Easy introduction to options and their determinants

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1 Introduction

The main goal of this paper is to give a brief and easy introduction to the world of the option instruments.
Throughout my academic career, it has always been challenging to comprehend the nature of these products, their functioning, and their significance, mainly due to the way in which these products presented and explained to me.
I will introduce you to the concept behind an option, what are the main factors that influence the price of the instrument and what are the main strategies that are applicable via a combination of them.

Let's first give the common definition for an option.

An **option** is a contract that gives the buyer (seller) the right (obligation) to buy or sell an underlying asset at a specific price (known as the **strike price**) on or before a certain date.

By an **underlying asset** we refer to the financial instrument on which the options price is based. This could be futures, stocks, commodities or currency, noting a change in the price of the underlying asset causes a change in the price of the option.
By the **premium of the option** we mean the cost, or the gain, depends on which side we are, deriving from entering in the contract. It's very important since, for the option buyer, this will define the **maximum loss** in which he may incur. Conversely, for the option seller, this sets the limit for the **maximum potential gain** that can be achieved."

2 Option class

An option classes refers to **all** the call options or **all** the put options listed on an exchange for a particular underlying asset, regardless of the position type (long or short).

2.1 Long options

A **long call** option is a contract that gives the buyer the **right**, to buy an underlying asset at a specific price on or before a certain date, known as the expiry date.

A **long put** option is a contract that gives the buyer the **right** to sell an underlying asset at a specific price on or before a certain date, known as the expiry date.

2.2 Short options

A **short call** option is a contract that **obligates** the seller to sell an underlying asset at a specific price on or before a certain date, known as the expiry date.

A **short put** option is a contract that **obligates** the seller to buy an underlying asset at a specific price on or before a certain date, known as the expiry date.

3 Factors that affect options value

The main factors that affect the value of an option, are six:

- the current underlying price, S_0;
- strike price, K;
- the volatility, σ;
- time to expiration, T;
- expected dividend during the life of the option, D.
- risk-free interest rate, r;

Let's now examine each type of option and how changes in the main factors affect their price.

4 Long CALL option

Let's define an option with strike price $K = 80$ and draw a picture of a long call option, considering a premium or a cost to enter in the position of 5 dollars.

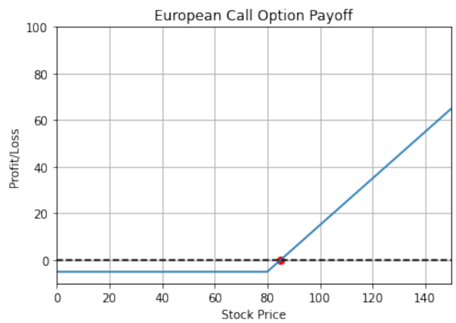

Figure 1: Payoff at the expiring date of a long Call option

In every book, the payoff at the expiring date is expressed as

$$Max(S_T - K, 0)$$

Unfortunately, there's a **premium** that we have to pay to enter in the position, leading the payoff to be:

$$Max(S_T - (K + premium), -premium)$$

When entering in a long call option, our **vision** is surely **bullish**.
Indeed, the only moment in which exercising the option would be convenient, is when the price of the underlying returns more than what we paid to buy the option, that is, in our example, when the underlying price goes above 85 dollar, given by $K + Premium = 80 + 5 = 85$.

IMPORTANT NOTE: The payoff showed you before and the canonical one shown in the books, is true **only at the expiring date**.
When facing with option, during their life, there are the so known "Greek letters" that can change sharply our position and our gains or losses, even blowing up position entailing a "margin call".

In the previous section we talked about the main factors that impact the price of an option, let's now explain why and how.

4.1 Strike price

When entering into an option, you can choose the strike price of the contract from among the various options available on the exchange.
The **lower** it is, in the case of a long call option, more convenient this will be to us and, therefore, **more expensive** the call option will be.
Why? Lowering the strike price will lead to "more room" for gains, since, by looking at the payoff previously wrote, we are increasing the gap between the underlying price S_0 and the strike price K. Let's show an example:

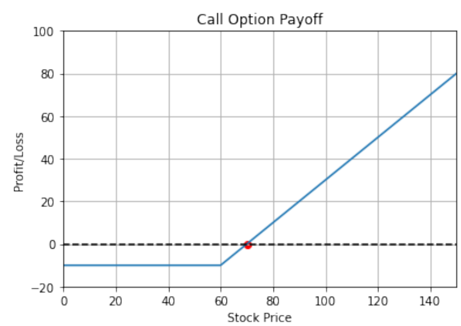

Figure 2: Payoff of a long Call option with K=60

In this case, we will exercise the option only if the price will be above $K + P = 60 + 10 = 70$, so we will be easier to be in gain, since we don't need anymore that the price goes above 85 as before, but only 70.

Conversely, if we choose an **higher** strike, this will not be convenient to us, so we are going to pay less for that option, since there's "smaller room" for gains.

From a mathematical point of view, we can say that the option price is **negatively related** to the strike Price.

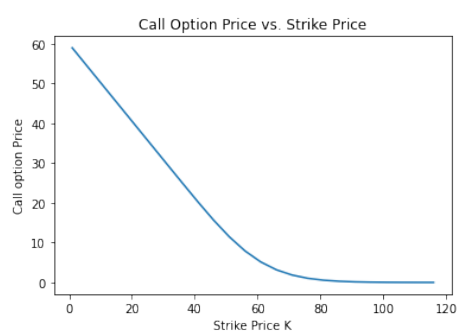

Figure 3: $S_0 = 60$, T = 1year, $\sigma = 20\%$, r= 3%

4.2 Volatility

Volatility is one of the most important factors when dealing with option, mostly during the life of the product. An **increase** of the volatility is interpreted as an increase of the fluctuation of the underlying price.

Let's suppose that the probability that the price goes up or down is 50% each.
If the price of the underlying goes **down**, it won't be good for us, but at least we know how much money we can lose.
If the price of the underlying goes **up**, we won't have any limits to the potential returns.
So the potential Risk/Reward is higher, since we can earn a potential unlimited amount of money and losing a fixed amount of money.
Therefore, an higher volatility will lead to something positive and we are going to pay more to enter in the contract.
Conversely, a smaller volatility, will lead to reduction of the probability that the underlying fluctuate, thus, it's negative and will decrease the option payoff.

From a mathematical point view, in the contrary of the strike price, the relationship between the option price and the volatility is **positively related**.

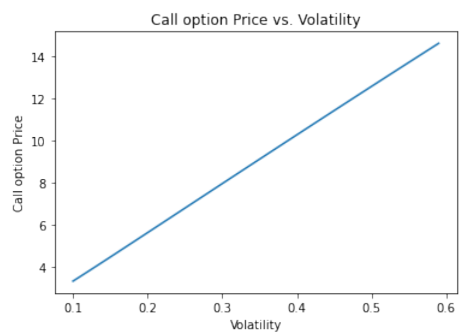

Figure 4: S_0 = 60, K = 60, T = 1year, r= 3%

4.3 Time to expiration

A premise is necessary. Option can be American or European. In the first case, them are exercisable at every moment during the life of the contract. Instead, in the European ones, them are exercisable only at the **expiring date**.

In the case of the **American** option, as the time to expiration increase, this will definitely lead to an increase of long call option, since we will have more opportunity to exercise the option rather than the one owning an option with a smaller time to expiration.
In the case of **the European** option, I'd like to report a simple example from the John Hull book.

Consider two European call options on a stock: one with an expiration date in 1 month, the other with an expiration date in 2 months.
Suppose that a very large dividend is expected in 6 weeks.
The dividend will cause the stock price to decline, so that the short-life option could be worth more than the long-life option.

From a mathematical point of view, the relationship between the call option price and the time to maturity, is positively related.

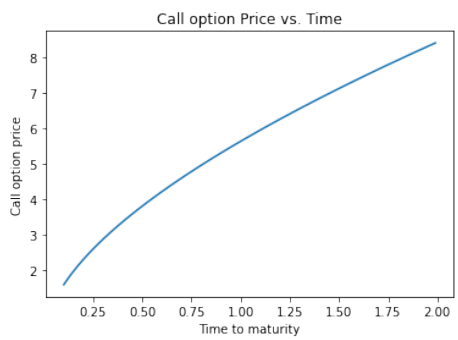

Figure 5: $S_0 = 60$, K = 60, sigma = 30%, r= 3%

4.4 Future dividends

Dividends have the effect of **reducing** the stock price on the ex-dividend date. This mean that this event will reduce the value of underlying, which is negative, since we are reducing the gap between the S_T and K.
Therefore, the value of a call option is negatively related to the size of the dividend.

4.5 Risk-Free Interest Rate

The impact of this factor is not so clear and depends in which way we consider it.
If we think about it ceteris paribus, thus if we assume that the interest rate change,while all the other variable stay the same, two things may happen:

1. As the risk-free interest rate increase, the expected return of the investors from the stocks tend to increase

2. The present value of any future cash flow received by the holder of the option, decreases.

This two, combined, will surely be beneficial and more convenient to the call option holder and therefore, will increase the price of the call.
However, in practice, an increase in interest rates typically leads to a decrease in stock prices, which in turn can negatively impact the option price, as previously discussed.

In conclusion, the impact of this factor depends on which of this two "forces" will be predominant. Sometimes a rise in the interest rates won't have a huge impact on the underlying price, but sometimes it may happen.

From a mathematical point of view, the price of a long call option is **positively related** with the interest rates.

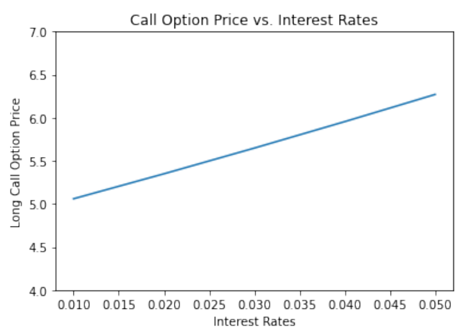

Figure 6: S_0 = 60, K = 60, sigma = 20%, T= 1year

5 Long PUT option

Let's define an option contract with a strike price $K = 80$ and let's draw a picture of a long put option, considering a premium, or a cost to enter in the position, of 5 dollars.

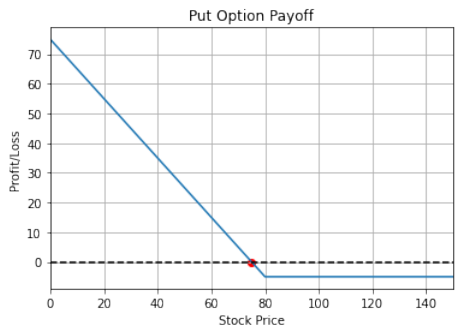

Figure 7: Payoff at the expiring date of a long Put option

In every book, the payoff at the expiring date* is expressed as

$$Max(K - S_T, 0)$$

Unfortunately, there's a **premium** that we have to pay to enter in the position, leading the payoff to be:

$$Max(K - (S_T + premium), -premium)$$

When entering in a put option, our **vision** is surely **bearish**.
Indeed, the only moment in which exercising the option would be convenient, is when the price of the underlying is smaller than what we paid to buy the option minus the strike price, that is, in our example, when the underlying price goes below 75 dollar, given by $K - Premium = 80 - 5 = 75$.

IMPORTANT NOTE: As in the case of the call option, the payoff we are facing now is **true at the expiring date**.

5.1 Strike Price

Conversely to what said in the option, this time, the **higher** the strike price is, more convenient this will be to us and, therefore, **more expensive** the put option will be.
Why? Choosing an higher strike price will lead to "more room" for gains, since, by looking at the payoff previously wrote, we are increasing the gap between the strike price K and the underlying price. Let's show an example:

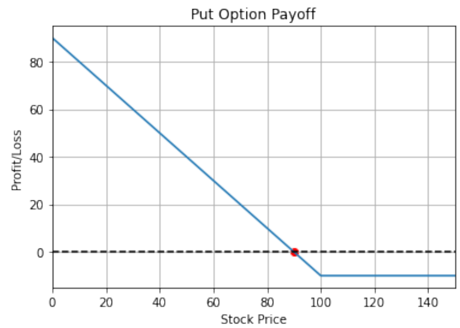

Figure 8: Payoff of a long put option with K=100

In this case, we will exercise the option only if the underlying price will be below $K - P = 100 - 10 = 90$, so will be easier to be in gain, since we don't need anymore that the price goes below 75 as before, but at least below 90.

Conversely, if we choose a **lower** strike, this will not be convenient to us, so we are going to pay less for that option, since there's "smaller room" for gains.

From a mathematical point of view, conversely to what said for the long call option, the long put option price has a negative relationship with the strike price.

Figure 9: $S_0 = 60$, T = 1, sigma = 20%, r= 3%

5.2 Volatility

For the long put option, is the same as in the long call option. An **increase** of the volatility is interpreted as an increase of the fluctuation of the underlying price.

Let's suppose that the probability that the price goes up or down is 50% each. If the price of the underlying goes **down**, it won't be good for us, but at least we know how much money we can lose.

If the price of the underlying goes **up**, we won't have any limits to the potential returns.
So the potential Risk/Reward is higher, since we can earn a potential unlimited amount of money and losing a fixed amount of money.
Therefore, an higher volatility will lead to something positive and we are going to pay more to enter in the contract.
Conversely, a smaller volatility, will lead to reduction of the probability that the underlying fluctuate, thus, it's negative and will decrease the option payoff.

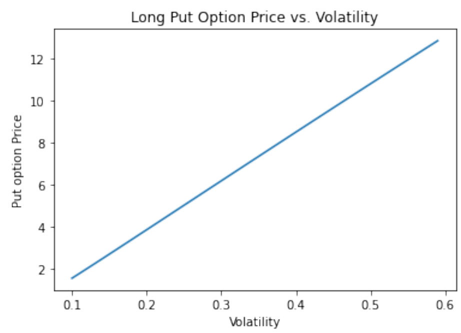

Figure 10: $S_0 = 60$, K = 60, T = 1, r= 3%

5.3 Future dividends

As we said before, dividends have the effect of **reducing** the stock price on the ex-dividend date. This mean that this event will reduce the value of underlying, which is positive, since we are increasing the gap between the K and the S_T.

5.4 Time to expiration

In the case of the **American** option, as the time to expiration increase, this will definitely lead to an increase of long put option, since we will have more opportunity to exercise the option rather than the one owning an option with a smaller time to expiration.
In the case of **the European** option, instead, this is not so immediate as already expressed for the long call option.

If during the life of the option there's a large dividend expectation, which is nice to remind that lead to a decrease in the underlying price when dealing with stocks*, this increase the value of the option.
If we pick again the same example as before, conversely we would have: Consider two European put options on a stock: one with an expiration date in 1 month, the other with an expiration date in 2 months.
Suppose that a very large dividend is expected in 6 weeks.

The dividend will cause the stock price to decline, so that the short-life option could be worth **less** than the long-life option.

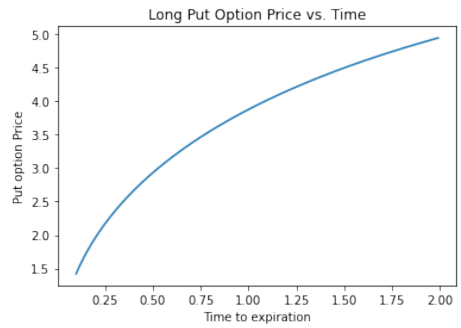

Figure 11: $S_0 = 60$, K = 60, $\sigma = 20\%$, r= 3%

5.5 Risk-Free Interest Rate

The impact of this factor is not so clear and depends in which way we consider it.
If we think about it ceteris paribus, thus if we assume that the interest rate change, while all the other variables stay the same, two things may happen:

1. As the risk-free interest rate increase, the expected return of the investors from the stocks tend to increase

2. The present value of any future cash flow received by the holder of the option, decreases.

This two, combined, will surely not be beneficial to the long put option holder and therefore, will decrease the option price.
However, in practice, a rise in the interest rates lead to a fall in the stock prices, that is good for the put holder, and lead to an increase in the option price.

In conclusion, the impact of this factor depends on which of this two "forces" will be predominant. Sometimes a rise in the interest rates won't have a huge impact on the underlying price, but sometimes it may happen.

From a mathematical point of view, the price of a long put option is **negatively related** with the interest rates.

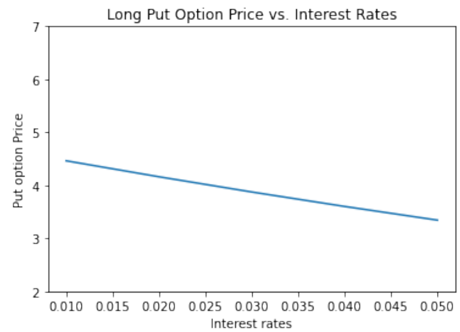

Figure 12: $S_0 = 60$, K = 60, sigma = 20%, T= 1year