Theory section, Jaco Brandsen

1. Continuous formulation of problem

1.1. Governing equations

Consider the domain, Ω, of the FSI problem illustrated in figure 1, which is set in a d-dimensional space. The domain contains a moving solid body, Ω_s, surrounded by an incompressible Newtonian fluid, Ω_f, such that $\Omega = \Omega_\mathrm{s} \cup \Omega_\mathrm{f}$. The boundaries of Ω_s and Ω are denoted by $\partial\Omega_\mathrm{s}$ and $\partial\Omega$, respectively. In the fictitious domain method, the governing equations of the fluid are solved on the entire domain Ω. The region Ω_s is therefore treated as though it is filled with the same fluid as Ω_f. If the body is rigid, the governing quations of the fluid can be formulated as:

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\nabla\mathbf{u})\mathbf{u} = -\nabla p + \mu\Delta\mathbf{u} + \mathbf{b} + \mathbf{f} \qquad \text{in } \Omega \times (0,T] \tag{1a}$$

$$\mathrm{div}\ \mathbf{u} = 0 \qquad \text{in } \Omega \times (0,T] \tag{1b}$$

$$\mathbf{u}(\mathbf{x},t) = \mathbf{U}(\mathbf{X}(t)) + \boldsymbol{\Omega}(t) \times (\mathbf{x} - \mathbf{X}(t)) \qquad \text{in } \Omega_s \times (0,T] \tag{1c}$$

$$\mathbf{u}(\mathbf{x},t) = \hat{\mathbf{u}}(\mathbf{x},t) \qquad \text{on } \partial\Omega_u \times (0,T] \tag{1d}$$

$$p(\mathbf{x},t) = \hat{p}(\mathbf{x},t) \qquad \text{on } \partial\Omega \setminus \partial\Omega_u \times (0,T] \tag{1e}$$

$$\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}) \qquad \text{in } \Omega \setminus \Omega_\mathrm{s} \tag{1f}$$

$$\mathbf{u}(\mathbf{x},0) = \mathbf{U}(\mathbf{X}(0)) + \boldsymbol{\Omega}(0) \times (\mathbf{x} - \mathbf{X}(0)) \qquad \text{in } \Omega_\mathrm{s} \tag{1g}$$

$$p(\mathbf{x},0) = p_0(\mathbf{x}) \qquad \text{in } \Omega \tag{1h}$$

where $\mathbf{u}$ is the velocity of the fluid, p is its pressure, ρ is its density, μ is its dynamic viscosity, and $\mathbf{b}$ is a body-force such as gravity. The quantity $\partial\Omega_u$ represents the portion of $\partial\Omega$ on which the Dirichlet boundary conditions for $\mathbf{u}$ are imposed. The vectors $\mathbf{X}$ and $\mathbf{U}$ denote the spatial coordinates and linear velocity of the center of gravity of the body, respectively, and $\boldsymbol{\Omega}$ is the body's angular velocity. The variable $\mathbf{f}$ is an artificial body-force that accounts for the effect of the motion of the body on the fluid by indirectly enforcing the no-slip condition (equation (1c)). The formulation of $\mathbf{f}$ will be addressed in section 1.2 for the immersed body method and section 1.3 for the distributed Lagrange multiplier method.

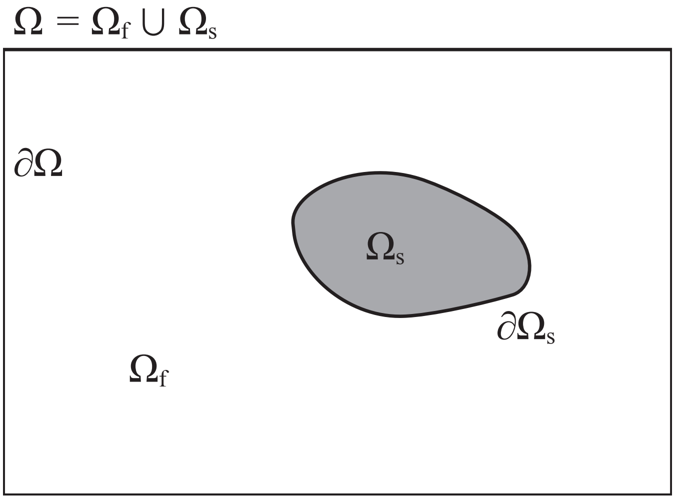

Figure 1: Domain of a fluid-structure interaction problem, Ω, consisting of a solid body, Ω_s, surrounded by an incompressible fluid, Ω_f.

In this article, the formulations presented will only be tested on one-way FSI problems, in other words, problems in which the solid body executes a prescribed motion. However, these formulations are general enough that they can also be applied to FSI problems where the motion of the body depends on the dynamics of the fluid. If the motion of the body is prescribed, equations (1a) through (1h) fully capture the dynamics of the FSI problem.

1.2. Penalty approach

In the immersed body method [1, 2], the artificial body-force term has the form

$$\mathbf{f} = \beta\alpha(\mathbf{U}(\mathbf{X}(t)) + \boldsymbol{\Omega}(t) \times (\mathbf{x} - \mathbf{X}(t)) - \mathbf{u}) \tag{2}$$

where β is a constant penalty factor and α is the solid concentration field [1, 2]. The solid concentration field tracks the volume fraction of solid material at each point in Ω and is defined according to

$$\alpha(\mathbf{x}, t) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega_{\mathrm{s}}, \\ 0 & \text{if } \mathbf{x} \notin \Omega_{\mathrm{s}} \end{cases} \tag{3}$$

The value of β is selected to be much larger than the magnitude of the other terms in equation (1a). In order to satisfy the no-slip condition (equation (1c)),the velocity of the fluid, $\mathbf{u}$, must be equal to the velocity of the solid body, $(\mathbf{U}(\mathbf{X}(t)) + \mathbf{\Omega}(t) \times (\mathbf{x} - \mathbf{X}(t)))$, on Ω_{s}. The large value of β enables $\mathbf{f}$ to apply the required forcing to the fluid, whilst allowing the difference between $(\mathbf{U}(\mathbf{X}(t)) + \mathbf{\Omega}(t) \times (\mathbf{x} - \mathbf{X}(t)))$ and $\mathbf{u}$ to be small.

The artificial body-force can be divided into two parts:

$$\mathbf{f} = \mathbf{f}_1 - \mathbf{f}_2 \tag{4}$$

The first part, $\mathbf{f}_1 = \beta\alpha(U(\mathbf{X}(t)) + \mathbf{\Omega}(t) \times (\mathbf{x} - \mathbf{X}(t)))$ contains the velocity of the body. Since, the motion of the body is prescribed, the value of $\mathbf{f}_1$ is known at any moment in time. The second part, $\mathbf{f}_2 = \beta\alpha\mathbf{u}$, is calculated by Fluidity when solving the balance of linear momentum for $\mathbf{u}$.

The immersed body method is simpler than the distributed Lagrange multiplier method as it does not require any modifications to the governing equations of the fluid, other than the inclusion of the artificial body force. However, notice from equation (3) that the difference between $(\mathbf{U}(\mathbf{X}(t)) + \mathbf{\Omega}(t) \times (\mathbf{x} - \mathbf{X}(t)))$ and $\mathbf{u}$ can never be exactly zero as this would result in $\mathbf{f}$ also being zero, which is quivalent to the body vanishing. The immersed body method is therefore incapable of imposing the no-slip condition exactly. Of course, this limitation is acceptable as long as the difference between the velocity of solid body and the velocity of the fluid can be made small enough to accurately account for the influence of the body.

References

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