation of their model uses a Kalman filter. They perform a numerical illustration on the French market with a regime switching vector autoregressive model. The dynamic of the spot price depends on a Markov chain with three states and an unobservable continuous latent variable, which makes their model three dimensional. To provide an example of their approach, the form of the spot price $S_t = \exp s_t$ is:

$$s_{t+1} = \mu' z_{t+1} + \sum_{i=1}^{3} \phi_{1i}(s_{t-i+1} - \mu' z_{t-i+1}) + \phi_{21} y_t + \sigma z_{t+1} \varepsilon^s_{t+1}$$

$$y_{t+1} = \psi_1(s_t - \mu' z_t) + \psi_2 y_t + \varepsilon^y_{t+1},$$

where $\mu = [\mu_1, \mu_2, \mu_3]$ and $\sigma = [\sigma_1, \sigma_2, \sigma_3]$ while z_t takes its value from the columns of the identity matrix of size three, and y_t is a hidden continuous state that helps the fit with the forward prices. One trick of this modelling approach is in the fact that the dynamic is written as $s_t - \mu' z_t$. Thus, if z_t is in a spiky state and has a low probability of staying in that state, then in the next step, $s_{t+1} - \mu' z_{t+1}$ forgets the spike. With this model, the authors succeed in reconstructing the different forward prices (week, month, quarter, and year) with good precision.

To conclude this section and to make a transition into the next one, I have to mention that multi-factor models do not exclude the use of observable variables. Indeed, the model developed by Burger et al. [48] represents a multi-factor model mixing both observed variables (the load and its ratio with the maximum available capacity) and two non-observable factors to take into account the long-term trend of the prices and the short-term volatility that cannot be explained by the load.

3.4.3 Conclusion

Multi-factor models of the spot price supply a great set of possibilities. Although there are not many papers which present the whole modelling and calibration process of the spot and forward prices, there is little doubt that they are able to produce a fit with the observed forward prices. However, with this class of models, the difficulty moves from the computation of the forward prices to the estimation and calibration procedures. As indicated, the statistical methods which are involved to perform the estimation always require some filtering methods. If the use of a Kalmant filter in finance is not new, some models presented here require filtering methods that are beyond the common toolbox of the financial engineers.

3.5 Structural Models

Because more than one factor is needed to properly represent the spot price while having a realistic dynamic of the forward curve, *structural models* use the available information on the electric system to get the extra factors needed. For instance, the approach based on the multi-factor models in Sect. 3.4 is justified for markets where little information is available on the supply and demand side or on the causes of the prices. This is not the case for electricity markets. It is the opposite. In each country, regulators impose the publication of a large amount of data. A quick look at the NordPool website is convincing: for example, hourly historical consumption per geographic area, detailed production and planned outages, exchanges, and the levels of the reservoirs.

The recent survey by Carmona and Coulon [51] clearly presents the advantages and the drawbacks of this approach compared to reduced-form models, that is, the models described in Sects. 3.3 and 3.4. The structural models move the modelling difficulty of electricity prices away from the statistical methods involved in multi-factor models to a focus on the dependencies and the price formation mechanism. These models find a certain echo in the electrical engineering community. For example, the financial engineers in electric utilities generally find themselves more comfortable using Gaussian models for the observed data and building a price as a function of those bricks than implementing a customised filtering method for a multi-factor model.

In this section, I call structural all of the models that use only some observable variables as extra factors. The depth of this structure can vary a lot amongst the models. I sort them from the lighter to the stronger structure. I begin first with the seminal structural model designed by Barlow [10] (Sect. 3.5.1). Although he does not do the computation of the forward prices in his paper, the idea of this paper deserves special attention. Then I turn to the spot price models which use the available information as auxiliary variables (Sect. 3.5.2). In this class, I examine Cartea and Villaplana's [64] model which uses demand and capacity; Pirrong and Jermakayan's [144] model which uses demand and gas prices; and the models which use co-integration techniques such as Frikha and Lemaire [92], Dejong and Schneider [72], and Benmenzer et al. [17]. Then, I review the structural models which implement a simplified stack curve mechanism (Sect. 3.5.3).

3.5.1 The Mother of All Structural Models

Barlow's paper [10] shows that the spiky behaviour of electricity spot prices can be obtained by using an inverse demand function applied to an Ornstein-Uhlenbeck process. In his model, the spot price S_t is determined by the equilibrium between an increasing supply function $u_t(x)$ and a decreasing demand function $d_t(x)$, $u_t(S_t) = d_t(S_t)$. Assuming a constant supply function $u_t = g$ and an inelastic demand $d_t(x) = D_t$ with D_t for the electricity consumption, the spot price is given by $S_t = g^{-1}(D_t)$. He uses a basic non-linear form for $g(x) = a_0 - b_0 x^\alpha$ with $\alpha < 0$. The price is caped if the demand exceeds the maximum capacity a_0. The spot price is thus given by the relation:

$$S_t = \begin{cases} \left(\frac{a_0 - D_t}{b_0}\right)^{1/\alpha} & D_t \le a_0 - \varepsilon_0 b_0 \\ \varepsilon_0^{1/\alpha} & D_t \ge a_0 - \varepsilon_0 b_0 \end{cases} \tag{3.31}$$

where ε_0 is determined by the level of the price cap A_0.

Remark 3.5 The inverse demand function can be rewritten $g^{-1}(D_t) \approx \frac{C}{(a_0 - D_t)}$ by considering that $\alpha \approx -1$ (see the estimation below). In this regard, it appears that the price is inversely proportional to a quantity that can be interpreted as a residual capacity if a_0 is the maximum available capacity in the system.

After some algebraic simplifications, the spot price dynamic results in a non-linear Ornstein-Uhlenbeck process that can be expressed as:

$$S_t = \begin{cases} (1 + \alpha X_t)^{1/\alpha} & 1 + \alpha X_t \le \varepsilon_0 \\ \varepsilon_0^{1/\alpha} & 1 + \alpha X_t \ge \varepsilon_0 \end{cases} \tag{3.32}$$

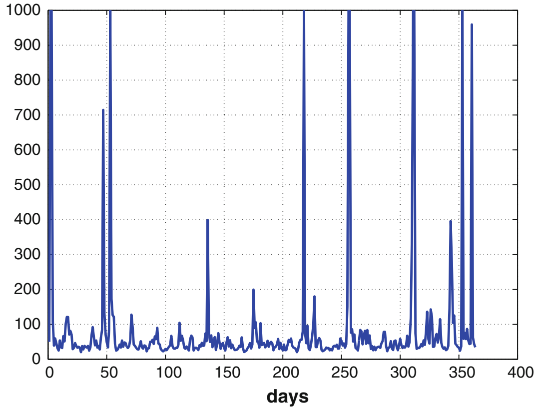


Fig. 3.9 Simulated daily spot price trajectory obtained with Barlow's model [10] with parameter values of $\alpha = -1.08$, $\lambda = 172$, $a = 0.91$, and $\sigma = 0.91$

$$dX_t = -\lambda (X_t - a)\, dt + \sigma dW_t, \tag{3.33}$$

where $A_0 = \varepsilon_0^{1/\alpha}$ and the remaining parameters λ, α, a, and σ have to be estimated.

Figure 3.9 presents an example of a spot price trajectory with the values of the parameters obtained by the author on Alberta's market data. Although the modelling framework makes use only of a continuous process, the trajectory exhibits a striking spiky behaviour.

Nevertheless, despite this success, there are still some problems left in this model. As Carmona and Coulon [51] point out, the estimation parameter leads to a negative $\alpha = -1.08$ which makes the supplys curve a steep function near the breaking point given by $a_0 - \varepsilon_0 b_0$. Thus, the model tends to produce a lot of prices equal to the price cap. Indeed, the steepness needed to produce spikes leads to a very short range of demand where prices can explode. Hence, when a spike occurs, it is more likely to reach the cap. Some improvements have been made to Barlow's model. In particular, Kanamura and Ohashi [110] and Kanamura [109] improve the offer curve to make it closer to a hockey stick to avoid the concentration of prices at the price cap.

But, this model is only a one-factor model. Thus, even if no attempt is made to compute and analyse the forward price induced by the relation (3.32), it is predictable that the forward prices do not avoid the drawback of the one-factor model presented in Sect. 3.3. And the model exhibits a flat dynamic, as shown in Fig. 3.10.

3.5.2 Auxiliary Variables

The first approach in using auxiliary variables is to rely on the statistical relation between the electricity prices and the variables that cause them, such as fuel prices.

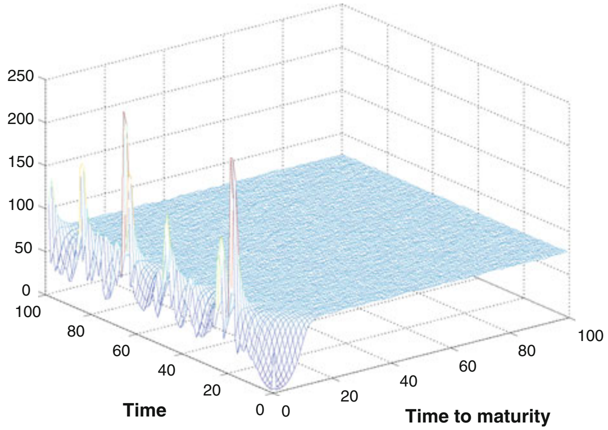


Fig. 3.10 Dynamic of the forward curve obtained with Barlow's (2002) model with parameter values identical to Fig. 3.9 and prices cut at 250 for aesthetic reasons

In particular, co-integration represents the dependence of the gas prices on the oil prices in Benmenzer et al. [17]. This idea is developed in the case of the relation between the electricity prices and the gas prices in continental Europe in Dejong and Schneider [72] and Frikha and Lemaire [92]. Unfortunately, the computation of the futures prices and the calibration of the forward curve is left for future work.

Close to this approach but with a more practical design is the model by Grine and Diko [95]. Their main idea is to use a multi-layered structure to reproduce the intuitive relationship between fuel dependencies. If there are m fuels, then the fuel price i is given by

$$S^i(t) = \left(\sum_{j \in D^i} \gamma^i_j S^j(t) + \eta^i(t) \right) \times \hat{\omega}^i(t) \times exp(b^i(t) X^i(t)).$$

The time varying functions η^i and $\hat{\omega}^i$ represent the yearly and weekly pattern, while X is an affine jump-diffusion process (drift and volatility are an affine function of X), and γ^i_j, $b^i(t)$ are the parameters. Each X^i has to be independent of the price processes of the underlying energies to avoid cycles (X depends on Y which depends on Z which depends on X). Thus the summation is made over a set D^i that depends on fuel i. The interest of this modelling is that the relation between the spot prices is transferred straightforwardly to the futures:

$$F^i(t,T) = \left(\sum_{j \in D^i} \gamma^i_j F^j(t,T) + \eta^i(T) \right) \times \hat{\omega}^i(T) \times \mathbb{E}^{\mathbb{Q}}_t \left[\exp(b^i(T) X^i(T)) \right].$$

In this framework, the authors succeed in providing an analytical approximation of the forward prices and the vanilla spread options.

Nevertheless, the approach above introduces known dependencies between the electricity prices and the fuel prices by using only a statistical relation and not a microeconomic model. This is the solution developed in Cartea and Villaplana [64] and Lyle and Elliott [129] with demand and capacity, and Pirrong and Jermakyan [144] with the load and the gas price.

I illustrate the use of the observable auxiliary variables with Cartea and Villaplana's model [64]. In their model, the spot price P_t is defined by an equilibrium function ϕ such that

$$P_t = \phi(D_t, C_t)$$

where D is the electricity consumption and C is the available capacity. The equilibrium function ϕ increases with D and decreases with C. As in Barlow's model, the relation between the observed variables and the price is deterministic. The function ϕ and the dynamic on the demand D_t and the production capacity C_t are chosen to allow for analytical computation of the conditional expectations of the spot price:

$$P_t = \beta \exp(\gamma C_t + \alpha D_t) \tag{3.34}$$

with positive α and β but negative γ. For the demand and capacity dynamic, the authors use a deterministic seasonal part plus Ornstein-Uhlenbeck noise. They set:

$$D_t = g_t^D + X_t^D \quad dX_t^D = -\kappa^D X_t^D dt + \sigma_t^D dW_t^D, \tag{3.35}$$

$$C_t = g_t^C + X_t^C \quad dX_t^C = -\kappa^C X_t^C dt + \sigma_t^C dW_t^C, \tag{3.36}$$

where W^D and W^C are independent. The estimation of the parameters of these dynamics is done by applying the maximum likelihood principle to the historical data on demand and capacity.

Because of these choices, the forward prices can be analytically computed. Adding two time-dependent market prices of risk $\phi^D(t)$ and $\phi^C(t)$ for each un-hedgeable factor D and C, the dynamic of X^C and X^D reads as

$$dX_t^D = -\kappa^D \left(X_t^D + \theta^D(t)\right) dt + \sigma^D(t) dW_t^D,$$

$$dX_t^C = -\kappa^C \left(X_t^C + \theta^C(t)\right) dt + \sigma^C(t) dW_t^C,$$

with $\theta^D(t) = \dfrac{\sigma^D(t)\phi^D(t)}{\kappa^D}$ and $\theta^C(t) = \dfrac{\sigma^C(t)\phi^C(t)}{\kappa^C}$.

And thus, the futures write as

$$\begin{aligned}\ln F(t,T) &= h(T) \\ &\quad + \gamma\left(e^{-\kappa^C(T-t)}X_t^C + \kappa^C\int_t^T e^{-\kappa^C(T-s)}\theta^C(s)ds \right.\\ &\quad\left. + \frac{1}{2}\int_t^T e^{-2\kappa^C(T-s)}\sigma_C^2(s)ds\right) \\ &\quad + \alpha\left(e^{-\kappa^D(T-t)}X_t^D + \kappa^D\int_t^T e^{-\kappa^D(T-s)}\theta^D(s)ds \right.\\ &\quad\left. + \frac{1}{2}\int_t^T e^{-2\kappa^D(T-s)}\sigma_D^2(s)ds\right)\end{aligned} \tag{3.37}$$

Figures 3.11 and 3.12 provide illustrations of the behaviours of the spot price and of the forward curve dynamic. I use the parameter values provided by the authors for the English and Wales market. As expected, the spot price does not exhibit any spiky behaviour. This is because of the choices of a smooth equilibrium function and the Gaussian factors. But, the forward curve dynamic presents the floating behaviour of the long-term maturities.

3.5.3 Stack Curve Models

The idea of a structural model based on a stack curve is to simplify the merit order models used in generation management enough to ensure computational tractability of the futures prices. As an example, I follow the model develop by Aïd et al. [2, 3].

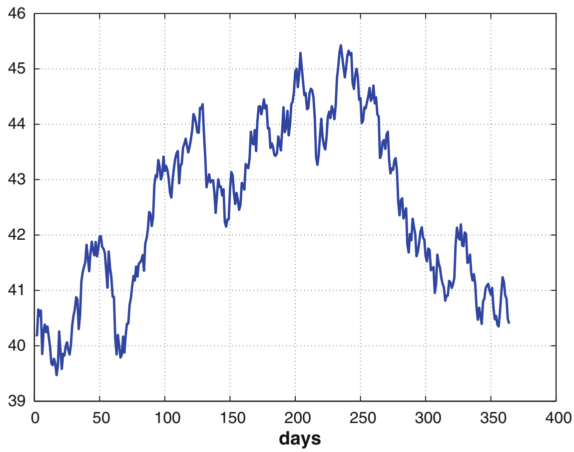


Fig. 3.11 Simulated daily spot price obtained with Cartea and Villaplana [64] in the case of constant volatilities with parameter values of $\kappa_D = 0.33$, $\sigma_D = 1580$, $\kappa_C = 0.37$, $\sigma_C = 2056$ and $g_t^D = 3.5e4 + 1e3\cos\left(\frac{2\pi(t+7)}{364}\right)$ $g_t^C = 3.7e4 + 1e3\cos\left(\frac{2\pi(t+7)}{364}\right)$

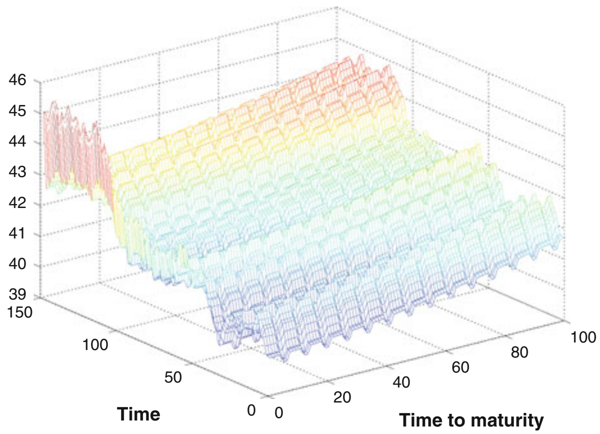
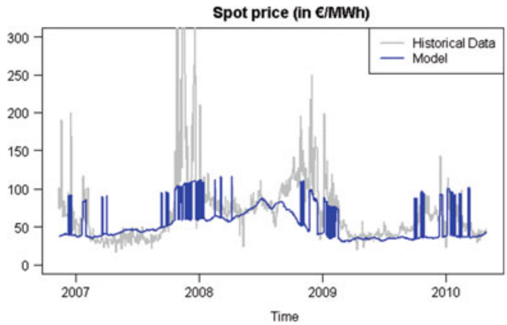


Fig. 3.12 Dynamic of the forward curve obtained with Cartea and Villaplana's (2008) model with the same parameter values as in Fig. 3.11

Fig. 3.13 Reconstructed marginal fuel price on the French market according to Aïd et al. model [2]



The idea behind this structural model is to deduce the no-arbitrage condition for the electricity prices from the no-arbitrage condition for storable fuel prices. The authors suppose that the electricity demand D_t can be satisfied with n different technologies by using all of the fuels. Each technology i has an available capacity C_t^i and its fuel cost is given by $h_i\ S_t^i$ where S_t^i is the fuel price and h_i is the heat rate of the technology. The technology i capacity interval is $I_t^i = \left(\sum_{k=1}^{i-1} C_t^k, \sum_{k=1}^{i} C_t^k\right]$. Thus, the marginal fuel of the system is given by:

$$\widehat{P}_t = \sum_{i=1}^{n} h_i S_t^i \mathbb{1}_{\{D_t \in I_t^i\}} \tag{3.38}$$

The price of the marginal fuel is an (important) approximation of the electricity spot price. However, it does not take into account the effect of the bidding strategies that lead to prices disconnected from the marginal fuel price. The marginal fuel price given by relation (3.38) is illustrated on Fig. 3.13 with French data and an approximation using only two fuels: coal and oil prices (estimation details can be found in [2]).

The spiky behaviour of electricity prices can be obtained using a scarcity function. The high prices are highly correlated with situations where the reserve margin is low (as described in Sect. 2.1.1). I denote the overall capacity with $\overline{C}_t := \sum_{k=1}^{n} C_t^k$ and the reserve margin with $R_t = \overline{C}_t - D_t$. Statistical analyses show that high spot prices correspond to low values of R_t. Thus, the authors set:

$$P_t = g\left(R_t\right) \times \widehat{P_t} \tag{3.39}$$

with g the scarcity function given by:

$$g\left(x\right) := \min\left(\frac{\gamma}{x^{\nu}}, M\right) \mathbb{1}_{\{x>0\}} + M\mathbb{1}_{\{x\leqslant 0\}}. \tag{3.40}$$

This relation says that the spot price is the marginal fuel price multiplied by a factor that is inversely proportional to the reserve margin. This is an extension of Barlow's model. Figure 3.14 presents the effect of the scarcity function on the modelling of the French electricity spot prices at the 19th hour (the peak hour). This figure represents a back-testing of the model (3.39). The historical demand, coal prices, oil prices, and the available capacity are substituted in relation (3.39) after performing the estimation of the parameters γ and ν. The fit is not perfect. But, the behaviour is properly reproduced and the phenomenon in Barlow's model of too many spikes at the price cap no longer occurs here. This is due to the fact that the two fuels are represented by the existence of small jumps when the demand changes from one fuel to another.

The model given by relation (3.39) is an incomplete market model because the price depends on un-hedgeable factors such as demand and capacity. The definition of the futures prices requires the choice of a risk-neutral measure. The authors choose to use the risk-neutral measure given by the local risk minimisation criterion (see Pham [141]). An alternative criterion that consists in indifference utility pricing with the exponential utility is studied in Benedetti and Campi [16]. The benefit of the local

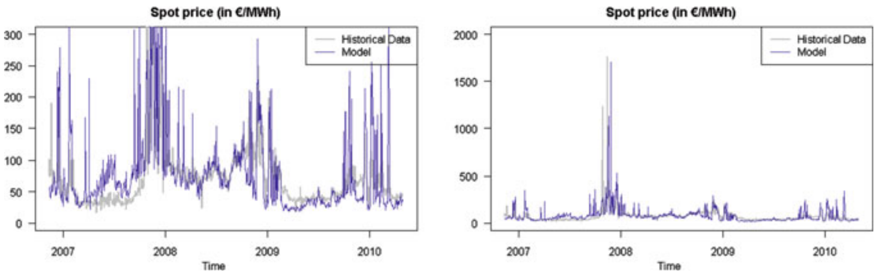


Fig. 3.14 Back-tested spot price on the French electricity market at the 19th hour according to Aïd et al. (2013) model [2]

risk minimisation is that it allows a decomposition of the derivative prices in the hedgeable and un-hedgeable parts. Moreover, it allows explicit formulas. Further, the future price $F^e(t, T)$ at time t for delivery at hour T of 1 MWh of power is given by

$$F^e(t, T) = \sum_{i=1}^{n} h_i G_i^T(t, C_t, D_t) F^i(t, T), \tag{3.41}$$

where $F^i(t, T)$ is the quoted futures prices for the fuel i at time t for delivery at time T, and G_i is the weights that depend on demand and capacity, but not on fuel prices. Observe that $F_t^i(T)$ are known at time t and no model is needed for them to compute the electricity forward price at that time. The relation (3.41) states that the electricity futures price is a linear combination of the fuel prices. However, this relation tells that in an economy where electricity is produced only by coal-fired plants, the electricity futures prices are note a basic reproduction of the coal futures prices. The examination of the weights G_i shows a little more of the relative importance of the fuel prices. The weights are given by:

$$G_i^T(t, C_t, D_t) = \mathbb{E}_t^{\mathbb{P}}\left[g(R_T)\mathbb{1}_{\{D_T \in I_T^i\}}\right], \tag{3.42}$$

where the conditional expectation is taken under the historical probability. The relation (3.42) states that the weights are not only given by the expected probability that the fuel i will be the marginal fuel, that is, $\mathbb{E}_t^{\mathbb{P}}\left[\mathbb{1}_{\{D_T \in I_T^i\}}\right]$. But, this marginality should be corrected by the tension in the system when it occurs, that is, $g(R_T)$.

However, the calibration of this model to the observed forward curve is left open in Aïd et al. [2]. Further, once the risk-neutral measure is chosen by using the local risk minimisation criterion, there are no more parameters left to fit the theoretical prices to the observed futures prices. They should perfectly fit. This lack is filled by Féron and Daboussi [86]. I reproduce a small part of their results with their kind permission.

They reconstruct the French futures prices by using Ornstein-Uhlenbeck models with seasonal components for the demand and the capacity. They take into account three capacities (nuclear, coal, and oil). They estimate the parameters of their four processes based on historical data. The realised availability of the power plants is published by the French TSO. The weights G_i are then computed for all of the delivery periods involved in the electricity futures prices. The futures prices for coal and oil are taken from the historical data. The nuclear fuel cost could be neglected here because nuclear power is rarely marginal. The theoretical futures prices are then computed by using the relation (3.41) for the delivery period of the quoted futures prices. A comparison is then made with the observed futures prices.

Figure 3.15 compares the real market quotations and the reconstructed futures prices for electricity for the year-ahead base-load contract from 2009 to 2012 with and without the effect of the scarcity function. During all of the study period, the variations between the observed prices and the reconstructed prices are correct. But, it is also clear that there are two periods. During the first three years, an important error is made by the model (10 Euros on a price of 55 Euros) while the fit is quite good in the remaining years. The effect of the scarcity function is also important. With the scarcity function, the error is less than 5 % in the remaining years.

But a decision-maker cannot get satisfied with a 5 % misfit between quotations and modelled prices. However, the framework of the local risk minimisation can be saved despite the observed error. The error can be attributed to the models of demand and capacity. They are over-simplified and do not capture the fine structure of their real dynamic. But, instead of trying to improve their modelling, an estimation of the *implied demand* to be added or subtracted from the input factors is computed to obtain a perfect fit. The value of this implied demand is estimated to be $\approx$1 GW, which is less than 1 % of the installed capacity of the French market. Thus, even if there is no market price of risk to be used to fit the price model with the observed quotation, it is still possible to correct the models of the input factors with a small perturbation to obtain this perfect fit.

But, the structural model given by relations (3.39–3.41) suffers some non-negligible drawbacks.

First, even if it is not reported in their paper, the computation of a year-ahead contract with the relation (3.41) at one date still consumes too much time. For a year-ahead contract, this relation should be used 8,760 times. Despite the development of the Taylor expansion approximation for the weights G_i, the Monte-Carlo method is still better for computing the values (3.42). But, it will still take one minute to compute one quotation of a year-ahead contract, which is far from an industrial standard.

Second, the model (3.38) makes the hypothesis that power plant efficiencies can be sorted out once and for all by looking at their fuel prices. Some old and inefficient coal-fired plants might be more costly than recent efficient combined gas cycles. Further, if the carbon price is high enough, then the coal-fired plant becomes less

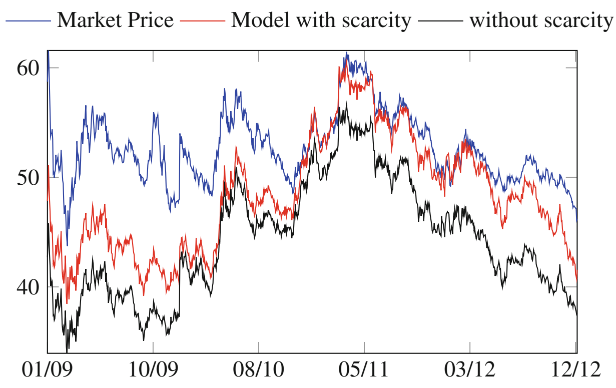


Fig. 3.15 Reconstruction of the French Year-Ahead Base-Load Contract during the period 2009–2012 using the model (3.41)

efficient than the gas-fired plants, which leads to a switch between the two fuels. Aïd et al. [3] take this point into account, but at the expense of having to compute the probability of the switch at future dates.

To deal with these aspects, Carmona et al. [53] develop a structural model not only based on the modelling of a stack curve, but on a *bid curve* model. Their model allows for switches in the merit order between fuels. The spot price is defined by

$$P_t = \max\left(\min_i \underline{b}_i(S_t^i), \sup\left\{p : \sum_i \hat{b}_i^{-1}(p, S_t^i) < D_t\right\}\right) \tag{3.43}$$

where D_t is the demand, S_t^i is the fuel price i, $\hat{b}_i = b_i^{-1}(., s)(p)$ is the bid curve for fuel i, and $\underline{b}_i = b_i(0, s)$. In this setting, with two fuels (gas and coal for instance), P_t is determined by five cases that can then be reduced to three, in the case of a bid stack curve model as an exponential $b_i(x, s) = sexp(k_i + m_i x)$.

In this model, because the fuels can be mixed, it might be that more than one generation capacity of the same fuel is not fully used. The model provides a formula for the spot price of the two fuels and an extension to capture the spikes as well as the negative prices. The forward prices are obtained in the absence of the hypothesis on the demand probability distribution. And, with a Gaussian hypothesis, an analytical formula is provided. Even if those formulas are quite long, they avoid the time consuming computation of the relation (3.41).

Table 3.2 A classification of the references in this chapter on the electricity price models for pricing derivatives

	HJM style	One-factor spot	Multi-factor spot	Structural
2000		[34]		
2001				
2002		[127]	[107, 127]	[10]
2003	[87]	[22]		
2004		[28]	[11, 48, 115]	
2005	[101, 121]	[61, 75]		
2006	[41, 102]	[94]		
2007	[26]		[23, 168]	[110]
2008	[25, 89]	[46]	[134]	[64, 144]
2009	[90, 118]	[62]	[31] [72]	[129] [3, 67]
2010	[91]		[84, 95]	
2011				
2012			[24, 135]	
2013			[92]	[2, 51]

3.6 Conclusion

More than 40 papers were published in the last 15 years on modelling electricity prices just for futures valuation purposes. And each time, most of them propose a new modelling framework. The Table 3.2 tries to illustrate this richness by putting the papers in the categories used in this chapter. Even if I see that the HJM framework has some difficulties in tackling the delivery periods of the electricity futures, some developments based on the ambit fields by the Norwegian School of Mathematics might reveal themselves as quite efficient in the future. The research on the one-factor spot model seems to have found its limits, and the trend clearly is towards multi-factor models. For their part, the structural models should still be considered in their prime even if their first occurrence goes back to Barlow [10].

This diversity also reflects the lack of a reference model for market players. If traders accommodate a simple HJM two-factor model because they can avoid making transactions on the spot market, most of the actors involved with the generation management of physical assets find themselves in need of joint hourly spot prices and futures prices. The lack of a reference model has an impact on the perception of the values of the options by market agents. It leads to divergence in perceptions, higher bid-ask spreads, and low liquidity. Thus, this lack is an obstacle to the development of the market. For this reason, even if it might seem that the fields are quite well explored now, there is still room for the search of a realistic, consistent, efficient, robust, and generic model for electricity prices that can be adopted by a large portion of the electricity market players.

Chapter 4
Derivatives

In this chapter, I present the derivatives that can be considered to be particular to electricity. In this regard, the spread options described in Sect. 4.1 should be considered as the basis for more complex derivatives such as tolling contracts or swing options which were first mentioned in Sect. 2.3. Thus, I limit this section on spread options to a presentation of their different forms and on the cases where there is a closed-form formula or an analytical approximation. Because the spread options are also common in other markets, I also refer the reader to the concise and yet complete survey on spread options both on their taxonomy and their valuation by Carmona and Durrleman [57].

Sections 4.2 and 4.3 are devoted to what are maybe the most difficult derivatives to evaluate and hedge on the electricity markets, namely power plants and tolling contracts on the one hand, and storage and swings on the other hand. As surprising as it might seem, although particularly difficult, these options receive important attention from the academic literature. Furthermore, an important panel of sophisticated numerical methods has been developed which has not been necessarily common knowledge amongst financial engineers in energy utilities. The description of the different numerical methods is beyond the scope of this book. For details on them, I refer the reader to Carmona et al. [53] book. In this chapter, I limit the presentation of the reported performance of these methods.

Further, I examine more exotic derivatives. Section 4.4 presents the case of retail contracts. I describe the different approaches that are proposed to put a price to the load curve of a customer. This problem is far more basic in its standard formulation than the valuation problems of the real derivatives above. Nevertheless, it is important because it is the main source of revenues for electricity utilities. Then, I present in Sect. 4.5 the weather derivatives. These derivatives are supposed to offer a hedge to firms whose profits depend on weather conditions. I describe how they work and propose reasons for why they have not yet met the commercial success that they should have in the energy business, despite its high dependency on temperature, sunshine, and rainfall.

R. Aïd, *Electricity Derivatives*, SpringerBriefs in Quantitative Finance,
DOI 10.1007/978-3-319-08395-7_4

4.1 Spreads

The most commonly used spread options in the electricity markets are fuel-spread options and locational spread options. A fuel-spread option consists of an owner with the right to buy electricity and sell a fuel (either gas, coal, or oil). And the holder of a locational spread has the right to buy power from one zone and to sell it to another at the cost of its transportation. The fuel-spread options are embedded in thermal power plants. A power plant owner who wants to hedge the generation value just has to sell the fuel spread at different future maturities. As Sect. 2.3 shows, the payoff of the fuel-spread options that are of some interest to power plants are of the form:

$$\left(S_T^e - hS_T^f - gS_T^c - K\right)^+$$

with S_T^e the price of electricity, S_T^f the price of fuel, S_T^c the price of carbon at time T, and K the start-up cost. The parameters h and g are respectively the heat rate of the power plant and its emission factor. So, there are the three assets involved in this payoff. It might be that more assets are involved when the power is quoted in euros while gas is quoted in pounds and coal in US dollars. In this situation, the fuel-spread option includes four assets (three prices and one exchange rate).

It is not possible to neglect the strike price induced by the start-up cost. Here is an example with the order of magnitude of the costs and the prices:

- For a coal-fired plant, the fixed start-up cost is ≈50,000 €. Running 12 h/day with a capacity of 500 MW leads to a $K = 8$ €/MWh $(50.000/(500 \times 12))$. The dark spread at the time this book is written is around 4 €/MWh and it was 30 €/MWh at its recent peak.
- For a gas-fired plant, the fixed start-up cost is ≈15,000 €. Running 6 h/day with a capacity of 500 MW leads to a $K = 5$ €/MWh. The crack spread at the time this book is written is -15 €/MWh.

The situation is not better for the locational spread options. In Europe for instance, the transportation cost for generation is around 2 €/MWh, and the spread between France and Germany is around 4 €/MWh.

Thus, the managers of generation assets are interested in the valuation and hedging of options whose payoff can be summarised as:

$$p(T, S_0^a, S_0^b, K) = \left(S_T^a - S_T^b - K\right)^+ \tag{4.1}$$

with S^a and S^b the price of the two assets, K the strike, and T the maturity.

Since Margrabe's paper [130] on the options to exchange one asset against the other, important studies have been written on spread options. Despite the intense research activity in this field, there is little hope to expect closed-formed formulas for spread options in situations with a non-zero strike price. Nor is there much hope for more than two assets and for asset prices following a more general dynamic than a geometric Brownian motion.

I begin here by reporting some of the most commonly used formulas for spread options, which are Margrabe's formulas and their derivatives. These formulas provide a first approximation of the value of the spread options and can be used as benchmarks for the more complex models. The formulas reported here are not based on an asymptotic expansion with respect to some parameter that is supposedly small. They are empirical formulas. I then turn to the valuation formulas in the models which are closer to the observed dynamic of the commodities and which are applicable to electricity. Further, I say a word on the papers which provide the valuations of the spread options for specific electricity price models.

Black and Scholes assets model. I assume $dS^i = S^i\left[rdt + \sigma^i dW^i\right]$, $i = a, b$ with $dW^a dW^b = \rho dt$. When $K = 0$, the value of the spread option whose payoff is given by (4.1) is:

$$p = e^{-rT}\left[S_0^a N(d_1) - S_0^b N(d_2)\right] \tag{4.2}$$

with

$$d_1 = \frac{\log\left(S_0^a / S_0^b\right) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

and

$$\sigma^2 = \sigma_a^2 - 2\rho\sigma_a\sigma_b + \sigma_b^2 \tag{4.3}$$

Margrabe's formula is an application of the Black and Scholes formula for the asset S^a/S^b and the equivalent volatility (4.3). Margrabe's formula ceases to be true for a non-zero strike price. Only approximations are available. Kirk's formula (1995) and Eydeland and Wolynieck [85] are the most commonly cited. Another approximation formula can be found in the technical report by Bjerksund and Stensland [36]. To avoid the multiplication of formulas, I limit the chapter to three of them.

Kirk's formula (1995) reads as

$$\hat{p}^K = e^{-rT}\left[S_0^a N(d_1^K) - (S_0^b + K)N(d_2^K)\right] \tag{4.4}$$

with

$$d_1^K = \frac{\log\left(S_0^a / (S_0^b + K)\right) + \frac{\sigma_K^2 T}{2}}{\sigma_K\sqrt{T}} \qquad d_2^K = d_1^K - \sigma_K\sqrt{T}$$

and the equivalent volatility is:

$$\sigma_K^2 = \sigma_a^2 - 2\rho\sigma_a\sigma_b\frac{S_0^b}{S_0^b + K} + \sigma_b^2\left(\frac{S_0^b}{S_0^b + K}\right)^2 \tag{4.5}$$

Kirk's formula consists of using Margrabe's formula with a strike equal to $S_0^b + K$ and of changing the volatility for S^b accordingly. Here, the reader should note the non-symmetric role of the two assets in the equivalent volatility σ_K. This is not the case in the variant proposed by Eydeland and Wolynieck's formula [85, pp. 345–346]:

$$\hat{p}^E = e^{-rT}\left[S_0^a N(d_1^E) - (S_0^b + K)N(d_2^E)\right] \tag{4.6}$$

with

$$d_1^E = \frac{\log\left(S_0^a/(S_0^b + K)\right) + \frac{\sigma_E^2 T}{2}}{\sigma_E\sqrt{T}} \qquad d_2^E = d_1^E - \sigma_E\sqrt{T}$$

and

$$\sigma_E^2 = \sigma_a^2 - 2\rho\sigma_a\sigma_b\frac{S_0^a}{S_0^b + K} + \sigma_b^2\left(\frac{S_0^a}{S_0^b + K}\right)^2.$$

In this last formula, it is possible to exchange an asset b at a lower price S_0^b for a higher strike K and still have the same value for the spread option.

Another basic and yet efficient way to approximate the value of the spread options is to approximate the density of the spread at maturity with a Gaussian distribution and by matching its two first moments. This is the method proposed in Carmona and Durrlemann [57, Proposition 4.1]. This method leads to the following approximation:

$$\hat{p}^M = \left(m - Ke^{-rT}\right)N(d_1^M) - \sigma_A N'(d_1^M) \tag{4.7}$$

with

$$d_1^M = \frac{m - Ke^{-rT}}{\sigma_M} \qquad m = (S_0^a - S_0^b)exp(-rT)$$

and

$$\sigma_M^2 = e^{-2rT}\left[(S_0^a)^2(e^{\sigma_a^2 T} - 1) + (S_0^b)^2(e^{\sigma_b^2 T} - 1) - 2S_0^a S_0^b(e^{\rho\sigma_a\sigma_b T} - 1)\right]. \tag{4.8}$$

A systematic analysis of the relative precision of these different methods is too long for this review. But, it is possible to quickly illustrate them with some numerical computations and at the same time give the reader some reasoning behind the behaviour reflected by the value of the spread options. Specifically, I take a null interest rate, a maturity of one year, and a reference volatility of 10 % for asset a and 15 % for asset b. Further, I choose S_0^b and K such that $S_0^b + K = S_0^a$ so that the initial spread value is always zero when K changes. I consider only the case with a strong correlation of either 80 or −80 %. The resulting value for the spread option for the different approximation formulas are plotted on Fig. 4.1.

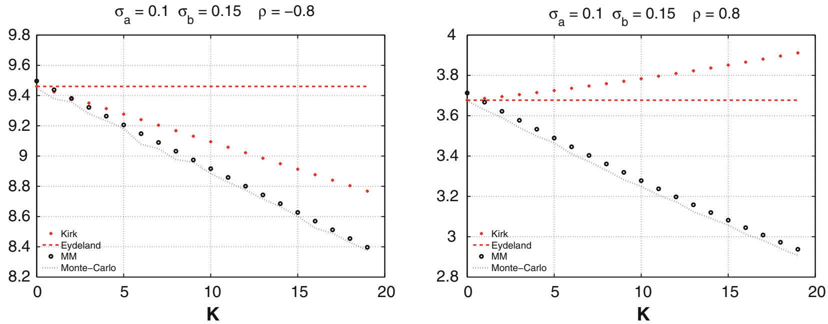


Fig. 4.1 Different approximation formulas for the value of a spread option: Kirk, Eydeland and Wolyniec, Moment Matching (MM) and a Monte-Carlo estimation (MC). *Left* with strong negative correlation. *Right* with strong positive correlation

Several comments can be made about Fig. 4.1. First, the value of the option spread can be significantly positive although the initial spread is zero. The value decreases when the correlation between the assets increases: the stronger the correlation the less probable the spread will increase. Regarding the approximation formulas, Eydeland and Wolyniec's formula provides a constant value in this situation because when $S_b + K$ is constant, so is the equivalent volatility. The moment matching method provides a precise and constant estimation of the value of the option, while the error in Kirk's formula tends to increase as the strike price increases and the initial price of the asset b decreases. The reason comes from the fact that Kirk's equivalent volatility decreases when S_b is substituted with K, which makes the second asset less likely to reach the threshold $S_a + K$. Again, the estimation obtained by the moment matching method is correct for both negative and positive correlations. But, Kirk's formula strongly favours positively correlated assets.

Regarding the Greeks, I illustrate the effect of the volatility of the asset a on the delta. However, I change the conditions of the computations because when the strike is zero, the volatility of the assets has the same effect. Thus, as before, I take a null interest rate and an one-year maturity option but now, $S_0^a = 100$, $S_0^b = 80$, and $K = 10$. At time zero, the spread has a positive value. Figure 4.2 presents the deltas obtained for the different approximation formulas above. For both types of correlations, the approximations remain good for the volatilities in a reasonable range. For high volatility, the moment matching method ceases to provide a reliable estimate.

In the case where more than two assets are taken into account but where the dynamic of their prices are still driven by the geometric Brownian motions, several developments are done. The most basic one is the extension of Kirk's formula to three assets by Alos et al. [6]. Considering that the asset prices S^a, S^b, and S^c follow the geometric Brownian motions with the correlations ρ_{ab}, ρ_{ac}, and ρ_{bc}, the value of the call option with payoff $(S_T^a - S_T^b - S_T^c - K)^+$ is approximated for small maturities T by

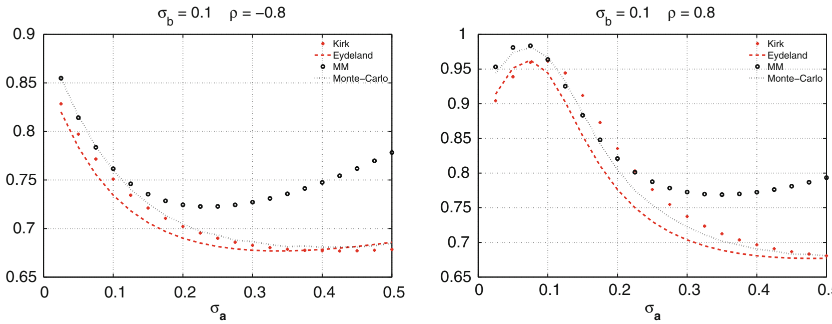


Fig. 4.2 Delta obtained for the different approximation formulas for the values of the spread options: Kirk, Eydeland and Wolyniec, Moment Matching (MM) and a Monte-Carlo estimation (MC). *Left* with strong negative correlation. *Right* with strong positive correlation

$$\hat{p}^L = e^{-rT}\left[S_0^a N(d_1^L) - (S_0^b + S_0^c + K)N(d_2^L)\right] \tag{4.9}$$

with

$$d_1^L = \frac{\log\left(S_0^a/(S_0^b + S_0^c + K)\right) + \frac{\sigma_L^2 T}{2}}{\sigma_L\sqrt{T}} \qquad d_2^L = d_1^L - \sigma_L\sqrt{T}$$

and

$$\sigma_L^2 = \sigma_a^2 + \sigma_b^2\pi_b^2 + \sigma_c^2\pi_c^2 - 2\rho\sigma_a\sigma_b\pi_b - 2\rho\sigma_a\sigma_c\pi_c + 2\rho\sigma_b\sigma_c\pi_b\pi_c,$$

where $\pi_b = S_0^b/(S_0^b + S_0^c + K)$ and $\pi_c = S_0^c/(S_0^b + S_0^c + K)$.

The numerical illustrations reported by the authors show that relation (4.9) can provide precise results even for the spread options with a maturity of one year.

The creation of approximation formulas for more general spread options (more assets and more complex price dynamics) is still an active field of research. An example is provided by Landon's PhD thesis [122, Chap. 8] which provides formulas for general spread options with n assets based on asymptotic expansions.

Beyond Black and Scholes. Several authors propose valuation formulas for the spread options in mean-reversion models. Some formulas are provided in Tsitakis et al. [157] for two-asset spread options with an Ornstein-Uhlenbeck dynamic and a zero strike price. A more general situation is addressed in Hikspoors and Jaimungal [100] where two-factor mean-reversion models are used for each asset. The closed-form formulas are provided up to a specific change of measure to preserve the mean-reversion in the risk-neutral measure.

Another approach is to directly model the spread dynamic. Dempster et al. [74] develop this approach with co-integration, or in Benth and Benth [29] and Cartea and González-Pedraz [63] with a mean-reverting jump-diffusion process. With this simplification, the problem is brought back to the valuation of a call option on an asset following a mean-reverting process, problems for which the closed-form solutions are available when the dynamic has no jump. The introduction of jumps leads all of the authors to rely on the fast Fourier transform methods to compute the option price. For an introduction to Fourier methods for pricing derivatives in the context of jump processes, see Eberlein [80].

The direct modelling of the spread allows the reduction of the dimension of the problem to one. But this simplification is made at a cost. One loses the capacity to differentiate the deltas for each asset that compose the spread. When the spread is modelled directly, there is only one volatility, and the same absolute value has to be used for the deltas of each asset. It can be misleading in the case of a spread where the two assets have significantly different volatilities. For instance, consider the case of the dark spread with the volatility in the electricity spot price at 50 % and the volatility in the coal price at 10 %. For a spread option with a one-year maturity, the delta for electricity is around 70 %, while the delta for the coal is closer to 50 %.

In the case of electricity, the valuation of the spread option is also analysed with structural models. In particular, Carmona et al. [52] offer a comparison between the valuation of the spread in this setting and other modelling approaches (the exponential Ornstein-Uhlenbeck). Their comparison includes Margrabe's benchmark formula and a formula derived in a co-integration setting. Amongst their findings, they show that the highly negative correlation between fuel does not imply a high payoff in the structural models contrary to the prediction of Margrabe's formula because a structural link imposes the dependence of the electricity price on its fuel, which lowers the density of the spreads.

4.2 Power Plants and Tollings

With the development of spot and forward markets, tolling contracts have emerged as a financial component of physical power plants. Tolling contracts are a risk transfer mechanism by which the owner of a power plant can transfer the market price risks (fuel and power) to an investor who is ready to take these risks against a fixed premium. They are over-the-counter contracts, and many forms are possible. For a description of tolling contracts, the reader is referred to Eydeland and Wolyniec[85] and Baldi [7]. But, their general form is to stay close to the operation of a power plant. In particular, they consist in the daily exercise of the spread option between the power price and the fuel price during a period of time of at least a year. In this regard, tolling contracts consist of an approximation of a power plant as a strip of spread options (hence, the strong and stressed interest of electricity utilities on an efficient method to value the spread options).

Unfortunately, this proxy neglects several factors. First, power plants are subject to a list of operational constraints which prevents them from capturing all of the possible hourly spreads. For example, they have maximum and minimum generation capacities, minimum stopping time, minimum running time, ramping capacity, and maximum numbers of start-ups and shutdowns per year. They are also subject to outages which are both planned for maintenance purposes and unplanned. Unplanned outages have the unpleasant effect of leaving the owner who thought he or she was long in electricity short of the asset. Moreover, the approximation of the value of a power plant by a strip of spread options neglects the incompleteness of the market. The possible exercises involved with a gas-fired plant, for instance, are at least daily while the forward curve provides hedging instruments for only a grossly aggregated period of time (week, month, quarter, and year).

Taking into account the different constraints of power plants in its valuation leads to stochastic control problems. The valuation requires more than the computation of an expectation. It needs to solve an optimisation problem. Moreover, the dimension of the problem measured by the state of the system might be large. The conditional expectations of the daily payoff might be three dimensional (power, fuel, and emission) if the one-factor models are considered for each asset, or six dimensional in the case of the more realistic two-factor models. The controlled state might itself be in a dimension higher than one when dynamical constraints are taken into account. Moreover, this problem only allows for the determination of the power plant's operation. Tolling contracts say when the plant should be started up and shut down. They do not say anything about the actions the operator has to do on the forward market to hedge the expected value of the power plant. Taking into account the hedging problem leads to a more complex optimal control problem where both the hedge and the operation of the power plant have to be determined simultaneously. In this context, there is no hope to succeed in getting a closed-form formula, which means having to use numerical methods.

I address here an overview of the different possible formulations of the problems and of the numerical methods used to solve them. Most papers in the literature deal with the operation problem, which is already a non-negligibly difficult problem. This class contains in chronological order Thompson et al. [156], Hlouskova et al. [103], Deng and Xia [76], Carmona and Ludkovski [58], Pirrong and Jermakayan [144], Adkins and Paxon [4], and Ryabchenko and Uryasez [151]. On the other hand, there are few attempts to solve the operation and hedging problems together. In this class, I find only Ludkovski [128] and Porchet et al. [145].

Formulation of the operation problem: Formulations for the operation problem can vary according to the constraints taken and to the price dynamic chosen. I follow the optimal switching formulation provided by Carmona and Ludkovski [58] as a reference to which the other papers are compared by highlighting the differences. Indeed, Carmona and Ludkovski [58] offer an exhaustive presentation of the valuation problem of a power plant which is very close to the problem confronted by the decision-makers in electricity utilities, and it provides a numerical method to solve it.

The ingredients of their formulation are the following: The vector X_t represents the market prices of electricity and fuel (gas or coal). It is assumed to be Markov. The power plant can be in $M+1$ different states (including zero and maximum generation). To each state m, a payoff is associated $\phi(t, X_t, m)$ at time t when the prices are X_t. For example, when the plant is not running, the payoff is $\phi(t, X_t, 0) = -1$ which represents the fixed operating cost; and when the plant is running in a first state the payoff is $\phi(t, X_t, 1) = Q_1 \times (P_t - S_t)$, with $X_t = (P_t, S_t)$, and Q_1 is the level of generation of the power plant. The decision-maker can switch from state i to state j by incurring a cost $C_{i,j}$. The switching cost can depend on X_t, without changing either the formulation or the numerical scheme. The control of the decision-maker is given by a discrete sequence $u = (\xi_k, \tau_k)$ of stopping times τ_k at which the decision-maker makes the power plant switch to state ξ_k which takes values in $\{0, \ldots, M\}$. The control u_t which denotes the operation mode at time t should be seen as a piece-wise constant control process $u_t = \sum_{\tau_k < T} \xi_k \mathbb{1}_{[\tau_k, \tau_{k+1})}$.

I denote by $\mathscr{U}(t)$ the set of admissible control strategies from t to T. The objective of the decision-maker is to maximise the expected value of the power plant over the time horizon $[0, T]$:

$$J(t, x, i) = \sup_{u \in \mathscr{U}(t)} \mathbb{E}\left[\int_t^T \phi(s, X_s, u_s)ds - \sum_{\tau_k \leq T} C_{u_{\tau_k^-}, u_{\tau_k}}\right]. \tag{4.10}$$

Moreover, in this context where only the operations of the power plant are looked at, the expectation is taken under the risk-neutral probability because it is not the core difficulty. This formulation is named an *optimal switching problem* because the objective is to find the optimal strategy to switch from one state to another according the state of the system. For an introduction to optimal switching problems, the reader can refer to Pham [142, Chap. 5].

The mainstream numerical method used to solve problem (4.10) is dynamic programming. Here, to be able to solve the problem, the authors introduce a series of problems where the number of switches in (4.10) are limited to k. Denoting $J^k(t, x, i)$ as the value function when the number of switches is limited to k and the initial condition at time t is (x, i), the algorithm takes the following form:

$$J^0(t, x, i) = \mathbb{E}\left[\int_t^T \phi(s, X_s, i)ds | X_t = x\right] \tag{4.11}$$

$$J^k(t, x, i) = \sup_{\tau} \mathbb{E}\left[\int_t^\tau \phi(s, X_s, i)ds + \mathbf{M}^{k,i}(\tau, X_\tau^x) | X_t = x\right] \tag{4.12}$$

$$\mathbf{M}^{k,i}(t, X_t^x) = \max_{j \neq i} \left(-C_{i,j} + J^{k-1}(t, x, j)\right). \tag{4.13}$$

The first relation (4.12) is the necessary terminal condition for the recursive relation (4.13). This relation gives the value one can expect when there is no possible switch left. The relation (4.13) indicates whether it is worth paying the cost $C_{i,j}$ to leave state i at time t while the prices are x. When these values are computed, the relation (4.13) provides the answer of whether it is interesting to stay in state i between t and τ (the expectation is greater than the second term) or to leave state i.

The main numerical difficulty in this algorithm is the computation of the conditional expectations $\mathbb{E}[h(X)|X = x]$ that appear in Eq. (4.13). Many numerical schemes have been proposed since Longstaff and Schwartz's [125] seminal work on American options with regression methods. Quantisation [138] and Malliavin calculus [88] are amongst the most cited alternatives. The schemes based on the interacting particle models are also proposed and analysed by Del Moral et al. [73]. For a comparison of the efficiency of these different schemes, the reader might be interested in the results of Bouchard and Warin [43] which shows the increased performance of the local regression scheme with the dimension of the state. For an exhaustive presentation of the main numerical schemes with an application to energy, I refer the reader to Carmona et al. [55].

Alternative methods to dynamic programming: The problem (4.10) admits alternative representations which are often cited in the quantitative finance literature and for which intensive mathematical research is ongoing. As in the case of the classical valuation of a call option in the Black and Scholes model, it is possible to associate a partial differential equation to problem (4.10). Nevertheless, it takes the more complex form of a *quasi-variational inequality* (QVI).

I denote $\mathbf{M}v(t,x,i) = \max_j\{-C_{ij} + v(t,x,j)\}$ as the intervention operator and $\mathscr{L}_X$ as the infinitesimal generator of the process X_t. In the case when $dX_t = \mu(X_t)dt + \sigma(X_t)dW_t$, the operator $\mathscr{L}_X$ is $\mathscr{L}_X v(t,x) = \mu(x)v_x + \frac{1}{2}\sigma^2(x)v_{xx}$. With some not so restrictive regularity conditions, the value function (4.10) is the solution of:

$$\max\{\mathscr{L}_X v + \phi; -\psi + \mathbf{M}v\} = 0. \tag{4.14}$$

Relation (4.14) shows that the maximum of the two terms should always be zero. Thus, if the first term is zero, the second is negative, and it states that $M\psi < \psi$, which means that it is not worth making an intervention. Thus, in this situation, the evolution of the system is driven by the partial differential equation $\mathscr{L}_X\psi + \phi = 0$. Thus, the optimal solution consists in doing nothing as long as the value of X_t belongs to a certain region and to change the state when X_t touches a certain boundary.

This is the approach developed in Thompson et al. [156]. In this study, the electricity spot price is modelled with a mean-reversion with jumps. Thus, the operator $\mathscr{L}_X$ is a little more complex and takes the form of an integro-differential equation. The QVI involves then a partial integro-differential equation. It is solved by an adapted finite difference scheme.

A second formulation can be associated to the problem (4.10): a *backward stochastic differential equation* (BSDE). By defining the process $Y_t^{k,i} = J^k(t, X_t^x, i)$, the BSDE satisfies the system:

$$Y_t^{k,i} = \int_t^T \phi(s, X_s^{t,x}, i)ds + A_T^{k,i} - A_t^{k,i} - \int_t^T Z_s^{k,i} dW_s, \tag{4.15}$$

$$\mathbf{M}^{k,i}(t, X_t^x) \leq Y_t^{M,i}, \tag{4.16}$$

$$\int_0^T (Y_t^{k,i} - \mathbf{M}^{k,i}(t, X_t^x))dA_t^{k,i} = 0, \qquad A_0^{k,i} = 0. \tag{4.17}$$

In the system (4.15), one has to find $Y_t^{M,i}$ and also the two auxiliary processes $Z_t^{M,i}$ and $A_t^{M,i}$. I will not enter into the interpretation of these processes here and refer the reader to Pham [142, Chap. 6] for an introduction to the subject. Although apparently complex, this approach has been used in Hamadène and Jeanblanc [97] for the valuation of a power plant in the case of a basic on-off regime and by Porchet et al. [145] to take into account the market's incompleteness and its hedging problems.

Formulation of the hedging problem: To illustrate the formulation of the valuation of a power plant in an incomplete market but with the ability to hedge, I follow Ludkovski's formulation [128]. As above, a finite number I of generation regimes is possible. The output price is Y_t following

$$dY_t = a(Y_t, \xi_t)dt + b(Y_t, \xi_t) \cdot (\rho dW_t^1 + \sqrt{1-\rho^2}dW_t^2).$$

It leads to an income flow $\psi(t, Y_t, i)$. An operational strategy is a sequence (τ_k, ξ_k) of switching times τ_k to the states ξ_k. It is possible to switch state at costs $C_{i,j}(t, Y_t)$. The revenue of the power plant is thus

$$B(\xi) = \int_0^T e^{-rt}\psi(t, Y_t, \xi_t)dt - \sum_{\tau_k} e^{-r\tau_k} C_{\xi_{k-1},\xi_k}.$$

The manager is supposed to be able to hedge in continuous time with a storable asset with price S_t. The storable asset can be the futures contract or any other assets which present the highest correlation with the power plant's revenues. The storable asset price follows $dS_u = \mu S_u du + \sigma S_u dW_u^1$.

When applying the operation strategy ξ and the hedging strategy π, the wealth at time t of the owner of the power plant is $X_t^{t,x,\pi,\xi}$ with x the initial wealth. The wealth satisfies the dynamic:

$$dX_u^{t,x,\pi,\xi} = \pi_u \frac{dS_u}{S_u} + r\left(X_u^{t,x,\pi,\xi} - \pi_u\right)du + \psi(\xi_u, u, Y_u)du - \sum_k C_{\xi_{k-1},\xi_k} \mathbb{1}_{\tau_k = u}.$$

The amount π_u of money invested in the hedging provides a risky return dS/S while the remaining part provides a constant risk-free return. Moreover, the wealth is accrued by the operational income from the power plant $\psi_{\xi_u}(u, Y_u)du$ and is decreased by the cost of the changed states $\sum_k C_{\xi_{k-1},\xi_k} \mathbb{1}_{\tau_k = u}$.

Because the market is incomplete, it is necessary to provide a way to specify what the value of an asset is for the agent. The author's choice relies on the indifference price. Supposing that the risk preferences of the owner are given by an exponential utility function $U(w) = -\exp(-\gamma w)$, the purpose of the owner is to maximise the expected utility of his or her terminal wealth over all of the admissible operation and hedging strategies:

$$V(t, y, x, i) = \sup_{\pi,\xi} \mathbb{E}_{t,x,y,i}\left[U(X_T^{t,x,\pi,\xi})\right]. \tag{4.18}$$

However, in this case, the value function is not measured in dollars or euros but in "utils", a currency that does not exist on corporate balance sheets. The value of the power plant is thus defined as an indifference price.

Taking the point of view of the owner of the power plant, the indifference price corresponds to the cash needed to increase the owner's initial endowment for giving up the operation of the power plant. I denote $U^0(t, x) = \sup_\pi \mathbb{E}_{t,x}\left[U(X_T^{t,x,\pi})\right]$ as the utility when the owner is not operating the asset. The indifference price $p_{t,T}(x, y, i)$ which is measured in currency is defined as

$$V(t, y, x, i) = U^0(t, x + p_{t,T}(x, y, i)). \tag{4.19}$$

The indifference price measures the amount of cash that makes the owner indifferent between having a higher initial wealth but no power plant and having a lower initial wealth but having the power plant. Because of the use of an exponential utility, the indifference price also corresponds to the value an owner is ready to receive to give up the operation of the power plant. Note that in this framework, risk management (the hedging strategy) has a clear and measurable effect on the value of the asset.

The complexity of the problem (4.18) and (4.19) cannot be understated. Two optimal controls, ξ and π, have to be found in a problem with a four-dimensional state. However, the choice of the utility function as well as the criterion of terminal wealth maximisation allows to first compute the operational strategy ξ and then the hedging strategy π. The choice of an exponential utility function also has an important property. The indifference price is time additive in the sense that for any intermediate time τ, it can be expressed as a function of the current accumulated income between t and τ, and the indifference price running from τ to T. This property can be desirable to the asset owners.

The hedging strategy is given by:

$$\pi_t^* = -\rho \frac{b(Y_t)}{\sigma} \frac{\partial p}{\partial y}(t, Y_t, \xi_t^*). \tag{4.20}$$

The term $\frac{\partial p}{\partial y}(t, Y_t, \xi_t^*)$ corresponds to the standard delta of the pricing in a complete market that is modified by a ratio which takes into account the efficiency of the hedging instrument. With no correlation, that is, the hedging market has no relation with the local price, the owner does not use it. Because the indifference price increases with the local price, the more the hedging market is correlated to the local price, the more the owner sells the hedging asset.

The optimal switching times are given by:

$$\tau_{k+1}^* = \inf \left\{ s > \tau_k^* \quad : p(s, Y_s, \xi_k^*) = \max_{j \neq \xi_k^*} p(s, Y_s, j) - C_{\xi *_k, j} \right\}$$

The owner switches as soon as there is some state j where the indifference price $p(s, Y_s, j)$ minus the switching cost again reaches the indifference price $p(s, Y_s, \xi_k^*)$ in the current state. In this situation, the operational strategy depends on the hedging strategy because the switching times are given by the evolution of the indifference price which depends on the efficiency of the hedging instrument. In the case of the power plants, this result might seem surprising because usually the scheduling of the power plant can be determined independently from the hedging strategy. Indeed, for electricity that might be the case if the available forward contracts present little relation with the hourly spot price. Therefore, either the hedging instrument has some efficiency and the hedging and operation strategies have to be coordinated or the hedging instrument has little efficiency and it is useless to hedge.

To illustrate his model, the author gives a basic but realistic case study of the operation of an oil platform. The facility has three operating modes: 0, 5 Mbl/y, 10 Mbl/y. The hedging asset's price is given by a geometric Brownian motion with a 5 % growth rate and a 40 % volatility. The local price (Y) is a geometric Brownian motion with a 5 % growth rate and a 40 % volatility and a 90 % correlation with the hedging asset. The cash-flows in the different states are $\psi_0(y) = 0$, $\psi_1(y) = 5(y - 50)$, and $\psi_2(y) = 10(y - 56)$; and the switching cost is $C_{i,j} = 0.25|i - j|$ M\$. The interest rate is 5 %, the time horizon is six months and the time discretisation is one decision per day.

As for American options, the optimal policy is given by switching thresholds:

- Switch from 0 to 1 if $Y \geq 53$\$/bbl (greater than the break-even price of 50 in mode 1)
- Switch from 1 to 2 if $Y \geq 65$\$/bbl (greater than the equal price of 62)
- Switch back from 2 to 1 if $Y \leq 60$\$/bbl (lower than the break-even price of 60 in mode 2)
- Switch back from 1 to 0 if $Y \leq 47$\$/bbl (lower than the break-even price of 50 in mode 1)

This hedging strategy operates as if the owner holds a portfolio with two continuously paying calls: Call-1 with pay-off $5(Y - 50)^+$ and Call-2 with pay-off $5(2Y - 112)^+$. When Y is small, then $\pi^* \approx 0$. When Y is large, then $\pi^* \approx \pi^{Call2}$. When Y is in between, then $\pi^* \approx \pi^{Call1}$.

The effect of the operational constraints and the incompleteness of the market can be precisely evaluated in this framework. The value of a strip of options with daily exercise is 12.4 M\$. The indifference price without risk aversion or correlation but with switching costs is 11.6 M\$, which results in an overestimation of 7 %. On the contrary, neglecting the operational constraints but taking into account the risk aversion or the market's incompleteness leads to a value of 9.72 M\$, or a 20 %

overestimation. Taking into account both effects leads to a value of 8.8 M$, which results in an overestimation of 40 %.

Numerical efficiency: Because the valuation problem of the power plants relies heavily on the numerical algorithm, I conclude this section with numerical efficiency considerations. I summarize here the numerical efficiencies obtained by different authors together with a short description of the difficulty of their problems and the numerical method used.

In Porchet et al. [145], the valuation of the power plant takes into account the switching costs, minimum run time, ramp rates, and the non-constant heat rate. The market is incomplete, and the utility indifference pricing method is used as in Ludkovski [128]. Both authors solve the problem with two representations: a partial differential equation (PDE) and a system of forward backward stochastic differential equations (FBSDE). Each time they use finite difference schemes and compare them to a Monte-Carlo regression method. They use a two-factor model for electricity and gas and an un-hedgeable additive noise to the electricity forward price to represent the non-convergence of that price to the spot price. The authors report that the finite difference method for the PDE with a six month time horizon in an incomplete market with three dimensions takes around one week of computation. For the same valuation with a four-day horizon, the FBSDE takes around 16 min.

In Carmona and Ludkovski [60], by using dynamic programming with a Monte-Carlo regression for the valuation of a power plant with several constraints on a three-month horizon and a six-hour time step takes around five minutes on a desktop computer using Matlab.

In Ryabchenko and Uryasev [151], the authors report the valuation of a tolling contract with several constraints close to a power plant with a ten-year horizon and a one-day time step took around 17 s. Their method is based on the approximation of the optimal stationary switching boundaries which use *heuristics* and linear programming.

These three examples show two things. First, the large difference in computation time that can be obtained with different methods. Second, the difficulty in comparing those results because each time everything changed: the definition of the problem, the market model, and the efficiency of the computer itself. Making numerical experiments more standard would considerably help the assessment of the progress in this field which is of utmost concern to electricity utilities.

4.3 Storage and Swings

Because electricity cannot be stored, utilities have found ways to overcome the necessity of having flexibility in the generation management system. Energy is stored in different forms. The main form of storage is a water reservoir. Another form consists of a demand-side management contract which ensures that the customers will give up their consumption in certain situations. A typical example of a demand-side management contract was developed by EDF in the 1980s. The customers holding this special contract enjoy a lower tariff throughout the year, except during a certain number N of days (22) of the year (1 November to 31 March) where any kWh consumed cost the customer a dissuasive price. The important thing here is that the days with the dissuasive price are not known in advance by the customer or the utility. The utility warns the client the day before. Thus, the electricity utility actually holds an option contract: it has the right to select 22 days during the year where the customer's price is different. This is the first example of a *swing* option.

Further, with the development of electricity markets, swing options have emerged as a financial standardised representation for the physical and complex hydroelectric storage held by utilities. Swing options are contracts where the buyer has the right to receive a certain amount of energy q between certain fixed bounds 0 and $\overline{q}$ on a certain number of days N during a certain period of time T at a price fixed in advance. But, the contract has a final constraint which states that the entire amount of energy should be delivered between a lower bound $\underline{Q}$ and an upper bound $\overline{Q}$.

These contracts reproduce the management of an hydroelectric reservoir while getting rid of most of the operational constraints involved in the management of a real hydraulic turbine and also getting rid of the randomness of the inflows. Compared to the demand-side management contract described above, the swing option has no uncertainty on the volume consumed at each exercise.

As mentioned in Jaillet et al. [105], swing options offer the right level of protection against high-peaking days during the year. Imagine that there is a particular month of the year with high demand where the producer might be short. To cover the risk of the 22 business days of this month, the producer may want to buy a strip of 22 call options. But, this protection might be excessive because there is little likelihood that all of the days of the month will be peaking. And 10 American options covering the period of the 22 days would still be excessive because they have the same optimal exercise time. Thus, buying a swing contract with 10 dates out of 22 would be the right level of protection.

As Sect. 2.3 shows, the valuation of an hydroelectric storage system with a single independent reservoir can be formulated as:

$$V(t, s, a, x) = \sup_{q_u \in [0, \overline{q}, \delta_t \geq 0]} \mathbb{E}\left[\int_t^T q_u S_u^{t,s} du + g(S_T, X_T^{t,x})\right], \tag{4.21}$$

where $S_u^{t,s}$ is the electricity spot price—supposed here to be Markov—which takes the value s at time t, and $X_t^{t,x}$ is the level of the reservoir which takes value x at time t. This level satisfies the following dynamic:

$$dX_s^{t,x} = (A_s^{t,a} - q_s - \delta_s) ds, \tag{4.22}$$

where $A_s^{t,a}$ is a random Markov inflow which takes the value a at time t. Moreover, X is subject to level constraints and should stay within $[\underline{x}, \overline{x}]$. The control variable δ_s is the spilling variable that allows to throw away a possible excessive amount of water. The function g represents a reward function for keeping water at the final time. Without this reward function, the reservoir will be empty at time T.

For more standard swing options, there is no inflow and, following Barrera-Esteve [13], the valuation problem writes as

$$V(t, s, Q) = \sup_{\underline{q} \leq q \leq \overline{q}} \mathbb{E}\left[\sum_{0 \leq i \leq N-1} \psi(t_i, q(t_i), S(t_i), Q(t_i)) + P(T, S_T, Q_T)\right] \tag{4.23}$$

where S_t is the market spot price—still supposed here to be Markov—which takes value s at time t. Moreover, the energy purchased at time t, $q(t)$, has to be between $\underline{q}$ and $\overline{q}$. The total purchase $Q(t) = \sum_i q(t_i)$ has to be between Q_m and Q_M. The function $P(T, S, Q)$ represents a penalty function. It can be a basic linear function of the over- or under-consumption at maturity or a quadratic penalty to get a smooth function. The reward function ψ can take different forms. The most basic case is $\psi(t, q, S, Q) = q \times (S - K)$. For gas market applications, the reward functions are of the form:

$$\psi(t, q, S, Q) = -q(S + c_I)\mathbb{1}_{q \geq 0} - q(S - c_w)\mathbb{1}_{q \leq 0}$$

where c_I is an injection cost, and c_w is a withdrawal cost.

Owners are not only interested in the value itself $V(0, S_0, 0)$, but also in the optimal control strategy q, that is, how to optimally manage the storage or the swing contract, and how to hedge the value. In the case of a multi-factor price model, the number of factors increases the dimension of the state. However, there is no closed-form solutions for these valuation problems. But, some works have explored the quantitative analysis of swings options without numerical methods, like Bardou and Bouthemy [8] who show under which conditions swings are bang-bang. In the cases of linear diffusion and multiple stopping-time problems, Carmona and Dayanik [54] propose approximation formulas, whereas Keppo [113] and Rodriguez [149] propose an analysis that states in which cases swing options can be replicated by a combination of futures and call options. These results offer a better understanding of the behaviour of the swing options but do not eliminate the need for numerical methods in the general case.

Literature review. Even more than the problem of valuation and the hedging of power plants, the case of swing contracts and storage valuation has drawn the attention of the academic community. The fact that these problems are shared with the gas market certainly increases the possibility of applying the proposed methods. Thirty papers have been published in the last decade on this problem. Clearly, a swing valuation requires different ingredients:

- the asset (constrained, reward function, penalty function)
- the price model

Table 4.1 Classification of the research papers on storage and swing valuations

	References	Asset	Price model	Formulation	Numerical method	Performance
Jaillet (2004)	[105]	SW	1-MR	DP	TR	–
Dahlgren (2005)	[69]	SW	BS	VI	FD	
Dahlgren (2005)	[68]					
Zeghal (2006)	[170]	MS	Lévy	DP	–	
Barrera-Esteve (2006)	[13]	SW	MR	DP	TR, RM, NN	
Carmona (2008)	[60]					
Carmona (2008)	[54]					
Wilhelm (2008)	[167]	SW	BS		TR, RM, FE	1 h (RM), 16 s (TR), 25 s (FE)
Kjaer (2008)	[119]	SW	MRJ	PIDE	FD	–
Boogert (2008a)	[39]	GS	MR	PDE	FD	
Pflug (2009)	[140]					
Hambly (2009)	[98]	SW	2-MRJ			10 mn
Haarbrucker (2009)	[96]					
Bardou (2009)	[9]	GS	2-MR	DP	Q	1 mn
Wahab (2010)	[163]	SW	RS	DP	TR	
Bardou (2010)	[8]					
Becker (2010)	[15]					
Bronstein (2010)	[45]	SW		DP	Q	30 s
Kiesel (2010)	[117]	SW	MR	PDE, DP	FD, RM	
Carmona (2010)	[59]	GS		DP	RM	30 mn
Wahab (2010)	[163]	SW	RS	DP	TR	5 mn
Benth (2011)	[27]	SW	MR	PDE	FD	15 s
Boogert (2011)	[40]	GS	3-GBM	DP	RM	–
Rodriguez (2011)	[149]	SW				
Marshall (2011)	[131]	SW	5-GBM	DP	TR	20 s
Wahab (2011)	[162]	SW	RS	DP	TR	
Bernhart (2012)	[32]	SW	MRJ	BSDE	RM	
Turboult (2012)	[158]	SW	GBM		RM	
Warin (2012)	[164]	GS	2-MR	DP	RM	
Wiebauer (2012)	[166]	SW	GBM		RM	
Edoli (2013)	[81]	SW	MR	DP	TR	1 s
Eriksson (2014)	[123]	SW	MDL	PDE	FD	

Whenever possible, a report of the numerical efficiency is reported. Legend: *Asset* multiple stopping-time (MS), swing contract (SW), gas storage (GS), hydroelectric reservoir (HR); *Price model* Black and Scholes (BS), Gaussian mean-reversion (MR), mean-reversion with jumps (MRJ), Lévy (Lévy), regime-switching (RS); *Formulation* dynamic programming (DP), variational inequality (VI), partial differential equation (PDE), partial integro-differential equation (PIDE), backward stochastic differential equation (BSDE); *Numerical method* finite difference (FD), finite element (FE), Monte-Carlo regression (RM), neural network (NN), quantisation (Q), Malliavin calculus (Mal), trees or lattices (TR)

- the mathematical representation of the solution
- the numerical methods.

This number of ingredients makes it difficult to find a guideline and to summarise the different findings. Moreover, many alternatives have been developed for each mainstream method, which makes exposing the different solutions proposed in the literature more difficult. Here, I only provide a map of this literature and report any numerical efficiency results with the caveat that no standardised procedure exists to allow sound comparisons. The resulting map is given in Table 4.1.

As Table 4.1 shows, the literature mainly concentrates on swing contracts with Gaussian models. The reason is that this case is close to what trading desks are used to and because it also concentrates on the efficiency of numerical methods. However, it is not easy to assess the relative efficiency of those different methods. The column performance presents a sample of the efficiencies reported. It is clear that the variations are quite large ranging from a few seconds to an hour. Because all of these different works take different specifications of the asset and of the market price model, I rely on the papers which perform the comparison of different methods applied to the same problems. The results reported in Wilhelm and Winter [167] are of much interest. For a swing with five exercises, the authors find a computational time of more than one hour for a Monte-Carlo regression based on Carmona and Touzi [54] to less than ten seconds for a finite difference scheme and less than a second for a lattice method. In the same vein, the comparison performed in Bardou et al. [9] between a quantisation method and a Monte-Carlo regression exhibits a large efficiency difference in favour of the quantisation method. However, the test is performed on a one-factor model for which the quantisation is an advantage that tends to disappear with more dimensions, as shown in Bouchard and Warin [43]. Thus, despite being an important research effort, it is still difficult to decide on which numerical methods a pricing and risk management system for electricity derivatives should be built on.

Moreover, regarding the computation of the hedge on swing options, the literature is less prolific. For example, Warin [164] is the only paper dealing explicitly on the problem of the efficient computation of the deltas for the simulation of hedging strategies. The author proposes a method that reduces by a factor 80 the computational time of the deltas compared to the most efficient methods.

4.4 Retail Contracts

The complex electricity derivatives described in the preceding sections and exchanged on the wholesale market constitute only a very small part of the source of the income of utilities. Most of their revenue comes from the selling of electricity to final customers with very different terms than the complex options presented earlier. Further, retail customers are mainly looking for a fixed price contract for their variable consumption. The problem for the utility is to find a fair price for taking the risks involved with these contracts. The most important uncertainties are the consumption of the customer and the realised spot prices.

The problem faced by the utilities varies with the segment of the markets, whether they are industrial, large, professional, small, or households. The load curve of industrial and large customers are generally precisely metered. Industrial customers ask for a quotation from electricity providers through a call for tender in which they provide their past load curve. The quotation can be just for the next year or for a longer term. These contracts are tailor-made and can contain embedded options to take into account the industrial process of the customers. The offers also come with an expiry date. Further, during the customer's decision time, the prices can vary which puts the providers at greater risk. Either prices drop and the customer asks for a new quotation, or they go up and the customer exercises the free options embedded in the offer. Thus, there is always a premium added to the price of the contract to reflect this option value. The competitive pressure is strong in this segment of customers because it is not very costly—compared to the households—to gain customers because their decision is mostly driven by the price. For small firms, things are a little different. It is no longer possible to design an offer and a price for each individual firm. Moreover, only some of them have a metered load curve (think of the office of your dentist or florist). This is even more true for households which only have two measures per year in countries without smart metering. In this situation, the prices should apply to a large set of customers for which only aggregated information is known. Moreover, the competition for households is not as strong as for the industrial segment: it is much more difficult and costly to increase one's market share of this segment, because it implies spending an important amount of cash on advertising and maintaining important salesmen.

Although important for utilities and electricity providers, the problem of pricing retail contracts has received little attention from the literature. Indeed, most of papers addressing the problem concentrate on the industrial customers. This is the case in Keppo and Räsänen [114], Karandikar et al. [111], Prokopczuk et al. [148], Karandikar et al. [112], and Burger and Müller [49].

A brief summary of these approaches relies on the allocation of required capital to cover the price risk and the load risk involved with the customers' electricity consumption. If l_t is the load of the customer at hour t of the year of delivery, S_t is the spot price for the same hour, then the expected cash-flow of the provider from this customer is $\Pi(p) = \mathbb{E}[R(p)]$ with $R(p) = \int_0^T (p - S_t) l_t dt$ where p is the selling price. The criterion chosen in this literature is to find a price that satisfies a risk adjusted return on capital (RAROC). The RAROC is the ratio between the expected return of an investment and the capital used. It is fixed *ex ante* by the utility as a hurdle rate μ to be achieved by its transactions. In this situation, the return is identified as the expected cash flow from the contract. To take into account risk, the definition of the capital used requires a risk measure. In this context, the most common risk measure proposed is the cash-flow-at-risk which is a quantile of the distribution of the cash-flow. Thus, noting $q_\alpha(X)$ as the α-quantile of the random variable X, the problem consists in finding p such that

$$\mu = \frac{\Pi(p)}{\Pi(p) - q_\alpha(R(p))}. \tag{4.24}$$

Variations of this idea are developed in the literature above to take into account the correlation between the customer and the system loads, or to take into account that part of the customer's future consumption that can be hedged. The literature also develops the modelling of the spot price in the context of spikes.

The fact that this formulation is far less attractive than the nice optimal control problems arising from the valuation of power plants and swing options has certainly something to do with the relative lack of interest in this topic. Nevertheless, as Burger and Müller [49] point out, the price proposed to the customer includes a margin decided by the management of the utility. Thus, although useful for risk management purposes, the former approach does not exhaust the problem posed by the fact that retail prices are the result of a competitive process.

Further, if I consider the UK market at the time this book is written, there is a significant number of large electricity providers (Centrica, EDF Energy, E.ON, Npower, SSE, and Scottish Power) involved in a fierce competition process. They might change the prices in their contracts whenever they want. But, the change in prices affects their customers who can switch to another provider, leading to a loss of market share. Knowing this, retailers do not change their prices unless they have a good reason. And that reason comes from the increase in their sourcing costs on the wholesale market. Thus, when the wholesale forward prices increase, their retail margin decreases. At some point in time, some retailers will not take the losses anymore and will increase their prices. I illustrate the kind of dynamic that can be observed on the UK retail market on Fig. 4.3 with two retailers. In this picture, one is the leader who moves first while the other waits for the leader's move before acting. It is only a simulation but it reproduces the shape and structure of the observed data.

This situation finds its place in game theory. This game is a mixture between a *revenue management* problem and an *attrition game*. The revenue management is a

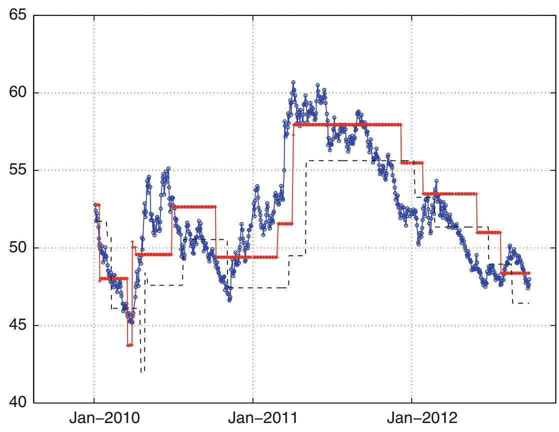


Fig. 4.3 Illustration of the dynamic of retail prices. *Blue dotted line* wholesale year-ahead electricity price, *Red crossed line* retail price of the leader, *Black dotted line* retail price performed by the follower

marketing science developed to increase revenues in the airline or the hotel industry to maximise their profit with a smart pricing policy depending on their booking rate and on the time left before reservation. It leads to dynamic control problems (see Bitran and Caldenty's [35] survey on this topic). But the situation faced by utilities is in a sense more basic: the product they sell is hardly different from their competitors and the competition is basically done on the price. Financial and market share losses are the only motivation for a move on prices. Those situations relate to the attrition game, which goes back to Maynar [133]. In this game, the players are facing a choice between staying in the game and enduring a cost or stopping the game and incurring an income. In the case of an electricity or a commodity market, the study of these situations with a tractable dynamic stochastic model is yet to be done.

4.5 Weather Derivatives

With weather derivatives, I come to a class of financial products for which the underlying cannot be held nor even produced. Since 1997, financial products have been based on the temperature, precipitation, and now the wind. Because a significant portion of the economy is sensitive to climate conditions (1/7 of the US economy according to Cao and Wei [50]), it is natural to think of insurance contracts to immunise industries from bad weather conditions. The cash flows of electric utilities are particularly sensitive to temperature. In southern countries, the hot weather during summer leads to an increase in air conditioning whereas in countries with electrical heating, cold waves lead to increases in electricity consumption. And, now, with the increase in wind generation, electricity utilities are also sensitive to the wind.

But, it is not obvious how to design a standardised financial product whose underlying is a non-storable, non-producible asset like a weather condition. And it is even more challenging to price and hedge such a contract. The typical way a contract on temperature is structured has four ingredients: a temperature index, a delivery period, a location, and a tick size. Regarding the index, the most commonly used is heating degree days (HDD or in short η) and cooling degree days (CDD) on day t. The calculation for HDD is:

$$\eta_t = \left(\theta - \frac{T_t^M + T_t^m}{2}\right)^+ ,$$

with T_t^M the maximum temperature of the day, T_t^m the minimum temperature of the day, and θ a temperature threshold often equal to 18 °C. To write a contract, a definition is needed of the location where the temperature station is located (Chicago Airport, Paris Orly, Essen…) and a period of time. Generally, the accumulation of degree days are considered over a month or a season (winter for HDD and summer for CDD). Further, one has to convert the index quoted in a physical measure (degree, mm of rain…) into a currency. This is done with the tick size. Each degree day

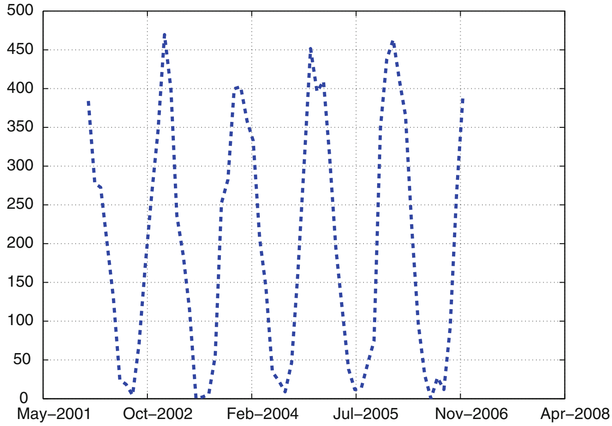


Fig. 4.4 Heating degree days for the Paris Orly airport from 2002 to 2006 as used by the Chicago Mercantile Exchange (CME) weather index

corresponds to a certain amount of cash. Thus, if the tick size is 100 €, the buyer of a forward contract on temperatures in December 2014 with HDD as an index, agrees to pay at maturity $100 \times \sum_{1 \le t \le 31} \eta_t$. Figure 4.4 shows that for a winter month, a typical value of the HDD in Paris is approximately 400. Thus, each contract leads to the payment of $400 \times 100 = 40{,}000$ €. The options can be defined as well. For example, a European call option on the temperature with a strike K in degrees Celsius for a delivery month has the payoff of

$$A \left(\sum_{1 \le t \le 31} \eta_t - K \right)^+ ,$$

where A is the value of the tick.

The problem with these derivatives is to find a principle to price them. Indeed, they belong to a frontier between insurance products and financial market products. But as Bouchard and Elie [42] point out, those derivatives cannot be priced by the actuarial pricing method where a price is determined by the diversification principal amongst the customers nor by the principle of temporal diversification (hedging). For instance, regarding the temperature outcome in winter for electricity utilities in Europe, the outcome affects all of them in the same way, which seriously limits the diversification effect for the seller of the option on temperature.

But, the literature does not seem to be afraid by these difficulties and offers several approaches to tackle this problem. Basically, people either refer to different ways to maximise utility or to estimate a market price of risk for temperatures by using data on option prices. Regarding utility maximisation, Cao and Wei [50] propose to deduce the price of a weather derivative by using Lucas's representative agent of inter-temporal utility maximisation. Davis [70] argues that risk preferences towards weather derivatives are genuinely specific to economic agents, and thus their prices could not be extracted by using the utility of a representative agent. He proposes as an alternative to define the price as the marginal value of the substitution for a utility maximising agent which is between seeing the agent's initial wealth decrease by the price of the derivatives and taking the claim. This point of view only deals with one agent and defines a fair price as relating to that agent's perception of risk. A similar approach is undertaken by Carmona and Diko [56] for precipitation derivatives where the authors use the more classical approach with the valuation method of the indifference price. However, for a transaction to occur, another agent is needed for which this price is also admissible. Barrieu and El Karoui [14] performs the study of this equilibrium by analysing the conditions under which two agents, the writer of the option and the buyer, will transact on an un-hedgeable claim. This approach helps in understanding the conditions under which weather derivatives might be exchanged. But, the utilities in the market might be better off to turn towards the more usual approach contained in Alaton et al. [5] and Benth et al. [30] for temperature derivatives on the Chicago Mercantile Exchange or Benth et al. [30] for wind futures [18]. This approach is nothing unusual from what was presented in Chap. 3 in fitting the forward curve by using a market price of risk estimated with option data. Thus, I will not enter into the detailed implementation here because the problem boils down to the modelling of the underlying risk factor (either the temperature per station, the wind, or the precipitation).

I will conclude this section on weather derivatives with comments on the development of this market. Despite the efforts of the many actors, in particular the *Weather Risk Management Association*, it is difficult to claim that the weather derivatives market has known the growth expected since its beginning in 1997. In the United States, weather derivatives are still quoted on the Chicago Mercantile Exchange, but in Europe the successive attempts to develop such a market has not been successful. Indeed, the study of the weather derivatives market performed by Huault and Rainelli [104] clearly shows that the weather derivatives market which represents only a thin portion of the derivative markets has the capacity to attract attention, but somehow fails to meet the expectations of the economic agents. The idea of protecting industries' incomes from changing weather conditions is not new. But the development of *standardised* weather products that will have the required impact on the balance sheets of a given industry is yet another problem. Most of the deals involving weather indices are performed on an over-the-counter basis with customised contracts. The reason is that a temperature measured at some point in the country might not be representative of the effect on a specific business. For example, the accumulated temperature in Paris Orly during winter might not tell much about the presence of snow in the Alps, thus making it difficult to construct an exchange between an actor positively impacted by a low temperature and one negatively impacted.

Because there is a need in many industries to find protection against bad weather outcomes for low prices, the market for weather derivatives should see its development continue. Nevertheless, at this stage, it seems that research is needed less on the pricing side than on the design of the contracts. The development of the market should not only rely on the exchange between industries looking for protection against an undesirable weather outcome and speculators ready to bet on the temperature. An equilibrium between actors having opposite sensitivities to weather conditions should exist even without speculators. This last class of economic agents only provides an excess of liquidity.

Chapter 5
Conclusion

After more than 20 years, research has proposed many alternative models or evaluation methods to address the problems in electricity derivatives. Indeed, from Gaussian mean-reversion processes to the cutting edge ambit fields, it seems that no modeling framework has escaped implementation in the electricity markets. But, I want to use this opportunity to propose some guidelines for future research. I currently see four major research streams.

1. Despite the large number of existing models, I think that there is still room for creation and innovation. However, research should not focus on power only, but should tackle the joint modelling of electricity prices and commodity prices, including carbon prices. Further, the structural models presented in Chap. 3 are just some methods to capture this dependence. Other techniques, maybe more efficient, could be applied with greater success. Moreover, the massive introduction of renewable energy and its impact has led to the need for price models which can suitably take into account this increasing phenomenon which leads to negative prices. Further, little has been done to jointly model prices in interconnected areas. And, new markets have appeared that should attract the attention of academics as well, such as the intraday market. Those who look at the dynamic of the 32 h traded forward might find it challenging to model this market. The intraday market also offers an opportunity to test economic theory on the relation between spot and forward prices because intensive data are available for this market and exogenous random events are precisely known. Other markets such as capacity markets are in their infancy in Europe and should also have their share of challenges.
2. The ordering and classifying of the zoology of models would be of great help to the industry. To use the criteria cited in the introduction of Chap. 3, it would be useful to assess them according to their realism, consistency, efficiency, robustness, and generality. Amongst these criteria, efficiency and robustness are of major importance to the development of an operational system of risk management and valuation. They should be given an unambiguous meaning to allow sound comparisons. This research would be useful in identifying a model that can be

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used as a reference or a benchmark in order to form a consensus on the pricing of electricity derivatives, such as power plants, tolling contracts, and swings. In the same vein, standardization efforts should be made so that the various numerical methods developed to evaluate these complex options can be compared.

3. In the coming years, due to the development in households of smart metering, the questions regarding the pricing of household consumptions will be of major importance. The effect of competition in the retail market could lead to interesting applications of option games. Moreover, the value of the flexibilities in household appliances should also deserves some attention.
4. More research could be devoted to the optimal hedging of physical assets by using real futures contracts available on the market. Indeed, the rare works presented in Sect. 4.2 that tackled the joint problem of physical operation of a power plant and its hedging relied on a simplified version of the electricity forward curve. More realistic optimal hedging models of physical assets might help to assess the real efficiency of the hedging strategies.
5. In connection with the preceding point, I wonder if the present structure of the electricity derivative market is the best suited to achieve its purpose, namely helping operators to hedge the value of their assets. Indeed, the present structure with its decomposition into year, quarter, month, week, day and hours results from a sound decomposition of information and is a good way to avoid a dispersion of liquidity. It seems quite natural now but, maybe, there are alternative structures that might allow better hedges. Here, it is not a question of market design, but a question of the optimal microstructure design: what series of futures and options contracts might be optimal by offering the best trade-off between liquidity and hedging efficiency?
6. The problem of valuation and hedging have been addressed at the level of an individual asset but not often at the level of the electric utility itself. Indeed, the question of the efficiency of or even the interest in risk management for a nonfinancial institution is an important field of research in the quantitative corporate finance literature. In the case of the energy markets, the problem could be addressed from the perspective of the new European financial regulation that constrains the use of derivatives by energy firms (REMIT, Regulation on Energy Market Integrity and Transparency and EMIR, European Market Infrastructure Regulation).

There is more to be done in the future than just improving the models developed in the preceding decade. More than 30 years after its birth, the field of electricity derivatives still needs innovation.

References

1. R. Adkins, D. Paxson, Reciprocal energy-switching options. J. Energy Markets **4**(1), 91–120 (2011)
2. R. Aïd, L. Campi, N. Langrené, A structural risk neutral model for pricing and hedging power derivatives. Math. Financ. **23**(3), 387–438 (2013)
3. R. Aïd, L. Campi, A. Nguyen Huu, N. Touzi, A structural risk neutral model of electricity price. Int. J. Theor. Appl. Financ. **12**(7), 925–947 (2009)
4. R. Aïd, G. Chemla, A. Porchet, N. Touzi, Hedging and vertical integration in electricity markets. Manag. Sci. **57**(8), 1438–1452 (2011)
5. P. Alaton, B. Djehiche, D. Stillberger, On modeling and pricing of weather derivatives. Appl. Math. Financ. **9**(1), 1–20 (2002)
6. E. Alos, A. Eydeland, P. Laurence, A Kirk's and a Bachelier's formula for three asset spread options. Energy Risk **9**, 52–57 (2011)
7. F. Baldi, Switch, switch, switch! A regime-switching option-based model for valuing a tolling agreement. Eng. Econ. **55**(3), 268–304 (2010)
8. O. Bardou, S. Bouthemy, When are swing options bang-bang? Int. J. Theor. Appl. Financ. **13**(6), 867–899 (2010)
9. O. Bardou, S. Bouthemy, G. Pages, Optimal quantization for the pricing of swing options. Appl. Math. Financ. **16**(2), 183–217 (2009)
10. M. Barlow, A diffusion model for electricity prices. Math. Financ. **12**(4), 287–298 (2002)
11. M. Barlow, Y. Gusef, M. Lai, Calibration of multifactor models in electricity markets. Int. J. Theor. Appl. Financ. **7**(2), 101–120 (2004)
12. O.E. Barndorff-Nielsen, F.E. Benth, A.E.D. Veraart, Modelling electricity futures by ambit fields. Adv. Appl. Probab. **46**(3), 719–745 (2014)
13. C. Barrera-Esteve, F. Bergeret, C. Dossal, E. Gobet, A. Meziou, R. Munos, D. Reboul-Salze, Numerical methods for the pricing of swing options: a stochastic control approach. Methodol. Comput. Appl. Probab. **8**(4), 517–540 (2006)
14. P. Barrieu, N. El Karoui, Optimal design of derivatives in illiquid markets. Quant. Financ. **2**, 181–188 (2002)
15. M. Becker, Unbiased Monte-Carlo valuation of lookback, swing and barrier options with continuous monitoring under variance gamma models. J. Comput. Financ. **13**(4), 35–61 (2010)
16. G. Benedetti, L. Campi, Utility indifference valuation for non-smooth payoffs with an application to power derivatives (2014 submitted)
17. G. Benmenzer, E. Gobet, C. Jérusalem, Arbitrage free cointegrated models in gas and oil futures markets (2007), http://hal.archives-ouvertes.fr/hal-00200422/fr/

R. Aïd, *Electricity Derivatives*, SpringerBriefs in Quantitative Finance,
DOI 10.1007/978-3-319-08395-7

18. F.E. Benth, J.Ł. Benth, Dynamic pricing of wind futures. Energy Econ. **31**(1), 16–24 (2009)
19. F.E. Benth, J.S. Benth, S. Koekebakker, *Stochastic Modeling of Electricity and Related Markets* (World Scientific Publishing Company, Singapore, 2008)
20. F.E. Benth, R. Biegler-König, R. Kiesel, An empirical study of the information premium on electricity markets. Energy Econ. **36**, 55–77 (2013)
21. F.E. Benth, A. Cartea, R. Kiesel, Pricing forward contracts in power markets by the certainty equivalence principle: explaining the sign of the market risk premium. J. Bank. Finance **32**(10), 2006–2021 (2008)
22. F.E. Benth, L. Ekeland, R. Hauge, B.F. Nielsen, A note on arbitrage-free pricing of forward contracts in energy markets. Appl. Math. Financ. **10**(4), 325–336 (2003)
23. F.E. Benth, J. Kallsen, T. Meyer-Brandis, A non-Gaussian Ornstein-Uhlenbeck process for electricity spot price modeling and derivatives pricing. Appl. Math. Financ. **14**(2), 153–169 (2007)
24. F.E. Benth, R. Kiesel, A. Nazarova, A critical empirical study of three electricity spot price models. Energy Econ. **34**(5), 1589–1616 (2012)
25. F.E. Benth, S. Koekebakker, Stochastic modeling of financial electricity contracts. Energy Econ. **30**, 1116–1157 (2008)
26. F.E. Benth, S. Koekebakker, F. Ollmar, Extracting and applying smooth forward curves from average-based commodity contracts with seasonal variation. J. Deriv. **15**(1), 52–66 (2007)
27. F.E. Benth, J. Lempa, T.K. Nilssen, On the optimal exercise of swing options in electricity markets. J. Energy Markets **4**(4), 3–28 (2011)
28. F.E. Benth, J. Saltyte-Benth, The normal inverse Gaussian distribution and spot price modelling in energy markets. Int. J. Theor. Appl. Financ. **7**(2), 177–192 (2004)
29. F.E. Benth, J. Saltyte-Benth, Analytical approximation for the price dynamics of spark spread options. Stud. Nonlinear Dyn. Econom. **10**(3), 1–28 (2006)
30. F.E. Benth, J. Saltyte-Benth, T. Koekebakker, Putting a price on temperature. Scand. J. Stat. **34**(4), 746–767 (2007)
31. F.E. Benth, L. Vos, A multivariate non-Gaussian stochastic volatility model with leverage for energy markets. Technical report 20, Department of Mathematic CMA (2009)
32. M. Bernhart, H. Pham, P. Tankov, X. Warin, Swing Options Valuation: A BSDE with Constrained Jumps Approach, ed. by R. Carmona, P. Del Moral, P. Hu, N. Oudjane. Springer Proceedings in Mathematics, vol. 12 (Springer, Berlin, 2012), pp. 379–400
33. H. Bessembinder, M.L. Lemmon, Equilibrium pricing and optimal hedging in electricity forward markets. J. Financ. **23**, 1347–1382 (2002)
34. K. Bhanot, Behavior for power prices: implications for the valuation and hedging of financial contracts. J. Risk **2**(3), 43–62 (2000)
35. G. Bitran, R. Caldentey, An overview of pricing models for revenue management. Manuf. Serv. Oper. Manage. **3**(5), 203–229 (2003)
36. P. Bjerksund, G. Stensland, Closed form spread option valuation. Working paper, Department of Finance NHH, Oct 2006
37. F. Black, The pricing of commodity contracts. J. Financ. Econ. **3**(12), 167–179 (1976)
38. M. Boiteux, Peak-load pricing. J. Bus. **33**(2), 157–179 (1960)
39. A. Boogert, C. De Jong, Gas storage valuation using a Monte-Carlo method. J. Deriv. **15**(3), 81–98 (2008)
40. A. Boogert, C. de Jong, Gas storage valuation using a multifactor price process. J. Energy Markets **4**(4), 29–52 (2011)
41. S. Borovkova, H. Geman, Seasonal and stochastic effects in commodity forward curves. Rev. Deriv. Res. **9**(2), 167–186 (2006)
42. B. Bouchard, R. Elie, L. Moreau, A note on utility based pricing and asymptotic risk diversification. Math. Financ. Econ. **6**(1), 59–74 (2011)
43. B. Bouchard, X. Warin, Monte-Carlo Valuation of American Options: Facts and New Algorithms to Improve Existing Methods, ed. by R. Carmona, P. Del Moral, P. Hu, N. Oudjane. Springer Proceedings in Mathematics, vol. 12 (Springer, Berlin, 2012), pp. 215–255
44. D. Brigo, F. Mercurio, *Interest Rate Models Theory and Practice* (Springer, New York, 2001)

45. A.L. Bronstein, G. Pages, B. Wilbertz, How to speed up the quantization tree algorithm with an application to swing options? Quant. Financ. **10**(9), 995–1007 (2010)
46. E. Broszkiewicz-Suwaj, A. Jurlewicz, Pricing on electricity market based on coupled-continuous-time-random-walk concept. Phys. A Stat. Mech. Appl. **387**(22), 5503–5510 (2008)
47. M. Burger, B. Graeber, G. Schindlmayr, *Managing Energy Risk: An Integrated View on Power and Other Energy Markets* (Wiley, Hoboken, 2008)
48. M. Burger, B. Klar, A. Müller, G. Schlindmayr, A spot market model for pricing derivatives in electricity markets. Quant. Financ. **4**, 109–122 (2004)
49. M. Burger, J. Müller, Risk-adequate pricing of retail power contracts. J. Energy Markets **4**(4), 53–75 (2011)
50. M. Cao, J. Wei, Weather derivatives valuation and market price of weather risk. J. Futures Markets **11**(24), 1065–1089 (2004)
51. R. Carmona, M. Coulon, A Survey of Commodity Markets and Structural Models for Electricity Prices, ed. by F. E. Benth, V. Kholodny, P. Laurence. Volume Quantitative Energy Finance: Modeling, Pricing and Hedging in Energy and Commodity Markets (Springer, Berlin, 2013), pp. 41–83
52. R. Carmona, M. Coulon, D. Schwarz, The valuation of clean spread options: linking electricity, emissions and fuels. Working paper available on ssrn (2012)
53. R. Carmona, M. Coulon, D. Schwarz, Electricity price modeling and asset valuation: a multi-fuel structural approach. Math. Financ. Econ. **7**(2), 167–202 (2013)
54. R. Carmona, S. Dayanik, Optimal multiple stopping of linear diffusion. Math. Oper. Res. **33**(2), 446–460 (2008)
55. R. Carmona, P. Del Moral, N. Oudjane, P. Hu, *Numerical Methods in Finance* (Springer, New York, 2012)
56. R. Carmona, P. Diko, Pricing precipitation based derivatives. Int. J. Theor. Appl. Financ. **8**(7), 959–988 (2005)
57. R. Carmona, V. Durrleman, Pricing and hedging spread options. SIAM Rev. **45**, 627–685 (2003)
58. R. Carmona, M. Ludkovski, Pricing asset scheduling flexibility using optimal switching. Math. Oper. Res. **15**(5–6), 405–447 (2008)
59. R. Carmona, M. Ludkovski, Valuation of energy storage: an optimal switching approach. Quant. Financ. **10**(4), 359–374 (2010)
60. R. Carmona, N. Touzi, Optimal multiple stopping and valuation of swing options. Math. Financ. **18**(2), 239–268 (2008)
61. A. Cartea, M.G. Figueroa, Pricing in electricity markets: a mean reverting jump diffusion model with seasonality. Appl. Math. Financ. **12**(4), 313–335 (2005)
62. A. Cartea, M.G. Figueroa, H. Geman, Modelling electricity prices with forward looking capacity constraints. Appl. Math. Financ. **16**(2), 103–122 (2009)
63. A. Cartea, C. González-Pedraz, How much should we pay for interconnecting electricity markets? A real options approach. Energy Econ. **34**(1), 14–30 (2012)
64. A. Cartea, P. Villaplana, Spot price modeling and the valuation of electricity forward contracts: the role of demand and capacity. J. Bank. Finance **32**(12), 2501–2519 (2008)
65. L. Clewlow, C. Strickland, *Energy Derivatives* (Lacima Group, London, 2000)
66. G. Cortazar, E. Schwartz, The valuation of commodity contingent claims. J. Deriv. **1**, 27–39 (1994)
67. M. Coulon, S. Howison, Stochastic behaviour of the electricity bid stack: from fundamental drivers to power prices. J. Energy Markets **2**(1), 29–69 (2009)
68. M. Dahlgren, A continuous time model to price commodity-based swing options. Rev. Deriv. Res. **8**(1), 27–47 (2005)
69. M. Dahlgren, R. Korn, The swing option on the stock market. Int. J. Theor. Appl. Financ. **8**(1), 123–139 (2005)
70. M.H.A. Davis, Pricing weather derivatives by marginal value. Quant. Financ. **1**, 305–308 (2001)

71. C. De Franco, P. Tankov, X. Warin, Numerical methods for the quadratic hedging problem in Markov models with jumps. J. Comput. Financ. (2015 to appear)
72. C. De Jong, S. Schneider, Cointegration between gas and power spot prices. J. Energy Markets **2**(3), 27–46 (2009)
73. P. Del Moral, P. Hu, O. Oudjane, B. Rémillard, On the robustness of the Snell envelope. SIAM J. Financ. Math. **2**, 587–626 (2011)
74. M.A.H. Dempster, E. Medova, K. Tang, Long term spread option valuation and hedging. J. Bank. Finance **32**(12), 2530–2540 (2008)
75. S.-J. Deng, W. Jiang, Levy process-driven mean-reverting electricity price model: the marginal distribution analysis. Decis. Support Syst. **40**(3–4), 483–494 (2005)
76. S.-J. Deng, Z. Xia, A real options approach for pricing electricity tolling agreements. Int. J. Inf. Technol. Decis. Making **5**(3), 421–436 (2006)
77. P. Diko, S. Lawford, V. Limpens, Risk premia in electricity forward prices. Stud. Nonlinear Dyn. Econ. 10(3), 1–25 (2006) (paper 7)
78. R. Djabali, J. Hoeksema, Y. Langer, *COSMOS Description CWE Market Coupling Algorithm Document* (APX Endex, AelPex and EpexSpot, Jan 2011)
79. R. Döttling, P. Heider, Spread volatility of co-integrated commodity pairs. J. Energy markets **55**(2), 251–276 (2013)
80. E. Eberlein, Fourier-based Valuation Methods in Mathematical Finance, ed. by F.E. Benth, V. Kholodnyi, P. Laurence. Quantitative Energy Finance: Modeling, Pricing and Hedging in Energy and Commodity Markets (Springer, Berlin, 2013), pp. 85–114
81. E. Edoli, S. Fiorenzani, S. Ravelli, T. Vargiolu, Modeling and valuing make-up clauses in gas swing contracts. Energy Econ. **35**, 58–73 (2013)
82. A. Ehrenmann, K. Neuhoff, A comparison of electricity market designs in networks. Oper. Res. **57**(2), 274–286 (2009)
83. R.F. Engle, C.W.J. Granger, Co-integration and error correction: representation, estimation, and testing. Econometrica **55**(2), 251–276 (1987)
84. C. Erlwein, F.E. Benth, R. Mamon, HMM filtering and parameter estimation of an electricity spot price model. Energy Econ. **32**(5), 1034–1043 (2010)
85. A. Eydeland, K. Wolyniec, *Energy and Power Risk Management: New Developments in Modeling, Pricing and Hedging* (Wiley, Hoboken, 2002)
86. O. Féron, E. Daboussi, Calibration of Electricity Price Models. Commodities, Energy and Environmental Finance, ed. by M. Ludkovski, R. Sircar, R. Aïd (Springer, Berlin, 2013)
87. S.E. Fleten, J. Lemming, Constructing forward price curves in electricity markets. Energy Econ. **25**, 409–424 (2003)
88. E. Fournié, J.M. Lasry, J. Lebuchoux, P.-L. Lions, N. Touzi, Some applications of Malliavin calculus to Monte-Carlo methods in finance. Financ. Stoch. **3**, 391–412 (1999)
89. D. Frestad, Common and unique factors influencing daily swap returns in the Nordic electricity market, 1997–2005. Energy Econ. **30**(3), 1081–1097 (2008)
90. D. Frestad, Correlations among forward returns in the Nordic electricity market. Int. J. Theor. Appl. Financ. **12**(5), 589–603 (2009)
91. D. Frestad, F.E. Benth, S. Koekebakker, Modeling term structure dynamics in the Nordic electricity swap market. Energy J. **31**(2), 53–86 (2010)
92. N. Frikha, V. Lemaire, Joint modelling of gas and electricity spot prices. Appl. Math. Financ. **20**(1), 69–93 (2013)
93. H. Geman, *Commodities and Commodity Derivatives: Modelling and Pricing for Agriculturals, Metals and Energy* (Wiley, Chichester, 2007)
94. H. Geman, A. Roncoroni, Understanding the fine structure of electricity prices. J. Bus. **79**(3), 1225–1262 (2006)
95. S. Grine, P. Diko, Multi-layer model of correlated energy prices. J. Comput. Appl. Math. **233**(10), 2590–2610 (2010)
96. G. Haarbrücker, D. Kuhn, Valuation of electricity swing options by multistage stochastic programming. Automatica **45**(4), 889–899 (2009)

97. S. Hamadéne, M. Jeanblanc, On the starting and stopping problem: application in reversible investments. Math. Oper. Res. **32**(1), 182–192 (2007)
98. B. Hambly, S. Howison, T. Kluge, Modelling spikes and pricing swing options in electricity markets. Quant. Financ. **9**(8), 937–949 (2009)
99. D. Heath, R. Jarrow, A. Morton, Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation. Econometrica **60**(1), 77–105 (1992)
100. S. Hikspoors, S. Jaimungal, Energy spot price models and spread options pricing. Int. J. Theor. Appl. Financ. **10**(7), 1111–1135 (2007)
101. J. Hinz, L. von Grafenstein, M. Verschuere, M. Wilhelm, Pricing electricity risk by interest rate methods. Quant. Financ. **5**(1), 49–60 (2005)
102. J. Hinz, M. Wilhelm, Pricing flow commodity derivatives using fixed income market techniques. Int. J. Theor. Appl. Financ. **9**(8), 1299–1321 (2006)
103. J. Hlouskova, S. Kossmeier, M. Obersteiner, A. Schnabl, Real options and the value of generation capacity in the German electricity market. Rev. Financ. Econ. **14**(3–4), 297–310 (2005)
104. I. Huault, H. Rainelli-Weiss, A market for weather risk? conflicting metrics, attempts at compromise and limits to commensuration. Organ. Stud. **10**(32), 1395–1419 (2011)
105. P. Jaillet, E.I. Ronn, S. Tompaidis, Valuation of commodity-based swing options. Manag. Sci. **50**(7), 909–921 (2004)
106. F. Jamshidian, Commodity options valuation in the Gaussian futures term structure model. Rev. Futures Markets **10**(2), 324–346 (1993)
107. K. Kaaresen, E. Husby, A joint state-space model for spot and futures power. Energy Power Risk Manag. **7**(8), 14 (2002)
108. N. Kaldor, A note on the theory of the forward market. Rev. Econ. Stud. **7**(3), 196–201 (1940)
109. T. Kanamura, A supply and demand based volatility model for energy prices. Energy Econ. **31**(5), 736–747 (2009)
110. T. Kanamura, K. Ohashi, A structural model for electricity prices with spikes: measurement of spike risk and optimal policies for hydropower plant operation. Energy Econ. **29**(5), 1010–1032 (2007)
111. R.G. Karandikar, S.A. Khaparde, S.V. Kulkarni, Quantifying price risk of electricity retailer based on CAPM and RAROC methodology. Int. J. Electr. Power Energy Syst. **29**(10), 803–809 (2007)
112. R.G. Karandikar, S.A. Khaparde, S.V. Kulkarni, Strategic evaluation of bilateral contract for electricity retailer in restructured power market. Int. J. Electr. Power Energy Syst. **32**(5), 457–463 (2010)
113. J. Keppo, Pricing of electricity swing options. J. Deriv. **11**(3), 26–43 (2004)
114. J. Keppo, M. Räsänen, Pricing of electricity tariffs in competitive markets. Energy Econ. **21**(3), 213–223 (1999)
115. V. Kholodnyi, Valuation and hedging of European contingent claims on power with spikes: a non-Markovian approach. J. Eng. Math. **49**(3), 233–252 (2004)
116. V. Kholodnyi, Modelling Power Forward Prices for Positive and Negative Power Spot Prices with Upward and Downward Spikes in the Framework of the Non-Markovian Approach, ed. by F.E. Benth, V. Kholodny, P. Laurence, Quantitative Energy, Finance: Modeling, Pricing and Hedging, in Energy and Commodity Markets, (Springer, Berlin, 2013), pp. 189–211
117. R. Kiesel, J. Gernhard, S.-O. Stoll, Valuation of commodity-based swing options. J. Energy Markets **3**(3), 91–112 (2010)
118. R. Kiesel, G. Schindlmayr, R.H. Borger, A two-factor model for the electricity forward market. Quant. Financ. **9**(3), 279–287 (2009)
119. M. Kjaer, Pricing of swing options in a mean reverting model with jumps. Appl. Math. Financ. **15**(5–6), 479–502 (2008)
120. P.J. Knez, R. Litterman, J. Scheinkman, Common factors affecting bond returns. J. Financ. **49**(5), 1861–1882 (1994)
121. S. Koekebakker, F. Ollmar, Forward curve dynamics in the Nordic electricity market. Manag. Financ. **31**(6), 73–94 (2005)

122. N. Landon, Almost sure optimal stopping times: theory and applications, Ph.D. thesis, Ecole Polytechnique, 2013 (Available on Hal)
123. J. Lempa, M. Eriksson, T.K. Nilssen, Swing options in commodity markets: a multidimensional Lévy diffusion model. Math. Methods Oper. Res. **79**(1), 31–67 (2014)
124. R. Litterman, J. Scheinkman, Common factors affecting bond returns. J. Fixed Income **1**(1), 54–61 (1991)
125. F.A. Longstaff, E.S. Schwartz, Valuing American options by simulation: a simple least-squares approach. Rev. Financ. **14**, 113–147 (2001)
126. F.A. Longstaff, A.W. Wang, Electricity forward prices: a high-frequency empirical analysis. J. Financ. **59**(4), 1877–1900 (2004)
127. J. Lucia, E.S. Schwartz, Electricity prices and power derivatives: evidence from the Nordic power exchange. Rev. Deriv. Res. **5**(1), 5–50 (2002)
128. M. Ludkovski, Financial hedging of operational flexibility. Int. J. Theor. Appl. Financ. **11**(8), 799–839 (2008)
129. M.R. Lyle, R.J. Helliott, A “simple” hybrid model for power derivatives. Energy Econ. **31**, 757–767 (2009)
130. W. Margrabe, The value of an option to exchange one asset for another. J. Financ. **33**(1), 177–186 (1978)
131. T.J. Marshall, R.M. Reesor, Forest of stochastic meshes: a new method for valuing high-dimensional swing options. Oper. Res. Lett. **39**(1), 17–21 (2011)
132. P. Massé, *Les réserves et la régulation de l’avenir dans la vie économique (Reservoirs and regulation for future economic activity)* (Herman, Paris, 1947)
133. S. Maynard, The theory of games and the evolution of animal conflicts. J. Theor. Biol. **47**, 209–221 (1974)
134. T. Meyer-Brandis, P. Tankov, Multifactor jump-diffusion models of electricity prices. Int. J. Theor. Appl. Financ. **11**(5), 503–528 (2008)
135. A. Monfort, O. Féron, Joint econometric modeling of spot electricity prices, forwards and options. Rev. Deriv. Res. **15**(3), 217–256 (2012)
136. M. Musiela, M. Rutkowski, *Martingale Methods in Financial Modelling* (Springer, Berlin, 2005)
137. S. Ohana, Modeling global and local dependence in a pair of commodity forward curves with an application to the US natural gas and heating oil markets. Energy Econ. **32**(2), 373–388 (2010)
138. G. Pagés, J. Printemps, Functional quantization for numerics with an application to option pricing. Monte Carlo Methods Appl. **11**(4), 407–446 (2005)
139. M.V.F. Pereira, Optimal stochastic operations scheduling of large hydroelectric systems. Int. J. Electr. Power Energy Syst. **11**, 161–169 (1989)
140. G.C. Pflug, N. Broussev, Electricity swing options: behavioral models and pricing. Eur. J. Oper. Res. **197**(3–16), 1041–1050 (2009)
141. H. Pham, On quadratic hedging in continuous time. Math. Methods Oper. Res. **51**, 315–319 (2000)
142. H. Pham, *Continuous-time Stochastic Control and Optimization with Financial Applications* (Springer, Berlin, 2011)
143. D. Pilipovic, *Energy Risk: Valuing and Managing Energy Derivatives*, 2nd edn. (McGraw Hill, New York, 2007)
144. G. Pirrong, M. Jermakyan, The price of power–the valuation of power and weather derivatives. J. Bank. Finance **32**(12), 2520–2529 (2008)
145. A. Porchet, N. Touzi, X. Warin, Valuation of power plants by utility indifference and numerical computation. Math. Methods Oper. Res. **70**(1), 47–75 (2009)
146. A. Prekopa, J. Mayer, B. Strazicky, I. Deak, J. Hoffer, A. Nemeth, B. Potecz, *Scheduling of Power Generation: A Large-Scale Mixed-Variable Model* (Springer, Berlin, 2014)
147. H. Prevot, B. de Juvigny, F. Lehmann, M. Louvot, C. Izart, *Rapport d’enquête sur les prix de l’électricité (report on the prices of electricity)* (Document, French Ministry of Economy, Finance and Industry, 2004)

148. M. Prokopczuk, S.T. Rachev, G. Schindlmayr, S. Trück, Quantifying risk in the electricity business: a raroc-based approach. Energy Econ. **29**(5), 1033–1049 (2007)
149. J.F. Rodriguez, Hedging swing options. Int. J. Theor. Appl. Financ. **14**(2), 295–312 (2011)
150. RTE, Power System Reliability Memento RTE, 2004
151. V. Ryabchenko, S. Uryasev, Pricing energy derivatives by linear programming: tolling agreement contracts. J. Comput. Financ. **14**(3), 73–126 (2011)
152. E. Schwartz, The stochastic behavior of commodity prices: implications for valuation and hedging. J. Financ. **52**(3), 923–973 (1997)
153. F.P. Sioshansi, W. Pfaffenberger, *Electricity Market Reform: An International Perspective* (Elsevier, Amsterdam, 2006)
154. S. Stoft, *Power System Economics: Designing Markets for Electricity* (Wiley-IEEE Press, New York, 2002)
155. G. Swindle, *Valuation and Risk Management in Energy Markets* (Cambridge University Press, Cambridge, 2014)
156. M. Thompson, M. Davison, H. Rasmussen, Valuation and optimal operation of electric power plants in competitive markets. Oper. Res. **52**(4), 546–562 (2004)
157. D. Tsitakis, S. Xanthopoulos, A.N. Yannacopoulos, A closed-form solution for the price of cross-commodity electricity derivatives. Phys. A Stat. Mech. Appl. **371**(2), 543–551 (2006)
158. F. Turboult, Y. Youlal, Swing Option Pricing by Optimal Exercise Boundary Estimation, ed. by R. Carmona, P. Del Moral, P. Hu, N. Oudjane. Springer Proceedings in Mathematics, vol. 12 (Springer, Berlin, 2012), pp. 400–419
159. O.A. Vasicek, An equilibrium characterization of the term structure. J. Financ. Econ. **5**, 177–188 (1977)
160. J. Viehmann, Risk premiums in the German day-ahead electricity market. Energy Policy **39**(1), 386–394 (2011)
161. N.-H.M. von der Fehr, P.V. Hansen, Electricity retailing in Norway. Energy J. **13**(1), 25–45 (2010)
162. M. Wahab, C.-G. Lee, Pricing swing options with regime switching. Ann. Oper. Res. **185**(1), 139–160 (2011)
163. M.I.M. Wahab, Z. Yin, N.C.P. Edirisinghe, Pricing swing options in the electricity markets under regime-switching uncertainty. Quant. Financ. **10**(9), 975–994 (2010)
164. X. Warin, Gas Storage Hedging, ed. by R. Carmona, P. Del Moral, P. Hu, N. Oudjane. Springer Proceedings in Mathematics, vol. 12 (Springer, Berlin, 2012), pp. 421–445
165. R. Weron, *Modeling and Forecasting Electricity Loads and Prices–A Statistical Approach* (Wiley, Chichester, 2006)
166. K. Wiebauer, A Practical View on Valuation of Multi-Exercise American Style Options in Gas and Electricity Markets, ed. by R. Carmona, P. Del Moral, P. Hu, N. Oudjane, Springer Proceedings, in Mathematics, vol. 12, (Springer, Berlin, 2012), pp. 353–378
167. M. Wilhelm, C. Winter, Finite element valuation of swing options. J. Comput. Financ. **11**(3), 107–132 (2008)
168. S. Wilkens, J. Wimschulte, The pricing of electricity futures: evidence from the European energy exchange. J. Futures Markets **27**(4), 387–410 (2007)
169. A.J. Wood, B.F. Wollenberg, G.B. Sheblé, *Power Generation, Operation and Control*, 3rd edn. (Wiley, New York, 2013)
170. A.B. Zeghal, M. Mnif, Optimal multiple stopping and valuation of swing options in Levy models. Int. J. Theor. Appl. Financ. **9**(8), 1267–1297 (2006)