POD-based deflation techniques for two-phase flow simulation in large and highly heterogeneous porous media.

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Introduction

Reservoir simulation involves the solution of a set of partial differential equations which, often, lead to a system of linear equations. When simulating two phases, the solution of the pressure equation is the most time-consuming part, especially in the cases where the system is large or ill-conditioned. Furthermore, if it is required to compute a large number of simulations, the solution of this problem becomes expensive. Therefore, techniques to improve the simulation speed are required.

Reduced Order Models (ROM) have been studied to reduce the dimension of the system, by capturing relevant information and using it to project high-dimensional data into a lower-dimension space [1, 2, 3, 4, 5, 6]. These methods show that essential information of the system can be captured by computing a set of basis functions based on solutions of the system. These solutions are known as 'snapshots', where the relevant information of the system is stored for later use.

Proper Orthogonal Decomposition (POD) is a ROM method that has been studied in recent years to accelerate the solution of the pressure equation, resulting from reservoir simulation [7, 8, 9, 10, 11, 12] among other applications. With POD procedures, only few basis functions are necessary. The basis is such that it contains most of the variation of the original system [10, 2]. Hence, the system can be represented in terms of this basis. Once the basis is obtained, the POD method can be used in different ways. For the solution of a large-scale system, Markovinovic et al. [8] propose the use of POD techniques to compute a good initial guess that accelerates the solution of an iterative method. The solution of the problem in the small-scale domain and the projection back to the large-scale system is also approached by Astrid et al. [7]. Pasetto et al. [3] propose the use of this basis for the construction of a preconditioner. Diaz Cortes et al. [13] propose the use of the POD basis as deflation matrix.

If the system is not only large but also ill-conditioned (containing large variations in the coefficients), some Krylov subspace iterative methods are also used[1]. The speed of convergence of these methods depends on the condition number and the right-hand side of the system. If the condition number is very large, some techniques can be implemented to reduce it. Commonly, preconditioners (matrices modifying the original system) are used. These matrices, in general, are cheap to compute and they cluster the eigenvalues of the original system, which transforms the system into a better conditioned one.

In recent years, deflation techniques have been approached for the acceleration of the convergence of Krylov subspace methods [15, 16, 17, 18, 19]. For a good performance of this method, a deflation subspace needs to be found, such that, the smallest eigenvalues of the system are no longer present and the iterative method is be accelerated. In this work, we use POD techniques to construct the above-mentioned deflation subspace. We use this methodology to solve two-phase flow problems in large-scale, highly heterogeneous porous media.

[1]Given a linear system $\mathbf{Ax} = \mathbf{b}$, and the initial residual $\mathbf{r}^0 = \mathbf{b} - \mathbf{Ax}^0$, with $\mathbf{x}^0$ an initial guess of $\mathbf{x}$, we define the Krylov subspace as $\mathcal{K}_k(\mathbf{A}, \mathbf{r}^0) = span\{\mathbf{r}^0, \mathbf{Ar}^0, \ldots, \mathbf{A}^{k-1}\mathbf{r}^0\}$ [14]. That is, the set of linear combinations of powers of $\mathbf{A}$ times $\mathbf{r}^0$.

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