$$\int \frac{\operatorname{arctg}^2 x + \operatorname{arctg} x}{1+x^2}\,dx$$

$\frac{1}{1+x^2}$ = Derivata de $\operatorname{arctg} x$

$t = \operatorname{arctg} x \Rightarrow x = \operatorname{tg} t$

$$\left[\int \frac{t^2 - t}{(1-\operatorname{tg}^2 t)}(1+\operatorname{tg}^2 t)\,dt\right]_{t=\operatorname{arctg}} = \left[\int t^2 - t\,dt\right]_{t=\operatorname{arctg} x} =$$

$$= \left[\frac{t^3}{3} - \frac{t^2}{2} + c\right]_{t=\operatorname{arctg} x} = \frac{\operatorname{arctg}^3 x}{3} - \frac{\operatorname{arctg}^2 x}{2} + c$$

$$\int \frac{\log^2 x - 3\log x + 1}{x}\,dx$$

$D(\log x) = \frac{1}{x}$

$$= \left[\int t^2 - 3t + 1\,dt\right]_{t=\log x} = \left[\frac{t^3}{3} - \frac{3}{2}t^2 + t + c\right]_{t=\log x}$$

$$= \frac{\log^3 x}{3} - \frac{3}{2}\log^2 x + \log x + c$$

$$\int \frac{\operatorname{sen} x}{1+\cos^2 x}\,dx = -\int -\frac{\operatorname{sen} x}{1+\cos^2 x}\,dx =$$

$$= -\left[\int \frac{1}{1-t^2}\,dt\right]_{t=\cos x} =$$

$$= -\operatorname{arctg} t = -\operatorname{arctg}(\cos x) + C$$

$$\int x \underset{\substack{\downarrow \\ FD}}{\operatorname{arctg} x}\,dx = x\,\frac{1}{x^2+1} - \int \frac{1}{x^2+1}\,dx$$

$$= x\,\frac{1}{x^2+1} - \operatorname{arctg} x + C$$

$$\int \frac{x+1}{x^3-6x^2+9x}\,dx \qquad x(x^2-6x+9)$$

$$\Delta = 0 \qquad x(x-3)^2$$

$$\frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2} = \frac{Ax^2-6Ax+9A+Bx^2-3Bx+Cx}{x(x-3)^2}$$

$$\begin{cases} A+B=0 \\ -6A+C-3B=1 \\ 9A=1 \end{cases} \qquad \begin{cases} B=-\frac{1}{9} \\ C=\frac{4}{3} \\ A=\frac{1}{9} \end{cases}$$

$$I = \frac{1}{9}\int \frac{1}{x}\,dx - \frac{1}{9}\int \frac{1}{x-3}\,dx + \frac{4}{3}\int \frac{1}{(x-3)^2}\,dx =$$

$$= \frac{1}{9}\log|x| - \frac{1}{9}\log|x-3| - \frac{4}{3}\,\frac{1}{x-3} + c$$

$$\int (x-3)^{-2} \Downarrow -\frac{(x-3)^{-1}}{1} \Downarrow -\frac{1}{x-3}$$

$$\int \frac{x+4}{x^2-x-6}\,dx \qquad \Delta = 25 \qquad x = \begin{cases} 3 \\ -2 \end{cases}$$

$$\frac{x+4}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2} = \frac{Ax - 3A + Bx + 2B}{(x+2)(x-3)}$$

$$\begin{cases} A + B = 1 \\ -3A + 2B = 4 \end{cases} \qquad \begin{cases} A = \frac{7}{5} \\ B = -\frac{2}{5} \end{cases}$$

$$I = \frac{7}{5}\int \frac{1}{x-3}\,dx - \frac{2}{5}\int \frac{1}{x+2}\,dx = \frac{7}{5}\log|x-3| - \frac{2}{5}\log|x+2| + c$$

$$\int \frac{x+1}{x^2+6x+10}\,dx \qquad \Delta < 0 \qquad \Rightarrow \quad (x+3)^2+1$$

$$\frac{1}{2}\int \frac{2(x+1)+4-4}{(x+3)^2+1}\,dx = \frac{1}{2}\int \frac{2x+6}{(x+3)^2+1}\,dx - 2\int \frac{1}{(x+3)^2+1}\,dx$$

$$= \frac{1}{2}\log|(x+3)^2+1| - 2\arctan x+3 + K$$

$$\int \frac{1}{\sqrt{x}+1}\,d\tau \qquad \tau = \sqrt{x} \Rightarrow x = \tau^2$$

$$\left[\int \frac{d\tau}{\tau+1}\cdot 2\tau\right]_{\tau=\sqrt{x}} = 2\left[\int \frac{\tau+1-1}{\tau+1}\right]_{\tau=\sqrt{x}} =$$

$$= 2\left[\int d\tau - \int \frac{d\tau}{\tau+1}\right]_{\tau=\sqrt{x}} =$$

$$= \left[2\tau - 2\log|\tau+1| + c\right]_{\tau=\sqrt{x}}$$

$$= 2\sqrt{x} - 2\log|\sqrt{x}+1| + c$$

$$f(x) = \frac{1}{\sqrt{x}+1} : [0+\infty[\to \mathbb{R}$$

$g = \sqrt{x}$

$g : [0,+\infty[\to \underbrace{[0,+\infty[}$

g deriv. in $[0,+\infty[$ Immagine = Dominio

crescente $\Rightarrow$ invert.

Inversa $\Rightarrow$ $\sqrt{x} = t$

$x = \underbrace{t^2}_{g^{-1}(t)}$

$$\int \cos(\log x)\,dx$$

$$\Downarrow$$

$$= \left[\int \cos t \cdot e^{t}\, dt\right]_{t=\log x}$$

FD

$$= \cos t\, e^{t} - \int -\operatorname{sen} t\, e^{t}\, dt$$

$$\cos t\, e^{t} - \left[\left(-\operatorname{sen} t\, e^{t}\right) - \int -\cos t\, e^{t}\, dt\right] =$$

$$\cos t\, e^{t} + \operatorname{sen} t\, e^{t} - \int \cos t\, e^{t}\, dt =$$

$$= \left[\frac{\cos t\, e^{t} + \operatorname{sen} t\, e^{t}}{2}\right]_{t=\log x}$$

$$= \frac{1}{2}\left(\cos(\log x)\, e^{\log x} + \operatorname{sen}(\log x)\, e^{\log x}\right) + K$$

$f(x) : \mathbb{R} \to \left]-1, 1\right[$

$g(x) = \log x : [0, +\infty[\Rightarrow \mathbb{R}$ ✓

L'Im di g deve coincidere con il dominio della funzione f

Cons $g_{|]1,+\infty[} = \log x :]1, +\infty[\Rightarrow \mathbb{R}$ ✓

g è deriv. in $]1, +\infty[$

g è invert.

$\Downarrow$

$\log x = t$

$x = e^{t}$

$$\int \frac{e^{x}}{e^{2x}+e^{x}-12}\,dx$$

$$\int \frac{e^{x}\cdot e^{x}}{(e^{2x}+e^{x}-12)e^{x}}\,dx \Rightarrow \left[\int \frac{\cancel{t}}{(t^{2}+t-12)\cancel{t}}\,dt\right]_{t=e^{x}}$$

$$= \frac{1}{t^{2}+t-12} = \frac{A}{t-3} + \frac{B}{t+4} = \frac{At+4A+Bt-3B}{(t-3)(t+4)}$$

$$\begin{cases} A+B=0 \\ 4A-3B=1 \end{cases} \qquad \begin{cases} B=-\frac{1}{7} \\ A=\frac{1}{7} \end{cases}$$

$$\left[\int \frac{1}{t^{2}+t-12}\,dt\right] = \frac{1}{7}\log|t-3| - \frac{1}{7}\log|t+4| + K$$

$$= \frac{1}{7}\log|e^{x}-3| - \frac{1}{7}\log(e^{x}+4) + K$$

$$\int \underset{\substack{\downarrow \\ FD}}{x} \operatorname{arctg} x \, dx = \frac{x^2}{2}\operatorname{arctg} x - \underbrace{\int \frac{x^2}{2} \frac{1}{x^2+1}\, dx}_{J}$$

$$= \frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} - \frac{1}{2}\operatorname{arctg} x + K$$

$$J = \int \frac{x^2+1}{x^2+1}\, dx - \int \frac{dx}{x^2+1} = x - \operatorname{arctg} x$$

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1) Enunciare e dimostrare la formula di integraz. definita per parti

1) Determinare la funzione F primitiva in $[0,4]$ di

$$f(x) = |x^2-2x| + 4x - 1 \quad \text{t.c.} \quad F(3) = 10$$

2) Scrivere la tangente al grafico di

$$F(x) = \int_1^{2x} \frac{\cos(\pi t)}{t^2}\, dt \qquad \text{in} \qquad x = 1$$

A) Metodo di Integrazione per parti

f, g derivabili e tali che $f'(x)\, g(x)$ sia dotata di primitive

Allora anche $f(x)\, g'(x)$ è dotata di primitive e si ha

$$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$$

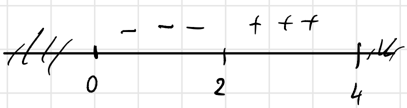
DIM

$$\int f g'\, dx = \int (f g' + f' g - f' g) = \int \overbrace{(f g' + f' g)}^{D(fg)} - \int f' g =$$

Decompongo in somma

$$= f g - \int f' g \qquad \text{CVD}$$

1) $f(x) = |x^2 - 2x| + 4x - 1$



$$f(x) = \begin{cases} x^2 + 2x - 1 & \text{in } [2, 4] \\ -x^2 + 6x - 1 & \text{in } [0, 2] \end{cases}$$

$$F(x) = \begin{cases} \dfrac{x^3}{3} + x^2 - \dfrac{x^2}{2} + c & \text{in } [0, 2] \\ -\dfrac{x^3}{3} + 3x^2 - x + k & \text{in }]2, 4] \end{cases}$$

impongo la cont. $\lim_{x \to 2^-} F(x) = \lim_{x \to 2^+} F(x)$

$$-\frac{8}{3} + 10 + K = \frac{8}{3} + 2 + c$$

$$c = K + \frac{8}{3}$$

$$F(x) = \begin{cases} -\frac{x^3}{3} + 3x^2 - x + K + \frac{8}{3} & \text{in } [0,2] \\ \frac{x^3}{3} + x^2 - \frac{x^2}{2} + c & \text{in } [2,4] \end{cases}$$

$F(3) = 10$ $9 + 9 - 3 + c = 10$

$$K = -5$$

$$F(x) = \begin{cases} -\frac{x^3}{3} + 3x^2 - x - 5 - \frac{8}{3} & \text{in } [0,2] \\ \frac{x^3}{3} + x^2 - \frac{x^2}{2} - 5 & \text{in } [2,4] \end{cases}$$

$$F(x) = \int_1^{2x} \frac{\cos(\pi t)}{t^2}\, dt \qquad \text{in} \qquad x=1$$

$$\boxed{y = f(c) + f'(c)(x-c)}$$

Calcolo $F'(x) = 2\,\frac{\cos(2\pi x)}{(2x)^2} = \frac{1}{2}\,\frac{\cos(2\pi x)}{x^2}$

$F'(1) = \frac{1}{2}$

$$F(1) = \int_1^2 \frac{\cos(\pi t)}{t^2}\, dt =$$

$$\int_1^x f t \; dt \qquad F(1) = 0$$

Invece

$$\int_1^{2x} f t \; dt \qquad F(1) = \int_1^2 f t \; dt$$