

RESTRIZIONI (VEDI PURE "FOGLIO 5")

IN GENERALE, $E_m = \{(x, y) \in \mathbb{R}^2 : \boxed{y = mx}\}$ $m \in \mathbb{R}$

$h(x) = f(x, mx)$, SI FA $\lim_{x \rightarrow 0}$, SE ESSO DIPENDE DA m IL LIMITE

$$\frac{xy}{x^2+y^2} \quad h_x = \frac{mx^2}{x^2+m^2x^2} = \frac{m}{1+m^2} \quad \text{DIPENDE DA } m$$

SE IL LIMITE NON DIPENDE DA m ALLORA IL LIMITE POTREBBE ESSERE LUI

$$f(x, y) = \frac{x^2 y}{x^2 + y^2} \quad h(x) = f(x, mx) = \frac{mx^3}{x^2 + m^2 x^2} = \frac{m}{1+m} x \rightarrow 0 \Rightarrow \text{FORSE IL LIMITE È ZERO}$$

$$0 \leq |f(x, y)| \leq \text{QUALCOSA CHE TENDE A 0}$$

$$0 \leq |f(x, y)| \leq \frac{x^2}{x^2+y^2} |y| \xrightarrow{\downarrow 0} 0 \quad \Rightarrow \lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$$

$$f(x, y) = \frac{xy}{\sqrt{x^2+y^2}} \quad h(x, mx) = \frac{mx^2}{\sqrt{x^2+m^2x^2}} = \frac{mx^2}{\sqrt{(1+m^2)x^2}} = \frac{m}{\sqrt{1+m^2}} x^{2-\frac{1}{2}} \rightarrow 0 \quad \text{IL LIMITE POTREBBE ESSERE 0}$$

$$0 \leq |f(x, y)| \leq \frac{|x|}{\sqrt{x^2+y^2}} |y| \xrightarrow{\downarrow 0} 0 \quad \lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$$

DERIVATE PARZIALI

$$f(x, y) = 2x^3y^2 - 2xy + 3x - 2y$$

$$f_x(1, 2)$$

$$g(x) = 2x^3(2^2) - 2x(2) + 3x - 2(2)$$

$$\uparrow$$
$$g(x) = f(x, y_0)$$

$$g(x) = 8x^3 - 4x + 3x - 4$$

$$g'(x) = 24x^2 - 4 + 3 = 24x^2 - 1$$

$$f_x(1, 2) = 23$$

$$f(x, y) = 6xy^2 - 3x + 4y^2 + x^3y$$

IN UN GENERICO P SI DERIVA RISPETTO ALLA x
CON y COSTANTE

$$f_x(x, y) = 6y^2 - 3 + 0 + 3x^2y$$

$$f(x, y) = 2x^2y^2 - 3xy^2 + x^2y - 3x^4y$$

$$f_y(x, y) = 4x^2y - 6xy + x^2 - 0 + 1 = 4x^2y - 6xy + x^2 + 1$$

GRADIENTE

$$f(x, y) = \frac{2x^2y - 3x + y^2}{x + y}$$

$$\nabla f(2, 0)$$



$$f_x(x, y) = \frac{(4xy - 3)(x + y) - (2x^2y - 3x + y^2)}{(x + y)^2}$$

$$\nabla f = (f_x, f_y) = (0, 11/2)$$

$$f_y(x, y) = \frac{(2x^2 + 2y)(x + y) - (2x^2y - 3x + y^2)}{(x + y)^2}$$

DERIVATE SECONDE

$$f(x, y) = 2x^2y - 3xy^3$$

$$f_x = 4xy - 3y^3$$

$$f_y = 2x^2 - 9xy^2$$

$$f_{xy} = 4x - 9y^2$$

$$f_{yx} = 4x - 9y^2$$

$$f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

$$(x, y) \neq (0, 0)$$

$$\begin{aligned} \bullet f_x &= y \frac{x^2 - y^2}{x^2 + y^2} + xy \frac{2x(x^2 + y^2) - (x^2 - y^2)(2x)}{(x^2 + y^2)^2} = y \frac{x^2 - y^2}{x^2 + y^2} + xy \frac{\cancel{2x^3} + 2xy^2 - 2x^3 + 2xy^2}{(x^2 + y^2)^2} = \\ &= \frac{x^2y - y^3}{x^2 + y^2} + \frac{4x^2y^2}{(x^2 + y^2)^2} = \frac{x^4 - y^5 + 4x^2y^3}{(x^2 + y^2)^2} \end{aligned}$$

$$\exists f_x(0, 0)? \quad g(x) = f(x, 0) = 0 \quad \forall x \Rightarrow f_x(0, 0) = 0$$

$$\exists f_{xy}(0, 0)? \quad h(y) = f_x(0, y) = -y \Rightarrow f_{xy}(0, 0) = -1$$

$$\bullet f_y = x \frac{x^2 - y^2}{x^2 + y^2} + xy \frac{-2x^2y - 2y^3 - 2xy^2 + 2y^3}{(x^2 + y^2)^2} = \frac{x^5 - xy^4 - 4x^3y^2}{(x^2 + y^2)^2} \quad -1 \neq 1$$

$$\exists f_y(0, 0)? = 0$$

$$\exists f_{yx}(0, 0)? \quad h(y) = f_y(x, 0) = x \Rightarrow f_{yx}(0, 0) = 1$$

TESTA 124001

$$\lim_{(x,y) \rightarrow (1,0)} \frac{\sin((x-1)^2 + y^2)}{\sqrt{(x-1)^2 + y^2}}$$

$$\bullet E_1 = \{(x,y) : y = 0\}$$

$$f|_{E_1} = \frac{\sin(x-1)^2}{\sqrt{(x-1)^2}}$$

$$\lim_{(x,y) \rightarrow (1,0)} \frac{\sin(x-1)^2}{\sqrt{(x-1)^2}} = \frac{\sin(x-1)^2}{(x-1)^2} \cdot \frac{(x-1)^2}{\sqrt{(x-1)^2}} \rightarrow 0$$

$\frac{\sin t}{t} = \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$ NOT.

$\frac{(x-1)^2}{|x-1|} = |x-1|$

$$\bullet E_2 = \{(x,y) : y = -x+1\}$$

$$f|_{E_2} = \frac{\sin((x-1)^2 + (-x+1)^2)}{\sqrt{(x-1)^2 + (-x+1)^2}}$$

SE IL LIMITE ESISTE SARA' 0

$$\lim_{(x,y) \rightarrow (1,0)} \frac{\sin((x-1)^2 + (-x+1)^2)}{(x-1)^2 + (-x+1)^2} \cdot \frac{(x-1)^2 + (-x+1)^2}{\sqrt{(x-1)^2 + (-x+1)^2}} \rightarrow 0$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 + y^2}$$

$$E_m = \{(x,y) : y = mx\}$$

$$f|_{E_m} = \frac{x^2 mx}{x^2 + m^2 x^2} = \frac{mx^3}{x^2(1+m^2)} = \frac{m}{1+m^2} x$$

$$\lim_{f|_{E_m}} = 0 \quad \forall m$$

$$0 \leq |f(x,y)| \leq \frac{x^2}{x^2 + y^2} |y|$$

PER IL TEOREMA DEL CONFRONTO IL LIMITE TENDE A 0

TEOREMA SULLA DIFFERENZIABILITÀ

$$1) f(x,y) = \sqrt{x^2 + y^2} \quad \text{CONTINUA IN } (0,0)$$

$$f(x) = f(x,0) = |x| \quad \nexists f'(x,0)$$

$$h(y) \quad \nexists f'(0,y)$$

• f CONTINUA MA NON DIFF.
(SE LO FOSSE AVEREBBE LE DERIVATE)

$$2) f(x,y) = \begin{cases} 1 & x=y=0 \\ 0 & x \neq y=0 \end{cases}$$

$$f_x(0,0) = f_y(0,0) = 0$$

• f È DER. MA NON DIFFERENZIABILE
(SE LO FOSSE SAREBBE CONTINUA)

$$3) f(x,y) = \sqrt{|xy|} \quad \text{È CONT. IN } (0,0)$$

$$\exists f_x(0,0)? \quad g(x) = 0 \Rightarrow f_x(0,0) = 0 \quad df = 0$$

$$f \text{ DIFF. IN } (0,0)? \quad \frac{\Delta f - df}{\sqrt{\Delta x^2 + \Delta y^2}} = \frac{\sqrt{|\Delta x \Delta y|}}{\sqrt{\Delta x^2 + \Delta y^2}} \quad \begin{matrix} h=0 \rightarrow 0 \\ h=k \rightarrow \frac{1}{\sqrt{2}} \end{matrix} > \nexists \lim \Rightarrow f \text{ NON DIFF.}$$

DERIVATE DIREZIONALI

$$1) f(x, y) = 3x^2y - y + x^3$$

$$f_r(1, 2)$$

$$r = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$f_r(P_0) = \nabla f(P_0) \cdot v$$

$$\nabla f(x, y) = (6xy + 3x^2, 3x^2 - 1)$$

$$f_r(1, 2) = \textcircled{15} \cdot \frac{1}{\sqrt{2}} + \textcircled{2} \cdot \frac{1}{\sqrt{2}} = \frac{17}{\sqrt{2}}$$

$$2) f(x, y) = \frac{3xy^2}{x^2 + y^2}$$

$$P_0(1, 1)$$

$$f_r(1, 1)$$

$$v = (2, 3) \quad |v| = \sqrt{13}$$

NON È UN VETTORE

COSTRUISCO IL VETTORE CHE RAPPRESENTA LA DIREZ.

$$DI \ v \Rightarrow w = \left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)$$

$$y \neq -x^2$$

$$\nabla f(x, y) = \left(\frac{3x^2y^2 + 3y^3 - 6x^2y^2}{(x^2 + y^2)^2}, \frac{6x^3y + 6xy^2 - 3xy^2}{(x^2 + y^2)^2} \right) = \left(\frac{3y^3 - 3x^2y^2}{(x^2 + y^2)^2}, \frac{6x^3y + 3xy^2}{(x^2 + y^2)^2} \right)$$

$$f_w(1, 1) = 0 \cdot \frac{2}{\sqrt{13}} + \frac{9}{4} \cdot \frac{3}{\sqrt{13}} = \frac{27}{4\sqrt{13}}$$

$$3) f(x, y) = \frac{2xy^3}{x^2 + 3y}$$

$$P_0(1, 1)$$

$$f_r(1, 1)$$

DOVE v È LA DIR. DELLA RETTA

$$2: 2x - y + 3 = 0$$

$$v = (1, 2)$$

$$|v| = \sqrt{5} \neq 1$$

$$w = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$\nabla f(x, y) = \left(\frac{-2x^2y^3 + 6y^4}{(x^2 + 3y)^2}, \frac{6x^3y^2 + 12xy^3}{(x^2 + 3y)^2} \right)$$

$$f_w(1, 1) = \frac{4}{16} \cdot \frac{1}{\sqrt{5}} + \frac{18}{16} \cdot \frac{2}{\sqrt{5}} = \frac{1}{4\sqrt{5}} + \frac{9}{8\sqrt{5}} = \frac{10}{8\sqrt{5}} = \frac{5}{4\sqrt{5}} = \frac{5}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{10} = \frac{\sqrt{5}}{2}$$

$$4) f(x,y) = 2x^2y + 3xy^3$$

$$f_v(2,1) \quad z: x+3y-2=0 \\ v(3,-1) \quad |v| = \sqrt{10} \quad w = \left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)$$

$$\triangleright f(x,y) = (4xy + 3y^3, 2x^2 + 9xy^2)$$

$$f_w(2,1) = \frac{33}{\sqrt{10}} - \frac{26}{\sqrt{10}} = \frac{7}{\sqrt{10}}$$

RICERCA ESTREMI RELATIVI

$$1) f(x,y) = 2x^2 + xy^2 + x + \frac{1}{2}y^2 \quad x \in \mathbb{R}^2$$

$$f_x = 4x + y^2 + 1$$

$$f_y = 2xy + y$$

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \Rightarrow \begin{cases} 4x + y^2 + 1 = 0 \\ 2xy + y = 0 \end{cases} \Rightarrow \begin{cases} 4x = -y^2 - 1 \\ // \end{cases} \Rightarrow \begin{cases} x = \frac{-y^2 - 1}{4} \\ y(y^2 - 1) = 0 \end{cases}$$

$$\begin{cases} x = -1/4 \\ y = 0 \end{cases} ; \begin{cases} x = -1/2 \\ y = 1 \end{cases} ; \begin{cases} x = -1/2 \\ y = -1 \end{cases}$$

$$P_1(-1/4, 0) \quad P_2(-1/2, 1)$$

$$P_3(-1/2, -1) \quad \text{PUNTI STAZIONARI}$$

$$f_{xx} = 4 \quad f_{xy} = 2y \quad f_{yy} = 2x + 1$$

$$H(x,y) = f_{xx}f_{yy} - f_{xy}^2$$

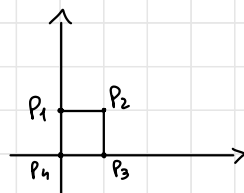
$$H(P_1) = 2 - 0 > 0 \quad f_{xx}(P_1) > 0 \Rightarrow P_1 \quad \text{PUNTO MIN. REL.}$$

$$H(P_2) = 0 - 4 < 0 \Rightarrow P_2 \quad \text{PUNTO DI SELLA}$$

$$H(P_3) = 0 - 4 < 0 \Rightarrow P_3 \quad \text{PUNTO DI SELLA}$$

ESTREMI ASSOLUTI

$$f(x,y) = 2x^2 + xy - x - y \quad P_1(0,1) \quad P_2(1,1) \quad P_3(1,0) \quad P_4(0,0)$$



$$f_x = 4x + y - 1$$

$$f_y = x - 1$$

$$P(1,-3) \notin X$$

$$X_1 = \emptyset$$

$$X_2 = \emptyset$$

STUDIO LA RESTRIZIONE A $X_3 = F(x)$

$$P_1 P_2 \quad y=1 \quad g(x) = f(x,1) = 2x^2 - 1 \quad 0 \leq x \leq 1$$

$$g'(x) = 4x = 0 \quad \text{PER } x=0 \quad \text{PRENDO IN CONSID. SOLO } P_1 P_2$$

$$P_2 P_3 \quad x=1 \quad h(y) = f(1,y) = 1 \quad \text{PRENDO IN CONSID. SOLO } P_2 P_3$$

$$P_3 P_4 \quad y=0 \quad l(x) = 2x^2 - x \quad 0 \leq x \leq 1$$

$$l'(x) = 4x - 1 = 0 \quad x = \frac{1}{4} \quad P(\frac{1}{4}, 0)$$

$$P_1 P_4 \quad x=0 \quad m(y) = -y \quad 0 \leq y \leq 1 \quad \text{PRENDO } P_1 P_4$$

$$f(0,1) = -1$$

$$f(1,1) = 3$$

$$f(0,0) = 0$$

$$f(1,0) = 1$$

$$f(\frac{1}{4}, 0) = -\frac{1}{8}$$

$$\min_x f = -1 = f(0,1)$$

$$\max_x f = 3 = f(1,1)$$