Simulation with a Block-Centered Grid

4.1 Introduction

This chapter presents discretization of 1D, 2D, and 3D reservoirs using block-centered grids in Cartesian and radial-cylindrical coordinate systems. As the name implies, the gridblock dimensions are selected first, followed by the placement of points in central locations of the blocks. In this, the distance between block boundaries is the defining variable in space. In contrast, the gridpoints (or nodes) are selected first in the point-distributed grid, which is discussed in Chapter 5. Chapter 2 introduced the terminology for reservoir discretization into blocks. This chapter describes the construction of a block-centered grid for a reservoir and the relationships between block sizes, block boundaries, and distances between points representing blocks. The resulting gridblocks can be classified into interior and boundary gridblocks. Chapter 2 also derived the flow equations for interior gridblocks. However, the boundary gridblocks are subject to boundary conditions and thus require special treatment. This chapter presents the treatment of various boundary conditions and introduces a general flow equation that is applicable for interior blocks as well as boundary blocks. This chapter also presents the equations for directional transmissibilities in both Cartesian and radial-cylindrical coordinate systems and discusses the use of symmetry in reservoir simulation.

4.2 Reservoir Discretization

Reservoir discretization means that the reservoir is described by a set of gridblocks whose properties, dimensions, boundaries, and locations in the reservoir are well-defined. Figure 4–1 shows a block-centered grid for a 1D reservoir in the direction of the x axis. The grid is constructed by choosing n_x gridblocks that span the entire reservoir length in the x direction. The gridblocks are assigned predetermined dimensions (Δx_i, $i = 1, 2, 3 \ldots n_x$) that are not necessarily equal. Then the point that represents each gridblock is subsequently located at the center of that gridblock. Figure 4–2 focuses on gridblock i and its neighboring gridblocks in the x direction. It shows how the gridblocks are related to each other, gridblock dimensions (Δx_{i-1}, Δx_i, Δx_{i+1}), gridblock boundaries ($x_{i-1/2}$, $x_{i+1/2}$), distances between the point that represents gridblock i and gridblock boundaries (δx_{i^-}, δx_{i^+}), and distances between the points representing these gridblocks ($\Delta x_{i-1/2}$, $\Delta x_{i+1/2}$).

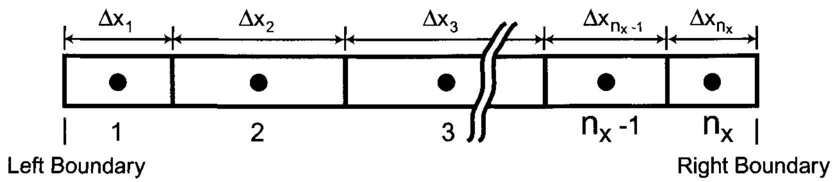


Figure 4–1 Discretization of a 1D reservoir using a block-centered grid.

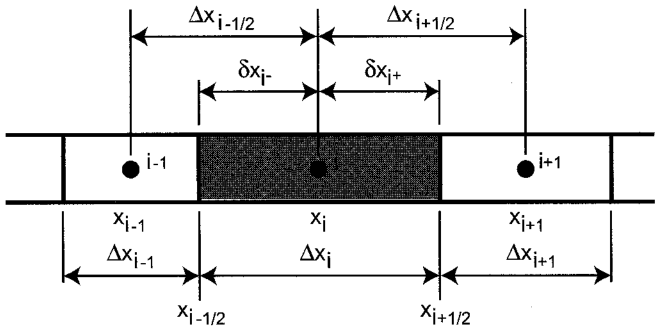


Figure 4–2 Gridblock *i* and its neighboring gridblocks in the *x* direction.

Gridblock dimensions, boundaries, and locations satisfy the following relationships:

$$\sum_{i=1}^{n_x} \Delta x_i = L_x \ ,$$

$$\delta x_{i^-} = \delta x_{i^+} = \tfrac{1}{2}\Delta x_i \ , \ i = 1,2,3 \ldots n_x \ ,$$

$$\Delta x_{i-1/2} = \delta x_{i^-} + \delta x_{i-1^+} = \tfrac{1}{2}(\Delta x_i + \Delta x_{i-1}) \ , \ i = 2,3 \ldots n_x \ ,$$

$$\Delta x_{i+1/2} = \delta x_{i^+} + \delta x_{i+1^-} = \tfrac{1}{2}(\Delta x_i + \Delta x_{i+1}) \ , \ i = 1,2,3 \ldots n_x - 1 \tag{4.1}$$

$$x_{i+1} = x_i + \Delta x_{i+1/2} \ , \ i = 1,2,3 \ldots n_x - 1 \ , \ x_1 = \tfrac{1}{2}\Delta x_1$$

$$x_{i-1/2} = x_i - \delta x_{i^-} = x_i - \tfrac{1}{2}\Delta x_i \ , \ i = 1,2,3 \ldots n_x$$

$$x_{i+1/2} = x_i + \delta x_{i^+} = x_i + \tfrac{1}{2}\Delta x_i \ , \ i = 1,2,3 \ldots n_x$$

Figure 4–3 shows the discretization of a 2D reservoir into a 5×4 irregular grid. An irregular grid implies that block sizes in the direction of the *x* axis (Δx_i) and the *y* axis (Δy_j) are neither equal nor constant. Discretization using a regular grid means that block sizes in the *x* and *y* directions are constants but not necessarily equal. The discretization in the *x* direction uses the procedure just mentioned and the relationships presented in Eq. 4.1. The discretization in the *y* direction uses a procedure and relationships similar to those for the *x* direction, and the same can be said for the *z* direction for a 3D reservoir. Inspection of Figure 4–1 and

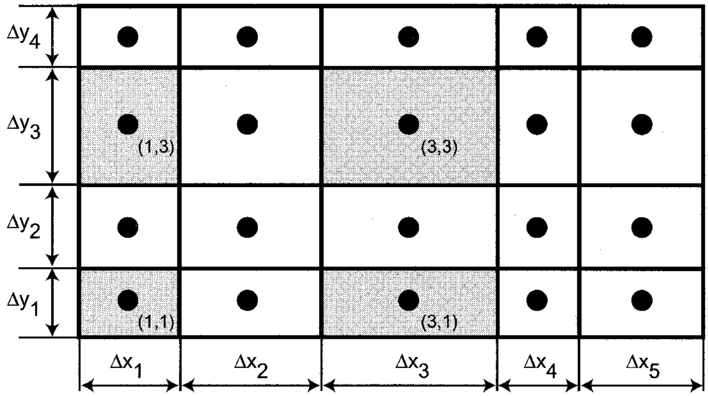


Figure 4–3 Discretization of a 2D reservoir using a block-centered grid.

Figure 4–3 shows that the point that represents a gridblock falls in the center of that block and that all points representing gridblocks fall inside reservoir boundaries.

Example 4.1 A 5000 ft×1200 ft×75 ft horizontal reservoir contains oil that flows along its length. The reservoir rock porosity and permeability are 0.18 and 15 md, respectively. The oil FVF and viscosity are 1 RB/STB and 10 cp, respectively. The reservoir has a well located at 3500 ft from the reservoir left boundary and produces oil at a rate of 150 STB/D. Discretize the reservoir into five equal blocks using a block-centered grid and assign properties to the gridblocks comprising this reservoir.

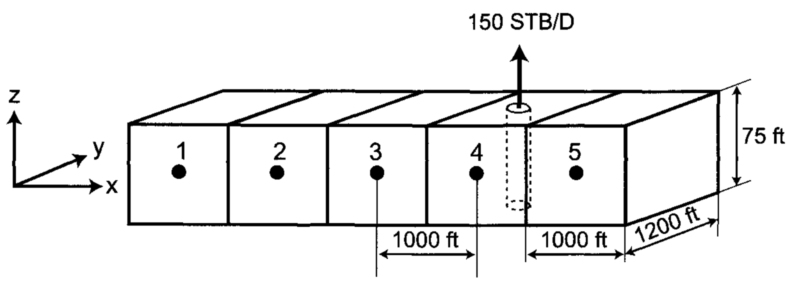


Figure 4–4 Discretized 1D reservoir in Example 4.1.

Solution

Using a block-centered grid, the reservoir is divided along its length into five equal blocks. Each block is represented by a point at its center. Therefore, $n_x = 5$ and $\Delta x = L_x / n_x = 5000/5 = 1000$ ft. Gridblocks are numbered from 1 to 5 as shown in Figure 4–4. Now the reservoir is described through assigning properties to its five gridblocks ($i = 1,2,3,4,5$). All the gridblocks (or the points that represent them) have the same elevation because the reservoir is horizontal. Each gridblock has the dimensions of $\Delta x = 1000$ ft, $\Delta y = 1200$ ft, and $\Delta z = 75$ ft and properties of $k_x = 15$ md and $\phi = 0.18$. The points representing gridblocks are equally spaced; i.e., $\Delta x_{i+1/2} = \Delta x = 1000$ ft

and $A_{x_{i\mp 1/2}} = A_x = \Delta y \times \Delta z = 1200 \times 75 = 90000$ ft^2. Gridblock 1 falls on the reservoir left boundary and gridblock 5 falls on the reservoir right boundary. Gridblocks 2, 3, and 4 are interior gridblocks. In addition, gridblock 4 hosts a well with $q_{sc_4} = -150$ STB/D. Fluid properties are $B = 1$ RB/STB and $\mu = 10$ cp.

4.3 Flow Equation for Boundary Gridblocks

In this section, we present a form of the flow equation that applies to interior blocks as well as boundary blocks. This means that the proposed flow equation reduces to the flow equations presented in Chapters 2 and 3 for interior blocks, but it also includes the effects of boundary conditions for boundary blocks. Figure 4–1 shows a discretized 1D reservoir in the direction of the x axis. Gridblocks 2, 3, …n_x–1 are interior blocks, whereas gridblocks 1 and n_x are boundary blocks, each of which falls on one reservoir boundary. Figure 4–3 shows a discretized 2D reservoir. This figure highlights an interior gridblock, gridblock (3,3), two boundary gridblocks, each of which falls on one reservoir boundary, gridblocks (1,3) and (3,1), and a gridblock that falls on two reservoir boundaries, gridblock (1,1). In 3D reservoirs, there are interior gridblocks and boundary gridblocks. Boundary gridblocks may fall on one, two, or three reservoir boundaries. Figure 4–5 demonstrates the terminology used in this book for the reservoir boundaries in the negative and positive directions of the x, y, and z axes. Reservoir boundaries along the x axis are termed the reservoir west (b_W) and the reservoir east boundary (b_E), and those along the y axis are termed the reservoir south boundary (b_S) and the reservoir north boundary (b_N). Reservoir boundaries along the z axis are termed the reservoir lower boundary (b_L) and the reservoir upper boundary (b_U).

The characteristic forms of the difference equations for interior and boundary gridblocks differ in the terms dealing with space variables, i.e., the flow terms. The production (injection) term and the accumulation term are the same for both interior and boundary gridblocks. The engineering approach involves replacing the boundary condition with a no-flow boundary plus a fictitious well having a flow rate $q^m_{sc_{b,bB}}$ that reflects fluid transfer between the reservoir boundary itself (b) and the boundary block (bB). In other words, a fictitious well having flow rate of $q^m_{sc_{b,bB}}$ replaces the flow term that represents fluid transfer across a reservoir boundary between a boundary block and a block exterior to the reservoir. The number of flow terms in the flow equation for an interior gridblock equals the number of neighboring gridblocks (two, four, or six terms for 1D, 2D, or 3D reservoir, respectively). For the flow equation for a boundary gridblock, the number of flow terms equals the number of existing neighboring gridblocks in the reservoir and the number of fictitious wells equals the number of reservoir boundaries adjacent to the boundary gridblock.

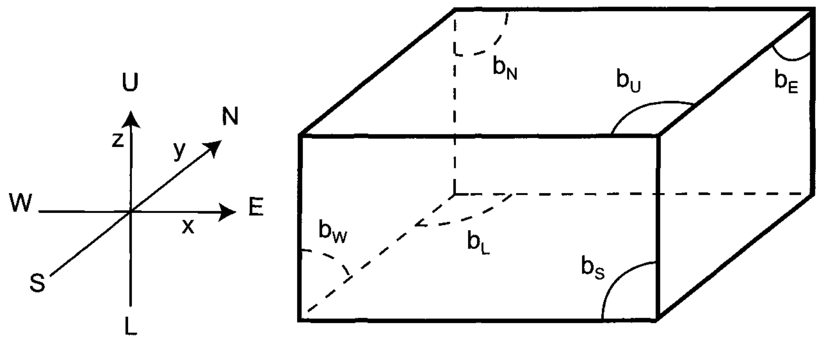


Figure 4-5 Definition of left and right boundaries in 3D reservoirs.

A general form of the flow equation that applies to boundary gridblocks as well as interior gridblocks in 1D, 2D, or 3D flow in both Cartesian and radial-cylindrical coordinates can be expressed best using CVFD terminology. The use of summation operators in CVFD terminology makes it flexible and suitable for describing flow terms in the equation of any gridblock sharing none or any number of boundaries with the reservoir. The general form for gridblock n can be written as

$$\sum_{l\in\psi_n} T_{l,n}^m[(p_l^m - p_n^m) - \gamma_{l,n}^m(Z_l^m - Z_n^m)] + \sum_{l\in\xi_n} q_{sc_{l,n}}^m + q_{sc_n}^m = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})_n^{n+1} - (\frac{\phi}{B})_n^n] \tag{4.2a}$$

or, in terms of potentials, as

$$\sum_{l\in\psi_n} T_{l,n}^m(\Phi_l^m - \Phi_n^m) + \sum_{l\in\xi_n} q_{sc_{l,n}}^m + q_{sc_n}^m = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})_n^{n+1} - (\frac{\phi}{B})_n^n] \tag{4.2b}$$

where ψ_n = the set whose elements are the existing neighboring gridblocks in the reservoir, ξ_n = the set whose elements are the reservoir boundaries ($b_L, b_S, b_W, b_E, b_N, b_U$) that are shared by gridblock n , and $q_{sc_{l,n}}^m$ = flow rate of the fictitious well representing fluid transfer between reservoir boundary l and gridblock n as a result of a boundary condition. For a 3D reservoir, ξ_n is either an empty set for interior gridblocks or a set that contains one element for gridblocks that fall on one reservoir boundary, two elements for gridblocks that fall on two reservoir boundaries, or three elements for gridblocks that fall on three reservoir boundaries. An empty set implies that the gridblock does not fall on any reservoir boundary; i.e., gridblock n is an interior gridblock, and hence $\sum_{l \in \xi_n} q^m_{sc_{l,n}} = 0$. In engineering notation, $n \equiv (i,j,k)$ and Eq. 4.2a becomes

$$\sum_{l \in \psi_{i,j,k}} T^m_{l,(i,j,k)}[(p^m_l - p^m_{i,j,k}) - \gamma^m_{l,(i,j,k)}(Z_l - Z_{i,j,k})] + \sum_{l \in \xi_{i,j,k}} q^m_{sc_{l,(i,j,k)}} + q^m_{sc_{i,j,k}}$$
$$= \frac{V_{b_{i,j,k}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{i,j,k} - (\frac{\phi}{B})^n_{i,j,k}]. \tag{4.2c}$$

It must be mentioned that reservoir blocks have a three-dimensional shape whether fluid flow is 1D, 2D, or 3D. The number of existing neighboring gridblocks and the number of reservoir boundaries shared by a reservoir gridblock add up to six as is the case in 3D flow. Existing neighboring gridblocks contribute to flow to or from the gridblock, whereas reservoir boundaries may or may not contribute to flow depending on the dimensionality of flow and the prevailing boundary conditions. The dimensionality of flow implicitly defines those reservoir boundaries that do not contribute to flow at all. In 1D flow problems, all reservoir gridblocks have four reservoir boundaries that do not contribute to flow. In 1D flow in the x direction, the reservoir south, north, lower, and upper boundaries do not contribute to flow to any reservoir gridblock, including boundary gridblocks. These four reservoir boundaries (b_L, b_S, b_N, b_U) are discarded as if they did not exist. As a result, an interior reservoir gridblock has two neighboring gridblocks and no reservoir boundaries, whereas a boundary reservoir gridblock has one neighboring gridblock and one reservoir boundary. In 2D flow problems, all reservoir gridblocks have two reservoir boundaries that do not contribute to flow at all. For example, in 2D flow in the x-y plane the reservoir lower and upper boundaries do not contribute to flow to any reservoir gridblock, including boundary gridblocks. These two reservoir boundaries (b_L, b_U) are discarded as if they did not exist. As a result, an interior reservoir gridblock has four neighboring gridblocks and no reservoir boundaries, a reservoir gridblock that falls on one reservoir boundary has three neighboring gridblocks and one reservoir boundary, and a reservoir gridblock that falls on two reservoir boundaries has two neighboring gridblocks and two reservoir boundaries. In 3D flow problems, any of the six reservoir boundaries may contribute to flow depending on the specified boundary condition. An interior gridblock has six neighboring gridblocks. It does not share any of its boundaries with any of the reservoir boundaries. A boundary gridblock may fall on one, two, or three of the reservoir boundaries. Therefore, a boundary gridblock that falls on one, two, or three reservoir boundaries has five, four, or three neighboring gridblocks, repectively. The above discussion leads to a few conclusions related to the number of elements contained in sets ψ and ξ.

(1) For an interior reservoir gridblock, set ψ contains two, four, or six elements for a 1D, 2D, or 3D flow problem, respectively, and set ξ contains no elements or, in other words, is empty.

(2) For a boundary reservoir gridblock, set ψ contains less than two, four, or six elements for a 1D, 2D, or 3D flow problem, respectively, and set ξ is not empty.

(3) The sum of the number of elements in sets ψ and ξ for any reservoir gridblock is a constant that depends on the dimensionality of flow. This sum is two, four, or six for a 1D, 2D, or 3D flow problem, respectively.

For 1D reservoirs, the flow equation for interior gridblock i is given by Eq. 2.32 or 2.33,

$$T^m_{x_{i-1/2}}(\Phi^m_{i-1}-\Phi^m_i)+T^m_{x_{i+1/2}}(\Phi^m_{i+1}-\Phi^m_i)+q^m_{sc_i}=\frac{V_{b_i}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_i-(\frac{\phi}{B})^n_i] \quad \textbf{(4.3)}$$

The above flow equation can be obtained from Eq. 4.2b for $n=i$, $\psi_i=\{i-1,i+1\}$, $\xi_i=\{\}$, and by observing that $\sum_{l\in\xi_i} q^m_{sc_{l,i}}=0$ for an interior gridblock and $T^m_{i\mp1,i}=T^m_{x_{i\mp1/2}}$.

The flow equation for boundary gridblock 1, which falls on the reservoir west boundary in Figure 4–6, can be written as

$$T^m_{x_{1-1/2}}(\Phi^m_0-\Phi^m_1)+T^m_{x_{1+1/2}}(\Phi^m_2-\Phi^m_1)+q^m_{sc_1}=\frac{V_{b_1}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_1-(\frac{\phi}{B})^n_1] \quad \textbf{(4.4a)}$$

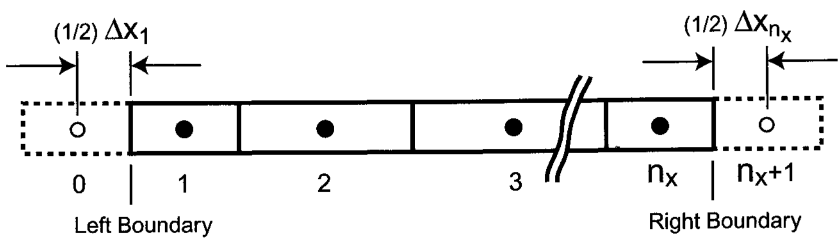


Figure 4–6 Boundary gridblocks at the left and right boundaries of a 1D reservoir (dashed lines represent fictitious reflective blocks).

The first term on the LHS of Eq. 4.4a represents the rate of fluid flow across the reservoir west boundary (b_W). This term can be replaced with the flow rate of a fictitious well ($q^m_{sc_{bW,1}}$) that transfers fluid across the reservoir west boundary to gridblock 1; i.e.,

$$q^m_{sc_{bW,1}}=T^m_{x_{1-1/2}}(\Phi^m_0-\Phi^m_1) \quad \textbf{(4.5a)}$$

Substitution of Eq. 4.5a into Eq. 4.4a yields

$$q^m_{sc_{bW,1}}+T^m_{x_{1+1/2}}(\Phi^m_2-\Phi^m_1)+q^m_{sc_1}=\frac{V_{b_1}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_1-(\frac{\phi}{B})^n_1] \quad \textbf{(4.4b)}$$

The above flow equation can be obtained from Eq. 4.2b for $n=1$, $\psi_1=\{2\}$, $\xi_1=\{b_W\}$, and by observing that $\sum_{l\in\xi_1} q^m_{sc_{l,1}} = q^m_{sc_{b_W,1}}$ and $T^m_{2,1}=T^m_{x_{1+1/2}}$.

The flow equation for boundary gridblock n_x, which falls on the reservoir east boundary in Figure 4–6, can be written as

$$T^m_{x_{n_x-1/2}}(\Phi^m_{n_x-1}-\Phi^m_{n_x})+T^m_{x_{n_x+1/2}}(\Phi^m_{n_x+1}-\Phi^m_{n_x})+q^m_{sc_{n_x}}=\frac{V_{b_{n_x}}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_{n_x}-(\frac{\phi}{B})^n_{n_x}] \tag{4.5a}$$

The second term on the LHS of Eq. 4.6a represents the rate of fluid flow across the reservoir east boundary (b_E). This term can be replaced with the flow rate of a fictitious well ($q^m_{sc_{b_E,n_x}}$) that transfers fluid across the reservoir east boundary to gridblock n_x ; i.e.,

$$q^m_{sc_{b_E,n_x}}=T^m_{x_{n_x+1/2}}(\Phi^m_{n_x+1}-\Phi^m_{n_x}) \tag{4.6a}$$

Substitution of Eq. 4.7a into Eq. 4.6a yields

$$T^m_{x_{n_x-1/2}}(\Phi^m_{n_x-1}-\Phi^m_{n_x})+q^m_{sc_{b_E,n_x}}+q^m_{sc_{n_x}}=\frac{V_{b_{n_x}}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_{n_x}-(\frac{\phi}{B})^n_{n_x}] \tag{4.6b}$$

The above flow equation can also be obtained from Eq. 4.2b for $n=n_x$, $\psi_{n_x}=\{n_x-1\}$, $\xi_{n_x}=\{b_E\}$, and by observing that $\sum_{l\in\xi_{n_x}} q^m_{sc_{l,n_x}}=q^m_{sc_{b_E,n_x}}$ and $T^m_{n_x-1,n_x}=T^m_{x_{n_x-1/2}}$.

For 2D reservoirs, the flow equation for interior gridblock (i,j) is given by Eq. 2.37,

$$\begin{aligned}&T^m_{y_{i,j-1/2}}(\Phi^m_{i,j-1}-\Phi^m_{i,j})+T^m_{x_{i-1/2,j}}(\Phi^m_{i-1,j}-\Phi^m_{i,j})+T^m_{x_{i+1/2,j}}(\Phi^m_{i+1,j}-\Phi^m_{i,j})\\&+T^m_{y_{i,j+1/2}}(\Phi^m_{i,j+1}-\Phi^m_{i,j})+q^m_{sc_{i,j}}=\frac{V_{b_{i,j}}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_{i,j}-(\frac{\phi}{B})^n_{i,j}].\end{aligned} \tag{4.8a}$$

The above flow equation can be obtained from Eq. 4.2b for

$$n\equiv(i,j),\quad \psi_{i,j}=\{(i,j-1),(i-1,j),(i+1,j),(i,j+1)\},\quad \xi_{i,j}=\{\}$$

and by observing that $\sum_{l\in\xi_{i,j}} q^m_{sc_{l,(i,j)}}=0$ for an interior gridblock, $T^m_{(i,j\mp1),(i,j)}=T^m_{y_{i,j\mp1/2}}$, and $T^m_{(i\mp1,j),(i,j)}=T^m_{x_{i\mp1/2,j}}$.

For a gridblock that falls on one reservoir boundary, like gridblock (3,1), which falls on the reservoir south boundary in Figure 4–3, the flow equation can be written as

$$T^m_{y_{3,1-1/2}}(\Phi^m_{3,0}-\Phi^m_{3,1})+T^m_{x_{3-1/2,1}}(\Phi^m_{2,1}-\Phi^m_{3,1})+T^m_{x_{3+1/2,1}}(\Phi^m_{4,1}-\Phi^m_{3,1})$$
$$+T^m_{y_{3,1+1/2}}(\Phi^m_{3,2}-\Phi^m_{3,1})+q^m_{sc_{3,1}}=\frac{V_{b_{3,1}}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_{3,1}-(\frac{\phi}{B})^n_{3,1}]. \quad \textbf{(4.9a)}$$

The first term on the LHS of Eq. 4.9a represents the rate of fluid flow across the reservoir south boundary (b_S). This term can be replaced with the flow rate of a fictitious well ($q^m_{sc_{b_S,(3,1)}}$) that transfers fluid across the reservoir south boundary to gridblock (3,1); i.e.,

$$q^m_{sc_{b_S,(3,1)}}=T^m_{y_{3,1-1/2}}(\Phi^m_{3,0}-\Phi^m_{3,1}) \quad \textbf{(4.10)}$$

Substitution of Eq. 4.10 into Eq. 4.9a yields

$$q^m_{sc_{b_S,(3,1)}}+T^m_{x_{3-1/2,1}}(\Phi^m_{2,1}-\Phi^m_{3,1})+T^m_{x_{3+1/2,1}}(\Phi^m_{4,1}-\Phi^m_{3,1})$$
$$+T^m_{y_{3,1+1/2}}(\Phi^m_{3,2}-\Phi^m_{3,1})+q^m_{sc_{3,1}}=\frac{V_{b_{3,1}}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_{3,1}-(\frac{\phi}{B})^n_{3,1}]. \quad \textbf{(4.9b)}$$

The above flow equation can be obtained from Eq. 4.2b for

$$n\equiv(3,1)\ ,\quad \psi_{3,1}=\{(2,1),(4,1),(3,2)\}\ ,\quad \xi_{3,1}=\{b_S\}$$

and by observing that $\sum_{l\in\xi_{3,1}} q^m_{sc_{l,(3,1)}}=q^m_{sc_{b_S,(3,1)}}$, $T^m_{(2,1),(3,1)}=T^m_{x_{3-1/2,1}}$, $T^m_{(4,1),(3,1)}=T^m_{x_{3+1/2,1}}$, and $T^m_{(3,2),(3,1)}=T^m_{y_{3,1+1/2}}$.

For a gridblock that falls on two reservoir boundaries, like boundary gridblock (1,1), which falls on the reservoir south and west boundaries in Figure 4–3, the flow equation can be written as

$$T^m_{y_{1,1-1/2}}(\Phi^m_{1,0}-\Phi^m_{1,1})+T^m_{x_{1-1/2,1}}(\Phi^m_{0,1}-\Phi^m_{1,1})+T^m_{x_{1+1/2,1}}(\Phi^m_{2,1}-\Phi^m_{1,1})$$
$$+T^m_{y_{1,1+1/2}}(\Phi^m_{1,2}-\Phi^m_{1,1})+q^m_{sc_{1,1}}=\frac{V_{b_{1,1}}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_{1,1}-(\frac{\phi}{B})^n_{1,1}]. \quad \textbf{(4.11a)}$$

The first term on the LHS of Eq. 4.11a represents fluid flow rate across the reservoir south boundary (b_S). This term can be replaced with the flow rate of a fictitious well ($q^m_{sc_{b_S,(1,1)}}$) that transfers fluid across the reservoir south boundary to gridblock (1,1); i.e.,

$$q^m_{sc_{b_S,(1,1)}}=T^m_{y_{1,1-1/2}}(\Phi^m_{1,0}-\Phi^m_{1,1}) \quad \textbf{(4.12)}$$

The second term on the LHS of Eq. 4.11a represents fluid flow rate across the reservoir west boundary (b_W). This term can also be replaced with the flow rate of another fictitious well ($q^m_{sc_{b_W,(1,1)}}$) that transfers fluid across the reservoir west boundary to gridblock (1,1); i.e.,

$$q^m_{sc_{b_W,(1,1)}} = T^m_{x_{1-1/2,1}}(\Phi^m_{0,1} - \Phi^m_{1,1}) \tag{4.13}$$

Substitution of Eqs. 4.12 and 4.13 into Eq. 4.11a yields

$$\begin{aligned} &q^m_{sc_{b_S,(1,1)}} + q^m_{sc_{b_W,(1,1)}} + T^m_{x_{1+1/2,1}}(\Phi^m_{2,1} - \Phi^m_{1,1}) \\ &+T^m_{y_{1,1+1/2}}(\Phi^m_{1,2} - \Phi^m_{1,1}) + q^m_{sc_{1,1}} = \frac{V_{b_{1,1}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{1,1} - (\frac{\phi}{B})^n_{1,1}]. \end{aligned} \tag{4.11b}$$

The above flow equation can also be obtained from Eq. 4.2b for $n \equiv (1,1)$, $\psi_{1,1} = \{(2,1),(1,2)\}$, $\xi_{1,1} = \{b_S, b_W\}$, and by observing that $\sum_{l \in \xi_{1,1}} q^m_{sc_{l,(1,1)}} = q^m_{sc_{b_S,(1,1)}} + q^m_{sc_{b_W,(1,1)}}$, $T^m_{(2,1),(1,1)} = T^m_{x_{1+1/2,1}}$, and $T^m_{(1,2),(1,1)} = T^m_{y_{1,1+1/2}}$.

The following example demonstrates the use of the general equation, Eq. 4.2b, to write the flow equations for interior gridblocks in a 1D reservoir.

Example 4.2 For the 1D reservoir described in Example 4.1, write the flow equations for interior gridblocks 2,3, and 4.

Solution

The flow equation for gridblock *n*, in a 1D horizontal reservoir, is obtained by neglecting the gravity term in Eq. 4.2a, yielding

$$\sum_{l \in \psi_n} T^m_{l,n}(p^m_l - p^m_n) + \sum_{l \in \xi_n} q^m_{sc_{l,n}} + q^m_{sc_n} = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_n - (\frac{\phi}{B})^n_n] \tag{4.14}$$

For interior gridblocks *n*, $\psi_n = \{n-1, n+1\}$ and $\xi_n = \{\}$. Therefore, $\sum_{l \in \xi_n} q^m_{sc_{l,n}} = 0$.

The gridblocks in this problem are equally spaced; therefore, $T^m_{l,n} = T^m_{x_{n\mp1/2}} = T^m_x$, where

$$T^m_x = \beta_c \frac{k_x A_x}{\mu B \Delta x} = 0.001127 \times \frac{15 \times (1200 \times 75)}{10 \times 1 \times 1000} = 0.1521 \text{ STB/D-psi} \tag{4.15}$$

For gridblock 2, $n = 2$, $\psi_2 = \{1,3\}$, $\xi_2 = \{\}$, $\sum_{l \in \xi_2} q^m_{sc_{l,2}} = 0$, and $q^m_{sc_2} = 0$.

Therefore, Eq. 4.14 becomes

$$(0.1521)(p_1^m - p_2^m) + (0.1521)(p_3^m - p_2^m) = \frac{V_{b_2}}{\alpha_c \Delta t}[(\frac{\phi}{B})_2^{n+1} - (\frac{\phi}{B})_2^n]. \quad \textbf{(4.16)}$$

For gridblock 3, $n = 3$, $\psi_3 = \{2,4\}$, $\xi_3 = \{\}$, $\sum_{l \in \xi_3} q_{sc_{l,3}}^m = 0$, and $q_{sc_3}^m = 0$.

Therefore, Eq. 4.14 becomes

$$(0.1521)(p_2^m - p_3^m) + (0.1521)(p_4^m - p_3^m) = \frac{V_{b_3}}{\alpha_c \Delta t}[(\frac{\phi}{B})_3^{n+1} - (\frac{\phi}{B})_3^n]. \quad \textbf{(4.17)}$$

For gridblock 4, $n = 4$, $\psi_4 = \{3,5\}$, $\xi_4 = \{\}$, $\sum_{l \in \xi_4} q_{sc_{l,4}}^m = 0$,

and $q_{sc_4}^m = -150$ STB/D. Therefore, Eq. 4.14 becomes

$$(0.1521)(p_3^m - p_4^m) + (0.1521)(p_5^m - p_4^m) - 150 = \frac{V_{b_4}}{\alpha_c \Delta t}[(\frac{\phi}{B})_4^{n+1} - (\frac{\phi}{B})_4^n]. \quad \textbf{(4.18)}$$

4.4 Treatment of Boundary Conditions

A reservoir boundary can be subject to one of four conditions: 1) no-flow boundary, 2) constant flow boundary, 3) constant pressure gradient boundary, and 4) constant pressure boundary. In fact, the first three boundary conditions reduce to the specified pressure gradient condition (the Neumann boundary condition) and the fourth boundary condition is the Dirichlet boundary condition (constant pressure value). This section presents in detail the treatment of boundary conditions for 1D flow in the x direction, followed by generalizations for the treatment of boundary conditions in multidimensional reservoirs. In this section, we refer to reservoir boundaries as left and right boundaries because the lower, south, and west boundaries can be considered left boundaries, while the east, north, and upper boundaries can be considered right boundaries in 3D reservoirs. The flow rate of the fictitious well ($q_{sc_{b,bB}}^m$) reflects fluid transfer between the boundary block (bB) (e.g., gridblock 1 for the reservoir left boundary and gridblock n_x for the reservoir right boundary in Figure 4–1) and the reservoir boundary itself (b), or between the boundary block and the block next to the reservoir boundary that falls outside the reservoir (bB^{**}) (e.g., gridblock 0 for the reservoir left boundary and gridblock $n_x + 1$ for the reservoir right boundary in Figure 4–6). Eq. 4.4b expresses the flow equation for boundary gridblock 1, which falls on the reservoir left boundary, and Eq. 4.6b expresses the equation for boundary gridblock n_x which falls on the reservoir right boundary.

For boundary gridblock 1, which falls on the reservoir left boundary, the rate of fictitious well is expressed by Eq. 4.5a, which states

$$q_{sc_{bW,1}}^m = T_{x_{1-1/2}}^m (\Phi_0^m - \Phi_1^m) \quad \textbf{(4.5a)}$$

Since there is no geologic control for areas outside the reservoir, including aquifers, it is not uncommon to assign reservoir rock properties to those areas in the neighborhood of the reservoir under consideration. Therefore, we use the reflection technique at left boundary of the reservoir, shown in Figure 4–6, with regards to transmissibility only (i.e., $T^m_{0,b_W} = T^m_{b_W,1}$) and evaluate $T^m_{x_{1-1/2}}$ in terms of the transmissibilities between gridblock 0 and the reservoir west boundary b_W and between gridblock 1 and the reservoir west boundary b_W. The result is

$$T^m_{x_{1/2}} = [\beta_c \frac{k_x A_x}{\mu B \Delta x}]^m_{1/2} = [\beta_c \frac{k_x A_x}{\mu B \Delta x_1}]^m_1 = \tfrac{1}{2}[\beta_c \frac{k_x A_x}{\mu B(\Delta x_1 / 2)}]^m_1 = \tfrac{1}{2} T^m_{b_W,1} = \tfrac{1}{2} T^m_{0,b_W} \quad \textbf{(4.19a)}$$

or

$$T^m_{0,b_W} = T^m_{b_W,1} = 2T^m_{x_{1/2}} \quad \textbf{(4.19b)}$$

Substitution of Eq. 4.19b into Eq. 4.5a gives

$$q^m_{sc_{b_W,1}} = \tfrac{1}{2} T^m_{b_W,1}(\Phi^m_0 - \Phi^m_1) \quad \textbf{(4.5b)}$$

Similarly, for boundary gridblock n_x, which falls on the reservoir right boundary,

$$q^m_{sc_{b_E,n_x}} = T^m_{x_{n_x+1/2}}(\Phi^m_{n_x+1} - \Phi^m_{n_x}) \quad \textbf{(4.7a)}$$

and

$$q^m_{sc_{b_E,n_x}} = \tfrac{1}{2} T^m_{b_E,n_x}(\Phi^m_{n_x+1} - \Phi^m_{n_x}) \quad \textbf{(4.7b)}$$

In other words, the flow term between a boundary gridblock and the gridblock located immediately on the other side of the reservoir boundary can be replaced by a fictitious well having a flow rate $q^m_{sc_{b,bB}}$. The general form for $q^m_{sc_{b,bB}}$ is

$$q^m_{sc_{b,bB}} = T^m_{bB,bB^{**}}(\Phi^m_{bB^{**}} - \Phi^m_{bB}) \quad \textbf{(4.20a)}$$

or

$$q^m_{sc_{b,bB}} = \tfrac{1}{2} T^m_{b,bB}(\Phi^m_{bB^{**}} - \Phi^m_{bB}), \quad \textbf{(4.20b)}$$

where, as shown in Figure 4–7, $q^m_{sc_{b,bB}}$ = flow rate of a fictitious well representing flow across reservoir boundary (b) between boundary block (bB) and the block that is exterior to the reservoir and located immediately next to reservoir boundary (bB^{**}), $T^m_{bB,bB^{**}}$ = transmissibility between boundary gridblock bB and gridblock bB^{**}, and $T^m_{b,bB}$ = transmissibility between reservoir boundary (b) and boundary gridblock bB.

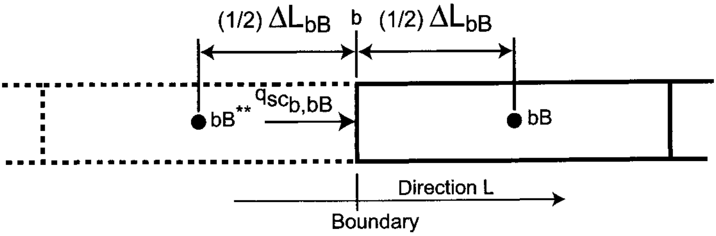


Figure 4–7 Definition of terminology used in Eq. 4.20.

In the following sections, we derive expressions for $q^m_{sc_{b,bB}}$ under various boundary conditions for a block-centered grid in Cartesian coordinates. We stress that this rate must produce the same effects as the specified boundary condition. In Cartesian coordinates, real wells have radial flow and fictitious wells have linear flow, whereas in radial-cylindrical coordinates in single-well simulation both real wells and fictitious wells have radial flow. Therefore, in single-well simulation, 1) the equations for the flow rate of real wells presented in Sections 6.2.2 and 6.3.2 can be used to estimate the flow rate of fictitious wells representing boundary conditions in the radial direction only, 2) the flow rate equations of fictitious wells in the z-direction are similar to those presented next in this section because flow in the vertical direction is linear, and 3) there are no reservoir boundaries and hence no fictitious wells in the θ direction. The flow rate of a fictitious well is positive for fluid gain (injection) or negative for fluid loss (production) across a reservoir boundary.

4.4.1 Specified Pressure Gradient Boundary Condition

For boundary gridblock 1 shown in Figure 4–8, which falls on the left boundary of the reservoir, Eq. 4.20a reduces to Eq. 4.5a that can be rewritten as

$$q^m_{sc_{bW},1} = T^m_{x_{1/2}}(\Phi^m_0 - \Phi^m_1) = [\beta_c \frac{k_x A_x}{\mu B \Delta x}]^m_{1/2}(\Phi^m_0 - \Phi^m_1) = [\beta_c \frac{k_x A_x}{\mu B}]^m_{1/2} \frac{(\Phi^m_0 - \Phi^m_1)}{\Delta x_{1/2}}$$

$$\cong [\beta_c \frac{k_x A_x}{\mu B}]^m_{1/2}[-\frac{\partial \Phi}{\partial x}\Big|^m_{b_W}] = -[\beta_c \frac{k_x A_x}{\mu B}]^m_{1/2} \frac{\partial \Phi}{\partial x}\Big|^m_{b_W} = -[\beta_c \frac{k_x A_x}{\mu B}]^m_1 \frac{\partial \Phi}{\partial x}\Big|^m_{b_W}. \tag{4.21}$$

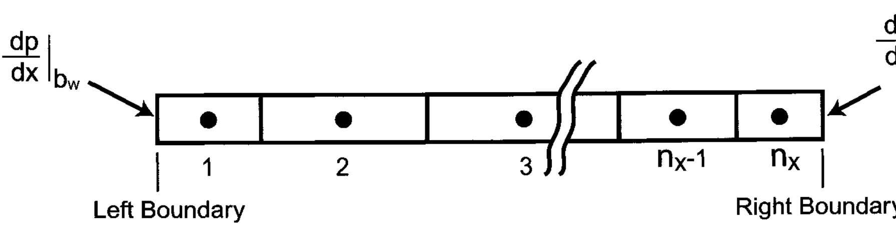


Figure 4–8 Specified pressure gradient condition at reservoir boundaries in a block-centered grid.

Note that in arriving at the above equation, we used the reflection technique shown in Figure 4–6 with respect to transmissibility and used the central-difference approximation of first order derivative of potential.

Similarly for gridblock n_x, which falls on the reservoir right boundary, Eq. 4.20a reduces to Eq. 4.7a that can be rewritten as

$$\begin{aligned} q^m_{sc_{b_E,n_x}} &= T^m_{x_{n_x+1/2}}(\Phi^m_{n_x+1} - \Phi^m_{n_x}) = [\beta_c \frac{k_x A_x}{\mu B \Delta x}]^m_{n_x+1/2}(\Phi^m_{n_x+1} - \Phi^m_{n_x}) \\ &= [\beta_c \frac{k_x A_x}{\mu B}]^m_{n_x+1/2} \frac{(\Phi^m_{n_x+1} - \Phi^m_{n_x})}{\Delta x_{n_x+1/2}} \cong [\beta_c \frac{k_x A_x}{\mu B}]^m_{n_x+1/2}[\frac{\partial \Phi}{\partial x}\bigg|^m_{b_E}] \\ &= [\beta_c \frac{k_x A_x}{\mu B}]^m_{n_x+1/2} \frac{\partial \Phi}{\partial x}\bigg|^m_{b_E} = [\beta_c \frac{k_x A_x}{\mu B}]^m_{n_x} \frac{\partial \Phi}{\partial x}\bigg|^m_{b_E}. \end{aligned} \tag{4.22}$$

Here again, we used the reflection technique shown in Figure 4–6 with respect to transmissibility and used the central-difference approximation of first order derivative of potential.

In general, for specified pressure gradient at the reservoir left (lower, south, or west) boundary,

$$q^m_{sc_{b,bB}} \cong -[\beta_c \frac{k_l A_l}{\mu B}]^m_{bB} \frac{\partial \Phi}{\partial l}\bigg|^m_b \tag{4.23a}$$

or, after combining with Eq. 2.10,

$$q^m_{sc_{b,bB}} \cong -[\beta_c \frac{k_l A_l}{\mu B}]^m_{bB}[\frac{\partial p}{\partial l}\bigg|^m_b - \gamma^m_{bB} \frac{\partial Z}{\partial l}\bigg|_b] \tag{4.23b}$$

and at the reservoir right (east, north, or upper) boundary,

$$q^m_{sc_{b,bB}} \cong [\beta_c \frac{k_l A_l}{\mu B}]^m_{bB} \frac{\partial \Phi}{\partial l}\bigg|^m_b \tag{4.24a}$$

or, after combining with Eq. 2.10,

$$q^m_{sc_{b,bB}} \cong [\beta_c \frac{k_l A_l}{\mu B}]^m_{bB}[\frac{\partial p}{\partial l}\bigg|^m_b - \gamma^m_{bB} \frac{\partial Z}{\partial l}\bigg|_b] \tag{4.24b}$$

where l is the direction normal to the boundary.

4.4.2 Specified Flow Rate Boundary Condition

The specified flow rate boundary condition arises when the reservoir near the boundary has higher or lower potential than that of a neighboring reservoir or aquifer. In this case, fluids move across the reservoir boundary. Methods such as water influx calculations and classical material balance in reservoir engineering can be used to estimate fluid flow rate, which we term here as specified (q_{spsc}). Therefore, Eq. 4.5a for boundary gridblock 1 becomes

$$q^m_{sc_{bW},1} = T^m_{x_{1/2}}(\Phi^m_0 - \Phi^m_1) = q_{spsc} \tag{4.25}$$

and Eq. 4.7a for boundary gridblock n_x becomes

$$q^m_{sc_{bE},n_x} = T^m_{x_{n_x+1/2}}(\Phi^m_{n_x+1} - \Phi^m_{n_x}) = q_{spsc} \tag{4.26}$$

In general, for a specified flow rate boundary condition, Eq. 4.20a becomes

$$q^m_{sc_{b,bB}} = q_{spsc} \tag{4.27}$$

In multidimensional flow with q_{spsc} specified for the whole reservoir boundary, $q^m_{sc_{b,bB}}$ for each boundary gridblock is obtained by prorating q_{spsc} among all boundary gridblocks that share that boundary; i.e.,

$$q^m_{sc_{b,bB}} = \frac{T^m_{b,bB}}{\sum\limits_{l \in \psi_b} T^m_{b,l}} q_{spsc} \tag{4.28}$$

where ψ_b is the set that contains all boundary gridblocks that share the reservoir boundary in question, $T_{b,l}$ = transmissibility between the reservoir boundary and boundary gridblock l, which is a member of the set ψ_b , and $T^m_{b,bB}$ is defined as

$$T^m_{b,bB} = [\beta_c \frac{k_l A_l}{\mu B(\Delta l/2)}]^m_{bB} \tag{4.29}$$

The length l and subscript l in Eq. 4.29 are replaced with x, y, or z depending on the boundary face of boundary block. It should be mentioned that Eq. 4.28 incorporates the assumption that the potential drops across the reservoir boundary for all gridblocks sharing that boundary are equal.

4.4.3 No-flow Boundary Condition

The no-flow boundary condition results from vanishing permeability at a reservoir boundary (e.g., $T^m_{x_{1/2}} = 0$ for the left boundary of gridblock 1, and $T^m_{x_{n_x+1/2}} = 0$ for the right boundary of gridblock n_x) or because of symmetry about the reservoir boundary (e.g., $\Phi^m_0 = \Phi^m_1$ for gridblock 1 and $\Phi^m_{n_x} = \Phi^m_{n_x+1}$ for gridblock n_x). In either case, Eq. 4.5a for boundary gridblock 1 reduces to

$$q^m_{sc_{bW},1} = T^m_{x_{1/2}}(\Phi^m_0 - \Phi^m_1) = 0(\Phi^m_0 - \Phi^m_1) = T^m_{x_{1/2}}(0) = 0 \tag{4.30}$$

and Eq. 4.7a for boundary gridblock n_x reduces to

$$q^m_{sc_{bE,n_x}} = T^m_{x_{n_x+1/2}}(\Phi^m_{n_x+1} - \Phi^m_{n_x}) = 0(\Phi^m_{n_x+1} - \Phi^m_{n_x}) = T^m_{x_{n_x+1/2}}(0) = 0 \tag{4.31}$$

In general, for a reservoir no-flow boundary, Eq. 4.20a becomes

$$q^m_{sc_{b,bB}} = 0 \tag{4.32}$$

For multidimensional flow, $q^m_{sc_{b,bB}}$ is set to zero, as Eq. 4.32 implies, for each boundary gridblock that falls on a no-flow boundary in the x, y, or z direction.

4.4.4 Specified Boundary Pressure Condition

This condition arises when the reservoir is in communication with a strong water aquifer, or when wells on the other side of the reservoir boundary operate to maintain voidage replacement and as a result keep boundary pressure (p_b) constant. Figure 4–9 shows this boundary condition at the reservoir left and right boundaries.

Eq. 4.5a for boundary gridblock 1 can be rewritten as

$$\begin{aligned} q^m_{sc_{bW,1}} &= T^m_{x_{1/2}}(\Phi^m_0 - \Phi^m_1) = T^m_{x_{1/2}}[\Phi^m_0 - \Phi_{b_W} + \Phi_{b_W} - \Phi^m_1] \\ &= T^m_{x_{1/2}}[(\Phi^m_0 - \Phi_{b_W}) + (\Phi_{b_W} - \Phi^m_1)] = T^m_{x_{1/2}}(\Phi^m_0 - \Phi_{b_W}) + T^m_{x_{1/2}}(\Phi_{b_W} - \Phi^m_1). \end{aligned} \tag{4.33}$$

Combining the above equation and Eq. 4.19b yields

$$q^m_{sc_{bW,1}} = \tfrac{1}{2}T^m_{0,b_W}(\Phi^m_0 - \Phi_{b_W}) + \tfrac{1}{2}T^m_{b_W,1}(\Phi_{b_W} - \Phi^m_1). \tag{4.34}$$

In order to keep the potential at the left boundary of gridblock 1 constant, the fluid leaving the reservoir boundary to one side (point 1) has to be equal to the fluid entering the reservoir boundary from the other side (point 0); see Figure 4–6. That is,

$$T^m_{0,b_W}(\Phi^m_0 - \Phi_{b_W}) = T^m_{b_W,1}(\Phi_{b_W} - \Phi^m_1). \tag{4.35}$$

Substitution of Eq. 4.35 into Eq. 4.34 and making use of Eq. 4.19b yields

$$q^m_{sc_{bW,1}} = T^m_{b_W,1}(\Phi_{b_W} - \Phi^m_1). \tag{4.36}$$

Keeping the potential at any point constant implies the pressure is kept constant because potential minus pressure is constant as required by Eq. 2.11.

In general, for a specified pressure boundary, Eq. 4.20a becomes

$$q^m_{sc_{b,bB}} = T^m_{b,bB}(\Phi_b - \Phi^m_{bB}) \tag{4.37a}$$

Eq. 4.37a can be rewritten in terms of pressure as

$$q^m_{sc_{b,bB}} = T^m_{b,bB}[(p_b - p^m_{bB}) - \gamma^m_{b,bB}(Z_b - Z_{bB})] \tag{4.37b}$$

where $\gamma^m_{b,bB}$ is nothing but fluid gravity in boundary block bB and $T^m_{b,bB}$ = transmissibility between the reservoir boundary and the point representing the boundary gridblock and is given by Eq. 4.29,

$$T^m_{b,bB} = [\beta_c \frac{k_l A_l}{\mu B(\Delta l / 2)}]^m_{bB} \tag{4.29}$$

Combining Eqs. 4.29 and 4.37b gives

$$q^m_{sc_{b,bB}} = [\beta_c \frac{k_l A_l}{\mu B(\Delta l / 2)}]^m_{bB}[(p_b - p^m_{bB}) - \gamma^m_{b,bB}(Z_b - Z_{bB})] \tag{4.37c}$$

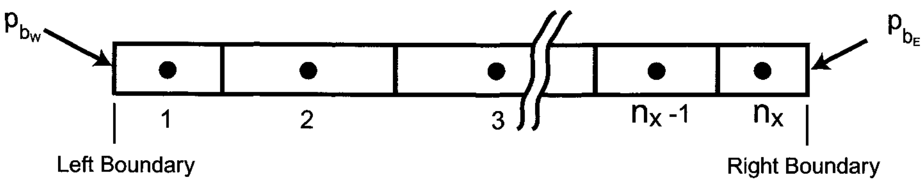


Figure 4–9 Specified pressure condition at reservoir boundaries in a block-centered grid.

Substitution of Eq. 4.37c in the flow equation for boundary gridblock bB maintains a second-order correct finite-difference flow equation in the mathematical approach (see Exercise 4-7). Abou-Kassem, Osman, and Islam (2006) have recently proved that such a treatment of this boundary condition is second-order correct. In multidimensional flow, $q^m_{sc_{b,bB}}$ for a boundary gridblock falling on a specified pressure boundary in the x, y, or z direction is estimated using Eq. 4.37c with the corresponding x, y, or z replacing l.

4.4.5 Specified Boundary Block Pressure

This condition arises if one makes the mathematical assumption that the boundary pressure is displaced half a block to coincide with the center of the boundary gridblock; i.e., $p_1 \cong p_{b_W}$ or $p_{n_x} \cong p_{b_E}$. This approximation is first-order correct and produces results that are less accurate than the treatment that uses Eq. 4.37c. Currently available books on reservoir simulation use this treatment to deal with the specified boundary pressure condition. Following this treatment, the problem reduces to finding the pressure of other gridblocks in the reservoir as demonstrated in Example 7.2.

The following examples demonstrate the use of the general equation, Eq. 4.2a, and the appropriate expressions for $q^m_{sc_{b,bB}}$ to write the flow equations for boundary gridblocks in 1D and 2D reservoirs that are subject to various boundary conditions.

Example 4.3 For the 1D reservoir described in Example 4.1, the reservoir left boundary is kept at a constant pressure of 5000 psia, and the reservoir right boundary is a no-flow (sealed) boundary as shown in Figure 4–10. Write the flow equations for boundary gridblocks 1 and 5.

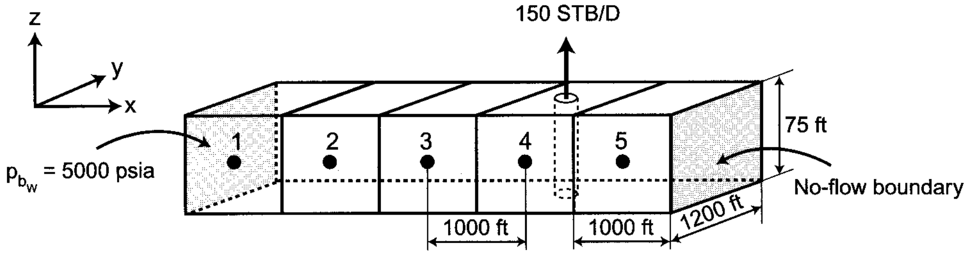


Figure 4–10 Discretized 1D reservoir in Example 4.3.

Solution

The flow equation for gridblock n in a 1D horizontal reservoir is obtained from Eq. 4.2a by neglecting the gravity term, resulting in

$$\sum_{l\in\psi_n} T_{l,n}^m (p_l^m - p_n^m) + \sum_{l\in\xi_n} q_{sc_{l,n}}^m + q_{sc_n}^m = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})_n^{n+1} - (\frac{\phi}{B})_n^n] \tag{4.14}$$

From Example 4.2, $T_{l,n}^m = T_x^m = 0.1521$ STB/D-psi.

For boundary gridblock 1, $n=1$, $\psi_1 = \{2\}$, $\xi_1 = \{b_W\}$, $\sum_{l\in\xi_1} q_{sc_{l,1}}^m = q_{sc_{b_W,1}}^m$, and

$q_{sc_1}^m = 0$.

Therefore, Eq. 4.14 becomes

$$0.1521(p_2^m - p_1^m) + q_{sc_{b_W,1}}^m = \frac{V_{b_1}}{\alpha_c \Delta t}[(\frac{\phi}{B})_1^{n+1} - (\frac{\phi}{B})_1^n] \tag{4.38}$$

where the rate of flow across the reservoir left boundary is given by Eq. 4.37c,

$$q_{sc_{b_W,1}}^m = [\beta_c \frac{k_x A_x}{\mu B(\Delta x/2)}]_1^m [(p_{b_W} - p_1^m) - \gamma_{b_W,1}(Z_{b_W} - Z_1)] \tag{4.39}$$

$$= 0.001127 \times \frac{15\times(1200\times 75)}{10\times 1\times(1000/2)}[(5000 - p_1^m) - \gamma_{b_W,1}\times 0]$$

or

$$q_{sc_{b_W,1}}^m = (0.3043)(5000 - p_1^m). \tag{4.40}$$

Substitution of Eq. 4.40 into Eq. 4.38 results in the flow equation for boundary gridblock 1,

$$(0.1521)(p_2^m - p_1^m) + (0.3043)(5000 - p_1^m) = \frac{V_{b_1}}{\alpha_c \Delta t}[(\frac{\phi}{B})_1^{n+1} - (\frac{\phi}{B})_1^n] \quad \textbf{(4.41)}$$

For boundary gridblock 5, $n = 5$, $\psi_5 = \{4\}$, $\xi_5 = \{b_E\}$, $\sum_{l \in \xi_5} q_{sc_{l,5}}^m = q_{sc_{b_E,5}}^m$, and $q_{sc_5}^m = 0$. Therefore, Eq. 4.14 becomes

$$(0.1521)(p_4^m - p_5^m) + q_{sc_{b_E,5}}^m = \frac{V_{b_5}}{\alpha_c \Delta t}[(\frac{\phi}{B})_5^{n+1} - (\frac{\phi}{B})_5^n] \quad \textbf{(4.42)}$$

where the flow rate across the reservoir right boundary (no-flow boundary) is given by Eq. 4.32. For the reservoir right boundary, $b \equiv b_E$, $bB \equiv 5$, and

$$q_{sc_{b_E,5}}^m = 0 \quad \textbf{(4.43)}$$

Substitution into Eq. 4.42 results in the flow equation for boundary gridblock 5,

$$(0.1521)(p_4^m - p_5^m) = \frac{V_{b_5}}{\alpha_c \Delta t}[(\frac{\phi}{B})_5^{n+1} - (\frac{\phi}{B})_5^n] \quad \textbf{(4.44)}$$

Example 4.4 For the 1D reservoir described in Example 4.1, the reservoir left boundary is kept at a constant pressure gradient of –0.1 psi/ft and the reservoir right boundary is supplied with fluid at a rate of 50 STB/D as shown in Figure 4–11. Write the flow equations for boundary gridblocks 1 and 5.

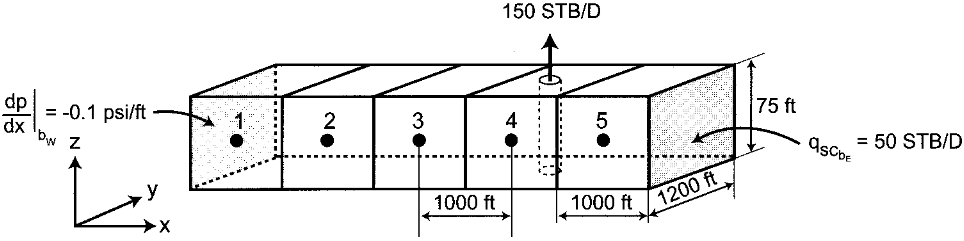


Figure 4–11 Discretized 1D reservoir in Example 4.4.

Solution

The flow equation for gridblock n in a 1D horizontal reservoir is obtained from Eq. 4.2a by neglecting the gravity term, resulting in

$$\sum_{l \in \psi_n} T_{l,n}^m (p_l^m - p_n^m) + \sum_{l \in \xi_n} q_{sc_{l,n}}^m + q_{sc_n}^m = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})_n^{n+1} - (\frac{\phi}{B})_n^n] \quad \textbf{(4.14)}$$

From Example 4.2, $T_{l,n}^m = T_x^m = 0.1521$ STB/D-psi.

For boundary gridblock 1, $n=1$, $\psi_1 = \{2\}$, $\xi_1 = \{b_W\}$, $\sum_{l \in \xi_1} q_{sc_{l,1}}^m = q_{sc_{b_W,1}}^m$, and

$q_{sc_1}^m = 0$. Therefore, Eq. 4.14 becomes

$$(0.1521)(p_2^m - p_1^m) + q_{sc_{b_W,1}}^m = \frac{V_{b_1}}{\alpha_c \Delta t}[(\frac{\phi}{B})_1^{n+1} - (\frac{\phi}{B})_1^n] \tag{4.38}$$

where the flow rate of a fictitious well for the specified pressure gradient at the reservoir left boundary is estimated using Eq. 4.23b,

$$q_{sc_{b_W,1}}^m = -[\beta_c \frac{k_x A_x}{\mu B}]_1^m [\frac{\partial p}{\partial x}\Big|_{b_W} - \gamma_1^m \frac{\partial Z}{\partial x}\Big|_{b_W}] \tag{4.45}$$

$$= -[0.001127 \times \frac{15 \times (1200 \times 75)}{10 \times 1}][-0.1 - 0] = -152.145 \times (-0.1)$$

or

$$q_{sc_{b_W,1}}^m = 15.2145. \tag{4.46}$$

Substitution of Eq. 4.46 into Eq. 4.38 results in the flow equation for boundary gridblock 1,

$$(0.1521)(p_2^m - p_1^m) + 15.2145 = \frac{V_{b_1}}{\alpha_c \Delta t}[(\frac{\phi}{B})_1^{n+1} - (\frac{\phi}{B})_1^n] \tag{4.47}$$

For boundary gridblock 5, $n=5$, $\psi_5 = \{4\}$, $\xi_5 = \{b_E\}$, $\sum_{l \in \xi_5} q_{sc_{l,5}}^m = q_{sc_{b_E,5}}^m$, and

$q_{sc_5}^m = 0$. Therefore, Eq. 4.14 becomes

$$(0.1521)(p_4^m - p_5^m) + q_{sc_{b_E,5}}^m = \frac{V_{b_5}}{\alpha_c \Delta t}[(\frac{\phi}{B})_5^{n+1} - (\frac{\phi}{B})_5^n] \tag{4.42}$$

where the flow rate of a fictitious well for a specified rate boundary is estimated using Eq. 4.27; i.e.,

$$q_{sc_{b_E,5}}^m = 50 \text{ STB/D} \tag{4.48}$$

Substitution of Eq. 4.48 into Eq. 4.42 results in the flow equation for boundary gridblock 5,

$$(0.1521)(p_4^m - p_5^m) + 50 = \frac{V_{b_5}}{\alpha_c \Delta t}[(\frac{\phi}{B})_5^{n+1} - (\frac{\phi}{B})_5^n] \tag{4.49}$$

Example 4.5 Consider single-phase fluid flow in the 2D horizontal reservoir shown in Figure 4–12. A well located in gridblock 7 produces at a rate of 4000 STB/D. All gridblocks have $\Delta x = 250$ ft, $\Delta y = 300$ ft, $h = 100$ ft, $k_x = 270$ md, and $k_y = 220$ md. The FVF and viscosity of the flowing fluid are 1.0 RB/STB and 2 cp, respectively. The reservoir south boundary is maintained at 3000 psia, the reservoir west boundary is sealed off to flow, the reservoir east boundary is kept at a constant pressure gradient of 0.1 psi/ft, and the reservoir loses fluid across its north boundary at a rate of 500 STB/D. Write the flow equations for boundary gridblocks 2, 5, 8, and 11.

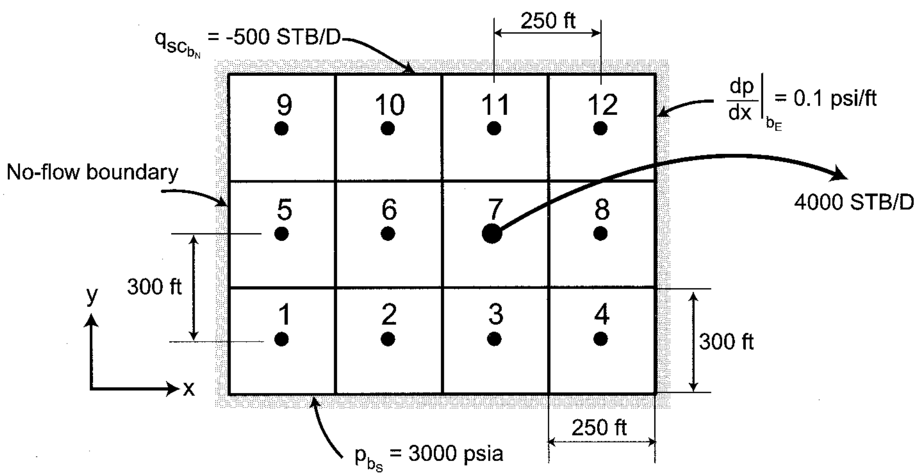


Figure 4–12 Discretized 2D reservoir in Examples 4.5 and 4.6.

Solution

The general flow equation for a 2D horizontal reservoir is obtained from Eq. 4.2a by neglecting the gravity term, resulting in

$$\sum_{l\in\psi_n} T^m_{l,n}(p^m_l - p^m_n) + \sum_{l\in\xi_n} q^m_{sc_{l,n}} + q^m_{sc_n} = \frac{V_{b_n}}{\alpha_c \Delta t}\left[\left(\frac{\phi}{B}\right)^{n+1}_n - \left(\frac{\phi}{B}\right)^n_n\right] \tag{4.14}$$

Note that $\Delta x_{l,n} = \Delta x = 250$ ft, $\Delta y_{l,n} = \Delta y = 300$ ft, and k_x, k_y, μ, and B are constant. Therefore,

$$T^m_x = \beta_c \frac{k_x A_x}{\mu B \Delta x} = 0.001127 \times \frac{270 \times (300 \times 100)}{2 \times 1 \times 250} = 18.2574 \text{ STB/D-psi} \tag{4.50}$$

$$T^m_y = \beta_c \frac{k_y A_y}{\mu B \Delta y} = 0.001127 \times \frac{220 \times (250 \times 100)}{2 \times 1 \times 300} = 10.3308 \text{ STB/D-psi} \tag{4.51}$$

For boundary gridblock 2, $n=2$, $\psi_2=\{1,3,6\}$, $\xi_2=\{b_S\}$, and $q^m_{sc_2}=0$.

$\sum_{l\in\xi_2} q^m_{sc_{l,2}} = q^m_{sc_{b_S,2}}$, where $q^m_{sc_{b_S,2}}$ is obtained from Eq. 4.37c after discarding the gravity term, resulting in

$$q^m_{sc_{b_S,2}} = [\beta_c \frac{k_y A_y}{\mu B(\Delta y/2)}]^m_2 (p_{b_S} - p^m_2) \tag{4.52}$$

$$= [0.001127 \times \frac{220\times(250\times100)}{2\times1\times(300/2)}](3000-p^m_2)$$

or

$$q^m_{sc_{b_S,2}} = (20.6617)(3000-p^m_2) \tag{4.53}$$

Substitution into Eq. 4.14 results in the flow equation for boundary gridblock 2,

$$\begin{aligned}&(18.2574)(p^m_1-p^m_2)+(18.2574)(p^m_3-p^m_2)+(10.3308)(p^m_6-p^m_2)\\&+(20.6617)(3000-p^m_2)=\frac{V_{b_2}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_2-(\frac{\phi}{B})^n_2].\end{aligned} \tag{4.54}$$

For boundary gridblock 5, $n=5$, $\psi_5=\{1,6,9\}$, $\xi_5=\{b_W\}$, and $q^m_{sc_5}=0$.

$\sum_{l\in\xi_5} q^m_{sc_{l,5}} = q^m_{sc_{b_W,5}}$, where $q^m_{sc_{b_W,5}}$ is obtained from Eq. 4.32 for a no-flow boundary;

i.e., $q^m_{sc_{b_W,5}}=0$.

Substitution into Eq. 4.14 results in the flow equation for boundary gridblock 5,

$$\begin{aligned}&(10.3308)(p^m_1-p^m_5)+(18.2574)(p^m_6-p^m_5)+(10.3308)(p^m_9-p^m_5)\\&=\frac{V_{b_5}}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_5-(\frac{\phi}{B})^n_5].\end{aligned} \tag{4.55}$$

For boundary gridblock 8, $n=8$, $\psi_8=\{4,7,12\}$, $\xi_8=\{b_E\}$, and $q^m_{sc_8}=0$.

$\sum_{l\in\xi_8} q^m_{sc_{l,8}} = q^m_{sc_{b_E,8}}$, where $q^m_{sc_{b_E,8}}$ is estimated using Eq. 4.24b for the reservoir east boundary,

$$\begin{aligned}q^m_{sc_{b_E,8}} &= [\beta_c \frac{k_x A_x}{\mu B}]^m_8 [\frac{\partial p}{\partial x}\bigg|^m_{b_E} - \gamma^m_8 \frac{\partial Z}{\partial x}\bigg|_{b_E}] = [0.001127\times\frac{270\times(300\times100)}{2\times1}][0.1-0]\\&= 4564.35\times(0.1) = 456.435.\end{aligned} \tag{4.56}$$

Substitution into Eq. 4.14 results in the flow equation for boundary gridblock 8,

$$\begin{aligned}&(10.3308)(p_4^m - p_8^m) + (18.2574)(p_7^m - p_8^m) + (10.3308)(p_{12}^m - p_8^m)\\ &+456.435 = \frac{V_{b_8}}{\alpha_c \Delta t}[(\frac{\phi}{B})_8^{n+1} - (\frac{\phi}{B})_8^n].\end{aligned} \quad \textbf{(4.57)}$$

For boundary gridblock 11, $n = 11$, $\psi_{11} = \{7,10,12\}$, $\xi_{11} = \{b_N\}$, and $q_{sc_{11}}^m = 0$.

$\sum_{l \in \xi_{11}} q_{sc_{l,11}}^m = q_{sc_{b_N,11}}^m$, where $q_{sc_{b_N,11}}^m$ is estimated using Eq. 4.28 because

$q_{spsc} = -500$ STB/D is specified for the whole reservoir north boundary. This rate has to be prorated among all gridblocks sharing that boundary. Therefore,

$$q_{sc_{b_N,11}}^m = \frac{T_{b_N,11}^m}{\sum_{l \in \psi_{b_N}} T_{b_N,l}^m} q_{spsc} \quad \textbf{(4.58)}$$

where $\psi_{b_N} = \{9,10,11,12\}$.

Using Eq. 4.29,

$$T_{b_N,l}^m = T_{b_N,11}^m = [\beta_c \frac{k_y A_y}{\mu B(\Delta y/2)}]_{11}^m = [0.001127 \times \frac{220 \times (250 \times 100)}{2 \times 1 \times (300/2)}] = 20.6616 \quad \textbf{(4.59)}$$

for all values of $l \in \psi_{b_N}$.

Substitution of Eq. 4.59 into Eq. 4.58 yields

$$q_{sc_{b_N,11}}^m = \frac{20.6616}{4 \times 20.6616} \times (-500) = -125 \text{ STB/D} \quad \textbf{(4.60)}$$

Substitution into Eq. 4.14 results in the flow equation for boundary gridblock 11,

$$\begin{aligned}&(10.3308)(p_7^m - p_{11}^m) + (18.2574)(p_{10}^m - p_{11}^m) + (18.2574)(p_{12}^m - p_{11}^m)\\ &-125 = \frac{V_{b_{11}}}{\alpha_c \Delta t}[(\frac{\phi}{B})_{11}^{n+1} - (\frac{\phi}{B})_{11}^n].\end{aligned} \quad \textbf{(4.61)}$$

Example 4.6 Consider single-phase fluid flow in the 2D horizontal reservoir described in Example 4.5. Write the flow equations for gridblocks 1, 4, 9, and 12, where each gridblock falls on two reservoir boundaries.

Solution

The general flow equation for a 2D horizontal reservoir is obtained from Eq. 4.2a by neglecting the gravity term, resulting in Eq. 4.14,

$$\sum_{l\in\psi_n} T_{l,n}^m (p_l^m - p_n^m) + \sum_{l\in\xi_n} q_{sc_{l,n}}^m + q_{sc_n}^m = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})_n^{n+1} - (\frac{\phi}{B})_n^n] \tag{4.14}$$

The data necessary to write flow equations for any boundary gridblock were calculated in Example 4.5. The following is a summary:

$$T_x^m = 18.2574 \text{ STB/D-psi}; \tag{4.62a}$$

$$T_y^m = 10.3308 \text{ STB/D-psi}; \tag{4.62b}$$

$$q_{sc_{b_S,bB}}^m = (20.6617)(3000 - p_{bB}^m) \text{ STB/D} \tag{4.62c}$$

for $bB = 1,2,3,4$;

$$q_{sc_{b_W,bB}}^m = 0 \text{ STB/D for } bB = 1,5,9 ; \tag{4.62d}$$

$$q_{sc_{b_E,bB}}^m = 456.435 \text{ STB/D for } bB = 4,8,12 ; \tag{4.62e}$$

and

$$q_{sc_{b_N,bB}}^m = -125 \text{ STB/D for } bB = 9,10,11,12 . \tag{4.62f}$$

For boundary gridblock 1, $n = 1$, $\psi_1 = \{2,5\}$, $\xi_1 = \{b_S, b_W\}$, $q_{sc_1}^m = 0$, and

$$\sum_{l\in\xi_1} q_{sc_{l,1}}^m = q_{sc_{b_S,1}}^m + q_{sc_{b_W,1}}^m = (20.6617)(3000 - p_1^m) + 0 = (20.6617)(3000 - p_1^m) \text{ STB/D.}$$

Substitution into Eq. 4.14 results in the flow equation for boundary gridblock 1,

$$\begin{aligned}&(18.2574)(p_2^m - p_1^m) + (10.3308)(p_5^m - p_1^m) + (20.6617)(3000 - p_1^m)\\&= \frac{V_{b_1}}{\alpha_c \Delta t}[(\frac{\phi}{B})_1^{n+1} - (\frac{\phi}{B})_1^n].\end{aligned} \tag{4.63}$$

For boundary gridblock 4, $n = 4$, $\psi_4 = \{3,8\}$, $\xi_4 = \{b_S, b_E\}$, $q_{sc_4}^m = 0$, and

$$\sum_{l\in\xi_4} q_{sc_{l,4}}^m = q_{sc_{b_S,4}}^m + q_{sc_{b_E,4}}^m = (20.6617)(3000 - p_4^m) + 456.435 \text{ STB/D.}$$

Substitution into Eq. 4.14 results in the flow equation for boundary gridblock 4,

$$\begin{aligned}&(18.2574)(p_3^m - p_4^m) + (10.3308)(p_8^m - p_4^m) + (20.6617)(3000 - p_4^m) + 456.435\\&= \frac{V_{b_4}}{\alpha_c \Delta t}[(\frac{\phi}{B})_4^{n+1} - (\frac{\phi}{B})_4^n].\end{aligned} \tag{4.64}$$

For boundary gridblock 9, $n = 9$, $\psi_9 = \{5,10\}$, $\xi_9 = \{b_W, b_N\}$, $q_{sc_9}^m = 0$, and

$$\sum_{l\in\xi_9} q^m_{sc_{l,9}} = q^m_{sc_{b_W,9}} + q^m_{sc_{b_N,9}} = 0 - 125 = -125 \text{ STB/D.}$$

Substitution into Eq. 4.14 results in the flow equation for boundary gridblock 9,

$$(10.3308)(p^m_5 - p^m_9) + (18.2574)(p^m_{10} - p^m_9) - 125 = \frac{V_{b_9}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_9 - (\frac{\phi}{B})^n_9]. \quad \textbf{(4.65)}$$

For boundary gridblock 12, $n = 12$, $\psi_{12} = \{8,11\}$, $\xi_{12} = \{b_E, b_N\}$, $q^m_{sc_{12}} = 0$, and

$$\sum_{l\in\xi_{12}} q^m_{sc_{l,12}} = q^m_{sc_{b_E,12}} + q^m_{sc_{b_N,12}} = 456.435 - 125 = 331.435 \text{ STB/D.}$$

Substitution into Eq. 4.14 results in the flow equation for boundary gridblock 12,

$$(10.3308)(p^m_8 - p^m_{12}) + (18.2574)(p^m_{11} - p^m_{12}) + 331.435 = \frac{V_{b_{12}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{12} - (\frac{\phi}{B})^n_{12}]. \quad \textbf{(4.66)}$$

4.5 Calculation of Transmissibilities

Eq. 2.39 in Chapter 2 defines the transmissibilities in the flow equations in Cartesian coordinates. The definitions of transmissibility in the x, y, and z directions are expressed as

$$T_{x_{i\mp1/2,j,k}} = G_{x_{i\mp1/2,j,k}} (\frac{1}{\mu B})_{x_{i\mp1/2,j,k}} \quad \textbf{(4.67a)}$$

$$T_{y_{i,j\mp1/2,k}} = G_{y_{i,j\mp1/2,k}} (\frac{1}{\mu B})_{y_{i,j\mp1/2,k}} \quad \textbf{(4.67b)}$$

$$T_{z_{i,j,k\mp1/2}} = G_{z_{i,j,k\mp1/2}} (\frac{1}{\mu B})_{z_{i,j,k\mp1/2}} \quad \textbf{(4.67c)}$$

where the geometric factors G for anisotropic porous media and irregular gridblock distribution are given in Table 4–1 (Ertekin, Abou-Kassem, and King 2001). The treatment of the pressure dependent term (μB) in Eq. 4.67 is discussed in detail under linearization in Chapter 8 (Section 8.4.1).

Example 4.7 Derive the equation for the geometric factor of transmissibility in the x direction between gridblocks i and $i+1$ in 1D flow using

(a) Table 4–1 and

(b) Darcy's Law

Solution

(a) The geometric factor of transmissibility in the x direction is given as

Table 4–1 Geometric Factors in Rectangular Grids (Ertekin et al. 2001)

Direction	Geometric Factor
x	$G_{x_{i\mp1/2,j,k}} = \dfrac{2\beta_c}{\Delta x_{i,j,k}/(A_{x_{i,j,k}} k_{x_{i,j,k}}) + \Delta x_{i\mp1,j,k}/(A_{x_{i\mp1,j,k}} k_{x_{i\mp1,j,k}})}$
y	$G_{y_{i,j\mp1/2,k}} = \dfrac{2\beta_c}{\Delta y_{i,j,k}/(A_{y_{i,j,k}} k_{y_{i,j,k}}) + \Delta y_{i,j\mp1,k}/(A_{y_{i,j\mp1,k}} k_{y_{i,j\mp1,k}})}$
z	$G_{z_{i,j,k\mp1/2}} = \dfrac{2\beta_c}{\Delta z_{i,j,k}/(A_{z_{i,j,k}} k_{z_{i,j,k}}) + \Delta z_{i,j,k\mp1}/(A_{z_{i,j,k\mp1}} k_{z_{i,j,k\mp1}})}$

$$G_{x_{i\mp1/2,j,k}} = \frac{2\beta_c}{\Delta x_{i,j,k}/(A_{x_{i,j,k}} k_{x_{i,j,k}}) + \Delta x_{i\mp1,j,k}/(A_{x_{i\mp1,j,k}} k_{x_{i\mp1,j,k}})} \tag{4.68}$$

For flow between gridblocks i and $i+1$ in a 1D reservoir, $j = 1$ and $k = 1$. Discarding these subscripts and the negative sign in Eq. 4.68 yields the sought geometric factor,

$$G_{x_{i+1/2}} = \frac{2\beta_c}{\Delta x_i/(A_{x_i} k_{x_i}) + \Delta x_{i+1}/(A_{x_{i+1}} k_{x_{i+1}})} \tag{4.69}$$

(b) Consider the steady-state flow of incompressible fluid ($B = 1$ and μ = constant) in incompressible porous media between gridblocks i and $i+1$. Gridblock i has cross-sectional area A_{x_i} and permeability k_{x_i}, and gridblock $i+1$ has cross-sectional area $A_{x_{i+1}}$ and permeability $k_{x_{i+1}}$. Boundary $i+1/2$ between the two blocks is δx_{i^+} away from point i and δx_{i+1^-} away from point $i+1$ as shown in Figure 4–13. Fluid flows from gridblock i to block boundary i+½ and then from block boundary i+½ to gridblock $i+1$. The rate of fluid flow from the center of gridblock i to block boundary i+½ is given by Darcy's Law as

$$q_{i,i+1/2} = \frac{\beta_c k_{x_i} A_{x_i}}{B\mu\delta x_{i^+}}(p_i - p_{i+1/2}) \tag{4.70}$$

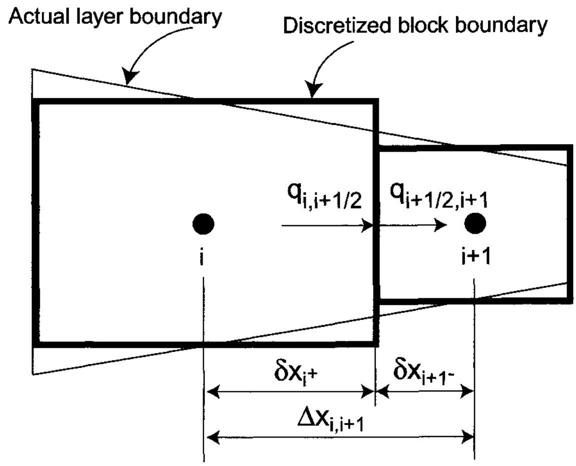


Figure 4–13 Transmissibility between two adjacent blocks.

Similarly, the rate of fluid flow from block boundary $i+½$ to the center of gridblock $i+1$ is given by Darcy's Law as

$$q_{i+1/2,i+1} = \frac{\beta_c k_{x_{i+1}} A_{x_{i+1}}}{B\mu \delta x_{i+1^-}} (p_{i+1/2} - p_{i+1}) \tag{4.71}$$

In this flow system, there is neither fluid accumulation nor fluid depletion. Therefore, the rate of fluid leaving gridblock i ($q_{i,i+1/2}$) has to be equal to the rate of fluid entering gridblock $i+1$ ($q_{i+1/2,i+1}$); i.e.,

$$q_{i,i+1/2} = q_{i+1/2,i+1} = q_{i,i+1} \tag{4.72}$$

The rate of fluid flow between the centers of gridblocks i and $i+1$ is given by Darcy's Law as

$$q_{i,i+1} = \frac{G_{x_{i+1/2}}}{B\mu} (p_i - p_{i+1}) \tag{4.73}$$

The pressure drop between the centers of gridblocks i and $i+1$ is equal to the sum of the pressure drops between the block centers and the block boundary between them; i.e.,

$$(p_i - p_{i+1}) = (p_i - p_{i+1/2}) + (p_{i+1/2} - p_{i+1}) \tag{4.74}$$

Substituting for pressure drops in Eq. 4.74 using Eqs. 4.70, 4.71, and 4.73 yields

$$\frac{q_{i,i+1} B\mu}{G_{x_{i+1/2}}} = \frac{q_{i,i+1/2} B\mu \delta x_{i^+}}{\beta_c k_{x_i} A_{x_i}} + \frac{q_{i+1/2,i+1} B\mu \delta x_{i+1^-}}{\beta_c k_{x_{i+1}} A_{x_{i+1}}} \tag{4.75}$$

Combining Eqs. 4.75 and 4.72 and dividing by flow rate, FVF, and viscosity yields

$$\frac{1}{G_{x_{i+1/2}}} = \frac{\delta x_{i^+}}{\beta_c k_{x_i} A_{x_i}} + \frac{\delta x_{i+1^-}}{\beta_c k_{x_{i+1}} A_{x_{i+1}}} \tag{4.76}$$

Eq. 4.76 can be solved for $G_{x_{i+1/2}}$. The resulting equation is

$$G_{x_{i+1/2}} = \frac{\beta_c}{\delta x_{i^+} /(A_{x_i} k_{x_i}) + \delta x_{i+1^-} /(A_{x_{i+1}} k_{x_{i+1}})} \tag{4.77}$$

Observing that $\delta x_{i^+} = \frac{1}{2}\Delta x_i$ and $\delta x_{i+1^-} = \frac{1}{2}\Delta x_{i+1}$ for a block-centered grid, Eq. 4.77 becomes

$$G_{x_{i+1/2}} = \frac{2\beta_c}{\Delta x_i /(A_{x_i} k_{x_i}) + \Delta x_{i+1} /(A_{x_{i+1}} k_{x_{i+1}})} \tag{4.78}$$

Eqs. 4.69 and 4.78 are identical.

Eq. 2.69 in Chapter 2 defines the transmissibilities in the flow equations in radial-cylindrical coordinates. The definitions of transmissibility in the r, θ, and z directions are expressed as

$$T_{r_{i\mp1/2,j,k}} = G_{r_{i\mp1/2,j,k}} (\frac{1}{\mu B})_{r_{i\mp1/2,j,k}} \tag{4.79a}$$

$$T_{\theta_{i,j\mp1/2,k}} = G_{\theta_{i,j\mp1/2,k}} (\frac{1}{\mu B})_{\theta_{i,j\mp1/2,k}} \tag{4.79b}$$

$$T_{z_{i,j,k\mp1/2}} = G_{z_{i,j,k\mp1/2}} (\frac{1}{\mu B})_{z_{i,j,k\mp1/2}} \tag{4.79c}$$

where the geometric factors G for anisotropic porous media and irregular gridblock distribution are given in Table 4–2 (Farouq Ali 1986). Note that in this table, r_i and $r_{i\mp1/2}$ depend on the value of subscript i only for $j = 1,2,3...n_\theta$ and $k = 1,2,3...n_z$; $\Delta\theta_j$ and $\Delta\theta_{j\mp1/2}$ depend on the value of subscript j only for $i = 1,2,3...n_r$ and $k = 1,2,3...n_z$; and Δz_k and $\Delta z_{k\mp1/2}$ depend on the value of subscript k only for $i = 1,2,3...n_r$ and $j = 1,2,3...n_\theta$. The treatment of the pressure dependent term (μB) in Eq. 4.79 is discussed in detail under linearization in Chapter 8 (Section 8.4.1).

Table 4–2 uses gridblock dimensions and block boundaries in the z direction as in Eq. 4.1, with z replacing x. Those in the θ direction are defined in a similar way. Specifically,

$$\sum_{j=1}^{n_\theta} \Delta\theta_j = 2\pi \ , \ \Delta\theta_{j+1/2} = \frac{1}{2}(\Delta\theta_{j+1} + \Delta\theta_j) \ , j = 1, 2, 3\ldots\ n_\theta - 1 \ ,$$

Table 4-2 Geometric Factors in Cylindrical Grids (Farouq Ali 1986)

Direction	Geometric Factor
r	$G_{r_{i-1/2,j,k}} = \dfrac{\beta_c \Delta\theta_j}{\log_e(r_i / r^L_{i-1/2})/(\Delta z_{i,j,k} k_{r_{i,j,k}}) + \log_e(r^L_{i-1/2} / r_{i-1})/(\Delta z_{i-1,j,k} k_{r_{i-1,j,k}})}$
	$G_{r_{i+1/2,j,k}} = \dfrac{\beta_c \Delta\theta_j}{\log_e(r^L_{i+1/2} / r_i)/(\Delta z_{i,j,k} k_{r_{i,j,k}}) + \log_e(r_{i+1} / r^L_{i+1/2})/(\Delta z_{i+1,j,k} k_{r_{i+1,j,k}})}$
θ	$G_{\theta_{i,j\mp1/2,k}} = \dfrac{2\beta_c \log_e(r^L_{i+1/2} / r^L_{i-1/2})}{\Delta\theta_j /(\Delta z_{i,j,k} k_{\theta_{i,j,k}}) + \Delta\theta_{j+1} /(\Delta z_{i,j\mp1,k} k_{\theta_{i,j\mp1,k}})}$
z	$G_{z_{i,j,k\mp1/2}} = \dfrac{2\beta_c (\tfrac{1}{2}\Delta\theta_j)(r^2_{i+1/2} - r^2_{i-1/2})}{\Delta z_{i,j,k} / k_{z_{i,j,k}} + \Delta z_{i,j,k\mp1} / k_{z_{i,j,k\mp1}}}$

$$\theta_{j+1} = \theta_j + \Delta\theta_{j+1/2}\ , j = 1,2,3\ldots\ n_\theta - 1\ ,\ \ \theta_1 = \tfrac{1}{2}\Delta\theta_1 \tag{4.80}$$

$$\theta_{j\mp1/2} = \theta_j \mp \tfrac{1}{2}\Delta\theta_j\ ,\ \ i\ = 1, 2, 3\ldots\ n_\theta\ .$$

In the r direction, however, the points representing gridblocks are spaced such that the pressure drops between neighboring points are equal (see Example 4.8). Block boundaries for transmissibility calculations are spaced logarithmically in r to warrant that the radial flow rates between neighboring points using the integrated continuous and discretized forms of Darcy's Law are identical (see Example 4.9). Block boundaries for bulk volume calculations are spaced logarithmically in r^2 to warrant that the actual and discretized bulk volumes of gridblocks are equal. Therefore, the radii for the pressure points ($r_{i\mp1}$), transmissibility calculations ($r^L_{i\mp1/2}$), and bulk-volume calculations ($r_{i\mp1/2}$) are as follows (Aziz and Settari 1979, Ertekin et al. 2001):

$$r_{i+1} = \alpha_{lg} r_i \tag{4.81}$$

for $i = 1,2,3 \ldots n_r - 1$;

$$r^L_{i+1/2} = \frac{r_{i+1} - r_i}{\log_e(r_{i+1} / r_i)} \tag{4.82a}$$

for $i = 1,2,3 \ldots n_r - 1$;

$$r^L_{i-1/2} = \frac{r_i - r_{i-1}}{\log_e(r_i / r_{i-1})} \tag{4.83a}$$

for $i = 2,3 \ldots n_r$;

$$r_{i+1/2}^2 = \frac{r_{i+1}^2 - r_i^2}{\log_e(r_{i+1}^2 / r_i^2)} \tag{4.84a}$$

for $i = 1,2,3...n_r - 1$;

$$r_{i-1/2}^2 = \frac{r_i^2 - r_{i-1}^2}{\log_e(r_i^2 / r_{i-1}^2)} \tag{4.85a}$$

for $i = 2,3...n_r$;

where

$$\alpha_{lg} = \left(\frac{r_e}{r_w}\right)^{1/n_r} \tag{4.86}$$

$$r_1 = [\alpha_{lg} \log_e(\alpha_{lg})/(\alpha_{lg} - 1)]r_w \tag{4.87}$$

Note that the reservoir internal boundary (r_w) and the reservoir external boundary (r_e) through which fluid may enter or leave the reservoir are respectively the internal boundary of gridblock 1 and the external boundary of gridblock n_r that are used to calculate transmissibility. That is to say, $r_{1/2}^L = r_w$ and $r_{n_r+1/2}^L = r_e$ by definition for a block-centered grid (Ertekin, Abou-Kassem, and King 2001).

The bulk volume of gridblock (i,j,k) is calculated from

$$V_{b_{i,j,k}} = (r_{i+1/2}^2 - r_{i-1/2}^2)(\tfrac{1}{2}\Delta\theta_j)\Delta z_{i,j,k} \tag{4.88a}$$

for $i = 1,2,3...n_r - 1$, $j = 1,2,3...n_\theta$, $k = 1,2,3...n_z$; and

$$V_{b_{n_r,j,k}} = (r_e^2 - r_{n_r-1/2}^2)(\tfrac{1}{2}\Delta\theta_j)\Delta z_{n_r,j,k} \tag{4.88c}$$

for $j = 1,2,3...n_\theta$, $k = 1,2,3...n_z$.

Example 4.8 Prove that the grid spacing in the radial direction defined by Eqs. 4.81 and 4.86 satisfies the condition of constant and equal pressure drops between successive points in steady-state radial flow of incompressible fluid.

Solution

The stead-state flow of incompressible fluid towards a well with radius r_w in a horizontal reservoir with an external radius r_e is expressed by Darcy's Law,

$$q = \frac{-2\pi\beta_c k_H h}{B\mu \log_e(\frac{r_e}{r_w})}(p_e - p_w). \tag{4.89}$$

The pressure drop across the reservoir is obtained from Eq. 4.89 as

$$(p_e - p_w) = \frac{-qB\mu \log_e(\frac{r_e}{r_w})}{2\pi\beta_c k_H h}. \tag{4.90}$$

Let the reservoir be divided into n_r radial segments that are represented by points $i = 1,2,3...n_r$ placed at $r_1, r_2, r_3, ... r_{i-1}, r_i, r_{i+1}, ... r_{n_r}$. The location of these points will be determined later (Eq. 4.81). For steady-state radial flow between points $i+1$ and i,

$$q = \frac{-2\pi\beta_c k_H h}{B\mu \log_e(\frac{r_{i+1}}{r_i})}(p_{i+1} - p_i). \tag{4.91}$$

The pressure drop between points $i+1$ and i is obtained from Eq. 4.91 as

$$(p_{i+1} - p_i) = \frac{-qB\mu \log_e(\frac{r_{i+1}}{r_i})}{2\pi\beta_c k_H h}. \tag{4.92}$$

If the pressure drop over each of the radial distances $(r_{i+1} - r_i)$ for $i = 1,2,3...n_r - 1$ is chosen to be constant and equal, then

$$(p_{i+1} - p_i) = \frac{(p_e - p_w)}{n_r} \tag{4.93}$$

for $i = 1,2,3...n_r - 1$.

Substituting Eqs. 4.90 and 4.92 into Eq. 4.93 yields

$$\log_e(\frac{r_{i+1}}{r_i}) = \frac{1}{n_r}\log_e(\frac{r_e}{r_w}) \tag{4.94}$$

or

$$(\frac{r_{i+1}}{r_i}) = (\frac{r_e}{r_w})^{1/n_r} \tag{4.95a}$$

for $i = 1,2,3...n_r - 1$.

For the convenience of manipulation, define

$$\alpha_{lg} = \left(\frac{r_e}{r_w}\right)^{1/n_r} \tag{4.86}$$

then Eq. 4.95a becomes

$$(\frac{r_{i+1}}{r_i}) = \alpha_{lg} \tag{4.95b}$$

or

$$r_{i+1} = \alpha_{lg} r_i \tag{4.81}$$

for $i = 1,2,3...n_r - 1$.

Eq. 4.81 defines the locations of the points in the *r* direction that result in equal pressure drops between any two successive points.

Example 4.9 Show that the block boundaries defined by Eq. 4.82a ensure that the flow rate across a block boundary is identical to that obtained from Darcy's Law.

Solution

From Example 4.8, for steady-state radial flow of incompressible fluid between points $i+1$ and i ,

$$q = \frac{-2\pi\beta_c k_H h}{B\mu \log_e(\frac{r_{i+1}}{r_i})}(p_{i+1} - p_i). \tag{4.91}$$

The steady-state fluid flow rate across a block boundary is also expressed by the differential form of Darcy's Law at block boundary $r^L_{i+1/2}$,

$$q_{r^L_{i+1/2}} = \frac{-2\pi\beta_c k_H h r^L_{i+1/2}}{B\mu} \left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_{r^L_{i+1/2}} \tag{4.96}$$

The pressure gradient at a block boundary can be approximated, using central differencing, as

$$\left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_{r^L_{i+1/2}} \cong \frac{p_{i+1} - p_i}{r_{i+1} - r_i} \tag{4.97}$$

Substitution of Eq. 4.97 into Eq. 4.96 results in

$$q_{r^L_{i+1/2}} = \frac{-2\pi\beta_c k_H h r^L_{i+1/2}}{B\mu} \frac{p_{i+1} - p_i}{r_{i+1} - r_i}. \tag{4.98}$$

If the flow rate calculated from Darcy's Law (Eq. 4.91) is identical to the flow rate calculated from the discretized Darcy's Law (Eq. 4.98), then

$$\frac{-2\pi\beta_c k_H h}{B\mu \log_e(\frac{r_{i+1}}{r_i})}(p_{i+1} - p_i) = \frac{-2\pi\beta_c k_H h r^L_{i+1/2}}{B\mu} \frac{p_{i+1} - p_i}{r_{i+1} - r_i} \tag{4.99}$$

which simplifies to give

$$r^L_{i+1/2} = \frac{r_{i+1} - r_i}{\log_e(\frac{r_{i+1}}{r_i})} \tag{4.82a}$$

Eqs. 4.82a, 4.83a, 4.84a, 4.85a, 4.88a and 4.88c can be expressed in terms of r_i and α_{lg} as

$$r^L_{i+1/2} = \{(\alpha_{lg} - 1)/[\log_e(\alpha_{lg})]\}r_i \tag{4.82b}$$

for $i = 1,2,3...n_r - 1$;

$$r^L_{i-1/2} = \{(\alpha_{lg} - 1)/[\alpha_{lg}\log_e(\alpha_{lg})]\}r_i = (1/\alpha_{lg})r^L_{i+1/2} \tag{4.83b}$$

for $i = 2,3...n_r$;

$$r^2_{i+1/2} = \{(\alpha^2_{lg} - 1)/[\log_e(\alpha^2_{lg})]\}r^2_i \tag{4.84b}$$

for $i = 1,2,3...n_r - 1$;

$$r^2_{i-1/2} = \{(\alpha^2_{lg} - 1)/[\alpha^2_{lg}\log_e(\alpha^2_{lg})]\}r^2_i = (1/\alpha^2_{lg})r^2_{i+1/2} \tag{4.85b}$$

for $i = 2,3...n_r$;

$$V_{b_{i,j,k}} = \{(\alpha^2_{lg} - 1)^2/[\alpha^2_{lg}\log_e(\alpha^2_{lg})]\}r^2_i(\tfrac{1}{2}\Delta\theta_j)\Delta z_{i,j,k} \tag{4.88b}$$

for $i = 1,2,3,...n_{r-1}$; and

$$V_{b_{n_r,j,k}} = \{1 - [\log_e(\alpha_{lg})/(\alpha_{lg} - 1)]^2(\alpha^2_{lg} - 1)/[\alpha^2_{lg}\log_e(\alpha^2_{lg})]\}r^2_e(\tfrac{1}{2}\Delta\theta_j)\Delta z_{n_r,j,k} \tag{4.88d}$$

for $i = n_r$.

Example 4.10 Prove that Eqs. 4.82b, 4.83b, 4.84b, 4.85b, and 4.88b are equivalent to Eqs. 4.82a, 4.83a, 4.84a, 4.85a, and 4.88a, respectively. In addition, express the arguments of the log terms that appear in Table 4–2 and the gridblock bulk volume in terms of α_{lg} .

Solution

Using Eq. 4.81, we obtain

$$r_{i+1} - r_i = \alpha_{lg}r_i - r_i = (\alpha_{lg} - 1)r_i \tag{4.100}$$

$$r_{i+1}/r_i = \alpha_{lg} \tag{4.101}$$

Substitution of Eqs. 4.100 and 4.101 into Eq. 4.82a yields

$$r_{i+1/2}^{L} = \frac{r_{i+1} - r_i}{\log_e(r_{i+1}/r_i)} = \frac{(\alpha_{lg} - 1)r_i}{\log_e(\alpha_{lg})}_i = \{(\alpha_{lg} - 1)/\log_e(\alpha_{lg})\}r_i \tag{4.102}$$

Eq. 4.102 can be rearranged to give

$$r_{i+1/2}^{L}/r_i = (\alpha_{lg} - 1)/\log_e(\alpha_{lg}) \tag{4.103}$$

from which

$$\log_e(r_{i+1/2}^{L}/r_i) = \log_e[(\alpha_{lg} - 1)/\log_e(\alpha_{lg})]\ ) \tag{4.104}$$

Eqs. 4.101 and 4.102 can be combined by eliminating r_i, yielding

$$r_{i+1/2}^{L} = \frac{1}{\log_e(\alpha_{lg})}(\alpha_{lg} - 1)(r_{i+1}/\alpha_{lg}) = \{(\alpha_{lg} - 1)/[\alpha_{lg}\log_e(\alpha_{lg})]\}r_{i+1} \tag{4.105}$$

Eq. 4.105 can be rearranged to give

$$r_{i+1}/r_{i+1/2}^{L} = [\alpha_{lg}\log_e(\alpha_{lg})]/(\alpha_{lg} - 1) \tag{4.106}$$

from which

$$\log_e(r_{i+1}/r_{i+1/2}^{L}) = \log_e\{[\alpha_{lg}\log_e(\alpha_{lg})]/(\alpha_{lg} - 1)\} \tag{4.107}$$

Using Eq. 4.81 and replacing subscript i with $i-1$ yields

$$r_i = \alpha_{lg} r_{i-1} \tag{4.108}$$

$$r_i/r_{i-1} = \alpha_{lg} \tag{4.109}$$

Substitution of Eqs. 4.108 and 4.109 into Eq. 4.83a yields

$$r_{i-1/2}^{L} = \frac{r_i - r_{i-1}}{\log_e(r_i/r_{i-1})} = \frac{r_i - r_i/\alpha_{lg}}{\log_e(\alpha_{lg})} = \{(\alpha_{lg} - 1)/[\alpha_{lg}\log_e(\alpha_{lg})]\}r_i \tag{4.110}$$

Eq. 4.110 can be rearranged to give

$$r_i/r_{i-1/2}^{L} = [\alpha_{lg}\log_e(\alpha_{lg})]/(\alpha_{lg} - 1) \tag{4.111}$$

from which

$$\log_e(r_i/r_{i-1/2}^{L}) = \log_e\{[\alpha_{lg}\log_e(\alpha_{lg})]/(\alpha_{lg} - 1)\} \tag{4.112}$$

Eqs. 4.108 and 4.110 can be combined by eliminating r_i, yielding

$$r_{i-1/2}^{L} = \frac{1}{\log_e(\alpha_{lg})}[(\alpha_{lg} - 1)/\alpha_{lg}](\alpha_{lg} r_{i-1}) = [(\alpha_{lg} - 1)/\log_e(\alpha_{lg})]r_{i-1} \tag{4.113}$$

Eq. 4.113 can be rearranged to give

$$r_{i-1/2}^L / r_{i-1} = (\alpha_{lg} - 1) / \log_e(\alpha_{lg}) \tag{4.114}$$

from which

$$\log_e(r_{i-1/2}^L / r_{i-1}) = \log_e[(\alpha_{lg} - 1) / \log_e(\alpha_{lg})] \tag{4.115}$$

Eqs. 4.102 and 4.110 are combined to get

$$r_{i+1/2}^L / r_{i-1/2}^L = \frac{\{(\alpha_{lg} - 1) / \log_e(\alpha_{lg})\} r_i}{\{(\alpha_{lg} - 1) / [\alpha_{lg} \log_e(\alpha_{lg})]\} r_i} = \alpha_{lg} \tag{4.116}$$

from which

$$\log_e(r_{i+1/2}^L / r_{i-1/2}^L) = \log_e(\alpha_{lg}) \tag{4.117}$$

Substitution of Eqs. 4.81 and 4.101 into Eq. 4.84a yields

$$r_{i+1/2}^2 = \frac{r_{i+1}^2 - r_i^2}{\log_e(r_{i+1}^2 / r_i^2)} = \frac{(\alpha_{lg}^2 - 1) r_i^2}{\log_e(\alpha_{lg}^2)} = [(\alpha_{lg}^2 - 1) / \log_e(\alpha_{lg}^2)] r_i^2 \tag{4.118}$$

Substitution of Eqs. 4.108 and 4.109 into Eq. 4.85a yields

$$r_{i-1/2}^2 = \frac{r_i^2 - r_{i-1}^2}{\log_e(r_i^2 / r_{i-1}^2)} = \frac{(1 - 1/\alpha_{lg}^2) r_i^2}{\log_e(\alpha_{lg}^2)} = \{(\alpha_{lg}^2 - 1) / [\alpha_{lg}^2 \log_e(\alpha_{lg}^2)]\} r_i^2 \tag{4.119}$$

Subtraction of Eq. 4.119 from Eq. 4.118 yields

$$\begin{aligned} r_{i+1/2}^2 - r_{i-1/2}^2 &= \frac{(\alpha_{lg}^2 - 1)}{\log_e(\alpha_{lg}^2)} r_i^2 - \frac{[(\alpha_{lg}^2 - 1) / \alpha_{lg}^2]}{\log_e(\alpha_{lg}^2)} r_i^2 \\ &= \frac{(\alpha_{lg}^2 - 1)(1 - 1/\alpha_{lg}^2)}{\log_e(\alpha_{lg}^2)} r_i^2 = \{(\alpha_{lg}^2 - 1)^2 / [\alpha_{lg}^2 \log_e(\alpha_{lg}^2)]\} r_i^2. \end{aligned} \tag{4.120}$$

Combining Eqs. 4.88a and 4.120 yields

$$V_{b_{i,j,k}} = \{(\alpha_{lg}^2 - 1)^2 / [\alpha_{lg}^2 \log_e(\alpha_{lg}^2)]\} r_i^2 (\tfrac{1}{2} \Delta\theta_j) \Delta z_{i,j,k} \tag{4.121}$$

Eq. 4.121 can be used to calculate bulk volumes of gridblocks other than those that fall on the reservoir external boundary in the *r* direction. For blocks with $i = n_r$, Eq. 4.88d is used and the proof is left as an exercise (Exercise 4-13).

Example 4.10 demonstrates that quotients $r_i / r_{i-1/2}^L$, $r_{i-1/2}^L / r_{i-1}$, $r_{i+1/2}^L / r_i$, $r_{i+1} / r_{i+1/2}^L$, and $r_{i+1/2}^L / r_{i-1/2}^L$ are functions of the logarithmic spacing constant α_{lg} only as expressed in the following equations:

$$r_i / r_{i-1/2}^L = [\alpha_{lg} \log_e(\alpha_{lg})] / (\alpha_{lg} - 1) \tag{4.111}$$

$$r^L_{i-1/2} / r_{i-1} = (\alpha_{lg} - 1) / \log_e(\alpha_{lg}) \quad \textbf{(4.114)}$$

$$r^L_{i+1/2} / r_i = (\alpha_{lg} - 1) / \log_e(\alpha_{lg}) \quad \textbf{(4.103)}$$

$$r_{i+1} / r^L_{i+1/2} = [\alpha_{lg} \log_e(\alpha_{lg})] / (\alpha_{lg} - 1) \quad \textbf{(4.106)}$$

$$r^L_{i+1/2} / r^L_{i-1/2} = \alpha_{lg} \quad \textbf{(4.116)}$$

By substituting the above five equations into the equations in Table 4–2 and observing that $(\frac{1}{2}\Delta\theta_j)(r^2_{i+1/2} - r^2_{i-1/2}) = V_{b_{i,j,k}} / \Delta z_{i,j,k}$ using Eq. 4.88a, Table 4–3 is obtained.

Now, the calculation of geometric factors and pore volumes can be simplified using the following algorithm.

(1) Define $\alpha_{lg} = \left(\dfrac{r_e}{r_w}\right)^{1/n_r}$ **(4.86)**

(2) Let $r_1 = [\alpha_{lg} \log_e(\alpha_{lg}) / (\alpha_{lg} - 1)] r_w$ **(4.87)**

(3) Set $r_i = \alpha_{lg}^{i-1} r_1$ **(4.122)**

where $i = 1,2,3,...n_r$.

(4) For $j = 1,2,3,...n_\theta$ and $k = 1,2,3,...n_z$ set

$$V_{b_{i,j,k}} = \{(\alpha^2_{lg} - 1)^2 / [\alpha^2_{lg} \log_e(\alpha^2_{lg})]\} r_i^2 (\tfrac{1}{2}\Delta\theta_j) \Delta z_{i,j,k} \quad \textbf{(4.88b)}$$

for $i = 1,2,3,...n_r - 1$; and

$$V_{b_{n_r,j,k}} = \{1 - [\log_e(\alpha_{lg}) / (\alpha_{lg} - 1)]^2 (\alpha^2_{lg} - 1) / [\alpha^2_{lg} \log_e(\alpha^2_{lg})]\} r_e^2 (\tfrac{1}{2}\Delta\theta_j) \Delta z_{n_r,j,k} \quad \textbf{(4.88d)}$$

for $i = n_r$.

(5) Estimate the geometric factors using the equations in Table 4–3. Note that in the calculation of $G_{z_{1/2,j,k}}$, $G_{z_{n_r+1/2,j,k}}$, $G_{z_{i,j,1/2}}$, or $G_{z_{i,j,n_z+1/2}}$, terms that describe properties of blocks that fall outside the reservoir ($i = 0$, $i = n_r + 1$, $k = 0$, and $k = n_z + 1$) are discarded.

Examples 4.11 and 4.12 show that reservoir discretization in the radial direction can be accomplished using either the traditional equations reported in the previous literature (Eqs. 4.81, 4.82a, 4.83a, 4.84a, 4.85a, 4.86, 4.87, 4.88a, and 4.88c) or those reported in this book (Eqs. 4.81, 4.82b, 4.83b, 4.84b, 4.85b, 4.86, 4.87, 4.88b, and 4.88d) that led to Table 4–3. The equations reported in this book, however, are easier and less confusing because they only use r_i and α_{lg} . In Example 4.13, we demonstrate how to use Eq. 4.2a

Table 4–3 Geometric Factors in Cylindrical Grids

Direction	Geometric Factor
r	$G_{r_{i-1/2,j,k}} = \dfrac{\beta_c \Delta\theta_j}{\{\log_e[\alpha_{lg}\log_e(\alpha_{lg})/(\alpha_{lg}-1)]/(\Delta z_{i,j,k} k_{r_{i,j,k}}) + \log_e[(\alpha_{lg}-1)/\log_e(\alpha_{lg})]/(\Delta z_{i-1,j,k} k_{r_{i-1,j,k}})\}}$
	$G_{r_{i+1/2,j,k}} = \dfrac{\beta_c \Delta\theta_j}{\{\log_e[(\alpha_{lg}-1)/\log_e(\alpha_{lg})]/(\Delta z_{i,j,k} k_{r_{i,j,k}}) + \log_e[\alpha_{lg}\log_e(\alpha_{lg})/(\alpha_{lg}-1)]/(\Delta z_{i+1,j,k} k_{r_{i+1,j,k}})\}}$
θ	$G_{\theta_{i,j\mp1/2,k}} = \dfrac{2\beta_c \log_e(\alpha_{lg})}{\Delta\theta_j/(\Delta z_{i,j,k} k_{\theta_{i,j,k}}) + \Delta\theta_{j\mp1}/(\Delta z_{i,j\mp1,k} k_{\theta_{i,j\mp1,k}})}$
z	$G_{z_{i,j,k\mp1/2}} = \dfrac{2\beta_c (V_{b_{i,j,k}}/\Delta z_{i,j,k})}{\Delta z_{i,j,k}/k_{z_{i,j,k}} + \Delta z_{i,j,k\mp1}/k_{z_{i,j,k\mp1}}}$

and the appropriate expressions for $q^m_{sc_{b,bB}}$, along with Table 4–3, to write the flow equations for boundary and interior gridblocks in a 2D single-well simulation problem.

Example 4.11 Consider the simulation of a single-well in 40-acre spacing. Wellbore diameter is 0.5 ft. The reservoir thickness is 100 ft. The reservoir can be simulated using a single layer discretized into five gridblocks in the radial direction.

(a) Find the gridblock spacing in the r direction.

(b) Find the gridblock boundaries in the r direction for transmissibility calculations.

(c) Calculate the arguments of the $\log_e$ terms in Table 4–2.

(d) Find the gridblock boundaries in the r direction for bulk volume calculations and calculate bulk volumes.

Solution

(a) The reservoir external radius can be estimated from well spacing,

$r_e = \sqrt{43{,}560 \times 40/\pi} = 744.73$ ft, and well radius is given as $r_w = 0.25$ ft.

First, estimate α_{lg} using Eq. 4.86,

$$\alpha_{lg} = \left(\frac{r_e}{r_w}\right)^{1/n_r} = \left(\frac{744.73}{0.25}\right)^{1/5} = 4.9524 \cdot$$

Second, let $r_1 = [(4.9524)\log_e(4.9524)/(4.9524-1)](0.25) = 0.5012$ ft according to Eq. 4.87. Third, calculate the location of the gridblocks in the *r* direction using Eq. 4.122, $r_i = \alpha_{lg}^{i-1} r_1$. As for example, for $i = 2$,

$r_2 = (4.9524)^{2-1} \times 0.5012 = 2.4819$ ft. Table 4–4 shows the location of the other gridblocks along the *r*-direction.

(b) Block boundaries for transmissibility calculations ($r^L_{i-1/2}, r^L_{i+1/2}$) are estimated using Eqs. 4.82a and 4.83a.

For $i = 2$,

$$r^L_{2+1/2} = \frac{r_3 - r_2}{\log_e(r_3 / r_2)} = \frac{12.2914 - 2.4819}{\log_e(12.2914/2.4819)} = 6.1315 \text{ ft.} \tag{4.123}$$

$$r^L_{2-1/2} = \frac{r_2 - r_1}{\log_e(r_2 / r_1)} = \frac{2.4819 - 0.5012}{\log_e(2.4819/0.5012)} = 1.2381 \text{ ft.} \tag{4.124}$$

Table 4–4 shows the boundaries for transmissibility calculations for other gridblocks.

(c) Table 4–4 shows the calculated values for $r_i / r^L_{i-1/2}$, $r_{i+1} / r^L_{i+1/2}$, $r^L_{i-1/2} / r_{i-1}$, $r^L_{i+1/2} / r_i$, and $r^L_{i+1/2} / r^L_{i-1/2}$, which appear in the argument of $\log_e$ terms in Table 4–2.

(d) The block boundaries for bulk volume calculations ($r_{i-1/2}, r_{i+1/2}$) are estimated using Eqs. 4.84a and 4.85a.

Table 4–4 r_i , $r^L_{i\mp1/2}$ **and** $\log_e$ **Arguments in Table 4–2 for Example 4.11**

i	r_i	$r^L_{i-1/2}$	$r^L_{i+1/2}$	$r_i / r^L_{i-1/2}$	$r_{i+1} / r^L_{i+1/2}$	$r^L_{i-1/2} / r_{i-1}$	$r^L_{i+1/2} / r_i$	$r^L_{i+1/2} / r^L_{i-1/2}$
1	0.5012	0.25[a]	1.2381	2.005	2.005	2.47	2.47	4.9528
2	2.4819	1.2381	6.1315	2.005	2.005	2.47	2.47	4.9524
3	12.2914	6.1315	30.3651	2.005	2.005	2.47	2.47	4.9524
4	60.8715	30.3651	150.379	2.005	2.005	2.47	2.47	4.9524
5	301.457	150.379	744.73[b]	2.005	2.005	2.47	2.47	—

[a] $r^L_{1-1/2} = r_w = 0.25$

[b] $r^L_{5+1/2} = r_e = 744.73$

Table 4–5 Gridblock Boundaries and Bulk Volumes for Gridblocks in Example 4.11

i	r_i	$r_{i-1/2}$	$r_{i+1/2}$	V_{b_i}
1	0.5012	0.2744	1.3589	556.4939
2	2.4819	1.3589	6.7299	13648.47
3	12.2914	6.7299	33.3287	334739.9
4	60.8715	33.3287	165.056	8209770
5	301.4573	165.056	744.73[a]	165681140

[a] $r_{5+1/2} = r_e = 744.73$

For $i = 2$,

$$r^2_{2+1/2} = \frac{r_3^2 - r_2^2}{\log_e(r_3^2 / r_2^2)} = \frac{(12.2914)^2 - (2.4819)^2}{\log_e[(12.2914)^2 / (2.4819)^2]} = 45.2906 \text{ ft}^2 \tag{4.125}$$

$$r^2_{2-1/2} = \frac{r_2^2 - r_1^2}{\log_e(r_2^2 / r_1^2)} = \frac{(2.4819)^2 - (0.5012)^2}{\log_e[(2.4819)^2 / (0.5012)^2]} = 1.8467 \text{ ft}^2. \tag{4.126}$$

Therefore, the gridblock boundaries for bulk volume calculations are

$$r_{2+1/2} = \sqrt{45.2906} = 6.7298 \text{ ft}$$

$$r_{2-1/2} = \sqrt{1.8467} = 1.3589 \text{ ft.}$$

The bulk volume for the gridblocks can be calculated using Eqs. 4.88a and 4.88c.

For $i = 2$,

$$V_{b_2} = (r^2_{2+1/2} - r^2_{2-1/2})(\tfrac{1}{2}\Delta\theta)\Delta z_2 \tag{4.127}$$

$$= [(6.7299)^2 - (1.3589)^2](\tfrac{1}{2} \times 2\pi) \times 100 = 13648.47 \text{ ft}^3.$$

For $i = 5$,

$$V_{b_5} = (r_e^2 - r^2_{5-1/2})(\tfrac{1}{2}\Delta\theta)\Delta z_5 \tag{4.128}$$

$$= [(744.73)^2 - (165.056)^2](\tfrac{1}{2} \times 2\pi) \times 100 = 165.68114 \times 10^6 \text{ ft}^3.$$

Table 4–5 shows the gridblock boundaries and the bulk volumes for other gridblocks.

Example 4.12 Solve Example 4.11 again, this time using Eqs. 4.82b, 4.83b, 4.84b, 4.85b, 4.88d, which make use of r_i and α_{lg}, and Eq. 4.88d.

Solution

(a) From Example 4.11, $r_e = 744.73$ ft, $r_w = 0.25$ ft, $r_1 = 0.5012$ ft, and $\alpha_{lg} = 4.9524$. In addition, Table 4–4 reports radii of points representing gridblocks (r_i) calculated using Eq. 4.122.

(b) Block boundaries for transmissibility calculations ($r^L_{i-1/2}, r^L_{i+1/2}$) are estimated using Eqs. 4.82b and 4.83b, yielding

$$r^L_{i+1/2} = \{(\alpha_{lg} - 1)/[\log_e(\alpha_{lg})]\}r_i = \{(4.9524 - 1)/[\log_e(4.9524)]\}r_i \\ = 2.47045 r_i \tag{4.129}$$

$$r^L_{i-1/2} = \{(\alpha_{lg} - 1)/[\alpha_{lg} \log_e(\alpha_{lg})]\}r_i = \{(4.9524 - 1)/[4.9524 \log_e(4.9524)]\}r_i \\ = 0.49884 r_i. \tag{4.130}$$

Substitution of the values of r_i into Eqs. 4.129 and 4.130 produces the results reported in Table 4–4.

(c) The ratios $r_i / r^L_{i-1/2}$, $r_{i+1} / r^L_{i+1/2}$, $r^L_{i-1/2} / r_{i-1}$, $r^L_{i+1/2} / r_i$, and $r^L_{i+1/2} / r^L_{i-1/2}$ as functions of α_{lg} were derived in Example 4.10 as Eqs. 4.111, 4.106, 4.114, 4.103, and 4.116, respectively. Substitution of $\alpha_{lg} = 4.9524$ in these equations, we obtain:

$$r_i / r^L_{i-1/2} = [\alpha_{lg} \log_e(\alpha_{lg})]/(\alpha_{lg} - 1) = [4.9524 \log_e(4.9524)]/(4.9524 - 1) = 2.005, \tag{4.131}$$

$$r_{i+1} / r^L_{i+1/2} = [\alpha_{lg} \log_e(\alpha_{lg})]/(\alpha_{lg} - 1) = 2.005 \tag{4.132}$$

$$r^L_{i-1/2} / r_{i-1} = (\alpha_{lg} - 1)/\log_e(\alpha_{lg}) = (4.9524 - 1)/\log_e(4.9524) = 2.470 \tag{4.133}$$

$$r^L_{i+1/2} / r_i = (\alpha_{lg} - 1)/\log_e(\alpha_{lg}) = 2.470 \tag{4.134}$$

$$r^L_{i+1/2} / r^L_{i-1/2} = \alpha_{lg} = 4.9524 \tag{4.135}$$

Note that the values of the above ratios are the same as those reported in Table 4–4.

(d) Block boundaries for bulk volume calculations ($r_{i-1/2}, r_{i+1/2}$) are estimated using Eqs. 4.84b and 4.85b.

$$r_{i+1/2}^2 = \{(\alpha_{lg}^2 - 1)/[\log_e(\alpha_{lg}^2)]\}r_i^2 = \{\left((4.9524)^2 - 1\right)/[\log_e\left((4.9524)^2\right)]\}r_i^2 = (7.3525)r_i^2 \quad (4.136)$$

$$r_{i-1/2}^2 = \{(\alpha_{lg}^2 - 1)/[\alpha_{lg}^2 \log_e(\alpha_{lg}^2)]\}r_i^2 = \{7.3525/(4.9524)^2\}r_i^2 = (0.29978)r_i^2 \quad (4.137)$$

Therefore,

$$r_{i+1/2} = \sqrt{(7.3525)r_i^2} = (2.7116)r_i \quad (4.138)$$

$$r_{i-1/2} = \sqrt{(0.29978)r_i^2} = (0.54752)r_i \quad (4.139)$$

The bulk volume associated with each gridblock can be calculated using Eqs. 4.88b and 4.88d.

For $i = 1,2,3,4$;

$$V_{b_i} = \{(\alpha_{lg}^2 - 1)^2 /[\alpha_{lg}^2 \log_e(\alpha_{lg}^2)]\}r_i^2[\tfrac{1}{2}(2\pi)]\Delta z$$
$$= \{[(4.9524)^2 - 1]^2 /[(4.9524)^2 \log_e(4.9524)^2]\}r_i^2[\tfrac{1}{2}(2\pi)]\times 100 = 2215.7r_i^2. \quad (4.140)$$

For $i = 5$;

$$V_{b_5} = \{1 - [\log_e(4.9524)/(4.9524 - 1)]^2 \times [(4.9524)^2 - 1]$$
$$/[(4.9524)^2 \times \log_e\left((4.9524)^2\right)]\} \times (744.73)^2(\tfrac{1}{2}\times 2\pi)\times 100 = 165.681284\times 10^6. \quad (4.141)$$

Note that the values of estimated bulk volumes slightly differ from those reported in Table 4–5 due to round-off errors resulting from approximations in the various stages of calculations.

Example 4.13 A 0.5-ft diameter water well is located in 20-acre spacing. The reservoir thickness, horizontal permeability, and porosity are 30 ft, 150 md, and 0.23, respectively. The (k_V / k_H) for this reservoir is estimated from core data as 0.30. The flowing fluid has a density, FVF, and viscosity of 62.4 lbm/ft^3, 1 RB/B, and 0.5 cp, respectively. The reservoir external boundary in the radial direction is a no-flow boundary, and the well is completed in the top 20 ft only and produces at a rate of 2000 B/D. The reservoir bottom boundary is subject to influx such that the boundary is kept at 4000 psia. The reservoir top boundary is sealed to flow. Assuming the reservoir can be simulated using three equal gridblocks in the vertical direction and four gridblocks in the radial direction, as shown in Figure 4–14, write the flow equations for gridblocks 1, 3, 5, 7, and 11.

Solution

In order to write the flow equations, the gridblocks are first ordered using natural ordering ($n = 1,2,3,...10,11,12$) as shown in Figure 4–14, in addition to being identified using the engineering notation along the radial direction ($i = 1,2,3,4$) and the vertical direction ($k = 1,2,3$). This is followed by the estimation of reservoir rock and fluid property data, the determination of the location of points representing gridblocks in the radial direction, and the calculation of gridblock sizes and elevation in the vertical direction. Next, bulk volumes and transmissibilities in the *r* and *z* directions are calculated. We demonstrate in this example that block boundaries for transmissibility calculations and for bulk volume calculations are not needed if we use Eqs. 4.88b and 4.88d for bulk volume calculations and Table 4–3 for transmissibility calculations. Making use of the above information, we estimate the contributions of the gridblocks to well rates and fictitious well rates resulting from reservoir boundary conditions.

Reservoir rock and fluid data are restated as follows: $h = 30$ ft, $\phi = 0.23$,

$k_r = k_H = 150$ md, $k_z = k_H(k_V / k_H) = 150 \times 0.30 = 45$ md, $B = 1$ RB/B,

$\mu = 0.5$ cp, $\gamma = \gamma_c \rho g = 0.21584 \times 10^{-3}(62.4)(32.174) = 0.4333$ psi/ft,

$r_w = 0.25$ ft, and the reservoir external radius is estimated from well spacing as

$r_e = (20 \times 43560 / \pi)^{1/2} = 526.60$ ft. The reservoir east (external) and upper (top) boundaries are no-flow boundaries, the reservoir lower (bottom) boundary has $p_{b_L} = 4000$ psia, and the reservoir west (internal) boundary has $q_{spsc} = -2000$ B/D to reflect the effect of the production well (i.e., the well is treated as a boundary condition).

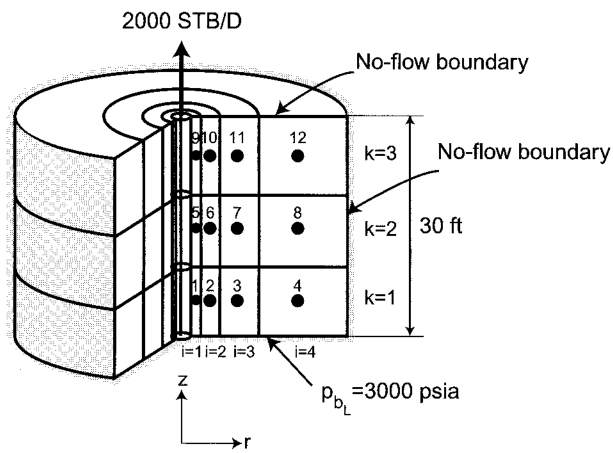


Figure 4–14 Discretized 2D radial-cylindrical reservoir in Example 4.13.

For the block-centered grid shown in Figure 4–14, $n_r = 4$, $n_z = 3$, $\Delta z_k = h / n_z = 30/3 = 10$ ft for $k = 1,2,3$; hence, $\Delta z_n = 10$ ft for $n = 1,2,3,4,5,6,7,8,9,10,11,12$ and $\Delta z_{k+1/2} = 10$ ft for $k = 1,2$. Assuming the top of the reservoir as the reference level for elevation, $Z_n = 5$ ft for $n = 9,10,11,12$; $Z_n = 15$ ft for $n = 5,6,7,8$; $Z_n = 25$ ft for $n = 1,2,3,4$; and $Z_{b_L} = 30$ ft.

The locations of gridblocks in the radial direction are calculated using Eqs. 4.86, 4.87, and 4.122; i.e.,

$$\alpha_{lg} = (526.60 / 0.25)^{1/4} = 6.7746 ,$$

$$r_1 = [(6.7746)\log_e(6.7746)/(6.7746-1)]\times 0.25 = 0.56112 \text{ ft},$$

$$r_i = (6.7746)^{(i-1)}(0.56112)$$

for $i = 2,3,4$ or $r_2 = 3.8014$ ft, $r_3 = 25.753$ ft, and $r_4 = 174.46$ ft.

Eq. 4.88b is used to calculate bulk volume for gridblocks that have $i = 1,2,3$,

$$\begin{aligned} V_{b_{i,k}} &= \{(\alpha_{lg}^2 - 1)^2 / [\alpha_{lg}^2 \log_e(\alpha_{lg}^2)]\} r_i^2 (\tfrac{1}{2}\Delta\theta)\Delta z_{i,k} \\ &= \{[(6.7746)^2 - 1]^2 / [(6.7746)^2 \log_e\big((6.7746)^2\big)]\} r_i^2 (\tfrac{1}{2}\times 2\pi)\Delta z_k \\ &= (36.0576) r_i^2 \Delta z_k \end{aligned}$$

and Eq. 4.88d for gridblocks that have $i = n_r = 4$,

$$\begin{aligned} V_{b_{n_r,k}} &= \{1 - [\log_e(\alpha_{lg})/(\alpha_{lg} - 1)]^2 (\alpha_{lg}^2 - 1)/[\alpha_{lg}^2 \log_e(\alpha_{lg}^2)]\} r_e^2 (\tfrac{1}{2}\Delta\theta_j)\Delta z_{n_r,k} \\ &= \{1 - [\log_e(6.7746)/(6.7746 - 1)]^2 [(6.7746)^2 - 1]/[(6.7746)^2 \log_e\big((6.7746)^2\big)]\} \\ &\times (526.60)^2 (\tfrac{1}{2}\times 2\pi)\Delta z_k \\ &= (0.846740\times 10^6)\Delta z_k . \end{aligned}$$

Eq. 4.79c defines the transmissibility in the vertical direction, resulting in

$$T_{z_{i,k\mp1/2}} = G_{z_{i,k\mp1/2}} \left(\frac{1}{\mu B}\right) = G_{z_{i,k\mp1/2}} \left(\frac{1}{0.5\times 1}\right) = (2) G_{z_{i,k\mp1/2}} \qquad \textbf{(4.142)}$$

where $G_{z_{i,k\mp1/2}}$ is defined in Table 4–3 as

$$G_{z_{i,k\mp1/2}} = \frac{2\beta_c (V_{b_{i,k}} / \Delta z_k)}{\Delta z_k / k_{z_{i,k}} + \Delta z_{k\mp1} / k_{z_{i,k\mp1}}} \qquad \textbf{(4.143)}$$

For this problem, gridblock spacing, thickness, and permeability in the vertical direction are constants. Therefore, Eq. 4.143 reduces to

$$G_{z_{i,k\mp1/2}} = \frac{\beta_c k_z (V_{b_{i,k}} / \Delta z_k)}{\Delta z_k}$$

or after substitution for values it becomes

$$G_{z_{i,k\mp1/2}} = \frac{(1.127\times10^{-3})(45)(36.0576\times r_i^2)}{10} = (0.182866)r_i^2 \tag{4.144a}$$

for $i = 1,2,3$ and $k = 1,2,3$ and

$$G_{z_{i,k\mp1/2}} = \frac{(1.127\times10^{-3})(45)(0.846740\times10^6)}{10} = 4294.242 \tag{4.144b}$$

for $i = 4$ and $k = 1,2,3$.

Substituting Eq. 4.144 into Eq. 4.142 results in

$$T_{z_{i,k\mp1/2}} = 2(0.182866)r_i^2 = (0.365732)r_i^2 \tag{4.145a}$$

for $i = 1,2,3$ and $k = 1,2,3$ and

$$T_{z_{i,k\mp1/2}} = 2(4294.242) = 8588.484 \tag{4.145b}$$

for $i = 4$ and $k = 1,2,3$.

Eq. 4.79a defines the transmissibility in the r direction, yielding

$$T_{r_{i\mp1/2,k}} = G_{r_{i\mp1/2,k}}\left(\frac{1}{\mu B}\right) = G_{r_{i\mp1/2,k}}\left(\frac{1}{0.5\times1}\right) = (2)G_{r_{i\mp1/2,k}} \tag{4.146}$$

where $G_{r_{i\mp1/2,k}}$ is defined in Table 4–3. With $\Delta\theta = 2\pi$ and constant radial permeability, the equation for the geometric factor reduces to

$$\begin{aligned} G_{r_{i\mp1/2,k}} &= \frac{2\pi\beta_c k_r \Delta z_k}{\log_e\{[\alpha_{lg}\log_e(\alpha_{lg})/(\alpha_{lg}-1)]\times[(\alpha_{lg}-1)/\log_e(\alpha_{lg})]\}} \\ &= \frac{2\pi\beta_c k_r \Delta z_k}{\log_e(\alpha_{lg})} = \frac{2\pi(0.001127)(150)\Delta z_k}{\log_e(6.7746)} = (0.5551868)\Delta z_k. \end{aligned} \tag{4.147}$$

Therefore, transmissibility in the radial direction can be estimated by substituting Eq. 4.147 into Eq. 4.146,

$$T_{r_{i\mp1/2,k}} = (2)G_{r_{i\mp1/2,k}} = (2)(0.5551868)\Delta z_k = (1.1103736)\Delta z_k \tag{4.148}$$

Table 4–6 lists the estimated transmissibilities in the radial and vertical directions and bulk volumes. Before writing the flow equation, the well production rate (the specified rate for the reservoir west boundary) must be prorated between gridblocks 5 and 9 using Eq. 4.28,

Table 4–6 Gridblock Location, Bulk Volume, and Radial and Vertical Transmissibilities for Example 4.13

n	i	k	r_i (ft)	Δz_n (ft)	Z_n (ft)	V_{b_n} (ft³)	$T_{r_{i\pm1/2,k}}$ (B/D-psi)	$T_{z_{i,k\pm1/2}}$ (B/D-psi)
1	1	1	0.56112	10	25	113.5318	11.10374	0.115155
2	2	1	3.8014	10	25	5210.583	11.10374	5.285098
3	3	1	25.753	10	25	239123.0	11.10374	242.5426
4	4	1	174.46	10	25	8467440.	11.10374	8588.532
5	1	2	0.56112	10	15	113.5318	11.10374	0.115155
6	2	2	3.8014	10	15	5210.583	11.10374	5.285098
7	3	2	25.753	10	15	239123.0	11.10374	242.5426
8	4	2	174.46	10	15	8467440.	11.10374	8588.532
9	1	3	0.56112	10	5	113.5318	11.10374	0.115155
10	2	3	3.8014	10	5	5210.583	11.10374	5.285098
11	3	3	25.753	10	5	239123.0	11.10374	242.5426
12	4	3	174.46	10	5	8467440.	11.10374	8588.532

$$q^m_{sc_{b,bB}} = \frac{T^m_{b,bB}}{\sum\limits_{l\in\psi_b} T^m_{b,l}} q_{spsc} \tag{4.28}$$

where $T^m_{b,bB}$ = transmissibility in the radial direction between reservoir boundary b and gridblock bB with the well being the reservoir internal boundary and $\psi_b = \psi_w = \{5,9\}$. Note that gridblock 1 has a no-flow boundary because it is not penetrated by the well; i.e., $q^m_{sc_{b_W,1}} = 0$.

From Table 4–6,

$$T^m_{b_W,5} = T^m_{r_{5,6}} = 11.10374 \text{ B/D-psi}$$

$$T^m_{b_W,9} = T^m_{r_{9,10}} = 11.10374 \text{ B/D-psi.}$$

The application of Eq. 4.28 results in

$$q^m_{sc_{bW},9} = \frac{11.10374}{11.10374 + 11.10374} \times (-2000) = -1000 \text{ B/D}$$

$$q^m_{sc_{bW},5} = \frac{11.10374}{11.10374 + 11.10374} \times (-2000) = -1000 \text{ B/D.}$$

With this treatment of the production well, $q^m_{sc_n} = 0$ for each gridblock (including gridblocks 1, 5, and 9).

For the reservoir lower boundary, $p_{b_L} = 4000$ psia. The flow rates of the fictitious wells in gridblocks 1, 2, 3, and 4 are estimated using Eq. 4.37c, yielding

$$q^m_{sc_{b_L},n} = T^m_{b_L,n}[(4000 - p_n) - (0.4333)(30 - 25)] \text{ B/D} \tag{4.149}$$

where $T^n_{b_L,n}$ is estimated using Eq. 4.29 and $A_{z_n} = V_{b_n} / \Delta z_n$,

$$T^m_{b_L,n} = \beta_c \frac{k_{z_n} A_{z_n}}{\mu B(\Delta z_n / 2)} = 0.001127 \times \frac{45 \times (V_{b_n} / \Delta z_n)}{0.5 \times 1 \times (10/2)} \tag{4.150}$$
$$= (0.0020286) V_{b_n}.$$

For the reservoir east and upper (no-flow) boundaries, $q^m_{sc_{b_E},n} = 0$ for $n = 4,8,12$

and $q^m_{sc_{b_U},n} = 0$ for $n = 9,10,11,12$. Table 4–7 summarizes the contributions of gridblocks to well rates and fictitious well rates.

The general form of the flow equation for gridblock n is obtained from Eq. 4.2a,

$$\sum_{l \in \psi_n} T^m_{l,n}[(p^m_l - p^m_n) - \gamma^m_{l,n}(Z^m_l - Z^m_n)] + \sum_{l \in \xi_n} q^m_{sc_{l,n}} + q^m_{sc_n} = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_n - (\frac{\phi}{B})^n_n] \tag{4.2a}$$

For gridblock 1, $n = 1$, $i = 1$, $k = 1$, $\psi_1 = \{1,5\}$, $\xi_1 = \{b_L, b_W\}$, and

$\sum_{l \in \xi_1} q^m_{sc_{l,1}} = q^m_{sc_{b_L},1} + q^m_{sc_{bW},1}$, where from Table 4–7,

$q^m_{sc_{b_L},1} = (0.23031)[(4000 - p^m_1) - (0.4333)(30 - 25)]$ B/D and $q^m_{sc_{bW},1} = 0$, and

$q^m_{sc_1} = 0$. Therefore, substitution into Eq. 4.2a yields

$$(11.10374)[(p^m_2 - p^m_1) - (0.4333)(25 - 25)]$$
$$+(0.115155)[(p^m_5 - p^m_1) - (0.4333)(15 - 25)]$$
$$+(0.23031)[(4000 - p^m_1) - (0.4333)(30 - 25)] + 0 + 0 = \frac{113.5318}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_1 - (\frac{\phi}{B})^n_1].$$

(4.151)

Table 4–7 Contribution of Gridblocks to Well Rates and Fictitious Well Rates

n	i	k	$q^m_{sc_n}$ (B/D)	$q^m_{sc_{b_L,n}}$ (B/D)	$q^m_{sc_{b_W,n}}$ (B/D)	$q^m_{sc_{b_E,n}}$ (B/D)	$q^m_{sc_{b_U,n}}$ (B/D)
1	1	1	0	$(0.23031)[(4000-p^m_1)-(0.4333)(30-25)]$	0		
2	2	1	0	$(10.5702)[(4000-p^m_2)-(0.4333)(30-25)]$			
3	3	1	0	$(485.085)[(4000-p^m_3)-(0.4333)(30-25)]$			
4	4	1	0	$(17177.1)[(4000-p^m_4)-(0.4333)(30-25)]$		0	
5	1	2	0		−1000		
6	2	2	0				
7	3	2	0				
8	4	2	0			0	
9	1	3	0		−1000		0
10	2	3	0				0
11	3	3	0				0
12	4	3	0			0	0

For gridblock 3, $n=3$, $i=3$, $k=1$, $\psi_3=\{2,4,7\}$, $\xi_3=\{b_L\}$, and

$\sum_{l\in\xi_3} q^m_{sc_{l,3}} = q^m_{sc_{b_L,3}}$, where from Table 4–7,

$q^m_{sc_{b_L,3}} = (485.085)[(4000-p^m_3)-(0.4333)(30-25)]$ B/D, and $q^m_{sc_3}=0$ (no wells). Therefore, substitution into Eq. 4.2a yields

$$\begin{aligned}&(11.10374)[(p^m_2-p^m_3)-(0.4333)(25-25)]\\&+(11.10374)[(p^m_4-p^m_3)-(0.4333)(25-25)]\\&+(242.5426)[(p^m_7-p^m_3)-(0.4333)(15-25)]\\&+(485.0852)[(4000-p^m_3)-(0.4333)(30-25)]+0=\frac{239123.0}{\alpha_c\Delta t}[(\frac{\phi}{B})^{n+1}_3-(\frac{\phi}{B})^n_3].\end{aligned}\tag{4.152}$$

For gridblock 5, $n=5$, $i=1$, $k=2$, $\psi_5=\{1,6,9\}$, $\xi_5=\{b_W\}$, and

$\sum_{l\in\xi_5} q^m_{sc_{l,5}} = q^m_{sc_{b_W,5}}$, where from Table 4–7, $q^m_{sc_{b_W,5}}=-1000$ B/D, and $q^m_{sc_5}=0$ (the well is treated as a boundary condition). Therefore, substitution into Eq. 4.2a yields

$$(0.115155)[(p_1^m - p_5^m) - (0.4333)(25-15)]$$
$$+(11.10374)[(p_6^m - p_5^m) - (0.4333)(15-15)]$$
$$+(0.115155)[(p_9^m - p_5^m) - (0.4333)(5-15)] - 1000 + 0 = \frac{113.5318}{\alpha_c \Delta t}[(\frac{\phi}{B})_5^{n+1} - (\frac{\phi}{B})_5^n]. \quad \textbf{(4.153)}$$

For gridblock 7, $n = 7$, $i = 3$, $k = 2$, $\psi_7 = \{3,6,8,11\}$, $\xi_7 = \{\}$, $\sum_{l \in \xi_7} q_{sc_{l,7}}^m = 0$ (interior gridblock), and $q_{sc_7}^m = 0$ (no wells). Therefore, substitution into Eq. 4.2a yields

$$(242.5426)[(p_3^m - p_7^m) - (0.4333)(25-15)]$$
$$+(11.10374)[(p_6^m - p_7^m) - (0.4333)(15-15)]$$
$$+(11.10374)[(p_8^m - p_7^m) - (0.4333)(15-15)]$$
$$+(242.5426)[(p_{11}^m - p_7^m) - (0.4333)(5-15)] + 0 + 0 = \frac{239123.0}{\alpha_c \Delta t}[(\frac{\phi}{B})_7^{n+1} - (\frac{\phi}{B})_7^n]. \quad \textbf{(4.154)}$$

For gridblock 11, $n = 11$, $i = 3$, $k = 3$, $\psi_{11} = \{7,10,12\}$, $\xi_{11} = \{b_U\}$, $\sum_{l \in \xi_{11}} q_{sc_{l,11}}^m = q_{sc_{b_U,11}}^m$, $q_{sc_{b_U,11}}^m = 0$ (no-flow boundary), and $q_{sc_{11}}^m = 0$ (no wells). Therefore, substitution into Eq. 4.2a yields

$$(242.5426)[(p_7^m - p_{11}^m) - (0.4333)(15-5)]$$
$$+(11.10374)[(p_{10}^m - p_{11}^m) - (0.4333)(5-5)]$$
$$+(11.10374)[(p_{12}^m - p_{11}^m) - (0.4333)(5-5)] + 0 + 0 = \frac{239123.0}{\alpha_c \Delta t}[(\frac{\phi}{B})_{11}^{n+1} - (\frac{\phi}{B})_{11}^n]. \quad \textbf{(4.155)}$$

4.6 Symmetry and Its Use in Solving Practical Problems

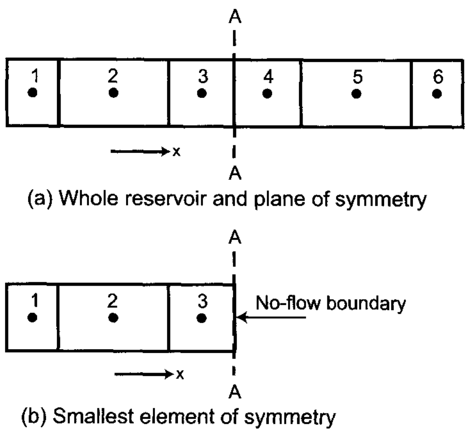
Reservoir rock properties are heterogeneous, and reservoir fluids as well as fluid-rock properties vary from one region to another within the same reservoir. In other words, it is rare to find a petroleum reservoir that has constant properties. The literature, however, is rich in study cases in which homogeneous reservoirs were modeled to study flood patterns such as five-spot and nine-spot patterns. In teaching reservoir simulation, educators as well as textbooks in this area make use of homogeneous reservoirs most of the time. If reservoir properties vary spatially region-wise, then symmetry may exist. The use of symmetry reduces the efforts to solve a problem by solving a modified problem for one element of symmetry in the reservoir, usually the smallest element of symmetry (Abou-Kassem, Ertekin, and Lutchmansingh 1991). The smallest element of symmetry is a segment of the reservoir that is a mirror image of the rest of reservoir segments. Before solving the modified problem for one element of symmetry, however, symmetry must first be established. For symmetry to exist about a plane, there must be symmetry with regard to (1) the number of gridblocks and gridblock dimensions, (2) reservoir rock properties, (3) physical wells, (4) reservoir boundaries, and (5) initial conditions. Gridblock dimensions deal with gridblock size (Δx , Δy , Δz) and gridblock elevation (Z). Reservoir rock properties deal with gridblock porosity (ϕ) and permeability in the various directions (k_x , k_y , k_z). Wells deal with well location, well type (injection or production), and well operating condition. Reservoir boundaries deal with the geometry of boundaries and boundary conditions. Initial conditions deal with initial pressure and fluid saturation distributions in the reservoir. Failing to satisfy symmetry with respect to any of the items mentioned above means there is no symmetry about that plane. The formulation of the modified problem for the smallest element of symmetry involves replacing each plane of symmetry with a no-flow boundary and determining the new interblock geometric factors, bulk volume, wellblock rate, and wellblock geometric factor for those gridblocks that share their boundaries with the planes of symmetry. To elaborate on this point, we present a few possible cases. In the following discussion, we use bold numbers to identify the gridblocks in the element of symmetry which require determining new values for their bulk volume, wellblock rate, wellblock geometric factor, and interblock geometric factors.

The first two examples show planes of symmetry that coincide with the boundaries between gridblocks. Figure 4–15a presents a 1D flow problem in which the plane of symmetry A-A, which is normal to the flow direction (x direction) and coincides with the boundary between gridblocks 3 and 4, divides the reservoir into two symmetrical elements. Consequently, $p_1 = p_6$, $p_2 = p_5$, and $p_3 = p_4$. The modified problem is represented by the element of symmetry shown in Figure 4–15b, with the plane of symmetry being replaced with a no-flow boundary.

Figure 4–16a presents a 2D horizontal reservoir with two vertical planes of symmetry A-A and B-B. Plane of symmetry A-A is normal to the x direction and coincides with the boundaries between gridblocks 2, 6, 10, and 14 on one side and gridblocks 3, 7, 11, and 15 on the other side. Plane of symmetry B-B is normal to the y direction and coincides with the boundaries between gridblocks 5, 6, 7, and 8 on one side and gridblocks 9, 10, 11, and 12 on the other side. The two planes of symmetry divide the reservoir into four symmetrical elements. Consequently,

$$p_1 = p_4 = p_{13} = p_{16} \text{ , } p_2 = p_3 = p_{14} = p_{15} \text{ , } p_5 = p_8 = p_9 = p_{12} \text{ , and } p_6 = p_7 = p_{10} = p_{11} \text{ .}$$

The modified problem is represented by the smallest element of symmetry shown in Figure 4–16b, with each plane of symmetry being replaced with a no-flow boundary.



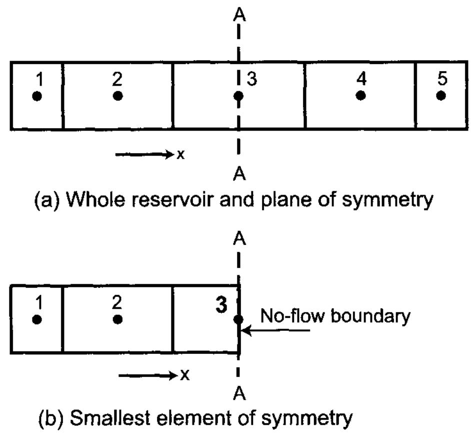
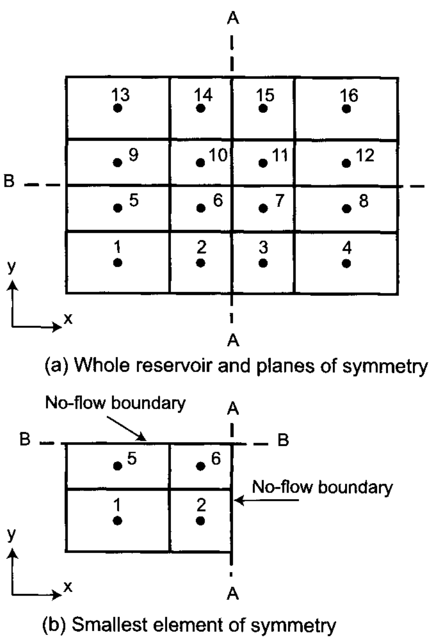


Figure 4–15 Reservoir with even gridblocks exhibiting a vertical plane of symmetry (left).
Figure 4–17 Reservoir with odd gridblocks exhibiting a vertical plane of symmetry (right).



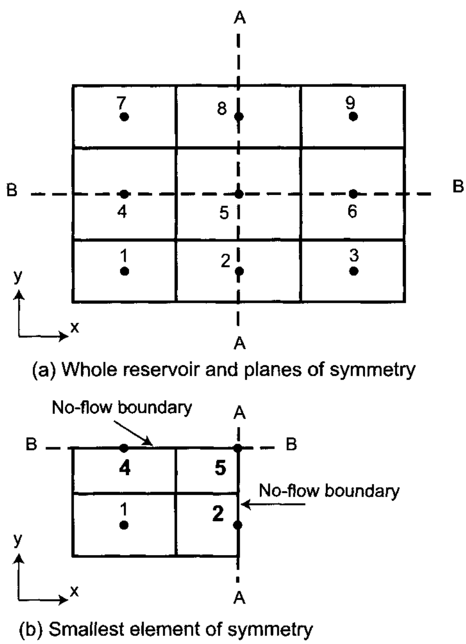


Figure 4–16 Reservoir with even gridblocks in the *x* and *y* directions exhibiting two vertical planes of symmetry (left).
Figure 4–18 Reservoir with odd gridblocks in the *x* and *y* directions exhibiting two vertical planes of symmetry (right).

The second two examples show planes of symmetry that pass through the centers of gridblocks. Figure 4–17a presents a 1D flow problem where the plane of symmetry A-A, which is normal to the flow direction (x direction) and passes through the center of gridblock 3, divides the reservoir into two symmetrical elements. Consequently, $p_1 = p_5$ and $p_2 = p_4$. The modified problem is represented by the element of symmetry shown in Figure 4–17b, with the plane of symmetry being replaced with a no-flow boundary. This plane of symmetry bisects the gridblock bulk volume, wellblock rate, and wellblock geometric factor for gridblock **3** in Figure 4–17a. Therefore, for gridblock **3**, $V_{b_3} = \frac{1}{2} V_{b_3}$, $q_{sc_3} = \frac{1}{2} q_{sc_3}$, and $G_{w_3} = \frac{1}{2} G_{w_3}$. Note that the interblock geometric factor in the direction normal to the plane of symmetry ($G_{x_{2,3}}$) is not affected. Figure 4–18a presents a 2D horizontal reservoir with two vertical planes of symmetry A-A and B-B. Plane A-A is a vertical plane of symmetry that is parallel to the *y-z* plane (normal to the *x* direction) and passes through the centers of gridblocks 2, 5, and 8. Note that gridblocks 1, 4, and 7 are mirror images of gridblocks 3, 6, and 9. Plane B-B is a vertical plane of symmetry that is parallel to the *x-z* plane (normal to the *y* direction) and passes through the centers of gridblocks 4, 5, and 6. Note that gridblocks 1, 2, and 3 are mirror images of gridblocks 7, 8, and 9. The two planes of symmetry divide the reservoir into four symmetrical elements. Consequently, $p_1 = p_3 = p_7 = p_9$, $p_4 = p_6$, and $p_2 = p_8$. The modified problem is represented by the smallest element of symmetry shown in Figure 4–18b, with each plane of symmetry being replaced with a no-flow boundary. Each plane of symmetry bisects the block bulk volume, wellblock rate, and wellblock geometric factor of the gridblock it passes through and bisects the interblock geometric factors in the directions that are parallel to the plane of symmetry. Therefore, $V_{b_2} = \frac{1}{2} V_{b_2}$, $q_{sc_2} = \frac{1}{2} q_{sc_2}$, $G_{w_2} = \frac{1}{2} G_{w_2}$; $V_{b_4} = \frac{1}{2} V_{b_4}$, $q_{sc_4} = \frac{1}{2} q_{sc_4}$, $G_{w_4} = \frac{1}{2} G_{w_4}$; $V_{b_5} = \frac{1}{4} V_{b_5}$, $q_{sc_5} = \frac{1}{4} q_{sc_5}$, $G_{w_5} = \frac{1}{4} G_{w_5}$; $G_{y_{2,5}} = \frac{1}{2} G_{y_{2,5}}$; and $G_{x_{4,5}} = \frac{1}{2} G_{x_{4,5}}$. Because gridblocks **2**, **4**, and **5** fall on the boundaries of the element of symmetry, they can be looked at as if they were gridpoints as in Chapter 5, and the same bulk volumes, wellblock rates, wellblock geometric factors, and interblock geometric factors will be calculated as those reported above. Note also that a plane of symmetry passing through the center of a gridblock results in a factor of $\frac{1}{2}$, as in gridblocks 2 and 4. Two planes of symmetry passing through the center of a gridblock result in a factor of $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, as in gridblock 5.

The third example presents two planes of symmetry, one coinciding with the boundaries between the gridblocks and the other passing through the centers of the gridblocks. Figure 4–19a presents a 2D horizontal reservoir with two vertical planes of symmetry A-A and B-B.

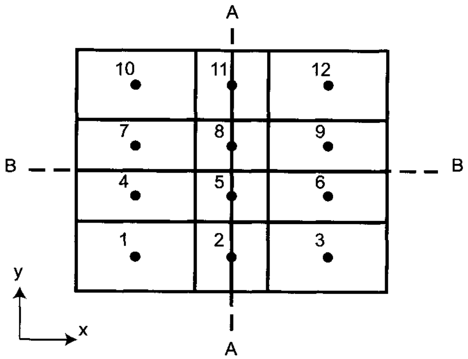
Plane A-A is a vertical plane of symmetry that is parallel to the *y-z* plane (normal to the *x* direction) and passes through the centers of gridblocks 2, 5, 8, and 11. Note that gridblocks 1, 4, 7, and 10 are mirror images of gridblocks 3, 6, 9, and 12. Plane B-B is a vertical plane of symmetry that is parallel to the *x-z* plane (normal to the *y* direction) and coincides with the boundaries between gridblocks 4, 5, and 6 on one side and gridblocks 7, 8, and 9 on the other side. Note that gridblocks 1, 2, and 3 are mirror images of gridblocks 10, 11, and 12. Addionally, gridblocks 4, 5, and 6 are mirror images of gridblocks 7, 8, and 9. The two planes of symmetry divide the reservoir into four symmetrical elements. Consequently, $p_1 = p_3 = p_{10} = p_{12}$, $p_4 = p_6 = p_7 = p_9$, $p_2 = p_{11}$, and $p_5 = p_8$. The modified problem is represented by the smallest element of symmetry shown in Figure 4–19b, with each plane of symmetry being replaced with a no-flow boundary. Plane of symmetry A-A bisects the block bulk volume, wellblock rate, and wellblock geometric factor of the gridblocks it passes through and bisects the interblock geometric factors in the directions that are parallel to the plane of symmetry (y direction in this case). Therefore, $V_{b_2} = \frac{1}{2}V_{b_2}$, $q_{sc_2} = \frac{1}{2}q_{sc_2}$, $G_{w_2} = \frac{1}{2}G_{w_2}$; $V_{b_5} = \frac{1}{2}V_{b_5}$, $q_{sc_5} = \frac{1}{2}q_{sc_5}$, $G_{w_5} = \frac{1}{2}G_{w_5}$; $V_{b_8} = \frac{1}{2}V_{b_8}$, $q_{sc_8} = \frac{1}{4}q_{sc_8}$, $G_{w_8} = \frac{1}{4}G_{w_8}$; $V_{b_{11}} = \frac{1}{2}V_{b_{11}}$, $q_{sc_{11}} = \frac{1}{4}q_{sc_{11}}$, $G_{w_{11}} = \frac{1}{4}G_{w_{11}}$; $G_{y_{2,5}} = \frac{1}{2}G_{y_{2,5}}$; $G_{y_{5,8}} = \frac{1}{2}G_{y_{5,8}}$; and $G_{y_{8,11}} = \frac{1}{2}G_{y_{8,11}}$. Because gridblocks **2**, **5**, **8**, and **11** fall on the boundaries of the element of symmetry, they can be looked at as if they were gridpoints as in Chapter 5, and the same bulk volumes, wellblock rates, wellblock geometric factors, and interblock geometric factors will be calculated as those reported above. Note also that a plane of symmetry passing through the center of a gridblock results in a factor of $\frac{1}{2}$, as in gridblocks 2, 5, 8, and 11 in Figure 4–19a.

The fourth set of examples show oblique planes of symmetry. Figure 4–20a shows a reservoir similar to that depicted in Figure 4–16a, but the present reservoir has two additional planes of symmetry C-C and D-D. The four planes of symmetry divide the reservoir into eight symmetrical elements, each with a triangular shape as shown in Figure 4–20b. Consequently,

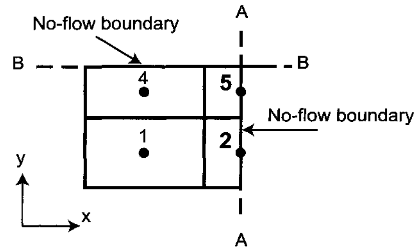
$$p_1 = p_4 = p_{13} = p_{16},\ p_6 = p_7 = p_{10} = p_{11}, \text{ and } p_2 = p_3 = p_{14} = p_{15} = p_5 = p_8 = p_9 = p_{12}.$$

The modified problem is represented by the smallest element of symmetry shown in Figure 4–20b, with each plane of symmetry being replaced with a no-flow boundary. Figure 4–21a shows a reservoir similar to that depicted in Figure 4–18a, but the present reservoir has two additional planes of symmetry C-C and D-D. The four planes of symmetry divide the reservoir into eight symmetrical elements, each with a triangular shape as shown in Figure 4–21b. Consequently, $p_1 = p_3 = p_7 = p_9$ and $p_4 = p_6 = p_2 = p_8$. The modified problem is represented by the smallest element of symmetry shown in Figure 4–21b, with each plane of symmetry being replaced with a no-flow boundary. A vertical plane of symmetry C-C or D-D that passes through the center of a gridblock but is neither parallel to the x axis nor the y axis (oblique plane), as shown in Figure 4–20a and Figure 4–21a, bisects the gridblock bulk volume, wellblock rate, and wellblock geometric factor of the gridblock it passes through. An oblique plane does not affect the interblock geometric factors in the x axis or the y axis. In reference to gridblocks 1, 6, and 5 in Figure 4–20 and Figure 4–21,

$V_{b_1} = \frac{1}{2}V_{b_1}$, $q_{sc_1} = \frac{1}{2}q_{sc_1}$, $G_{w_1} = \frac{1}{2}G_{w_1}$; $V_{b_6} = \frac{1}{2}V_{b_6}$, $q_{sc_6} = \frac{1}{2}q_{sc_6}$, $G_{w_6} = \frac{1}{2}G_{w_6}$; $V_{b_5} = \frac{1}{8}V_{b_5}$,



(a) Whole reservoir and planes of symmetry



(b) Smallest element of symmetry

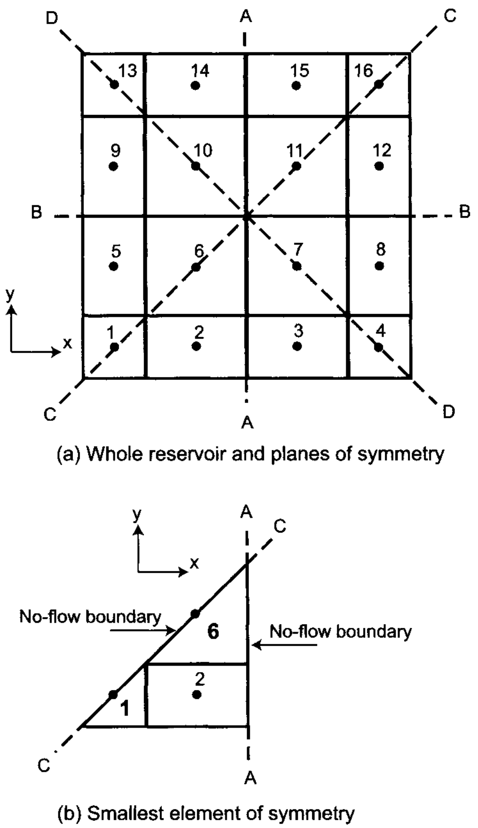
Figure 4–19 Reservoir with even gridblocks in the *y* direction and odd gridblocks in the *x* direction exhibiting two vertical planes of symmetry.

$q_{sc_5} = \frac{1}{8} q_{sc_5}$, $G_{w_5} = \frac{1}{8} G_{w_5}$; $G_{y_{2,5}} = \frac{1}{2} G_{y_{2,5}}$; and $G_{x_{2,6}} = G_{x_{2,6}}$. Note that the four planes of symmetry (A-A, B-B, C-C, and D-D) passing through the center of gridblock **5** in Figure 4–21a result in the factor of $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ used to calculate the actual bulk volume, wellblock rate, and wellblock geometric factor for gridblock 5 in Figure 4–21b. That is to say, the modifying factor equals $\frac{1}{n_{vsp}} \times \frac{1}{2}$, where n_{vsp} is the number of vertical planes of symmetry passing through the center of a gridblock.

It should be mentioned that set ξ_n for gridblocks in the modified problem might include new elements such as $b_{SW}, b_{NW}, b_{SE}, b_{NE}$ that reflect oblique boundaries such as plane C-C or D-D. The flow rates across such boundaries ($q^m_{sc_{l,n}}$) are set to zero because these boundaries represent no-flow boundaries.

4.7 Summary

This chapter presents reservoir discretization in Cartesian and radial-cylindrical coordinates using a block-centered grid. For the Cartesian coordinate system, equations similar to those represented by Eq. 4.1 define gridblock locations and the relationships between gridblock sizes, gridblock boundaries, and distances between points representing gridblocks in the *x*, *y*, and *z* directions, and Table 4–1 presents equations for the calculation of the transmissibility geometric factors in the three directions. For the radial-cylindrical coordinate system used for single-well simulation, the equations that define block locations and the relationships between gridblock sizes, gridblock boundaries, and distances between points representing blocks in the *r* direction are given by Eqs. 4.81 through 4.88, Eq. 4.80 in the θ direction, and an equation similar to Eq. 4.1 for the *z* direction. The equations in either Table 4–2 or Table 4–3 can be used to calculate transmissibility geometric



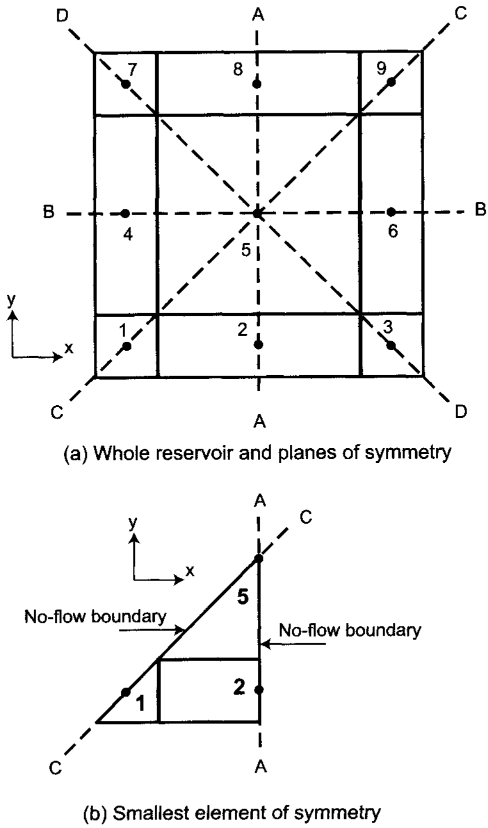


Figure 4–20 Reservoir with even gridblocks in the *x* and *y* directions exhibiting four vertical planes of symmetry (left).
Figure 4–21 Reservoir with odd gridblocks in the *x* and *y* directions exhibiting four vertical planes of symmetry (right).

factors in the r, θ, and z directions. Eq. 4.2 expresses the general form of the flow equation that applies to boundary gridblocks as well as interior gridblocks in 1D, 2D, or 3D flow in both Cartesian and radial-cylindrical coordinates. The flow equation for any gridblock has flow terms equal to the number of existing neighboring gridblocks and fictitious wells equal to the number of boundary conditions. Each fictitious well represents a boundary condition. The flow rate of a fictitious well is given by Eq. 4.24, 4.27, 4.32, or 4.37 for a specified pressure gradient, specified flow rate, no-flow, or specified pressure boundary condition, respectively.

If reservoir symmetry exists, it can be exploited to define the smallest element of symmetry. Planes of symmetry may pass along block boundaries or through block centers. To simulate the smallest element of symmetry, planes of symmetry are replaced with no-flow boundaries and new interblock geometric factors, bulk volume, wellblock rate, and wellblock geometric factors for boundary blocks, in the element of symmetry, are calculated prior to simulation.

4.8 Exercises

4-1 What is the meaning of reservoir discretization into gridblocks?

4-2 Using your own words, describe how you discretize a reservoir of length L_x along the x direction using n gridblocks.

4-3 Figure 4–5 shows a reservoir with regular boundaries.

(a) How many boundaries does this reservoir have along the x direction? Identify and name these boundaries.

(b) How many boundaries does this reservoir have along the y direction? Identify and name these boundaries.

(c) How many boundaries does this reservoir have along the z direction? Identify and name these boundaries.

(d) How many boundaries does this reservoir have along all directions?

4-4 Consider the 2D reservoir described in Example 4.5 and shown in Figure 4–12.

(a) Identify the interior and boundary gridblocks in the reservoir.

(b) Write the set of neighboring gridblocks (ψ_n) for each gridblock in the reservoir.

(c) Write the set of boundaries (ξ_n) for each gridblock in the reservoir.

(d) How many boundary conditions does each boundary gridblock have? How many fictitious wells does each boundary gridblock have? Write the terminology for the flow rate of each fictitious well.

(e) How many flow terms does each boundary gridblock have?

(f) Add the number of flow terms and number of fictitious wells for each boundary gridblock. Do they add up to four for each boundary gridblock?

(g) How many flow terms does each interior gridblock have?

(h) What can you conclude from your results of (f) and (g) above?

4-5 Consider fluid flow in the 1D horizontal reservoir shown in Figure 4–22.

(a) Write the appropriate flow equation for gridblock n in this reservoir.

(b) Write the flow equation for gridblock 1 by finding ψ_1 and ξ_1 and then using them to expand the equation in (a).

(c) Write the flow equation for gridblock 2 by finding ψ_2 and ξ_2 and then using them to expand the equation in (a).

(d) Write the flow equation for gridblock 3 by finding ψ_3 and ξ_3 and then using them to expand the equation in (a).

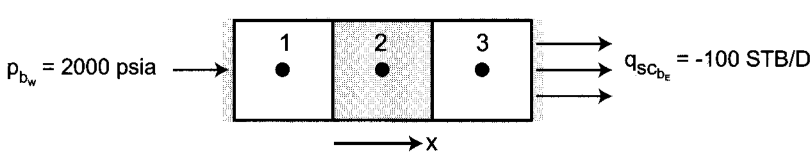


Figure 4–22 1D reservoir in Exercise 4-5.

4-6 Consider fluid flow in the 2D horizontal reservoir shown in Figure 4–23.

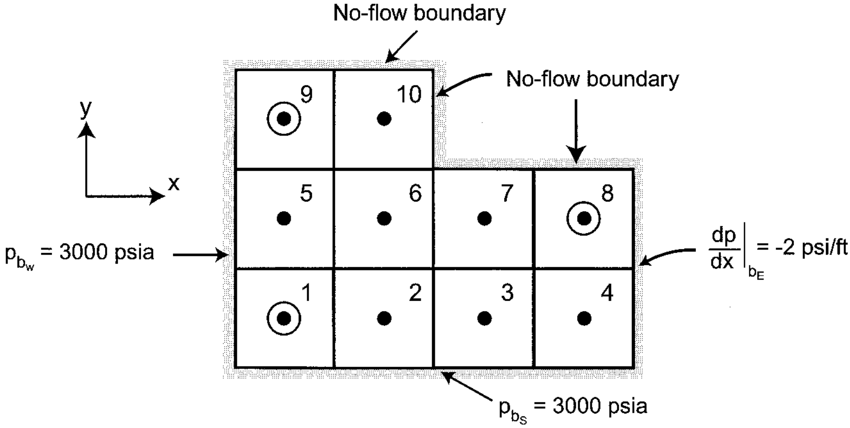


Figure 4–23 2D reservoir for Exercise 4-6.

(a) Write the appropriate flow equation for gridblock n in this reservoir.
(b) Write the flow equation for gridblock 1 by finding ψ_1 and ξ_1 and then using them to expand the equation in (a).
(c) Write the flow equation for gridblock 3 by finding ψ_3 and ξ_3 and then using them to expand the equation in (a).
(d) Write the flow equation for gridblock 5 by finding ψ_5 and ξ_5 and then using them to expand the equation in (a).
(e) Write the flow equation for gridblock 9 by finding ψ_9 and ξ_9 and then using them to expand the equation in (a).

4-7 Consider single-phase flow in a homogeneous, 1D reservoir with constant pressure specification at the reservoir left boundary. The reservoir is discretized using a regular grid. Write the flow equation for gridblock 1, which shares its left boundary with the reservoir, and prove that $p_b = \frac{1}{2}(3p_1 - p_2)$. Aziz and Settari (1979) claim that the above equation represents a second-order correct approximation for boundary pressure.

4-8 A single-phase oil reservoir is described by four equal gridblocks as shown in Figure 4–24. The reservoir is horizontal and has $k = 25$ md. Gridblock dimensions are $\Delta x = 500$ ft, $\Delta y = 700$ ft, and $h = 60$ ft. Oil properties are $B = 1$ RB/STB and $\mu = 0.5$ cp. The reservoir left boundary is maintained at constant pressure of 2500 psia, and the reservoir right boundary is sealed off to flow. A well in gridblock 3 produces 80 STB/D of oil. Assuming that the reservoir rock and oil are incompressible, calculate the pressure distribution in the reservoir.

4-9 A 1D horizontal oil reservoir shown in Figure 4–25 is described by four equal gridblocks. Reservoir blocks have $k = 90$ md, $\Delta x = 300$ ft,

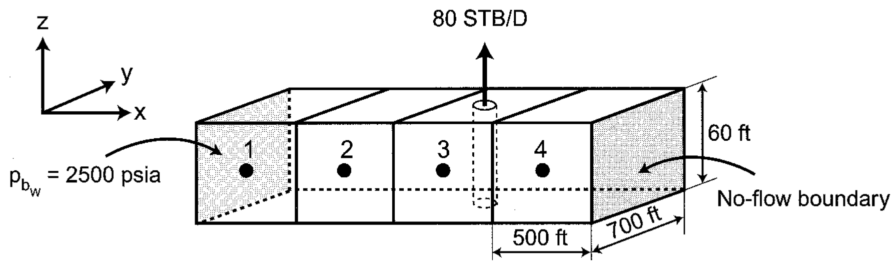


Figure 4–24 Discretized 1D reservoir in Exercise 4-8.

$\Delta y = 250$ ft, and $h = 45$ ft. Oil FVF and viscosity are 1 RB/STB and 2 cp, respectively. The reservoir left boundary is maintained at constant pressure of 2000 psia, and the reservoir right boundary has a constant influx of oil at a rate of 80 STB/D. A well in gridblock 3 produces 175 STB/D of oil. Assuming that the reservoir rock and oil are incompressible, calculate the pressure distribution in the reservoir.

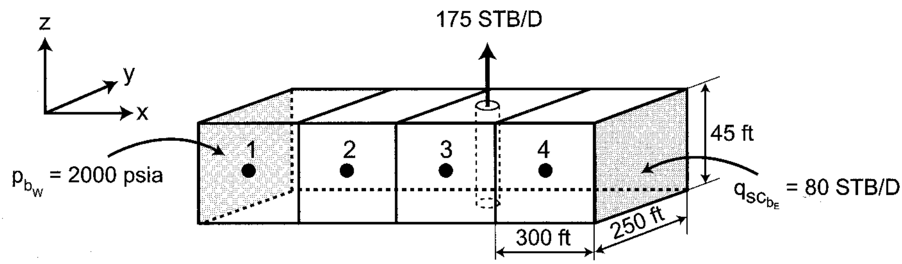


Figure 4–25 Discretized 1D reservoir in Exercise 4-9.

4-10 A 1D horizontal oil reservoir shown in Figure 4–26 is described by four equal gridblocks. Reservoir blocks have $k = 120$ md, $\Delta x = 500$ ft, $\Delta y = 450$ ft, and $h = 30$ ft. Oil FVF and viscosity are 1 RB/STB and 3.7 cp, respectively. The reservoir left boundary is subject to a constant pressure gradient of –0.2 psi/ft, and the reservoir right boundary is a no-flow boundary. A well in gridblock 3 produces oil at a rate such that the pressure of gridblock 3 is maintained at 1500 psia. Assuming that the reservoir rock and oil are incompressible, calculate the pressure distribution in the reservoir. Then estimate the well production rate.

4-11 A 1D horizontal oil reservoir shown in Figure 4–27 is described by four equal gridblocks. Reservoir blocks have $k = 70$ md, $\Delta x = 400$ ft, $\Delta y = 660$ ft, and $h = 10$ ft. Oil FVF and viscosity are 1 RB/STB and 1.5 cp, respectively. The reservoir left boundary is maintained at constant pressure of 2700, while the boundary condition at the reservoir right boundary is not known but the pressure of gridblock 4 is maintained at 1900 psia. A well in gridblock 3 produces 150 STB/D of oil. Assuming that

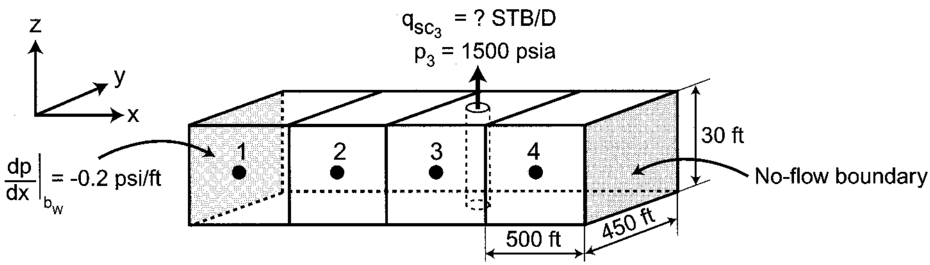


Figure 4–26 Discretized 1D reservoir in Exercise 4-10.

the reservoir rock and oil are incompressible, calculate the pressure distribution in the reservoir. Estimate the rate of oil that crosses the reservoir right boundary.

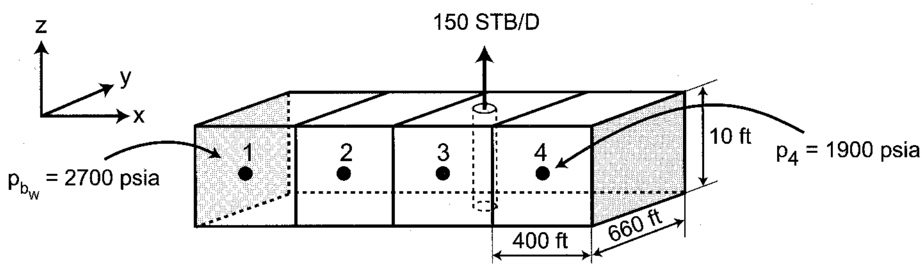


Figure 4–27 Discretized 1D reservoir in Exercise 4-11.

4-12 Consider the 2D horizontal oil reservoir shown in Figure 4–28. The reservoir is described using a regular grid. Reservoir gridblocks have $\Delta x = 350$ ft, $\Delta y = 300$ ft, $h = 35$ ft, $k_x = 160$ md, and $k_y = 190$ md. Oil FVF and viscosity are 1 RB/STB and 4.0 cp, respectively. Boundary conditions are specified as shown in the figure. A well in gridblock 5 produces oil at a rate of 2000 STB/D. Assume that the reservoir rock and oil are incompressible. Write the flow equations for all gridblocks. Do not solve the equations.

4-13 Starting with Eq. 4.88c, which expresses the bulk volume of gridblock (n_r, j, k) in terms of r_e and $r_{n_r-1/2}$, derive Eq. 4.88d, which expresses the bulk volume in terms of α_{lg} and r_e.

4-14 A 6-inch vertical well producing 500 STB/D of oil is located in 16-acre spacing. The reservoir is 30 ft thick and has horizontal permeability of 50 md. The oil FVF and viscosity are 1 RB/B and 3.5 cp, respectively. The reservoir external boundaries are no-flow boundaries. The reservoir is simulated using four gridblocks in the radial direction as shown in Figure 4–29. Write the flow equations for all gridblocks. Do not substitute for values on the RHS of equations.

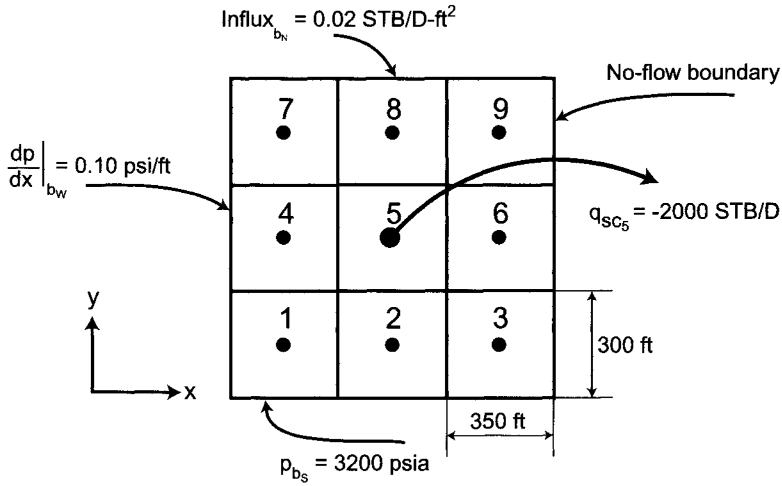


Figure 4–28 Discretized 2D reservoir in Exercise 4-12.

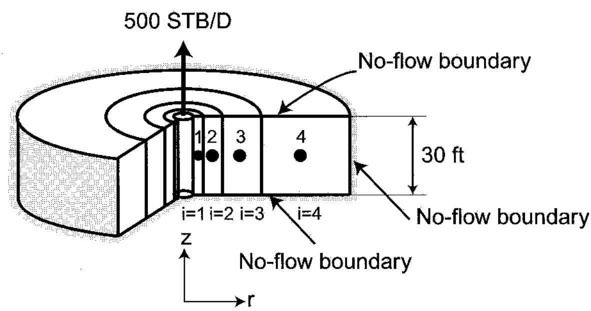


Figure 4–29 Discretized reservoir in Exercise 4-14.

4-15 A 9 ⅝ -inch vertical well is located in 12-acre spacing. The reservoir thickness is 50 ft. Horizontal and vertical reservoir permeabilities are 70 md and 40 md, respectively. The flowing fluid has a density, FVF, and viscosity of 62.4 lbm/ft^3, 1 RB/B, and 0.7 cp, respectively. The reservoir external boundary in the radial direction is a no-flow boundary, and the well is completed in the top 20 ft only and produces at a rate of 1000 B/D. The reservoir bottom boundary is subject to influx such that the boundary is maintained at 3000 psia. The reservoir top boundary is sealed off to flow. Assuming the reservoir can be simulated using two gridblocks in the vertical direction and four gridblocks in the radial direction as shown in Figure 4–30, write the flow equations for all gridblocks in this reservoir.

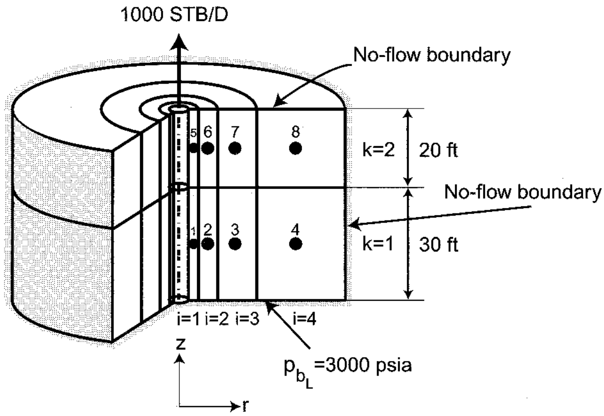


Figure 4–30 Discretized 2D radial-cylindrical reservoir in Exercise 4-15.