PHYS 512 Assignment 2

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Due Date: Sept. 25, 2020.

Problem 1

Refer to `A2_Q1_1.py`
We are using five points, so in the lazy method we make five function evaluations every time we split the interval in half. For my method, we start with five function evaluations, but we only make two function evaluations every time we split the interval in half. For the Lorentzian function, the old way used 35 function evaluations, where as my method used 17. For the exponential function, the old way used 25 function evaluations, where as my method used 13.

Problem 2

Refer to `A2_Q2_1.py`
The linear transformation (as seen in Burden's Numerical Analysis) for this interval is:

$$\bar{x} = \frac{1}{2}\left[(b-a)\,x + a + b\right]$$

where $a = 0.5$ and $b = 1$.
We needed seven terms, i.e. a sixth order polynomial, to get the desired error of 10^{-6}. This because the eight Chebyshev coefficient is approximately 1.8×10^{-6}.

```
rms error for chebyshev interpolation is  1.3746700693144719e-06  with max error  2.1237406535590253e-06
rms error for Legendre polyfit is  1.3352359837841491e-06  with max error  2.7615277148340667e-06
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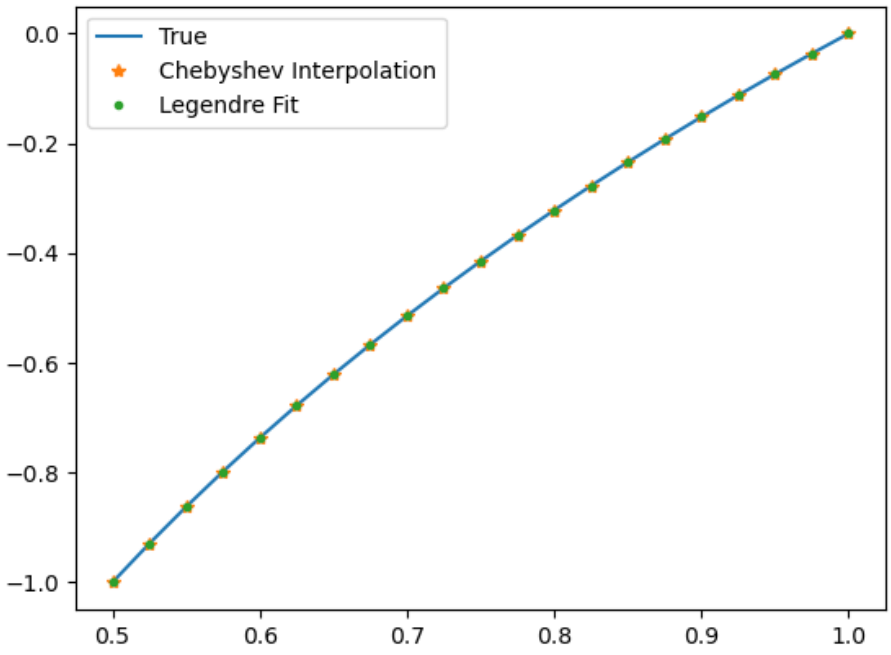
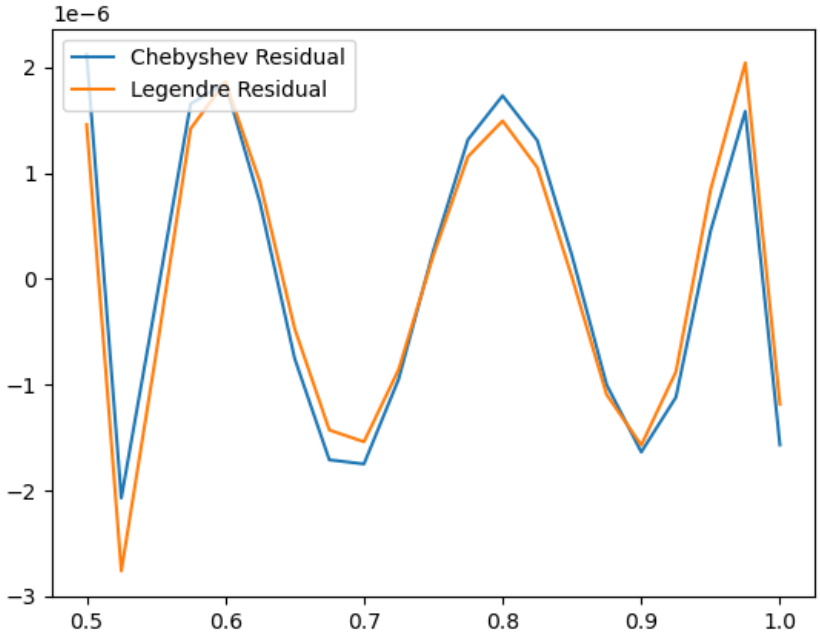
The Chebyshev fit has a larger RMS error, which makes sense because this fit spreads the error evenly throughout the whole function. The polynomial fit has the larger max error for the aforementioned reason.





Problem 3

Refer to `A2_Q3_1.py`
We need to use the solver that is for stiff equations. This is because the wide range of half-life values makes it a stiff system.

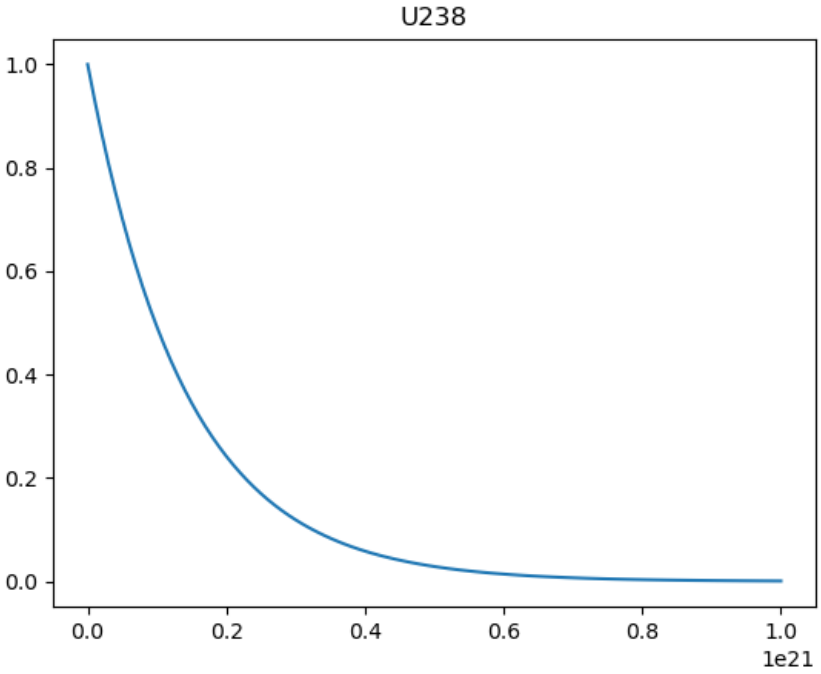


Figure 1: We see the expected exponential decay because this is the first element.

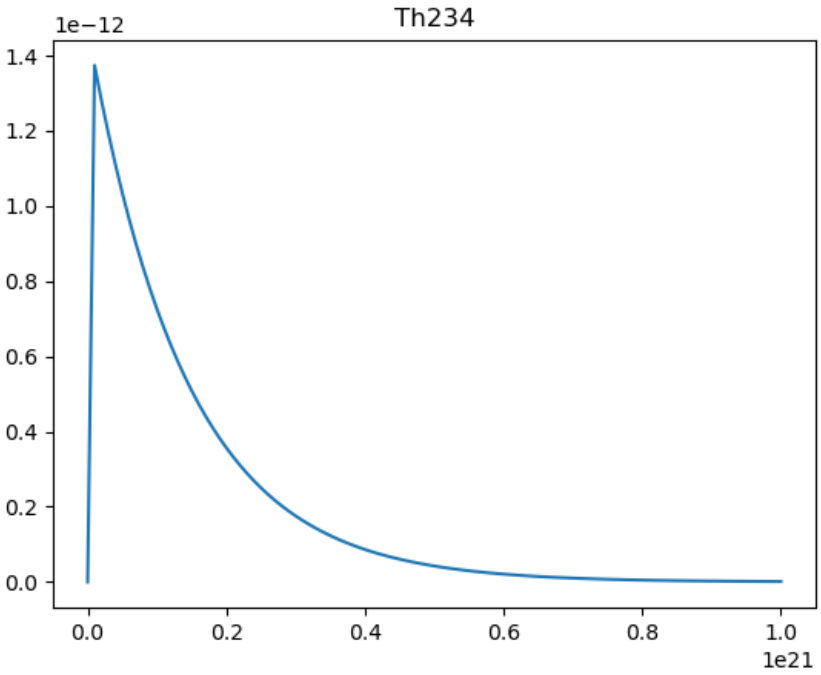


Figure 2: We see an increase in Th234 from when U238 was decaying. Then Th234 has exponential decay into the next element.

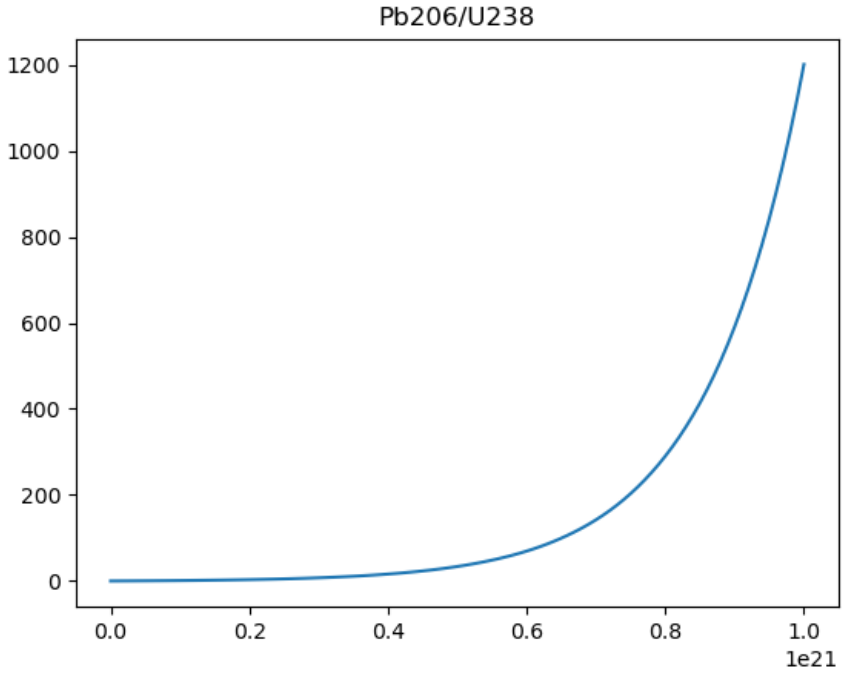


Figure 3: U238 decays into Pb206 so we expect to see growth. The solution is some combination of exponentials, so it makes sense that this growth looks exponential.

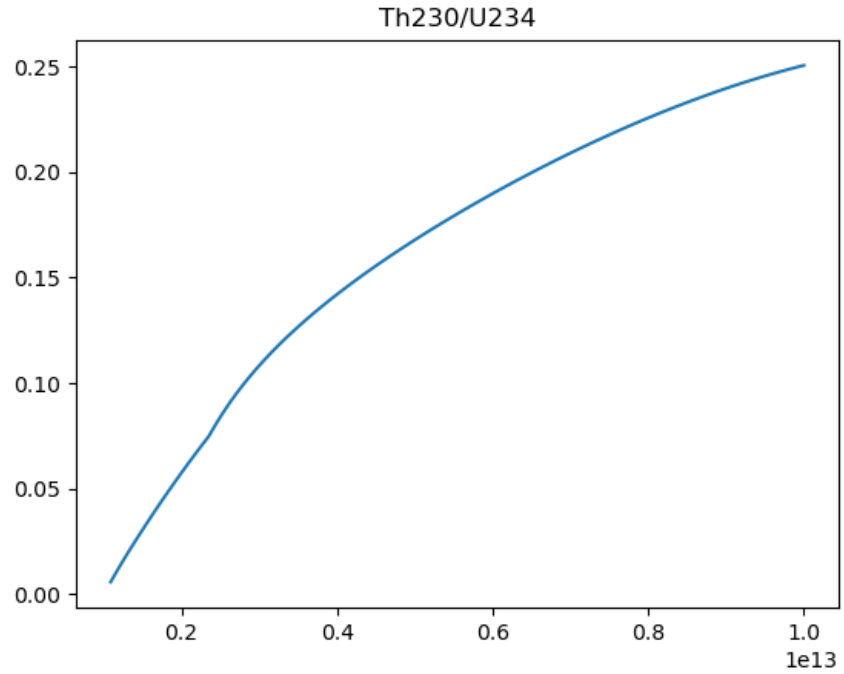


Figure 4: I'm not sure why the second derivative is negative here.