

# Problem 1

(1)

Given:

$$V(x, y, z) = \pi \rho abc \int_0^{\infty} \left[ 1 - \frac{x^2}{a^2+s} - \frac{y^2}{b^2+s} - \frac{z^2}{c^2+s} \right] \frac{ds}{\sqrt{(a^2+s)(b^2+s)(c^2+s)}}$$

Compute:

$$V(x, y, z) = \pi \rho abc \int_0^{\infty} \left[ 1 - \frac{x^2}{a^2+s} - \frac{y^2}{b^2+s} - \frac{z^2}{c^2+s} \right] \frac{ds}{\sqrt{(a^2+s)(b^2+s)(c^2+s)}}$$

More Given:

$$[\lambda] = M^2$$

$$E_x = -\frac{\partial V}{\partial x} = 2\pi \rho abc x \int_0^{\infty} \frac{ds}{(a^2+s)\sqrt{(a^2+s)(b^2+s)(c^2+s)}} = 2\pi \rho M_x x$$

$$E_y = -\frac{\partial V}{\partial y} = 2\pi \rho abc y \int_0^{\infty} \frac{ds}{(b^2+s)\sqrt{(a^2+s)(b^2+s)(c^2+s)}} = 2\pi \rho M_y y$$

$$E_z = -\frac{\partial V}{\partial z} = 2\pi \rho abc z \int_0^{\infty} \frac{ds}{(c^2+s)\sqrt{(a^2+s)(b^2+s)(c^2+s)}} = 2\pi \rho M_z z$$

More compute:

$$E_x = 2\pi \rho abc \cdot x \int_0^{\infty} \frac{1}{(a^2+s)\sqrt{(a^2+s)(b^2+s)(c^2+s)}} ds = 2\pi \rho M_x(\lambda) \cdot x$$

$$E_y = \dots = 2\pi \rho M_y(\lambda) \cdot y$$

$$E_z = \dots = 2\pi \rho M_z(\lambda) \cdot z$$

$$\lambda = \text{dimensionless parameter}$$

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} + \frac{z^2}{c^2+\lambda} = 1$$

$$M_x(\lambda) = abc \int_1^{\infty} \frac{ds}{(a^2+s)\sqrt{a^2+s}(b^2+s)(c^2+s)} = \frac{abc}{a^5} \int_1^{\infty} \frac{ds}{(1+\frac{s}{a^2})\sqrt{(1+\frac{s}{a^2})(\frac{b^2}{a^2}+\frac{s}{a^2})/(\frac{c^2}{a^2}+\frac{s}{a^2})}} = \quad (2)$$

$$= \frac{b}{a} \frac{c}{a} \int_{\lambda/a^2}^{\infty} \frac{ds}{(1+s)\sqrt{(1+s)(\frac{b^2}{a^2}+s)(\frac{c^2}{a^2}+s)}} \quad \frac{b}{a} = \frac{R_y}{R_x} \quad \frac{c}{a} = \frac{R_z}{R_x}$$

$$M_y(\lambda) = abc \int_1^{\infty} \frac{ds}{(b^2+s)\sqrt{a^2+s}(b^2+s)(c^2+s)} = \frac{abc}{a^5} \int_1^{\infty} \frac{ds}{(\frac{b^2}{a^2}+\frac{s}{a^2})\sqrt{(1+\frac{s}{a^2})(\frac{b^2}{a^2}+\frac{s}{a^2})/(\frac{c^2}{a^2}+\frac{s}{a^2})}} =$$

$$= \frac{b}{a} \frac{c}{a} \int_{\lambda/a^2}^{\infty} \frac{ds}{(b^2/a^2+s)\sqrt{(1+s)(b^2/a^2+s)(c^2/a^2+s)}}$$

$$M_z(\lambda) = \frac{b}{a} \frac{c}{a} \int_{\lambda/a^2}^{\infty} \frac{ds}{(c^2/a^2+s)\sqrt{(1+s)(b^2/a^2+s)(c^2/a^2+s)}}$$

$\frac{1}{a^2}$  y qobuvost bo'lsa

$$1 = \frac{x^2/a^2}{1+\lambda/a^2} + \frac{y^2/a^2}{b^2/a^2+\lambda/a^2} + \frac{z^2/a^2}{c^2/a^2+\lambda/a^2} = \frac{x^2/a^2}{1+\lambda/a^2} + \frac{y^2/a^2 \cdot b^2/a^2}{b^2/a^2+\lambda/a^2} + \frac{z^2/a^2 \cdot c^2/a^2}{c^2/a^2+\lambda/a^2}$$

qaytadan