RESULTS *

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1. Results. We study single and two-phase flow. For the single-phase case, we simulate water flow with a density of $\rho_w = 1000\ kg/m^3$ and a viscosity of $\mu_w = 1\ cp$. For the two-phase problem, the second fluid is oil with a density of $\rho_o = 700\ kg/m^3$ and a viscosity of $\mu_o = 10\ cp$, the relative permeability functions are $k_{r(w)} = (S_w)^2$ and $k_{r(o)} = (1 - S_w)^2$.

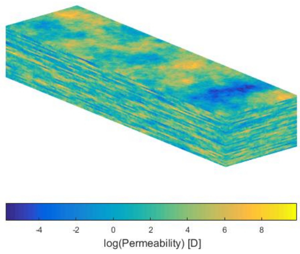

Fig. 1: Permeability field, 35 layers.

We study the SPE 10 benchmark consisting on 60 x 220 x 85 cells. In this work we study 2 cases: a 2D problem consisting on one layer (60 x 220 cells); and a 3D case containing 35 layers (60 x 220 x 35 cells) and gravity terms. The permeability field for the 3D case is presented in Figure 1. We consider injection through the boundary and injection through wells. The wells setup consists on one injector and four producers. A description of the time parameters, and initial pressures and saturations is presented for each case.

1.1. Single-phase problem. For these experiments, we use the *l.i. systems* approach to obtain the deflation vectors. In this case, we have 5 wells: four producers with a pressure of $P = 275$ bars and an injector $I = 4 * P = 1100$ bars. This means 4 l.i. systems. Each system consists of 3 active producers with pressure $P = 275$ bars, and the injector also active with pressure $I = 3 * P = 825$ bars. The initial pressure on the reservoir is $P_0 = 500$ bars. The number of iterations for the ICCG and DICCG methods are presented in Table 1, and the pressure field for the 3D case is presented in Figure 2.

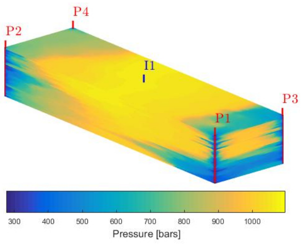

Fig. 2: Pressure field, 35 layers.

Layers	ICCG Iterations	DICCG Iterations
1	251	2
35	507	2

Table 1: Comparison between the average number of linear iteration of the ICCC and DICCG methods.

We observe that the number of iterations increases with the size of the problem for the ICCG method. By contrast, it remains constant (2 iterations) for the DICCG method. It has been shown that if the deflation vectors consist of all the possible solutions of the l.i. systems, the convergence of the DICCG method is achieved in one iteration [?]. If the solutions of the l.i. systems are not accurate enough, the number of iterations can increase slightly [?]. For the 2D problem, the effective condition number of the deflated systems is $\kappa_{eff}(PM^{-1}A) = 10^2$. If we want an error of $\varepsilon = 10^{-8}$ in the approximation obtained with the deflated method, the necessary accuracy of the snapshots is $\epsilon = \varepsilon/\kappa_{eff}(A) = 10^{-8}/10^2 = 10^{-10}$ (see Appendix **??**). As this accuracy is not fulfilled, convergence of the DICCG method in one iteration is not achieved. However, the accuracy of the snapshots is close to the required one, thus, the number of iterations is only two.

1.2. Two-phase problem. In this section, we study waterflooding with injection through the left boundary and through wells. To show the applicability of both approaches we obtain the POD basis using the *moving window* in the case of injection through the left boundary; and the *training run* approach for the injection through wells. The applicability of both approaches does not depend on the injection strategy (*rhs*).

Injection through left boundary. Water is injected through the left boundary at a constant rate of 40 m^3/day into a reservoir initially filled with oil. The initial pressure of the reservoir is 100 bars, and the pressure at the right boundary is set to 0 bars. We run the simulation for 200

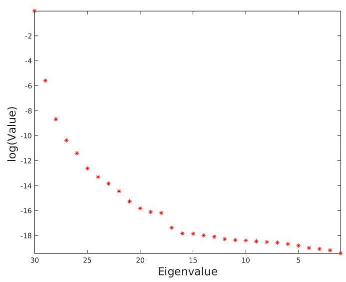

Fig. 3: Eigenvalues of the correlation matrix, 35 layers.

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timesteps, with a time step of 20 days. The eigenvalues of the correlation matrix are presented in Figure 3 for the last time step of the 3D problem. The results are presented in Table 2 and the pressure and water saturation at various timesteps are shown in Figure 4 and Figure 5 for the 2D problem.

Total ICCG Iterations	DICCG Method	ICCG Iterations (snapshots)	DICCG Iterations	Total ICCG + DICCG Iterations	% of Total ICCG Iterations
1 layer					
42062	$DICCG_{10}$	2309	8153	10462	25
42062	$DICCG_{30}$	6923	4035	10958	26
35 layers					
66728	$DICCG_{10}$	2759	17190	19949	30
66728	$DICCG_{30}$	8535	11798	20333	30

Table 2: Comparison between the ICCC and DICCG methods of the average number of linear iterations. Injection through left boundary,

Using the *moving window* approach to obtain the POD basis, we note that the number of ICCG iterations is reduced to around 25% for the DICCG method with 10 or 30 deflation vectors (see Table 2). This reduction does not change significantly when using one or 35 layers (which also includes gravity terms for the 3D case). We note that around 12 eigenvalues are larger than the rest (see Figure 3), this implies that most of the information is contained in these larger eigenvalues. As a consequence, we do not have an important decrease in the number of iterations if we use 30 instead of 10 deflation vectors.

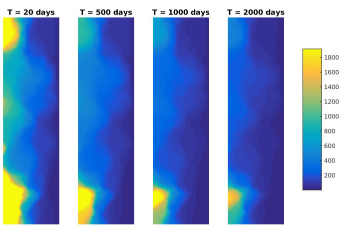

Fig. 4: Pressure, 1 layer.

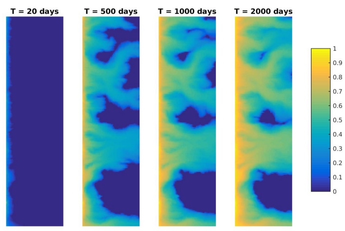

Fig. 5: Water Saturation, 1 layer.

Wells. In this section, we perform a series of experiments injecting water through wells. For these experiments, the POD basis and deflation matrix are obtained offline in a *training phase* run with the ICCG method. Once the POD basis is obtained, a series of simulations are performed with the DICCG method for diverse values of *bhp* in the producers.

The pressure of the production wells is varied randomly every 2 time steps during the *training phase* between $P = 137.5$ and $P = 275$ bars. The pressure in the injection well is maintained constant at $I = 1100$ bars for all cases. For the examples solved with DICCG, we use different pressures in the producers. In the first experiment, the pressure is $P = 275$ bars in all the producers, an extreme value of the *training phase* run. The pressure of the producers in the second experiment is $P = 200$ bars, a value inside the pressure range of the *training phase*. And the final experiment has a pressure of $P = 400$ bars, a pressure outside the *training phase* pressure range.

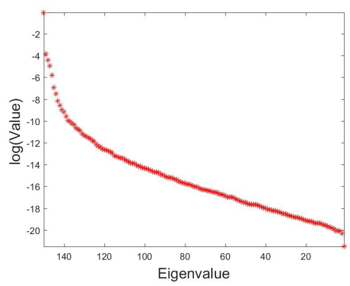

Fig. 6: Eigenvalues of the correlation matrix, 1 layer.

The eigenvalues of the correlation matrix are presented in Figure 6 for the last time step of the 2D problem. The simulations are performed during 150 timesteps with a time step of 50 days for the 2D problem and 10 days for 3D. The resulting number of iterations for the ICCG and DICCG methods are presented in Table 3 for one and 35 layers. The pressure and water saturation at diverse timesteps are shown in Figure 7 and Figure 8.

From Table 3, we observe that for the 2D problem, using 30 deflation vectors only requires $\sim 16\%$ of the number ICCG iterations for the three cases, even when the pressure of the producers is outside

1 layer			
Total ICCG	DICCG Method	Iter	% of ICCG Iter
P_{bhp} = 275 [bars]			
32237	DICCG$_{30}$	5503	17
32237	DICCG$_{10}$	8811	27
P_{bhp} = 200 [bars]			
32237	DICCG$_{30}$	5794	18
32237	DICCG$_{10}$	9207	29
P_{bhp} = 400 [bars]			
32237	DICCG$_{30}$	4818	15
32237	DICCG$_{10}$	8094	25

35 layers			
Total ICCG	DICCG Method	Iter	% of ICCG Iter
P_{bhp} = 275 [bars]			
59806	DICCG$_{30}$	13093	22
59806	DICCG$_{10}$	22577	38
P_{bhp} = 200 [bars]			
59806	DICCG$_{30}$	13256	22
59806	DICCG$_{10}$	23529	39
P_{bhp} = 400 [bars]			
59806	DICCG$_{30}$	12959	22
59806	DICCG$_{10}$	21526	36

Table 3: Comparison between the ICCC and DICCG methods of the average number of linear iterations for diverse bhp in the production wells.

the *training phase* range. When we decrease the number of deflation vectors to 10, the percentage of ICCG iterations increases to $\sim 27\%$, which is still an important reduction. In Figure 6, we observe that the 30 first eigenvalues are larger than the rest, e.g., they contain more information about the system. Therefore, using 30 instead of 10 deflation vectors gives a better performance for the DICCG method. However, the first ten eigenvalues contain the main information; Thus, using 30 or 10 does not change significantly the number of iterations.

Changing the size of the system to 35 layers and including gravity terms (see Table 3), the percentage of the number of iterations increases slightly to $\sim 20\%$ of the number of ICCG iterations when using 30 deflation vectors and to $\sim 35\%$ when using 10. This is a small increase taking into account the change of the problem size.

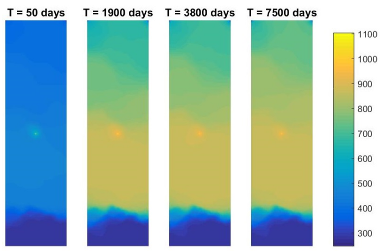

Fig. 7: Pressure, 1 layer.

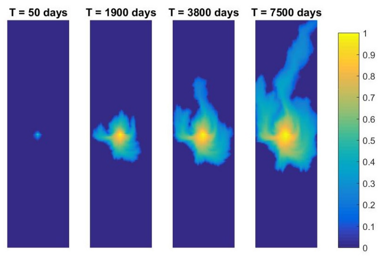

Fig. 8: Water Saturation, 1 layer.

The number of flops per iteration of the ICCG method is around $(4m+11)N$, and $(4m+4p+11)N$ for the DICCG method, where N is the size of the problem, m is the sparcity of the matrix and p is the number of deflation vectors [**?**]. For $p = 10$, the ratio between the flops of the ICCG and the DICCG methods is 2.3 (2D case, $m = 5$) and 2 (3D case, $m = 7$). For $p = 30$, the ratio is 4.8 (2D) and 4 (3D). Then, we need from 2 to 4 the number of operations when using the DICCG method. We note that, as the sparcity of the problem decreases, the ratio between flops is reduced. Therefore, this method can present a better performance for non-sparse matrices. Further reduction on the flops of the DICCG method can also be achieved by reducing the number of deflation vectors or making the *deflation-subspace matrix* ($\mathbf{Z}$) sparse.