Gilmor : Chapter 7.2

Equation (7-1) ((own magnetic field of the beam is neglected!)): (1)

extern field!

$$\ddot{r} - r\dot{\theta}^2 = -\frac{e}{m}\left(E_r + \frac{Br\dot{\theta}}{c}\right), \qquad (7\text{-}1)$$

where Bush's theoreme is (7-2):

$$\dot{\theta} = \frac{e}{2mc}\left(B - B_c\frac{r_c^2}{r^2}\right) = \frac{eB}{2mc}\left(1 - \frac{B_c}{B}\frac{r_c^2}{r^2}\right) =$$

$$= \omega_L\left(1 - \frac{B_c}{B}\frac{r_c^2}{r^2}\right) \qquad (7\text{-}2)$$

where ω_L is larmor frequency, which a half of the cyclotron frequency $\omega_{cyc} = \frac{eH}{mc}$; r_c, B_c – cathod radiu and magnetic field on the cathod.

~~theorem~~ Gauss's theoreme: on the boundary of the beam:

$$2\pi b E_r = -4\pi\rho = -4\pi\frac{I}{u_0},$$

where u_0 – longitudinal velocity of the electrons.

So,

$$E_r = -\frac{2I}{b u_0} \qquad (7\text{-}3)$$

Now the equation for „scallop" of the beam (from (7-1))

$$0 = \ddot{b} - b\dot{\theta}^2 + \frac{e}{m}\left(E_r + \frac{Bb\dot{\theta}}{c}\right) = \ddot{b} - b\omega_L^2\left(1 - \frac{B_c}{B}\frac{r_c^2}{b^2}\right)^2 +$$

$$+ \frac{eB}{mc}b\omega_L\left(1 - \frac{B_c}{B}\frac{r_c^2}{b^2}\right) - \frac{e}{m}\frac{2I}{bu_0} =$$

$$= \ddot{b} + \omega_L^2 b\left[-1 + \cancel{2\frac{B_c}{B}\frac{r_c^2}{b^2}} - \left(\frac{B_c}{B}\frac{r_c^2}{b^2}\right)^2 + 2 - \cancel{2\frac{B_c}{B}\frac{r_c^2}{b^2}}\right] - \frac{2eI}{mbu_0} =$$

$$= \ddot{b} + b\omega_L^2\left[1 - \left(\frac{B_c}{B}\frac{r_c^2}{b^2}\right)^2\right] - \frac{2eI}{mbu_0} =$$

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$$= \ddot{b} + \omega_L^2 \left\{ b\left[1-\left(\frac{B_c}{B}\frac{r_c^2}{b^2}\right)^2\right] - \underbrace{\frac{2eI/mU_0}{\omega_L^2}}_{=a^2}\frac{1}{b}\right\} =$$

$$= \ddot{b} + \omega_L^2 \left\{ b\left[\quad\right] - \frac{a^2}{b}\right\} =$$

$$\Rightarrow \frac{\ddot{b}}{a} + \omega_L^2 \left\{ \frac{b}{a}\left[\quad\right] - \frac{1}{b/a}\right\}, \qquad (7\text{-}6)$$

where confinement factor a is

$$a = \sqrt{\frac{2eI/mU_0}{\omega_L^2}} = \sqrt{\frac{2eI}{mU_0}\frac{4m^2c^2}{e^2H^2}} = \frac{2}{B}\sqrt{\frac{2I/e}{U_0/mc^2}} \qquad (1)$$

The equilibrium radius of the beam b_e is defined by equation (from (7-6), when $\ddot{b}_e = 0$)

$$0 = \frac{b_e}{a}\left[1-\left(\frac{B_c}{B}\frac{r_c^2}{b_e^2}\right)^2\right] - \frac{1}{b_e/a} =$$

$$= \left(\frac{b_e}{a}\right)^2\left[1-\left(\frac{B_c}{B}\frac{r_c^2}{a^2}\frac{a^2}{b_e^2}\right)^2\right] - 1 =$$

$$= \left(\frac{b_e}{a}\right)^2 - \left(\frac{B_c}{B}\frac{r_c^2}{a^2}\right)^2\frac{a^2}{b_e^2} - 1 = \qquad (2)$$

$$= \frac{b_e^2}{a^2} - \left(\frac{B_c}{B}\frac{r_c^2}{a^2}\right)^2\frac{1}{b_e^2/a^2} - 1 =$$

$$= \left(\frac{b_e}{a}\right)^4 - \left(\frac{b_e}{a}\right)^2 - \left(\frac{B_c}{B}\frac{r_c^2}{a^2}\right)^2.$$

Solution:

$$\left(\frac{b_e}{a}\right)^2 = \frac{1 \pm \sqrt{1+4\left(\frac{B_c}{B}\frac{r_c^2}{a^2}\right)^2}}{2} \rightarrow \text{only sign „+"!}$$

$$\rightarrow \frac{1+\sqrt{\quad}}{2}$$

So

$$m = \frac{b_e}{a} = \frac{1}{\sqrt{2}}\sqrt{1+\sqrt{1+4\left(\frac{B_c}{B}\frac{r_c^2}{a^2}\right)^2}} \qquad (3)$$

For small a:

$$m = \frac{1}{\sqrt{2}}\sqrt{1+\sqrt{1+4(\ )^2}} \simeq \frac{1}{\sqrt{2}}\sqrt{1+2\frac{B_c}{B}\frac{r_c^2}{a^2}} \simeq$$

$$\simeq \sqrt{\frac{B_c}{B}}\frac{r_c}{a} \gg 1 \qquad (4)$$

Torqe

$$b_e = ma = r_c\sqrt{\frac{B_c}{B}} \qquad (5)$$