

## DISCUSSÃO SOBRE SISTEMAS LINEARES

Tarefa Básica

1)  $\begin{cases} ax + 4y = 1 \\ x + 2y = b \end{cases}$   $D = \begin{vmatrix} a & 4 \\ 1 & 2 \end{vmatrix} = 2a - 4$   $DX = \begin{vmatrix} 1 & 4 \\ b & 2 \end{vmatrix} = 2 - 4b$

$a = 2 \Rightarrow 0$   $2 \cdot 2 - 4 = 0$   $DX = 2 - 4b = 0$

$b = 1 \Rightarrow 0$   $2 - 4 \cdot 1 = 0$   $D = 2 - 4b = 0$   $B)$

$2$   $2$

$D \neq 0 \rightarrow SPD$

$D = 0 \rightarrow SPI$  ou  $SI$

$\downarrow$   $\downarrow$

$DX = 0$   $DX \neq 0$

$DY = 0$   $DY \neq 0$

2)  $\begin{cases} x + ky = 1 \\ kx + y = 1 - k \end{cases}$   $\rightarrow \begin{pmatrix} 1 & k & : & 1 \\ k & 1 & : & 1 - k \end{pmatrix} \rightarrow \begin{pmatrix} 1 & k & : & 1 \\ 0 & -k^2 + 1 & : & -2k + 1 \end{pmatrix}$

$k = -1 \Rightarrow y = (-2k + 1) \Rightarrow y = (-2(-1) + 1) = 1$

$k = 1 \Rightarrow 0 \quad (-k^2 + 1)$

S. Impensável

3)  $\begin{cases} x + 2y + cz = 1 \\ y + z = 2 \\ 3x + 2y + 2z = -1 \end{cases}$

$3c + 2 + 0 = 1$

a)  $A = \begin{vmatrix} 1 & 2 & c & | & 1 \\ 0 & 1 & 1 & | & 2 \\ 3 & 2 & 2 & | & -1 \end{vmatrix}$   $\det = 8 - 3c - 2$   $b) \quad 6 - 3c \neq 0$

$\det = 6 - 3c$   $6 = 3c$

$-3$

$-2 = c$

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4)  $\begin{cases} x - y = k \\ 12x - ky + z = 1 \\ 36x + kz = 2 \end{cases}$

$0 + 0 + (-12k) = -12k$

$D = \begin{vmatrix} 1 & -1 & 0 & 1 & -k \\ 12 & -k & 1 & 12 & -k \\ 36 & 0 & k & 36 & 0 \end{vmatrix} = \det = -k^2 - 36 = 12k$

$-x^2 + (-36) + 0 = -x^2 - 36$

$D \neq 0 \rightarrow -k^2 - 36 - 12k (-1)$

$k^2 + 12k + 36 \quad 6 \cdot 6 = 36 \rightarrow 6 + 6 = 12 \quad k \neq 6$

$\uparrow \quad \uparrow \quad \nabla \quad 6 + 6 = 12 \quad E)$

$S \quad (P) \quad 6 \cdot 6 = 36$

$$5) \begin{cases} x - y + z = 6 \\ 2x + y - z = -3 \\ x + 2y - z = -5 \end{cases} \quad \begin{cases} x - y + z = 6 \\ 2x + y - z = -3 \\ 3y - 2z = -11 \end{cases} \quad \begin{cases} x - y + z = 6 \\ 3y - 3z = -15 \\ 0 - z = -4 \end{cases}$$

$$z = 4$$

$$3y - 2 \cdot 4 = -11 \quad | \quad 3y - 8 = -11$$

x, y, z

$$1 \cdot -1 \cdot 4 = -4$$

B)

$$3y = -3$$

$$y = -1$$

$$x + 1 + 4 = 6$$

$$x + 5 = 6$$

$$x = 6 - 5$$

$$x = 1$$

$$6) \begin{cases} x + y + z = k \\ kx + y + z = 1 \\ x + y - z = k \end{cases} \quad D = \begin{vmatrix} 1 & 1 & 1 \\ k & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2k - 2$$

$$k = 1 \rightarrow D = 0$$

$$\begin{cases} x + y + z = 1 \\ x + y + z = 1 \\ x + y - z = 1 \end{cases} \quad \begin{cases} x + y + z = 1 \\ -x + y - z = 1 \\ 0 + 0 - 2z = 0 \end{cases} \quad \begin{cases} x + y = 1 \\ x = 1 - y \\ z = 0 \end{cases} \quad k = 1, (1 - y, y, 0)$$

$$7) \begin{cases} x + y + z = 1 \\ mx - 2y + 4z = 5 \\ m^2x + 4y + 16z = 25 \end{cases} \quad D = \begin{vmatrix} 1 & 1 & 1 \\ m & -2 & 4 \\ m^2 & 4 & 16 \end{vmatrix} = 6m^2 - 12m - 48$$

$$6m^2 - 12m - 48 = 0$$

$$\Delta = 12^2 - 4 \cdot 6 \cdot (-48)$$

$$\Delta = 144 + 1152$$

$$\Delta = 1296$$

$$x_1 = 12 + 36 = 48 = 4$$

$$2 \cdot 6$$

$$12$$

$$4 - 2 = 2$$

$$x_2 = 12 - 36 = -24 = -2$$

$$2 \cdot 6$$

$$12$$

## SISTEMAS LINEARES HOMOGÊNEOS

### Tarefa Básica - Sistemas Lineares Homogêneos

$$1) \begin{bmatrix} 1 & 7 \\ 7 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = k \cdot \begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{cases} x + 7y = kx \\ 7x + y = ky \end{cases}$$

$$D = \begin{bmatrix} 1 & 7 \\ 7 & 1 \end{bmatrix} = 1 - 49 = -48$$

$$DX = \begin{bmatrix} k & 7 \\ k & 1 \end{bmatrix} = k - 7k = -6k$$

$$\rightarrow \begin{pmatrix} 1 & 7 & k \\ 7 & 1 & k \end{pmatrix} \sim \begin{pmatrix} : \\ 0 & -48 & -6k \end{pmatrix}$$

$$\rightarrow z = -48 = -6k$$

$$D \neq 0 \rightarrow -48 \neq -6k$$

$$-48 = k$$

$$-6$$

$$8 = k$$

$$2) \begin{cases} 3x + 4y - z = 0 \\ 2x - y + 3z = 0 \\ x + y = 0 \end{cases}$$

$$D = \begin{bmatrix} 3 & 4 & -1 \\ 2 & -1 & 3 \\ 1 & 1 & 0 \end{bmatrix}$$

$$1 + 9 + 0 = 10$$

$$2 - 1 = 10 - 10 = 0$$

$$DX = \begin{bmatrix} 0 & 4 & -1 \\ 0 & -1 & 3 \\ 0 & 1 & 0 \end{bmatrix}$$

$$x = Dx = 0$$

$$0 + 12 + (-2) = 10$$

$$y = Dy = 0, z = Dz = 0 \quad \text{S.P.T}$$

$$D \quad 0 \quad 0 \quad 0 \quad D)$$

$$Dy = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix}$$

$$Dz = \begin{bmatrix} 3 & 4 & 0 \\ 2 & -1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

tilibra



$$3) \begin{cases} x+y+z=0 \\ kx+3y+4z=0 \\ x+ky+3z=0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ k & 3 & 4 \\ 1 & k & 3 \end{vmatrix} = 3+4k+3k = 3+7k$$

$$\text{Soma } k \quad 9+4+k^2 = 13+k^2$$

$$13+k^2 - 13+7k$$

$$2+5=7$$

$$k^2+7k+10=0$$

$$2+5=7$$

$$2 \cdot 5 = 10$$

$$4) \begin{cases} x+kz=0 \\ kx+y=0 \\ x+ky=0 \end{cases}$$

$$\begin{cases} DX=0 \\ DY=0 \\ DZ=0 \end{cases} \begin{cases} 1 & 0 & k=0 \\ k & 1 & 0=0 \\ 1 & k & 0=0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 0 & k \\ k & 1 & 0 \\ 1 & k & 0 \end{vmatrix} = k+0+0$$

$$k^3-k$$

$$k(k-1)(k+1)=0 \quad k \neq 0, k \neq 1, k \neq -1$$

$$k=-1, k=0, k=1$$

$$V = \{(k \in \mathbb{R} \mid k \neq 0, k \neq 1, k \neq -1)\}$$

A)

$$5) \begin{cases} -x+2y-3=0 \\ 3x-y+3=0 \\ 2x-4y+6=0 \end{cases}$$

$$D = \begin{vmatrix} -1 & 2 & -3 \\ 3 & -1 & 3 \\ 2 & -4 & 6 \end{vmatrix} = -6+(-12)+36 = 18$$

$$6+12+36=54$$

$$D \neq 0 \rightarrow 36$$

B)