

## MATRIZ INVERSA

## EXERCÍCIOS:

**TAREFA BÁSICA**

1)  $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$   $B = \begin{bmatrix} 3 & -1 \\ 7 & 2 \end{bmatrix}$   $\begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{cases} 3x + 7 = 1 \\ 15 + 3 = 0 \end{cases} \quad \begin{cases} -x + 2 = 0 \\ -5 + 6 = 1 \end{cases}$$

$$\begin{aligned} 3x + 7 &= 1 & 15 + 3 &= 0 & -x + 2 &= 0 \\ 3x &= 1 - 7 & & & -x &= -2 \\ 3x &= -6 & & & x &= 2 \end{aligned}$$

$$\begin{aligned} 3x + 7 &= 1 & & & & \\ 3(2) + 7 &= 1 & & & & \\ 6 + 7 &= 1 & & & & \\ 13 &= 1 & & & & \end{aligned}$$

$$\begin{aligned} 15 + 3 &= 0 & & & & \\ 15 + 3 &= 0 & & & & \\ 18 &= 0 & & & & \end{aligned}$$

$$\begin{aligned} -x + 2 &= 0 & & & & \\ -2 + 2 &= 0 & & & & \\ 0 &= 0 & & & & \end{aligned}$$

$$\begin{aligned} x &= 2 & & & & \\ x &= 2 & & & & \end{aligned}$$

$$\begin{aligned} x + y &= -3 & & & & \\ 2 + y &= -3 & & & & \\ y &= -5 & & & & \end{aligned}$$

2)  $A = \begin{pmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{pmatrix}$   $\begin{pmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ k & 1 \\ 1 & k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$

$$\begin{aligned} 1 + 3k &= 0 \\ 3 + k^2 - 3k - 1 &= 0 \\ k^2 - 3k + 2 &= 0 \end{aligned}$$

$$\Delta = (-3)^2 - 4 \cdot 1 \cdot 2$$

$$\Delta = 9 - 8$$

$$\Delta = 1$$

$$x = \frac{-(-3) \pm \sqrt{1}}{2 \cdot 1} = \frac{3 \pm 1}{2} = 2$$

$$\begin{aligned} 3 - 1 &= 2 = 1 & c) \\ 2 \cdot 1 &= 2 \end{aligned}$$

3) B inversa de  $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$   $\det A = 12 - 10 = 2 \neq 0$

$$B = A^{-1} \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} : 2$$

$$= \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix} \quad c)$$

$$20 + 2x + 3x = 5x - 20$$

4)  $\begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix}$   $\det =$

$$= 26 + x^2 - 5x - 20 = x^2 - 5x + 6 = 0$$

$$\Delta = (5)^2 - 4 \cdot 1 \cdot 6 = 25 - 24 = 1$$

$$x = \frac{5 \pm \sqrt{1}}{2} = \frac{5 \pm 1}{2} = 3 \text{ or } 2$$

$\{x=3 \text{ or } x=2\}$

A)  $5 - 1 = 4 = 2$

2 + 2 + 2 = 6

5)  $A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}$   $\det = 7 - 6 = 1$

1 + 2 + 4 = 7

$A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix}$

$A = \begin{pmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{pmatrix} + A^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{pmatrix} = A + A^{-1} = \begin{pmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{pmatrix}$

6)  $(X \cdot A)^T = B$

$X \cdot A \cdot A^{-1} = B \cdot A^{-1}$

$X = B^T \cdot A^{-1}$

7)  $B = \begin{bmatrix} x \\ y \end{bmatrix}$  and  $C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$

$A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix}$   $B = \begin{bmatrix} x \\ y \end{bmatrix}$   $C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$

$\det = 24 - 25 = -1$

$A^{-1} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div -1 \rightarrow A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$

$$8) A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix}$$

$$2 \cdot 1 - (-2)k$$

$$2k + 2 = \det A$$

$$2k + 2 = -1$$

$$2k = -3$$

$$k = -3/2$$

$$2k_1 + 2 = 1$$

$$2k_1 = -1$$

$$k_1 = -1/2$$

$$-\frac{1}{2} + \frac{(-3)}{2} = -\frac{4}{2} = -2$$

$$9) a) (A+B) \cdot (A-B)$$

$$A^2 - AB + BA - B^2$$

$$\neq 0$$

$$b) (A+B)^2 =$$

$$a^2 + ab + ba + b^2$$

$$ab = ba$$

$$c) \frac{\det(A)}{\det(-A)} = \frac{1}{1} = 1$$

$$\det(-A) = 1/1$$

$$d) B = A^{-1}$$

$$B = I$$

$$\det A$$

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