October 5th 2023

Physics Concerto Seminar Series

Supported by the Weinberg Institute for Theoretical Physics, Department of Physics, University of Texas

Beyond the Big Bang: Delving into Inflationary Cosmology

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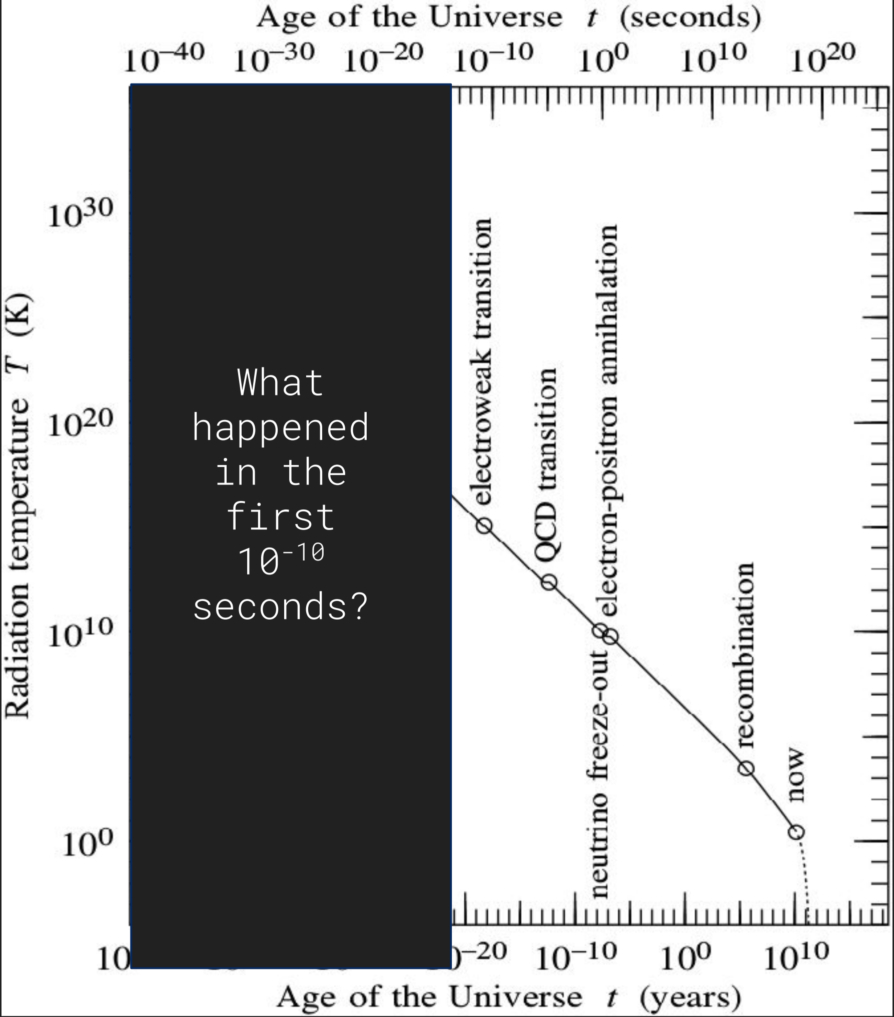
PHYSICS CONCERTO

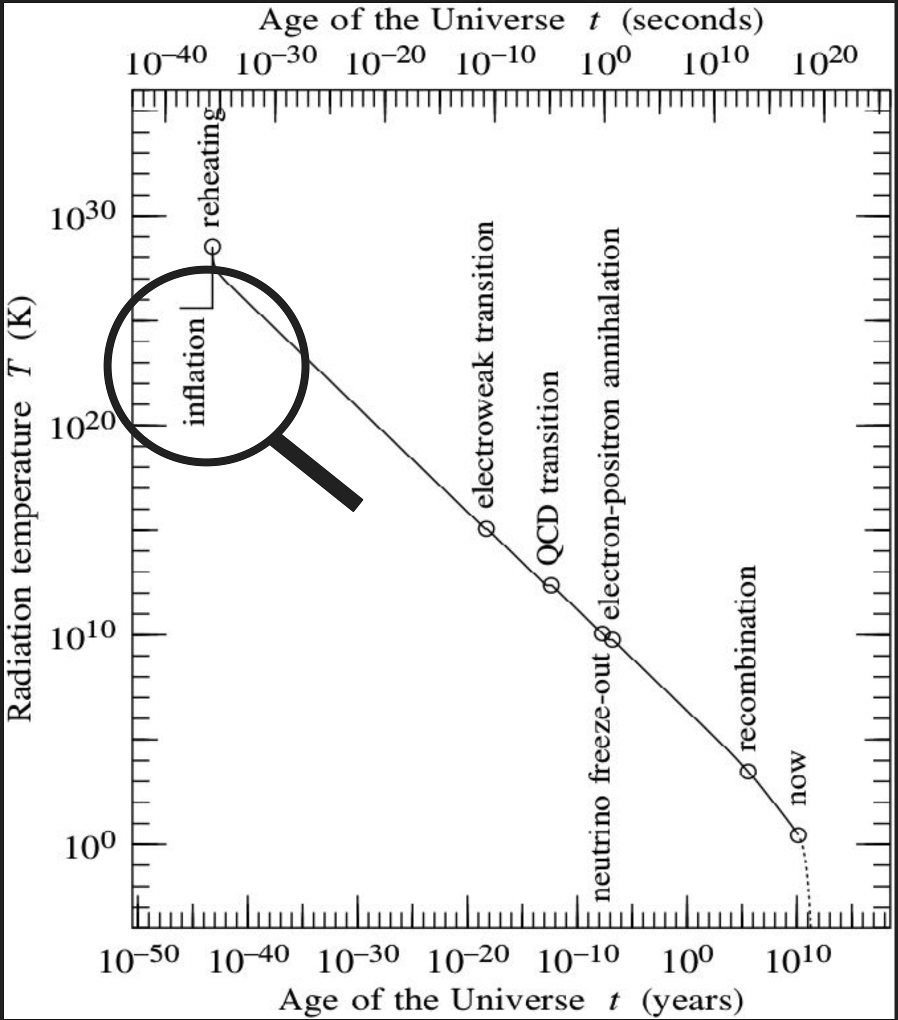
BRIDGING PHYSICS SPECIALTIES THROUGH PEER TO PEER SEMINARS

History of the universe



A.J.S. Hamilton, Modern Cosmology
https://jila.colorado.edu/~ajsh/courses/astr2010_22/index.html

History of the universe



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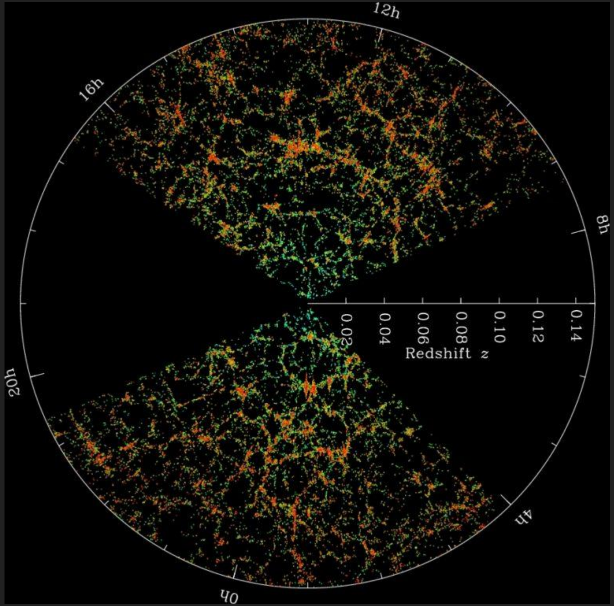
Outline

- properties of our universe and big bang cosmology
- the shortcomings of the big bang model
- cosmic inflation
- challenges and shortcomings of the inflationary picture

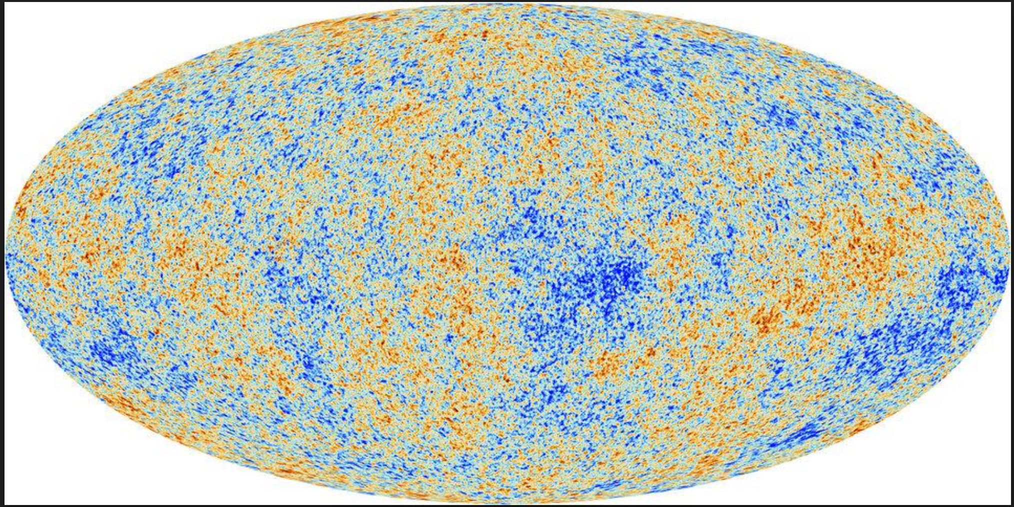
The Cosmological Principle

The universe is homogeneous and isotropic at the largest cosmological scales,
i.e. we are NOT special!

DESI Collaboration
https://www.darkenergysurvey.org/the-des-project/science/



Planck Collaboration
https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB



General Relativity in Cosmology

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- describes the geometry and curvature of spacetime
- it is a function of the metric $g_{\mu\nu}$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

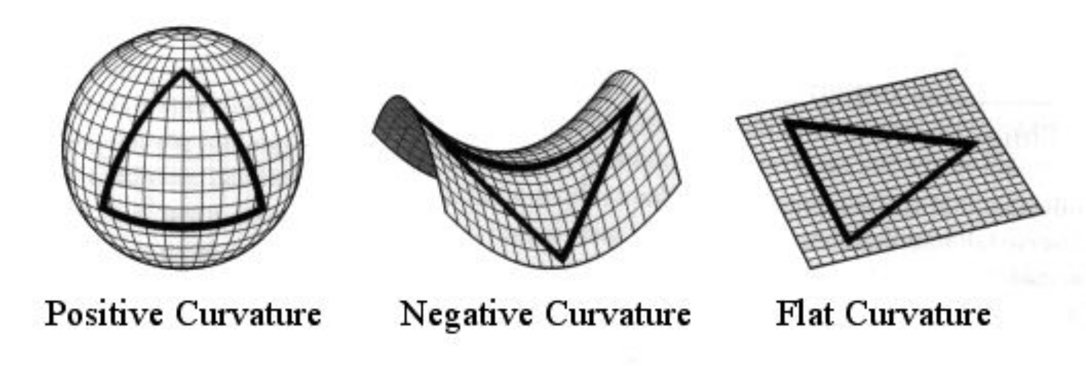
encodes how much “stuff” (matter, energy, momentum, pressure, etc.) is present at each spacetime point

General Relativity in Cosmology

- Cosmological principle → FRW metric:

$$ds^2 = -dt^2 + \underbrace{a(t)^2}_{\text{the scale factor}} \cdot \left(\frac{1}{1 - kr^2} dr^2 + r^2 d\Omega^2 \right)$$

- Only three possibilities: k={1,0,-1}



J. Schombert, Cosmology
https://pages.uoregon.edu/jschombe/cosmo/lectures/lec15.html

General Relativity in Cosmology

- Cosmological principle + perfect fluid → Diagonal $\mathbf{T}_{\mu\nu}$

$$T^{00} = -\rho, \qquad T^{jj} = p.$$

- Friedmann Equations:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho(1+3\omega)$$

$$\underbrace{\left(\frac{\dot{a}}{a}\right)}^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\frac{\dot{a}}{a} \equiv H$$

(The Hubble parameter)

- Fluid Equation:

$$\frac{\dot{\rho}}{\rho} = -3H(1+\underbrace{w}_{\omega \equiv p/\rho})$$

(the equation of state parameter)

The universe accelerates if $\omega < -1/3$!

General Relativity in Cosmology

- Matter: $p_m \ll \rho_m \rightarrow \omega_m \simeq 0: \quad a(t) \sim t^{2/3}$
- Radiation: $p_r = \frac{1}{3}\rho_r \rightarrow \omega_r = \frac{1}{3}: \quad a(t) \sim t^{1/2}$
- Vacuum: $p_v = -\rho_v \rightarrow \omega_v = -1: \quad a(t) \sim e^{Ht}$

Note, a convenient notation: $\Omega_j = \rho_j/\rho_c$ (the jth energy density parameter)

$$\rho_c \equiv \frac{3H^2}{8\pi G}$$ (the critical energy density)

General Relativity in Cosmology

- Friedmann Equations:

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\rho(1-\epsilon)$$

$$\epsilon \equiv \frac{3}{2}(1+\omega)$$ (The Hubble slow-roll parameter)

The universe accelerates if ε<1!

General Relativity in Cosmology

- The Friedmann equation can be rewritten as:

$$1 - \underbrace{\sum_j \Omega_j}_{\Omega \equiv \sum_j \Omega_j} = \underbrace{\Omega_k}_{\Omega_k \equiv -\frac{k}{a^2 H^2}}$$

Note, a convenient notation: $\Omega_j = \rho_j / \rho_c$ (the jth energy density parameter)

$$\rho_c \equiv \frac{3H^2}{8\pi G}$$ (the critical energy density)

General Relativity in Cosmology

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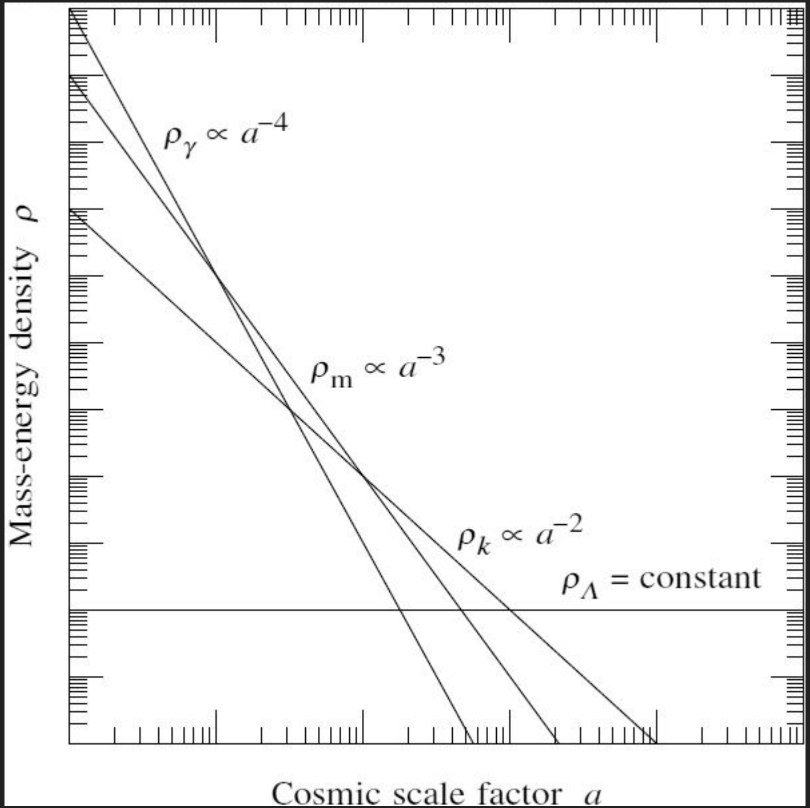
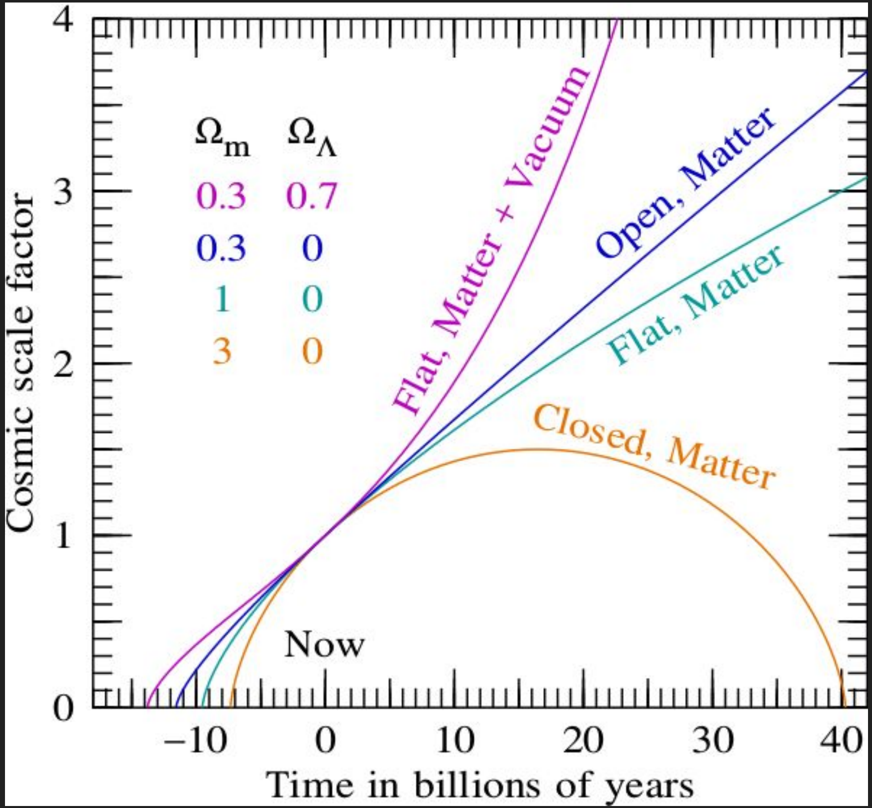




The shortcomings of Big Bang Cosmology
The flatness problem

- We live in a FLAT universe
  - Present observations suggest that $|\Omega_0 - 1| \lesssim 10^{-3}$

- Necessity of an extreme *fine tuning* of the initial value of Ω.

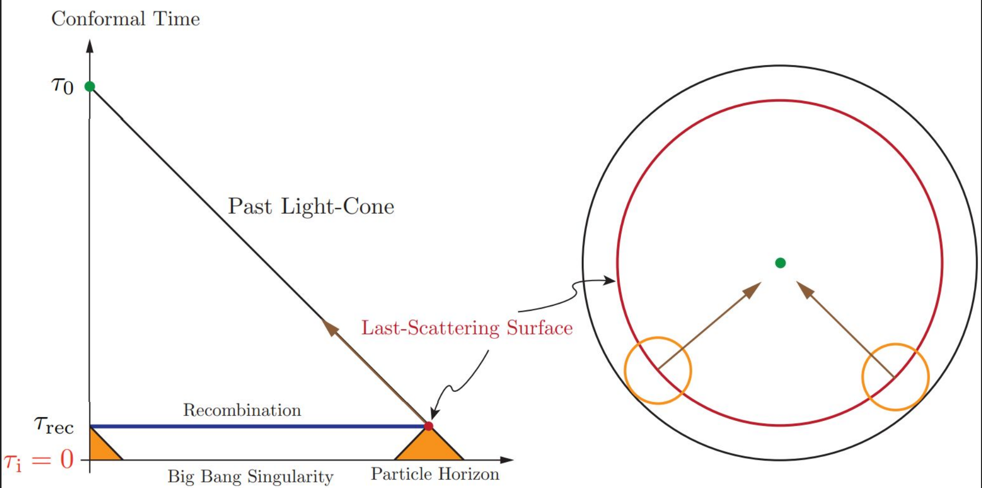
$$|\Omega - 1| \propto t \quad \text{(radiation domination)}$$

$$|\Omega - 1| \propto t^{\frac{2}{3}} \quad \text{(matter domination)}$$

  - this implies $|\Omega - 1| \lesssim 10^{-16}$ at nucleosynthesis epoch, and $|\Omega - 1| \lesssim 10^{-64}$ at Planck epoch.

The shortcomings of Big Bang Cosmology
The Horizon problem

- The universe at the time of decoupling was in *thermal equilibrium*, yet there had not been enough time for distant regions to be in casual contact.
  - CMB consist of ~10^5 causally disconnected regions.



D. Baumann, TASI Lectures on Inflation, arXiv0907.5424

The shortcomings of Big Bang Cosmology
The Monopole problem

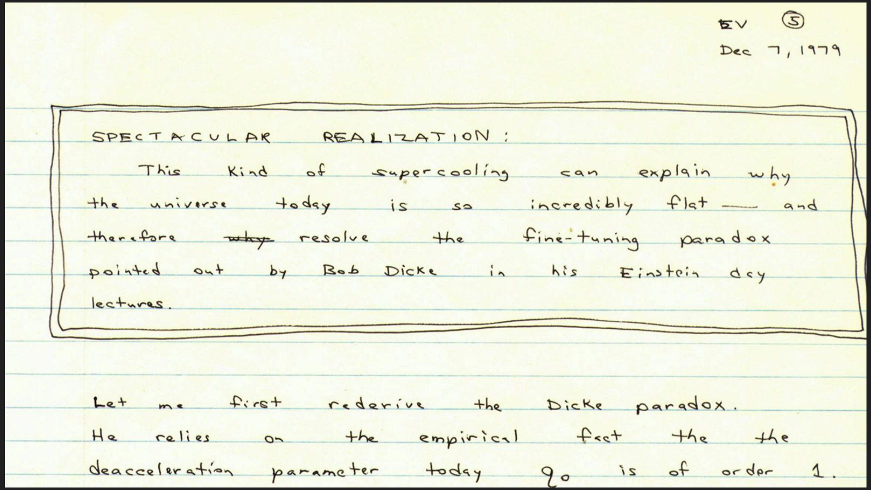
- All Grand Unified Theories predict the existence of magnetic monopoles, extremely heavy particles with net magnetic charge.

- If these particles exist in the early universe, they could be the dominant materials in the universe, yet we do not observe them.

Cosmic Inflation

- A period of accelerated expansion in the early universe
- Explains the observed flatness, homogeneity, and the lack of relic monopoles
- Provides with a mechanism for generating the inhomogeneities observed in the Cosmic Microwave Background

https://breakthroughprize.org/Laureates/1/L2



EV 5
Dec 7, 1979

SPECTACULAR REALIZATION:
This kind of supercooling can explain why the universe today is so incredibly flat — and therefore ~~why~~ resolve the fine-tuning paradox pointed out by Bob Dicke in his Einstein day lectures.

Let me first rederive the Dicke paradox. He relies on the empirical fact the the deacceleration parameter today q_0 is of order 1.

https://www.symmetrymagazine.org/article/december-2004january-2005/inflation?language_content_entity=und

Cosmic Inflation

- Single scalar field minimally coupled to gravity

$$S_\phi = \int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

- Slowly-rolling homogeneous field that dominates the energy density of the universe induces an *exponential* expansion

$$V(\phi) \gg \dot{\phi}^2$$

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} \left\{ \begin{array}{l} \rho_\phi = -T_{00} = \cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi) + \cancel{\frac{\nabla^2\phi}{2}} \\ p_\phi = \frac{1}{3}T^j_j = \cancel{\frac{1}{2}\dot{\phi}^2} - V(\phi) - \cancel{\frac{\nabla^2\phi}{6}} \end{array} \right. \longrightarrow \quad \epsilon_\phi \equiv \frac{\cancel{\frac{3}{2}\dot{\phi}^2}}{\cancel{\frac{\dot{\phi}^2}{2}} + V(\phi)}$$

$$\boxed{\begin{array}{rr} \ddot{a} > 0 : & \epsilon_\phi < 1 \\ & \simeq 0 \end{array}}$$

Cosmic Inflation

$$\boxed{H^2 \simeq \frac{8\pi G}{3} V(\phi) \longrightarrow a(t) \sim e^{Ht}}$$

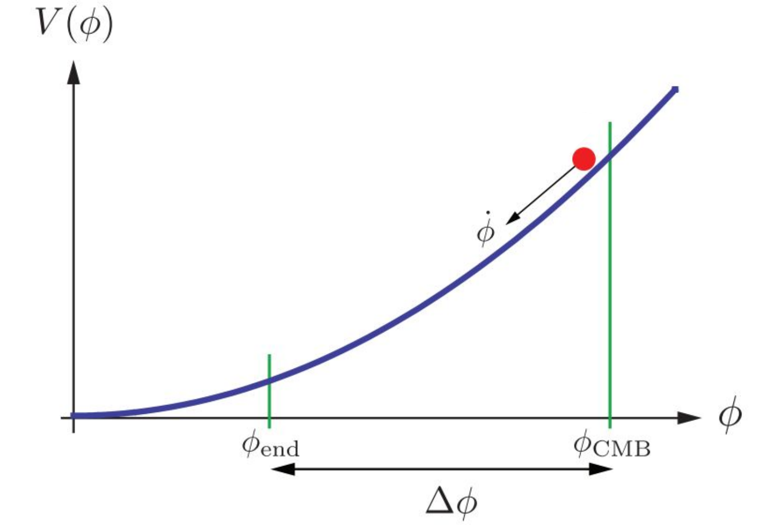
- Accelerated expansion will only be sustained if the second time derivative of the field is small enough

$$|\ddot{\phi}| \ll |3H\dot{\phi}|, |V_{,\phi}| .$$

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} = \varepsilon - \frac{1}{2\varepsilon}\frac{d\varepsilon}{\underbrace{dN}} < 1$$

(The number of e-foldings)

$$N \equiv \ln(a_f/a_i) = \int_{t_i}^{t_f} H\,dt$$



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The successes of Inflation

The flatness problem

J. Schombert, Cosmology
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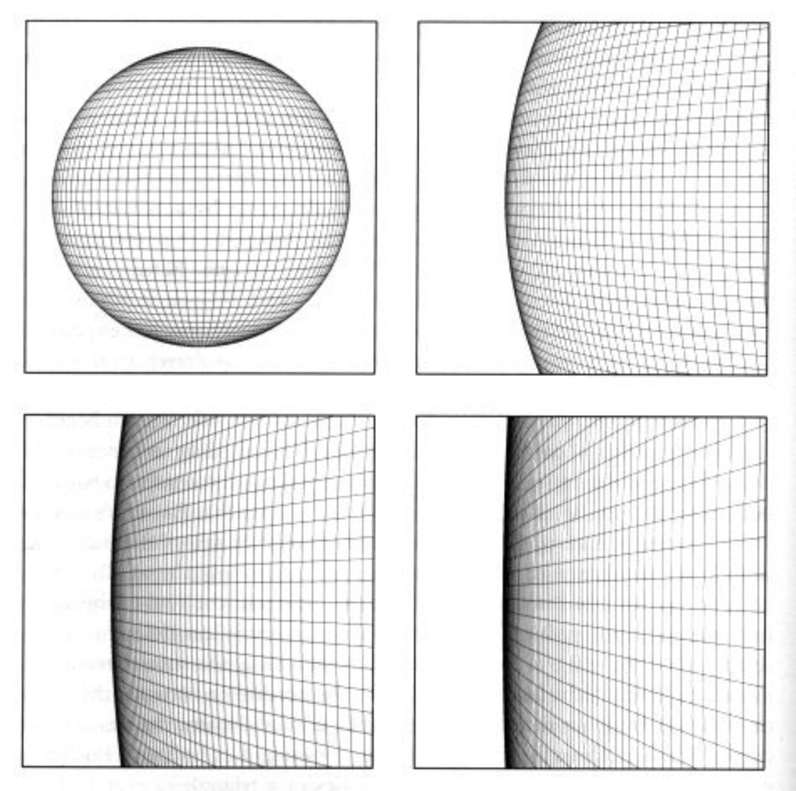
- During inflation: $|\Omega - 1| \propto e^{-2Ht}$

- To solve the flatness problem we need at the end of inflation:

$$|\Omega_f - 1| \lesssim 10^{-60}$$

$$\frac{|\Omega_f - 1|}{|\Omega_i - 1|} \simeq \left(\frac{a_i}{a_f}\right)^2 = e^{-2N}$$

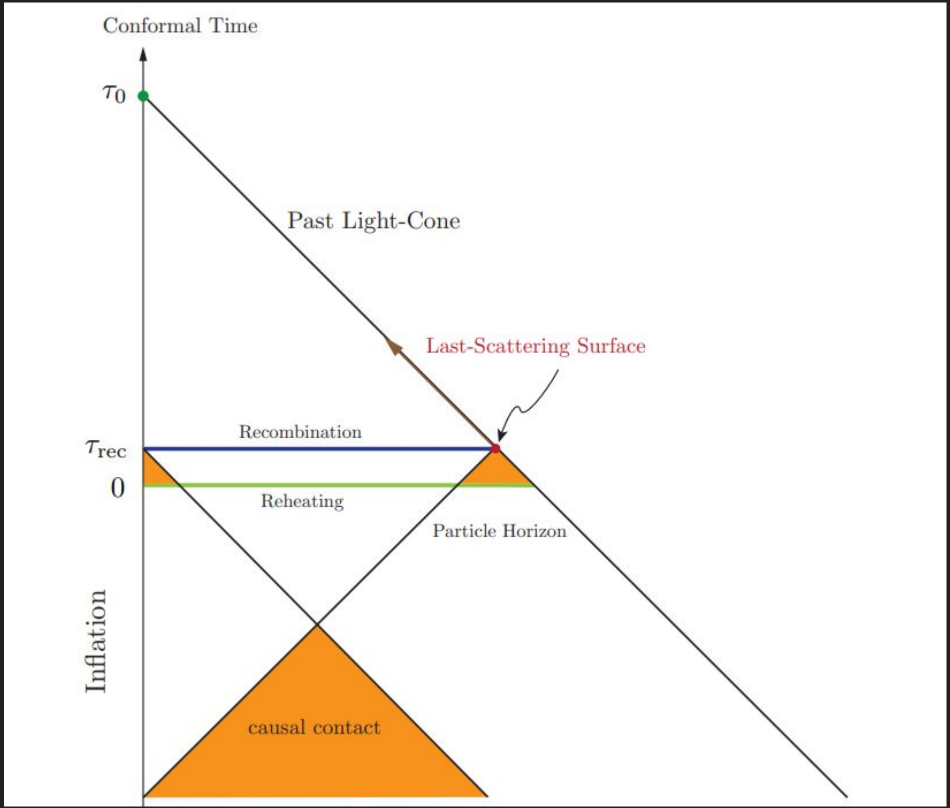
- **Roughly 70 e-folds of inflation solve this issue!**



The successes of Inflation
The Horizon problem

- The *superluminal* accelerated expansion stretches a small causally connected patch, to large cosmological scales works.

- Once again, **roughly 70 e-folds of inflation are sufficient to solve this issue.**



D. Baumann, TASI Lectures on Inflation, arXiv0907.5424

The successes of Inflation

The Monopole problem

- Simply arrange the parameters such that inflation takes place after (or during) monopole production, so the monopole density is *diluted* to a completely negligible level.

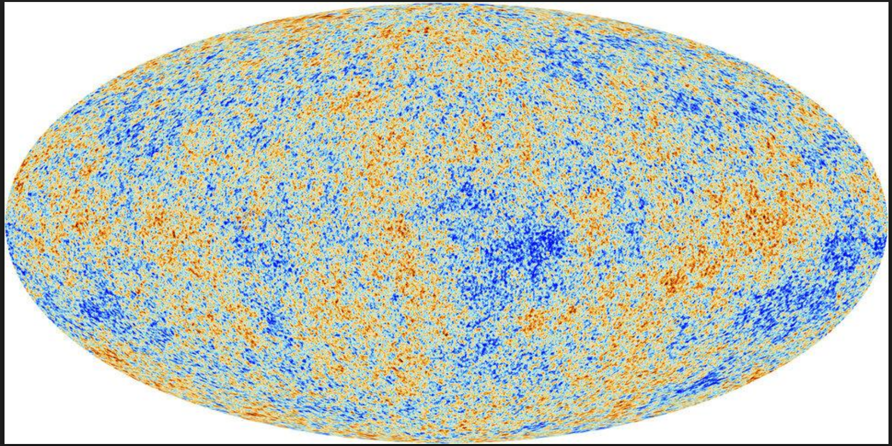
The successes of Inflation
CMB anisotropies

- Provides a mechanism for generating the inhomogeneities observed in the Cosmic Microwave Background

- Quantum fluctuations are driven to cosmological scales via the expansion

$$\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} \propto \langle \delta\phi^2 \rangle^{1/2}$$

Planck Collaboration
https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB



Summary

- Inflation is a cosmological theory proposing a *rapid and exponential* expansion of the universe in its early moments, resolving several long-standing problems in cosmology (homogeneity, flatness, unwanted relics,origin of cosmic structures)

- Inflation is simple: a single scalar field, minimally coupled to gravity, and slowly-rolling down a nearly flat potential, does the job.

Maybe not so simple?

A list of long-standing concerns

- **Multiverse Hypothesis**, i.e. eternal inflation
- **Measure problem:** are we the most likely patch of the universe?
- **Initial conditions problem:** are these generic or need to be fine-tuned?
- **Tuning of the Inflationary model:** for some, a high-degree of fine-tuning is needed to fit observations
- **Quantum gravity concerns:** Inflation's early moments involve extremely high energies, where the effects of quantum gravity may not be negligible
- How do we actually **reheat** the universe?