PRIMITIVE DI DERIVATE DI FUNZIONI COMPOSTE / PRIMITIVE "ELEMENTARI" GENERALIZZATE

Titolo nota 29/12/2013

SIA $F(x)$ UNA PRIMITIVA DI $f(x)$ E SIA $g(x)$ UNA FUNZIONE DERIVABILE E TALE CHE SIA POSSIBILE COSTRUIRE LA FUNZIONE COMPOSTA $F(g(x))$. IN QUESTO CASO

$$\left[F(g(x))\right]' = f(g(x))\cdot g'(x) \quad \Longrightarrow \quad \int f(g(x))\, g'(x)\, dx = F(g(x)) + c$$

ESEMPIO 1

$$\int \underbrace{3x^2}_{g'(x)} \underbrace{\sin(x^3)}_{f(g(x))} dx = \underbrace{-\cos(x^3)}_{F(g(x))} + c$$

ESEMPIO 2

$$\int \underbrace{e^{\sin x}}_{f(g(x))} \underbrace{\cos x}_{g'(x)} dx = \underbrace{e^{\sin x}}_{F(g(x))} + c$$

ESEMPIO 3

RICORDANDO CHE $\int x^m dx = \frac{x^{m+1}}{m+1}$

$$\int x \underbrace{(x^2-1)^{2014}}_{f(g(x))} dx = \frac{1}{2}\int \underbrace{2x}_{g'(x)} \underbrace{(x^2-1)^{2014}}_{f(g(x))} dx = \frac{1}{2}\,\frac{(x^2-1)^{2015}}{2015} + c = \frac{(x^2-1)^{2015}}{4030} + c$$

1 / 1

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

es $$\int x e^x dx = xe^x - \int e^x dx = xe^x - e^x + c$$

es
$$\begin{aligned}\int x^4 e^x dx &= x^4 e^x - \int 4x^3 e^x dx = \\ &= x^4 e^x - 4x^3 e^x + \int 12x^2 e^x dx = \\ &= x^4e^x - 4x^3 e^x + 12x^2 e^x - \int 24x e^x dx = \\ &= x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24x e^x + \int 24 e^x dx = \\ &= x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24x e^x + 24 e^x + K\end{aligned}$$

$$\int \frac{3x+2}{x^2+1}\,dx = \frac{3}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{2}{x^2+1}\,dx$$

$$\Downarrow$$

$$\frac{3}{2}\log(x^2+1) + 2\arctg x + K$$

$$\int x \underset{\substack{\downarrow \\ FD}}{e^x}\,dx = xe^x - \int e^x\,dx = xe^x - e^x + K = e^x(x-1) + K$$

$$\int x^4 \underset{\substack{\downarrow \\ FD}}{e^x}\,dx = x^4e^x - \int 4x^3 \underset{\substack{\downarrow \\ FD}}{e^x}\,dx =$$

$$= x^4e^x - 4x^3e^x + \int 12x^2 e^x\,dx =$$

$$= x^4e^x - 4x^3e^x + 12x^2e^x - \int 24x e^x\,dx =$$

$$= x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + \int 24e^x\,dx =$$

$$= x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24e^x + K =$$

$$\int x^2 \log x\,dx = \log x \cdot \frac{x^3}{3} - \int \frac{1}{\cancel{x}}\frac{x^{\cancel{3}}}{3}\,dx =$$

$$= \log x\,\frac{x^3}{3} - \frac{x^3}{9} + K$$

$$= \frac{x^3}{3}\left(\log x - \frac{1}{3}\right) + K$$

$$\int \operatorname{sen} x\, e^x\,dx = \operatorname{sen} x\, e^x - \int \cos x\, e^x\,dx =$$

$$= \operatorname{sen} x\, e^x - \cos x\, e^x + \int -\operatorname{sen} x\, e^x\,dx =$$

$$= \operatorname{sen} x\, e^x - \cos x\, e^x - \int \operatorname{sen} x\, e^x\,dx =$$

$$\int \operatorname{sen} x\, e^x = \frac{1}{2}\left(\operatorname{sen} x\, e^x - \cos x\, e^x + K\right)$$

$$\int \cos^3 x \, dx = \int \cos x \cdot \cos^2 x \, dx =$$

$$= \int \cos x \left(1 - \operatorname{sen}^2 x\right) dx =$$

$$= \left[\int (1-t^2)\, dt\right]_{t = \operatorname{sen} x}$$

$$= \left[t - \frac{1}{3} t^3 + K\right]_{t = \operatorname{sen} x}$$

$$= \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + K$$

$$\int \frac{\operatorname{arctg}^2 x + 2 \operatorname{arctg} x}{1+x^2} dx = \left[\int t^2 + 2t \, dt\right]_{t = \operatorname{arctg} x}$$

$$= \left[\frac{t^3}{3} + t^2 + K\right]_{t = \operatorname{arctg} x}$$

$$= \frac{\operatorname{arctg}^3 x}{3} + \operatorname{arctg}^2 x + K$$

$$\int \frac{\log^3 x - \log x + 4}{x} dx = \left[\int t^3 - t + 4 \, dt\right]_{t = \log x}$$

$$= \left[\frac{t^4}{4} - \frac{t^2}{2} + 4t\right]_{t = \log x}$$

$$= \frac{\log^4 x}{4} - \frac{\log^2 x}{2} + 4 \log x$$

$$\int \sin^4 x \cos^2 x \, dx =$$

$$\int \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1+\cos 2x}{2}\right) dx =$$

$$\frac{1}{8} \int \left(1 + \cos^2 2x - 2\cos 2x\right)\left(1+\cos 2x\right) dx =$$

$$\frac{1}{8} \int 1 + \cos^2 2x - 2\cos 2x + \cos 2x + \cos^3 2x - 2\cos^2 2x \; dx =$$

Integrale di funzione raz. fratte

$$\int \frac{3x+1}{x^2+x}\,dx$$

$$\frac{A}{x} + \frac{B}{(x+1)} = \frac{Ax+A+Bx}{x(x+1)}$$

$$\begin{cases} A+B=3 \\ A=1 \end{cases} \quad \begin{cases} A=1 \\ B=2 \end{cases}$$

$$I = \int \frac{dx}{x} + 2\int \frac{dx}{x+1} = \lg|x| + 2\lg|x+1| + K$$

② $$\int \frac{3x+1}{x^2-10x+25}\,dx$$

$$x^2-10x+25 = (x-5)^2$$

$$\frac{A}{(x-5)} + \frac{B}{(x-5)^2} = \frac{A(x-5)+B}{(x-5)^2}$$

$$\begin{cases} A=3 \\ -5A+B=1 \end{cases} \quad \begin{cases} A=3 \\ B=16 \end{cases}$$

$$\int \frac{3}{x-5}\,dx + \int \frac{16}{(x-5)^2}\,dx =$$

$$= 3\lg|x-5| - \frac{16}{x-5} + K$$

③ $\int \frac{3x+1}{x^2+x+3}\,dx$ $\Delta < 0$

Devo ottenere al numerat. la deriv. del denom.

$$3\int \frac{x+\frac{1}{3}}{x^2+x+3} = \frac{3}{2}\int \frac{2x+1}{x^2+x+3}\,dx - \frac{1}{2}\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{11}{4}}$$

$q = \frac{\sqrt{11}}{2}$

$$= \frac{3}{2}\log\left(x^2+x+3\right) - \frac{1}{2}\,\frac{2}{\sqrt{11}}\,\operatorname{arctg}\frac{x+\frac{1}{2}}{\frac{\sqrt{11}}{2}} + K$$

$$\int \frac{x-3}{(x-2)^2+9}\,dx = \frac{1}{2}\int \frac{2x-4-2}{(x-2)^2+9}\,dx =$$

$$D\left[x^2-4x+13\right] = 2x-4$$

$$= \frac{1}{2}\int \frac{2x-4}{(x-2)^2+9}\,dx + \int \frac{-2}{(x-2)^2+9}\,dx$$

$$= \frac{1}{2}\log\left((x-2)^2+9\right) - \frac{2}{3}\operatorname{arctg}\frac{x-2}{3}$$

$$I \int \frac{3x+5}{x^2+6x+8}\,dx$$

$$\Delta = 36-32=4 \ (>0)$$

$$x = \frac{-6 \pm 2}{2} = \begin{cases} \to -4 \\ \downarrow \\ -2 \end{cases}$$

$$\frac{3x+5}{x^2+6x+8} = \frac{A}{(x+2)} + \frac{B}{(x+4)} = \frac{Ax+4A+Bx+2B}{(x+2)(x+4)}$$

$$\begin{cases} A+B=3 \\ 4A+2B=5 \end{cases} \quad \begin{cases} A=3-B \\ 4(3-B)+2B=5 \end{cases} \quad \begin{cases} A=-\frac{1}{2} \\ B=\frac{7}{2} \end{cases}$$

$$I = -\frac{1}{2}\int \frac{dx}{(x+2)} + \frac{7}{2}\int \frac{dx}{(x+4)} = -\frac{1}{2}\log|x+2| + \frac{7}{2}\log|x+4| + K$$

$$I \int \frac{4x+1}{x^2-6x+9} \quad (x-3)^2$$

$$\Delta = 36-4(9) = 0$$

$$\frac{4x+1}{x^2-6x+9} = \frac{A}{(x-3)} + \frac{B}{(x-3)^2} = \frac{Ax-3A+B}{(x-3)^2}$$

$$\begin{cases} A=4 \\ B-3A=1 \end{cases} \quad \begin{cases} A=4 \\ B=13 \end{cases}$$

$$I = 4\int \frac{1}{x-3}\,dx + 13\int \frac{1}{(x-3)^2}\,dx$$

$$= 4\log|x-3| - \frac{13}{x-3} + K$$

$$\int \frac{2x+1}{x^2+2x+6}\,dx =$$

$\Delta = 4 - 4(6) = -20\ (<0)$

$\Downarrow$

Completamento dei quadrati

$$x^2+2x+6 = (x+1)^2+5$$

$\downarrow$

x^2+2x+1.

$$\int \frac{2x+1+1-1}{x^2+2x+6}\,dx \Rightarrow \int \frac{2x+2}{x^2+2x+6}\,dx - \int \frac{dx}{x^2+2x+6} =$$

$x^2+2x+6 = (x+1)^2+5$

$\downarrow$

$$D[x^2+2x+6] = 2x+2$$

$\downarrow$

Devo ottenere $2x+2$ al numeratore

$$\log(x^2+2x+6) - \frac{1}{\sqrt{5}}\,\text{arctg}\,\frac{x+1}{\sqrt{5}} + K$$

$$\int \frac{x-3}{(x-2)^2+9}\,dx = \frac{1}{2}\int \frac{2x-4-2}{(x-2)^2+9}\,dx = \frac{1}{2}\int \frac{2x-4}{(x-2)^2+9}\,dx - \int \frac{dx}{(x-2)^2+9}$$

$$= \frac{1}{2}\log\left((x-2)^2+9\right) - \frac{1}{3}\operatorname{arctg}\frac{(x-2)}{3}$$

$$D\left[x^2-4x+13\right] = 2x-4$$

Trovare f prim. di $\frac{2x-3}{x^2+4x+4}$ t.c. $f(1)=1$ $(\Delta=0)$

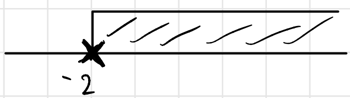
$$\frac{2x-3}{x^2+4x+4} = \frac{A}{x+2} + \frac{B}{(x+2)^2} = \frac{Ax+2A+B}{(x+2)^2}$$

$$\begin{cases} A=2 \\ 2A+B=-3 \end{cases} \qquad \begin{cases} A=2 \\ B=-7 \end{cases}$$

$$\int \frac{2x-3}{x^2+4x+4} = 2\int \frac{dx}{x+2} - 7\int \frac{dx}{(x+2)^2}$$

$]-2, +\infty[$

$$F(x) = 2\log|x+2| + \frac{7}{x+2} + K$$



$$F(1) = 2\log 3 + \frac{7}{3} + K = 1 \qquad K = -2\log 3 - \frac{4}{3}$$

$$2\log|x+2| + \frac{7}{x+2} - 2\log 3 - \frac{4}{3}$$

Trovare f prima di $\dfrac{x}{x^2+3x+4}$ t.c $f(0)=2$

$$\Delta = 9 - 16 = (<0) \qquad \left(x+\frac{3}{2}\right)^2 = x^2 + 3x + \frac{9}{4} = \left(x+\frac{3}{2}\right)^2 + \frac{7}{4}$$

$$\frac{1}{2}\int \frac{2x+3-3}{x^2+3x+4}\,dx = \frac{1}{2}\log|x^2+3x+4| - \frac{3}{\frac{\sqrt{7}}{2}} \operatorname{arctg} \frac{x+\frac{3}{2}}{\frac{\sqrt{7}}{2}} + K$$

$$F(x) = \frac{1}{2}\log|x^2+3x+4| - \frac{3}{\sqrt{7}} \operatorname{arctg} \frac{x+\frac{3}{2}}{\frac{\sqrt{7}}{2}} + K$$

$$\frac{1}{2}\log 4 - \frac{3}{\sqrt{7}} \operatorname{arctg} \frac{3}{\sqrt{7}} + K = 2$$

$$K = 2 - \log 2 - \frac{3}{\sqrt{7}} \operatorname{arctg} \frac{3}{\sqrt{7}}$$

$$\frac{1}{2}\log|x^2+3x+4| - \frac{3}{\sqrt{7}} \operatorname{arctg} \frac{x+\frac{3}{2}}{\frac{\sqrt{7}}{2}} + 2 - \log 2 - \frac{3}{\sqrt{7}} \operatorname{arctg} \frac{3}{\sqrt{7}}$$

$$I\int \frac{\operatorname{tg} x}{\operatorname{tg}^2 x - \operatorname{tg} x - 2}\,dx = \int (1+\operatorname{tg}^2 x)\frac{\operatorname{tg} x}{(1+\operatorname{tg}^2 x)(\operatorname{tg}^2 x - \operatorname{tg} x - 2)}\,dx$$

$$= \Bigg[\underbrace{\int \frac{t}{(1+t^2)(t^2-t-2)}\,dt}_{J}\Bigg]_{t=\operatorname{tg} x}$$

$$t^2 - t - 2 = 0 \qquad \Delta = 1 - 4(-2) = 9 \qquad \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

$$J = \frac{At+B}{1+t^2} + \frac{C}{t+1} + \frac{D}{t-2} =$$

$$\frac{(At+B)(t^2-t-2) + C(t^2+1)(t-2) + D(t^2+1)(t+1)}{(t^2+1)(t+1)(t-2)} =$$

$$\begin{cases} A = -\frac{9}{10} \\ C = \frac{1}{6} \\ D = \frac{2}{15} \\ B = -\frac{1}{10} \end{cases}$$

$$J = \int \frac{-\frac{9}{10}t - \frac{1}{10}}{t^2+1}\,dt + \int \frac{\frac{1}{6}}{t+1}\,dt + \int \frac{\frac{2}{15}}{t-2}\,dt =$$

$$= -\frac{9}{20}\int \frac{2t}{t^2+1}\,dt - \frac{1}{10}\int \frac{dt}{t^2+1} + \frac{1}{6}\int \frac{dt}{t+1} + \frac{2}{15}\int \frac{dt}{t-2} =$$

$$= -\frac{9}{20}\log(t^2+1) - \frac{1}{10}\operatorname{arctg} t + \frac{1}{6}\log|t+1| + \frac{2}{15}\log|t-2| + K$$

$$-\frac{9}{20}\log(\operatorname{tg}^2 x + 1) - \frac{1}{10}x + \frac{1}{6}\log(\operatorname{tg} x + 1) + \frac{2}{15}\log(\operatorname{tg} x - 2) + K$$

$$I = \int \frac{x+3}{(x^2+4)^2} \mathrm{d}x \qquad D(x^2+4) = 2x$$

$$I = \frac{1}{2}\int \frac{2x+6}{(x^2+4)^2}\,dx = \frac{1}{2}\left[\int \frac{2x}{(x^2+4)^2}\,dx\right]_{t=x^2+4} + 3\int \frac{dx}{(x^2+4)^2}$$

$$= -\frac{1}{2(x^2+4)} + 3\underbrace{\int \frac{dx}{(x^2+4)^2}}_{J}$$

$$J = \frac{3}{4}\int \frac{4+x^2-x^2}{(x^2+4)^2}\,dx = \frac{3}{4}\int \frac{\cancel{x^2+4}}{(x^2+4)^{\cancel{2}}}\,dx - \frac{3}{8}\int x\,\frac{2x}{(x^2+4)^2}\,dx$$

$$\downarrow$$

$$= \frac{3}{8}\operatorname{arctg}\frac{x}{2} + \frac{3}{8}\,\frac{x}{x^2+4} + K$$

$$I = -\frac{1}{2(x^2+4)} + \frac{3}{8}\operatorname{arctg}\frac{x}{2} + \frac{3}{8}\,\frac{x}{x^2+4} + K$$

$$\int x \arctan x \, dx, = \operatorname{arctg} x \cdot \frac{x^2}{2} - \frac{1}{2}\int \frac{x^2+1-1}{x^2+1}\,dx$$

$\downarrow$
FD

$$= \operatorname{arctg} x\,\frac{x^2}{2} - \frac{1}{2}\int \frac{x^2+1}{x^2+1}\,dx + \frac{1}{2}\int \frac{dx}{x^2+1}$$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{x}{2} + \frac{\operatorname{arctg} x}{2} + K$$

$$\int (x+2)\cos x\,\mathrm{d}x, = \int \underset{\text{FD}}{\underbrace{x\cos x}}\,dx + 2\int \cos x\,dx =$$

$$= x\,\text{sen}\,x - \int \text{sen}\,x\,dx + 2\,\text{sen}\,x =$$

$$= x\,\text{sen}\,x + \cos x + 2\,\text{sen}\,x + K$$

$$I = \int \frac{x+1}{x^3-6x^2+9x}\,\mathrm{d}x,$$

$$x\left(x^2-6x+9\right) = x(x-3)^2$$

$$\frac{x+1}{x^3-6x^2+9x} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

→ Mi son magnata la X

$$= \frac{Ax^2-6Ax+9A+Bx-3B+Cx}{x(x-3)^2}$$

$$\begin{cases} A=0 \\ -6A+B+C=1 \\ 9A-3B=1 \end{cases} \qquad \begin{cases} A=0 \\ B=-\frac{1}{3} \\ C=\frac{2}{3} \end{cases}$$

$$I = -\frac{1}{3}\int \frac{dx}{x-3} + \frac{2}{3}\int \frac{dx}{(x-3)^2}$$

$$= -\frac{1}{3}\log|x-3| - \frac{2}{3}\,\frac{1}{(x-3)}$$

$$\int (\sin^3 x)\,(\cos^4 x)\,\mathrm{d}x,$$

$$\int \operatorname{sen} x \;\operatorname{sen}^2 x \;(\cos^2 x)^2 \,dx = \left[\int (1 - t^2) \cdot t^4 \;dt\right]_{t=-\cos x}$$

$$= \left[\int t^4 - t^6\right]_{t=\cos x}$$

$$= \left[\frac{t^5}{5} - \frac{t^7}{7} + K\right]_{t=-\cos x}$$

$$-\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + K$$

$$I = \int \frac{2e^x+1}{e^x+3}\,dx = \int \frac{e^x}{e^x}\,\frac{2e^x+1}{e^x+3}\,dx = \left[\int \frac{2t+1}{t(t+3)}\,dt\right]_{t=e^x}$$

$$\frac{2t+1}{t(t+3)} = \frac{A}{t} + \frac{B}{t+3} = \frac{At+3A+Bt}{t(t+3)} \qquad \begin{cases} A+B=2 \\ 3A=1 \end{cases} \quad \begin{cases} B=\frac{5}{3} \\ A=\frac{1}{3} \end{cases}$$

$$I = \left[\frac{1}{3}\int \frac{1}{t}\,dt + \frac{5}{3}\int \frac{1}{t+3}\,dt\right]_{t=e^x}$$

$$= \left[\frac{1}{3}\log|t| + \frac{5}{3}\log|t+3|\;dt\right]_{t=e^x}$$

$$= \frac{x}{3} + \frac{5}{3}\log(e^x+3) + K$$

$$\int \frac{x^2 - x + 6}{x+1}\,dx = \int \frac{x^2 - x + 6 - 1 + 1}{x+1}\,dx$$

$$\int \frac{x^2 - 1}{x+1}\,dx - \int \frac{x-7}{x+1}\,dx$$

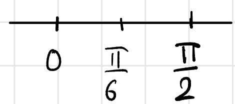
$$\int \frac{(x-1)(x+1)}{x+1}\,dx - \int \frac{x-7}{x+1}\,dx$$

$$\int x - 1\,dx - \int \frac{x+1-8}{x+1}\,dx$$

$$\frac{x^2}{2} - 2x + 8\log|x+1| + K$$

Determ f prim. di $\left|\operatorname{sen} x - \frac{1}{2}\right|$ in $\left[0, \frac{\pi}{2}\right]$ t.c. $f\left(\frac{\pi}{4}\right) = \frac{\pi}{24}$

$$f(x) \begin{cases} \operatorname{sen} x - \frac{1}{2} & \text{in } \left]\frac{\pi}{6}, \frac{\pi}{2}\right] \\ -\operatorname{sen} x + \frac{1}{2} & \text{in } \left[0, \frac{\pi}{6}\right] \end{cases}$$



$$F(x) \begin{cases} -\cos x - \frac{x}{2} + K' & \text{in } \left]\frac{\pi}{6}, \frac{\pi}{2}\right] \\ \cos x + \frac{x}{2} + c & \text{in } \left[0, \frac{\pi}{6}\right] \end{cases}$$

Impongo la continuità

$$\lim_{x \to \frac{\pi}{6}^-} F(x) = \lim_{x \to \frac{\pi}{6}^+} F(x)$$

$$-\frac{\sqrt{3}}{2} - \frac{\pi}{12} + K = \frac{\sqrt{3}}{2} + \frac{\pi}{12} + c$$

$$K = \sqrt{3} + \frac{\pi}{6} + c$$

$$F(x) \begin{cases} -\cos x - \frac{x}{2} + \sqrt{3} + \frac{\pi}{6} + c & \text{in } \left]\frac{\pi}{6}, \frac{\pi}{2}\right] \\ \cos x + \frac{x}{2} + c & \text{in } \left[0, \frac{\pi}{6}\right] \end{cases}$$

$$F\left(\frac{\pi}{4}\right) = \frac{\pi}{24}$$

$$-\cos\frac{\pi}{4} - \frac{\pi}{8} + \sqrt{3} + \frac{\pi}{6} + c = \frac{\pi}{24}$$

$$c = +\frac{\sqrt{2}}{2} + \frac{\pi}{8} - \sqrt{3} - \frac{\pi}{6} + \frac{\pi}{24}$$

la richiesta è

$$F(x) = \begin{cases} -\cos x - \frac{x}{2} + \sqrt{3} + \frac{\pi}{6} + \frac{\sqrt{2}}{2} + \frac{\pi}{8} - \sqrt{3} - \frac{\pi}{6} + \frac{\pi}{24} & \text{in } \left]\frac{\pi}{6}, \frac{\pi}{2}\right] \\ \cos x + \frac{x}{2} + c & \text{in } \left[0, \frac{\pi}{6}\right] \end{cases}$$

$$\int \frac{2x}{\sqrt{x-1}}\,dx$$

IP $f: (a,b) \Rightarrow \mathbb{R}$ dotata di primitive

$g: (c,d) \Rightarrow (a,b)$ su tutta, derivabile e invertibile

TS $\underline{\int f(x)\,dx = \left[\int f(g(t))\,g'(t)\,dt\right]_{t=g^{-1}(x)}}$

$$\begin{cases} \sqrt{x-1} \neq 0 \\ x-1 \geq 0 \end{cases} \Rightarrow \quad x > 1$$

1) $(a,b) = \left]1, +\infty\right[$

$g(t) \Rightarrow \quad \sqrt{x-1} = t \qquad t>0 \Rightarrow \quad x = t^2+1$

2) $g: [1, +\infty[\; \Rightarrow [1, +\infty[\quad \Rightarrow (c,d) = [1, +\infty[$

$\Downarrow$

contiene (a,b)

3) $g' = D[t^2+1] = 2t > 0 \quad$ se $t>0 \quad \Rightarrow$ deriv. in (a,b)

$\Downarrow$

monotona quindi invertibile

4) $g^{-1} = \sqrt{x-1}$

$$\int x^2 \cos 4x \, dx = \frac{1}{4}\int 4\, x^2 \cos 4x = \left[\frac{1}{4}\int \frac{t^2}{16}\cos t \, dt\right]_{t=4x}$$

$$D(4x) = 4$$

$$= \left[\frac{1}{64}\int t^2 \cos t \, dt\right]_{t=4x}$$

$$= \left[\frac{1}{64}\left(t^2 \operatorname{sen} t - \int 2t \operatorname{sen} t \, dt\right)\right]_{t=4x}$$

$$= \left[\frac{1}{64}\left(t^2 \operatorname{sen} t + 2t\cos t - 2\int \cos t \, dt\right)\right]_{t=4x}$$

$$= \left[\frac{1}{64}\left(t^2 \operatorname{sen} t + 2t\cos t - 2\operatorname{sen} t + K\right]_{t=4x}$$

$$= \frac{1}{64}\left(16x^2 \operatorname{sen} 4x + 8x\cos 4x - 2\operatorname{sen} 4x\right) + K$$

$$= \frac{x^2 \operatorname{sen} 4x}{4} + \frac{x\cos 4x}{8} - \frac{\operatorname{sen} 4x}{32} + K$$

$$\underset{\underset{FD}{\downarrow}}{\int (x-2)} \log \frac{x+3}{x}\, dx = \left(\frac{x^2}{2} - 2x\right)\log\frac{x+3}{x} - \int \frac{x^2-4x}{2}\, \frac{\cancel{x}}{x+3}\, \frac{\cancel{x}-\cancel{x}-3}{x^{\cancel{2}}} =$$

$$= \left(\frac{x^2}{2} - 2x\right)\log\frac{x+3}{x} + \frac{3}{2}\underbrace{\int \frac{x^2-4x}{x^2+3x}\, dx}_{J}$$

$$\frac{x^2-4x}{x^2+3x} = 1 - \frac{7}{x+3}$$

$$\Downarrow$$

$$J = \int\left(1 - \frac{7}{x+3}\right) dx = x - 7\log|x+3| + K$$

$$\int \frac{1}{\operatorname{tg}^2 x}\, dx = \int \frac{1+\operatorname{tg}^2 x}{\operatorname{tg}^2 x\,(1+\operatorname{tg}^2 x)}\, dx = \left[\int \frac{dt}{t^2(1+t^2)} \right]_{t = \operatorname{tg} x}$$

$$\frac{1}{t^2(1+t^2)} = \frac{A}{t} + \frac{B}{t^2} + \frac{Ct+D}{1+t^2}$$

$$\int \frac{2x}{\sqrt{x-1}}\, dx$$

1) $(a,b) = \left]1, +\infty\right[$

$g(t) \Rightarrow \sqrt{x-1} = t \quad t>0 \Rightarrow x = t^2+1$

2) $g : \left[1, +\infty\right[\Rightarrow \left[1, +\infty\right[\Rightarrow (c,d) = \left[1, +\infty\right[$

$\Downarrow$

contiene (a,b)

3) $g' = D[t^2+1] = 2t > 0$ se $t>0 \Rightarrow$ deriv. in (a,b)

$\Downarrow$

monotona quindi invertibile

4) $g^{-1} = \sqrt{x-1}$

$$\lim_{x\to 0} \frac{1}{x^3} \int_0^{\sin x} (e^{t^2} - 1)dt\,.$$

$$\frac{\int_0^{\sin x} (e^{t^2}-1)\,dt}{x^3} = \left[\frac{0}{0}\right]$$

Uso de l'Hospital per scrivere il rapporto delle derivate

$$\frac{\left(e^{\sin^2 x} - 1\right)\cos x}{3x^2} = \underbrace{\frac{e^{\sin^2 x} - 1}{\sin^2 x}}_{\downarrow \atop 1} \; \underbrace{\frac{\sin^2 x}{x^2}}_{\downarrow \atop 1} \; \underbrace{\frac{\cos x}{3}}_{\downarrow \atop \frac{1}{3}} \to \frac{1}{3}$$

17 Data la funzione definita dalla legge

$$G(x) = \int_1^{1+x^2} \sqrt{3+t^2}\mathrm{dt}$$

i) calcolarne la derivata prima;

ii) scrivere l'equazione della retta tangente al suo grafico nel punto di ascissa $x = 0$.

$$y = f(c) + f'(c)(x-c)$$

$$G(0) = 0$$

$$G'(x) = 2x\sqrt{3+(1+x^2)^2} \qquad G'(0) = 0$$

$y = 0$ eq. tangente

$$\int 1 \cdot \arctan(x)\, dx = x\arctan(x) - \frac{1}{2}\int \frac{2x}{x^2+1}\, dx$$

FD

$$= x\arctan(x) - \frac{1}{2}\log|x^2+1| + K$$

$$I = \int \frac{\log x + 3}{x(\log^3 x + \log^2 x)}\, dx = \left[\underbrace{\int \frac{t+3}{t^3 + t^2}\, dx}_{J} \right]_{t = \log x} =$$

$$t^3 + t^2 = t^2(t+1)$$

$$\frac{t+3}{t^3+t^2} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t+1} = \frac{At^2 + At + Bt + B + Ct^2}{t^2(t+1)}$$

$$\begin{cases} A + C = 0 \\ A + B = 1 \\ B = 3 \end{cases} \qquad \begin{cases} C = 2 \\ A = -2 \\ B = 3 \end{cases}$$

$$J = -2\int \frac{1}{t}\, dt + 3\int \frac{dt}{t^2} + 2\int \frac{dt}{t+1}$$

$$= -2 \log|t| - \frac{3}{t} + 2 \log|t+1| + K$$

$$I = -2 \log(\log x) - \frac{3}{\log x} + 2 \log(\log x + 1) + K$$

$f(x) = |x^2 - 4| - 2x + 3$ $\quad$ F prim in $[0, +\infty[$ $\quad$ t.c. $F(1) = 0$

$$f(x) = \begin{cases} x^2 - 2x - 1 & \text{in } [2, +\infty[\\ -x^2 - 2x + 7 & \text{in } [0, 2] \end{cases}$$

f è continua $\Rightarrow$ dotata di primitive

$$F(x) = \begin{cases} \dfrac{x^3}{3} - x^2 - x + K & \text{in } [2, +\infty[\\ -\dfrac{x^3}{3} - x^2 + 7x + c & \text{in } [0, 2[\end{cases}$$

Impongo la continuità in $x = 2$

$$\lim_{x \to 2^-} F(x) = \lim_{x \to 2^+} F(x)$$

$$-\frac{8}{3} - \cancel{4} + 14 + c = \frac{8}{3} - \cancel{4} - 2 + K$$

$$c = K + \frac{8}{3} + \frac{8}{3} - 14 - 2$$

$$K = c + \frac{32}{3}$$

$$F(x) = \begin{cases} \frac{x^3}{3} - x^2 - x + c + \frac{32}{3} & \text{in } [2, +\infty[\\ -\frac{x^3}{3} - x^2 + 7x + c & \text{in } [0, 2[\end{cases}$$

$$F(1) = -\frac{1}{3} - 1 + 7 + c = 0$$

$$\frac{-1 - 3 + 21}{3} + c = 0$$

$$c = \frac{17}{3}$$

$$F(x) = \begin{cases} \frac{x^3}{3} - x^2 - x + \frac{49}{3} & \text{in } [2, +\infty[\\ -\frac{x^3}{3} - x^2 + 7x + \frac{17}{3} & \text{in } [0, 2[\end{cases}$$

$$\int_0^2 |x^2 - 1| \, dx =$$

Trova una prim. in $[0, 2]$

$x^2 - 1 > 0$ per $x < -1 \vee x > 1$

$$\int_0^2 |x^2-1| \, dx = \int_0^1 1 - x^2 \, dx + \int_1^2 x^2 - 1 \, dx$$

$$= \left[x - \frac{x^3}{3} \right]_0^1 + \left[\frac{x^3}{3} - x \right]_1^2$$

$$= 1 - \frac{1}{3} + \frac{8}{3} - 2 - \frac{1}{3} + 1 = \frac{\cancel{3} - 1 + 8 - 6 \cancel{-1} \cancel{-3}}{3} = \frac{2}{3}$$

$$\int_0^{\frac{\pi}{2}} |2\operatorname{sen} x - 1| \, dx$$

$2\operatorname{sen} x - 1 = 0$

$\operatorname{sen} x = \frac{1}{2} \Leftrightarrow x = \frac{\pi}{6}$

$$\int_0^{\frac{\pi}{6}} 1 - 2\operatorname{sen} x \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\operatorname{sen} x - 1 \, dx$$

$$= \left[x + 2\cos x \right]_0^{\frac{\pi}{6}} + \left[2\cos x - x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{\pi}{6} + \cancel{2\frac{\sqrt{3}}{2}} - 2 - \frac{\pi}{2} - \cancel{2\frac{\sqrt{3}}{2}} + \frac{\pi}{6} = -\frac{\pi}{6} - 2$$

- Scrivere l'equaz. della tangente al grafico della funzione

$$F(x) = \int_2^x e^t \, dt \quad \text{nel punto } \underset{c}{x = 2}$$

$$\boxed{y = f(c) + f'(c)(x - c)}$$

$F(2) = 0$

Calcoliamo $F'(x) = e^x \rightarrow F'(2) = e^2$

allora $y = 0 + e^2(x - 2)$

eq. della tangente al grafico della funzione $F(x) = \int_1^{x^2} \frac{\operatorname{sen} t}{t} \, dt$ $\quad x=1$

$$y = f(c) + f'(c)(x-c)$$

$$F(1) = 0$$

$$F'(x) = \frac{\operatorname{sen} x^2}{x^2} \cdot 2x \quad \Rightarrow \quad F'(1) = 2\operatorname{sen}(1)$$

$$y = 2\operatorname{sen}(1)(x-1)$$

$$\int \frac{2x-1}{x^2+x+4} \, dx, \qquad \Delta < 0$$

$$= \int \frac{2x+1}{x^2+x+4} \, dx - 2 \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$= \log|x^2+x+4| - \frac{2}{\frac{\sqrt{3}}{2}} \operatorname{arctg} \frac{\left(x+\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}} + K$$

$$\int \frac{1}{\sqrt{x}+1}\,dx,$$

$(a,b) = [0, +\infty[$

$\sqrt{x} = t > 0$

$x = t^2 = g(t) :]-\infty,+\infty[\to [0+\infty[$

$g'(t) = 2t > 0$ monotona $\to$ invert.

$g^{-1}(t) = \sqrt{x}$

$$I = \left[\underbrace{\int \frac{2t}{t+1}\cdot dt}_{S}\right]_{t=\sqrt{x}}$$

$$\left[2\int \frac{t\,{+1-1}}{t+1}\cdot dt\right]_{t=\sqrt{x}}$$

$$\left[2t - 2\log|t+1| + K\right]_{t=\sqrt{x}}$$

$$2\sqrt{x} - 2\log(\sqrt{x}+1) + K$$

$$\int \cos(\log x)\,dx,$$

$(a,b) =]0, +\infty[$

$t = \log x \to x = e^t : \mathbb{R} \to]0,+\infty[$

$g'(t) = e^t \to$ invert.

$g^{-1}(t) = \log t$

$$= \left[\underbrace{\int \cos(t)\cdot e^t\,dt}_{S}\right]_{t=\log x}$$

$$= S = e^t\cos(t) - \int -\sin t\, e^t\,dt$$

$$= e^t\cos t + e^t\sin t - \int \cos t\, e^t\,dt =$$

$$= \frac{1}{2}\left(e^t\cos t + e^t\sin t\right) + K$$

$$I = [S]_{t=\log x} = \frac{1}{2}\left(e^{\log x}\cos(\log x) + e^{\log x}\sin(\log x)\right) + K$$

$$\int_1^4 \frac{|\log x - 1|}{x\log^2 x + x}\, dx,$$

$$\int \frac{\log x - 1}{x \log^2 x + x}\, dx = \left[\int \frac{t-1}{t^2+1}\, dt\right]_{t=\log x}$$

$$= \left[\frac{1}{2}\int \frac{2t}{t^2+1}\, dt\right]_{t=\log x} - \left[\int \frac{dt}{t^2+1}\right]_{t=\log x}$$

$$= \left[\frac{1}{2}\log|t^2+1| - \operatorname{arctg} x + K\right]_{t=\log x}$$

$$= \frac{1}{2}\log(\log^2(x)+1) - \operatorname{arctg}(\log(x)) + K$$

$$\Downarrow$$

$$\left[\frac{1}{2}\log(\log^2(x)+1) - \operatorname{arctg}(\log(x))\right]_0^4 = F(4) - F(0) = \ldots \in \mathbb{R}$$

$$\int \frac{\log(1+x)}{x^2}\,dx, \quad = -\frac{1}{x}\log(1+x) - \int -\frac{dx}{(1+x)x}$$

$$= -\frac{1}{x}\log(1+x) + \log|x| - \log|x+1| + K$$

$$J = \int \frac{dx}{(1+x)x} = \int \frac{dx}{x} - \int \frac{dx}{x+1} = \log|x| - \log|x+1| + K$$

$$\frac{A}{x} + \frac{B}{1+x} = \frac{Ax+A+Bx}{x(x+1)}$$

$$\begin{cases} A+B=0 \\ A=1 \end{cases} \begin{cases} A=1 \\ B=-1 \end{cases}$$

$$\int e^{\sqrt{x}}\,dx,$$

$(a,b) = [0,+\infty[$

$\sqrt{x} = t$

$x = t^2 \ge 0 \quad (c,d) : \mathbb{R} \to [0,+\infty[$

$g'(t) = 2t > 0 \to$ monotona $\to$ invertibile

$g^{-1}(t) = \sqrt{x}$

$$\left[\int e^t \cdot 2t\,dt\right]_{t=\sqrt{x}}$$

$\underset{FD}{\downarrow}$

$$= \left[2t e^t - 2\int e^t\,dt\right]_{t=\sqrt{x}}$$

$$= \left[2t \cdot e^t - 2e^t + K\right]_{t=\sqrt{x}}$$

$$= \left[2e^t(t-1) + K\right]_{t=\sqrt{x}}$$

$$= 2e^{\sqrt{x}}(\sqrt{x}-1) + K$$

$$\int_0^{\frac{3\pi}{4}} \frac{|\cos x|}{\sin^2 x + 2\sin x + 2}\,dx,$$

$$\downarrow$$

$$\int \frac{|\cos x|}{\sin^2 x + 2\sin x + 2}\,dx$$

$$f(x) = \begin{cases} \dfrac{-\cos x}{\sin^2 x + 2\sin x + 2} & \text{in } \left[\frac{\pi}{2}, \frac{3}{2}\pi\right] \\ \dfrac{\cos x}{\sin^2 x + 2\sin x + 2} & \text{altrimenti} \end{cases}$$

$$\left[\int \frac{\cos x}{\sin^2 x + 2\sin x + 2}\,dx\right]_{t=\sin x} = \left[\int \frac{dt}{t^2 + 2t + 2}\right]_{t=\sin x} = \left[\int \frac{1}{(t+1)^2+1}\,dt\right]_{t=\sin x} = \left[\arctan(t+1) + K\right]_{t=\sin x}$$

$$= \arctan(\sin(x) + 1) + K$$

$$F(x) = \begin{cases} -\arctan(\sin(x)+1) + K & \text{in } \left[\frac{\pi}{2}, \frac{3}{2}\pi\right] \\ \arctan(\sin(x)+1) + K & \text{altrimenti} \end{cases}$$

$$\Big[F(x)\Big]_0^{\frac{3}{4}\pi} = F\left(\tfrac{3}{4}\pi\right) - F(0) = \ldots$$

$$\int_0^2 \frac{|2x-3|}{x^2-5x+6}\,dx,$$

$$f(x) = \begin{cases} \dfrac{2x-3}{x^2-5x+6} & \text{in } \left[\frac{3}{2}, 2\right[\\ \dfrac{3-2x}{x^2-5x+6} & \text{in } \left]0, \frac{3}{2}\right] \end{cases}$$

$$\int \frac{2x-3-2+2}{x^2-5x+6}\,dx = \int \frac{2x-5}{x^2-5x+6}\,dx - 2\underbrace{\int \frac{dx}{x^2-5x+6}}_{J}$$

$$J = \int \frac{dx}{x^2-5x+6} = \int \frac{dx}{x-3} - \int \frac{dx}{x-2} = \log|x-3| - \log|x-2| + K$$

$$F(x) = \begin{cases} \log|x^2-5x+6| - 2\log|x-3| - 2\log|x-2| + K & \text{in } \left[\frac{3}{2}, 2\right[\\ -\log|x^2-5x+6| + 2\log|x-3| + 2\log|x-2| + K & \text{in } \left]0, \frac{3}{2}\right] \end{cases}$$

$$\left[F(x)\right]_0^2 = F(2) - F(0) = \ldots$$

Determinare la funzione F, primitiva della funzione nell'intervallo $]0, +\infty[$ della funzione definita dalla legge

$$f(x) = \frac{1}{e^x - 1}$$

e tale che $F(1) = \log(e-1)$

$$F(x) = \int \frac{1}{e^x - 1}\,dx = \int \frac{e^x}{e^x}\,\frac{1}{e^x - 1}\,dx$$

$$= \Big[\underbrace{\int \frac{dt}{t(t-1)}}_{I}\Big]_{t=e^x}$$

$$\int \frac{dt}{t(t-1)} = -\int \frac{dt}{t} + \int \frac{dt}{t-1} = -\log|t| + \log|t-1| + K$$

$$\frac{A}{t} + \frac{B}{t-1} = \frac{At - A + Bt}{t(t-1)} \qquad \begin{cases} A+B=0 \\ -A=1 \end{cases} \quad \begin{cases} A=-1 \\ B=1 \end{cases}$$

$$I = \Big[-\log|t| + \log|t-1| + K\Big]_{t=e^x} = -\log e^x + \log(e^x - 1) + K$$

$$F(1) = -\log(e) + \cancel{\log(e-1)} + K = \cancel{\log(e-1)}$$

$$K = 1$$

La funz. richiesta è $\underline{F(x) = -\log e^x + \log(e^x - 1) + 1}$

eq. della tangente al grafico della funzione $F(x) = \int_1^{x^2} \frac{\operatorname{sen}\tau}{\tau} d\tau \quad x=1$

$$y = f(c) + f'(c)(x-c)$$

$$F(1) = 0$$

$$F'(x) = \frac{\operatorname{sen} x^2}{x^2} \cdot 2x \quad \Rightarrow \quad F'(1) = 2\operatorname{sen}(1)$$

$$y = 0 + 2\operatorname{sen}(1)(x-1)$$

$\boxed{\textbf{16}}$ Calcolare il seguente integrale definito

$$\int_0^2 (x + |x-1|)\log(1+x)\mathrm{dx}.$$

$$\int (x + |x-1|)\log(1+x)\,dx$$

$$f(x) = \begin{cases} (2x-1)\log(1+x) & \text{in } [1, 2] \\ \log(1+x) & \text{in } [0, 1[\end{cases}$$

f è continua in $x=1$?

$$\lim_{x \to 1^-} f = \lim_{x \to 1^+} f$$

$\log(2) = \log(2) \Rightarrow f$ è cont in $x=1 \Rightarrow$ dotata di primit.

$$\int \underset{\Downarrow \, FD}{(2x-1)} \log(1+x) = \log(1+x)(x^2-x) - \int \frac{x^2-x}{x+1}\,dx$$

$$= \log(1+x)(x^2-x) - \frac{x^2}{2} - 2x + 2\log|x+1| + K$$

$$S = \int \frac{x^2-x}{x+1}\,dx = \int x-2\,dx + \int \frac{2}{x+1}\,dx$$

$$= \frac{x^2}{2} - 2x + 2\log|x+1| + K$$

$$\begin{array}{r|l} x^2 - x & x+1 \\ \hline -x^2 - x & x-2 \\ \hline / \; -2x & \\ +2x+2 & \\ \hline 2 & \end{array}$$

$$F(x) = \begin{cases} \log(1+x)(x^2-x) - \frac{x^2}{2} - 2x + 2\log|x+1| + K & \text{in } [1,2] \\ (1+x)\log(1+x) - (1+x) + C & \text{in } [0,1[\end{cases}$$

impongo la continuità

$$\lim_{x \to 1^-} F(x) = \lim_{x \to 1^+} F(x)$$

$$2 \cancel{\log 2} - 2 + c = -\frac{1}{2} - 2 + \cancel{2 \log 2} + K$$

$$K = c + \frac{1}{2}$$

$$F(x) = \begin{cases} \log(1+x)(x^2 - x) - \frac{x^2}{2} - 2x + 2\log|x+1| + c + \frac{1}{2} & \text{in } [1, 2] \\ (1+x)\log(1+x) - (1+x) + c & \text{in } [0, 1] \end{cases}$$

$$\Big[F(x)\Big]_0^2 = F(2) - F(0) = \ldots\ldots$$

$$\int \frac{2x}{\sqrt{x-1}}\,dx =$$

$$(a,b) =]1, +\infty[$$

$$t = \sqrt{x-1} \qquad t > 0$$

$$x = t^2+1 \qquad (c,d) = \mathbb{R}$$

$$g : (c,d) \to [1, +\infty[$$

$$g'(t) = 2t > 0 \Rightarrow \text{monotona} \Rightarrow \text{invertibile}$$

$$g^{-1} = \sqrt{x-1}$$

$$= \left[\int \frac{\cancel{2}(t^2+1)}{\cancel{t}} \cancel{2t}\,dt \right]_{t=\sqrt{x-1}}$$

$$\left[\frac{t^3}{3} + t \,dt \right]_{t=\sqrt{x-1}}$$

$$\frac{\left(\sqrt{x-1}\right)^3}{3} + \sqrt{x-1} + K .$$

$$\frac{1}{8}\int 8x \operatorname{arctg}(4x^2)\,dx = \left[\frac{1}{8} \int \operatorname{arctg}(t)\,dt \right]_{t=4x^2}$$

$$= \left[\frac{1}{8} \left(t \operatorname{arctg}(t) - \frac{1}{2} \log|t^2+1| + K \right) \right]_{t=4x^2}$$

$$= \frac{1}{8} \left(4x^2 \operatorname{arctg}(4x^2) - \frac{1}{2} \log(16x^4+1) \right) + K$$

$$\int \frac{1}{x\log x(1+\log^2 x)}\,dx = \underbrace{\left[\int \frac{dt}{t(1+t^2)}\right]_{t=\log x}}_{J} = \log(\log(x)) - \operatorname{arctg}(\log(x)) + K$$

$$J = \int \frac{1}{t(t^2+1)}\,dt = \int \frac{dt}{t} - \int \frac{dt}{t^2+1} = \log|t| - \operatorname{arctg} t + K$$

$$\frac{1}{t(t^2+1)} = \frac{A}{t} + \frac{Bt+C}{t^2+1} = \frac{At^2 + A + Bt^2 + Ct}{t\,(t^2+1)}$$

$$\begin{cases} A+B=0 \\ C=0 \\ A=1 \end{cases} \qquad \begin{cases} B=-1 \\ C=0 \\ A=1 \end{cases}$$

$$\int \frac{x-1}{1+\sqrt{x}}\,dx$$

$$\Downarrow$$

$$\left[\int \frac{t^2-1}{t+1}\,2t\right]_{t=\sqrt{x}}$$

$$\left[\int \frac{\cancel{(t+1)}(t-1)}{\cancel{t+1}}\,2t\,dt\right]_{t=\sqrt{x}}$$

$$\left[\int 2(t^2-t)\,dt\right]_{t=\sqrt{x}}$$

$$\left[\frac{2}{3}t^3 - t^2 + K\right]_{t=\sqrt{x}} = \frac{2}{3}(\sqrt{x})^3 - x + K$$

$(a,b) = [0, +\infty[$

$t = \sqrt{x} \qquad t>0$

$x = t^2 \qquad (c,d) = \mathbb{R}$

$g : \mathbb{R} \to [0, +\infty[$

$g' = 2t \quad > 0 \Rightarrow$ monotona $\Rightarrow$ invertibile

$g^{-1} = \sqrt{x}$

$$\int \frac{\tan x + 1}{(\tan x)\ (\tan^2 x + 1)}\,dx,$$

$$\int \frac{\tan x + 1}{(\tan x)\ (\tan^2 x + 1)} \frac{\tan^2 x + 1}{\tan^2 x + 1}\, dx = \Big[\underbrace{\int \frac{t+1}{t\ (t^2+1)(t^2+1)}\, dt}_{J} \Big]_{t = \tan(x)}$$

$$J = \int \frac{t+1}{t\ (t^2+1)(t^2+1)}\, dt =$$

$$\frac{t+1}{t\ (t^2+1)(t^2+1)} = \frac{A}{t} + \frac{Bt+C}{t^2+1} + \frac{Dt+E}{t^2+1} \quad - - - - - -$$

$$\int \frac{\log x + 1}{x(\log^2 x + 3)}\,dx, = \Big[\underbrace{\int \frac{t+1}{t^2+3}\, dt}_{J} \Big]_{t = \log x}$$

$$J = \frac{1}{2}\int \frac{2(t+1)}{t^2+3}\, dt = \frac{1}{2}\int \frac{2t}{t^2+3}\, dt + \int \frac{dt}{t^2+3} = \frac{1}{2}\log(t^2+3) + \sqrt{3}\,\operatorname{arctg}\frac{t}{\sqrt{3}} + K$$

$$D[t^2+3] = 2t$$

$$I = \Big[\frac{1}{2}\log(t^2+3) + \sqrt{3}\,\operatorname{arctg}\frac{t}{\sqrt{3}} + K \Big]_{t = \log x}$$

$$= \frac{1}{2}\log((\log(x))^2+3) + \sqrt{3}\,\operatorname{arctg}\frac{\log(x)}{\sqrt{3}} + K$$

$$\int \frac{x^5}{x^4-1}\, dx, = \int x + \frac{x}{\underset{S}{x^4-1}}\, dx = \frac{x^2}{2} - \frac{1}{4}\log(x^2+1) + \frac{1}{4}\log(x^2-1) + K$$

$$\begin{array}{r|l} x^5 & x^4-1 \\ \cline{2-2} -\; x^5 + x & x \\ \hline x & \end{array}$$

$$\frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2-1} = \frac{Ax^3-Ax+Bx^2-B+Cx^3+Cx+Dx^2+D}{(x^2+1)(x^2-1)}$$

$$\begin{cases} A+C=0 \\ B+D=0 \\ -A+C=1 \\ -B+D=0 \end{cases} \qquad \begin{cases} A=-\frac{1}{2} \\ B=0 \\ C=\frac{1}{2} \\ D=0 \end{cases}$$

$$S = \int \frac{x}{x^4-1}\, dx = -\frac{1}{2}\int \frac{x}{x^2+1}\, dx + \frac{1}{2}\int \frac{x}{x^2-1}\, dx$$

$$= -\frac{1}{4}\log(x^2+1) + \frac{1}{4}\log(x^2-1) + K$$

$$\int \frac{\sin x(\cos x + 1)}{\cos^3 x + \sin^2 x - 1}\, dx. \quad = -\int \frac{\sin x(\cos x + 1)}{\cos^3 x + 1 - \cos^2 x - 1}\, dx$$

$$\left[-\int \frac{t+1}{t^3 + \cancel{1} - t^2 - \cancel{1}}\, dt \right]_{t = \cos x} = \underline{-2 \log|\cos x| + \frac{1}{\cos x} + 2 \log|\cos x - 1| + K}$$

$$\mathcal{J}$$

$$\mathcal{J} = \int \frac{t+1}{t^2(t-1)}\, dt = -2 \log|t| + \frac{1}{t} + 2 \log|t-1| + K$$

$$\frac{A}{t} + \frac{B}{t^2} + \frac{C}{t-1} = \frac{At^2 - At + Bt - B + Ct^2}{t^2(t-1)}$$

$$\begin{cases} A + C = 0 \\ B - A = 1 \\ -B = 1 \end{cases} \qquad \begin{cases} C = 2 \\ A = -2 \\ B = -1 \end{cases}$$

$$\int (\cos^3 x)\,(\sin^6 x)\,\mathrm{d}x$$

$$\int \cos x \cdot \cos^2 x \cdot \sin^6 x \, dx = \int \cos x \cdot (1-\sin^2 x) \sin^6 x \, dx$$

$$= \left[\int (1-t^2)\, t^6 \, dt\right]_{t=\sin x}$$

$$= \left[\frac{1}{8} t^8 - \frac{1}{9} t^9 + K\right]_{t=\sin x}$$

$$= \frac{1}{8}\sin^8 x - \frac{1}{9}\sin^9 x + K$$

$$I = \int \frac{dx}{e^x + 2} = \int \frac{e^x}{e^x} \frac{dx}{e^x+2} = \left[\int \frac{dt}{t\,(t+2)}\right]_{t=e^x}$$

$$J = \int \frac{dt}{t\,(t+2)} = \frac{1}{2}\log|t| - \frac{1}{2}\log|t+2| + K$$

$$\frac{A}{t} + \frac{B}{t+2} = \frac{At + 2A + Bt}{t\,(t+2)}$$

$$\begin{cases} A + B = 0 \\ A = \frac{1}{2} \end{cases} \quad \begin{cases} B = -\frac{1}{2} \\ A = \frac{1}{2} \end{cases}$$

$$I = \frac{1}{2}\log(e^x) - \frac{1}{2}\log(e^x+2) + K$$

$\boxed{7}$ Determinare la funzione F, primitiva nell'intervallo $]0, \frac{\pi}{2}[$ della funzione definita dalla legge

$$f(x) = \frac{1}{\tan x + 1}$$

e tale che $F(\frac{\pi}{4}) = \frac{\pi}{8}$

$$I = \int \frac{1}{\operatorname{tg} x + 1} \cdot \frac{1+\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x} + K = \Big[\underbrace{\int \frac{dt}{(t+1)(t^2+1)}}_{J} \Big]_{t = \operatorname{tg} x}$$

$$\frac{1}{(t+1)(t^2+1)} = \frac{A}{t+1} + \frac{Bt+C}{t^2+1} = \frac{At^2 + A + Bt^2 + Bt + Ct + C}{(t+1)(t^2+1)}$$

$$\begin{cases} A+B=0 \\ B+C=0 \\ A+C=1 \end{cases} \qquad \begin{cases} C = \frac{1}{2} \\ B = -\frac{1}{2} \\ A = \frac{1}{2} \end{cases}$$

$$J = \frac{1}{2} \int \frac{dt}{t+1} - \frac{1}{2} \int \frac{t}{t^2+1} + \frac{\frac{1}{2}}{t^2+1} \, dt$$

$$= \frac{1}{2} \log|t+1| - \frac{1}{4} \log(t^2+1) + \frac{1}{4} \operatorname{arctg} t + K$$

$$F(x) = \frac{1}{2} \log|\operatorname{tg}(x)+1| - \frac{1}{4} \log(\operatorname{tg}(x)^2+1) + \frac{x}{4} + K$$

$$F\left(\frac{\pi}{4}\right) = \frac{1}{2} \log\left(\operatorname{tg}\left(\frac{\pi}{4}\right)+1\right) - \frac{1}{4} \log\left(\operatorname{tg}\left(\frac{\pi}{4}\right)^2+1\right) + \frac{\pi}{16} + K = \frac{\pi}{8}$$

- Trovo K e lo sostituisco a K nella primitiva generica

$$\int |x^2 - x| \, dx \qquad \text{in } [0, 2] \;|\; F\left(\frac{1}{2}\right) = \frac{1}{12}$$

$|\ldots| > 0$

$x(x-1) > 0 \iff x < 0 \lor x > 1$

$$f(x) = \begin{cases} -x^2 + x & \text{in } [0, 1] \\ x^2 - x & \text{in }]1, 2] \end{cases}$$

f è continua ?

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$0 = 0$ ✓ continua => dotata di primitive

$$F(x) = \begin{cases} -\frac{x^3}{3} + \frac{x^2}{2} + K & \text{in } [0, 1[\\ \frac{x^3}{3} - \frac{x^2}{2} + c & \text{in }]1, 2] \end{cases}$$

impongo la continuità

$$\lim_{x \to 1^-} F(x) = \lim_{x \to 1^+} F(x)$$

$$-\frac{1}{3} + \frac{1}{2} + K = \frac{1}{3} - \frac{1}{2} + c$$

$$c = K + \frac{1}{3}$$

$$F(x) = \begin{cases} -\frac{x^3}{3} + \frac{x^2}{2} + K & \text{in } [0,1] \\ \frac{x^3}{3} - \frac{x^2}{2} + K + \frac{1}{3} & \text{in }]1,2] \end{cases}$$

$$-\frac{x^3}{3} + \frac{x^2}{2} + K = \frac{1}{12}$$

$$-\frac{1}{24} + \frac{1}{8} + K = \frac{1}{2}$$

$$K = \frac{5}{12}$$

$$F(x) = \begin{cases} -\frac{x^3}{3} + \frac{x^2}{2} + \frac{5}{12} & \text{in } [0,1] \\ \frac{x^3}{3} - \frac{x^2}{2} + \frac{3}{4} & \text{in }]1,2] \end{cases}$$

$$f(x) = \begin{cases} \cos x & \text{se } x \geq 0 \\ x^2 + 1 & \text{se } x < 0 \end{cases}$$

$[-1, 1]$

$F\left(\frac{\pi}{6}\right) = \frac{3}{2}$

$$f(x) = \begin{cases} x^2 + 1 & \text{in } [-1, 0[\\ \cos x & \text{in } [0, 1] \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$0 = 0$$

$$F(x) = \begin{cases} \frac{x^3}{3} + x + K & \text{in } [-1, 0[\\ \operatorname{sen} x + c & \text{in }]0, 1] \end{cases}$$

impongo la cont.

$$\lim_{x \to 0^-} F(x) = \lim_{x \to 0^+} F(x)$$

$$K = c$$

$$F(x) = \begin{cases} \frac{x^3}{3} + x + c & \text{in } [-1, 0[\\ \operatorname{sen} x + c & \text{in } [0, 1] \end{cases}$$

$\operatorname{sen} \frac{\pi}{6} + c = \frac{3}{2}$ $\qquad c = 1$

$$F(x) = \begin{cases} \frac{x^3}{3} + x + 1 & \text{in } [-1, 0[\\ \operatorname{sen} x + 1 & \text{in } [0, 1] \end{cases}$$

$$f(x) = \begin{cases} 0 & \text{se} \quad x \leq 0 \\ x \arctan \frac{1}{x} & \text{se} \quad x > 0 \end{cases}$$

$F(0) = 2$

$$f(x) = \begin{cases} 0 &]-\infty, 0] \\ x \arctan \frac{1}{x} &]0, +\infty[\end{cases}$$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$ ✓ dotata di primitiva

$$F(x) = \begin{cases} K &]-\infty, 0[\\ & \end{cases}$$

$\int x \arctan \frac{1}{x} \, dx =$

$(a,b) = \mathbb{R} \setminus \{0\}$ $\quad (c,d) = \mathbb{R} \setminus \{0\} \to (a,b)$

$g(t) = \frac{1}{x}$

$x = \frac{1}{t}$ $\quad g'(t) = -\frac{1}{t^2} < 0$ monotona

$g^{-1}(t) = \frac{1}{x}$

$$\left[\int -\frac{1}{t^3} \operatorname{arctg}(t) \, dt \right]_{t = \frac{1}{x}}$$

$$= -\frac{1}{2t^2} \operatorname{arctg}(t) + \frac{1}{2} \underbrace{\int \frac{1}{(t^2+1)t^2} \, dx}_{S}$$

$$S = \int \frac{dt}{(t^2+1)t^2} = \int \frac{dt}{t^2} - \int \frac{dt}{t^2+1} = -\frac{1}{t} - \operatorname{arctg}(t) + K$$

$$\frac{1}{t^2(t^2+1)} = \frac{A}{t} + \frac{B}{t^2} + \frac{Ct+D}{t^2+1} = \frac{At^3 + At + Bt^2 + B + Ct^3 + Dt^2}{t^2\,(t^2+1)} =$$

$$= \frac{(A+C)t^3 + (B+D)t^2 + At + B}{t^2\,(t^2+1)}$$

$$\begin{cases} A + C = 0 \\ B + D = 0 \\ A = 0 \\ B = 1 \end{cases} \qquad \begin{cases} C = 0 \\ D = -1 \\ A = 0 \\ B = 1 \end{cases}$$

$$I = \left[-\frac{1}{2t^2}\,\mathrm{arctg}(t) - \frac{1}{2}\left(\frac{1}{t} + \mathrm{arctg}(t)\right) + K \right]_{y=\frac{1}{x}}$$

$$-\frac{1}{2x^2}\,\mathrm{arctg}\left(\frac{1}{x}\right) - \frac{1}{2}\left(x + \mathrm{arctg}\left(\frac{1}{x}\right)\right) + K$$

$$F(x) = \begin{cases} K &]-\infty, 0[\\ -\frac{1}{2x^2}\,\mathrm{arctg}\left(\frac{1}{x}\right) - \frac{1}{2}\left(x + \mathrm{arctg}\left(\frac{1}{x}\right)\right) + K &]0, +\infty[\end{cases}$$

impongo la continuità

$$\lim_{x \to 0^-} F(x) = \lim_{x \to 0^+} F(x)$$

$K = \ldots$ TO BE CONTINUED

$$\int_0^2 (x+|x-1|)\log(1+x)\mathrm{dx}. \Rightarrow I = \frac{3}{2} + 3\log 3 - 4\log 2 - 1 \in \mathbb{R}$$

$$= \int_0^1 (2x-1)\log(1+x)\,dx + \int_1^2 \log(1+x)\,dx$$

Trovo le primitive

$$\int (2x-1)\log(1+x)\,dx$$

$$= \left((x^2-x)\log(1+x) - \int \frac{x^2-x}{1+x}\,dx \right)$$

$$\left(- \text{-----} - \int x-2\,dx + 2\int \frac{dx}{x+1} \right)$$

$$(x^2-x)\log(1+x) + \frac{x^2}{2} - 2x + 2\log|x+1| + K$$

$x^2 - x$	$x+1$
$-x^2 - x$	$x-2$
$-2x$	
$+2x + 2$	
2	

$$\left[(x^2-x)\log(1+x) + \frac{x^2}{2} - 2x + 2\log|x+1| \right]_0^1 = (F(1) - F(0)) = \frac{3}{2} - 2\log 2$$

$$\int \log(1+x)\,dx = x\log(1+x) - \int \frac{x+1-1}{1+x}\,dx$$

$$= x\log(1+x) - x + \log|x+1| + K$$

$$\left[x\log(1+x) - x + \log|x+1| \right]_1^2 = F(2) - F(1) = 3\log 3 - 2\log 2 - 1$$

$\boxed{17}$ Data la funzione definita dalla legge

$$G(x) = \int_1^{1+x^2} \sqrt{3+t^2}\mathrm{dt}$$

i) calcolarne la derivata prima;

ii) scrivere l'equazione della retta tangente al suo grafico nel punto di ascissa $x = 0$.

i) $G'(x) = 2x\sqrt{3+(1+x^2)^2}$

ii) $y = G(c) + G'(c)(x-c)$

$= G(0) + G'(0)\,x$

$= 0 + 0 \cdot x = 0$

$\boxed{15}$ Determinare $F(x)$ primitiva in $]0, +\infty[$ della funzione definita dalla legge

$$f(x) = \frac{\sqrt{x+2}}{x}$$

e tale che $F(12) = 8$.

$f(x)$ continua $\Rightarrow$ ammette primitive

$F(x) = \int \frac{\sqrt{x+2}}{x}\,dx$

$(a,b) = \,]0, +\infty[$

$g(t) = \sqrt{x+2} \quad (t>0) \qquad (c,d) = [-2, +\infty[\to (a,b)$ ✓

$x = t^2 - 2$

$g'(t) = 2t > 0 \qquad g^{-1}(t) = \sqrt{x+2}$

$I = \left[\int \frac{t}{t^2-2}\, 2t\, dt\right]_{t=\sqrt{x+2}}$

$$J = 2\int \frac{t^2-2+2}{t^2-2} = 2x - \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + K$$

$$I = 2\sqrt{x+2} - \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\sqrt{x+2}}{\sqrt{2}} + K$$

$$F(12) = 8 \Rightarrow 2\sqrt{14} - \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\sqrt{14}}{\sqrt{2}} + K = 8$$

$$K = 8 - 2\sqrt{14} + \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\sqrt{14}}{\sqrt{2}}$$

$$F(x) = 2\sqrt{x+2} - \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\sqrt{x+2}}{\sqrt{2}} + 8 - 2\sqrt{14} + \frac{2}{\sqrt{2}} \operatorname{arctg} \frac{\sqrt{14}}{\sqrt{2}}$$

14 Determinare $F(x)$ primitiva in $[-1, 2]$ della funzione definita dalla legge

$$f(x) = |x-1| \arctan x$$

e tale che $F(0) = 0$.

$$f(x) = \begin{cases} (1-x)\arctan(x) & \text{in } [-1, 1[\\ (x-1)\arctan(x) & \text{in } [1, 2] \end{cases}$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \Rightarrow \text{continua} \Rightarrow \text{ammette primitiva}$$

Trova la primitiva

$$\int (1-x) \arctan x \, dx = \left(x - \frac{x^2}{2}\right) \arctan x + \frac{1}{2}\int \frac{x^2 - 2x + 1 - 1}{x^2 + 1} dx$$

$$= \left(x - \frac{x^2}{2}\right) \arctan x + \frac{1}{2}\left(x - \log\left(x^2+1\right)\right) + K$$

$$\int (x-1) \arctan(x) = \left(\frac{x^2}{2} - x\right) \arctan x - \frac{1}{2}\int \frac{x^2 - 2x + 1 - 1}{x^2+1} dx$$

$$= \left(\frac{x^2}{2} - x\right) \arctan x - \frac{1}{2}\left(x - \log\left(x^2+1\right)\right) + K$$

$$F(x) = \begin{cases} \left(x - \frac{x^2}{2}\right) \arctan x + \frac{1}{2}\left(x - \log\left(x^2+1\right)\right) + K & \text{in } [-1, 1[\\ \left(\frac{x^2}{2} - x\right) \arctan x - \frac{1}{2}\left(x - \log\left(x^2+1\right)\right) + c & \text{in } [1, 2] \end{cases}$$

impongo la cont. $\lim_{x \to 1^-} F(x) = \lim_{x \to 1^+} F(x)$

$$\frac{\pi}{8} + \frac{1}{2} - \frac{\log 2}{2} + K = -\frac{\pi}{8} - \frac{1}{2} + \frac{\log 2}{2} + c$$

$$c = \frac{\pi}{4} + 1 - \log 2 + K$$

$$F(x)=\begin{cases}\left(x-\frac{x^2}{2}\right)\arctan x+\frac{1}{2}\left(x-\log\left(x^2+1\right)\right)+K & [-1,1[\\ \left(\frac{x^2}{2}-x\right)\arctan x-\frac{1}{2}\left(x-\log\left(x^2+1\right)\right)+\frac{\pi}{4}+1-\log 2+K & [1,2]\end{cases}$$

$F(0)=0 \Rightarrow K=0$

$$\int_{-1}^{1}\frac{|x|}{x^3+8}dx$$

$$-\int_{-1}^{0}\frac{x}{x^3+8}dx+\int_{0}^{1}\frac{x}{x^3+8}dx$$

$$\int\frac{x}{x^3+8}dx=$$

$$=\int\frac{x}{\left(x^2-2x+4\right)(x+2)}dx$$

Ruffini - Paravini

	1	0	0	8
-2		-2	4	-8
	1	-2	4	0

$$\frac{x}{(x+2)(x-2)^2}=\frac{A}{x+2}+\frac{B}{x-2}+\frac{C}{(x-2)^2}=\frac{Ax^2-2Ax+4A+Bx^2-4Bx-4B+Cx+2C}{(x+2)(x-2)^2}$$

$$\begin{cases}A+B=0\\ -2A-4B+C=1\\ 4A-4B+2C=0\end{cases}\quad\begin{cases}A=-\frac{1}{2}\\ B=\frac{1}{2}\\ C=2\end{cases}$$

$$-\frac{1}{2}\int\frac{dx}{x+2}+\frac{1}{2}\int\frac{dx}{x-2}+2\int\frac{dx}{(x-2)^2}$$

$$=-\frac{1}{2}\log|x+2|+\frac{1}{2}\log|x-2|-\frac{2}{x-2}+K$$

$$=\left[-\frac{1}{2}\log|x+2|+\frac{1}{2}\log|x-2|-\frac{2}{x-2}\right]_{-1}^{0}+\left[-\frac{1}{2}\log|x+2|+\frac{1}{2}\log|x-2|-\frac{2}{x-2}\right]_{0}^{1}=$$

= ------ TO BE CONTINED

$$\int(\sin^4 x)(\cos^4 x)\,dx=\int\left(\frac{1-\cos 2x}{2}\right)^2\left(\frac{1+\cos 2x}{2}\right)^2dx$$

$$=\frac{1}{16}\int(1-2\cos 2x+\cos^2 2x)(1+2\cos 2x+\cos^2 2x)\,dx$$

$$=\frac{1}{16}\int$$ Svolgili i prodotti ...

$$\int \cos^2 x \cdot \operatorname{sen}^2 x \, dx \Rightarrow \int \frac{1+\cos(2x)}{2} \cdot \frac{1-\cos(2x)}{2} \, dx$$

$$= \frac{1}{4} \int \big(1+\cos(2x)\big) \cdot \big(1-\cos(2x)\big) \, dx$$

$$= \frac{1}{4} \int \left(1 - \frac{1+\cos(4x)}{2} \right) dx$$

$$= \frac{1}{4} \left(\int dx - \int \frac{1+\cos(4x)}{2} \, dx \right)$$

$$= \frac{1}{4} \left(\int dx - \frac{1}{2} \int dx + \frac{1}{4} \cdot \frac{1}{2} \int 4 \cos(4x) \, dx \right)$$

$$= \frac{1}{4} \left(\frac{x}{2} + \frac{1}{8} \operatorname{sen}(4x) + K \right)$$

$$\frac{x}{8} + \frac{1}{32} \operatorname{sen}(4x) + K$$

11 Determinare, se esistono, tutte le primitive nell'intervallo $[-1,1]$ della funzione definita dalla legge

$$f(x) = \begin{cases} \cos x & \text{se } x \geq 0 \\ x^2+1 & \text{se } x < 0 \end{cases}$$

e, fra esse, determinare la funzione F tale che $F(\frac{\pi}{6}) = \frac{3}{2}$

f è continua?

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$1 = 1$$

$$F(x) = \begin{cases} \operatorname{sen} x + K & \text{in }]0, 1] \\ \frac{x^3}{3} + x + c & \text{in } [-1, 0[\end{cases}$$

Impongo la continuità

$$\lim_{x \to 0^-} F(x) = \lim_{x \to 0^+} F(x)$$

$$c = K$$

$$F(x) = \begin{cases} \operatorname{sen} x + c & \text{in } [0, 1] \\ \frac{x^3}{3} + x + c & \text{in } [-1, 0[\end{cases}$$

$$F\left(\frac{\pi}{6}\right) = \frac{3}{2} \rightarrow \operatorname{sen}\frac{\pi}{6} + c = \frac{3}{2}$$

$$c = 1$$

$$F(x) = \begin{cases} \operatorname{sen} x + 1 & \ldots \\ \frac{x^3}{3} + x + 1 & \ldots \end{cases}$$

$$I = \int \frac{dx}{x^3+1} =$$

$$x^3+1 = (x+1)(x^2-x+1)$$

$$\Downarrow$$

$$\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\begin{array}{c|ccc|c} & 1 & 0 & 0 & 1 \\ -1 & & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

$$\frac{1}{x^3+1} = \frac{A}{x+1} + \frac{Bx+c}{x^2-x+1} = \frac{Ax^2-Ax+A+Bx^2+Bx+Cx+C}{(x+1)(x^2-x+1)}$$

$$\begin{cases} A+B=0 \\ -A+B+C=0 \\ A+C=1 \end{cases} \quad \begin{cases} B=-\frac{1}{3} \\ C=\frac{2}{3} \\ A=\frac{1}{3} \end{cases}$$

$$I = \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{3}\int \frac{x-2}{x^2-x+1}\,dx$$

$$= \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{6}\int \frac{2x-1-3}{x^2-x+1}\,dx$$

$$= \frac{1}{3}\int \frac{dx}{x+1} - \frac{1}{6}\left(\int \frac{2x-1}{x^2-x+1}\,dx - 3\int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}\right.$$

$$= \frac{1}{3}\log|x+1| - \frac{1}{6}\log|x^2-x+1| - \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{\left(x-\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}} + K$$

$$I = \int \underset{\substack{\downarrow \\ FD}}{\frac{1}{(x-3)^2}} \log(x+1)\, dx$$

$$FD: \; g'(x) = \frac{1}{(x-3)^2}$$

$$g(x) = -\frac{1}{x-3}$$

$$= -\frac{\log(x+1)}{x-3} - \underbrace{\int \frac{dx}{(x-3)(x-1)}}_{J}$$

$$J = \int \frac{dx}{(x-3)(x-1)} = \frac{1}{4}\log|x-3| - \frac{1}{4}\log|x-1| + K$$

$$\frac{1}{(x-3)(x-1)} = \frac{A}{x-3} + \frac{B}{x-1} = \frac{Ax + A + Bx - 3B}{(x-3)(x+1)}$$

$$\begin{cases} A+B=0 \\ A-3B=1 \end{cases} \quad \begin{cases} A = \frac{1}{4} \\ B = -\frac{1}{4} \end{cases}$$

$$I = -\frac{\log(x+1)}{x-3} + \frac{1}{4}\left(\log|x-3| - \log|x-1|\right) + K$$

$$\int \frac{x+4}{x^2-x-6}\,\mathrm{d}x,$$

$\Delta = 1 - 4(-6) = 25$

$x = \frac{1 \pm 5}{2} = \to 3$, $\downarrow -2$

$$\frac{x+4}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} = \frac{Ax+2A + Bx-3B}{(x-3)\quad(x+2}$$

$$\begin{cases} A+B=1 \\ 2A-3B=4 \end{cases} \quad \begin{cases} A = 1-B \\ 2-2B-3B=4 \end{cases} \quad \begin{cases} A = \frac{7}{5} \\ B = -\frac{2}{5} \end{cases}$$

$$\frac{7}{5}\int \frac{dx}{x-3} - \frac{2}{5}\int \frac{dx}{x+2} = \frac{7}{5}\lg|x-3| - \frac{2}{5}\lg|x+2| + k$$

$$\int_0^2 |x^2-1|\,dx =$$

prim in $[0,2]$

$$= -\int_0^1 x^2-1\,dx + \int_1^2 x^2-1\,dx =$$

Trovo le primitive $\Rightarrow \int x^2-1\,dx = \frac{1}{3}x^3 - x + K$

$$\left[-\frac{x^3}{3}+x\right]_0^1 + \left[\frac{x^3}{3}-x\right]_1^2 =$$

$$= -\frac{1}{3} \cancel{+1} + \frac{8}{3} \cancel{-2} - \frac{1}{3} \cancel{+1} = 2$$

$$\int_0^{\frac{\pi}{2}} |2\operatorname{sen}x-1|\,dx = -\int_0^{\frac{\pi}{6}} 2\operatorname{sen}x-1\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\operatorname{sen}x-1\,dx$$

Trovo le primitive

$$\int 2\operatorname{sen}x-1\,dx = -2\cos x - x + K$$

$$\Big[2\cos x + x\Big]_0^{\frac{\pi}{6}} + \Big[-2\cos x - x\Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} =$$

$$= \frac{\pi}{6} + \sqrt{3} - 2 - \frac{\pi}{2} + \sqrt{3} + \frac{\pi}{6}$$

---

eq. della tangente al grafico della funzione $F(x) = \int_1^{x^2} \frac{\operatorname{sen}t}{t}\,dt \quad x=1$

$$y = F(1) - F'(1)(x-1)$$

$$F(1) = 0$$

$$F'(x) = 2x\frac{\operatorname{sen}x^2}{x^2} \qquad F'(1) = 2\operatorname{sen}1$$

$$y = -2\operatorname{sen}1\,(x-1)$$

• Trovare F primit. di $f(x)e^x$ t.c. $F(3)=6$

$$\int_{x_0}^{x} e^{t}\,dt$$ Una generica prim. nasce in $x_0 = 3$

$$\int_{3}^{x} e^{t}\,dt + 6$$

F prim di $|x^2-4|$ t.c. $F(3)=1$

$$\int_{3}^{x} |t^2-4|\,dt + 1 = \left[\frac{t^3}{3} - 4t\,dt\right]_3^x + 1$$

$$\Downarrow$$

$$F(x) - F(3) + 1$$

$$F(x) = \begin{cases} \frac{x^3}{3} - 4x + c1 & \text{in }]-\infty, -2[\\ 4x - \frac{x^3}{3} + c2 & \text{in } [-2, 2] \\ \frac{x^3}{3} - 4x + c3 & \text{in }]2, +\infty[\end{cases}$$

impongo la cont.

$$-\frac{8}{3}+8+c1=-8+\frac{8}{3}+c2 \Rightarrow c2=c1-\frac{16}{3}+16=c1+\frac{32}{3}$$

$$8-\frac{8}{3}+c1+\frac{32}{3}=\frac{8}{3}-8+c3 \Rightarrow c3=c1+16-\frac{16}{3}+\frac{32}{3}=$$

$$=c1+\frac{64}{3}$$

$$F(x)=\begin{cases} \frac{x^3}{3}-4x+c1 & \text{in }]-\infty,-2[\\ 4x-\frac{x^3}{3}+c1+\frac{32}{3} & \text{in } [-2,2] \\ \frac{x^3}{3}-4x+c1+\frac{64}{3} & \text{in }]2,+\infty[\end{cases}$$

$$F(3)=1 \qquad 9-12+c1+\frac{64}{3}=1 \Rightarrow \quad c1=4-\frac{64}{3}=-\frac{48}{3}$$

Tang. de $F(x) = \int_{1}^{\frac{2x}{\pi}} \frac{\cos(\pi t)}{t^2}\, dt$ $x=1$

$$y = F(1) - F'(1)\,(x-1)$$

$$F(1) = \int_{1}^{\frac{2}{\pi}} \frac{\cos(\pi t)}{t^2}$$

$$F'(x) = \frac{\cos 2x}{\left(\frac{2x}{\pi}\right)^2} \cdot \frac{2}{\pi} \qquad F'(1) = \frac{\cos 2}{\frac{2}{\pi}}$$

$$y = \int_{1}^{\frac{2}{\pi}} \frac{\cos(\pi t)}{t^2} - \frac{\cos 2}{\frac{2}{\pi}} \left(x-1\right)$$

$$F(x) = \int_{1}^{x^2} \frac{\sin\frac{\pi t}{2}}{t^3}\, dt$$

tg in $x=1$

$F(1) = 0$

$$F'(x) = \frac{\sin\frac{\pi x^2}{2}}{x^6} \cdot 2x = \frac{2\sin\frac{\pi x^2}{2}}{x^5}$$

$y = 2(x-1$

$F'(1) = 2$

8 Determinare la funzione F, primitiva della funzione nell'intervallo $]0,+\infty[$ della funzione definita dalla legge

$$f(x) = \frac{1}{e^x - 1}$$

e tale che $F(1) = \log(e-1)$

f cont. $\Rightarrow$ f ammette primit.

$$F(x) = \int \frac{dx}{e^x - 1} \cdot \frac{e^x}{e^x}$$

$$= \Bigg[\underbrace{\int \frac{dt}{(t-1)\,t}}_{J} \Bigg]_{t=e^x}$$

$$\frac{1}{(t-1)t} = \frac{A}{t-1} + \frac{B}{t} = \frac{At + Bt - B}{(t-1)t}$$

$$\begin{cases} A + B = 0 \\ -B = 1 \end{cases} \qquad \begin{cases} A = +1 \\ B = -1 \end{cases}$$

$$J = \int \frac{dt}{t-1} - \int \frac{dt}{t}$$

$$\overset{!}{=} \log|t-1| - \log|t| + K$$

$$I = \log(e^x - 1) - \log(e^x) + K$$

$F(1) = \log(e-1)$ $\qquad \cancel{\log(e-1)} - \log(e) + K = \cancel{\log(e-1)}$

$$K = \log(e)$$

$$F(x) = -\log(e^x - 1) + \log(e^x) + \log(e)$$

$$\int \frac{\log x + 4}{x(\log^2 x - 2\log x - 3)}\, dx,$$

$$I = \left[\int \frac{t + 4}{\underbrace{t^2 - 2t - 3}} \, dt \right]_{t = \log x}$$

$$t^2 - 2t - 3 = 0$$

$$\Delta = 4 - 4(-3) = 4 + 12 = 16$$

$$x = \frac{2 \pm 4}{2} = \begin{matrix} \rightarrow 3 \\ \downarrow \\ -1 \end{matrix}$$

$$\frac{t + 4}{t^2 - 2t - 3} = \frac{A}{t - 3} + \frac{B}{t + 1} = \frac{At + A + Bt - 3B}{(t - 3)(t + 1)}$$

$$\begin{cases} A + B = 1 \\ A - 3B = 4 \end{cases} \quad \begin{cases} A = 1 - B \\ -4B = 3 \end{cases} \quad \begin{cases} A = \frac{1}{4} \\ B = -\frac{3}{4} \end{cases}$$

$$J = \frac{1}{4}\int \frac{dt}{t-3} - \frac{3}{4}\int \frac{dt}{t+1}$$

$$= \frac{1}{4}\log|t-3| - \frac{3}{4}\log|t+1| + K$$

$$I = \frac{1}{4}\log|\log(x)-3| - \frac{3}{4}\log|\log(x)+1| + K$$

$$\int \frac{\tan x + 2}{\tan^2 x + 4}\,dx, = \int \frac{\tan x + 2}{\tan^2 x + 4} \cdot \frac{\overset{D(\tan(x))}{\tan^2 x + 1}}{\tan^2 x + 1}\,dx$$

$$= \left[\underbrace{\int \frac{t+2}{(t^2+4)(t^2+1)}\,dt}_{J}\right]_{t=\tan(x)}$$

$$\frac{t+2}{(t^2+4)(t^2+1)} = \frac{At+B}{t^2+4} + \frac{Ct+D}{t^2+1} = \frac{\text{-------}}{(t^2+4)(t^2+1)}$$

$$\int_0^2 (x+|x-1|)\log(1+x)\,dx$$

f è continua? Dom $f =]-1, +\infty[$

$\Rightarrow$ cont. in $[0,2] \Rightarrow$ ha primit.

$$|x-1| = \begin{cases} x-1 & \text{se } 1 \le x \le 2 \\ 1-x & \text{se } 0 \le x < 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} (2x-1)\log(1+x) & \text{se } 1 \le x \le 2 \\ \log(1+x) & \text{se } 0 \le x < 1 \end{cases}$$

$$\int_0^1 \log(1+x)\,dx + \int_1^2 (2x-1)\log(1+x)\,dx$$

Trovo le primitive

$$\int \log(1+x)\,dx = x\log(1+x) - \int \frac{x+1-1}{1+x}\,dx$$

$$= x\log(1+x) - x + \log|1+x| + K$$

$$I = \int (2x-1)\,\log(1+x)\,dx = (x^2-x)\log(1+x) - \underbrace{\int \frac{x^2-x}{1+x}\,dx}_{J}$$

$$\begin{array}{r|l} x^2 - x & x+1 \\ \cline{2-2} -x - x & x-2 \\ \cline{1-1} -2x & \\ +2x+2 & \\ \cline{1-1} 2 & \end{array} \qquad \frac{x^2-x}{x+1} = (x-2) + \frac{2}{x+1}$$

$$J = \int x-2\,dx + 2\int \frac{dx}{x+1}$$

$$= \frac{x^2}{2} - 2x + 2\log|x+1| + K$$

$$I = (x^2-x)\log(1+x) - \frac{x^2}{2} + 2x - 2\log|x+1| + K$$

$$F(x) = \begin{cases} x\log(1+x) - x + \log|1+x| + K_1 & \text{se } 0 \le x < 1 \\ (x^2-x)\log(1+x) - \frac{x^2}{2} + 2x - 2\log|x+1| + K_2 & \text{se } 1 \le x \le 2 \end{cases}$$

$$\Big[x\log(1+x) - x + \log|1+x|\Big]_0^1 +$$

$$+\Big[(x^2-x)\log(1+x) - \frac{x^2}{2} + 2x - 2\log|x+1|\Big]_1^2 =$$

$= \log 2 + \log 2 + \frac{1}{2} - 1 + 2\log 2 = 4\log 2 - \frac{1}{2}$

$\int \frac{x}{\sqrt{1-x^4}}\,dx,$ $= \frac{1}{2}\int \frac{2x}{\sqrt{1-(x^2)^2}}\,dx$

$= \frac{1}{2}\arcsin(x^2) + K$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + c$

$\int \sqrt{x}\arctan\sqrt{x}\,dx.$

$(a,b) = [0, +\infty[$

$t = \sqrt{x} \quad t>0$

$x = t^2 = g(t) \quad (c,d) = \mathbb{R}$

$\operatorname{Im} g = (a,b)$

$g' = 2t$

$g > 0 \Rightarrow \text{in}.$

$g^{-1} = \sqrt{x}$

$$I = \Big[\underbrace{2\int t^2 \arctan t \, dt}_{J}\Big]_{t=\sqrt{x}}$$

$$J = \frac{t^3}{3}\arctan(t) - \frac{1}{3}\int \frac{t^3}{t^2+1}\,dt$$

$$\begin{array}{l|l} t^3 & t^2+1 \\ \hline -t^3 \quad -t & t \\ \hline -t & \end{array} \Rightarrow t - \frac{t}{t^2+1}$$

$$\int t\,dt - \frac{1}{2}\int \frac{2t}{t^2+1}\,dt = \frac{t^2}{2} - \frac{1}{2}\log(t^2+1) + K$$

$$J = \frac{t^3}{3} \arctan(t) - \frac{1}{3}\left(\frac{t^2}{2} - \frac{1}{2}\log(t^2+1)\right) + K$$

$$I = \left[\frac{2}{3}t^3 \arctan(t) - \frac{t^2}{3} + \frac{1}{3}\log(t^2+1) + K\right]_{t=\sqrt{x}}$$

$$\overset{!}{=} \frac{2}{3}(x\sqrt{x}) \arctan(\sqrt{x}) - \frac{x}{3} + \frac{\log(x+1)}{3} + K$$