$y' = X(x)Y(y)$

1) Separo le var. X e Y

2) Integriamo i due membri

3) Ricavo $y(x)$

1) $y' = y^2 \ln x$

$X(x) = \ln x \qquad (a,b) \Rightarrow]0,+\infty[$

$Y(y) = y^2 \qquad (c,d) \Rightarrow \mathbb{R}$

I categ

$H = \{0\}$ $\Vert$ Soluz $y(x) = 0$

$$\int \frac{y'(x)}{y^2(x)}\,dx = \int \ln x\,dx$$

$$\Rightarrow -\frac{1}{y(x)} = x\log|x| - x + K$$

$$\Rightarrow y(x) = -\frac{1}{x\log|x| - x + K}$$

$$y' = \frac{1+y^2}{x^2}$$

$X(x) = \frac{1}{x^2}$ $\quad (a,b) =]-\infty, 0[\cup]0 + \infty[$

$Y(y) = 1 + y^2$ $\quad (c,d) = \mathbb{R}$

I) $y^2 + 1 = 0 \quad \emptyset \qquad H = \{\emptyset\}$

II) $$\frac{y'(x)}{y^2(x)+1} = \frac{1}{x^2}$$

$$\Rightarrow \quad \operatorname{arctg}(y(x)) = -\frac{1}{x} + k$$

$$\operatorname{arctg}(y(x)) = \frac{kx-1}{x}$$

$$y(x) = \operatorname{tg}\left(\frac{kx-1}{x}\right)$$

$y'(x) = x y^2$

$X(x) = x \qquad (a,b) = \mathbb{R}$

$Y(x) = y^2 \qquad (c,d) = \mathbb{R}$

I) $H = \{0\}$ $\quad y(x) = 0$ é solur. di I categ

II)

$$\frac{y'(x)}{y^2(x)} = x \qquad -\frac{1}{y(x)} = \frac{x^2}{2} + K \Rightarrow y(x) = -\frac{2}{x^2 + 2K}$$

Se $y(x) > 0$

$$-\frac{2}{x^2 + 2K} > 0$$

$$x^2 + 2K < 0$$

se $K < 0$ le solur. sono $-\sqrt{-2K} < x < \sqrt{-2K}$

se $K \geq 0$ non esistono soluz.

Se $y(x) < 0$

$$-\frac{2}{x^2 + 2K} < 0$$

$$x^2 > -2K$$

se $K < 0$ $\quad x \in \left] \sqrt{-2K}, +\infty \right[$

se $K \geq 0$ $\quad x \in \mathbb{R}$

es $y' = \sqrt{|y|}$

$X(x) = 1 \quad (a,b) = \mathbb{R}$

$Y(y) = \sqrt{|y|} \quad (c,d) = \mathbb{R}$

I) $\sqrt{|y|} = 0 \qquad h = 0 \qquad H = \{0\}$

II) $\dfrac{y'(x)}{\sqrt{|y(x)|}} = 1$

Se $y(x) > 0$ $\qquad \dfrac{y'(x)}{2\sqrt{y(x)}} = \dfrac{1}{2}$

$D\left(\sqrt{y(x)}\right) = \dfrac{1}{2\sqrt{y(x)}}$

$\sqrt{y(x)} = \dfrac{x}{2} + k'$

$\sqrt{y(x)} = \dfrac{x+2k'}{2}$ $\qquad$ se $\dfrac{x+2k'}{2} > 0$

$\Downarrow$

$y(x) = \left(\dfrac{x+2k'}{2}\right)^2$ $\qquad x \in \left]-2k', +\infty\right[$

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$$\begin{cases} y' = e^x \\ y(x) = 1 \end{cases}$$

$y' = e^x$

$X(x) = e^x \quad (a,b) = \mathbb{R}$

$Y(y) = y \quad (c,d) = \mathbb{R}$

$$\frac{y'(x)}{y(x)} = e^x$$

$H = \{0\} \Rightarrow$ I caley.

$\ln|y(x)| = e^x + K$

$y(x) = e^{e^x} \cdot \underbrace{e^K}_{K'}$

$y(x) = Ke^{e^x}$

se $K>0$ $\quad y(x) = Ke^{e^x}$ e' solur. II caley.

se $K<0$ $\quad -y(x) = Ke^{e^x}$ e' solur. II caley.

$$\begin{cases} y' = \operatorname{sen} x \; e^{y} \\ y\left(\frac{\pi}{2}\right) = 1 \end{cases}$$

$$y' = \operatorname{sen} x \, e^{y}$$

$X(x) = \operatorname{sen} x \qquad (a,b) = \mathbb{R}$

$Y(y) = e^{y} \qquad (c,d) = \mathbb{R}$

I) $H = \{ \emptyset \}$

II) $\dfrac{y'(x)}{e^{y(x)}} = \operatorname{sen} x \quad \Rightarrow \quad + e^{-y(x)} = + \cos(x) + K$

$$y(x) = -\ln\left(\cos(x) + K\right)$$

Risolvo il PC

$$\begin{cases} y(x) = -\ln\left(\cos(x) + K\right) \\ y\left(\frac{\pi}{2}\right) = 1 \end{cases}$$

$$-\ln\left(\underbrace{\cos\left(\tfrac{\pi}{2}\right)}_{=0} + K\right) = 1$$

$$\ln(K) = -1 \quad \Rightarrow \quad K = \frac{1}{e}$$

$$y(x) = -\ln\left(\cos(x) + \frac{1}{e}\right) \Leftarrow$$ Soluz. al PC

$$y' + \cos(x)\, y = 0 \qquad X(x) = \cos(x) \qquad (a,b) = (c,d) = \mathbb{R}$$

$$Y(y) = y$$

$H = \{0\}$ $\quad h = 0$ è soluz. di prima categoria $\quad y(x) = 0$

$$\frac{y'}{y} = -\cos(t) \qquad y(x) = K e^{-\operatorname{sen}(x)} \text{ è soluz. dell'omogenea}$$

$$y' + xy = e^{-\frac{x^2}{2}} \qquad X(x) = -x$$

$$Y(y) = y$$

Scrivo l'omogenea $\qquad H = \{0\}$

$$\frac{y'(x)}{y(x)} = -x$$

Soluz. dell'omog. asso. $\quad y(x) = K e^{-\frac{x^2}{2}}$

Soluz. della completa

$$K \in \int e^{\frac{x^2}{2}} e^{-\frac{x^2}{2}} dx = K \in \int e^{A(x)} y(x)\, dx$$

$$K = x$$

$$y(x) = x e^{-\frac{x^2}{2}} + K e^{-\frac{x^2}{2}}$$

$y' - y = e^{2x}$ $\qquad X(x) = -1 \qquad (a,b) = (c,d) = \mathbb{R}$

$Y(x) = y$

$y' - y = 0$ Omogenea associata

Sol. dell'omogenea $\qquad y(x) = Ke^{x}$

Sol. part. della completa $\qquad \bar{y}(x) = K(x)\, e^{x}$

$$K = \int e^{A(x)} \cdot f(x)\, dx = \int e^{-x} e^{2x}\, dx = \int e^{x}\, dx = e^{x} + c$$

Sol. gen. della completa $\Rightarrow \bar{y}(x) = K(x)\, e^{-A(x)} + y(x) = e^{2x} + Ke^{x}$

$\Rightarrow W'(x) + a(x)\, W(x) = 0 \quad \Rightarrow W$ é soluz. dell'equaz.

Eq. diff. omogenee a coeff. costanti

1) $y'' + 2y' - 8y = 0$

Equaz. caratteristica $\alpha^2 + 2\alpha - 8 = 0$ $\Delta = 4 + 32 = 36$

$\alpha = \frac{-2 \pm 6}{2} = \begin{cases} -4 \\ 2 \end{cases}$

Integrale generale $y(x) = Ke^{2x} + Ke^{-4x}$

2) $y'' - 9y' + 18y = 0$

Eq. caratt. $\alpha^2 - 9\alpha + 18 = 0$ $\Delta = 81 - 72 = 9$

$\alpha = \frac{9 \pm 3}{2} = \begin{cases} \to 6 \\ \downarrow 3 \end{cases}$

Int. generale $y(x) = K_1^{3x} + K_2^{6x}$

3) $y'' - 10y' + 25y = 0$

Eq. caratt $\alpha^2 - 10\alpha + 25 = 0$ $\Delta = 0$

$\alpha = 5$ (molt. 2)

Int. generale $y(x) = K_1 e^{5x} - K_2 x e^{5x}$

1) $y'' - y' + 3y = 0$

Eq. caratt. $\quad \lambda^2 - \lambda + 3 = 0 \qquad \Delta = -11$

$$\lambda = \frac{1}{2} \pm i\frac{\sqrt{11}}{2}$$

Integ. generale $\quad y(x) = K_1 e^{\frac{1}{2}x} \cos \frac{\sqrt{11}}{2}x + K_2 e^{\frac{1}{2}x} \sin \frac{\sqrt{11}}{2}x$

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2) $y'' - y' = e^{2x}(x-1)$

Scrivo l'omogenea associata

$$y'' - y' = 0$$

Eq. caratt. $\quad \lambda^2 - \lambda = 0$

$$\lambda(\lambda - 1) = \begin{matrix} - 0 \\ - 1 \end{matrix}$$

Sol gen. dell'omogenea $\quad y(x) = K_1 + K_2 e^x$

Sol part della completa $\quad h = 2 \qquad s = 0 \qquad m = 1$

$$\bar{y}(x) = e^{h(x)} x^s q(x) = e^{2x}(ax+b)$$

$$\bar{y}'(x) = e^{2x}(2ax + 2b + a)$$

$$\bar{y}''(x) = e^{2x}(4ax + 4a + 4b)$$

$$\cancel{e^{2x}}(4ax + 4a + 4b - 2ax - 2b - a) = \cancel{e^{2x}}(x-1)$$

$$2ax + 3a + 2b = x - 1$$

$$\begin{cases} 2a = 1 \\ 3a + 2b = -1 \end{cases} \qquad \begin{cases} a = \frac{1}{2} \\ b = -\frac{5}{4} \end{cases}$$

Integ. generale della completa $\bar{y}(x) = K_1 + K_2 e^{x} + e^{2x}\left(\frac{x}{2} - \frac{5}{4}\right)$

$$y' = e^x\ (y-2)$$

$X(x) = e^x \qquad (a,b) = \mathbb{R}$

$Y(y) = y-2 \qquad (c,d) = \mathbb{R}$

Solur. I) categoria

$y-2=0 \qquad H = \{2\}$

II) categ. $\dfrac{y'(x)}{y(x)-2} = e^x$

$\Rightarrow \log|y(x)-2| = e^x + K$

$y(x) = Ke^{e^x} + 2$

$$y' - 2xy = x$$

$X(x) = x \qquad (a,b) = \mathbb{R}$

$Y(y) = 2y+1 \qquad (c,d) = \mathbb{R}$

$2y+1=0 \Rightarrow y = -\frac{1}{2} \qquad H = \{-\frac{1}{2}\}$ solur. di I categ.

$\dfrac{y'}{2y+1} = x \Rightarrow \log|2y(x)+1| = \dfrac{x^2}{2} + K$

$y(x) = \dfrac{Ke^{\frac{x^2}{2}} - 1}{2}$

$$y'' - y' - 2y = e^{2x}\ (x+3)$$

Scrivo l'omogenea associata

$$y'' - y' - 2y = 0$$

Equaz. caratt. $\lambda^2 - \lambda - 2 = 0$ $\quad \Delta = 1 - 4(-2) = 9$

$$\lambda = \frac{1 \pm 3}{2} = \begin{matrix} 2 \\ -1 \end{matrix}$$

Integrale gen. dell'omogenea

$$y(x) = K_1 e^{-1} + K_2 e^{2}$$

Integrale part. della completa $\quad h = 2 \quad m = 1 \quad s = 1$

$$\bar{y}(x) = e^{h(x)} x^s q(x) = e^{2x} \cdot x(ax+b) = e^{2x}(ax^2+bx)$$

$$\bar{y}'(x) = e^{2x}(2ax^2 + 2bx + 2ax + b)$$

$$\bar{y}''(x) = e^{2x}(4ax^2 + 4bx + 8ax + 4b + 2a)$$

$$\cancel{e^{2x}}(4\cancel{ax^2} + 4\cancel{bx} + 8ax + 4b + 2a - 2\cancel{ax^2} - 2\cancel{bx} - 2ax - b - 2\cancel{ax^2} - 2\cancel{bx}) = \cancel{e^{2x}}(x+3)$$

$$6ax + 3b + 2a = x + 3$$

$$\begin{cases} 6a = 1 \\ 3b + 2a = 3 \end{cases} \qquad \begin{cases} a = \frac{1}{6} \\ b = \frac{8}{9} \end{cases}$$

<u>Integ. generale della completa</u>

$$\bar{y}(x) = K_1 e^{-1} + K_2 e^{2} + e^{2x}\left(\frac{1}{6}x^2 + \frac{8}{9}x\right)$$

1) $$\begin{cases} y'' - y' = e^{2x}(x-3) + e^{x}x^{2} \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Scrivo l'eq. omogenea

$$y'' - y' = 0$$

eq. caratt. $\lambda^2 - \lambda = 0$ $\begin{cases} 0 \\ 1 \end{cases}$

integ. gen. omogenea $y(t) = K_1 + K_2 e^{x}$

Cons. $\underbrace{y'' - y' = e^{2x}(x-3)}_{(1)}$ $\qquad$ $\underbrace{y'' - y' = e^{x}x^{2}}_{(2)}$

- integ. particolare per (1) e (2) e li sommo

$h = 2 \qquad s = 0 \qquad m = 1$

$$\bar{y}(x) = e^{2x}(ax + b)$$

$$\bar{y}'(x) = e^{2x}(2ax + 2b + a)$$

$$\bar{y}''(x) = e^{2x}(4ax + 4b + 4a)$$

$$y'' - y' = e^{2x}(x-3)$$

$$e^{2x}(4ax + 4b + 4a - 2ax - 2b - a) = e^{2x}(x-3)$$

$$2ax + 2b + 3a = x - 3$$

$$\begin{cases} a = \frac{1}{2} \\ b = -\frac{9}{4} \end{cases}$$

$$\bar{y}(x) = e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right)$$

$$y'' - y' = e^{x}x^2$$

$$h = 1 \qquad s = 1 \qquad m = 2$$

$$\bar{y}(x) = e^{2x}(ax^3 + bx^2 + cx)$$

$$\bar{y}'(x) = e^{2x}(ax^3 + bx^2 + cx + 3ax^2 + 2bx + c)$$

$$\bar{y}''(x) = e^{2x}(ax^3 + bx^2 + cx + 6ax^2 + 4bx + 2c + 6ax + 2b)$$

$$e^{2x}(ax^3 + bx^2 + \cancel{cx} + 6ax^2 + 4bx + 2c + 6ax + 2b - ax^3 - bx^2 - \cancel{cx} - 3ax^2 - 2bx) = e^{x}x^2$$

$$3ax^2 + 6ax + 2bx + 2b + c = x^2$$

$$\begin{cases} a = \frac{1}{3} \\ b = -1 \\ c = 2 \end{cases}$$

$$\bar{z}(x) = e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$$

Sol. particolare della completa

$$\bar{y}(x) + \bar{z}(x) = e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right) + e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$$

$$\Downarrow$$

$$\bar{y}(x) = K_1 + K_2 e^{x} + e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right) + e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$$

Risolvo il PC

$$y'(x) = K_2 e^{x} + e^{2x}\left(x - \frac{9}{2} + \frac{1}{2}\right) + e^{x}\left(\frac{1}{3}x^3 \cancel{- x^2} \cancel{+ 2x} \cancel{+ x^2} \cancel{- 2x} + 2\right)$$

$$= e^{x}\left(\frac{1}{3}x^3 + 2 + K_2\right) + e^{2x}(x - 4)$$

$$\begin{cases} y(0) = 3 \\ y'(0) = 1 \end{cases} \quad \begin{cases} K_1 + K_2 - \frac{9}{4} = 3 \\ 2 + K_2 - 4 = 1 \end{cases} \quad \begin{cases} K_1 + K_2 = \frac{21}{4} \\ K_2 = 3 \end{cases} \quad \begin{cases} K_1 = \frac{9}{4} \\ K_2 = 3 \end{cases}$$

Sol. al PC $\quad y(x) = \frac{9}{4} + 3e^{x} + e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right) + e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$

$y'' + 4y = 2 \operatorname{sen} x$

omogenea $\quad y'' + 4y = 0$

caratt $\quad \alpha^2 + 4 = 0 \quad \pm 2i$

int gen omog. $\quad y(x) = K_1 \cos(2x) + K_2 \operatorname{sen}(2x)$

$e^{ix} = \cos x + i \operatorname{sen} x$

Cons. $\quad y'' + 4y = 2e^{ix}$

$e^{2ix} = \cos 2x + i \operatorname{sen} 2x$

Trovo un integ partic. di questa e prendo la parte immaginaria

$\star \quad h = i \qquad s = 0 \qquad m = 0$

$\bar{y}(x) = e^{ix}(K) = Ke^{ix}$

$\bar{y}'(x) = Ki\, e^{ix}$

$\bar{y}''(x) = -K e^{ix}$

$-Ke^{ix} + 4Ke^{ix} = 2e^{ix}$

$3K = 2 \Rightarrow K = \frac{2}{3}$

$\bar{y}(x) = \frac{2}{3} i e^{x} = \frac{2}{3}\cos(x) + \frac{2}{3} i \operatorname{sen}(x)$

int gen. omogenea $y(x) = K_1 \cos 2x + K_2 \sin 2x + \frac{2}{3} \sin x$

3) $y'' - y' - 6y = e^{3x} + x \sin x$

omog. $y'' - y' - 6y = 0$

caratt. $\lambda^2 - \lambda - 6 = 0$ $\lambda = \begin{cases} 3 \\ -2 \end{cases}$

int. gen. omogenea $y(x) = K_1 e^{3x} + K_2 e^{-2x}$

Cons. $y'' - y' - 6y = e^{3x}$ (1)

$y'' - y' - 6y = x \sin x$ (2)

$\qquad\qquad\qquad\qquad\quad \overset{\shortparallel}{x e^{ix}}$

(1) $h = 3$ $m = 0$ $s = 1$

Cerco $\bar{y}(x) = K e^{3x} x$

$\bar{y}'(x) = K e^{3x} (3x+1)$

$\bar{y}''(x) = K e^{3x} (9x+6)$

sost. nell' eq. $K \cancel{e^{3x}} (\cancel{9x} + 6 - \cancel{3x} - 1 - \cancel{6x}) = \cancel{e^{3x}}$

$$5K = 1 \to K = \frac{1}{5} \qquad \underline{\bar{y}(x) = \frac{1}{5} x e^{3x}}$$

(2) $\quad h = 1 \qquad s = 0 \qquad m = 1$

$$\bar{z}(x) = e^{ix}(ax+b)$$

$$\bar{z}'(x) = e^{ix}(iax + ib + a)$$

$$\bar{z}''(x) = e^{ix}(-ax - b + 2ia)$$

sost. nell' eq. $\quad \cancel{e^{ix}}(-ax - b + 2ia - iax - ib - a - 6ax - 6b) = x\cancel{e^{ix}}$

$$(-7a - ia)x + 2ia - a - 7b - ib = x$$

molt. e div. per $7-i$

$$\begin{cases} -7a - ia = 1 \\ 2ia - a - 7b = 0 \end{cases} \qquad \begin{cases} a = -\frac{1}{7+i} = -\frac{7-i}{50} = -\frac{7}{50} + \frac{1}{50}i \\ b = \frac{1}{125} - i\frac{1}{250} \end{cases}$$

$$\bar{z}(x) = \left(\frac{1}{125} + \frac{11}{250}i\right)(\cos x + i \sin x)$$

sol. di (2) è $\frac{1}{125} \operatorname{sen} x - \frac{11}{250} \cos x$

int generale di (1) $\quad y(x) = K_1 e^{3x} + K_2 e^{-2x} + \frac{1}{5} x e^{3x} + \frac{1}{125} \operatorname{sen} x - \frac{11}{250} \cos x$

$y'' - 4y' + 4y = 3e^{2x}$

omog. $\quad y'' - 4y' + 4y = 0$

caratt. $\quad \lambda^2 - 4\lambda + 4 = 0 \qquad \lambda = 2$ (doppia)

int gen. omogenea $\quad \tilde{y}(x) = K_1 e^{2x} + K_2 x e^{2x}$

$h = 2 \qquad \lambda = 2 \qquad m =$

cerco $\quad \bar{y}(x) = e^{2x} \cdot x^2 (K) = K e^{2x} x^2$

$\bar{y}'(x) = K e^{2x} (2x^2 + 2x)$

$\bar{y}''(x) = K e^{2x} (4x^2 + 8x + 2)$

sost. nell'eq. $\quad K \cancel{e^{2x}} (4x^2 + 8x + 2 - 8y^2 - 8x + 4x^2) = 3 \cancel{e^{2x}}$

$K = \frac{3}{2}$

int gen. della comp. $\quad \bar{y}(x) = K_1 e^{2x} + K_2 x e^{2x} + \frac{3}{2} x^2 e^{2x}$

$$\begin{cases} y'' - 4y' + 4y = 3e^{2x} \\ y(0) = 1 \\ y'(1) = 2 \end{cases}$$

$\bar{y}'(x) = 2K_1 e^{2x} + K_2 e^{2x} + 2K_2 x e^{2x} + 3x e^{2x} + 3x^2 e^{2x}$

$$\begin{cases} y(0) = K_1 \\ y'(0) = 2K_1 + K_2 \end{cases} \qquad \begin{cases} K = 1 \\ 2K_1 + K_2 = 2 \end{cases} \qquad \begin{matrix} K_1 = 1 \\ K_2 = 0 \end{matrix}$$

sol. $\quad y(x) = e^{2x} + \frac{3}{2} x^2 e^{2x}$

$$y'' + 3y' - 4y = 2x\, e^{3x}$$

Omog. ass. $\quad y'' + 3y' - 4y = 0$

Equaz. caratt $\quad \lambda^2 + 3\lambda - 4 = 0 \qquad \Delta = 9 + 16 = 25$

$$\lambda = \frac{-3 \pm 5}{2} = \begin{cases} 1 \\ -4 \end{cases}$$

Int gen. omogenea $\quad y(x) = Ke^{-4x} + Ke^{x}$

Metodo di somiglianza
(cerco l'integ. part. della completa)

$h = 3 \qquad s = 0 \qquad m = 1$

$\bar{y}(x) = e^{3x}(ax + b)$

$\bar{y}'(x) = e^{3x}(3ax + 3b + a)$

$\bar{y}''(x) = e^{3x}(9ax + 9b + 6a)$

Sostituisco nell'eq di partenza

$$\cancel{e^{3x}}(9ax + 9b + 6a + 9ax + 9b + 3a - 4ax - 4b) = 2x\,\cancel{e^{3x}}$$

$14ax - 14b + 9a = 2x$

$$\begin{cases} 14a = 2 \\ -14b + 9a = 0 \end{cases} \qquad \begin{cases} a = \frac{1}{7} \\ b = \frac{9}{98} \end{cases}$$

integrale part. della completa $\quad y(x) = e^{3x}\left(\frac{1}{7}x + \frac{9}{98}\right)$

integrale gen. della completa

$$y(x) = Ke^{-4x} + K e^{x} + e^{3x}\left(\frac{x}{7} + \frac{9}{98}\right)$$

$$y'' - 8y' + 16y = e^{-x}$$

Omogenea associata $\quad y'' - 8y' + 16y = 0$

Equaz. caratt. $\quad \lambda^2 - 8\lambda + 16 = 0 \qquad \Delta = 64 - 4(16) = 0 \qquad \lambda = 4$

Sol. gen. omogenea $\quad y(x) = K_1 e^{4x} + x K_2 e^{4x}$

Sol part completa $\quad h = -1 \qquad s = 0 \qquad m = 0$

$\bar{y}(x) = K e^{-x}$ $\qquad$ Sost. nell' equaz. = $\cancel{e^{-x}}(K + 8K + 16K) = \cancel{e^{-x}}$

$\bar{y}'(x) = -K e^{-x}$ $\qquad K = \frac{1}{25}$

$\bar{y}''(x) = K e^{-x}$ $\qquad \bar{y}(x) = -\frac{e^{-x}}{25}$

Int gen. della completa $y(x) = K_1 e^{4x} + x K_2 e^{4x} - \frac{e^{-x}}{25}$

$$\begin{cases} y'' - y' = e^{2x}(x-3) + e^{x} x^{2} \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Scrivo l'omogenea

$$y'' - y' = 0$$

Eq. caratt $\quad \alpha^2 - \alpha = 0 \quad \alpha = \begin{cases} 0 \\ 1 \end{cases}$

Integ. gener. omogenea $\quad y(x) = K_1 + K_2 e^{x}$

Cons. $\quad \underbrace{y'' - y' = e^{2x}(x-3)}_{(1)} \qquad \underbrace{y'' - y' = e^{x} x^{2}}_{(2)}$

$\Rightarrow$ Integ. particolare di (1) e (2) e li sommo

$h = 2 \qquad s = 0 \qquad m = 1$

$\bar{y}(x) = e^{2x}(ax + b)$

$\bar{y}'(x) = e^{2x}(2ax + 2b + a)$

$$\bar{y}''(x) = e^{2x}(4ax + 4b + 4a)$$

$$\cancel{e^{2x}}(4ax + 4b + 4a - 2ax - 2b - a) = \cancel{e^{2x}}(x-3)$$

$$2ax + 2b + 3a = x - 3$$

$$\begin{cases} 2a = 1 \\ 2b + 3a = -3 \end{cases} \quad \begin{cases} a = \frac{1}{2} \\ b = -\frac{9}{4} \end{cases}$$

int. part. della (1) $\quad \bar{y}(x) = e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right)$

$$y'' - y' = e^x x^2$$

$h = 1 \qquad s = 1 \qquad m = 2$

$$\bar{z}(x) = e^x x(ax^2 + bx + c) = e^x(ax^3 + bx^2 + cx)$$

$$\bar{z}'(x) = e^x(ax^3 + bx^2 + cx + 3ax^2 + 2bx + c)$$

$$\bar{z}''(x) = e^x(ax^3 + bx^2 + cx + 6ax^2 + 4bx + 2c + 6ax + 2b)$$

$$\cancel{e^x}(\cancel{ax^3} + \cancel{bx^2} + \cancel{cx} + 6ax^2 + 4bx + 2c + 6ax + 2b - \cancel{ax^3} - \cancel{bx^2} - \cancel{cx} - 3ax^2 - 2bx - c) = \cancel{e^x} x^2$$

$$3ax^2 + 2bx + c + 6ax + 2b = x^2$$

$$\begin{cases} 3a = 1 \\ 2b + 6a = 0 \\ 2b + c = 0 \end{cases} \quad \begin{cases} a = \frac{1}{3} \\ b = -1 \\ c = 2 \end{cases}$$

int. part. della (2) $\quad \bar{z}(x) = e^x\left(\frac{1}{3}x^3 - x^2 + 2x\right)$

Int particolare della completa

$$\bar{y}(x) = e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right) \qquad \bar{z}(x) = e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$$

$$y(x) = K_1 + K_2 e^{x} + e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right) + e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$$

$$\begin{cases} y'' - y' = e^{2x}(x-3) + e^{x}x^2 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

$$\frac{x}{2} = \frac{1 \cdot 2}{2}$$

$$\frac{x^3}{3} = \frac{3x^2 \cdot 3}{8} = \frac{8x}{8}$$

· Calcolo $y'(x) = K_2 e^{x} + e^{2x}\left(\cancel{2} \cdot \frac{1}{\cancel{2}}x - \cancel{2} \cdot \frac{\cancel{9}}{\cancel{4}_{2}} + \frac{1}{2}\right) + e^{x}\left(\frac{1}{3}x^3 \cancel{-x^2} \cancel{+2x} + \cancel{x^2} - \cancel{2x} + 2\right)$

$$y'(x) = K_2 e^{x} + e^{2x}(x - 4) + e^{x}\left(\frac{x^3}{3} + 2\right)$$

$$y'(0) = K_2 - 2$$

$$y(0) = K_1 + K_2 - \frac{9}{4}$$

$$\begin{cases} y(x) = \dots \\ K_2 - 2 = 0 \\ K_1 + K_2 - \frac{9}{4} = 1 \end{cases} \qquad \begin{cases} \dots \\ K_2 = 2 \\ K_1 = \frac{5}{4} \end{cases}$$

$$y(x) = \frac{5}{4} + 2e^{x} + e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right) + e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$$

$$y'' + y' - 6y = (x+2)\,e^{2x} + \frac{x}{e^x}$$

$$y'' + y' - 6y = (x+2)\,e^{2x} + x e^{-x}$$

Omog. ass. $\quad y'' + y' - 6y = 0$

Eq. Caratt. $\quad \lambda^2 + \lambda - 6 = 0 \qquad \Delta = 25$

$$\lambda = \frac{-1 \pm 5}{2} = \begin{cases} 2 \\ -3 \end{cases}$$

Sol gen. omog. $\quad y(x) = K_1 e^{-3x} + K_2 e^{2x}$

Cons. $\quad \underset{(1)}{y'' + y' - 6y = (x+2)\,e^{2x}} \qquad \underset{(2)}{y'' - y' - 6y = x e^{-x}}$

$(1) \Rightarrow \quad h = 2 \qquad s = 1 \qquad m = 1$

$$\bar{y}(x) = e^{2x}(ax^2 + bx)$$

$$\bar{y}'(x) = e^{2x}(2ax^2 + 2bx + 2ax + b)$$

$$\bar{y}''(x) = e^{2x}(4ax^2 + 4bx + 8ax + 4b + 2a)$$

Sol part (1) $\Rightarrow$

$$\cancel{e^{2x}}(\cancel{4ax^2} + \cancel{4bx} + 8ax + 4b + 2a + \cancel{2ax^2} + \cancel{2bx} + 2ax + b - \cancel{6ax^2} - \cancel{6bx}) = \cancel{e^{2x}}(x+2)$$

$$10ax + 5b + 2a = x + 2$$

$$\begin{cases} a = \frac{1}{10} \\ b = \frac{9}{25} \end{cases} \qquad \bar{y}(x) = e^{2x}\left(\frac{1}{10}x^2 + \frac{9}{25}x\right)$$

$$y'' + y' - 6y = x e^{-x} \qquad h = -1 \qquad s = 0 \qquad m = 1$$

$$\bar{z}(x) = e^{-x}(ax + b)$$

$$\bar{z}'(x) = e^{-x}(-ax - b + a)$$

$$\bar{z}''(x) = e^{-x}(ax + b - 2a)$$

$$\cancel{e^{-x}}(\cancel{ax} + \cancel{b} - 2a - \cancel{ax} - \cancel{b} - a - 6ax - 6b) = x\,\cancel{e^{-x}}$$

$$-a - 6ax - 6b = x$$

$$\begin{cases} a = -\frac{1}{6} \\ b = \frac{1}{36} \end{cases} \qquad \bar{z}(x) = e^{-x}\left(-\frac{1}{6}x + \frac{1}{36}\right)$$

Sol gen. completa

$$y(x) = K_1 e^{-3x} + K_2 e^{2x} + e^{2x}\left(\frac{1}{10}x^2 + \frac{9}{25}x\right) + e^{-x}\left(-\frac{1}{6}x + \frac{1}{36}\right)$$

$$y' = \frac{\sqrt{y^2+1}}{xy}$$

$X(x) = \frac{1}{x}$ $(a,b) = \mathbb{R} \setminus \{0\}$

$Y(y) = \frac{\sqrt{y^2+1}}{y}$ $(c,d) = \mathbb{R} \setminus \{0\}$

$$y' = \frac{1}{x} \frac{\sqrt{y^2+1}}{y}$$

- Cerco le sol. di I) categ.

$$\frac{\sqrt{y^2+1}}{y} = 0 \quad \text{mai} \qquad H = \{\emptyset\}$$

- II categ.

$$\frac{y'(x) \cdot y(x)}{\sqrt{y^2(x)+1}} = \frac{1}{x} \qquad \forall (x)$$

$$\Rightarrow \quad 2 \cdot \frac{y'(x)\, y(x)}{2\sqrt{y^2(x)+1}} = \frac{1}{x}$$

$$\Rightarrow \quad 2\sqrt{y^2(x)+1} = \log|x| + K$$

$$y'' - 2y' + y = e^x\ (x+3)$$

Omogenea associata $\quad y'' - 2y' + y = 0$

Equaz. caratt. $\quad \alpha^2 - 2\alpha + 1 = 0$

$$\Delta = 4 - 4(1) = 0 \qquad \alpha = 1 \text{ (molt. 2)}$$

Integ. gen. dell'omogenea $\quad y(x) = K_1 e^x + x K_2 e^x$

Somiglianza $\quad h = 1 \quad r = 2 \quad m = 1$

$$\bar{y}(x) = e^x x^2 (ax + b) = e^x (ax^3 + bx^2)$$

$$\bar{y}'(x) = e^x (ax^3 + bx^2 + 3ax^2 + 2bx)$$

$$\bar{y}''(x) = e^x (ax^3 + bx^2 + 6ax^2 + 4bx + 6ax + 2b)$$

$$\cancel{e^x}(\cancel{ax^3} + \cancel{bx^2} + \cancel{6ax^2} + \cancel{4bx} + 6ax + 2b - \cancel{2ax^3} - \cancel{2bx^2} - \cancel{6ax^2} - \cancel{4bx} + \cancel{ax^3} + \cancel{bx^2}) = \cancel{e^x}(x+3)$$

$$6ax + 2b = x + 3$$

$$\begin{cases} a = \frac{1}{6} \\ b = \frac{3}{2} \end{cases} \qquad \bar{y}(x) =$$

Integ. generale della completa $\quad y(x) = e^x \left(\frac{1}{6}x^3 + \frac{3}{2}x^2\right) + K_1 e^x + x K_2 e^x$

$y'' - 9y = x + 1$

Omogenea associata $\quad y'' - 9y = 0$

Eq. caratt. $\quad \lambda^2 - 9 = 0$

$\lambda = \pm 3$

Integ. gener. omogenea $\quad y(x) = K_1 e^{-3x} + K_2 e^{3x}$

$h = 0 \qquad s = 0 \qquad m = 1$

$\bar{y}(x) = ax + b$

$\bar{y}'(x) = a$

$y''(x) = 0$

Int part. della completa $\Rightarrow \bar{y}(x) = -9ax - 9b = x + 1$

$$\begin{cases} a = -\frac{1}{9} \\ b = -\frac{1}{9} \end{cases} \qquad \bar{y}(x) = -\frac{1}{9}x - \frac{1}{9}$$

Int gener. della completa $\Rightarrow \quad y(x) = -\frac{1}{9}x - \frac{1}{9} + K_1 e^{-3x} + K_2 e^{3x}$

$$\begin{cases} y'' + 4y = xe^x \\ y(0) = 0 \\ y'(0) = 3 \end{cases}$$

Risolvo $y'' + 4y = xe^x$

Omogenea ass. $y'' + 4y = 0$

Eq Caratt. $\lambda^2 + 4 = 0$

$\lambda = \pm 2i$

Integ. gen. dell'omogenea $e^{2ix} = \cos 2x + i \operatorname{sen} 2x$

$$y(x) = K_1(\cos(2x) + i \operatorname{sen}(2x)) + K_2(\cos(2x) - i \operatorname{sen}(2x))$$

$$= K_1 + K_2 (\cos(2x) + i (K_1 - K_2) \operatorname{sen}(2x))$$

$$= K_1 \cos(2x) + K_2 \operatorname{sen}(2x)$$

$$K_1(\cos 2x) + K_1 i \operatorname{sen}(2x) + K_2(\cos(2x)) - K_2 i \operatorname{sen}(2x)$$

$$\underbrace{(K_1 + K_2)}_{K_1}(\cos(2x)) + i \underbrace{(K_1 - K_2)}_{K_2}(\operatorname{sen}(2x))$$

$$K_1(\cos(2x)) + K_2(\operatorname{sen}(2x))$$

$$y'' + 4y = xe^x$$

$h = 1 \qquad s = 0 \qquad m = 1$

$$\bar{y}(x) = e^x(ax+b)$$
$$\bar{y}'(x) = e^x(ax+b+a)$$
$$\bar{y}''(x) = e^x(ax+b+2a)$$

$$\cancel{e^x}(5ax+2a+5b) = \cancel{e^x}\,x$$

$$\begin{cases} 5a = 1 \\ 2a+5b = 0 \end{cases} \qquad \begin{cases} a = \frac{1}{5} \\ b = -\frac{2}{25} \end{cases}$$

Integ. part. della completa $\quad \bar{y}(x) = e^x\left(\frac{1}{5}x - \frac{2}{25}\right)$

Integ. gen. della completa $\quad y(x) = K_1 \cos(2x) + K_2 \sin(2x) + e^x\left(\frac{1}{5}x - \frac{2}{25}\right)$

$$\begin{cases} y'' + 4y = xe^x \\ y(0) = 0 \\ y'(0) = 3 \end{cases} \qquad y'(x) = -2K_1 \sin 2x + 2K_2 \cos 2x + e^x\left(\frac{1}{5}x + \frac{3}{25}\right)$$

$$K_2 = \frac{72}{50}$$

$$y(0) = 0 \Rightarrow K_1 = \frac{2}{25}$$

$$y(x) = \frac{2}{25}\cos(2x) + \frac{72}{50}\sin(2x) + e^x\left(\frac{1}{5}x - \frac{2}{25}\right)$$

$$\begin{cases} y'' - 3y' = 3x + 1 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Risolvo l'equazione differenziale

$$y'' - 3y' = 3x + 1$$

Omogenea associata $\quad y'' - 3y' = 0$

Equaz. caratteristica $\quad \lambda^2 - 3\lambda = 0 \quad \lambda(\lambda - 3) = 0 \quad \lambda \to 0,\ 3$

Sol. gener. dell'omogenea $\quad y(x) = K_1 + K_2 e^{3x}$

Somiglianza

$$h = 0 \qquad s = 1 \qquad m = 1$$

$$\bar{y}(x) = ax^2 + bx$$

$$\bar{y}'(x) = 2ax + b$$

$$\bar{y}''(x) = 2a$$

Sostituisco nell'equazione $\quad 2a - 6ax - 3b = 3x + 1$

$$\begin{cases} -6a = 3 \\ 2a - 3b = 1 \end{cases} \qquad \begin{cases} a = -\frac{1}{2} \\ b = -\frac{2}{3} \end{cases}$$

Int. part. della completa $\quad \bar{y}(x) = -\frac{1}{2}x^2 - \frac{2}{3}x$

Int. gener. della completa $\quad y(x) = -\frac{1}{2}x^2 - \frac{2}{3}x + K_1 + K_2 e^{3x}$

$y'(0) = 0 \qquad -x - \frac{2}{3} + 3K_2 e^{3x}$

$$-\frac{2}{3} + 3K_2 = 0$$

$$K_2 = \frac{2}{9}$$

$y(0) = 1 \qquad K_1 = 1 - K_2$

$$K_1 = \frac{7}{9}$$

$$\underline{y(x) = -\frac{1}{2}x^2 - \frac{2}{3}x + \frac{7}{9} + \frac{2}{9}e^{3x}}$$

$$\begin{cases} y' - xy = 3x \\ y(1) = 0 \end{cases}$$

Risolvere $\quad y' - xy = 3x \quad (1)$

Equaz. differenziale lineare del primo ordine

$$y' - xy = 0 \qquad (2) \qquad X(x) = x \quad (a,b) = \mathbb{R}$$
$$Y(y) = y \quad (c,d) = \mathbb{R}$$

1° categoria $\quad y = 0 \quad H = \{0\}$

2° categoria $\quad \dfrac{y'(x)}{y(x)} = x$

Sol. gen. omogenea $\quad y(x) = K e^{\frac{x^2}{2}}$

Sol. part. omogenea $\quad y(x) = K(x)\, e^{\frac{x^2}{2}}$

con $K \in \displaystyle\int 3x \cdot e^{-\frac{x^2}{2}}\, dx \qquad K = -3 e^{\frac{x^2}{2}}$

Sol. gen. della completa $\quad y(x) = -3 e^{\frac{x^2}{2}} e^{-\frac{x^2}{2}} + K e^{\frac{x^2}{2}} = -3 + K e^{\frac{x^2}{2}}$

$y(1) = 0 \quad \Rightarrow \quad -3+K = 0$

$K = 3$

Soluzione del PC $\quad y(x) = -9 + e^{\frac{x^2}{2}}$

$$y' = \frac{1}{xy}$$

$X(x) = \frac{1}{x} \qquad (a,b) =]-\infty, 0[\text{ opp }]0, +\infty[$

$Y(y) = \frac{1}{y} \qquad (c,d) =]-\infty, 0[\text{ opp }]0, +\infty[$

Sol I categ $\Rightarrow H = \emptyset$

Sol II categ $\Rightarrow \quad y'(x) \cdot y(x) = \frac{1}{x}$

$y(x) = \sqrt{2 \ln|x| + K}$

Assumendo di lavorare in $]0, +\infty[$ le soluzioni sono

$K > 0 \qquad y(x) = \sqrt{2 \ln|x| + K}$

$$y' = \frac{\sqrt{y}}{\sqrt{x}}$$

$X(x) = \frac{1}{\sqrt{x}}$ $\quad (a,b) =]0, +\infty[$

$Y(y) = \sqrt{y}$ $\quad (c,d) = [0, +\infty[$

1° categoria $\quad H = \{0\}$

2° categoria $\quad 2 \cdot \frac{y'(x)}{2\sqrt{y(x)}} = 2 \cdot \frac{1}{2\sqrt{x}}$

$\underline{D(\sqrt{y})} = \frac{1}{2\sqrt{y}}$

$\underline{D(\sqrt{x})} = \frac{1}{2\sqrt{x}}$

$$2 \cdot \sqrt{y(x)} = 2\sqrt{x} + K$$

$y(x) = x + K$ $\quad$ (solo $K \geq 0$)

$$y' = -\frac{2x}{1+x^2}y + \frac{1}{x(1+x^2)}$$

$X(x) = -\dfrac{2x^2}{1+x^2}$

$(a,b) = \mathbb{R}$

$Y(y) = y$

$(c,d) = \mathbb{R}$

$H = \{\emptyset\}$

$$\frac{y'(x)}{y(x)} = -\frac{2x^2}{1+x^2}$$

$$\ln|y(x)| = -x - 2\,\mathrm{arctg}\,x + K$$

$$y(x) = Ke^{-x-2\,\mathrm{arctg}\,x}$$

$$\begin{cases} y'' + 2y' - 3y = 0 \\ y(1) = 0 \\ y'(1) = 1 \end{cases}$$

Risolvo $y'' + 2y' - 3y = 0$

Equaz. caratt. $\lambda^2 + 2\lambda - 3 = 0$ $\quad \Delta = 4 + 12 = 16$

$$\lambda = \frac{-2 \pm 4}{2} = \begin{cases} \to 1 \\ \downarrow -3 \end{cases}$$

Int. gen.

$$y(x) = K_1 e^x + K_2 e^{-3x}$$

$$y'(x) = K_1 e^x - 3K_2 e^{-3x}$$

$$\begin{cases} y(1) = 0 \\ y'(1) = 1 \end{cases} \qquad \begin{cases} K_1 e + K_2 e^{-3} = 0 \\ K_1 e - 3K_2 e^{-3} = 1 \end{cases} \qquad \begin{cases} K_2 = -\frac{e^3}{4} \\ K_1 = \frac{1}{4e} \end{cases}$$

Sol. al PC $\quad y(x) = \frac{e^x}{4e} - \frac{e^{3-3x}}{4}$

$$y' = \frac{\sqrt{y^2+1}}{x\,y}$$

$X(x) = \frac{1}{x}$ $\quad (a,b) =]-\infty, 0[$ opp. $]0, +\infty[$

$Y(y) = \frac{\sqrt{y^2+1}}{y}$ $\quad (c,d) =]-\infty, 0[$ opp. $]0, +\infty[$

Cerco le soluzioni di I categoria

$H = \{ \emptyset \}$

Sia $y : (\alpha, \beta) \to \mathbb{R}$ una soluzione di seconda categoria $y(x) \neq 0 \ \forall x \in (\alpha, \beta)$

$$\frac{y'(x)\, y(x)}{\sqrt{y^2(x)+1}} = \frac{1}{x}$$

$$D\left(\frac{1}{\sqrt{y(x)^2+1}}\right) = \frac{\cancel{2}\, y(x)}{\cancel{2}\sqrt{y(x)^2+1}}$$

$$\sqrt{y^2(x)+1} = \log|x| + K$$

$$y^2(x) = (\log|x| + K)^2 - 1$$

$(\alpha, \beta) \Rightarrow (\log|x| + K)^2 > 1$

$\log|x| > 1 - K$

$x > e^{1-K}$

quindi $(\alpha, \beta) =]e^{1-K}, +\infty[$

$$y(x) = \sqrt{(\log|x| + K)^2 - 1}$$

$y'' + 2y' - 8y = e^x\ (x^2 + 1)$

Omogenea an. $\quad y'' + 2y' - 8y = 0$

Eq. caratt. $\quad \lambda^2 + 2\lambda - 8 = 0$

$\Delta = 4 + 32 = 36$

$\lambda = -1 \pm 3 = \begin{cases} \to -4 \\ \to 2 \end{cases}$

Sol gen. omog. $\quad y(t) = K_1 e^{2t} + K_2 e^{-4x}$

Metodo di somiglianza

$h = 1 \qquad s = 0 \qquad m = 2$

$\bar{y}(x) = e^x (ax^2 + bx + c)$

$\bar{y}'(x) = e^x (ax^2 + bx + c + 2ax + b)$

$\bar{y}''(x) = e^x (ax^2 + bx + c + 4ax + 2b + 2a)$

Sost. nell' equaz.

$\cancel{e^x}(ax^2 + bx + c + 4ax + 2b + 2a + 2ax^2 + 2bx + 2c + 4ax + 2b - 8ax^2 - 8bx - 8c) = \cancel{e^x}(x^2 + 1)$

$-5ax^2 - 5bx - 5c + 8ax + 4b + 2a = x^2 + 1$

$\begin{cases} -5a = 1 \\ -5c + 4b + 2a = 1 \\ -5b + 8a = 0 \end{cases} \qquad \begin{cases} a = -\frac{1}{5} \\ b = -\frac{8}{25} \\ c = -\frac{42}{125} \end{cases}$

Sol. part. completa $\qquad \bar{y}(x) = e^{x}\left(-\frac{1}{5}x^2 - \frac{8}{25}x - \frac{42}{125}\right)$

Sol gen completa $\qquad y(x) = e^{x}\left(-\frac{1}{5}x^2 - \frac{8}{25}x - \frac{42}{125}\right) + K_1 e^{2x} + K_2 e^{-4x}$

$$\begin{cases} y'' - 3y' = 3x + 1 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Risolvo $\qquad y'' - 3y' = 3x + 1$

Omog. ass. $\qquad y'' - 3y' = 0$

Eq. caratt $\qquad \lambda^2 - 3\lambda = 0$

$\lambda(\lambda - 3) = 0 \qquad \lambda = 0, \; 3$

Sol gen. omog $\qquad y(x) = K_1 + K_2 e^{3x}$

$h = 0 \qquad s = 1 \qquad m = 1$

$\bar{y}(x) = ax^2 + bx$

$\bar{y}'(x) = 2ax + b$

$\bar{y}''(x) = 2a$

Sost. nell' eq. $y'' - 3y' = 3x + 1$

$$2a - 6ax - 3b = 3x + 1$$

$$\begin{cases} -6a = 3 \\ 2a - 3b = 1 \end{cases} \quad \begin{cases} a = -\frac{1}{2} \\ b = -\frac{2}{3} \end{cases}$$

Sol part compl $\bar{y}(x) = -\frac{x^2}{2} - \frac{2x}{3}$

Sol gen. completa $y(x) = -\frac{x^2}{2} - \frac{2x}{3} + K_1 + K_2 e^{3x}$

$$y'(x) = -x - \frac{2}{3} + 3K_2 e^{3x}$$

$$\begin{cases} y(0) = 1 \\ y'(0) = 0 \end{cases} \quad \begin{cases} K_1 + K_2 = 1 \\ -\frac{2}{3} + 3K_2 = 0 \end{cases} \quad \begin{cases} K_2 = \frac{2}{9} \\ K_1 = \frac{7}{9} \end{cases}$$

Sol. al PC $y(x) = -\frac{x^2}{2} - \frac{2x}{3} + \frac{7}{9} + \frac{2}{9} e^{3x}$

$$\begin{cases} y'' + 2y' - 3y = 0 \\ y(1) = 0 \\ y'(1) = 1 \end{cases}$$

Risolvo $\quad y'' + 2y' - 3y = 0$

Eq. caratt. $\quad \alpha^2 + 2\alpha - 3 = 0$

$\Delta = 4 + 12 = 16$

$\alpha = -1 \pm 2 = \rightarrow 1$, $\downarrow -3$

Int gen. $\quad y(x) = K_1 e^x + K_2 e^{-3x}$

$y'(x) = K_1 e^x - 3 K_2 e^{-3x}$

$$\begin{cases} y(1) = 0 \\ y'(1) = 1 \end{cases} \quad \begin{cases} K_1 e + K_2 e^{-3} = 0 \\ K_1 e - 3 K_2 e^{-3} = 1 \end{cases} \quad \begin{cases} K_1 = \frac{1}{4e} \\ K_2 = -\frac{e^3}{4} \end{cases}$$

Sol. del PC $\quad y(x) = \frac{1}{4e} e^x - \frac{e^3}{4} e^{-3x}$

$$y' = \frac{\sqrt{y^2+1}}{xy}$$

$X(x) = \frac{1}{x}$ $\quad (a,b) =]-\infty, 0[$ opp. $]0, +\infty[$

$Y(y) = \frac{\sqrt{y^2+1}}{y}$ $\quad (c,d) =]-\infty, 0[$ opp. $]0, +\infty[$

I categ. $\frac{\sqrt{y^2+1}}{y} = 0 \Rightarrow H\{\emptyset\}$

Sia ora y una sol. di II categoria $(\alpha, \beta) \subseteq (a,b)$

$y : (\alpha, \beta) \Rightarrow \mathbb{R}$

$$\frac{y'(x)\, y(x)}{\sqrt{y^2(x)+1}} = \frac{1}{x}$$

$$\sqrt{y^2(x)+1} = \log|x| + K \qquad \log|x| + K > 0$$

$$y^2(x) = (\log|x| + K)^2 - 1 \qquad (\log|x| + K)^2 > 1$$

$$\log|x| > 1 - K$$

$$y(x) = \sqrt{(\log|x| + K)^2 - 1} \qquad x > e^{1-K}$$

$$(\alpha, \beta) =]e^{1-K}, +\infty[$$

(c) $y' = -\frac{2x}{1+x^2}y + \frac{1}{x(1+x^2)}$

Scrivo l' omog. $y' = -\frac{2x}{1+x^2}y$

$X(x) = -\frac{2x}{1+x^2}$ $(a,b) = \mathbb{R}$

$Y(y) = y$ $(c,d) = \mathbb{R}$

$H = \{0\}$ $\nexists$ cost.

Sia $y(x)$ una sol. di $\nexists$ cost. $y : (\alpha,\beta) \subseteq (a,b) \to \mathbb{R}$

$\Rightarrow$ Int gen. omog. $y(x) = K e^{-\log|x^2+1|}$ $K \in \mathbb{R}$

$\bar{y}(x) = K(x) e^{-\log|x^2+1|}$ K deriv. in (α,β)

$$K(x) \in \int \frac{\cancel{x^2+1}}{x\cancel{(x^2+1)}} dx$$

$K(x) = \log|x| + c$

$\Rightarrow$ Int part completa $\bar{y}(x) = \log|x| e^{-\log|x^2+1|}$

Int gen. compl. $y(x) = \log|x| e^{-\log|x^2+1|} + K e^{-\log|x^2+1|}$

$$y'' + 2y' - 15y = (2x+1)\, e^{x}$$

Omogenea associata $\quad y'' + 2y' - 15y = 0$

Eq. caratt. $\quad \lambda^2 + 2\lambda - 15 = 0$

$\Delta = 4 + 4(15) = 4 + 60 = 64$

$\lambda = -1 \pm 4 = \begin{cases} 3 \\ -5 \end{cases}$

Sol gen. omog. $\quad y(x) = K_1 e^{3x} + K_2 e^{-5x}$

$h = 1 \qquad s = 0 \qquad m = 1$

$\bar{y}(x) = e^{x}(ax + b)$

$\bar{y}'(x) = e^{x}(ax + b + a)$

$\bar{y}''(x) = e^{x}(ax + b + 2a)$

Sostituisco $\quad \cancel{e^{x}}(ax + b + 2a + 2ax + 2b + 2a - 15ax - 15b) = \cancel{e^{x}}(2x+1)$

$$\begin{cases} -12a = 2 \\ 4a - 12b = 1 \end{cases} \quad \begin{cases} a = -\dfrac{1}{6} \\ b = \dfrac{5}{36} \end{cases}$$

Sol part comp $\bar{y}(x) = e^x\left(-\frac{1}{6}x + \frac{5}{36}\right)$

Sol gen comp $y(x) = e^x\left(-\frac{1}{6}x + \frac{5}{36}\right) + K_1 e^{3x} + K_2 e^{-5x}$

$$y'' + 3y' - 4y = x^2\, e^x$$

Omogenea associata $y'' + 3y' - 4y = 0$

Eq. caratt $\lambda^2 + 3\lambda - 4 = 0$

$\Delta = 9 + 16 = 25$ $\lambda = \frac{-3 \pm 5}{2} = \begin{cases} 1 \\ -4 \end{cases}$

Sol gen omog $y(x) = K_1 e^x + K_2 e^{-4x}$

$h = 1$ $s = 1$ $m = 2$

$\bar{y}(x) = e^x\left(ax^3 + bx^2 + cx\right)$

$\bar{y}'(x) = e^x\left(ax^3 + bx^2 + cx + 3ax^2 + 2bx + c\right)$

$\bar{y}''(x) = e^x\left(ax^3 + bx^2 + cx + 6ax^2 + 4bx + 2c + 2b + 6ax\right)$

Sost nell' equaz. $y'' + 3y' - 4y = x^2 e^x$

$$e^x\left(\cancel{ax^3}+\cancel{bx^2}+\cancel{cx}+6ax^2+4bx+2c\overset{+6ax}{}+2b+3\cancel{ax^3}+3\cancel{bx^2}+3\cancel{cx}+9ax^2+6bx+3c+\right.$$

$$\left.-\cancel{4ax^3}-4\cancel{bx^2}-4\cancel{cx}\right)=\cancel{e^x}x^2$$

$$15ax^2+6ax+10bx+2b+5c=x^2$$

$$\begin{cases}15a=1\\6a+10b=0\\2b+5c=0\end{cases}\qquad\begin{cases}a=\frac{1}{15}\\b=-\frac{6}{150}\\c=\frac{30}{75}\end{cases}$$

Sol part complete $\bar{y}(x)=e^x\left(\frac{1}{15}x^3-\frac{6}{150}x^2+\frac{30}{75}x\right)$

Sol gen compl $y(t)=e^x\left(\frac{1}{15}x^3-\frac{6}{150}x^2+\frac{30}{75}x\right)+K_1e^x+K_2e^{-4x}$

$$\begin{cases} y'' - y' = e^{2x}(x-3) + e^{x}x^2 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Risolvo $\quad y'' - y' = e^{2x}(x-3) + e^{x}x^2$

Omogenea associata $\quad y'' - y' = 0$

Equaz. caratt. $\quad \alpha^2 - \alpha = 0$

$\alpha(\alpha - 1) = 0 \qquad \alpha = \begin{matrix} 1 \\ 0 \end{matrix}$

Sol. gen. omogenea $\quad y(x) = K_1 + K_2 e^{x}$

cons $\quad y'' - y' = e^{2x}(x-3) \quad$ e $\quad y'' - y' = e^{x}x^2$

$(1) \qquad\qquad (2)$

1)

$h = 2 \qquad s = 0 \qquad m = 1$

$\bar{y}(x) = e^{2x}(ax + b)$

$\bar{y}'(x) = e^{2x}(2ax + 2b + a)$

$\bar{y}''(x) = e^{2x}(4ax + 4b + 4a)$

Sostituisco in (1)

$\cancel{e^{2x}}(4ax + 4b + 4a - 2ax - 2b - a) = \cancel{e^{2x}}(x-3)$

$2ax + 3a + 2b = x - 3$

$$\begin{cases} 2a = 1 \\ 3a + 2b = -3 \end{cases} \qquad \begin{cases} a = \frac{1}{2} \\ b = -\frac{9}{4} \end{cases}$$

$$\bar{y}(x) = e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right)$$

2) $h = 1 \qquad s = 1 \qquad m = 2$

$$\bar{y}(x) = e^{x}(ax^3 + bx^2 + cx)$$

$$\bar{y}'(x) = e^{x}(ax^3 + bx^2 + cx + 3ax^2 + 2bx + c)$$

$$\bar{y}''(x) = e^{x}(ax^3 + bx^2 + cx + 6ax^2 + 4bx + 2c + 6ax + 2b)$$

Sostituisco in (2)

$$\cancel{e^{x}}(\cancel{ax^3} + \cancel{bx^2} + \cancel{cx} + 6ax^2 + 4bx + 2c + 6ax + 2b - \cancel{ax^3} - \cancel{bx^2} - \cancel{cx} - 3ax^2 - 2bx - c) = \cancel{e^{x}}x^2$$

$$3ax^2 + 6ax + 2bx + c + 2b = x^2$$

$$\begin{cases} 3a = 1 \\ 6a + 2b = 0 \\ 2b + c = 0 \end{cases} \qquad \begin{cases} a = \frac{1}{3} \\ b = -1 \\ c = 2 \end{cases}$$

$$\bar{y}(x) = e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$$

$\Rightarrow$ Integrale particolare della completa

$$y(x) = K_1 + K_2 e^{x} + e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right) + e^{x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$$

$$y'(x) = K_2 e^x + e^{2x}(x-4) + e^x\left(\frac{1}{3}x^3 - \cancel{x^2} + \cancel{2x} + \cancel{x^2} - \cancel{2x} + 2\right)$$

$$\begin{cases} y'' - y' = e^{2x}(x-3) + e^x x^2 \\ y(0) = 3 \\ y'(0) = 1 \end{cases}$$

$$\begin{cases} K_1 + K_2 - \frac{9}{4} = 3 \\ K_2 - 4 + 2 = 1 \end{cases} \qquad \begin{cases} K_1 = -\frac{9}{4} \\ K_2 = 3 \end{cases}$$

Sol al PC $\qquad y(x) = -\frac{9}{4} + 3e^x + e^{2x}\left(\frac{1}{2}x - \frac{9}{4}\right) + e^x\left(\frac{1}{3}x^3 - x^2 + 2x\right)$

$$y'' - y' + 3y = 0$$

Eq. caratt $\qquad \lambda^2 - \lambda + 3 = 0$

$\Delta = 1 - 12 = -11 \qquad \lambda = \frac{1 \pm \sqrt{-11}}{2} = \frac{1}{2} \pm i\frac{\sqrt{11}}{2}$

Int gen $\qquad y(t) = K_1 e^{\frac{1}{2}x} \cos \frac{\sqrt{11}}{2}x + K_2 e^{\frac{1}{2}x} \operatorname{sen} \frac{\sqrt{11}}{2}x$