Some difficulties in quantizing gravity

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Table of Contents

Table of Contents

2 successful theories of nature:

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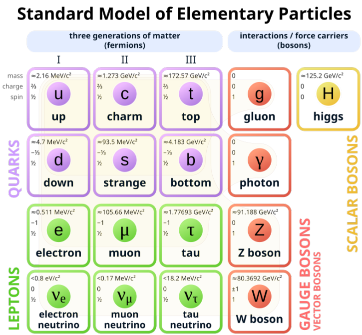
Quantum Mechanics

- valid at very small scales
- atoms, electromagnetism, standard model
- 3/4 fundamental forces



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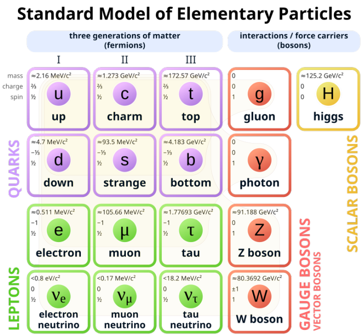
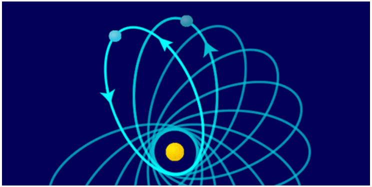


General Relativity (GR)

- works at large scales
- gravitational waves, lensing, perihelion precession of mercury



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- singularities (black holes, big bang)
- very, very small scales where gravity is strong

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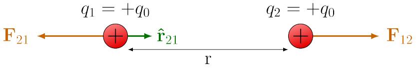
- it is simply not acceptable to have two theories of physics, one for small things and one for big things
- But reconciling quantum mechanics and gravity is hard :c

Table of Contents

Coulomb's Law



$$F = \frac{e^2}{4\pi\epsilon_0 r^2} = \frac{e^2}{4\pi(8.541878188 \times 10^{-12} Fm^{-1})}\frac{1}{r^2}$$

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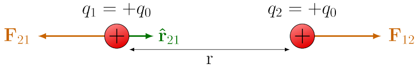

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- kind of stupid to write it like this, better in natural units

Dimensional Analysis

- "god given" units: $c = \hbar = k_B = 1$
- do everything in units of energy (E)

$$
\begin{aligned}
[\text{mass}] &= E \\
[\text{length}] &= E^{-1} \\
[\text{time}] &= E^{-1} \\
[\text{momentum}] &= E \\
[\text{force}] &= E^{2}
\end{aligned}
$$

.

Coulomb's Law

- now Coulomb's law is easy

$$F_e = \alpha \times \frac{1}{r^2}$$

where

$$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137} \,.$$

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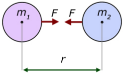
- we just "count" charges
- also keep in mind, in QM, perturbations should be small $H = H_0 + \lambda H_{int}$

Gravity



- play the same game with gravity

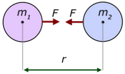
$$F_g = \frac{G_N E_1 E_2}{r^2} \implies [G_N] = E^{-2},$$

1

- $G_N \sim \frac{1}{{M_p}^2}$

[1] The E's are masses
[2] $M_P \sim 1.22 \times 10^{19}$ GeV

Gravity



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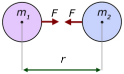

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- G_N having dimensions has very real consequences.

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Let's do an experiment

Let's do an experiment

- gravity is weak
- for e^-, 10^{40}ish times stronger.
- very hard to test experimentally

Table of Contents

What happens at small scales?

- want to query small length (high energy)

$$\ell \sim \frac{1}{E}.$$

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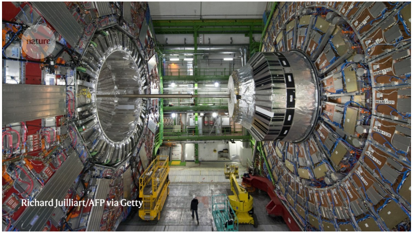

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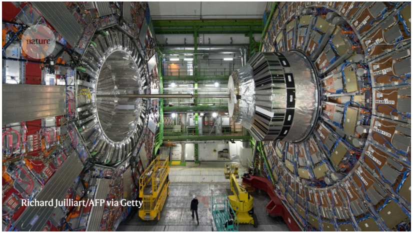

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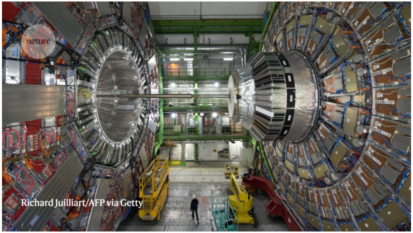

- natural bound: too much mass in too small area->black hole
- as long as the length I'm probing is bigger than Schwarschild radius $r_s = 2EG_N$ for a given energy, I'll be fine.

What happens at small scales?

- mathematically,

$$\ell > r_s$$
$$\frac{1}{E} > 2EG_N$$
$$\frac{1}{E^2} > 2\frac{1}{M_p^2}\,.$$

- takeaway: I can only ask questions up until the Planck mass until I get black holes forming.

Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}(+\Lambda g_{\mu\nu}) = 8\pi G_N T_{\mu\nu}\,.$$

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Einstein's equation

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- immediately see RHS is "wrong"
- $T_{\mu\nu}$ is a quantum operator
- can we set RHS $\sim \langle T_{\mu\nu} \rangle$ and deal with it

Unsatisfactory

- weird thing to do
- $\langle T_{\mu\nu} \rangle$ varies non-linearly with the state of the matter system
- H is part of $T_{\mu\nu}$
- $\implies$ H is no longer a linear operator on $\mathcal{H}$

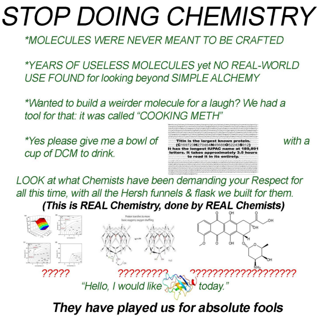
Unsatisfactory

- weird thing to do
- $\langle T_{\mu\nu} \rangle$ varies non-linearly with the state of the matter system
- H is part of $T_{\mu\nu}$
- $\implies H$ is no longer a linear operator on $\mathcal{H}$
- I really just need to quantize gravity to get graviton $g_{\mu\nu}$

Table of Contents

You don't need to solve schrodingers eq to build a bridge

- a chemist doesn't really need to know QM
- we don't need to know QED to make antennas



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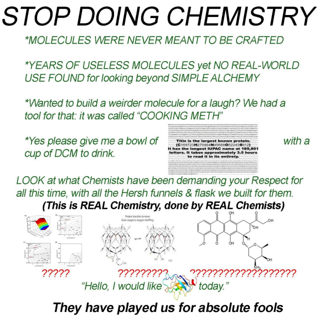
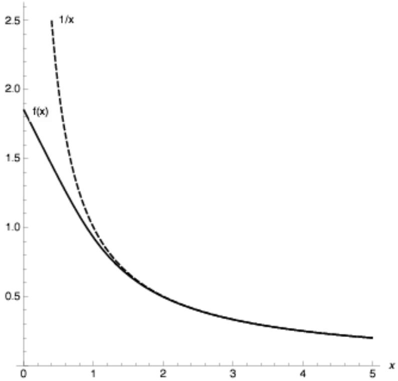


- "effective theories" are good enough for the scale they work at
- only good up until a certain point where they break down

Coulomb's law pt 2

From classical EM: $U = \frac{kq_1 q_2}{r}$



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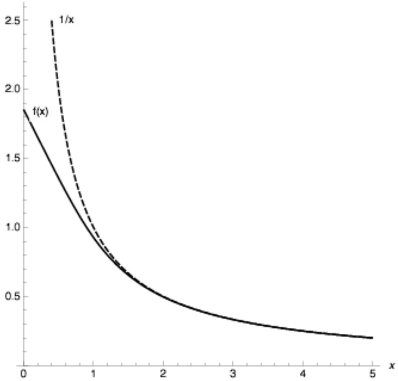

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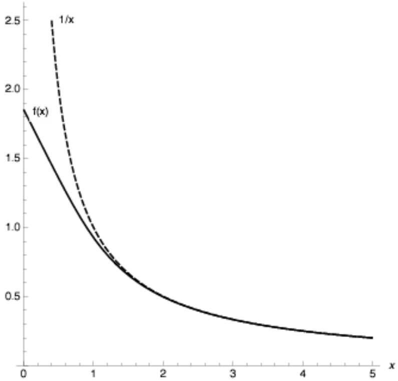

- implies $E \to \infty$ at $r \to 0$
- theory is telling me something stupid (it breaks)
- fixed in QED

Effective theories

- effective theories are very effective at scales they are designed for
- need to keep in mind they inherently break down at some scale

Back to gravity

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$$S = \int d^4x\sqrt{-g}\left(M_p^2 R + c_1 R^2 + c_2 R_{\mu\nu}^2 + \frac{c_3}{M_p^2} R^3 + \cdots + \mathcal{L}_{matter}\right) .$$

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- needs infinite number of terms
- at $E/M_p << 1$, still good perturbative expansion

Still not enough

- can get arbitrarily close just by including higher terms
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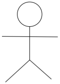
Still not enough

- can get arbitrarily close just by including higher terms
- need $E/M_p << 1$
- still can't ask questions in places where QM and gravity matter: $E \sim M_p$
- really should have a better theory that is good to higher energies

Table of Contents

Regions



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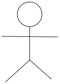

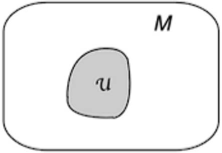

- background on which particles live is fluctuating
- need to describe how other fields affect gravity just as much as how gravity affects them
- no regime in which "pure" gravity matters and the other particle interactions don't

Observables

- good observables should be robust to the how we describe our system
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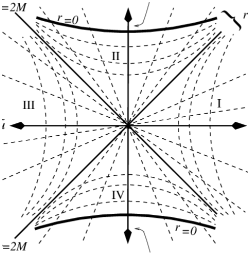
- good observables should be robust to the how we describe our system
- in other words, they should be *diffeomorphism* invariant
- can only do this in really simple cases: asymptotic flat space and asymptotic AdS
- in general regions, it is difficult to formulate
- usually look for extremal surfaces to use (surfaces that don't change to leading order fluctuations)



Summary

Problems in trying to do quantum gravity:

- Needs to include other particle interactions in addition to gravity
- Needs to be good up to arbitrarily high energies
- Needs to have good diffeomorphism invariant observables

Table of Contents

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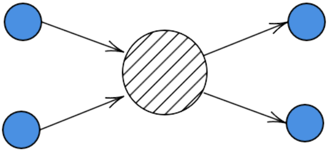
lets study a simple process in flat space and QFT



2->2 scattering observable $\mathcal{A}_4$

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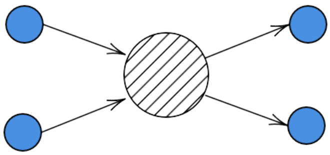

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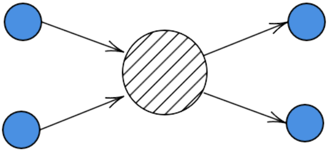

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- using QFT to write it down, will find it breaks down at high energy
- try to fix it while keeping special relativity and unitarity in mind
- this does work: get Virasoro-Shapiro amplitude

Virasoro-Shapiro

- reproduces behavior at low energies
- good high energy behavior
- no new species of particle
- follows unitarity, special relativity, etc.

$$\mathcal{A}_4 \sim G_N \frac{\langle 12 \rangle^4 [34]^4}{stu} \frac{\Gamma(1-s)\Gamma(1-t)\Gamma(1-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)} .$$

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- is very likely unique

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replaces fundamental object with strings:

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price to pay:

- needs 10 dimensions
- needs SUSY
- "nice" observables to study are at very high energies

nevertheless, is a good consistent theory of quantum gravity.