DEFINIZIONE f DIFFERENZIABILE

$f: X \to \mathbb{R}$ $\quad X \subseteq \mathbb{R}^2$ APERTO $\quad P_0 \in D(X)$

f SI DICE DIFFERENZIABILE IN P_0 SE $\exists\, l, m \in \mathbb{R}$:

$$\lim_{(h,k)\to(0,0)} \frac{\overbrace{f(x_0+h, y_0+k) - f(x_0,y_0)}^{\Delta f} - \overbrace{(lh+mk)}^{df}}{\sqrt{h^2+k^2}} = 0$$

$$df = f_x(x_0,y_0)h + f_y(x_0,y_0)k$$

TEOREMA SULLA DIFFERENZIABILITÀ $\Rightarrow$ COND. NECESSARIA

IP: $f: X \to \mathbb{R}$ $\quad X \subseteq \mathbb{R}^2$ APERTO $\quad P_0 \in D(X)$

f DIFFERENZIABILE IN P_0

TS: $\exists\, f_x(P_0) = l$

CONTROESEMPIO: $f(x,y) = \sqrt{|xy|}$
- CONTINUA IN $\mathbb{R}^2$
- $\exists\, f_x, f_y$ MA NON È DIFF.

$$\lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0)}{\sqrt{h^2}} = l$$

CONSIDERO $g(h,k) = \dfrac{f(x_0+h, y_0+k) - f(x_0,y_0) - (lh+mk)}{\sqrt{h^2+k^2}} = 0$ (PER IP.)

CONSIDERO $\displaystyle\lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0) - lh + lh}{|h|} = 0$

$$\lim_{h\to 0} \underbrace{\frac{f(x_0+h, y_0) - f(x_0,y_0) - lh}{|h|}}_{\downarrow\, 0} \cdot \underbrace{\frac{|h|}{h}}_{\pm 1} + \underbrace{\frac{lh}{h}}_{\downarrow\, l} = l$$

1 - NO $\quad y'' - xy = 0$

2 - SI $\quad y' - x^2y^2 = 0$

3 - NO $\quad y' = \log(xy)$

4 - SI $\quad y' - (\cos x)(\cos y) = 0$

5 - NO $\quad y' = \dfrac{y}{(x^2+3)(y^2-1)}$

6 - NO $\quad y' = \dfrac{y+xy}{1+y}$

7 - NO $\quad y' = \dfrac{y+xy}{1+xy}$

8 - NO $\quad y'' = \dfrac{1}{x^2y^2}$

$y' = x^2y^2 \qquad X(x) = x^2$

$\qquad\qquad\qquad Y(y) = y^2$

$y = (\cos x)(\cos y) \qquad X(x) = \cos x$

$\qquad\qquad\qquad Y(y) = \cos y$

DATE DUE FUNZIONI $X : (e, b) \to \mathbb{R}$

$Y : (e, d) \to \mathbb{R}$

SI DICE EQUAZIONE DIFFERENZIALE A VARIABILI SEPARABILI L'EQUAZIONE DIFFERENZIALE DEL PRIMO ORDINE IN CUI $F(x,y) = X(x)\,Y(y)$, OVVERO $y' = X(x)\,Y(y)$

ESSA È IL PROBLEMA DELLA RICERCA DI FUNZIONI $y : (\alpha, \beta) \to \mathbb{R}$: $i)$ y DERIVABILE IN (α, β)

$ii)$ $(\alpha, \beta) \subseteq (a, b)$ e $y(x) \in (c, d)$, $\forall x$

$iii)$ $y'(x) = X(x)\,Y(y(x))$, $\forall x \in (\alpha, \beta)$

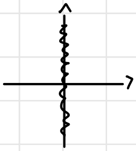
ES. DI VAR. SEP.

$y' = \dfrac{\sqrt{y^2+1}}{xy} \qquad X(x) = \dfrac{1}{x}$

$\qquad\qquad\qquad Y(y) = \dfrac{\sqrt{y^2+1}}{y}$

$f(x,y) = |x|(y^2+x)$ STUDIARE DER. PARZ. E DIFF. IN $\mathbb{R}^2$

$|0|$ NON E' DERIVABILE



$x>0$

xy^2+x^2

$x<0$

$-xy^2-x^2$

$f_x(x,y) = y^2+2x$

$f_y(x,y) = 2xy$

$f_x(x,y) = -y^2-2x$

$f_y(x,y) = -2xy$

STUDIO ASSE $\vec{x}$ $(y=0)$

$\exists f_x(a,0)$? $g(x) = f(x,0) = |x| \cdot x$ $\begin{cases} x^2 & x>0 \\ -x^2 & x<0 \end{cases}$

$x^2 = -x^2 \iff x=0$

$\nexists f_x$ SE $x \neq 0$

$\exists f_y(a,0)$? $h(y) = f(e,y) = |a|(y^2+a)$

NON POSSO APPLICARE IL TEOREMA DEL DIFF. TOTALE

$$\lim_{(h,k)\to(0,0)} \frac{|h|(k^2+h)}{\sqrt{h^2+k^2}} \to 0$$

$\to 0$ ≤ 1

f E' DIFF. PER DEFINIZIONE

$$\int_{-1}^{3} |x| \log(x^2+1)\,dx$$

PROPRIETÀ ADDITIVA

$$\int_{-1}^{0} -x\log(x^2+1)\,dx + \int_{0}^{3} x\log(x^2+1)\,dx$$

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SOSTITUZIONE

$$x^2+1=t \Rightarrow x=\sqrt{t-1}$$

$$dx = \frac{1}{2\sqrt{t-1}}$$

$\frac{1}{2} g(x) \cdot g'(x)$

$$\int_{-1}^{0} -\sqrt{t-1}\cdot \log(t)\,dt$$

INTEGRAZIONE PER PARTI

PERCHÉ PRODOTTO DI FUNZIONI

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$f(t) = \log t \qquad g'(t) = 1$

$f'(t) = \frac{1}{t} \qquad g(t) = t$

$$= -\frac{1}{2}\left[\log t \cdot t - \int \frac{1}{t}\cdot t\,dt\right] =$$

$$= \left[-\frac{1}{2}\log t\cdot t + \frac{1}{2}t + c\right]_{-1}^{0}$$

$$= \left[-\frac{1}{2}\log(x^2+1)\cdot(x^2+1) + \frac{1}{2}(x^2+1)\right]_{-1}^{0}$$

$$\int_{0}^{3} x\log(x^2+1)\,dx$$

SOSTITUISCO PURE QUI

$$x^2+1=t \Rightarrow x=\sqrt{t-1}$$

$$dx = \frac{1}{2\sqrt{t-1}}\,dt$$

$$\int_{0}^{3} \sqrt{t-1}\cdot\frac{1}{2\sqrt{t-1}}\cdot\log(t)\,dt = \frac{1}{2}\int \log(t)\,dt$$

INT. PER PARTI

$$\frac{1}{2}\left[\log t\cdot t - \int 1\,dt\right] = \left[\frac{1}{2}\log t\cdot t - t\right]_{0}^{3} =$$

$$= \left[\frac{1}{2}\log(x^2+1)(x^2+1) - \frac{1}{2}(x^2+1)\right]_{0}^{3}$$

$$\left[-\frac{1}{2}\log(x^2+1)\cdot(x^2+1) + \frac{1}{2}(x^2+1)\right]_{-1}^{0} + \left[\frac{1}{2}\log(x^2+1)(x^2+1) - \frac{1}{2}(x^2+1)\right]_{0}^{3} =$$

$$= \left[-\frac{1}{2}\log(1)\cdot 1 + \frac{1}{2}\right] - \left[-\frac{1}{\cancel{2}}\log(2)\cdot(\cancel{2}) + \frac{1}{\cancel{2}}(\cancel{2})\right] + \left[\frac{1}{2}\log(10)\cdot 10 - \frac{1}{2}\cdot \overset{5}{\cancel{10}}\right] -$$

$$\left[\frac{1}{2}\log(1)\cdot 1 - \frac{1}{2}\right] = \left[-\frac{1}{2}\cdot 0 + \frac{1}{2}\right] - \left[-\log(2)+1\right] + \left[\frac{1}{2}\cdot \overset{5}{10} - 5\right] - \left[-\frac{1}{2}\right] =$$

$$= \frac{1}{2} + \log(2) - 1 + \frac{1}{2} = \log(2)$$