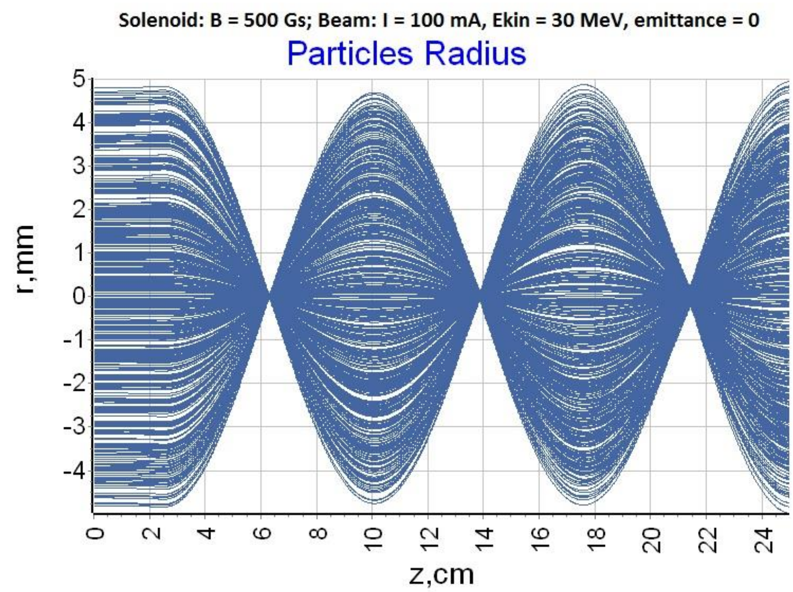
Brillouin Flow in Hellweg2d

1. Beam with zero emittance.

Hellweg2D result is:



Brillouin wave length is defined from equation (7-19) [1] for periodic fluctuation of the beam radius (nonrelativistic case!):

$$\lambda_{s,[\mathrm{m}]} = \frac{0.030 \cdot 10^{-3}}{B_{[\mathrm{T}]}} \sqrt{\frac{V_{[\mathrm{V}]}}{2 - 1/m^2}}, \qquad (1)$$

where m is the confinement factor. It can be shown (see Appendix), that usually $m >> 1$. So, for the electron beam with kinetic energy $E_{kin} = 30 \text{ keV}$ and solenoid with field $B = 500 \text{ Gs} = 0.05 \text{ T}$ one has $\lambda_s = 7.35 \text{ cm}$ in a good agreement with value $\lambda_s \approx 7.2 \text{ cm}$ from picture.

2. Beam with nonzero emittance.

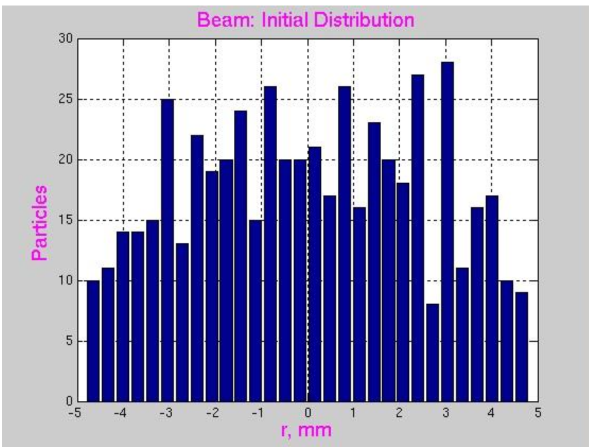
Firstly let rewrite formula (1), using expression for Larmor radius $\rho_L = \dfrac{pc}{eB}$:

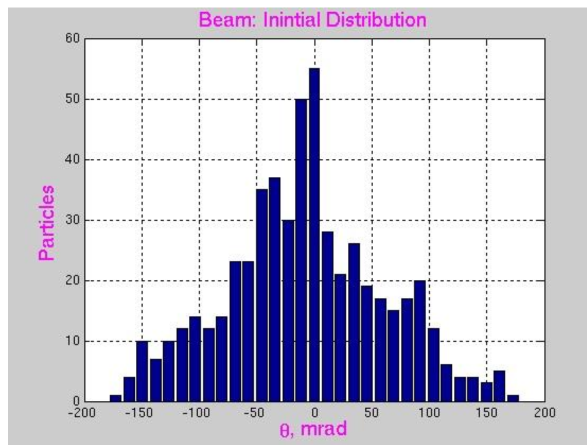
$$\lambda_s = 2\pi \frac{\rho_L}{\sqrt{2 - 1/m^2}}. \qquad (2)$$

It can be shown [2] that for the beam with emittance $\varepsilon = \sqrt{< r^2 > \cdot < r'^2 >}$ expression (2) moves to

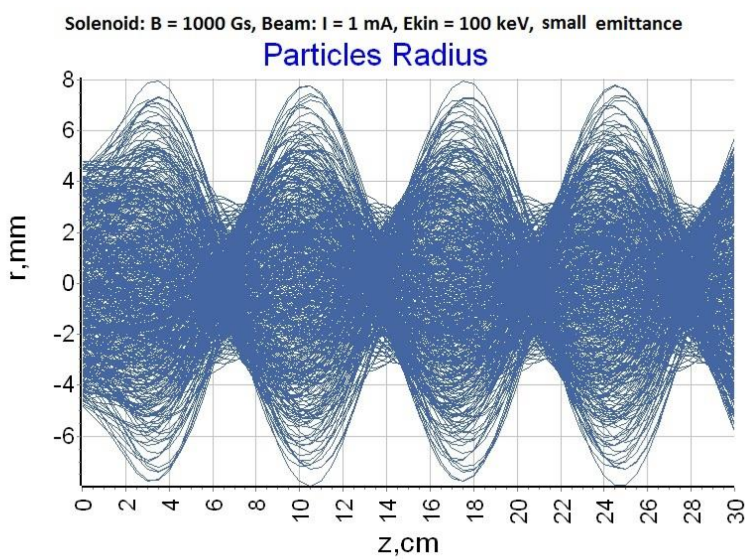
$$\lambda_s = \frac{2\pi}{\sqrt{2\left(\frac{1}{\rho_L^2} + \varepsilon^2\right)\left(1 - \frac{1}{2m^2}\right)}}. \tag{3}$$

It is possible again neglect the contribution of the confinement factor m. The following distributions are used in Hellweg2D simulation:





For these distributions $\sqrt{< r^2 >} = 0.2516\ \mathrm{cm}$ and $\sqrt{< r'^2 >} = 68.190\ \mathrm{mrad}$, so that $\varepsilon = 17.18\ \mathrm{cm} \cdot \mathrm{mrad}$. This emittance value is practically no effect on the length λ_s Hellweg2D simulation for this case confirms this conclusion:



For $E_{kin} = 100$ keV and $B = 1000$ Gs $= 0.1$ T one has $\lambda_s = 7.0$ cm in a excelent agreement with value $\lambda_s \approx 7$ cm from picture.

References

1. A.S. Gilmor. *Klystrons, traveling wave tubes, magnetrons, crossed-field amplifiers, and gyrotrons*. Artech House, 2011.
2. I.N. Meshkov. *Transportation of charged particle beams.* (IN Russian). Nauka, 1991.

Appendix: confinement Factor

Confinement factor m is defined in (7-12) as ratio of the equilibrium radius b_e and parameter a (I – current of the beam, e, m, u_0 – charge, mass, and longitudinal velocity of the electron and c – the speed of light) :

$$m = \frac{b_e}{a}, \quad \text{where (see formula (7-5))} \quad a = \frac{2}{B}\sqrt{\frac{2I}{q_e u_0 / mc^2}}. \tag{A1}$$

Equilibrium radius b_e can be found from equation (7-7):

$$\frac{b_e}{a}\left[1 - \left(\frac{B_c}{B}\frac{b_c^2}{b_e^2}\right)^2\right] - \frac{a}{b_e} = 0. \tag{A2}$$

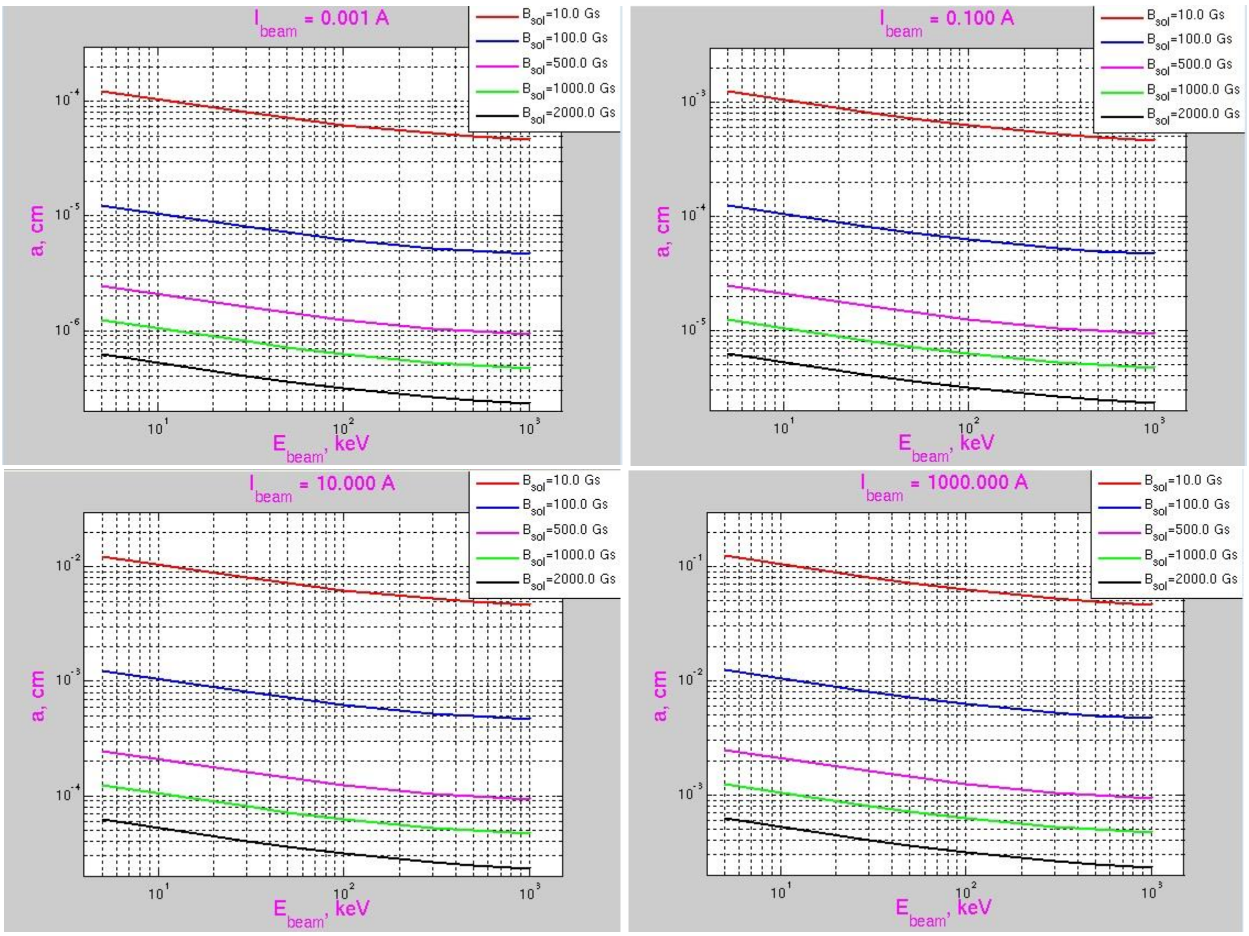
In this equation B_c, b_c are magnetic field and radius of the beam on the cathode. In our case ("cathode" is simply an entrance to the system) $B_c = B$ and $b_c = r_0$ – the initial radius of the beam. Then instead (A2) one has the following equation for m:

$$0 = \left(\frac{b_e}{a}\right)^2\left[1 - \left(\frac{r_0^2}{a^2}\frac{a^2}{b_e^2}\right)^2\right] - 1 = m^4 - m^2 - (r_0 / a)^4. \tag{A3}$$

Solution of this equation is

$$m = \sqrt{\frac{1}{2}\left(1 + \sqrt{1 + 4\frac{r_0^4}{a^4}}\right)}. \tag{A4}$$

Next figures show the dependence value of a from different parameters of the beam (energy, current and magnetic field).



Seen that usually $a >> r_0$, so that

$$m \approx \frac{r_0}{a} >> 1. \tag{A5}$$

Returning to the general case instead (A4) one has

$$m = \sqrt{\frac{1}{2}\left(1 + \sqrt{1 + 4\frac{B_c^2}{B^2}\frac{r_c^4}{a^4}}\right)} \tag{A6}$$

and for $a >> r_c$ instead (A5) one has

$$m \approx \frac{r_c}{a}\sqrt{\frac{B_c}{B}} >> 1, \tag{A7}$$

so that

$$b_e = ma = r_c\sqrt{\frac{B_c}{B}}. \tag{A8}$$