**Introduction (Jaco Brandsen)**

Structures that exhibit behaviour caused by fluid-structure interaction (FSI) are encountered throughout engineering. Examples of such behaviour include the aeroelastic vibration of wind turbine blades [1], the wave-induced motions of ships and floating platforms [2], and the opening and closing of prosthetic heart valves [3]. When designing each of these structures, analysing and understanding these behaviours is often critical. Simulations are an attractive option for this as a large number of different geometries, configurations and load cases can easily be considered, whilst minimising expensive laboratory and field testing. A popular approach to simulating FSI is to adopt a partitioned scheme. In this scheme the fluid dynamics and structural dynamics involved are each calculated separately using two different solvers. Each solver has usually been designed to efficiently perform calculations within the discipline to which it is being applied. The interaction between fluid and structure is accounted for by intermittently passing information between the solvers, with regards to the motion of the wetted boundary of the structure and the forces acting on it.

The approaches for achieving a partitioned scheme can be divided into two different groups. In the first group, which have been referred to as defined body (DB) methods by Vire et al. [4] or body conforming methods by Mittal et al. [5], the domain of the fluid dynamics model conforms to the shape of the structure. The wetted boundary is therefore also a boundary of the domain of the fluid dynamics model and the structure is located entirely outside of this domain. The no-slip condition that exists at the wetted boundary is imposed by specifying the displacement and velocity of this boundary as Dirichlet boundary conditions of the fluid dynamics model. Defined body methods have been used to investigate FSI in a wide variety of applications such as the vibration of wind turbine [1] and axial compressor rotors [6], the deployment of spacecraft parachutes [7] and the steering stability of tanker semi-trailers [8].

This article will focus on the second group, which will collectively be referred to as immersed methods. In an immersed method, the domain of the fluid dynamics model does not conform to the shape of the structure, but instead contains both the space occupied by structure and the space occupied by the fluid. Therefore, the wetted boundary no longer constitutes a boundary of the domain, but is located entirely within it. The no-slip condition at the wetted boundary is indirectly enforced by adding an artificial body force term to the governing equations of the fluid. This additional term causes the velocity of the fluid to approach that of the structure within the region of space that the structure occupies.

The origin of immersed methods can be traced back to the immersed boundary method of Peskin [9] who used it to simulate the motion of heart valve leaflets. Since then, the immersed boundary method has been extended by authors such as Taira and Colonius [10], who combined it with a predictor-corrector time integration scheme, and Lacis et al. [11], who adapted it for particulate flows. The development of a variant of the immersed boundary method, known as the distributed Lagrange multiplier (DLM) method, is a subject that is currently popular in the literature. Authors who have contributed to this subject include Glowinski et al. [15], Hou et al. [12], Kadapa et al. [13], and Boffi and Gastaldi [14]. The DLM method gets its name from the fact that it treats the artificial body force as a Lagrange multiplier field. A further group of immersed methods that are being developed formulate the artificial body force as a penalty term. Goldstein et al. [16] based the formulation of the penalty term on a feedback control system. Khadra et al. [17] computed its value by treating the structure as a porous medium. Two additional versions, namely the immersed body method [18, 19] and the immersed shell method [4], have been developed recently by Viré et al.

The purpose of this article is two-fold. Firstly, this article will present a unique implementation of the DLM method. Secondly, this article will provide an analysis comparing the performance of this version of the DLM method to that of a penalty formulation. The presented implementation differs from previous versions of the DLM method in the discretisations, and spatial and temporal integration schemes selected. The immersed boundary method was selected as the penalty formulation for the comparative analysis as, not only was it already available in Fluidity, it was felt that it is sufficiently representative of the majority of formulations in the literature. Similarly, it was felt that the implementation of the DLM method in Fluidity, despite the stated differences, was still similar enough to those in the literature to be representative of them. A comparative analysis of this nature has not, to the authors' knowledge, been carried out before and is of value as both of these approaches are active areas of research. Furthermore, this comparison is fitting as these two approaches have traditionally been in competition when it comes to enforcing constraints in numerical simulations and optimisation.

The article will begin by presenting the continuous formulation of both the DLM method and the immersed body method. This will be followed by the spatial and temporal discretisations of each of these methods, respectively. The results of the comparative analysis, in which DLM method and immersed body methods were used to conduct a number of benchmark simulations, will then be presented. Finally, the article will end with the conclusions and recommendations derived from the results.

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