Сретенский

Внутри:

$$V(x,y,z)=\pi\rho abc\int_0^{\infty}\left[1-\frac{x^2}{a^2+s}-\frac{y^2}{b^2+s}-\frac{z^2}{c^2+s}\right]\frac{ds}{\sqrt{(a^2+s)(b^2+s)(c^2+s)}}$$

Снаружи:

$$V(x,y,z)=\pi\rho abc\int_{\lambda}^{\infty}\left[\quad\right]\frac{ds}{\sqrt{\quad}};$$

$\boxed{\lambda \text{ — корень ур-ния } \frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}+\frac{z^2}{c^2+\lambda}=1}$

$[\lambda]=м^2$

Поле внутри

$$E_x=-\frac{\partial V}{\partial x}=2\pi\rho abc\cdot x\int_0^{\infty}\frac{ds}{(a^2+s)\sqrt{\quad}}=2\pi\rho M_x x$$

$$E_y=-\frac{\partial V}{\partial y}=2\pi\rho abc\cdot y\int_0^{\infty}\frac{ds}{(b^2+s)\sqrt{\quad}}=2\pi\rho M_y\cdot y$$

$$E_z=-\frac{\partial V}{\partial z}=2\pi\rho abc\cdot z\int_0^{\infty}\frac{ds}{(c^2+s)\sqrt{\quad}}=2\pi\rho M_z\cdot z$$

Поле снаружи:

$$E_x=2\pi\rho abc\cdot x\int_{\lambda}^{\infty}\frac{1}{(a^2+s)\sqrt{\quad}}=2\pi\rho M_x(\lambda)\cdot x$$

$$E_y=\quad\ldots\quad=2\pi\rho M_y(\lambda)\cdot y$$

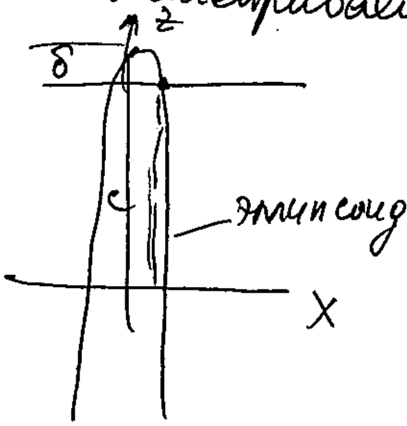
$$E_z=\quad\ldots\quad=2\pi\rho M_z(\lambda)\cdot z$$

Изменение поля в плоскости $y=0$ при переходе через поверхность эллипсоида (1)

Эллипсоид a, b, c. В плоскости $y=0$ его поверхность есть

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$$

Рассматриваем случай $c \gg a$ и $z \simeq c = c-\delta$



при $z=c-\delta$

$$x^* = a\sqrt{1-\left(\frac{c-\delta}{c}\right)^2} = a\left[1-\left(1-\frac{\delta}{c}\right)^2\right]^{1/2} = \cancel{\simeq a \left(1+\frac{2\delta}{c}\right)} = a\left(1-1+\frac{2\delta}{c}\right)^{1/2} =$$

$$= a\sqrt{\frac{2\delta}{c}}$$

Тогда

$$E_z\Big|_{x=0,\, z=c-\delta} = \frac{c-\delta}{(c-\delta)^3} = \frac{1}{(c-\delta)^2} = \frac{1}{c^2}\left(1-\frac{\delta}{c}\right)^{-2} = \frac{1}{c^2}\left(1+\frac{2\delta}{c}\right)$$

$$E_x\Big|_{x=0,\, z=c-\delta} = 0$$

$$E^2 = E_x^2 + E_z^2 = \frac{1}{c^4}\left(1+\frac{4\delta}{c}\right)$$

$$E_x = \Big|_{x=x^*,\, z=c-\delta} = \frac{x_*}{\left[(c-\delta)^2+x_*^2\right]^{3/2}}$$

$$E_z\Big|_{x=x^*,\, z=c-\delta} = \frac{c-\delta}{\left[(c-\delta)^2+x_*^2\right]^{3/2}}$$

Имеем

$$\frac{1}{[(c-\delta)^2+x_*^2]^{3/2}}=\frac{1}{c^3}\left[\left(1-\frac{\delta}{c}\right)^2+\left(\frac{x_*}{c}\right)^2\right]^{-3/2}\simeq$$

$$\simeq\frac{1}{c^3}\left[1-\frac{2\delta}{c}+\left(\frac{a}{c}\sqrt{\frac{2\delta}{c}}\right)^2\right]^{-3/2}=\frac{1}{c^3}\left[1-\frac{2\delta}{c}+\frac{2\delta}{c}\frac{a^2}{c^2}\right]^{-3/2}=$$

$$=\frac{1}{c^3}\left[1-\frac{2\delta}{c}\left(1-\frac{a^2}{c^2}\right)\right]^{-3/2}\simeq\frac{1}{c^3}\left[1+\frac{3\delta}{c}\left(1-\frac{a^2}{c^2}\right)\right]$$

Тогда

$$E_x\Big|_{x=x^*,\ z=c-\delta}=a\sqrt{\frac{2\delta}{c}}\,\frac{1}{c^3}\left[1+\frac{3\delta}{c}\left(1-\frac{a^2}{c^2}\right)\right]\simeq$$

$$\simeq\frac{1}{c^2}\frac{a}{c}\sqrt{\frac{2\delta}{c}}>0$$

$$\cancel{E_z\Big|_{x=x_*,\ z=c-\delta}=\frac{c-\delta}{[\ ]^{3/2}}\simeq c(1-\delta/c)\left[1+\frac{3\delta}{c}\left(1-\frac{a^2}{c^2}\right)\right]\simeq c\left[1-\frac{\delta}{c}+\frac{3\delta}{c}-\frac{3\delta}{c}\frac{a^2}{c^2}\right]=}$$

$$E_z\Big|_{x=x_*,\ z=c-\delta}=\frac{c-\delta}{[\ ]^{3/2}}\simeq c\left(1-\frac{\delta}{c}\right)\frac{1}{c^3}\left[1+\frac{3\delta}{c}\left(1-\frac{a^2}{c^2}\right)\right]$$

$$\simeq\frac{1}{c^2}\left[1-\frac{\delta}{c}+\frac{3\delta}{c}\left(1-\frac{a^2}{c^2}\right)\right]=\frac{1}{c^2}\left[1+\frac{2\delta}{c}\left(1-\frac{3a^2}{2c^2}\right)\right]$$

$$<E_z\Big|_{x=0,\ z=c-\delta}$$

$$E^2=E_x^2+E_y^2=\frac{1}{c^4}\left[\frac{a^2}{c^2}\frac{2\delta}{c}+1+\frac{4\delta}{c}\left(1-\frac{3}{2}\frac{a^2}{c^2}\right)\right]=$$

$$= \frac{1}{c^4}\left[1 + \frac{4\delta}{c}\left(1 - \frac{3}{2}\frac{a^2}{c^2} + \frac{1}{2}\frac{a^2}{c^2}\right)\right] =$$

$$= \frac{1}{c^4}\left[1 + \frac{4\delta}{c}\left(1 - \frac{a^2}{c^2}\right)\right] < E^2\Big/_{x=0,\, z=c-\delta}$$