Flow Equations Using CVFD Terminology

3.1 Introduction

The importance of the Control Volume Finite Difference (CVFD) method lies in its capacity to use the same form of flow equation for 1D, 2D, and 3D flow problems regardless of the ordering scheme of blocks. The only difference among 1D, 2D, and 3D flow equations is the definition of the elements for the set of neighboring blocks. The CVFD method is mainly used to write flow equations in a compact form that is independent of the dimensionality of flow, the coordinate system used, or the block ordering scheme. This chapter introduces the terminology used in the CVFD method and the relationship between this method and the traditional way of writing finite-difference equations presented in Chapter 2.

3.2 Flow Equations Using CVFD Terminology

In petroleum engineering, Aziz (1993) was the first author to refer to the CVFD method. However, the method had been developed and used by others without giving it a name (Abou-Kassem 1981, Lutchmansingh 1987, Abou-Kassem and Farouq Ali 1987). The terminology presented in this section is based on a 2001 work published by Ertekin, Abou-Kassem, and King. With this terminology, we can write the equations for 1D, 2D, and 3D flow in compact form, using Cartesian or radial-cylindrical coordinates. For the flow equation in Cartesian space, we define ψ_{x_n}, ψ_{y_n}, and ψ_{z_n} as the sets whose members are the neighboring blocks of block n in the directions of the x axis, y axis, and z axis, respectively. Then we define ψ_n as the set that contains the neighboring blocks in all flow directions as its members; i.e.,

$$\psi_n = \psi_{x_n} \cup \psi_{y_n} \cup \psi_{z_n} \tag{3.1a}$$

If there is no flow in a given direction, then the set for that direction is the empty set, { }. For the flow equation in radial-cylindrical space, the equation that corresponds to Eq. 3.1a is

$$\psi_n = \psi_{r_n} \cup \psi_{\theta_n} \cup \psi_{z_n} \tag{3.1b}$$

where ψ_{r_n}, ψ_{θ_n}, and ψ_{z_n} are the sets whose members are the neighboring blocks of block n in the r direction, θ direction, and z axis, respectively.

The following sections present the flow equations for blocks identified by engineering notation or by block ordering using the natural ordering scheme.

3.2.1 Flow Equations Using CVFD Terminology and Engineering Notation

For 1D flow in the direction of the x axis, block n is termed in engineering notation as block i (i.e., $n \equiv i$) as shown in Figure 3–1a. In this case,

$$\psi_{x_n} = \{(i-1),(i+1)\} \tag{3.2a}$$

$$\psi_{y_n} = \{\} \tag{3.2b}$$

$$\psi_{z_n} = \{\} \tag{3.2c}$$

Substitution of Eq. 3.2 into Eq. 3.1a results in

$$\begin{aligned}\psi_n &= \psi_i = \{(i-1),(i+1)\} \cup \{\} \cup \{\} \\ &= \{(i-1),(i+1)\}.\end{aligned} \tag{3.3}$$

The flow equation for block i expressed as Eq. 2.33,

$$\begin{aligned}&T^m_{x_{i-1/2}}[(p^m_{i-1} - p^m_i) - \gamma^m_{i-1/2}(Z_{i-1} - Z_i)] + T^m_{x_{i+1/2}}[(p^m_{i+1} - p^m_i) - \gamma^m_{i+1/2}(Z_{i+1} - Z_i)] \\ &+q^m_{sc_i} = \frac{V_{b_i}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_i - (\frac{\phi}{B})^n_i],\end{aligned} \tag{3.4a}$$

which can be written in CVFD form as

$$\sum_{l \in \psi_i} T^m_{l,i}[(p^m_l - p^m_i) - \gamma^m_{l,i}(Z_l - Z_i)] + q^m_{sc_i} = \frac{V_{b_i}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_i - (\frac{\phi}{B})^n_i], \tag{3.4b}$$

where

$$T^m_{i\mp1,i} = T^m_{i,i\mp1} \equiv T^m_{x_{i\mp1/2}} \tag{3.5}$$

and transmissibilities $T^m_{x_{i\mp1/2}}$ are defined by Eq. 2.39a. In addition,

$$\gamma^m_{i\mp1,i} = \gamma^m_{i,i\mp1} \equiv \gamma^m_{i\mp1/2} \tag{3.6}$$

For 2D flow in the x-y plane, block n is termed in engineering notation as block (i, j) , that is, $n \equiv (i, j)$, as shown in Figure 3–1b. In this case,

$$\psi_{x_n} = \{(i-1, j),(i+1, j)\} \tag{3.7a}$$

$$\psi_{y_n} = \{(i, j-1),(i, j+1)\} \tag{3.7b}$$

$$\psi_{z_n} = \{\} \tag{3.7c}$$

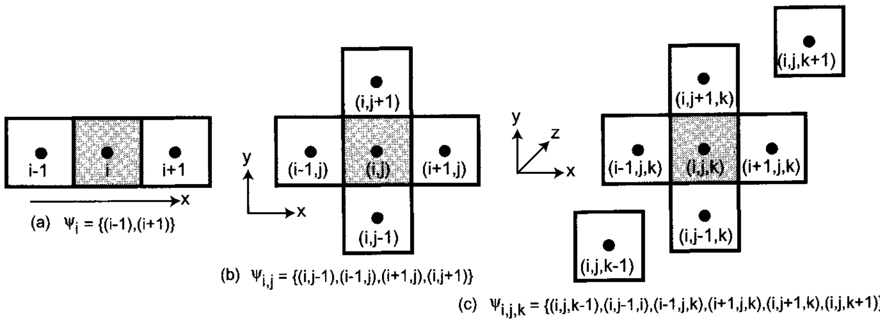


Figure 3–1 A block and its neighboring blocks in 1D, 2D, and 3D flow using engineering notation.

Substitution of Eq. 3.7 into Eq. 3.1a results in

$$\begin{aligned}\psi_n = \psi_{i,j} &= \{(i-1,j),(i+1,j)\} \cup \{(i,j-1),(i,j+1)\} \cup \{\} \\ &= \{(i,j-1),(i-1,j),(i+1,j),(i,j+1)\}.\end{aligned} \tag{3.8}$$

Eq. 2.37 expresses the flow equation for block (i,j) as

$$\begin{aligned}&T^m_{y_{i,j-1/2}}[(p^m_{i,j-1} - p^m_{i,j}) - \gamma^m_{i,j-1/2}(Z_{i,j-1} - Z_{i,j})] \\ &+T^m_{x_{i-1/2,j}}[(p^m_{i-1,j} - p^m_{i,j}) - \gamma^m_{i-1/2,j}(Z_{i-1,j} - Z_{i,j})] \\ &+T^m_{x_{i+1/2,j}}[(p^m_{i+1,j} - p^m_{i,j}) - \gamma^m_{i+1/2,j}(Z_{i+1,j} - Z_{i,j})] \\ &+T^m_{y_{i,j+1/2}}[(p^m_{i,j+1} - p^m_{i,j}) - \gamma^m_{i,j+1/2}(Z_{i,j+1} - Z_{i,j})] + q^m_{sc_{i,j}} = \frac{V_{b_{i,j}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{i,j} - (\frac{\phi}{B})^n_{i,j}],\end{aligned} \tag{3.9a}$$

which can be written in CVFD form as

$$\sum_{l \in \psi_{i,j}} T^m_{l,(i,j)}[(p^m_l - p^m_{i,j}) - \gamma^m_{l,(i,j)}(Z_l - Z_{i,j})] + q^m_{sc_{i,j}} = \frac{V_{b_{i,j}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{i,j} - (\frac{\phi}{B})^n_{i,j}] \tag{3.9b}$$

where

$$T^m_{(i\mp1,j),(i,j)} = T^m_{(i,j),(i\mp1,j)} \equiv T^m_{x_{i\mp1/2,j}} \tag{3.10a}$$

$$T^m_{(i,j\mp1),(i,j)} = T^m_{(i,j),(i,j\mp1)} \equiv T^m_{y_{i,j\mp1/2}} \tag{3.10b}$$

Transmissibilities, $T^m_{x_{i\mp1/2,j}}$ and $T^m_{y_{i,j\mp1/2}}$ have been defined by Eqs. 2.39a and 2.39b, respectively. In addition,

$$\gamma^m_{(i\mp1,j),(i,j)} = \gamma^m_{(i,j),(i\mp1,j)} \equiv \gamma^m_{i\mp1/2,j} \tag{3.11a}$$

$$\gamma^m_{(i,j\mp1),(i,j)} = \gamma^m_{(i,j),(i,j\mp1)} \equiv \gamma^m_{i,j\mp1/2} \tag{3.11b}$$

For 3D flow in the x-y-z space, block n is termed in engineering notation as block (i,j,k), that is, $n \equiv (i,j,k)$, as shown in Figure 3–1c. In this case,

$$\psi_{x_n} = \{(i-1,j,k),(i+1,j,k)\} \tag{3.12a}$$

$$\psi_{y_n} = \{(i,j-1,k),(i,j+1,k)\} \tag{3.12b}$$

$$\psi_{z_n} = \{(i,j,k-1),(i,j,k+1)\} \tag{3.12c}$$

Substitution of Eq. 3.12 into Eq. 3.1a results in

$$\begin{aligned} \psi_n &= \psi_{i,j,k} \\ &= \{(i-1,j,k),(i+1,j,k)\} \cup \{(i,j-1,k),(i,j+1,k)\} \cup \{(i,j,k-1),(i,j,k+1)\} \\ &= \{(i,j,k-1),(i,j-1,k),(i-1,j,k),(i+1,j,k),(i,j+1,k),(i,j,k+1)\}. \end{aligned} \tag{3.13}$$

The flow equation for block (i,j,k) expressed as Eq. 2.38,

$$\begin{aligned} & T^m_{z_{i,j,k-1/2}}[(p^m_{i,j,k-1} - p^m_{i,j,k}) - \gamma^m_{i,j,k-1/2}(Z_{i,j,k-1} - Z_{i,j,k})] \\ & + T^m_{y_{i,j-1/2,k}}[(p^m_{i,j-1,k} - p^m_{i,j,k}) - \gamma^m_{i,j-1/2,k}(Z_{i,j-1,k} - Z_{i,j,k})] \\ & + T^m_{x_{i-1/2,j,k}}[(p^m_{i-1,j,k} - p^m_{i,j,k}) - \gamma^m_{i-1/2,j,k}(Z_{i-1,j,k} - Z_{i,j,k})] \\ & + T^m_{x_{i+1/2,j,k}}[(p^m_{i+1,j,k} - p^m_{i,j,k}) - \gamma^m_{i+1/2,j,k}(Z_{i+1,j,k} - Z_{i,j,k})] \\ & + T^m_{y_{i,j+1/2,k}}[(p^m_{i,j+1,k} - p^m_{i,j,k}) - \gamma^m_{i,j+1/2,k}(Z_{i,j+1,k} - Z_{i,j,k})] \\ & + T^m_{z_{i,j,k+1/2}}[(p^m_{i,j,k+1} - p^m_{i,j,k}) - \gamma^m_{i,j,k+1/2}(Z_{i,j,k+1} - Z_{i,j,k})] \\ & + q^m_{sc_{i,j,k}} = \frac{V_{b_{i,j,k}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{i,j,k} - (\frac{\phi}{B})^n_{i,j,k}], \end{aligned} \tag{3.14a}$$

which can be written in CVFD form as

$$\sum_{l \in \psi_{i,j,k}} T^m_{l,(i,j,k)}[(p^m_l - p^m_{i,j,k}) - \gamma^m_{l,(i,j,k)}(Z_l - Z_{i,j,k})] + q^m_{sc_{i,j,k}} = \frac{V_{b_{i,j,k}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{i,j,k} - (\frac{\phi}{B})^n_{i,j,k}], \tag{3.14b}$$

where

$$T^m_{(i\mp1,j,k),(i,j,k)} = T^m_{(i,j,k),(i\mp1,j,k)} \equiv T^m_{x_{i\mp1/2,j,k}} \tag{3.15a}$$

$$T^m_{(i,j\mp1,k),(i,j,k)} = T^m_{(i,j,k),(i,j\mp1,k)} \equiv T^m_{y_{i,j\mp1/2,k}} \tag{3.15b}$$

$$T^m_{(i,j,k\mp1),(i,j,k)} = T^m_{(i,j,k),(i,j,k\mp1)} \equiv T^m_{z_{i,j,k\mp1/2}} \qquad \textbf{(3.15c)}$$

As mentioned earlier, transmissibilities $T^m_{x_{i\mp1/2,j,k}}$, $T^m_{y_{i,j\mp1/2,k}}$, and $T^m_{z_{i,j,k\mp1/2}}$ have been defined in Eq. 2.39. Also,

$$\gamma^m_{(i\mp1,j,k),(i,j,k)} = \gamma^m_{(i,j,k),(i\mp1,j,k)} \equiv \gamma^m_{i\mp1/2,j,k} \qquad \textbf{(3.16a)}$$

$$\gamma^m_{(i,j\mp1,k),(i,j,k)} = \gamma^m_{(i,j,k),(i,j\mp1,k)} \equiv \gamma^m_{i,j\mp1/2,k} \qquad \textbf{(3.16b)}$$

$$\gamma^m_{(i,j,k\mp1),(i,j,k)} = \gamma^m_{(i,j,k),(i,j,k+1)} \equiv \gamma^m_{i,j,k\mp1/2} \qquad \textbf{(3.16c)}$$

Eq. 3.4b for 1D flow, Eq. 3.9b for 2D flow, and Eq. 3.14b for 3D flow reduce to

$$\sum_{l\in\psi_n} T^m_{l,n}[(p^m_l - p^m_n) - \gamma^m_{l,n}(Z_l - Z_n)] + q^m_{sc_n} = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_n - (\frac{\phi}{B})^n_n], \qquad \textbf{(3.17)}$$

where, as mentioned before, $n \equiv i$ for 1D flow, $n \equiv (i, j)$ for 2D flow, and $n \equiv (i, j, k)$ for 3D flow, and the elements of set ψ_n are defined accordingly (Eq. 3.3, 3.8, or 3.13).

Note that the elements of the sets that contain the neighboring blocks given by Eqs. 3.3, 3.8, and 3.13 for 1D, 2D, and 3D, respectively, are ordered as shown in Figure 3–2. The following examples demonstrate the use of CVFD terminology to write the flow equations for an interior block identified by engineering notation in 1D and 2D reservoirs.

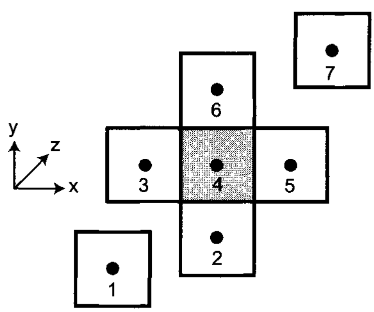


Figure 3–2 The sequence of neighboring blocks in the set $\psi_{i,j,k}$ or ψ_n .

Example 3.1 Consider the reservoir described in Example 2.4. Write the flow equation for interior block 3 using CVFD terminology.

Solution

We make use of Figure 2–12, which gives block representation of this reservoir.

For block 3, $\psi_{x_3} = \{2,4\}$, $\psi_{y_3} = \{\}$, and $\psi_{z_3} = \{\}$. Substitution into Eq. 3.1a gives $\psi_3 = \{2,4\} \cup \{\} \cup \{\} = \{2,4\}$. The application of Eq. 3.17 for $n \equiv 3$ produces

$$\sum_{l \in \psi_3} T_{l,3}^m [(p_l^m - p_3^m) - \gamma_{l,3}^m (Z_l - Z_3)] + q_{sc_3}^m = \frac{V_{b_3}}{\alpha_c \Delta t} [(\frac{\phi}{B})_3^{n+1} - (\frac{\phi}{B})_3^n], \tag{3.18}$$

which can be expanded as

$$T_{2,3}^m [(p_2^m - p_3^m) - \gamma_{2,3}^m (Z_2 - Z_3)] + T_{4,3}^m [(p_4^m - p_3^m) - \gamma_{4,3}^m (Z_4 - Z_3)] + q_{sc_3}^m = \frac{V_{b_3}}{\alpha_c \Delta t} [(\frac{\phi}{B})_3^{n+1} - (\frac{\phi}{B})_3^n]. \tag{3.19}$$

For this flow problem,

$$T_{2,3}^m = T_{4,3}^m = \beta_c \frac{k_x A_x}{\mu B \Delta x} = 0.001127 \times \frac{270 \times (900 \times 100)}{2 \times 1 \times 250} = 54.7722 \text{ STB/D-psi} \tag{3.20}$$

$Z_2 = Z_3 = Z_4$ for a horizontal reservoir, and $q_{sc_3}^m = -400$ STB/D.

Substitution into Eq. 3.19 yields

$$(54.7722)(p_2^m - p_3^m) + (54.7722)(p_4^m - p_3^m) - 400 = \frac{V_{b_3}}{\alpha_c \Delta t} [(\frac{\phi}{B})_3^{n+1} - (\frac{\phi}{B})_3^n]. \tag{3.21}$$

Eq. 3.21 is identical to Eq. 2.36, obtained in Example 2.4.

Example 3.2 Consider the reservoir described in Example 2.5. Write the flow equation for interior block (3,2) using CVFD terminology.

Solution

We make use of Figure 2–14, which gives block representation of this reservoir. For block (3,2), $\psi_{x_{3,2}} = \{(2,2),(4,2)\}$, $\psi_{y_{3,2}} = \{(3,1),(3,3)\}$, and $\psi_{z_{3,2}} = \{\}$.

Substitution into Eq. 3.1a gives

$$\psi_{3,2} = \{(2,2),(4,2)\} \cup \{(3,1),(3,3)\} \cup \{\} = \{(3,1),(2,2),(4,2),(3,3)\}.$$

The application of Eq. 3.17 for $n \equiv (3,2)$ produces

$$\sum_{l \in \psi_{3,2}} T_{l,(3,2)}^m [(p_l^m - p_{3,2}^m) - \gamma_{l,(3,2)}^m (Z_l - Z_{3,2})] + q_{sc_{3,2}}^m = \frac{V_{b_{3,2}}}{\alpha_c \Delta t} [(\frac{\phi}{B})_{3,2}^{n+1} - (\frac{\phi}{B})_{3,2}^n], \tag{3.22}$$

which can be expanded as

$$T_{(3,1),(3,2)}^m [(p_{3,1}^m - p_{3,2}^m) - \gamma_{(3,1),(3,2)}^m (Z_{3,1} - Z_{3,2})]$$
$$+ T_{(2,2),(3,2)}^m [(p_{2,2}^m - p_{3,2}^m) - \gamma_{(2,2),(3,2)}^m (Z_{2,2} - Z_{3,2})]$$

$$
\begin{aligned}
&+T^m_{(4,2),(3,2)}[(p^m_{4,2}-p^m_{3,2})-\gamma^m_{(4,2),(3,2)}(Z_{4,2}-Z_{3,2})] \\
&+T^m_{(3,3),(3,2)}[(p^m_{3,3}-p^m_{3,2})-\gamma^m_{(3,3),(3,2)}(Z_{3,3}-Z_{3,2})] \\
&+q^m_{sc_{3,2}} = \frac{V_{b_{3,2}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{3,2}-(\frac{\phi}{B})^{n}_{3,2}].
\end{aligned} \tag{3.23}
$$

For this flow problem,

$$
T^m_{(2,2),(3,2)} = T^m_{(4,2),(3,2)} = \beta_c \frac{k_x A_x}{\mu B \Delta x} = 0.001127 \times \frac{270\times(300\times100)}{2\times1\times250} = 18.2574 \tag{3.24}
$$

$$
T^m_{(3,1),(3,2)} = T^m_{(3,3),(3,2)} = \beta_c \frac{k_y A_y}{\mu B \Delta y} = 0.001127 \times \frac{220\times(250\times100)}{2\times1\times300} = 10.3308 \tag{3.25}
$$

$Z_{3,1} = Z_{2,2} = Z_{3,2} = Z_{4,2} = Z_{3,3}$ for a horizontal reservoir, and $q^m_{sc_{3,2}} = -400$ STB/D.

Substitution into Eq. 3.23 yields

$$
\begin{aligned}
&(10.3308)(p^m_{3,1}-p^m_{3,2})+(18.2574)(p^m_{2,2}-p^m_{3,2})+(18.2574)(p^m_{4,2}-p^m_{3,2}) \\
&+(10.3308)(p^m_{3,3}-p^m_{3,2})-400 = \frac{V_{b_{3,2}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{3,2}-(\frac{\phi}{B})^{n}_{3,2}].
\end{aligned} \tag{3.26}
$$

Eq. 3.26 is identical to Eq. 2.42, obtained in Example 2.5.

3.2.2 Flow Equations Using CVFD Terminology and the Natural Ordering Scheme

The flow equation in this case has one generalized form that is given by Eq. 3.17 with the corresponding definition of ψ_n for 1D, 2D, or 3D flow. Blocks in natural ordering can be ordered along rows or along columns. In this book, we adopt natural ordering along rows (with rows being parallel to the x axis) and refer to it, for short, as natural ordering. From this point on, all related discussions will use only natural ordering.

Figure 3–3a shows block n for 1D flow in the direction of the x axis. In this case,

$$
\psi_{x_n} = \{(n-1),(n+1)\} \tag{3.27a}
$$

$$
\psi_{y_n} = \{\} \tag{3.27b}
$$

$$
\psi_{z_n} = \{\} \tag{3.27c}
$$

Substitution of Eq. 3.27 into Eq. 3.1a results in

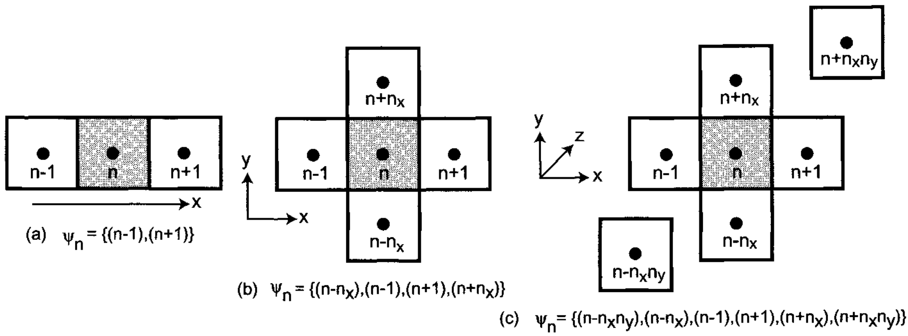


Figure 3–3 A block and its neighboring blocks in 1D, 2D, and 3D flow using natural ordering.

$$\begin{aligned}\psi_n &= \{(n-1),(n+1)\} \cup \{\,\} \cup \{\,\} \\ &= \{(n-1),(n+1)\}.\end{aligned} \tag{3.28}$$

Figure 3–3b shows block n for 2D flow in the x-y plane. In this case,

$$\psi_{x_n} = \{(n-1),(n+1)\} \tag{3.29a}$$

$$\psi_{y_n} = \{(n-n_x),(n+n_x)\} \tag{3.29b}$$

$$\psi_{z_n} = \{\,\} \tag{3.29c}$$

Substitution of Eq. 3.29 into Eq. 3.1a results in

$$\begin{aligned}\psi_n &= \{(n-1),(n+1)\} \cup \{(n-n_x),(n+n_x)\} \cup \{\,\} \\ &= \{(n-n_x),(n-1),(n+1),(n+n_x)\}.\end{aligned} \tag{3.30}$$

Figure 3–3c shows block n for 3D flow in the x-y-z space. In this case,

$$\psi_{x_n} = \{(n-1),(n+1)\} \tag{3.31a}$$

$$\psi_{y_n} = \{(n-n_x),(n+n_x)\} \tag{3.31b}$$

$$\psi_{z_n} = \{(n-n_x n_y),(n+n_x n_y)\} \tag{3.31c}$$

Substitution of Eq. 3.31 into Eq. 3.1a results in

$$\begin{aligned}\psi_n &= \{(n-1),(n+1)\} \cup \{(n-n_x),(n+n_x)\} \cup \{(n-n_x n_y),(n+n_x n_y)\} \\ &= \{(n-n_x n_y),(n-n_x),(n-1),(n+1),(n+n_x),(n+n_x n_y)\}.\end{aligned} \tag{3.32}$$

Note that the elements of the sets containing the neighboring blocks given by Eqs. 3.28, 3.30, and 3.32 for 1D, 2D, and 3D are ordered as shown in Figure 3–2. Now the flow equation for block n in 1D, 2D, or 3D can be written in CVFD form again as Eq. 3.17,

$$\sum_{l \in \psi_n} T_{l,n}^m [(p_l^m - p_n^m) - \gamma_{l,n}^m (Z_l - Z_n)] + q_{sc_n}^m = \frac{V_{b_n}}{\alpha_c \Delta t} [(\frac{\phi}{B})_n^{n+1} - (\frac{\phi}{B})_n^n], \tag{3.17}$$

where transmissibility $T_{l,n}^m$ is defined as

$$T_{n\mp1,n}^m = T_{n,n\mp1}^m \equiv T_{x_{i\mp1/2,j,k}}^m \tag{3.33a}$$

$$T_{n\mp n_x,n}^m = T_{n,n\mp n_x}^m \equiv T_{y_{i,j\mp1/2,k}}^m \tag{3.33b}$$

$$T_{n\mp n_x n_y,n}^m = T_{n,n\mp n_x n_y}^m \equiv T_{z_{i,j,k\mp1/2}}^m \tag{3.33c}$$

In addition, fluid gravity $\gamma_{l,n}^m$ is defined as

$$\gamma_{n,n\mp1}^m = \gamma_{n\mp1,n}^m \equiv \gamma_{i\mp1/2,j,k}^m \tag{3.34a}$$

$$\gamma_{n,n\mp n_x}^m = \gamma_{n\mp n_x,n}^m \equiv \gamma_{i,j\mp1/2,k}^m \tag{3.34b}$$

$$\gamma_{n,n\mp n_x n_y}^m = \gamma_{n\mp n_x n_y,n}^m \equiv \gamma_{i,j,k\mp1/2}^m \tag{3.34c}$$

We should mention here that, throughout this book, we use subscript n to refer to block order, while superscripts n and $n+1$ refer to old and new time levels, respectively. The following examples demonstrate the use of CVFD terminology to write the flow equations for an interior block identified by natural ordering in 2D and 3D reservoirs.

Example 3.3 As we did in Example 2.5, write the flow equations for interior block (3,2) using CVFD terminology, but this time use natural ordering of blocks as shown in Figure 3–4.

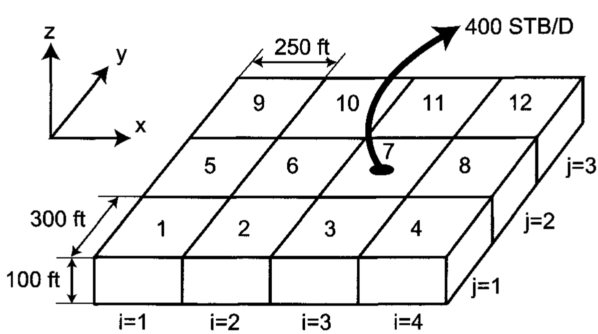


Figure 3–4 2D reservoir description in Example 3.3

Solution

Block (3,2) in Figure 2–14 corresponds to block 7 in Figure 3–4. Therefore, $n = 7$. For $n = 7$, $\psi_{x_7} = \{6,8\}$, $\psi_{y_7} = \{3,11\}$, and $\psi_{z_7} = \{\}$. Substitution into Eq. 3.1a results in $\psi_7 = \{6,8\} \cup \{3,11\} \cup \{\} = \{3,6,8,11\}$.

The application of Eq. 3.17 produces

$$\sum_{l \in \psi_7} T_{l,7}^m [(p_l^m - p_7^m) - \gamma_{l,7}^m (Z_l - Z_7)] + q_{sc_7}^m = \frac{V_{b_7}}{\alpha_c \Delta t}[(\frac{\phi}{B})_7^{n+1} - (\frac{\phi}{B})_7^n], \tag{3.35}$$

which can be expanded as

$$\begin{aligned} & T_{3,7}^m [(p_3^m - p_7^m) - \gamma_{3,7}^m (Z_3 - Z_7)] + T_{6,7}^m [(p_6^m - p_7^m) - \gamma_{6,7}^m (Z_6 - Z_7)] \\ & + T_{8,7}^m [(p_8^m - p_7^m) - \gamma_{8,7}^m (Z_8 - Z_7)] + T_{11,7}^m [(p_{11}^m - p_7^m) - \gamma_{11,7}^m (Z_{11} - Z_7)] \\ & + q_{sc_7}^m = \frac{V_{b_7}}{\alpha_c \Delta t}[(\frac{\phi}{B})_7^{n+1} - (\frac{\phi}{B})_7^n]. \end{aligned} \tag{3.36}$$

Here again,

$$T_{6,7}^m = T_{8,7}^m = \beta_c \frac{k_x A_x}{\mu B \Delta x} = 0.001127 \times \frac{270 \times (300 \times 100)}{2 \times 1 \times 250} = 18.2574 \text{ STB/D-psi} \tag{3.37}$$

$$T_{3,7}^m = T_{11,7}^m = \beta_c \frac{k_y A_y}{\mu B \Delta y} = 0.001127 \times \frac{220 \times (250 \times 100)}{2 \times 1 \times 300} = 10.3308 \text{ STB/D-psi} \tag{3.38}$$

$Z_3 = Z_6 = Z_7 = Z_8 = Z_{11}$ for a horizontal reservoir, and $q_{sc_7}^m = -400$ STB/D.

Substitution into Eq. 3.36 gives

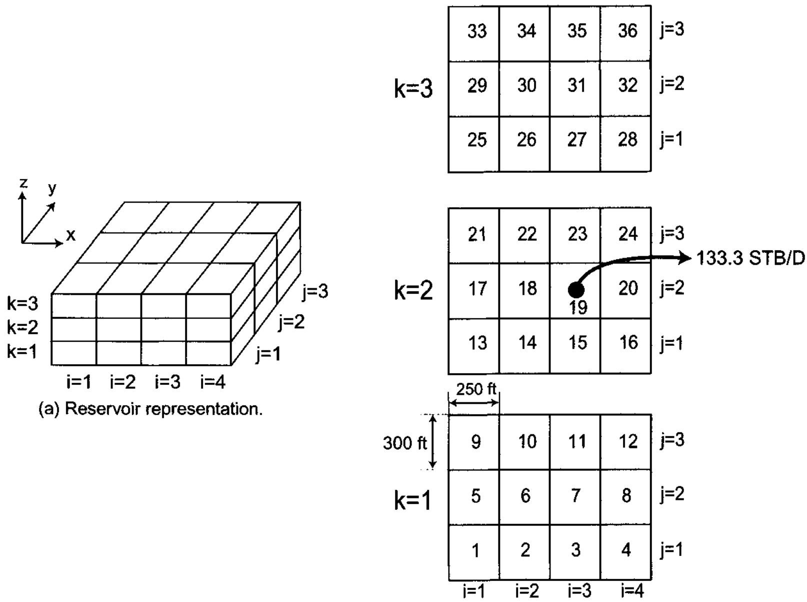
$$\begin{aligned} & (10.3308)(p_3^m - p_7^m) + (18.2574)(p_6^m - p_7^m) + (18.2574)(p_8^m - p_7^m) \\ & + (10.3308)(p_{11}^m - p_7^m) - 400 = \frac{V_{b_7}}{\alpha_c \Delta t}[(\frac{\phi}{B})_7^{n+1} - (\frac{\phi}{B})_7^n]. \end{aligned} \tag{3.39}$$

Eq. 3.39 corresponds to Eq. 2.42 in Example 2.5, which uses engineering notation.

Example 3.4 Consider single-phase fluid flow in the 3D horizontal reservoir in Example 2.6. Write the flow equation for interior block (3,2,2) using CVFD terminology, but this time use natural ordering of blocks as shown in Figure 3–5.

Solution

Block (3,2,2) in Figure 2–15 is block 19 in Figure 3–5. Therefore, $n = 19$. For $n = 19$, $\psi_{x_{19}} = \{18,20\}$, $\psi_{y_{19}} = \{15,23\}$, and $\psi_{z_{19}} = \{7,31\}$.



(a) Reservoir representation.

(b) Natural ordering of blocks.

Figure 3–5 3D reservoir described in Example 3.4.

Substitution into Eq. 3.1a gives

$\psi_{19} = \{18,20\} \cup \{15,23\} \cup \{7,31\} = \{7,15,18,20,23,31\}$.

The application of Eq. 3.17 produces

$$\sum_{l \in \psi_{19}} T_{l,19}^{m}[(p_l^m - p_{19}^m) - \gamma_{l,19}^m (Z_l - Z_{19})] + q_{sc_{19}}^m = \frac{V_{b_{19}}}{\alpha_c \Delta t}[(\frac{\phi}{B})_{19}^{n+1} - (\frac{\phi}{B})_{19}^{n}]. \tag{3.40}$$

This equation can be expanded as

$$\begin{aligned} & T_{7,19}^{m}[(p_7^m - p_{19}^m) - \gamma_{7,19}^m (Z_7 - Z_{19})] + T_{15,19}^{m}[(p_{15}^m - p_{19}^m) - \gamma_{15,19}^m (Z_{15} - Z_{19})] \\ & + T_{18,19}^{m}[(p_{18}^m - p_{19}^m) - \gamma_{18,19}^m (Z_{18} - Z_{19})] + T_{20,19}^{m}[(p_{20}^m - p_{19}^m) - \gamma_{20,19}^m (Z_{20} - Z_{19})] \\ & + T_{23,19}^{m}[(p_{23}^m - p_{19}^m) - \gamma_{23,19}^m (Z_{23} - Z_{19})] + T_{31,19}^{m}[(p_{31}^m - p_{19}^m) - \gamma_{31,19}^m (Z_{31} - Z_{19})] \\ & + q_{sc_{19}}^m = \frac{V_{b_{19}}}{\alpha_c \Delta t}[(\frac{\phi}{B})_{19}^{n+1} - (\frac{\phi}{B})_{19}^{n}]. \end{aligned} \tag{3.41}$$

For block 19, $Z_{15} = Z_{18} = Z_{19} = Z_{20} = Z_{23}$, $Z_7 - Z_{19} = 33.33$ ft,

$Z_{31} - Z_{19} = -33.33$ ft, $q_{sc_{19}}^m = -133.3$ STB/D. Since

$\Delta x_{18,19} = \Delta x_{20,19} = \Delta x = 250$ ft,

$\Delta y_{15,19} = \Delta y_{23,19} = \Delta y = 300$ ft, $\Delta z_{7,19} = \Delta z_{31,19} = \Delta z = 33.33$ ft, and μ , ρ , and B are constants, then

$$\gamma_{7,19}^{m} = \gamma_{31,19}^{m} = \gamma_c \rho g = 0.21584 \times 10^{-3} \times 55 \times 32.174 = 0.3819 \text{ psi/ft},$$

$$T_{18,19}^{m} = T_{20,19}^{m} = \beta_c \frac{k_x A_x}{\mu B \Delta x} = 0.001127 \times \frac{270 \times (300 \times 33.33)}{2 \times 1 \times 250} = 6.0857 \text{ STB/D-psi} \quad \textbf{(3.42)}$$

$$T_{15,19}^{m} = T_{23,19}^{m} = \beta_c \frac{k_y A_y}{\mu B \Delta y} = 0.001127 \times \frac{220 \times (250 \times 33.33)}{2 \times 1 \times 300} = 3.4436 \text{ STB/D-psi} \quad \textbf{(3.43)}$$

$$T_{7,19}^{m} = T_{31,19}^{m} = \beta_c \frac{k_z A_z}{\mu B \Delta z} = 0.001127 \times \frac{50 \times (250 \times 300)}{2 \times 1 \times 33.33} = 63.3944 \text{ STB/D-psi} \quad \textbf{(3.44)}$$

Substitution into Eq. 3.41 gives

$$\begin{aligned}&(63.3944)[(p_7^m - p_{19}^m) - 12.7287] + (3.4436)(p_{15}^m - p_{19}^m) + (6.0857)(p_{18}^m - p_{19}^m)\\&+(6.0857)(p_{20}^m - p_{19}^m) + (3.4436)(p_{23}^m - p_{19}^m) + (63.3944)[(p_{31}^m - p_{19}^m) + 12.7287]\\&-133.3 = \frac{V_{b_{19}}}{\alpha_c \Delta t}[(\frac{\phi}{B})_{19}^{n+1} - (\frac{\phi}{B})_{19}^{n}].\end{aligned} \quad \textbf{(3.45)}$$

Eq. 3.45 corresponds to Eq. 2.47 in Example 2.6, which uses engineering notation.

3.3 Flow Equations in Radial-Cylindrical Coordinates Using CVFD Terminology

The equations presented in Secs. 3.2.1 and 3.2.2 use Cartesian coordinates. The same equations can be made specific to radial-cylindrical coordinates by replacing the directions (and subscripts) x and y with the directions (and subscripts) r and θ, respectively. Table 3–1 lists the corresponding functions for the two coordinate systems. As such, we can obtain the generalized 3D flow equation in the r-θ-z space for block n—termed block (i, j, k) in engineering notation, meaning $n \equiv (i, j, k)$ —from those in the x-y-z space, Eqs. 3.12 through 3.16. Keep in mind that i, j, and k are counting indices in the r direction, θ direction, and z axis, respectively. Therefore, Eq. 3.12 becomes

$$\psi_{r_n} = \{(i-1, j, k), (i+1, j, k)\} \quad \textbf{(3.46a)}$$

$$\psi_{\theta_n} = \{(i, j-1, k), (i, j+1, k)\} \quad \textbf{(3.46b)}$$

$$\psi_{z_n} = \{(i, j, k-1), (i, j, k+1)\} \quad \textbf{(3.46c)}$$

Substitution of Eq. 3.46 into Eq. 3.1b produces

$$\begin{aligned}\psi_n &= \psi_{i,j,k} \\ &= \{(i-1,j,k),(i+1,j,k)\} \cup \{(i,j-1,k),(i,j+1,k)\} \cup \{(i,j,k-1),(i,j,k+1)\} \quad \textbf{(3.47)} \\ &= \{(i,j,k-1),(i,j-1,k),(i-1,j,k),(i+1,j,k),(i,j+1,k),(i,j,k+1)\},\end{aligned}$$

which is identical to Eq. 3.13.

Table 3-1 Functions in Cartesian and Radial-Cylindrical Coordinates

	Function in Cartesian Coordinates	Function in Radial-Cylindrical Coordinates
Coordinate	x	r
	y	θ
	z	z
Transmissibility	T_x	T_r
	T_y	T_θ
	T_z	T_z
Set of neighboring blocks along a direction	ψ_x	ψ_r
	ψ_y	ψ_θ
	ψ_z	ψ_z
Number of blocks along a direction	n_x	n_r
	n_y	n_θ
	n_z	n_z

The flow equation for block (i,j,k), represented by Eq. 3.14a, becomes

$$\begin{aligned}&T^m_{z_{i,j,k-1/2}}[(p^m_{i,j,k-1} - p^m_{i,j,k}) - \gamma^m_{i,j,k-1/2}(Z_{i,j,k-1} - Z_{i,j,k})] \\ &+T^m_{\theta_{i,j-1/2,k}}[(p^m_{i,j-1,k} - p^m_{i,j,k}) - \gamma^m_{i,j-1/2,k}(Z_{i,j-1,k} - Z_{i,j,k})]\end{aligned}$$

$$+T^m_{r_{i-1/2,j,k}}[(p^m_{i-1,j,k} - p^m_{i,j,k}) - \gamma^m_{i-1/2,j,k}(Z_{i-1,j,k} - Z_{i,j,k})]$$
$$+T^m_{r_{i+1/2,j,k}}[(p^m_{i+1,j,k} - p^m_{i,j,k}) - \gamma^m_{i+1/2,j,k}(Z_{i+1,j,k} - Z_{i,j,k})] \quad \textbf{(3.48a)}$$
$$+T^m_{\theta_{i,j+1/2,k}}[(p^m_{i,j+1,k} - p^m_{i,j,k}) - \gamma^m_{i,j+1/2,k}(Z_{i,j+1,k} - Z_{i,j,k})]$$
$$+T^m_{z_{i,j,k+1/2}}[(p^m_{i,j,k+1} - p^m_{i,j,k}) - \gamma^m_{i,j,k+1/2}(Z_{i,j,k+1} - Z_{i,j,k})]$$
$$+q^m_{sc_{i,j,k}} = \frac{V_{b_{i,j,k}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{i,j,k} - (\frac{\phi}{B})^n_{i,j,k}].$$

Eq. 3.14b, the flow equation in CVFD terminology, retains its form:

$$\sum_{l\in\psi_{i,j,k}} T^m_{l,(i,j,k)}[(p^m_l - p^m_{i,j,k}) - \gamma^m_{l,(i,j,k)}(Z_l - Z_{i,j,k})] + q^m_{sc_{i,j,k}} = \frac{V_{b_{i,j,k}}}{\alpha_c \Delta t}[(\frac{\phi}{B})^{n+1}_{i,j,k} - (\frac{\phi}{B})^n_{i,j,k}]. \quad \textbf{(3.48b)}$$

Eq. 3.15 , which defines transmissibilities, becomes

$$T^m_{(i\mp1,j,k),(i,j,k)} = T^m_{(i,j,k),(i\mp1,j,k)} \equiv T^m_{r_{i\mp1/2,j,k}} \quad \textbf{(3.49a)}$$

$$T^m_{(i,j\mp1,k),(i,j,k)} = T^m_{(i,j,k),(i,j\mp1,k)} \equiv T^m_{\theta_{i,j\mp1/2,k}} \quad \textbf{(3.49b)}$$

$$T^m_{(i,j,k\mp1),(i,j,k)} = T^m_{(i,j,k),(i,j,k\mp1)} \equiv T^m_{z_{i,j,k\mp1/2}} \quad \textbf{(3.49c)}$$

Transmissibilities in radial-cylindrical coordinates, $T^m_{r_{i\mp1/2,j,k}}$, $T^m_{\theta_{i,j\mp1/2,k}}$, and $T^m_{z_{i,j,k\mp1/2}}$ are defined by Eq. 2.69. Note that gravity terms, as described by Eq. 3.16, remain intact for both coordinate systems.

$$\gamma^m_{(i\mp1,j,k),(i,j,k)} = \gamma^m_{(i,j,k),(i\mp1,j,k)} \equiv \gamma^m_{i\mp1/2,j,k} \quad \textbf{(3.50a)}$$

$$\gamma^m_{(i,j\mp1,k),(i,j,k)} = \gamma^m_{(i,j,k),(i,j\mp1,k)} \equiv \gamma^m_{i,j\mp1/2,k} \quad \textbf{(3.50b)}$$

$$\gamma^m_{(i,j,k\mp1),(i,j,k)} = \gamma^m_{(i,j,k),(i,j,k\mp1)} \equiv \gamma^m_{i,j,k\mp1/2} \quad \textbf{(3.50c)}$$

For 3D flow in the r-θ-z space, if we desire to obtain the equations in CVFD terminology for block n with the blocks being ordered using natural ordering, we must write the equations that correspond to Eqs. 3.31 through 3.34 with the aid of Table 3–1 and then use Eq. 3.17. The resulting equations are listed below.

$$\psi_{r_n} = \{(n-1),(n+1)\} \quad \textbf{(3.51a)}$$

$$\psi_{\theta_n} = \{(n-n_r),(n+n_r)\} \quad \textbf{(3.51b)}$$

$$\psi_{z_n} = \{(n-n_r n_\theta),(n+n_r n_\theta)\} \quad \textbf{(3.51c)}$$

Substitution of Eq. 3.51 into Eq. 3.1b results in

$$\begin{aligned}\psi_n &= \{(n-1),(n+1)\}\cup\{(n-n_r),(n+n_r)\}\cup\{(n-n_r n_\theta),(n+n_r n_\theta)\} \\ &= \{(n-n_r n_\theta),(n-n_r),(n-1),(n+1),(n+n_r),(n+n_r n_\theta)\}.\end{aligned} \tag{3.52}$$

Now the flow equation for block n in 3D flow can be written again as Eq. 3.17,

$$\sum_{l\in\psi_n} T_{l,n}^m [(p_l^m - p_n^m) - \gamma_{l,n}^m (Z_l - Z_n)] + q_{sc_n}^m = \frac{V_{b_n}}{\alpha_c \Delta t}[(\frac{\phi}{B})_n^{n+1} - (\frac{\phi}{B})_n^n], \tag{3.17}$$

where transmissibility $T_{l,n}^m$ is defined as

$$T_{n\mp1,n}^m = T_{n,n\mp1}^m \equiv T_{r_{i\mp1/2,j,k}}^m \tag{3.53a}$$

$$T_{n\mp n_r,n}^m = T_{n,n\mp n_r}^m \equiv T_{\theta_{i,j\mp1/2,k}}^m \tag{3.53b}$$

$$T_{n\mp n_r n_\theta,n}^m = T_{n,n\mp n_r n_\theta}^m \equiv T_{z_{i,j,k\mp1/2}}^m \tag{3.53c}$$

In addition, fluid gravity $\gamma_{l,n}^m$ is defined as

$$\gamma_{n,n\mp1}^m = \gamma_{n\mp1,n}^m \equiv \gamma_{i\mp1/2,j,k}^m \tag{3.54a}$$

$$\gamma_{n,n\mp n_r}^m = \gamma_{n\mp n_r,n}^m \equiv \gamma_{i,j\mp1/2,k}^m \tag{3.54b}$$

$$\gamma_{n,n\mp n_r n_\theta}^m = \gamma_{n\mp n_r n_\theta,n}^m \equiv \gamma_{i,j,k\mp1/2}^m \tag{3.54c}$$

There are two distinct differences, however, between the flow equations in Cartesian (x-y-z) coordinates and radial-cylindrical (r-θ-z) coordinates. First, while reservoir external boundaries exist along the y axis at $j=1$ and $j=n_y$, there are no external boundaries in the θ direction because the blocks in this direction form a ring of blocks; i.e., block $(i,1,k)$ is preceded by block (i,n_θ,k) and block (i,n_θ,k) is followed by block $(i,1,k)$. Second, any block in Cartesian coordinates is a candidate to host (or contribute to) a well, whereas in radial-cylindrical coordinates only one well penetrates the inner circle of blocks parallel to the z direction and only blocks $(1,j,k)$ are candidates to contribute to this well.

3.4 Flow Equations Using CVFD Terminology in any Block Ordering Scheme

The flow equation using CFVD terminology for block n in any block ordering scheme is given by Eq. 3.17, where ψ_n is expressed by Eq. 3.1. The elements contained in sets ψ_{x_n}, ψ_{y_n}, and ψ_{z_n} are respectively the neighboring blocks of block n along the x axis, y axis, and z axis for Cartesian coordinates, and the elements contained in sets ψ_{r_n}, ψ_{θ_n}, and ψ_{z_n} are respectively the neighboring blocks of block n in the r direction, θ direction, and z axis for radial-cylindrical coordinates. The only difference between one ordering scheme and another is that the blocks in each scheme have different orders. Once reservoir blocks are ordered, the neighboring blocks are defined for each block in the reservoir, and finally the flow equation for any reservoir block can be written. This is in relation to writing the flow equations in a given reservoir; the method of solving the resulting set of equations is however another matter.

3.5 Summary

A flow equation in CVFD terminology has the same form regardless of the dimensionality of the flow problem or the coordinate system; hence, the objective of CVFD terminology is to write flow equations in compact form only. In CVFD terminology the flow equation for block n can be made to describe flow in 1D, 2D, or 3D reservoirs by defining the appropriate set of neighboring blocks (ψ_n). In Cartesian coordinates, Eqs. 3.3, 3.8, and 3.13 define the elements of ψ_n for 1D, 2D, and 3D reservoirs, respectively. Eq. 3.17 gives the flow equation, and transmissibilities and gravities are defined by Eqs. 3.15 and 3.16. Equivalent equations can be written for radial-cylindrical coordinates if subscript x is replaced with subscript r and subscript y is replaced with subscript θ.

3.6 Exercises

3-1 Is 0 the same as { }? If not, how does it differ?

3-2 Write the answers for 2 + 3 and $\{2\} \cup \{3\}$.

3-3 Using your own words, give the physical meanings conveyed by Eqs. 3.2a and 3.2b.

3-4 Consider the 1D reservoir representation in Figure 2–6b. Find ψ_1, ψ_2, ψ_3, and ψ_4.

3-5 Consider the 2D reservoir representation in Figure 3–4. Find ψ_n for $n = 1, 2, 3, \ldots 12$.

3-6 Consider the 3D reservoir representation in Figure 2–8c. Find ψ_n for $n = 1, 2, 3, \ldots 36$.

3-7 Consider the 3D reservoir representation in Figure 2–8b. Find $\psi_{(1,1,1)}$, $\psi_{(2,2,1)}$, $\psi_{(3,2,2)}$, $\psi_{(4,3,2)}$, $\psi_{(4,1,3)}$, $\psi_{(3,2,3)}$, and $\psi_{(1,3,3)}$.

3-8 Using the definitions of ψ_n, ψ_{x_n}, ψ_{y_n}, and ψ_{z_n} along with the aid of Figure 3–3c, prove that $\psi_n = \psi_{x_n} \cup \psi_{y_n} \cup \psi_{z_n}$.

3-9 Consider fluid flow in a 1D horizontal reservoir along the x axis. The reservoir left and right boundaries are closed to fluid flow. The reservoir consists of three blocks as shown in Figure 3–6.

(a) Write the appropriate flow equation for a general block n in this reservoir.
(b) Write the flow equation for block 1 by finding ψ_1 and then using it to expand the equation in (a).
(c) Write the flow equation for block 2 by finding ψ_2 and then using it to expand the equation in (a).
(d) Write the flow equation for block 3 by finding ψ_3 and then using it to expand the equation in (a).

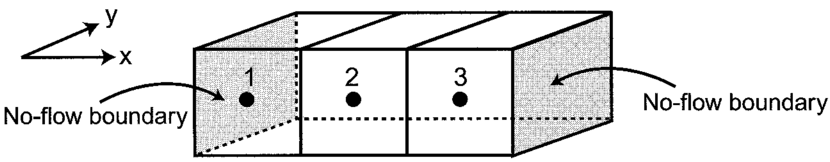


Figure 3–6 1D reservoir for Exercise 3-9.

3-10 Consider fluid flow in a 2D, horizontal, closed reservoir. The reservoir consists of nine blocks as shown in Figure 3–7.

(a) Write the appropriate flow equation for a general block n in this reservoir.
(b) Write the flow equation for block 1 by finding ψ_1 and then using it to expand the equation in (a).
(c) Write the flow equation for block 2 by finding ψ_2 and then using it to expand the equation in (a).
(d) Write the flow equation for block 4 by finding ψ_4 and then using it to expand the equation in (a).
(e) Write the flow equation for block 5 by finding ψ_5 and then using it to expand the equation in (a).

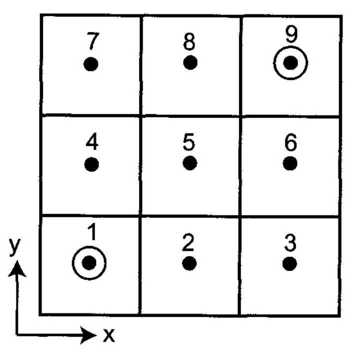


Figure 3–7 2D reservoir for Exercise 3-10.

3-11 A 2D oil reservoir is discretized into 4×4 blocks.

(a) Order the blocks in this reservoir using the natural ordering scheme, letting block 1 be the lower left corner block.

(b) Write the flow equation for each interior block in this reservoir.

3-12 A 2D oil reservoir is discretized into 4×4 blocks.

(a) Order the blocks in this reservoir using the D4 ordering scheme, letting block 1 be the lower left corner block.

(b) Write the flow equation for each interior block in this reservoir.

3-13 A single-phase oil reservoir is described by four equal blocks as shown in Figure 3–8. The reservoir is horizontal and has homogeneous and isotropic rock properties, $k = 150$ md and $\phi = 0.21$. Block dimensions are $\Delta x = 400$ ft, $\Delta y = 600$ ft, and $h = 25$ ft. Oil properties are $B = 1$ RB/STB and $\mu = 5$ cp. The pressures of blocks 1 and 4 are 2200 psia and 900 psia, respectively. Block 3 hosts a well that produces oil at a rate of 100 STB/D. Find the pressure distribution in the reservoir assuming that the reservoir rock and oil are incompressible.

3-14 A single-phase oil reservoir is described by five equal blocks as shown in Figure 3–9. The reservoir is horizontal and has $k = 90$ md and $\phi = 0.17$. Block dimensions are $\Delta x = 500$ ft, $\Delta y = 900$ ft, and $h = 45$ ft. Oil properties are $B = 1$ RB/STB and $\mu = 3$ cp. The pressures of blocks 1 and 5 are maintained at 2700 psia and 1200 psia, respectively. Gridblock 4 hosts a well that produces oil at a rate of 325 STB/D. Find the pressure distribution in the reservoir assuming that the reservoir oil and rock are incompressible.

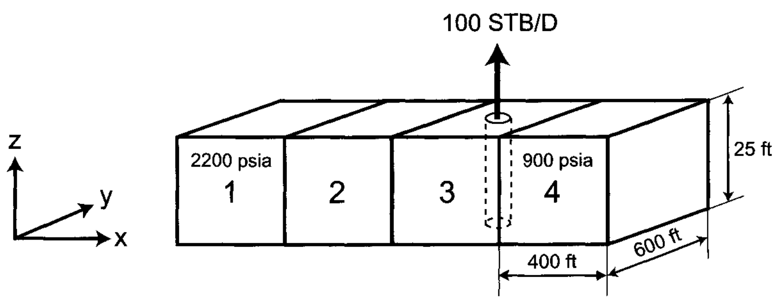


Figure 3–8 1D reservoir representation in Exercise 3-13.

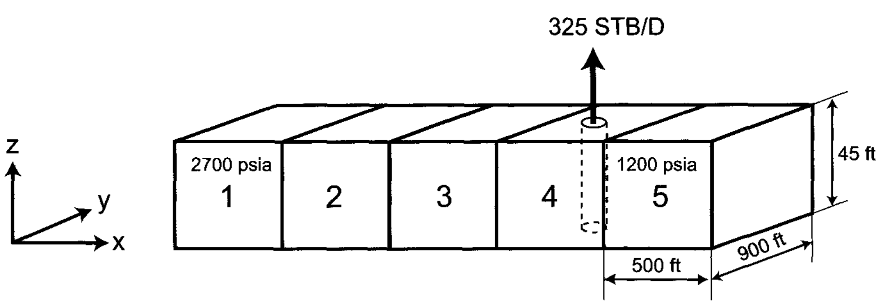


Figure 3–9 1D reservoir representation in Exercise 3-14.

3-15 Consider single-phase flow of oil in a 2D horizontal reservoir. The reservoir is discretized using 4×4 equal blocks as shown in Figure 3–10. Block (2,3) hosts a well that produces 500 STB/D of oil. All blocks have $\Delta x = \Delta y = 330$ ft, $h = 50$ ft, and $k_x = k_y = 210$ md. The oil FVF and viscosity are 1.0 RB/B and 2 cp, respectively. The pressures of reservoir boundary blocks are specified in Figure 3–10. Assuming that the reservoir oil and rock are incompressible, calculate the pressures of blocks (2,2), (3,2), (2,3), and (3,3).

3-16 Consider single-phase flow of oil in a 2D horizontal reservoir. The reservoir is discretized using 4×4 equal blocks as shown in Figure 3–11. Both blocks 6 and 11 host a well that produces oil at the rate shown in the figure. All blocks have $\Delta x = 200$ ft, $\Delta y = 250$ ft, $h = 60$ ft, $k_x = 80$ md, and $k_y = 65$ md. The oil FVF and viscosity are 1.0 RB/STB and 2 cp, respectively. The pressures of reservoir boundary blocks are specified in Figure 3–11. Assuming that the reservoir oil and rock are incompressible, calculate the pressures of blocks 6, 7, 10, and 11.

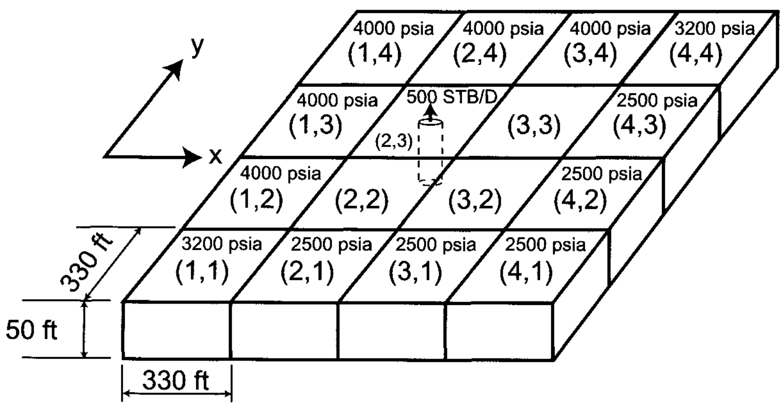


Figure 3–10 2D reservoir representation in Exercise 3-15.

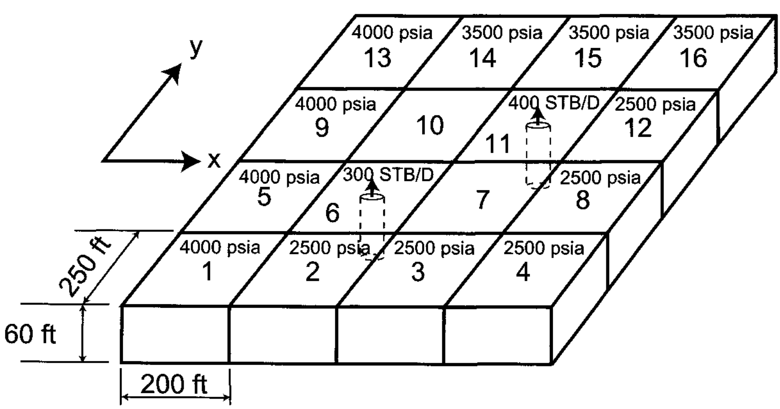


Figure 3–11 2D reservoir representation in Exercise 3-16.