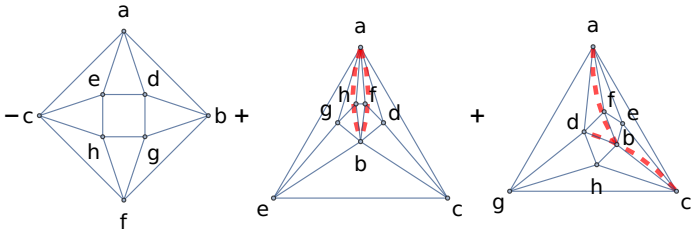
Predicting Vanishing Coefficients of f-Graphs with Machine Learning

1 Setup and Motivation

We consider the set of graph called f-graphs contributing to the four-point correlator at fixed loop order in planar $\mathcal{N} = 4$ supersymmetric Yang–Mills. Each f-graph is a has two types of edges which are graphically represented with solid and dashed lines. F-graphs are graphs with the following properties

- The solid lines form a planar graph.
- The solid lines cannot have multiple edges connecting a pair of vertices.
- For each vertex the number of solid edges minus the number of dashed edges is equal to 4.

The l-loop correlator is given by a linear combination with rational coefficients of f-graphs with $4 + l$ points. For example the 4-loop correlator is given by



where the sum over all possible labellings is understood. The space of f-graphs at 1,2 and 3 loop is one-dimensional. From 8 points onward, the space becomes non-trivial. At 6-loop the first zero-coefficient f-graphs appear. By 9-loop, zero-coefficient graphs form the majority, and this proportion grows

with the number of points. Our goal is to train machine learning models capable of predicting whether the coefficient associated with a given f-graph is zero or non-zero.

f-graph			
loop	**Total**	**#1**	**#0**
7	220	127	93
8	2,709	1,060	1,649
9	43,017	10,525	32,492
10	900,146	136,433	763,712

Table 1: Summary of f-graph data across loops.

2 Random forest for f-graphs

For each f-graph, we construct two main sets of features:

- **Solid line spectrum:** We compute the eigenvalue spectrum of the adjacency matrix associated with the subgraph defined by the solid lines. This captures global structural information about the propagator topology.
- **Difference spectrum:** Since different f-graphs can share the same denominator graph but still have different coefficients, we introduce a second matrix: the difference between the adjacency matrices of the solid line and the dashed lines. We compute the spectrum of this matrix as well.

Our classification target is the column `COEFFICIENT_DEN`, which takes binary values: 1 if the coefficient is non-zero, and 0 otherwise.

We evaluate each model using an 80/20 train-test split. That is, 80% of the full dataset is used for training, and the remaining 20% is reserved for testing the model's ability to classify unseen examples.

We also perform initial experiments on the full dataset, as our goal is to determine whether a perfect classifier exists. In such a context, overfitting is not a concern: we are not interested in generalization to unseen examples, but in whether a model can in principle learn the structure of the data.

Loop	Accuracy	Class 0 (Prec/Rec)	Class 1 (Prec/Rec)	Perfect Class.
7	0.89	0.89 / 0.84	0.88 / 0.92	yes
8	0.83	0.88 / 0.85	0.77 / 0.82	yes
9	0.87	0.89 / 0.94	0.78 / 0.65	yes
10	0.89	0.91 / 0.97	0.76 / 0.45	no

Table 2: Random Forest results for loops 7 to 10 using 0.8 training set and corresponding full set accuracies.

At 10-loop a perfect classification is not possible. The misclassified graphs indices are: [4724, 7075, 19810, 281029, 290977, 291224, 291226, 672010, 824379]

There are 3308 cospectral graphs. 4 out of 9 of the mismatches comes from the co-spectral graphs that have different coefficients, there are

$$(4656, 4724), (290977, 290994), (672010, 672028), (280495, 281029)$$

3 Solid graph classification

We now focus exclusively on the structure of the denominator graph. Rather than predicting the coefficient of each individual f-graph, we group all f-graphs that share the same denominator and ask a coarser question: *does at least one f-graph in the family contribute to the correlator?*

We assign a binary label to each denominator graph:

- **1** if at least one f-graph with this denominator has a non-zero coefficient,
- **0** if all f-graphs with this denominator have vanishing coefficients.

This shifts the focus from fine-grained coefficient prediction to identifying which denominator topologies are relevant for the correlator.

Loop	Total	#1	#0
7	164	107	57
8	1,432	729	703
9	13,972	5,661	8,311
10	153,252	51,263	101,989

Table 3: Summary of solid graph data across loops.

We trained a Random Forest classifier using only the spectral features of the denominator graph, and evaluated performance across three loop orders: 8, 9, and 10 loops. For each loop order, we report results using an 80/20 train-test split. For the 9-loop case, we also tested on the full dataset to assess whether perfect classification is possible.

Loop Order	**Accuracy (80/20 split)**	**Macro F1-score**	**Accuracy (full)**
7	0.7879	0.74	1
8	0.7770	0.78	1
9	0.7635	0.75	0.9998
10	0.7959	0.76	0.99986

Table 4: Random Forest classification performance using only denominator graph features. The macro F1-score averages performance across the two classes (0 and 1).