

July 5th 2024

Quantum Aspects of Inflationary Cosmology
Munich Institute of Astro-, Particle & BioPhysics

WIFI

A novel framework for Dark Matter production

Gabriele Montefalcone

Weinberg Institute for Theoretical Physics, University of Texas at Austin

Based on work with Katherine Freese & Barmak Shams Es Haghi (arXiv:2401.17371)



Outline of the Talk

- Dark Matter:

- What we know about it

- Production mechanisms from a thermal bath (Freeze-out & Freeze-in)

- Warm Inflation:

- What it is and the basics of how it works

- WIFI Framework:

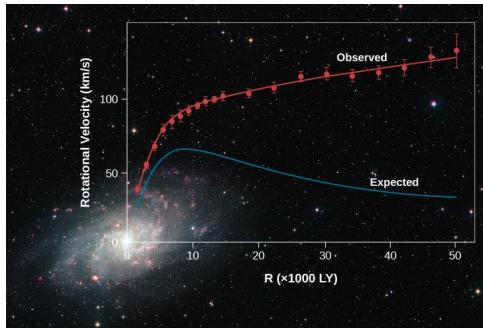
- General features and a representative example

DARK MATTER

Evidence for Dark Matter (DM)

Huge amount of evidence from **all scales** (only from **gravitational** interaction)

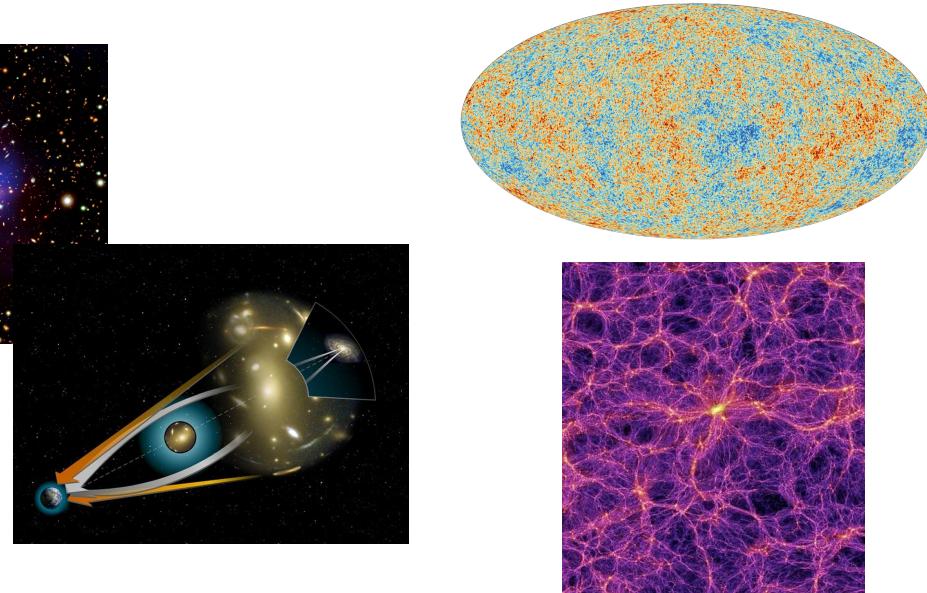
Galactic scales



Cluster scales



Cosmological scales



What we know about DM

- Cold and Massive
- Stable/long lived
- No/weak interactions with the Standard Model (SM)
- No/weak SM charge (electric and color)
- Abundance: DM corresponds to **%25** of the energy budget in the universe today
(**~5x** the amount of **ordinary matter**)

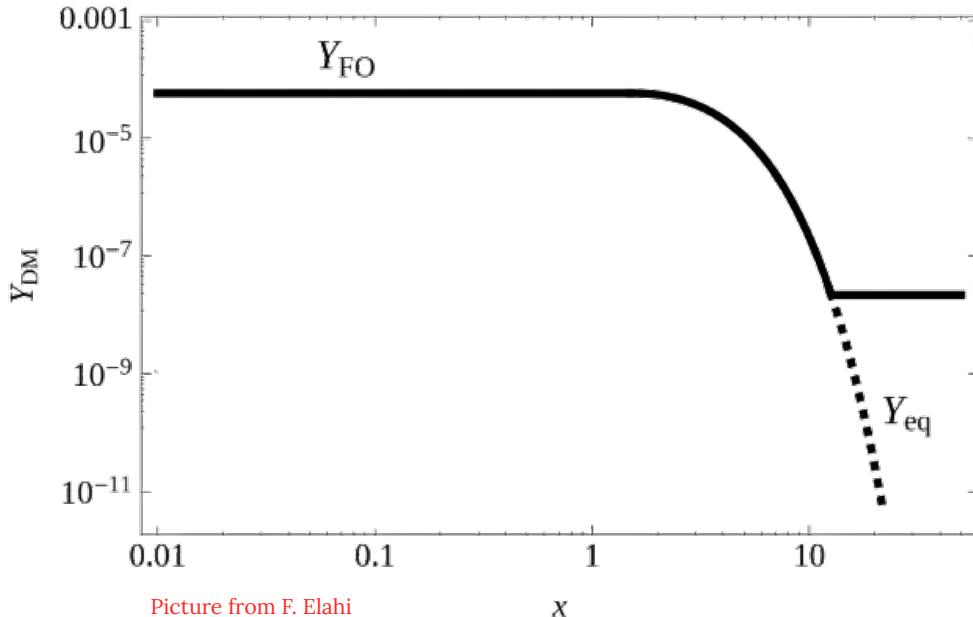
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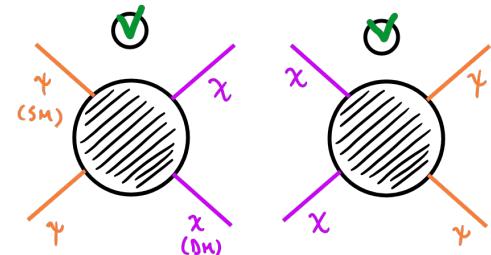
How was DM produced in the early universe?

The Canonical Freeze-out story

- DM is in thermal equilibrium with SM when $T \gg m_{\text{DM}}$
- DM freezes out at $T = m_{\text{DM}}/20$



BOTH REACTIONS OCCUR
AT THE SAME RATE



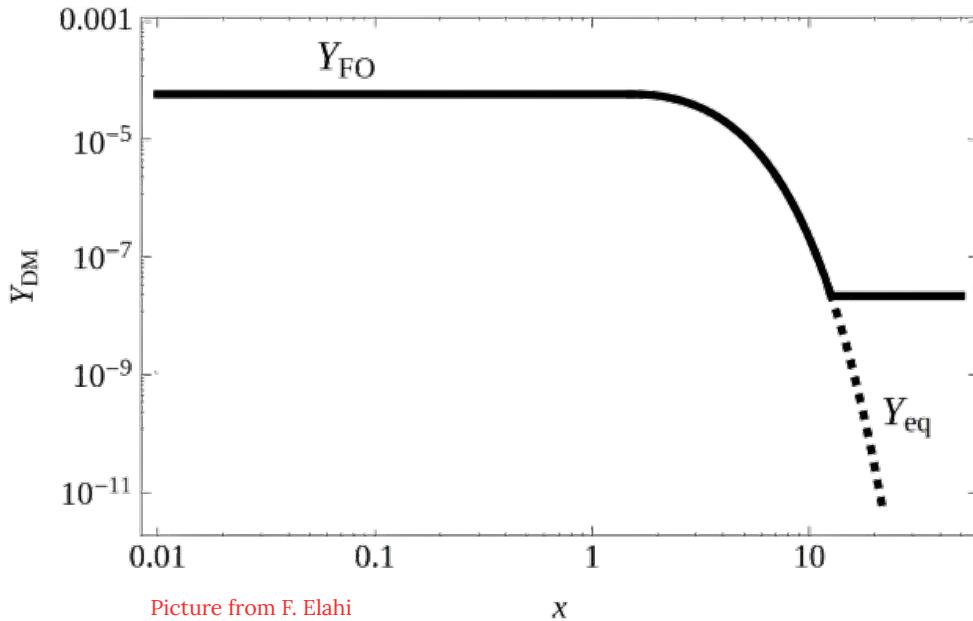
$$Y_{\text{DM}} \equiv \frac{n_{\text{DM}}}{s}$$

$$x \equiv \frac{m_{\text{DM}}}{T}$$

The Canonical Freeze-out story

The WIMP miracle!

$m_{\text{DM}} \square m_W$ and $\sigma_{\text{DM}} \square a_w^2 / m_w^2$ reproduces the observed DM abundance ($a_w \square 10^{-2}$, $m_w \square 100 \text{ GeV}$)



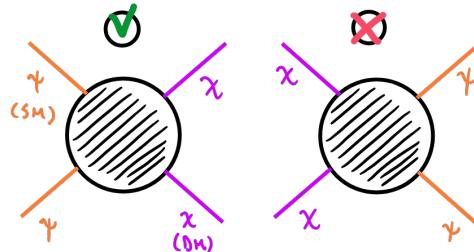
$$Y_{\text{DM}} \equiv \frac{n_{\text{DM}}}{s}$$

$$x \equiv \frac{m_{\text{DM}}}{T}$$

An alternative Scenario: Freeze-in

Hall, Jedamzik, March-Russell, West 2010

- **Feeble** interaction between DM and the SM so that DM is **never in thermal equilibrium** with the SM bath
- Initial DM abundance is negligible (i.e. inflaton reheats primarily the SM)
- The DM abundance is built up gradually (**no inverse process!**)

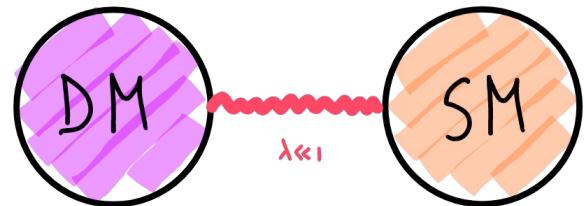


An alternative Scenario: Freeze-in DM from a feeble interaction with SM

The suppressed interaction with SM can arise from:

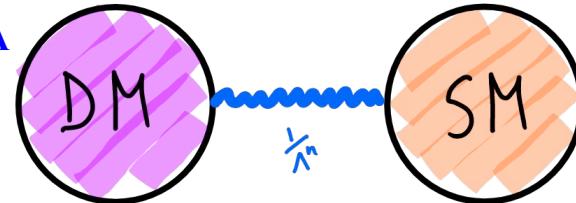
- **A very small** dimensionless coupling $\lambda_{\text{DM-SM}}$
 - renormalizable interaction

Hall, Jedamzik, March-Russell, West 2010



- Dimensionful coupling suppressed by a **heavy mass scale Λ**
 - non-renormalizable interaction of dimension $n+4$
 - Known as **UV freeze-in**

Elahi, Kolda, Unwin 2015

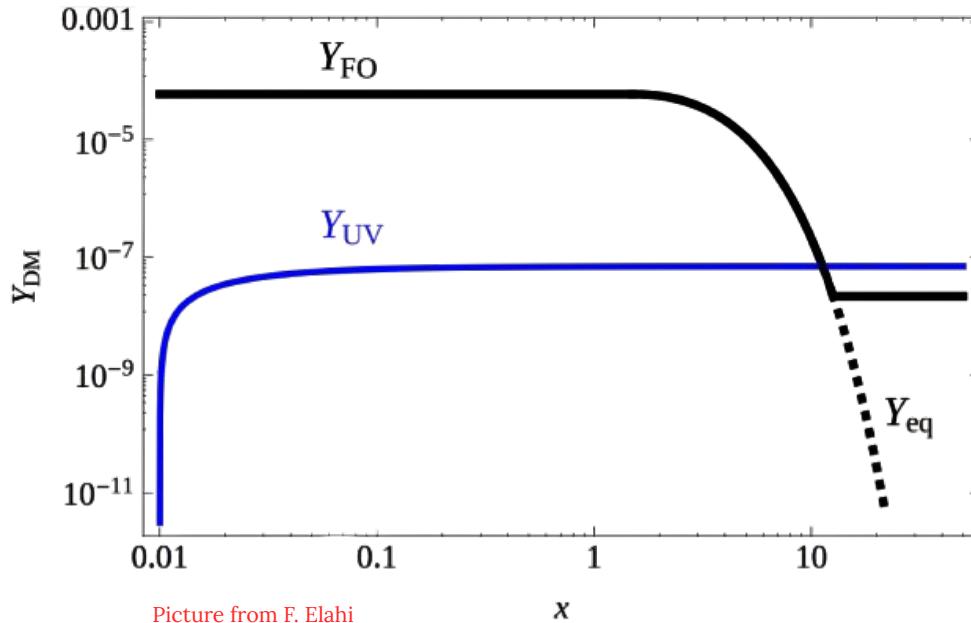


UV Freeze-in

Interaction through Non-Renormalizable operator

Elahi, Kolda, Unwin 2015

The DM abundance **freezes-in quickly** and depends on the **highest temperature**, i.e. T_{rh}



$$\mathcal{L} \supset \mathcal{O}_{n+4}/\Lambda^n$$

$$\dot{n}_\chi + 3Hn_\chi = T^{2n+4}/\Lambda^{2n}$$

$$Y_{\text{UV},\infty} \sim \frac{M_{\text{pl}} T_{\text{rh}}^{2n-1}}{\Lambda^{2n}}$$

UV Freeze-in

Beyond Instantaneous Reheating

So far we assumed **instantaneous** reheating to SM

→ What if we go beyond the instantaneous reheating approximation?

UV Freeze-in

Beyond Instantaneous Reheating

So far we assumed **instantaneous** reheating to SM

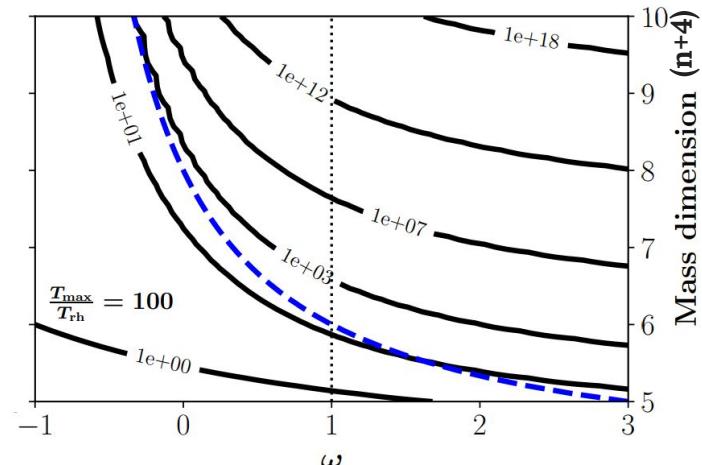
→ What if we go beyond the instantaneous reheating approximation?

Finite reheating can **enhance** final DM abundance ✓

Garcia, Mambrini, Olive, Peloso 2017; Chen, Kang 2018;
Bernal, Elahi, Maldonado, Unwin 2019; Barman, Bernal, Xu, Zapata 2022

- When $T_{\max} \gg T_{\text{rh}}$ Chung, Kolb, Riotto, 1998; Giudice, Kolb, Riotto, 2000; Kolb, Notari, Riotto, 2003
- The enhancement becomes relevant for $n \gtrsim 3$

$$Y_{\chi,\infty}/Y_{\chi,\infty}^{\text{RD}}(T_{\text{rh}}) \sim \left(\frac{T_{\max}}{T_{\text{rh}}}\right)^{n-n_c}, \quad n > n_c$$



UV Freeze-in

Beyond Instantaneous Reheating

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→ What if we consider the production during inflation?

UV Freeze-in

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→ What if we consider the production during inflation?

Requires a **thermal bath** within the **inflationary** phase, i.e. **Warm Inflation**

The WIFI framework Freese, GM, Shams 2024 

We show, for the first time, that inflation, specifically in a warm setting, can lead to substantial DM production via freeze-in.

In fact, **all of the DM in our universe can be produced during inflation** via this mechanism.

WARM INFLATION

Warm Inflation (WI)

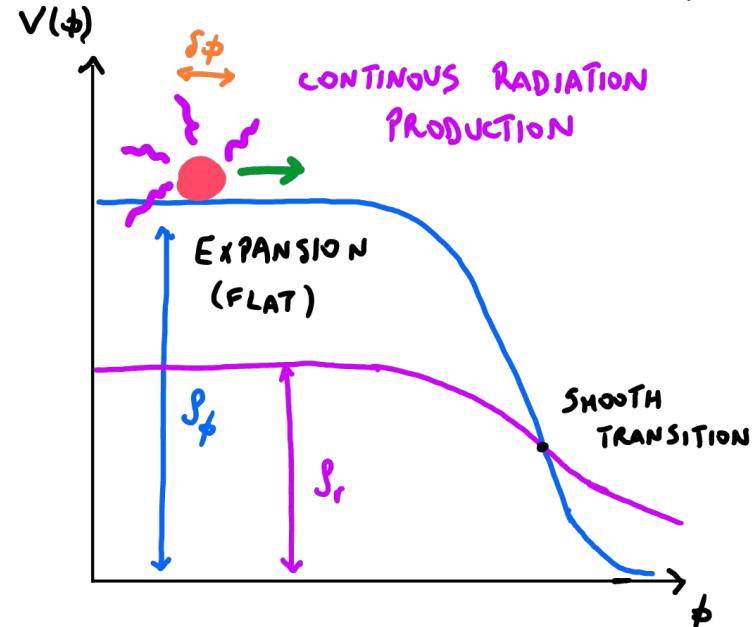
Subdominant radiation bath from dissipation

$T > H$

- Subdominant **radiation bath** at $T > H$ continuously sourced by dissipative interactions with ϕ .
- Dissipation rate Υ acts as additional **thermal friction**
 - This allows $\Delta\phi < M_{pl}$
- Smooth transition to radiation dominated (RD) universe after inflation

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_\phi = 0,$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2,$$



Warm Inflation (WI)

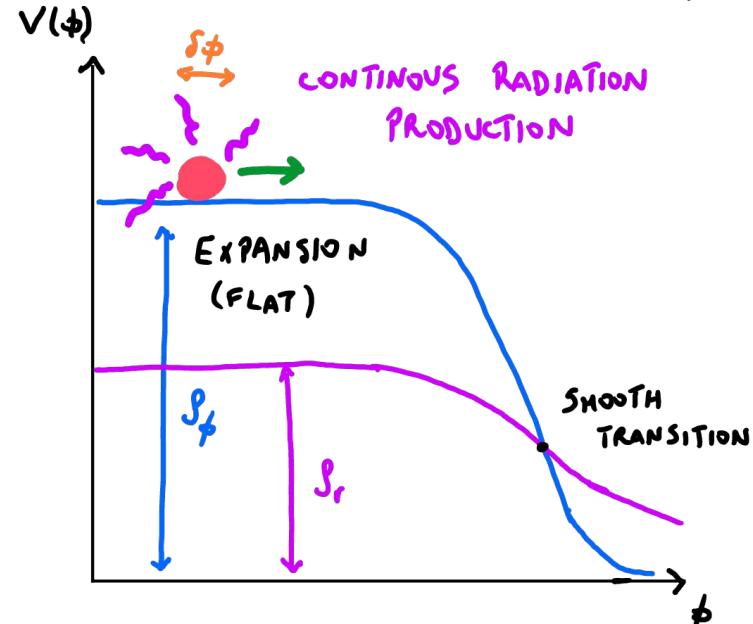
Subdominant radiation bath from dissipation

T>H

- Distinct observables due to thermal nature of perturbations
 - Tensor-to-scalar ratio r generically suppressed
 - Relatively large non-gaussianities

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_{\phi} = 0,$$

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Warm Inflation (WI)

Subdominant radiation bath from dissipation

T>H

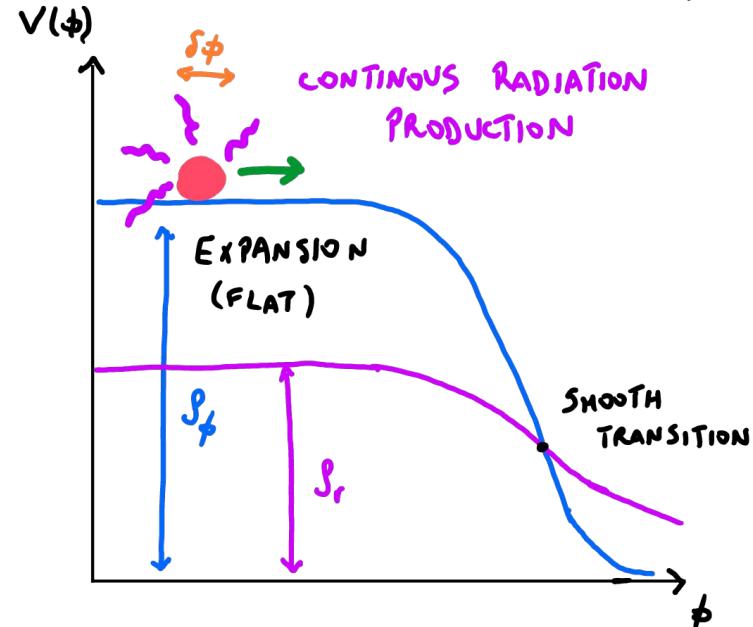
- Distinct observables due to thermal nature of perturbations
 - Tensor-to-scalar ratio r generically suppressed
 - Relatively large non-gaussianities

Recent development of minimal set-ups for a concrete particle physics realization of WI

- Warm Little Inflaton Bastero-Gil, Berera, Ramos, Rosa 2016
- Minimal Warm Inflation Berghaus, Graham, Kaplan 2019

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_\phi = 0,$$

$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2,$$



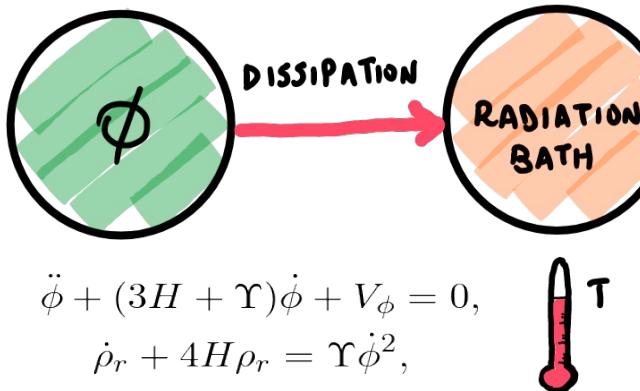
DM from Warm Inflation via UV Freeze-In

The WIFI framework

DM production during WI via UV Freeze-In

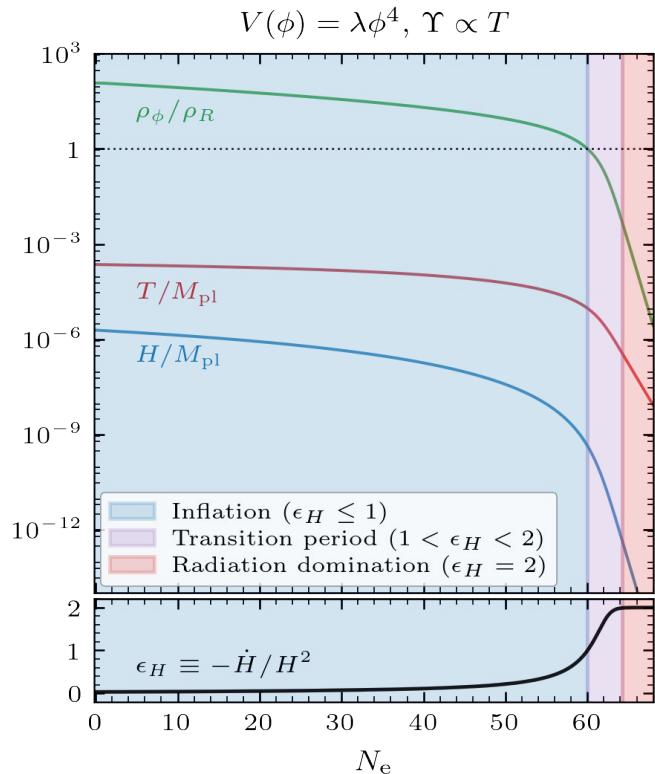
The WIFI framework

WARM INFLATION



Example motivated by Warm Little Inflaton

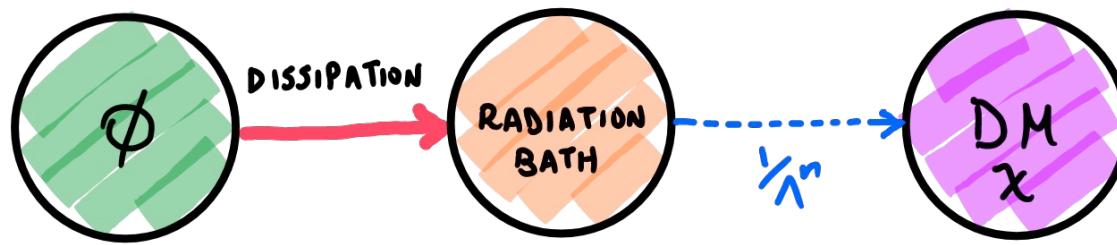
Bastero-Gil, Berera, Ramos, Rosa 2016



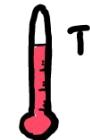
DM production during WI via UV Freeze-In

The WIFI framework

WARM INFLATION UV FREEZE IN



$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_\phi = 0,$$
$$\dot{\rho}_r + 4H\rho_r = \Upsilon\dot{\phi}^2,$$



$$\dot{n}_\chi + 3Hn_\chi = T^{2n+4}/\Lambda^{2n}$$

WIFI

DM production during WI via UV Freeze-In

The WIFI framework

DM production mostly occurs when the rate of change of the comoving DM number density $N_\chi \equiv e^{3N_e} n_\chi$ is peaked

$$\dot{n}_\chi + 3Hn_\chi = T^{2n+4}/\Lambda^{2n}$$

$$\mathcal{I}_\chi(N_e) = \frac{dN_\chi}{dN_e} \equiv e^{3N_e} \frac{T^{2n+4}(N_e)}{\Lambda^{2n} H(N_e)}$$

Number of e-folds
 $dN_e = H dt$

NOTATION REMINDER: Hereafter, we refer to DM by the greek letter χ

DM production during WI via UV Freeze-In

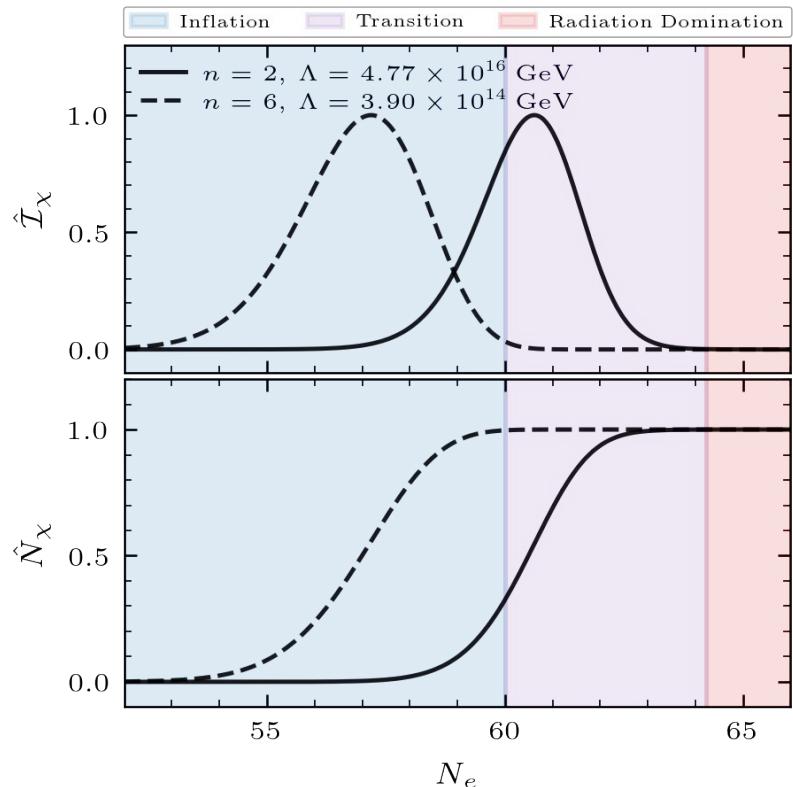
The WIFI framework

In WIFI \mathcal{I}_χ is **sharply peaked** at some e-fold N_e^{peak}

$$3 + (2n + 4) \frac{d \ln T(N_e)}{d N_e} - \frac{d \ln H(N_e)}{d N_e} = 0$$

- Deep in WI: $T, H \sim \text{const.} : \mathcal{I}_\chi \sim e^{3N_e}$
- In RD: $T \sim e^{-N_e}, H \sim T^2 : \mathcal{I}_\chi \sim e^{-(2n-1)N_e}$

$$\mathcal{I}_\chi(N_e) = \frac{dN_\chi}{dN_e} \equiv e^{3N_e} \frac{T^{2n+4}(N_e)}{\Lambda^{2n} H(N_e)}$$



DM production during WI via UV Freeze-In

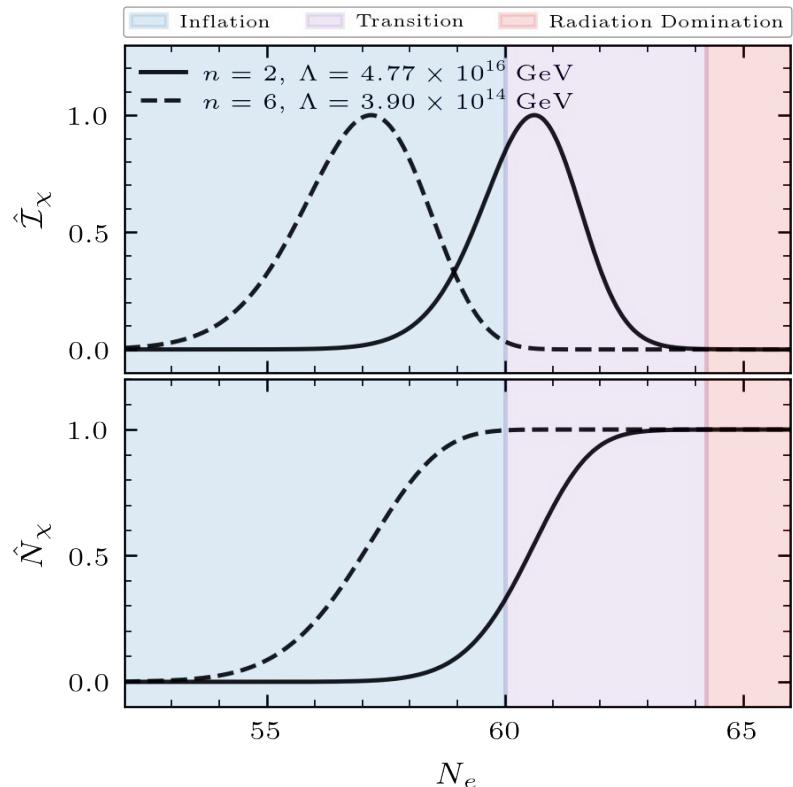
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**Key distinction from (standard)
radiation-dominated UV freeze-in:**

In WIFI, the relic DM yield is not set by the highest temperature of the bath, but rather in a short time interval around N_e^{peak}



DM production during WI via UV Freeze-In

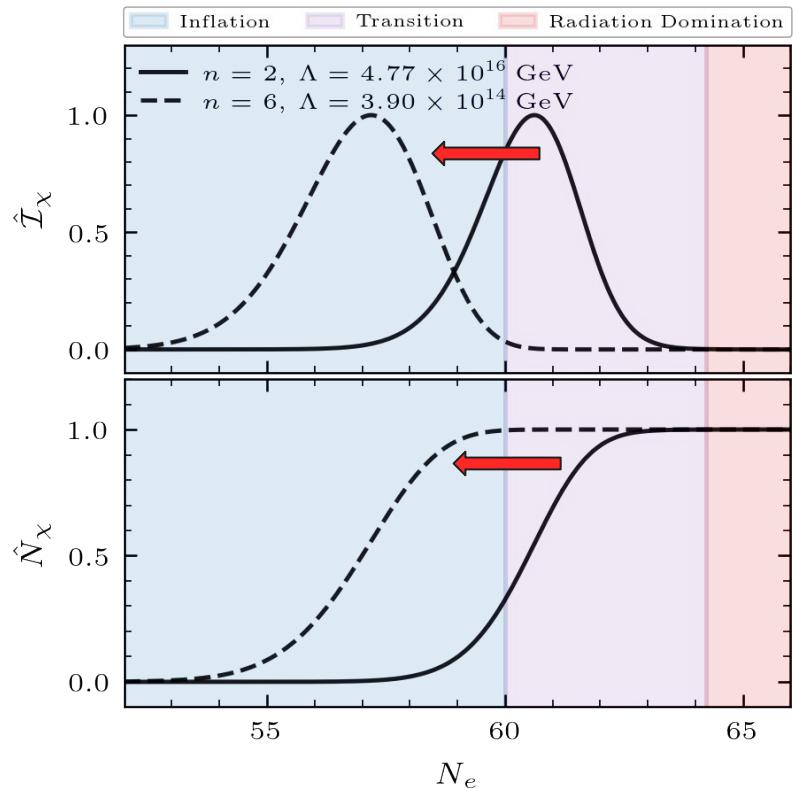
The WIFI framework

For increasing value of n , the peak of \mathcal{I}_χ moves to earlier times

$$\mathcal{I}_\chi(N_e) = \frac{dN_\chi}{dN_e} \equiv e^{3N_e} \frac{T^{2n+4}(N_e)}{\Lambda^{2n} H(N_e)}$$

Recall:

- Deep in WI: $\mathcal{I}_\chi \sim e^{3N_e}$
- In RD: $\mathcal{I}_\chi \sim e^{-(2n-1)N_e}$



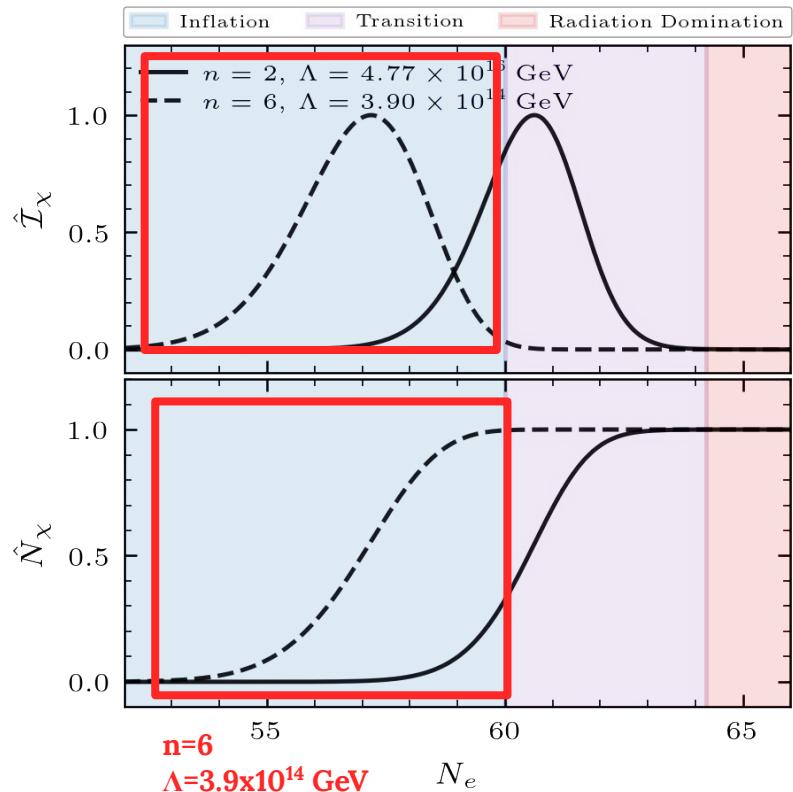
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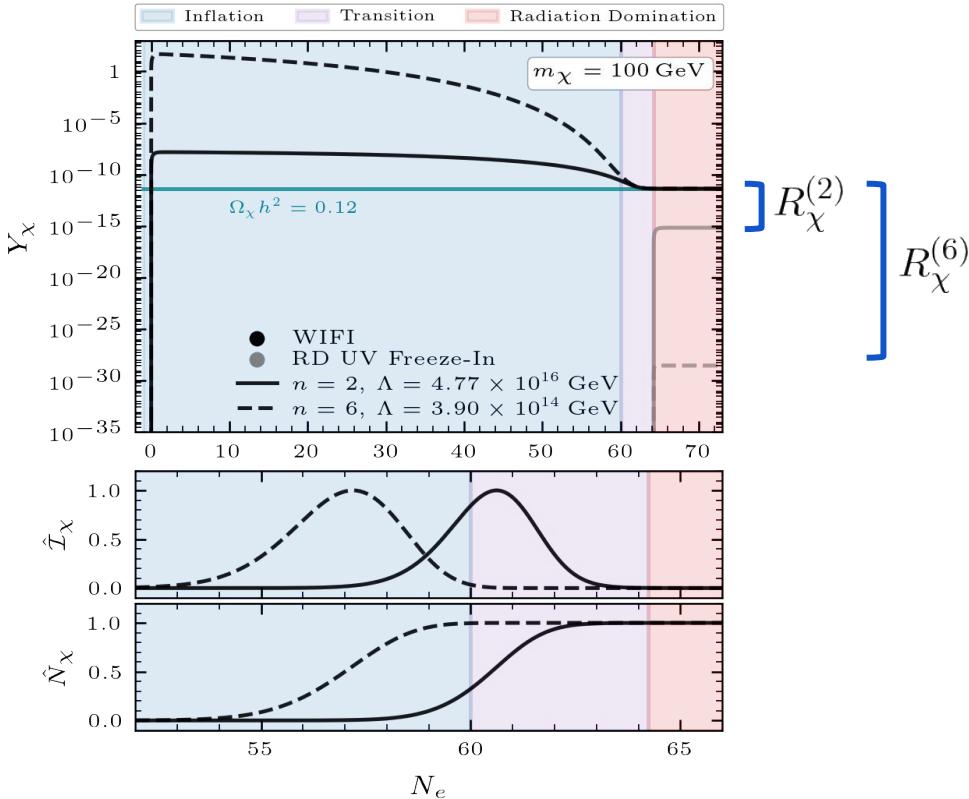
For sufficiently large values of n ,

the DM relic abundance is entirely created during
the inflationary phase



An example of DM production in WIFI

Evolution of DM abundance



The Enhancement Ratio:

$$R_\chi^{(n)} \equiv Y_{\chi,\infty}^{\text{WIFI}} / Y_{\chi,\infty}^{\text{UV}}$$

$$\simeq (2n - 1) \frac{\mathcal{I}_\chi(N_e^{\text{peak}})}{\mathcal{I}_\chi(N_e^{\text{RD}})} \Delta N_e^{\text{peak}}$$

RD condition:

$$\epsilon_H \equiv -\dot{H}/H^2$$

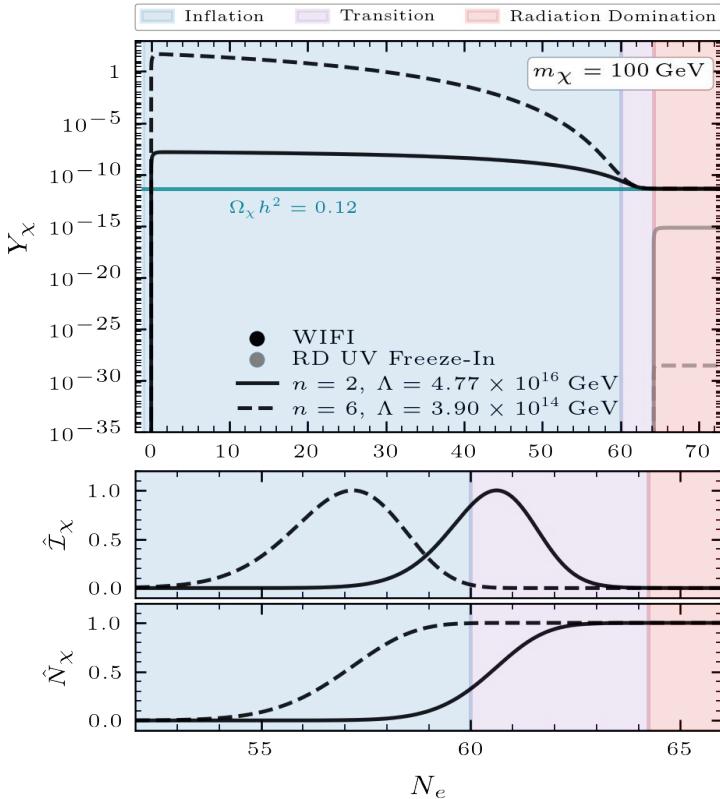
$$T_{\text{rh}} \equiv T(\epsilon_H = 2)$$

$$N_e^{\text{RD}} \equiv N_e(\epsilon_H = 2)$$

$$T(N_e^{\text{RD}}) = T_{\text{rh}}$$

An example of DM production in WIFI

Evolution of DM abundance



$$R_\chi^{(2)} \quad R_\chi^{(6)}$$

The Enhancement Ratio:

$$R_\chi^{(n)} \equiv Y_{\chi,\infty}^{\text{WIFI}} / Y_{\chi,\infty}^{\text{UV}}$$

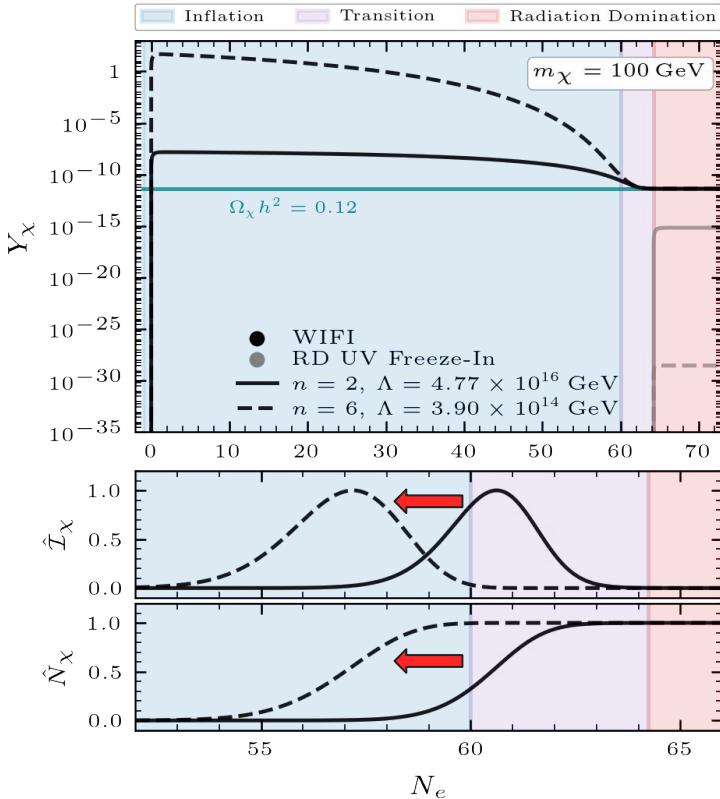
$$\simeq (2n - 1) \frac{\mathcal{I}_\chi(N_e^{\text{peak}})}{\mathcal{I}_\chi(N_e^{\text{RD}})} \Delta N_e^{\text{peak}}$$

$\gg 1$

The resulting DM yield in WIFI is **ALWAYS ENHANCED**
compared to the RD UV freeze-in scenario for the same
reheat temperature

An example of DM production in WIFI

Evolution of DM abundance



The Enhancement Ratio:

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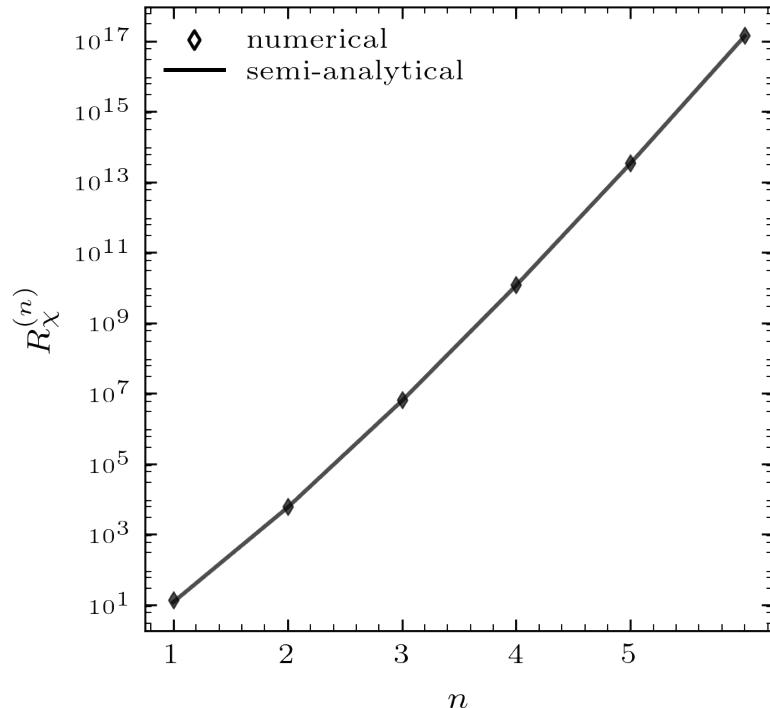
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An example of DM production in WIFI

The Enhancement Ratio

- The enhancement ratio increases **exponentially** with **n**
 - ~10 already for **n=1**
 - $\gg 10^3$ for **n≥3**
- For **sufficiently large n**, the DM abundance is fully determined during the **inflationary phase**, leading also to the **greatest enhancement in DM yield**.

$$R_\chi^{(n)} \simeq (2n - 1) \frac{\mathcal{I}_\chi(N_e^{\text{peak}})}{\mathcal{I}_\chi(N_e^{\text{RD}})} \Delta N_e^{\text{peak}}$$



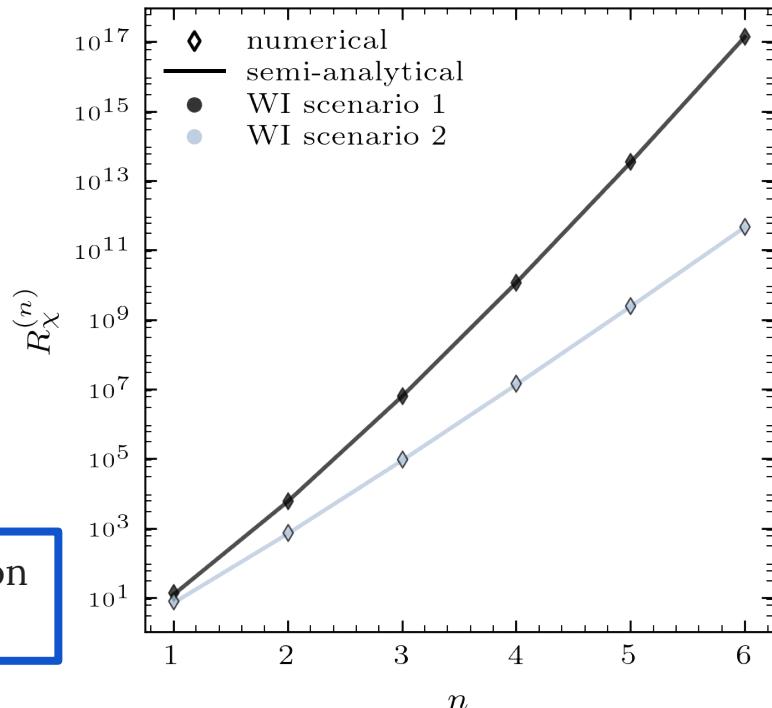
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NOTE: The specific WI dynamics has a significant effect on the size of overall enhancement

$$R_\chi^{(n)} \simeq (2n - 1) \frac{\mathcal{I}_\chi(N_e^{\text{peak}})}{\mathcal{I}_\chi(N_e^{\text{RD}})} \Delta N_e^{\text{peak}}$$



Conclusions

1. We showed, for the first time, that inflation, specifically in a **warm setting**, can lead to **substantial DM production** via **freeze-in**.
2. The DM yield in **WIFI** is **always significantly enhanced** compared to the **RD UV freeze-in** scenario for the **same reheat** temperature
3. **All of the DM in our universe** can be produced during **inflation** via the **WIFI** mechanism.

Outlook and Future Work

1. Tight link between inflationary dynamics and DM production
 - **Rich phenomenology** to be further explored
 - Possible **inprint in the phase-space distribution** of relic DM particles.
2. The WIFI framework can be also used to **produce other cosmological relics**, which could potentially play a significant role in the early Universe evolution.

Grazie per l'attenzione

Gabriele Montefalcone

Weinberg Institute for Theoretical Physics, University of Texas at Austin

Based on work with Katherine Freese & Barmak Shams Es Haghi (arXiv:2401.17371)

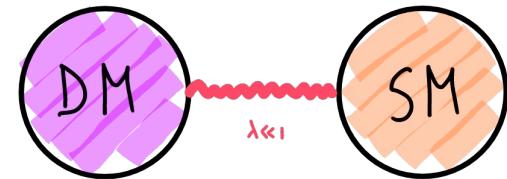
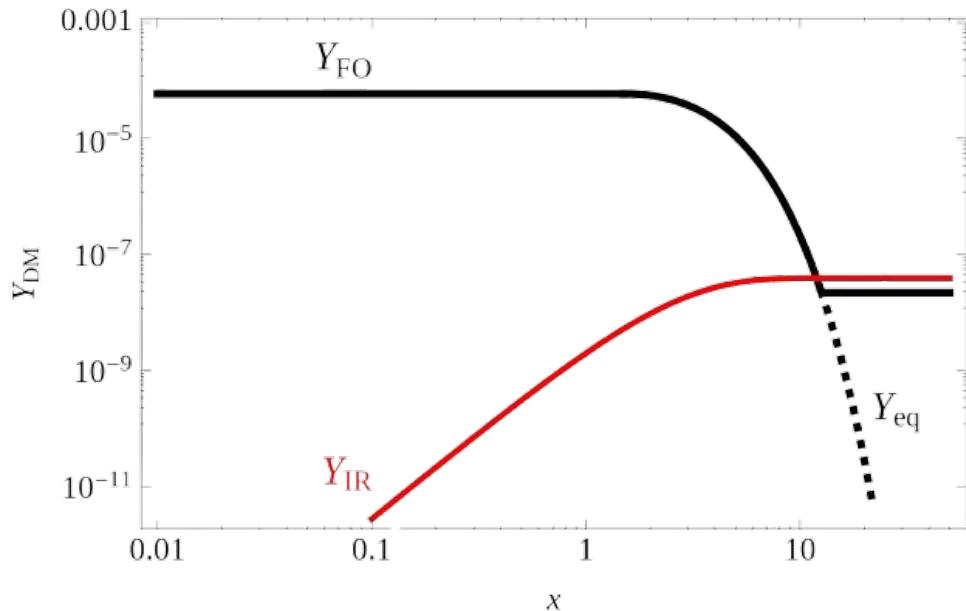
BACK-UP SLIDES

IR Freeze-in

Interaction through Renormalizable operator

Hall, Jedamzik, March-Russell, West 2010

- DM couples to SM through a renormalizable interaction with a very small coupling constant ($\lambda_{\text{DM-SM}} \sim 10^{-11}$)



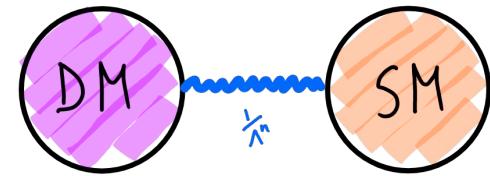
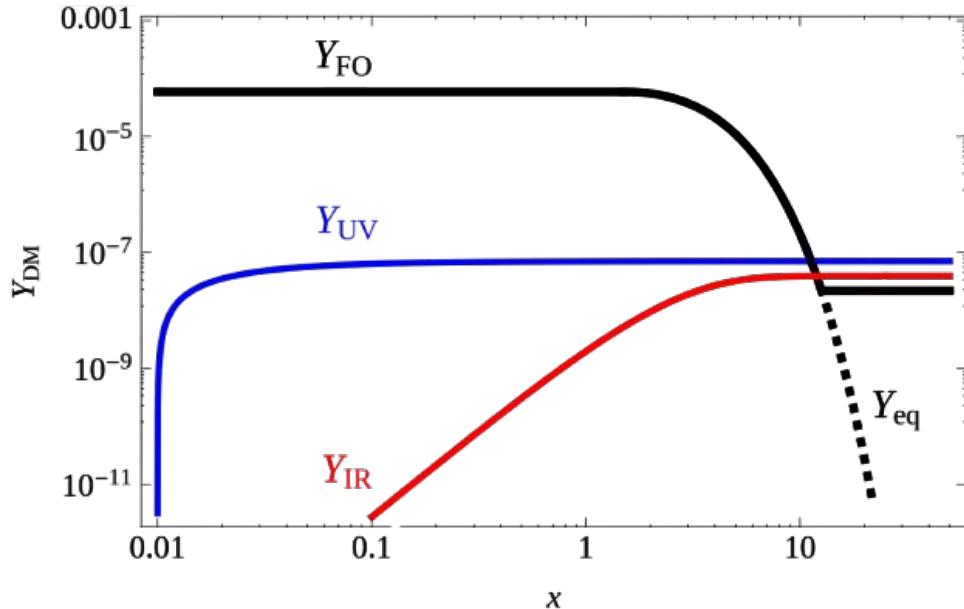
DM freezes-in gradually and depends on the lowest T, i.e. $T \square m_{\text{DM}}$

UV Freeze-in

Elahi, Kolda, Unwin 2015

Interaction through Non-Renormalizable operator

- DM couples to SM through a **non-renormalizable** interaction with a **heavy mass scale Λ**



DM freezes-in **quickly** and depends on the **highest T**, i.e. T_{rh}

UV Freeze-in

Elahi, Kolda, Unwin 2015

Interaction through Non-Renormalizable operator

$$\mathcal{L} \supset \mathcal{O}_{n+4}/\Lambda^n$$

$$\dot{n}_\chi + 3Hn_\chi = T^{2n+4}/\Lambda^{2n}$$

- Assumptions:

- Instantaneous Reheating (rh) to radiation-dominated (RD) epoch
- Vanishing initial DM abundance

$$Y_\chi \equiv \underbrace{\frac{n_\chi}{s}}$$

Bath entropy density

$$s = (2\pi^2/45)g_{*,S}(T)T^3$$

$$Y_{\chi,\infty}^{\text{RD}} \simeq \frac{1}{\sqrt{2}} \left(\underbrace{\frac{45}{\pi^2 g_\star}} \right)^{3/2} \frac{1}{2n-1} \frac{M_{\text{Pl}} T_{\text{rh}}^{2n-1}}{\Lambda^{2n}}.$$

(the effective number of relativistic degrees of freedom)

UV Freeze-In

A Concrete Example

- Extension of SM by a $\text{U}(1)$, broken at a high scale Λ
 - Both DM and SM fermions are charged under this new $\text{U}(1)'$ group

$$\mathcal{L} \supset i\bar{Q}\not{D}Q + i\bar{u}\not{D}u + i\bar{\chi}\not{D}\chi + \dots \quad \not{D} = \not{\partial} + \underbrace{iq'\not{Z}'}_{\text{Z}' \text{ and } q' \text{ are respectively the mediator and}} + \dots$$

Z' and q' are respectively the mediator and
the charge of the $\text{U}(1)'$

- For scales $\ll \Lambda \sim M_Z$, Z' is integrated out:

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{\Lambda^2} \bar{Q} \gamma_\mu Q \bar{\chi} \gamma^\mu \chi + \frac{1}{\Lambda^2} \bar{u}^c \gamma_\mu u^c \bar{\chi} \gamma^\mu \chi + \dots$$

UV Freeze-In

DM number density evolution

- Consider a dimension $(n+4)$ operator

$$\frac{1}{\Lambda^n} \phi_1 \phi_2 \cdots \phi_{n+3} \varphi$$

SM fields DM

$$\dot{n}_\varphi + 3Hn_\varphi = \int d\Pi_1 d\Pi_2 f_1 f_2 |\mathcal{M}|_{(n)}^2 \text{DLIPS}_{(n+2)}$$

$$\simeq \frac{2T}{(4\pi)^5 \Lambda^{2n}} \left[\frac{1}{4\pi^2} \right]^n \int_0^\infty ds s^{(2n+1)/2} K_1(\sqrt{s}/T)$$

$$\text{DLIPS}_{(n+2)} \sim \left[\frac{s}{4\pi^2} \right]^n \text{DLIPS}_{(2)}$$

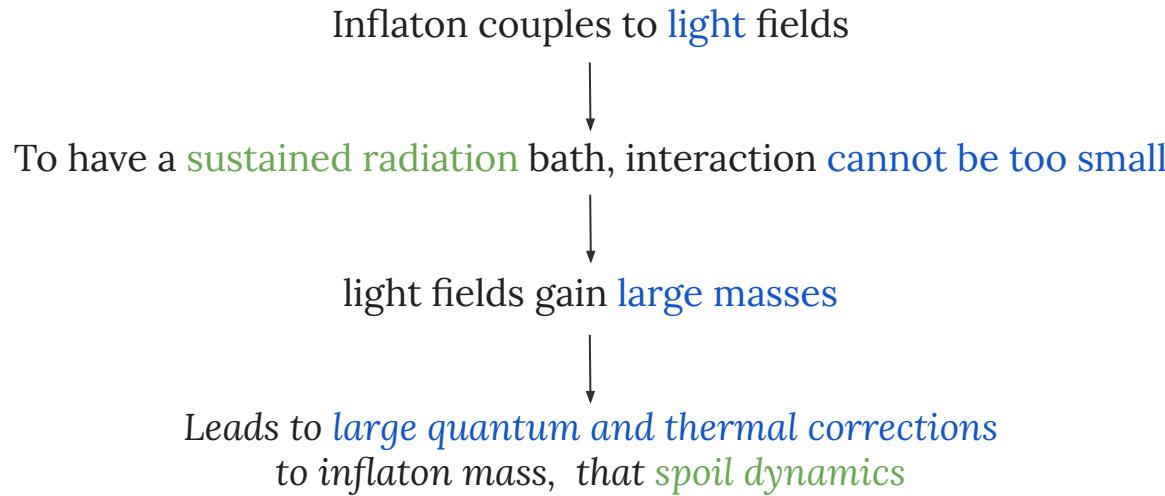
$$|\mathcal{M}|_{(n)}^2 \sim \left(\frac{1}{\Lambda^2} \right)^n$$

$$\dot{n}_\varphi + 3Hn_\varphi \simeq \underbrace{\frac{1}{(2\pi)^7} \left(\frac{n!(n+1)!}{\pi^{2n-2}} \right) \frac{T^{2n+4}}{\Lambda^{2n}}}$$

Numerical prefactor set to
1 in our work

Warm Inflation Model Building

The Challenge



J. Yokoyama, A. Linde, 1998

Warm Inflation Model Building

Early Attempts

- Make theory supersymmetric to control quantum corrections
Berera, Gleiser, Ramos, 1999
- Avoid **directly coupling** inflaton to radiation fields: only indirectly through **heavy mediators** so that thermal corrections are Boltzmann suppressed
Berera, Ramos 2001 - 2003; Moss, Xiong 2006; Bastero-Gil, Berera, Ramos 2011; Bastero-Gil, Berera, Ramos, Rosa 2014 - 2015

$$\mathcal{L} = V(\phi) - g^2 \phi^2 \chi^2 - h \phi \bar{\psi}_\chi \psi_\chi - f \chi \bar{\psi}_\sigma \psi_\sigma + g'^2 \chi^2 \sigma^2 + \dots$$

Heavy mediators Light Fermions
 $m_\chi = g\phi \gg H, T$ $m_\sigma \ll T$

- Need large number (**~10⁶**) of mediator fields though

Warm Inflation Model Building

Recent Developments

Warm Little Inflaton

Bastero-Gil, Berera, Ramos, Rosa 2016

$$-\mathcal{L} \supset V(\phi) + gM \cos(\phi/M) \bar{\psi}_1 \psi_1 + gM \sin(\phi/M) \bar{\psi}_2 \psi_2 + h\sigma (\bar{\psi}_j \chi + \bar{\chi} \psi_j)$$

- Inflaton is a PNGB coupled to a pair of light fermions
- Symmetry structure enforces (1) bounded fermion masses; (2) no thermal corrections

Minimal Warm Inflation

Berghaus, Graham, Kaplan 2019

$$\mathcal{L} \supset -V(\phi) + \frac{\phi}{f} \tilde{F}^{a\mu\nu} F_{\mu\nu}^a$$

- Inflaton is an axion-like particle protected by its shift symmetry
- Dissipation from sphaleron heating via coupling to Yang-Mills Fields

→ In this models, WI is almost inevitable DeRocco, Graham, Kalia 2021

Warm Inflation

Perturbation Spectra

Bastero-Gil, Berera, Ramos 2011
Graham, Moss 2009

- The scalar power spectrum is enhanced by thermal effects;
- The tensor power spectrum is unaltered;
- $G(Q)$ accounts for the direct coupling of the inflaton and radiation fluctuations due to a temperature dependent dissipative rate $\Gamma \propto T^c$
 - Approximated to a polynomial in Q
 - If $c > 0$: spectrum is further enhanced;
 - If $c < 0$: spectrum is suppressed;

$$\Delta_s^2 = \left(\frac{H^2}{2\pi\dot{\phi}} \right)^2 \left[1 + 2n_{BE} + \frac{2\sqrt{3}\pi Q}{\sqrt{3 + 4\pi Q}} \left(\frac{T}{H} \right) \right] G(Q)$$

} }
 } }

$\Delta_s^2 \text{ (vac, warm)}$
 $\Delta_s^2 \text{ (diss)}$

$$\Delta_s^{2 \text{ (vac, cold)}} = \left(\frac{H^2}{2\pi\dot{\phi}} \right)^2$$

$$n_{BE} = 1 / [\exp(H/T) - 1]$$

Warm Inflation

Computing Cosmological Observables

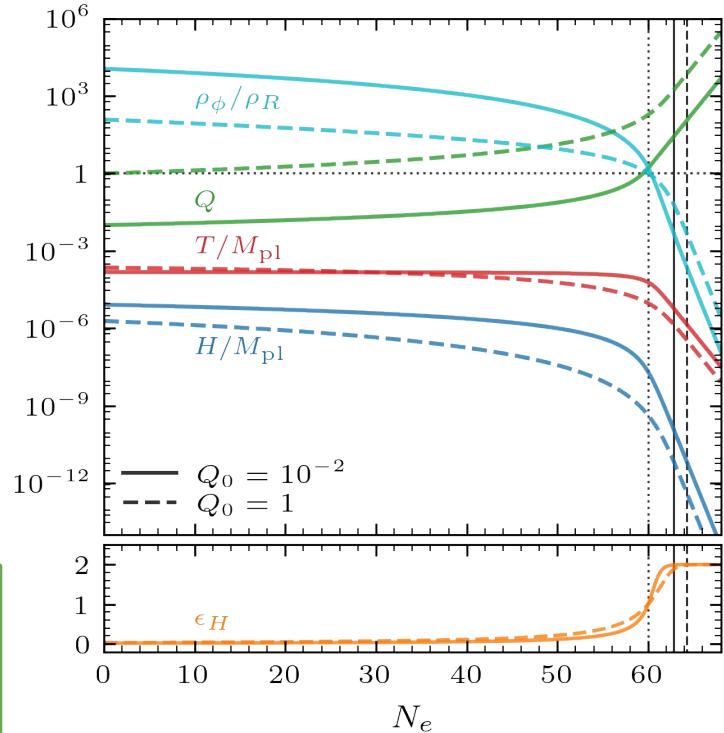
- We use WarmSPy to compute the background dynamics. GM, V. Aragam, L. Visinelli, K. Freese 2023
 - We fix the initial dissipation strength Q_0 (60 e-folds before the end of inflation)
 - We use an iterative algorithm to get the initial field value to ensure 60 e-folds of inflation, i.e. $\epsilon_H(N_e=60)=1$
 - We fix the height of the potential (in our case λ), to match the amplitude of the primordial power spectrum at the CMB pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$
 - We compute r and n_s and ensure they are within the CMB bounds, otherwise we repeat the process for a different Q_0

An example of DM production in WIFI WI dynamics

- We consider two scenarios: $V(\phi) = \lambda\phi^4$, $\Gamma \propto T$ with $Q_0 = 10^{-2}$ and $Q_0 = 1$.
 - Note $Q = \Gamma/(3H)$ defines the strength of the dissipation
- parameters of the inflaton potential set to enforce 60 e-folds and produce cosmological observables consistent with CMB data

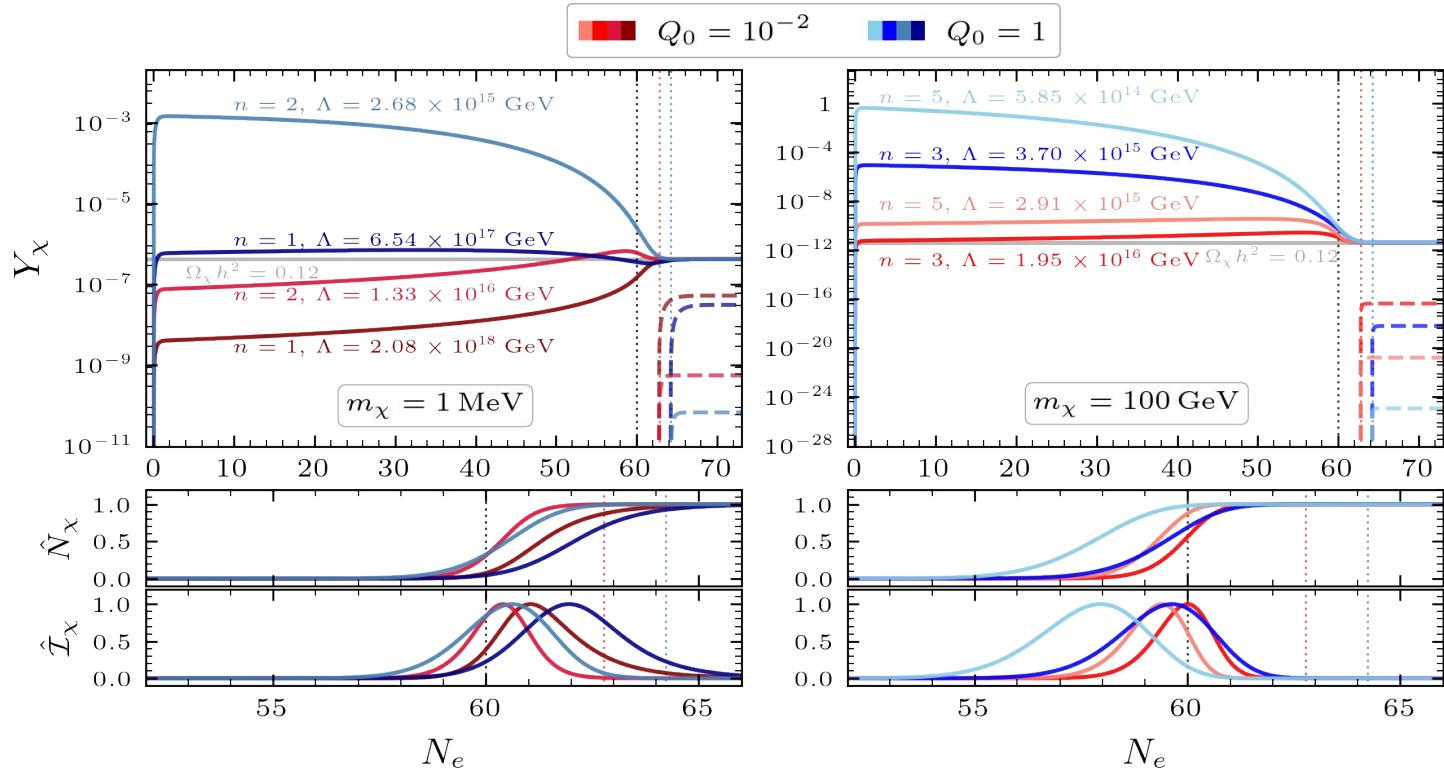
Dynamics computed with **WarmSPy!**
github.com/GabrieleMonte/WarmSPy

GM, V. Aragam, L. Visinelli, K. Freese 2023



An example of DM production in WIFI

The DM Yield Evolution



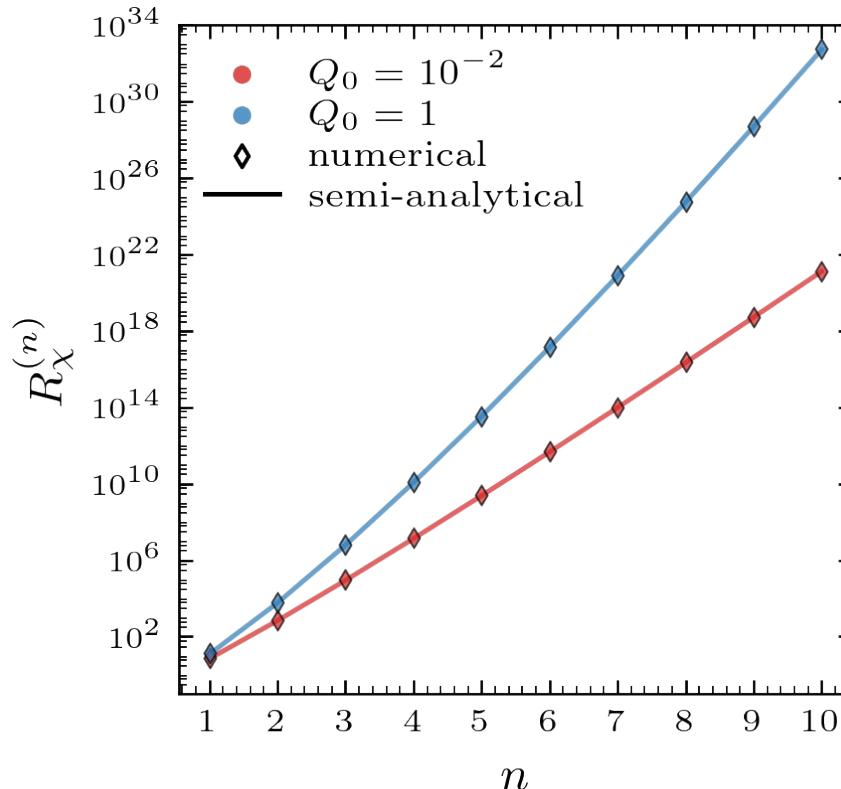
$$N_\chi = e^{3N_e} n_\chi$$

$$\begin{aligned} \mathcal{I}_\chi &\equiv e^{3N_e} \frac{T^{2n+4}}{H\Lambda^{2n}} \\ &= \frac{dN_\chi}{dN_e} \end{aligned}$$

An example of DM production in WIFI

The Enhancement Ratio

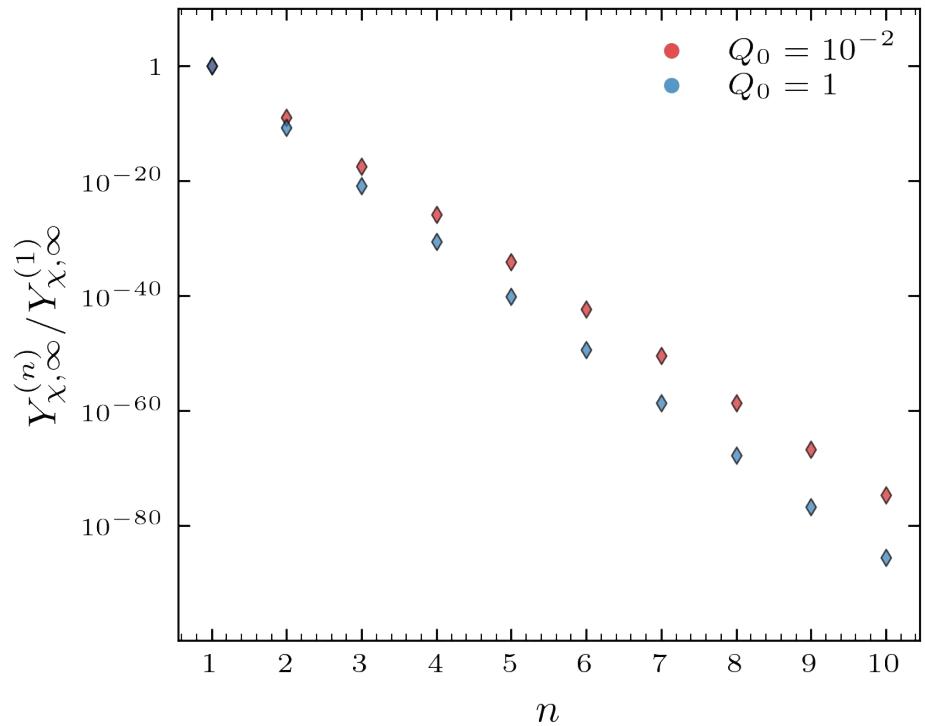
$$R_{\chi}^{(n)} \simeq (2n - 1) \frac{\mathcal{I}_{\chi}(N_e^{\text{peak}})}{\mathcal{I}_{\chi}(N_e^{\text{RD}})} \Delta N_e^{\text{peak}}$$



An example of DM production in WIFI

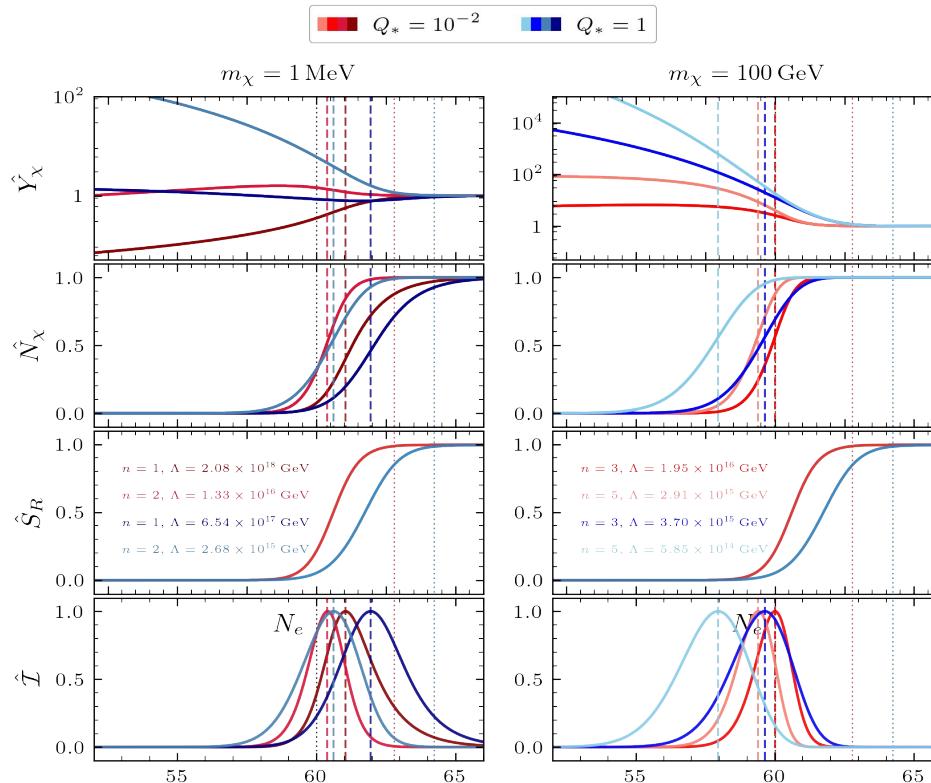
Comparison of DM yield for different n values

As expected for a fixed value of Λ ,
the relic DM yield is *exponentially suppressed*
at large values of n



An example of DM production in WIFI

More on the Yield Evolution



$$N_\chi = e^{3N_e} n_\chi$$

$$S_R = e^{3N_e} s_R$$

$$\begin{aligned} \mathcal{I}_\chi &\equiv e^{3N_e} \frac{T^{2n+4}}{H\Lambda^{2n}} \\ &= \frac{dN_\chi}{dN_e} \end{aligned}$$

An example of DM production in WIFI

Constraints on Λ and m_χ

$$m_\chi \cdot Y_{\chi,\infty}(\Lambda, n) = \left(\frac{\Omega_\chi h^2}{0.12} \right) \left(\frac{\rho_c}{1.0537 \times 10^{-5} h^2 \text{ GeV cm}^{-3}} \right) \left(\frac{2891.2 \text{ cm}^{-3}}{s_0} \right)$$

