

# On the value of relative flow data

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December 4, 2017

## 1 The importance of spatial-temporal data characteristics

We want to accurately estimate the change in cumulative flow, i.e.,  $\Delta N$ , over a link within a defined period of time, i.e., the space-time domain. As discussed above, we consider a model-based estimation approach and use traffic sensors which observes  $\Delta N$  over paths in space-time.

In model-based Traffic State Estimation (TSE), we assimilate different types of information to estimate the traffic state. To estimate the traffic state within a space-time domain without discontinuities, i.e., within a specific time-period on a link, we make a distinction between three information types. These are (1) sensor observations over paths within the space-time domain, (2) calibrated models and (3) external information related to the initial and boundary conditions of the space-time domain.

Sensing data influences all three information types. The sensing data which falls within the space-time domain yields the sensor observation paths (type 1). Historical data can be used to develop and calibrate models (type 2). Examples of these models are traffic flow models and observation models. In our case, the latter may describe the relation between the observed and true values of  $\Delta N$  for a road-side observer (which is certainly interesting in case of a structural error). External information (type 3) may be derived from sensing data which falls outside the space-time domain. For instance, data related to nearby links may provide valuable information for the considered link. In this case, information travels within the network over nodes from link to link.

The information related to the traffic state in the space-time domain will not be perfect: data can be incomplete and erroneous, and models do not perfectly describe relations between variables. Nevertheless, we desire accurate estimates. We can reach our main objective, that is accurate estimation of the traffic state, by:

1. Connecting all points in the space-time domain.
2. Limiting errors in information (prior to information assimilation).
3. Dealing with (remaining) information errors in assimilation.

These three points can be seen as sub-objectives to reach our main objective. The first sub-objective is a minimum requirement to estimate  $\Delta N$  between any two points in the space-time domain: without this sub-objective, the difference between two observations is unknown. The second and third sub-objectives should limit the negative effects of errors in the information. Sub-objective 2 aims to limit the error prior to information assimilation, whereas sub-objective 3 aims to deal with erroneous information in the assimilation process. Below we explain the three sub-objectives in more detail, and provide an illustration for each of the three conditions. In the illustrations we consider obtaining information on  $\Delta N$  for paths in space-time (resembling information from moving and road-side observers). Combined, these information paths form an information network. Depending on the spatial-temporal characteristics of the information, a network can satisfy the sub-objective. For simplicity and clarity, we exclude information in the form of a traffic flow model in the illustrations provided in this extended abstract.

## 1.1 Connecting all the points

To get the traffic state, we construct  $\Delta N$  over space-time. Let us consider information of  $\Delta N$  on paths in space-time. These individual pieces of information can be connected when the paths intersect in space-time as the value of  $N$  is equal at this point. In order to estimate  $\Delta N$  between any two points in the space-time domain, these two points have to be connected via the available information. All points in the space-time domain are connected if it is possible to travel from one point to any other point over information paths. This requirement is for instance satisfied if we have information related to the initial and boundary conditions in combination with a traffic flow model.

*Illustration:* Figure 1 shows four information networks, which consist of paths over which we know  $\Delta N$ . These paths are the only information we have on  $\Delta N$ . Our objective is to estimate  $\Delta N$  between the two points on the top right and bottom right corners of the network. In this illustration we thus consider connecting the two relevant points (instead of all points in space-time). As indicated by the question mark (?) between the relevant points, we cannot estimate  $\Delta N$  in the two left networks without adding information or assumptions. In contrast, estimation is possible in the two right networks. Here, it is possible to travel over the information paths from one point to the other, thereby connecting these points.

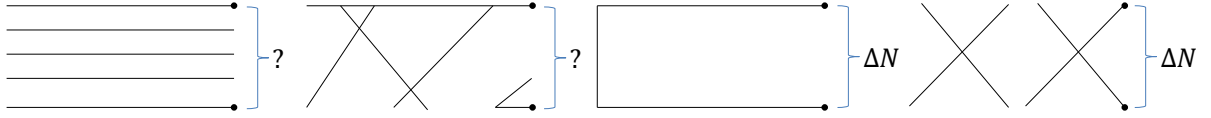


Figure 1: In space-time, information on  $\Delta N$  is known along the indicated paths. Connected paths lead to an estimation of  $\Delta N$  between the two points (indicated by the dots at the top and bottom right corners).

## 1.2 Limiting errors in information

It is valuable to understand the relation between important variables, e.g., between the observed and true values of  $\Delta N$  for a road-side observer. A relation between variables can be described using a model. We consider the following general model structure:  $y = f(x, \Theta)$ , where  $y$  and  $x$  are variables, and  $f()$  and  $\Theta$  respectively describe the model structure and parameters. Having a dataset which combines observations/estimates for the different variables (e.g.,  $y$  and  $x$ ) allows us to apply a supervised ML algorithm to evaluate different model structures and learn their parameters (calibration). The accuracy of the calibrated models, and thus the errors in information, depends on the available dataset of  $y$  and  $x$ . In turn, this dataset depends on the sensing data characteristics.

*Illustration:* Figure 2 shows an information network which contain areas which are enclosed by information paths. Enclosed areas allow us to compare one piece of information (e.g., one side of the area) with an estimate based on the other pieces of information (e.g., the other sides of the area). Here we use the fact that the sum of  $\Delta N$  over all sides of an area should be equal to zero (conservation-of-vehicles). Let us assume that we want to learn the structural error in the horizontal path (e.g., the road-side detector at the downstream link boundary) of the network. All enclosed areas including the top path (see bold blue paths) provide information along the horizontal path combined with an estimate based on the other pieces of information. This dataset would allow us to learn the relative structural error.

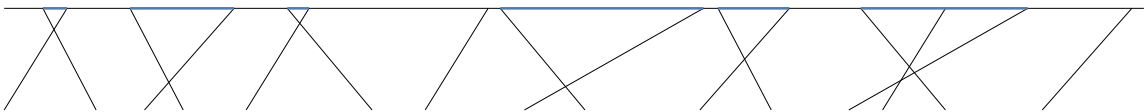


Figure 2: Information paths can create enclosed areas in space-time. These areas allow us to expose the relative structure error for the bold blue parts of the horizontal path.

### 1.3 Dealing with errors

It is valuable to understand that information is not perfect. It is even more valuable to have knowledge related to the information errors (e.g., in the form of an error distribution) and assimilate information based on this knowledge. In this way we can obtain the most reliable estimates. Information assimilation following this principle is not new in TSE, e.g., Wang & Papageorgiou (2005) and Herrera & Bayen (2010) apply an Extended Kalman Filter (EKF). Our ability to understand (learn) the error characteristics and to have alternative information depends on the data characteristics. As discussed above, the spatial-temporal characteristics affect our ability to learn important relations. This includes learning the error characteristics. Furthermore, depending on the spatial-temporal characteristics we may or may not have information alternatives.

*Illustration:* Let us consider a simple information network consisting of six pieces of information (see left most part of Figure 3). We assume that each piece contains information related to  $\Delta N$ . However, this information is subjected to errors, which follow an independent zero-mean Gaussian distribution with a known variance  $\sigma^2$ . Knowing the error variance allows us to fuse data by minimizing the error variance of our estimate (as is done using an EKF). This problem can be solved using the same equations as a resistor network. The resulting expected error variance of our estimate is depicted between the two points. This example shows that it is beneficial to use the most reliable information (option B instead of A), but that it is still beneficial to use all information alternatives (option C instead of B). This resembles having parallel paths in a resistor network.

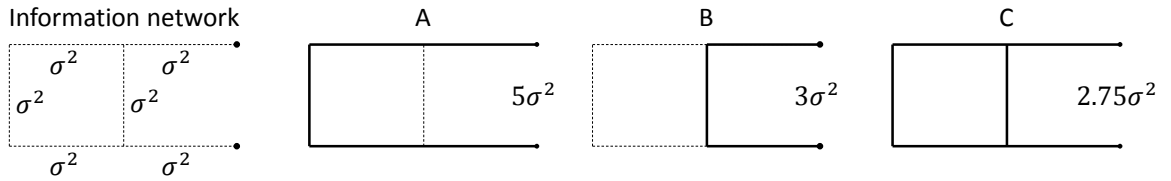


Figure 3: It is valuable to have information alternatives (parallel paths in the information network). Weighing these alternatives based on their individual reliability yields the most reliable estimate (option C).

## References

- Herrera, J. C., & Bayen, A. M. (2010). Incorporation of Lagrangian measurements in freeway traffic state estimation. *Transportation Research Part B: Methodological*, 44(4), 460–481.
- Wang, Y., & Papageorgiou, M. (2005, feb). Real-time freeway traffic state estimation based on extended Kalman filter: a general approach. *Transportation Research Part B: Methodological*, 39(2), 141–167.