(1)

Строгая теория потенциала эллипсоида с полуосями $a_1, a_2, a_3$, однородно заряженного с $\rho$.

1°. Для точек внутри эллипсоида

$$\varphi(x,y,z) = \pi\rho\left(I - A_1 x^2 - A_2 y^2 - A_3 z^2\right) \qquad (6.14)$$

где

$$A_i = a_1 a_2 a_3 \int_0^\infty \frac{du}{(a_i^2+u)\sqrt{(a_1^2+u)(a_2^2+u)(a_3^2+u)}} \qquad (6.17)$$

$$I = a_1 a_2 a_3 \int_0^\infty \frac{du}{\sqrt{(a_1^2+u)(a_2^2+u)(a_3^2+u)}} \qquad (6.16)$$

2°. Коэффициенты $I$ и $A_i$ выражаются через неполные эллиптические интегралы 1-го и 2-го рода:

$$I = \cancel{a_1 a_2 a_3} \frac{2a_1 a_2 a_3}{\sqrt{a_1^2 - a_3^2}} F(\varphi, k) \qquad (6.16)$$

$$A_1 = \frac{2a_1 a_2 a_3}{(a_1^2 - a_2^2)\sqrt{a_1^2 - a_3^2}}\left[F(\varphi,k) - E(\varphi,k)\right] \qquad (6.18)$$

$$(6.19)\quad A_2 = \frac{2a_1 a_2 a_3 \sqrt{a_1^2 - a_3^2}}{(a_1^2 - a_2^2)(a_2^2 - a_3^2)}\left[E(\varphi,k) - \frac{a_2^2 - a_3^2}{a_1^2 - a_3^2} F(\varphi,k) - \frac{a_3}{a_1 a_2}\frac{a_1^2 - a_2^2}{\sqrt{a_1^2 - a_3^2}}\right]$$

Здесь ошибка! Какая?

$$(6.20)\quad A_3 = \frac{2a_1 a_2 a_3 \sqrt{a_1^2 - a_3^2}}{(a_2^2 - a_3^2)\sqrt{a_1^2 - a_3^2}}\left[\frac{a_2\sqrt{a_1^2 - a_3^2}}{a_1 a_3} - E(\varphi,k)\right]$$

$$(6.22)\quad \varphi = \arcsin\frac{\sqrt{a_1^2 - a_3^2}}{a_1}, \quad k = \sqrt{\frac{a_1^2 - a_2^2}{a_1^2 - a_3^2}}$$

Проверил! (см. далее) $\sqrt{\ }$ в знаменателе лишний — это ошибка!

$$\left.\begin{aligned} F(\varphi,k) &= \int_0^{\varphi} \frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}} \\ E(\varphi,k) &= \int_0^{\varphi} \sqrt{1-k^2\sin^2\varphi}\, d\varphi \end{aligned}\right\} \quad (6,23)$$

Проверки

$$A_1 = \frac{2a_1a_2a_3 \prod \alpha_i(m)}{q^2\left(a_1^2 - a_3^2\, a_3^2(m)\right)^{3/2}}\left[F(\nu,q) - E(\nu,q)\right] \qquad (6.180)$$

$$q = \sqrt{\frac{a_1^2 - a_2^2\alpha_2^2(m)}{a_1^2 - a_3^2\alpha_3^2(m)}} \qquad \nu = \arcsin\sqrt{\frac{a_1^2 - a_3^2\alpha_3^2(m)}{\mu(m^2) + a_1^2}} \qquad (6.183)$$

$$\downarrow$$

$$k = \sqrt{\frac{a_1^2 - a_2^2}{a_1^2 - a_3^2}} \longrightarrow \alpha_2^2(m) = \alpha_3^2(m) = 1$$

$$\varphi = \arcsin\frac{\sqrt{a_1^2 - a_3^2}}{a_1} \longrightarrow \alpha_3^2(m) = 1 \quad \mu(m^2) = 0$$

$$\frac{2a_1a_2a_3\, \overbrace{\alpha_1(m)}^{=1?}}{\frac{a_1^2 - a_2^2}{a_1^2 - a_3^2}\left(a_1^2 - a_3^2\right)^{3/2}}\left[F(\varphi,k) - E(\varphi,k)\right] =$$

$$= \frac{2a_1a_2a_3}{(a_1^2 - a_2^2)\sqrt{a_1^2 - a_3^2}}\left[\quad\right]$$ совпадает с (6.18)

(4)

$$A_2 = \frac{2a_1a_2a_3}{\frac{a_1^2-a_2^2}{a_1^2-a_3^2}\underbrace{\left(1-\frac{a_1^2-a_2^2}{a_1^2-a_3^2}\right)}_{\frac{a_2^2-a_3^2}{a_1^2-a_3^2}}(a_1^2-a_3^2)^{3/2}}\left[E(\varphi,k)-\left(1-\frac{a_1^2-a_2^2}{a_1^2-a_3^2}\right)F(\varphi,k)-\right.$$

$$\left.-\frac{a_1^2-a_2^2}{a_1^2-a_3^2}\frac{\sin\nu\cos\nu}{\sqrt{\underbrace{1-\frac{a_1^2-a_3^2}{a_1^2-a_3^2}\sin^2\nu}_{1-q^2\sin^2\nu}}}\right]$$

$$\sin\nu = \sqrt{\frac{a_1^2-a_3^2}{a_1^2}} = \frac{\sqrt{a_1^2-a_3^2}}{a_1}$$

$$\cos^2\nu = 1-\frac{a_1^2-a_3^2}{a_1^2} = \frac{a_3^2}{a_1^2};$$

проверка

$$\cos^2\nu = \frac{\underbrace{\mu(m^2)}_{=0}+a_3^2\underbrace{\alpha_3^2(m)}_{=1}}{\underbrace{\mu(m^2)}_{=0}+a_1^2} = \frac{a_3^2}{a_1^2} \quad \text{ок}$$

$$1-q^2\sin^2\nu = \frac{\overbrace{\mu(m^2)}^{0}+a_2^2\overbrace{\alpha_2^2(m)}^{=1}}{\underbrace{\mu(m^2)}_{=0}+a_1^2} = \frac{a_2^2}{a_1^2}$$

$$A_2 = \frac{2a_1a_2a_3}{\frac{(a_1^2-a_2^2)(a_2^2-a_3^2)}{(a_1^2-a_3^2)^2}(a_1^2-a_3^2)^{3/2}}\left[E(\varphi,k)-\frac{a_2^2-a_3^2}{a_1^2-a_3^2}F(\varphi,k)-\right.$$

$$\left.-\frac{a_1^2-a_2^2}{a_1^2-a_3^2}\frac{\sqrt{a_1^2-a_3^2}}{a_1}\frac{a_3}{\cancel{a_1}}\frac{\cancel{a_1}}{a_2}\right]=$$

$$=\frac{2a_1a_2a_3\sqrt{a_1^2-a_3^2}}{(a_1^2-a_2^2)(a_2^2-a_3^2)}\left[E-\frac{a_2^2-a_3^2}{a_1^2-a_3^2}F-\frac{a_3(a_1^2-a_2^2)}{a_1a_2\sqrt{a_1^2-a_3^2}}\right]$$

ОК совпадает с 6.19

$$A_3 = \frac{2a_1 a_2 a_3 \prod \alpha_i}{q'^2 \left(a_1^2 - a_3^2 \alpha_3^2(m)\right)^{3/2}} \left[ \operatorname{tg} \nu \sqrt{1 - q^2 \sin^2 \nu} - E(\nu, q) \right] \quad (6.182)$$

$$q'^2 = 1 - q^2 = \frac{a_2^2 - a_3^2}{a_1^2 - a_3^2} \qquad \alpha_1 = \alpha_2 = \alpha_3 = 1$$

$$\sqrt{1 - q^2 \sin^2 \nu} = \frac{a_2}{a_1} \qquad \nu \to \varphi, \quad q \to K$$

$$\sin \nu = \frac{\sqrt{a_1^2 - a_3^2}}{a_1} \qquad \cos \nu = \frac{a_3}{a_1}$$

$$\operatorname{tg} \nu = \frac{\sin \nu}{\cos \nu} = \frac{\sqrt{a_1^2 - a_3^2}}{a_3}$$

$$A_3 = \frac{2a_1 a_2 a_3}{\frac{a_2^2 - a_3^2}{a_1^2 - a_3^2} \left(a_1^2 - a_3^2\right)^{3/2}} \left[ \frac{\sqrt{a_1^2 - a_3^2}}{a_3} \frac{a_2}{a_1} - E(\varphi, K) \right] =$$

$$= \frac{2a_1 a_2 a_3 \sqrt{a_1^2 - a_3^2}}{\left(a_2^2 - a_3^2\right)} \left[ \frac{a_2}{a_1 a_3} \sqrt{a_1^2 - a_3^2} - E(\varphi, K) \right] \quad \text{это } (6.20)!$$

Т.о. в (6.20) в знаменателе $\sqrt{a_1^2 - a_3^2}$ — ошибка!!!