

RESTRIZIONI (VEDI pure "FOGLIO 5")

IN GENERALE, $E_m = \{(x,y) \in \mathbb{R}^2 : y = mx\}$ $m \in \mathbb{R}$

$f(x) = f(x, mx)$, SI FA $\lim_{x \rightarrow 0} 0$, SE ESSO DIPENDE DA m . È IL LIMITE

$$\frac{xy}{x^2+y^2} \quad h(x) = \frac{mx^2}{x^2+m^2x^2} = \frac{m}{1+m} \quad \text{DIPENDE DA } m$$

SE IL LIMITE NON DIPENDE DA m ALLORA IL LIMITE POTREBBE ESSERE LUI

$$f(x,y) = \frac{x^2}{x^2+y^2} \quad h(x) = f(x, mx) = \frac{mx^3}{x^2+m^2x^2} = \frac{m}{1+m} \quad x \rightarrow 0 \Rightarrow \text{FORSE IL LIMITE È } \neq 0$$

$0 \leq |f(x,y)| \leq \text{QUALESOA CHE TENDE A } 0$

$$0 \leq |f(x,y)| \leq \frac{x^2}{x^2+y^2} \quad \begin{matrix} |y| \\ \downarrow 0 \end{matrix} \rightarrow 0 \quad \Rightarrow \lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$$

$$f(x,y) = \frac{xy}{\sqrt[3]{x^2+y^2}} \quad h(x, mx) = \frac{mx^2}{\sqrt[3]{x^2+m^2x^2}} = \frac{m x^2}{\sqrt[3]{(1+m^2)x^2}} = \frac{m}{\sqrt[3]{1+m^2}} x^{2-\frac{1}{3}} \rightarrow 0 \quad \text{IL LIMITE POTREBBE ESSERE } 0$$

$$0 \leq |f(x,y)| \leq \frac{|x|}{\sqrt[3]{x^2+y^2}} \quad \begin{matrix} |y| \\ \downarrow 0 \\ \leq 1 \end{matrix} \cdot \frac{\sqrt[3]{x^2+y^2}}{\sqrt[3]{x^2+y^2}} \rightarrow 0 \quad \lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$$

DERIVATE PARZIALI

$$f(x_1, y) = 2x^3y^2 - 2xy + 3x - 2y$$
$$g(x) = 2x^3(2^2) - 2x(2) + 3x - 2(2)$$
$$\uparrow$$
$$g(x) = f(x_1, y_0)$$

$$f_x(1, 2)$$

$$g(x) = 8x^3 - 4x + 3x - 4$$
$$g'(x) = 24x^2 - 4 + 3 = 24x^2 - 1$$
$$f_x(1, 2) = 23$$

$$f(x_1, y) = 6x^2y^2 - 3x + 4y^2 + x^3y$$
$$f_x(x_1, y) = 6y^2 - 3 + 0 + 3x^2y$$

IN UN GENERICO P SI DERIVA RISPETTO ALLA X
CON Y COSTANTE

$$f(x_1, y) = 2x^2y^2 - 3xy^2 + x^2y - 3x^4 + y$$
$$f_y(x_1, y) = 4x^2y - 6xy + x^2 - 0 + 1 = 4x^2y - 6xy + x^2 + 1$$

GRADIENTE

$$f(x_1, y) = \frac{2x^2y - 3x + y^2}{x+y}$$

$$\nabla f(2, 0)$$



$$f_x(x_1, y) = \frac{(4xy - 3)(x+y) - (2x^2y - 3x + y^2)}{(x+y)^2}$$

$$\nabla f = (f_x, f_y) = (0, 11/2)$$

$$f_y(x_1, y) = \frac{(2x^2 + 2y)(x+y) - (2x^2y - 3x + y^2)}{(x+y)^2}$$

DERIVATE SECONDE

$$f(x, y) = 2x^2y - 3xy^3$$

$$f_x = 4xy - 3y^3$$

$$f_y = 2x^2 - 9xy^2$$

$$f_{xy} = 4x - 9y^2$$

$$f_{yx} = 4x - 9y^2$$

$$f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

$$(x, y) \neq (0, 0)$$

$$\begin{aligned} f_x &= y \frac{x^2 - y^2}{x^2 + y^2} + xy \frac{2x(x^2 + y^2) - (x^2 - y^2)(2x)}{(x^2 + y^2)^2} = y \frac{x^2 - y^2}{x^2 + y^2} + xy \frac{2x^3 + 2xy^2 - 2x^2 + 2xy^2}{(x^2 + y^2)^2} = \\ &= \frac{x^2y - y^3}{x^2 + y^2} + \frac{4x^2y^3}{(x^2 + y^2)^2} = \frac{x^4 - y^5 + 4x^2y^3}{(x^2 + y^2)^2} \end{aligned}$$

$$\exists f_x(0, 0)? \quad g(x) = f(x, 0) = 0 \quad \forall x \Rightarrow f_x(0, 0) = 0$$

$$\exists f_{xy}(0, 0)? \quad h(y) = f_x(0, y) = -y \Rightarrow f_{xy}(0, 0) = -1$$

$$\begin{aligned} f_y &= x \frac{x^2 - y^2}{x^2 + y^2} + xy \frac{-2x^2y - 2y^3 - 2x^2y + 2y^3}{(x^2 + y^2)^2} = \frac{x^5 - xy^4 - 4x^3y^2}{(x^2 + y^2)^2} \quad - | \neq 1 \end{aligned}$$

$$\exists f_y(0, 0)? = 0$$

$$\exists f_{yx}(0, 0)? \quad h(y) = f_y(x, 0) = x \Rightarrow f_{yx}(0, 0) = 1$$

RESTRIZIONI

$$\lim_{(x,y) \rightarrow (1,0)} \frac{\sin((x-1)^2 + y^2)}{\sqrt{(x-1)^2 + y^2}}$$

- $E_1 = \{(x,y) : y = 0\}$

$$f|_{E_1} = \frac{\sin((x-1)^2)}{\sqrt{(x-1)^2}}$$

$$\lim_{(x,y) \rightarrow (1,0)} \frac{\sin((x-1)^2)}{\sqrt{(x-1)^2}} = \frac{\sin((x-1)^2)}{|(x-1)^2|} \cdot \frac{(x-1)^2}{\sqrt{(x-1)^2}} \xrightarrow{\substack{\sin t \approx t \\ t \rightarrow 0}} 1 \cdot |x-1| \xrightarrow{x \rightarrow 1} 0$$

- $E_2 = \{(x,y) : y = -x+1\}$

$$f|_{E_2} = \frac{\sin((x-1)^2 + (-x+1)^2)}{\sqrt{(x-1)^2 + (-x+1)^2}}$$

SE IL LIMITE ESISTE SARÀ 0

$$\lim_{(x,y) \rightarrow (1,0)} \frac{\sin((x-1)^2 + (-x+1)^2)}{(x-1)^2 + (-x+1)^2} \cdot \frac{(x-1)^2 + (-x+1)^2}{\sqrt{(x-1)^2 + (-x+1)^2}} \xrightarrow{\substack{(x-1)^2 + (-x+1)^2 \rightarrow 0 \\ \sin t \approx t}} 0$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^2+y^2}$$

$$E_m = \{(x,y) : y = mx\}$$

$$f|_{E_m} = \frac{x^2mx}{x^2+m^2x^2} = \frac{mx^3}{x^2(1+m^2)} = \frac{m}{1+m^2} x$$

$$\lim f|_{E_m} = 0 \quad \forall m$$

$$0 \leq |f(x,y)| \leq \frac{x^2}{x^2+y^2} |y|$$

PER IL TEOREMA DEL CONFRONTO IL LIMITE TENDE A 0

TEOREMA SULLA DIFFERENZIABILITÀ

$$1) f(x,y) = \sqrt{x^2+y^2} \text{ continua in } (0,0)$$

f continua ma non diff.
(SE LO FOSSE AUREBBERE LE DERIVATE)

$$f(x) = f(x,0) = |x| \not\in f_x(0,0)$$

$$f(y) \not\in f_y(0,0)$$

$$2) f(x,y) = \begin{cases} 1 & xy=0 \\ 0 & xy \neq 0 \end{cases}$$

f è der. ma non differenziabile
(SE LO FOSSE SAREBBERE CONTINUA)

$$f_x(0,0) = f_y(0,0) = 0$$

$$3) f(x,y) = \sqrt{|xy|} \quad \text{è cont. po}(0,0)$$

$$\exists f_x(0,0)? \quad g(x)=0 \Rightarrow f_x(0,0)=0 \quad df=0$$

$$f \text{ diff. in } (0,0)? \quad \frac{\Delta f - df}{\sqrt{h^2+k^2}} = \frac{\sqrt{|hk|}}{\sqrt{h^2+k^2}}$$

$$h=0 \rightarrow 0 \quad h=k \rightarrow \frac{1}{\sqrt{2}} > \lim \Rightarrow f \text{ non diff.}$$

DERIVATE DIREZIONALI

$$1) f(x,y) = 3x^2y - y + x^3$$

$$\begin{matrix} f_v(1,2) \\ v = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \end{matrix}$$

$$f_v(P_0) = \nabla f(P_0) \cdot v$$

$$\nabla f(x,y) = (6xy + 3x^2, 3x^2 - 1)$$

$$f_v(1,2) = (1) \cdot \frac{1}{\sqrt{2}} + (2) \cdot \frac{1}{\sqrt{2}} = \frac{14}{\sqrt{2}}$$

$$2) f(x,y) = \frac{3x^2y^2}{x^2+y}$$

$$\begin{matrix} P_0(1,1) \\ f_v(1,1) \end{matrix}$$

$$v = (2,1) \quad |v| = \sqrt{13}$$

NON E' UN VERSORE

$$y \quad f$$

COSTRUISSO IL VERSORE CHE RAPPRESENTA LA DIREZ.

$$\text{DI } v \Rightarrow w = \left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right)$$

$$y \neq -x^2$$

$$\nabla f(x,y) = \left(\frac{3x^2y^2 + 3y^3 - 6x^4y^2}{(x^2+y)^2}, \frac{6x^3y + 6x^2y^2 - 3xy^2}{(x^2+y)^2} \right) = \left(\frac{3y^3 - 3x^2y^2}{(x^2+y)^2}, \frac{6x^3y + 3xy^2}{(x^2+y)^2} \right)$$

$$f_w(1,1) = 0 \cdot \frac{2}{\sqrt{13}} + \frac{9}{4} \cdot \frac{3}{\sqrt{13}} = \frac{27}{4\sqrt{13}}$$

$$3) f(x,y) = \frac{2xy^3}{x^2+3y}$$

$$P_0(1,1) \quad f_v(1,1) \quad \text{DOVE } v \in \text{LA DIR. DELLA RETTA}$$

$$z: 2x - y + 3 = 0$$

$$z: ax + by + c = 0 \quad v \parallel z \quad v = (b, -a) \\ (-b, a)$$

$$v = (1, 2) \quad |v| = \sqrt{5} \neq 1 \\ w = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right)$$

$$\nabla f(x,y) = \left(\frac{-2x^2y^3 + 6y^4}{(x^2+3y)^2}, \frac{6x^3y^2 + 12x^2y^3}{(x^2+3y)^2} \right)$$

$$f_w(1,1) = \frac{1}{16} \cdot \frac{1}{\sqrt{5}} + \frac{9}{16} \cdot \frac{2}{\sqrt{5}} = \frac{1}{4\sqrt{5}} + \frac{9}{4\sqrt{5}} = \frac{10}{4\sqrt{5}} = \frac{5}{2\sqrt{5}} = \frac{\frac{1}{2}\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{5}}{2}$$

$$4) f(x,y) = 2x^2y + 3xy^3$$

$$\begin{aligned} & f_{xx}(2,1) \quad 2x+3y-2=0 \\ & V(3,-1) \quad |V| = \sqrt{10} \quad W = \left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right) \end{aligned}$$

$$\Rightarrow f(x,y) = (4xy + 3y^3, 2x^2 + 9xy^2)$$

$$f_W(2,1) = \frac{33}{\sqrt{10}} - \frac{26}{\sqrt{10}} = \frac{7}{\sqrt{10}}$$

Ricerca estremi relativi

$$1) f(x,y) = 2x^2 + xy^2 + x + \frac{1}{2}y^2 \quad X = \mathbb{R}^2$$

$$f_x = 4x + y^2 + 1 \quad f_y = 2xy + y$$

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{cases} 4x + y^2 + 1 = 0 \\ 2xy + y = 0 \end{cases} \Rightarrow \begin{cases} 4x = -y^2 - 1 = 0 \\ y(2x + 1) = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{-y^2 - 1}{4} \\ y(y^2 - 1) = 0 \end{cases}$$

$$\begin{cases} x = -1/4 \\ y = 0 \end{cases}; \quad \begin{cases} x = -1/2 \\ y = 1 \end{cases}; \quad \begin{cases} x = -1/2 \\ y = -1 \end{cases} \quad P_1(-\frac{1}{4}, 0) \quad P_2(-\frac{1}{2}, 0)$$

$P_3(-\frac{1}{2}, -1)$ PUNTI STAZIONARI

$$f_{xx} = 4 \quad f_{xy} = 2y \quad f_{yy} = 2x + 1$$

$$H(x,y) = f_{xx}f_{yy} - f_{xy}^2$$

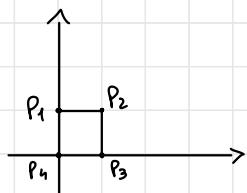
$$H(P_1) = 2 - 0 > 0 \quad f_{xx}(P_1) > 0 \Rightarrow P_1 \text{ PUNTO MIN. REL.}$$

$$H(P_2) = 0 - 4 < 0 \Rightarrow P_2 \text{ PUNTO DI SELLA}$$

$$H(P_3) = 0 - 4 < 0 \Rightarrow P_3 \text{ PUNTO DI SELLA}$$

ESTREMI ASSOLUTI

$$f(x,y) = 2x^2 + xy - x - y \quad P_1(0,1) \quad P_2(1,1) \quad P_3(1,0) \quad P_4(0,0)$$



$$X_1 = \emptyset$$

$$f_y = x - 1$$

$$P(1, -3) \notin X$$

$$X_2 = \emptyset$$

STUDIO LA RESTRIZIONE A $X_3 = F(x)$

$$P_1 P_2 \quad y=1 \quad g(x) = f(x, 1) = 2x^2 - 1 \quad 0 \leq x \leq 1$$

$$g'(x) = 4x = 0 \quad \text{PER } x=0 \quad \text{PRENDO IN CONSIDERAZIONE SOLO } P_1 P_2$$

$$P_2 P_3 \quad x=1 \quad h(y) = f(1, y) = 1 \quad \text{PRENDO IN CONSIDERAZIONE SOLO } P_2 P_3$$

$$P_3 P_4 \quad y=0 \quad l(x) = 2x^2 - x \quad 0 \leq x < 1$$

$$l'(x) = 4x - 1 = 0 \quad x = \frac{1}{4} \quad P\left(\frac{1}{4}, 0\right)$$

$$P_1 P_4 \quad x=0 \quad m(y) = -y \quad 0 \leq y \leq 1 \quad \text{PRENDO } P_1 P_4$$

$$\begin{aligned} f(0,1) &= -1 \\ f(1,1) &= 3 \\ f(0,0) &= 0 \\ f(1,0) &= 1 \\ f\left(\frac{1}{4}, 0\right) &= -\frac{1}{8} \end{aligned}$$

$$\min_x f = -1 = f(0,1)$$

$$\max_x f = 3 = f(1,1)$$