Linearization of Flow Equations

8.1 Introduction

The flow equations presented in Chapter 7 are generally nonlinear. To obtain the pressure distribution in the reservoir, these equations are linearized in order to use linear equation solvers. In this chapter, we aim at obtaining the linearized flow equation for an arbitrary gridblock (or gridpoint). To achieve this objective, we identify the nonlinear terms in the flow equations, present methods of linearizing these terms in space and time, and subsequently present the linearized flow equation for single-phase flow problems. In order to simplify the presentation of concepts, we use the implicit formulation of the 1D flow equation in the x direction and use a block-centered grid in discretizing the reservoir. We first discuss the incompressible fluid flow equation that exhibits linearity, then the implicit formulation for the slightly compressible fluid flow equation that exhibits very weak nonlinearity, and finally the implicit formulation for the compressible fluid flow equation that exhibits a higher degree of nonlinearity. Although single-phase flow equations exhibit different degrees of nonlinearity, these equations are usually classified as having weak nonlinearities.

8.2 Nonlinear Terms in Flow Equations

The terms composing any flow equation include interblock flow terms, the accumulation term, the well production rate term, and fictitious well rate terms reflecting flow across reservoir boundaries for boundary blocks. The number of interblock flow terms equals the number of all the existing neighboring blocks. The number of fictitious well rate terms equals the number of block boundaries that fall on reservoir boundaries. For any boundary block, the number of existing neighboring blocks and the number of fictitious wells always add up to two, four, or six for 1D, 2D, or 3D flow, respectively. In single-phase flow problems, if the coefficients of unknown block pressures in the flow equation depend on block pressure, the algebraic equation is termed nonlinear; otherwise, the equation is linear. Therefore, the terms that may exhibit pressure dependence include transmissibilities, the well production rate, fictitious well rates, and the coefficient of block pressure in the accumulation term. This is true for equations in the mathematical approach. In the engineering approach, however, interblock flow terms, the well production rate, and fictitious well rates receive the same treatment, i.e., block pressures contributing to flow potential (the pressure difference) in any term are treated implicitly as demonstrated in Chapter 7. Therefore, the nonlinear terms include transmissibilities in interblock flow terms and fictitious well rates, the coefficient of pressure drop in the well production rate term, and the coefficient of block pressure difference in the accumulation term.

8.3 Nonlinearity of Flow Equations For Various Fluids

In this section, we examine the nonlinearity of the flow equations for slightly compressible and compressible fluids. The flow equation for incompressible fluids is linear. We examine the pressure dependence of the various terms in a flow equation, namely, the interblock flow terms, the accumulation term, the well production rate term, and the fictitious well rate terms.

8.3.1 Linearity of the Incompressible Fluid Flow Equation

The 1D flow equation in the x direction for an incompressible fluid can be obtained from Eq. 7.16a, which states

$$\sum_{l\in\psi_n} T_{l,n}[(p_l - p_n) - \gamma_{l,n}(Z_l - Z_n)] + \sum_{l\in\xi_n} q_{sc_{l,n}} + q_{sc_n} = 0 \tag{8.1}$$

where $\psi_n = \{n-1, n+1\}$, $\xi_n = \{\}$, $\{b_W\}$, or $\{b_E\}$, and $n = 1, 2, 3, \ldots n_x$

For gridblock 1,

$$T_{x_{1+1/2}}[(p_2 - p_1) - \gamma_{1+1/2}(Z_2 - Z_1)] + q_{sc_{b_W,1}} + q_{sc_1} = 0 \tag{8.2a}$$

For gridblock $i = 2, 3, \ldots n_x - 1$,

$$T_{x_{i-1/2}}[(p_{i-1} - p_i) - \gamma_{i-1/2}(Z_{i-1} - Z_i)] + T_{x_{i+1/2}}[(p_{i+1} - p_i) - \gamma_{i+1/2}(Z_{i+1} - Z_i)] + q_{sc_i} = 0 \tag{8.2b}$$

For gridblock n_x,

$$T_{x_{n_x-1/2}}[(p_{n_x-1} - p_{n_x}) - \gamma_{n_x-1/2}(Z_{n_x-1} - Z_{n_x})] + q_{sc_{b_E,n_x}} + q_{sc_{n_x}} = 0 \tag{8.2c}$$

Transmissibility $T_{x_{i\mp1/2}}$ is expressed as Eq. 2.39a,

$$T_{x_{i\mp1/2}} = \left(\beta_c \frac{k_x A_x}{\mu B \Delta x}\right)\bigg|_{x_{i\mp1/2}} = G_{x_{i\mp1/2}}\left(\frac{1}{\mu B}\right)_{x_{i\mp1/2}} \tag{8.3a}$$

geometric factor $G_{x_{i\mp1/2}}$ is defined in Table 4–1 for a block-centered grid,

$$G_{x_{i\mp1/2}} = \frac{2\beta_c}{\Delta x_i /(A_{x_i} k_{x_i}) + \Delta x_{i\mp1} /(A_{x_{i\mp1}} k_{x_{i\mp1}})} \tag{8.4}$$

the well production rate (q_{sc_i}) is estimated according to the well operating condition as discussed in Chapter 6, and fictitious well rates ($q_{sc_{b_W,1}}, q_{sc_{b_E,n_x}}$) are estimated according to the type of boundary condition as discussed in Chapter 4. Note that $T_{x_{i\mp1/2}}$ and $G_{x_{i\mp1/2}}$ are functions of the space between gridblocks i and $i\mp1$ only. It should be mentioned that a numerical value for the well production rate could be calculated for well operating conditions other than a specified FBHP. Similarly, a numerical value for a fictitious well flow rate can be calculated for boundary conditions other than a specified pressure boundary. In such cases, both the well production rate and fictitious well rate are known quantities and, as a result, can be moved to the RHS of the flow equation (Eq. 8.2). Otherwise, the well production rate and fictitious well rate are functions of block pressure (p_i) and, as a result, part of the rate equations appear in the coefficient of p_i and the other part has to be moved to the RHS of the flow equation (Eq. 8.2). The FVF, viscosity, and gravity of an incompressible fluid are not functions of pressure. Therefore, transmissibilities and gravity are not functions of pressure; consequently, Eq. 8.2 represents a system of n_x linear algebraic equations. This system of linear equations can be solved for the unknown pressures ($p_1, p_2, p_3, ... p_{n_x}$) by the algorithm presented in Section 7.3.1.1.

8.3.2 Nonlinearity of the Slightly Compressible Fluid Flow Equation

The implicit flow equation for a slightly compressible fluid is expressed as Eq. 7.81a,

$$\sum_{l \in \psi_n} T_{l,n}^{n+1}[(p_l^{n+1} - p_n^{n+1}) - \gamma_{l,n}^n (Z_l - Z_n)] + \sum_{l \in \xi_n} q_{sc_{l,n}}^{n+1} + q_{sc_n}^{n+1} = \frac{V_{b_n} \phi_n^{\circ} (c + c_\phi)}{\alpha_c B^{\circ} \Delta t}[p_n^{n+1} - p_n^n] \tag{8.5}$$

where the FVF, viscosity, and density are described by Eqs. 7.5 through 7.7,

$$B = \frac{B^{\circ}}{[1 + c(p - p^{\circ})]} \tag{8.6}$$

$$\mu = \frac{\mu^{\circ}}{[1 - c_\mu (p - p^{\circ})]} \tag{8.7}$$

$$\rho = \rho^{\circ}[1 + c(p - p^{\circ})] \tag{8.8}$$

The numerical values of c and c_μ for slightly compressible fluids are in the order of magnitude of 10^{-6} to 10^{-5}. Consequently, the effect of pressure variation on the FVF, viscosity, and gravity can be neglected without introducing noticeable errors. Simply stated $B \cong B^{\circ}$, $\mu \cong \mu^{\circ}$, and $\rho \cong \rho^{\circ}$ and, in turn, transmissibilities and gravity are independent of pressure (i.e., $T_{l,n}^{n+1} \cong T_{l,n}$ and $\gamma_{l,n}^n \cong \gamma_{l,n}$). Therefore, Eq. 8.5 simplifies to

$$\sum_{l \in \psi_n} T_{l,n}[(p_l^{n+1} - p_n^{n+1}) - \gamma_{l,n} (Z_l - Z_n)] + \sum_{l \in \xi_n} q_{sc_{l,n}}^{n+1} + q_{sc_n}^{n+1} = \frac{V_{b_n} \phi_n^{\circ} (c + c_\phi)}{\alpha_c B^{\circ} \Delta t}[p_n^{n+1} - p_n^n] \tag{8.9}$$

Eq. 8.9 is a linear algebraic equation because the coefficients of the unknown pressures at time level $n+1$ are independent of pressure.

The 1D flow equation in the x direction for a slightly compressible fluid is obtained from Eq. 8.9 in the same way that was described in the previous section.

For gridblock 1,

$$T_{x_{1+1/2}}[(p_2^{n+1}-p_1^{n+1})-\gamma_{1+1/2}(Z_2-Z_1)]+q_{sc_{bW},1}^{n+1}+q_{sc_1}^{n+1}=\frac{V_{b_1}\phi_1^{\circ}(c+c_\phi)}{\alpha_c B^{\circ}\Delta t}[p_1^{n+1}-p_1^n] \quad \textbf{(8.10a)}$$

For gridblock $i=2,3,...n_x-1$,

$$T_{x_{i-1/2}}[(p_{i-1}^{n+1}-p_i^{n+1})-\gamma_{i-1/2}(Z_{i-1}-Z_i)]+T_{x_{i+1/2}}[(p_{i+1}^{n+1}-p_i^{n+1})-\gamma_{i+1/2}(Z_{i+1}-Z_i)]$$
$$+q_{sc_i}^{n+1}=\frac{V_{b_i}\phi_i^{\circ}(c+c_\phi)}{\alpha_c B^{\circ}\Delta t}[p_i^{n+1}-p_i^n]. \quad \textbf{(8.10b)}$$

For gridblock n_x,

$$T_{x_{n_x-1/2}}[(p_{n_x-1}^{n+1}-p_{n_x}^{n+1})-\gamma_{n_x-1/2}(Z_{n_x-1}-Z_{n_x})]+q_{sc_{bE},n_x}^{n+1}+q_{sc_{n_x}}^{n+1}$$
$$=\frac{V_{b_{n_x}}\phi_{n_x}^{\circ}(c+c_\phi)}{\alpha_c B^{\circ}\Delta t}[p_{n_x}^{n+1}-p_{n_x}^n]. \quad \textbf{(8.10c)}$$

In the above equation, $T_{x_{i\mp1/2}}$ and $G_{x_{i\mp1/2}}$ for a block-centered grid are defined by Eqs. 8.3a and 8.4,

$$T_{x_{i\mp1/2}}=\left(\beta_c\frac{k_xA_x}{\mu B\Delta x}\right)\Bigg|_{x_{i\mp1/2}}=G_{x_{i\mp1/2}}\left(\frac{1}{\mu B}\right)_{x_{i\mp1/2}} \quad \textbf{(8.3a)}$$

$$G_{x_{i\mp1/2}}=\frac{2\beta_c}{\Delta x_i/(A_{x_i}k_{x_i})+\Delta x_{i\mp1}/(A_{x_{i\mp1}}k_{x_{i\mp1}})} \quad \textbf{(8.4)}$$

Here again, the well production rate ($q_{sc_i}^{n+1}$) and fictitious well rates ($q_{sc_{bW},1}^{n+1}, q_{sc_{bE},n_x}^{n+1}$) are handled in exactly the same way as discussed in the previous section. The resulting set of n_x linear algebraic equations can be solved for the unknown pressures ($p_1^{n+1}, p_2^{n+1}, p_3^{n+1}, ...p_{n_x}^{n+1}$) by the algorithm presented in Section 7.3.2.2.

Although both Eqs. 8.2 and 8.10 represent a set of linear algebraic equations, there is a basic difference between them. While in Eq. 8.2 reservoir pressure depends on space (location) only, in Eq. 8.10 reservoir pressure depends on both the space and time. The implication of this difference is that the flow equation for an incompressible fluid (Eq. 8.2) has a steady-state solution (i.e., a solution that is independent of time), whereas the flow equation for a slightly compressible fluid (Eq. 8.10) has an unsteady-state solution (i.e., a solution that is dependent on time). It should be mentioned that the pressure solution for Eq. 8.10 at any time step is obtained without iteration because the equation is linear.

We must reiterate that the linearity of Eq. 8.9 is the result of neglecting the pressure dependence of FVF and viscosity in transmissibility, the well production rate, and the fictitious well rates on the LHS of Eq. 8.5. If Eqs. 8.6 and 8.7 are used to reflect such pressure dependence, the resulting flow equation becomes nonlinear. In conclusion, understanding the behavior of fluid properties has led to devising a practical way of linearizing the flow equation for a slightly compressible fluid.

8.3.3 Nonlinearity of the Compressible Fluid Flow Equation

The implicit flow equation for a compressible fluid is expressed as Eq. 7.162a,

$$\sum_{l\in\psi_n} T_{l,n}^{n+1}[(p_l^{n+1}-p_n^{n+1})-\gamma_{l,n}^n(Z_l-Z_n)]+\sum_{l\in\xi_n} q_{sc_{l,n}}^{n+1}+q_{sc_n}^{n+1}=\frac{V_{b_n}}{\alpha_c\Delta t}\left(\frac{\phi}{B_g}\right)_n'[p_n^{n+1}-p_n^n] \quad \textbf{(8.11)}$$

The pressure dependence of density is expressed as Eq. 7.9,

$$\rho_g=\frac{\rho_{sc}}{\alpha_c B_g} \quad \textbf{(8.12)}$$

in addition, gas FVF and viscosity are presented in a tabular form as functions of pressure at reservoir temperature,

$$B_g=\mathrm{f}(p) \quad \textbf{(8.13)}$$

$$\mu_g=\mathrm{f}(p) \quad \textbf{(8.14)}$$

As mentioned in Chapter 7, the density and viscosity of a compressible fluid increase as pressure increases but tend to level off at high pressures. The FVF decreases orders of magnitude as the pressure increases from low pressure to high pressure. Consequently, interblock transmissibilities, gas gravity, the coefficient of pressure in accumulation term, well production, and transmissibility in fictitious well terms are all functions of unknown block pressures. Therefore, Eq. 8.11 is nonlinear. The solution of this equation requires linearization of nonlinear terms in both space and time.

The 1D flow equation in the x direction for a compressible fluid can be obtained from Eq. 8.11 in the same way that was described in Section 8.3.1.

For gridblock 1,

$$T_{x_{1+1/2}}^{n+1}[(p_2^{n+1}-p_1^{n+1})-\gamma_{1+1/2}^n(Z_2-Z_1)]+q_{sc_{bW,1}}^{n+1}+q_{sc_1}^{n+1}=\frac{V_{b_1}}{\alpha_c\Delta t}\left(\frac{\phi}{B_g}\right)_1'[p_1^{n+1}-p_1^n] \quad \textbf{(8.15a)}$$

For gridblock $i=2,3,\ldots n_x-1$,

$$T_{x_{i-1/2}}^{n+1}[(p_{i-1}^{n+1}-p_i^{n+1})-\gamma_{i-1/2}^n(Z_{i-1}-Z_i)]+T_{x_{i+1/2}}^{n+1}[(p_{i+1}^{n+1}-p_i^{n+1})-\gamma_{i+1/2}^n(Z_{i+1}-Z_i)]$$
$$+q_{sc_i}^{n+1}=\frac{V_{b_i}}{\alpha_c\Delta t}(\frac{\phi}{B_g})_i'[p_i^{n+1}-p_i^n]. \tag{8.15b}$$

For gridblock n_x,

$$T_{x_{n_x-1/2}}^{n+1}[(p_{n_x-1}^{n+1}-p_{n_x}^{n+1})-\gamma_{n_x-1/2}^n(Z_{n_x-1}-Z_{n_x})]+q_{sc_{b_E,n_x}}^{n+1}+q_{sc_{n_x}}^{n+1}$$
$$=\frac{V_{b_{n_x}}}{\alpha_c\Delta t}(\frac{\phi}{B_g})_{n_x}'[p_{n_x}^{n+1}-p_{n_x}^n]. \tag{8.15c}$$

In the above equation, $T_{x_{i\mp1/2}}^{n+1}$ and $G_{x_{i\mp1/2}}$ for a block-centered grid are defined by Eqs. 8.3b and 8.4,

$$T_{x_{i\mp1/2}}^{n+1}=(\beta_c\frac{k_xA_x}{\mu B\Delta x})\Big|_{x_{i\mp1/2}}^{n+1}=G_{x_{i\mp1/2}}(\frac{1}{\mu B})_{x_{i\mp1/2}}^{n+1} \tag{8.3b}$$

$$G_{x_{i\mp1/2}}=\frac{2\beta_c}{\Delta x_i/(A_{x_i}k_{x_i})+\Delta x_{i\mp1}/(A_{x_{i\mp1}}k_{x_{i\mp1}})} \tag{8.4}$$

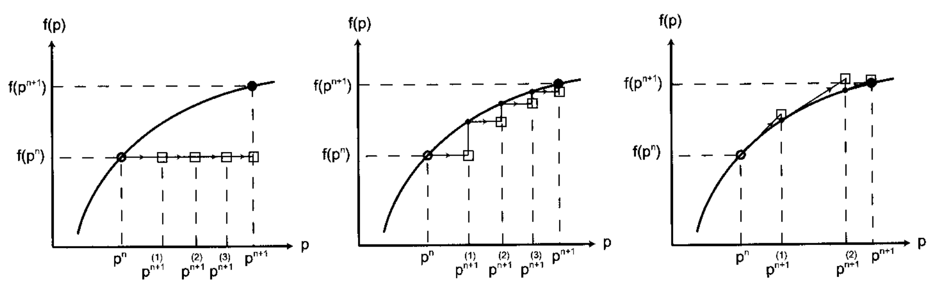
where B and μ stand for B_g and μ_g, respectively.

Here again, the well production rate ($q_{sc_i}^{n+1}$) and fictitious well rates ($q_{sc_{bW},1}^{n+1}$, q_{sc_{bE},n_x}^{n+1}) are handled in exactly the same way as discussed in Section 8.3.1. In addition, interblock transmissibility (Eq. 8.3b) is a function of the space between gridblocks i and $i\mp1$ and time. The resulting set of n_x nonlinear algebraic equations has to be linearized prior to being solved for the unknown pressures ($p_1^{n+1}, p_2^{n+1}, p_3^{n+1}, \ldots p_{n_x}^{n+1}$). The algorithm outlined in Section 7.3.3.2 uses simple iteration on transmissibility to linearize flow equations. This essentially involves transmissibility values being used from n th time step. The following section presents other methods of linearization. It should be mentioned that even though the solutions of Eqs. 8.10 and 8.15 are time-dependent, the solution of Eq. 8.10 requires no iteration because of the linearity of the equation, while the solution of Eq. 8.15 requires iteration to remove the nonlinearity due to time. In addition, while the pressure coefficients in Eq. 8.10 are constant (i.e., they do not change from one time step to another), the pressure coefficients in Eq. 8.15 are not constant and need to be updated at least once at the beginning of each time step.

8.4 Linearization of Nonlinear Terms

In this section, we present the various methods used to treat nonlinearities. Although the methods of linearization presented here may not be required because nonlinearities in single-phase flow are weak, these linearization methods are needed for the simulation of multiphase flow in petroleum reservoirs that is presented in Chapter 10. Nonlinear terms have to be approximated in both space and time. Linearization in space defines the location where the nonlinearity is to be evaluated and which reservoir blocks should be used in its estimation. Linearization in time implies how the term is approximated in order to reflect its value at the current time level where the pressure solution is unknown. Figure 8–1 sketches three commonly used linearization methods as they apply to a nonlinearity (f) that is a function of one variable (p): (a) the explicit method (Figure 8–1a), (b) the simple-iteration method (Figure 8–1b), and (c) the fully-implicit method (Figure 8–1c).

Each figure shows the improvements in the linearized value of the nonlinearity as iteration progresses from the first iteration $(\nu = 0)$ to the second iteration $(\nu = 1)$ and so on until the pressure converges to p^{n+1}. Iteration on pressure in the case of a compressible fluid only is necessary in order to satisfy the material balance and remove the nonlinearity of the accumulation term due to time. In Figure 8–1, the value of the nonlinearity at time level n (the beginning of the time step) is represented by an empty circle, its value at time level n+1 (after reaching convergence) is represented by a solid circle, and its value at any iteration is represented by an empty square at that iteration. Note that the explicit method, sketched in Figure 8–1a, does not provide for any improvement in the value of the nonlinearity as iteration progresses. The simple-iteration method, sketched in Figure 8–1b, provides for improvement in the value of nonlinearity in step-wise fashion. In the fully implicit treatment, presented Figure 8–1c, the improved value of the nonlinearity, as iteration progresses, falls on the tangent of the nonlinearity at the previous iteration. Other linearization methods, such as the linearized–implicit method (MacDonald and Coats 1970) and the semi-implicit method of Nolen and Berry (1972), are not applicable to single-phase flow. They are used in multiphase flow to deal with nonlinearities due to fluid saturation only. The treatments of the various nonlinear terms that appear in single-phase flow equations are presented in Sections 8.4.1 to 8.4.4.



(a) Explicit linearization. (b) Simple-iteration linearization. (c) Fully implicit linearization.

Figure 8–1 Convergence of different methods of linearization.

8.4.1 Linearization of Transmissibilities

Transmissibilities at time level n+1 are expressed by Eq. 8.3b,

$$T_{x_{i\mp1/2}}^{n+1} = (\beta_c \frac{k_x A_x}{\mu B \Delta x})\Big|_{x_{i\mp1/2}}^{n+1} = G_{x_{i\mp1/2}} (\frac{1}{\mu B})_{x_{i\mp1/2}}^{n+1} = G_{x_{i\mp1/2}} f_{p_{i\mp1/2}}^{n+1} \tag{8.16}$$

where $G_{x_{i\mp1/2}}$ is defined by Eq. 8.4 for a block-centered grid and $f_{p_{i\mp1/2}}^{n+1}$ is defined as

$$f_{p_{i\mp1/2}}^{n+1} = (\frac{1}{\mu B})_{x_{i\mp1/2}}^{n+1} \tag{8.17}$$

Therefore, linearization of transmissibility reduces to linearization of $f_{p_{i\mp1/2}}^{n+1}$. The function f_p is evaluated at block boundaries $x_{i\mp1/2}$ and at time level $n+1$, where the pressure solution is not known. Therefore, f_p needs to be expressed as a function of the pressure of the blocks on both sides of the specific block boundary and at some known time. These approximations are termed linearization in space and linearization in time.

8.4.1.1 Linearization of f_p in Space

There are several methods used to approximate f_p in space.

With single-point upstream weighting,

$$f_{p_{i\mp1/2}} = f_{p_i} \tag{8.18a}$$

if block i is upstream to block $i\mp1$, or

$$f_{p_{i\mp1/2}} = f_{p_{i\mp1}} \tag{8.18b}$$

if block i is downstream to block $i\mp1$. The potential difference between blocks i and $i\mp1$ is used to determine the upstream and downstream blocks.

With average function value weighting,

$$f_{p_{i\mp1/2}} = \overline{f} = \tfrac{1}{2}(f_{p_i} + f_{p_{i\mp1}}) \tag{8.19}$$

With average pressure value weighting,

$$f_{p_{i\mp1/2}} = f(\overline{p}) = 1/\mu(\overline{p})B(\overline{p}) \tag{8.20}$$

where

$$\overline{p} = \tfrac{1}{2}(p_i + p_{i\mp1}). \tag{8.21}$$

With average function components value weighting,

$$f_{p_{i\mp1/2}} = f(\overline{p}) = 1/\overline{\mu}\overline{B} \tag{8.22}$$

where

$$\bar{\mu} = \frac{\mu(p_i) + \mu(p_{i\mp1})}{2} \tag{8.23}$$

$$\bar{B} = \frac{B(p_i) + B(p_{i\mp1})}{2} \tag{8.24}$$

Once f_p is linearized in space as in Eqs. 8.18 through 8.24, then the space-linearized transmissibility is obtained by applying Eq. 8.16,

$$T_{x_{i\mp1/2}} = G_{x_{i\mp1/2}} f_{p_{i\mp1/2}} \tag{8.25}$$

8.4.1.2 Linearization of f_p in Time

The effect of the nonlinearity of f_p on the stability of the solution depends on the magnitude of the pressure change over a time step. The methods of time linearization presented earlier in Figure 8–1 may be used to approximate f_p in time. Note that f_p is a function of the pressures of the blocks that surround a block boundary as mentioned in the previous section; i.e., $f_p = \mathrm{f}(p_i, p_{i\mp1})$.

With the explicit method (see Figure 8–1a), the nonlinearity is evaluated at the beginning of the time step (at time level n),

$$f_{p_{i\mp1/2}}^{n+1} \cong f_{p_{i\mp1/2}}^{n} = \mathrm{f}(p_i^n, p_{i\mp1}^n) \tag{8.26}$$

With the simple-iteration method (see Figure 8–1b), the nonlinearity is evaluated one iteration behind the pressure solution,

$$f_{p_{i\mp1/2}}^{n+1} \cong \overset{(\nu)}{f}{}_{p_{i\mp1/2}}^{n+1} = \mathrm{f}(\overset{(\nu)}{p}{}_i^{n+1}, \overset{(\nu)}{p}{}_{i\mp1}^{n+1}) \, . \tag{8.27}$$

With the fully implicit method (see Figure 8–1c), the nonlinearity is approximated by its value at iteration level ν plus a term that depends on the rate of change of pressure over iteration,

$$\begin{aligned} f_{p_{i\mp1/2}}^{n+1} &\cong \overset{(\nu+1)}{f}{}_{p_{i\mp1/2}}^{n+1} \cong \mathrm{f}(\overset{(\nu)}{p}{}_i^{n+1}, \overset{(\nu)}{p}{}_{i\mp1}^{n+1}) + \left.\frac{\partial \mathrm{f}(p_i, p_{i\mp1})}{\partial p_i}\right|^{\overset{(\nu)}{n+1}} (\overset{(\nu+1)}{p}{}_i^{n+1} - \overset{(\nu)}{p}{}_i^{n+1}) \\ &+ \left.\frac{\partial \mathrm{f}(p_i, p_{i\mp1})}{\partial p_{i\mp1}}\right|^{\overset{(\nu)}{n+1}} (\overset{(\nu+1)}{p}{}_{i\mp1}^{n+1} - \overset{(\nu)}{p}{}_{i\mp1}^{n+1}). \end{aligned} \tag{8.28}$$

Once f_p is linearized in time as in Eqs. 8.26 through 8.28, then the time-linearized transmissibility is obtained by applying Eq. 8.16,

$$T_{x_{i\mp1/2}}^{n+1} = G_{x_{i\mp1/2}} f_{p_{i\mp1/2}}^{n+1} \tag{8.29}$$

8.4.2 Linearization of Well Production Rates

A wellblock production rate is evaluated in space at the gridblock (or gridpoint) for which the flow equation is written. Linearization in time of the wellblock production rate involves first linearizing the wellblock production rate equation and then substituting the result in the linearized flow equation for the wellblock. This method of linearization, which is usually used in reservoir simulation, parallels the linearization of transmissibility.

8.4.2.1 Well Production Rate Linearization in the Mathematical Approach

The following methods may be used to approximate the wellblock production rate in time. With the explicit method (see Figure 8–1a),

$$q_{sc_i}^{n+1} \cong q_{sc_i}^{n} \tag{8.30a}$$

With the simple-iteration method (see Figure 8–1b),

$$q_{sc_i}^{n+1} \cong q_{sc_i}^{\overset{(\nu)}{n+1}} \tag{8.31a}$$

With the fully-implicit method (see Figure 8–1c),

$$q_{sc_i}^{n+1} \cong q_{sc_i}^{\overset{(\nu+1)}{n+1}} \cong q_{sc_i}^{\overset{(\nu)}{n+1}} + \left.\frac{dq_{sc_i}}{dp_i}\right|^{\overset{(\nu)}{n+1}} (p_i^{\overset{(\nu+1)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) \tag{8.32}$$

Another method of wellblock production rate linearization in time involves substituting the appropriate wellblock production rate equation given in Chapter 6 into the flow equation for the wellblock prior to linearization and subsequently linearizing the resulting flow equation. That is to say, the wellblock production rate and interblock flow terms receive identical linearization treatments. For a well operating under a FBHP specification, this method results in the implicit treatment of wellblock pressure compared to the explicit treatments provided by the explicit method (Eq. 8.30a) and simple-iteration method (Eq. 8.31a). This method of linearization is identical to the linearization method used in the engineering approach and is presented in the next section.

8.4.2.2 Well Production Rate Linearization in the Engineering Approach

The following methods may be used to approximate a wellblock production rate in time. With the explicit transmissibility method,

$$q_{sc_i}^{n+1} \cong -G_{w_i} (\frac{1}{B\mu})_i^n (p_i^{n+1} - p_{wf_i}^n) \tag{8.30b}$$

With the simple iteration on transmissibility method,

$$q_{sc_i}^{n+1} \cong -G_{w_i} (\frac{1}{B\mu})_i^{\overset{(\nu)}{n+1}} (p_i^{n+1} - p_{wf_i}^n) \tag{8.31b}$$

With the fully-implicit method,

$$q_{sc_i}^{n+1} \cong \overset{(v+1)}{q_{sc_i}^{n+1}} \cong \overset{(v)}{q_{sc_i}^{n+1}} + \left.\frac{dq_{sc_i}}{dp_i}\right|^{\overset{(v)}{n+1}} (\overset{(v+1)}{p_i^{n+1}} - \overset{(v)}{p_i^{n+1}}) \tag{8.32}$$

In Eqs. 8.30b, 8.31b, and 8.32, the well is operated under a specified FBHP. Well production rate linearizations with Eqs. 8.30b and 8.31b provide tremendous improvement in stability over the linearization with Eqs. 8.30a and 8.31a. This is the case because the primary nonlinearity of the production rate is due to the ($p_i^{n+1} - p_{wf_i}^n$) term; the contribution of the $(\frac{1}{B\mu})_i^{n+1}$ term to nonlinearity is secondary.

8.4.3 Linearization of Fictitious Well Rates

In Chapters 4 and 5, we showed that a fictitious well rate is nothing but an interblock flow term between a reservoir boundary and a boundary block. Therefore, the linearization of fictitious well rates is similar to the linearization of interblock flow terms. For a block-centered grid, however, the coefficient of pressure drop depends on the pressure of the boundary block only; hence, a fictitious well rate can be linearized the same way as a well production rate (Section 8.4.2).

8.4.4 Linearization of Coefficients in Accumulation Term

The coefficient of pressure change in the accumulation term exhibits nonlinearity for a compressible fluid only (Eq. 8.11). This nonlinearity results from the pressure dependence of $\overset{(v)}{B_{g_n}^{n+1}}$ in the definition of $(\frac{\phi}{B_g})'_n$ given by Eq. 7.156a, as can be noted in Eq. 7.157. Linearization in space involves evaluating $\overset{(v)}{B_{g_n}^{n+1}}$ and hence $(\frac{\phi}{B_g})'_n$ at the pressure of the gridblock (or gridpoint) for which the flow equation is written (gridblock n). Linearization in time uses simple iteration; i.e., $\overset{(v)}{B_{g_n}^{n+1}}$ is evaluated at the current block pressure with one iteration lagging behind.

8.5 Linearized Flow Equations in Time

As mentioned earlier in this chapter, the flow equation for a compressible fluid exhibits the highest degree of nonlinearity among single-phase flow equations. Therefore, we choose this equation in 1D flow (Eq. 8.15) to demonstrate the various methods of linearizing flow equations. For each method presented in Section 8.4, we present the final form of the equation for an interior block (Eq. 8.15b). This final form is written for all gridblocks in the reservoir. For boundary blocks, the final form must be modified, as discussed in Chapters 4 and 5, to reflect specified boundary conditions for the given problem. The resulting system of equations is then solved using linear equation solvers such as those presented in Chapter 9.

8.5.1 Explicit Method

In the explicit method, transmissibilities, well production rates, gravities, and fictitious well rates if present are dated at the old time level (time level n). We still have to iterate on $(\frac{\phi}{B_g})'_i$, the pressure dependent function in the coefficient of the accumulation term.

Applying these assumptions, Eq. 8.15b becomes

$$\begin{aligned}&T^n_{x_{i-1/2}}[(p^{n+1}_{i-1}-p^{n+1}_i)-\gamma^n_{i-1/2}(Z_{i-1}-Z_i)]+T^n_{x_{i+1/2}}[(p^{n+1}_{i+1}-p^{n+1}_i)-\gamma^n_{i+1/2}(Z_{i+1}-Z_i)]\\&+q^n_{sc_i}=\frac{V_{b_i}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_i[p^{n+1}_i-p^n_i].\end{aligned} \tag{8.33}$$

Placing the iteration levels and rearranging terms, we obtain the final form of the flow equation for interior block i,

$$\begin{aligned}&T^n_{x_{i-1/2}}\overset{(v+1)}{p^{n+1}_{i-1}}-[T^n_{x_{i-1/2}}+T^n_{x_{i+1/2}}+\frac{V_{b_i}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_i]\overset{(v+1)}{p^{n+1}_i}+T^n_{x_{i+1/2}}\overset{(v+1)}{p^{n+1}_{i+1}}\\&=[T^n_{x_{i-1/2}}\gamma^n_{i-1/2}(Z_{i-1}-Z_i)+T^n_{x_{i+1/2}}\gamma^n_{i+1/2}(Z_{i+1}-Z_i)]-q^n_{sc_i}-\frac{V_{b_i}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_ip^n_i.\end{aligned} \tag{8.34}$$

The unknowns in Eq. 8.34, which reflects explicit treatment of transmissibilities and the well production rate in the flow equation for block i, are the pressures of blocks $i-1$, i, and $i+1$ at time level n+1 and current iteration $v+1$ ($\overset{(v+1)}{p^{n+1}_{i-1}}$, $\overset{(v+1)}{p^{n+1}_i}$, and $\overset{(v+1)}{p^{n+1}_{i+1}}$).

For the explicit treatment of nonlinearities, the general equation for block n in multidimensional flow has the form

$$\begin{aligned}&\sum_{l\in\psi_n}T^n_{l,n}\overset{(v+1)}{p^{n+1}_l}-\{\sum_{l\in\psi_n}T^n_{l,n}+\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n\}\overset{(v+1)}{p^{n+1}_n}\\&=\{\sum_{l\in\psi_n}[T^n_{l,n}\gamma^n_{l,n}(Z_l-Z_n)]-\sum_{l\in\xi_n}q^n_{sc_{l,n}}-q^n_{sc_n}-\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_np^n_n\}.\end{aligned} \tag{8.35}$$

8.5.2 Simple-Iteration Method

In the simple-iteration method, transmissibilities, well production rates, and fictitious well rates if present are dated at the current time level (time level n+1) with one iteration lagging behind. Gravities are dated at the old time level as mentioned in Chapter 7. We still have to iterate on $(\frac{\phi}{B_g})'_i$, the pressure dependent function in the coefficient of the accumulation term. Applying these assumptions and placing the iteration levels, Eq. 8.15b becomes

$$\overset{(v)}{T^{n+1}_{x_{i-1/2}}}[(\overset{(v+1)}{p^{n+1}_{i-1}} - \overset{(v+1)}{p^{n+1}_{i}}) - \gamma^n_{i-1/2}(Z_{i-1} - Z_i)] + \overset{(v)}{T^{n+1}_{x_{i+1/2}}}[(\overset{(v+1)}{p^{n+1}_{i+1}} - \overset{(v+1)}{p^{n+1}_{i}}) - \gamma^n_{i+1/2}(Z_{i+1} - Z_i)]$$
$$+\overset{(v)}{q^{n+1}_{sc_i}} = \frac{V_{b_i}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_i[\overset{(v+1)}{p^{n+1}_i} - p^n_i]. \tag{8.36}$$

Rearranging terms in Eq. 8.36, we obtain the final form of the flow equation for interior block i,

$$\overset{(v)}{T^{n+1}_{x_{i-1/2}}}\overset{(v+1)}{p^{n+1}_{i-1}} - [\overset{(v)}{T^{n+1}_{x_{i-1/2}}} + \overset{(v)}{T^{n+1}_{x_{i+1/2}}} + \frac{V_{b_i}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_i]\overset{(v+1)}{p^{n+1}_i} + \overset{(v)}{T^{n+1}_{x_{i+1/2}}}\overset{(v+1)}{p^{n+1}_{i+1}}$$
$$= [\overset{(v)}{T^{n+1}_{x_{i-1/2}}}\gamma^n_{i-1/2}(Z_{i-1} - Z_i) + \overset{(v)}{T^{n+1}_{x_{i+1/2}}}\gamma^n_{i+1/2}(Z_{i+1} - Z_i)] - \overset{(v)}{q^{n+1}_{sc_i}} - \frac{V_{b_i}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_i p^n_i. \tag{8.37}$$

The unknowns in Eq. 8.37, which reflects the simple-iteration treatment of transmissibilities and the well production rate in the flow equation for block i, are the pressures of blocks $i-1$, i, and $i+1$ at time level n+1 and current iteration $v+1$ ($\overset{(v+1)}{p^{n+1}_{i-1}}$, $\overset{(v+1)}{p^{n+1}_{i}}$, and $\overset{(v+1)}{p^{n+1}_{i+1}}$).

For the simple-iteration treatment of nonlinearities, the general equation for block n in multidimensional flow has the form

$$\sum_{l\in\psi_n}\overset{(v)}{T^{n+1}_{l,n}}\overset{(v+1)}{p^{n+1}_l} - \{\sum_{l\in\psi_n}\overset{(v)}{T^{n+1}_{l,n}} + \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n\}\overset{(v+1)}{p^{n+1}_n}$$
$$= \{\sum_{l\in\psi_n}[\overset{(v)}{T^{n+1}_{l,n}}\gamma^n_{l,n}(Z_l - Z_n)] - \sum_{l\in\xi_n}\overset{(v)}{q^{n+1}_{sc_{l,n}}} - \overset{(v)}{q^{n+1}_{sc_n}} - \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n p^n_n\} \tag{8.38}$$

8.5.3 Explicit Transmissibility Method

In the explicit transmissibility method, transmissibilities and coefficient of pressure drop in well production rate term are dated at old time level (time level n). One still has to iterate on $(\frac{\phi}{B_g})'_i$, the pressure dependent function in the coefficient of accumulation term. Applying these assumptions, Eq. 8.15b becomes

$$T^n_{x_{i-1/2}}[(p^{n+1}_{i-1} - p^{n+1}_i) - \gamma^n_{i-1/2}(Z_{i-1} - Z_i)] + T^n_{x_{i+1/2}}[(p^{n+1}_{i+1} - p^{n+1}_i) - \gamma^n_{i+1/2}(Z_{i+1} - Z_i)]$$
$$-G_{w_i}(\frac{1}{B\mu})^n_i(p^{n+1}_i - p^n_{wf_i}) = \frac{V_{b_i}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_i[p^{n+1}_i - p^n_i] \quad \textbf{(8.39)}$$

By placing the iteration levels and rearranging the terms, we obtain the final form of the flow equation for interior block i,

$$T^n_{x_{i-1/2}}\overset{(\nu+1)}{p^{n+1}_{i-1}} - [T^n_{x_{i-1/2}} + T^n_{x_{i+1/2}} + \frac{V_{b_i}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_i + G_{w_i}(\frac{1}{B\mu})^n_i]\overset{(\nu+1)}{p^{n+1}_i} + T^n_{x_{i+1/2}}\overset{(\nu+1)}{p^{n+1}_{i+1}}$$
$$= [T^n_{x_{i-1/2}}\gamma^n_{i-1/2}(Z_{i-1} - Z_i) + T^n_{x_{i+1/2}}\gamma^n_{i+1/2}(Z_{i+1} - Z_i)] - G_{w_i}(\frac{1}{B\mu})^n_i p^n_{wf_i} - \frac{V_{b_i}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_i p^n_i. \quad \textbf{(8.40)}$$

The wellblock in Eq. 8.40 operates under a specified FBHP. The unknowns in Eq. 8.40 are the pressures of blocks $i-1$, i, and $i+1$ at time level n+1 and current iteration $\nu+1$ ($\overset{(\nu+1)}{p^{n+1}_{i-1}}$, $\overset{(\nu+1)}{p^{n+1}_i}$, and $\overset{(\nu+1)}{p^{n+1}_{i+1}}$).

The general flow equation for interior block n in multidimensional flow using explicit transmissibilities can be expressed as

$$\sum_{l \in \psi_n} T^n_{l,n}\overset{(\nu+1)}{p^{n+1}_l} - [\sum_{l \in \psi_n} T^n_{l,n} + \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n + G_{w_n}(\frac{1}{B\mu})^n_n]\overset{(\nu+1)}{p^{n+1}_n}$$
$$= \sum_{l \in \psi_n} T^n_{l,n}\gamma^n_{l,n}(Z_l - Z_n) - G_{w_n}(\frac{1}{B\mu})^n_n p^n_{wf_n} - \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n p^n_n. \quad \textbf{(8.41)}$$

8.5.4 Simple Iteration on Transmissibility Method

In the simple iteration on transmissibility method, transmissibilities and the coefficient of pressure drop in a production rate term are dated at the current time level (time level n+1) with one iteration lagging behind. Gravities are dated at the old time level as mentioned in Chapter 7. We still have to iterate on $(\frac{\phi}{B_g})'_i$, the pressure dependent function in the coefficient of the accumulation term. Applying these assumptions and placing the iteration levels, Eq. 8.15b becomes

$$\overset{(\nu)}{T^{n+1}_{x_{i-1/2}}}[(\overset{(\nu+1)}{p^{n+1}_{i-1}} - \overset{(\nu+1)}{p^{n+1}_i}) - \gamma^n_{i-1/2}(Z_{i-1} - Z_i)] + \overset{(\nu)}{T^{n+1}_{x_{i+1/2}}}[(\overset{(\nu+1)}{p^{n+1}_{i+1}} - \overset{(\nu+1)}{p^{n+1}_i}) - \gamma^n_{i+1/2}(Z_{i+1} - Z_i)]$$
$$-G_{w_i}\overset{(\nu)}{(\frac{1}{B\mu})^{n+1}_i}(\overset{(\nu+1)}{p^{n+1}_i} - p^n_{wf_i}) = \frac{V_{b_i}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_i[\overset{(\nu+1)}{p^{n+1}_i} - p^n_i]. \quad \textbf{(8.42)}$$

The final form of the flow equation for interior block i is obtained by rearranging the terms, yielding

$$\begin{aligned}&\overset{(\nu)}{T}{}^{n+1}_{x_{i-1/2}}\overset{(\nu+1)}{p}{}^{n+1}_{i-1}-[\overset{(\nu)}{T}{}^{n+1}_{x_{i-1/2}}+\overset{(\nu)}{T}{}^{n+1}_{x_{i+1/2}}+\frac{V_{b_i}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_i+G_{w_i}(\frac{1}{B\mu})^{\overset{(\nu)}{n+1}}_i]\overset{(\nu+1)}{p}{}^{n+1}_i+\overset{(\nu)}{T}{}^{n+1}_{x_{i+1/2}}\overset{(\nu+1)}{p}{}^{n+1}_{i+1}\\&=[\overset{(\nu)}{T}{}^{n+1}_{x_{i-1/2}}\gamma^n_{i-1/2}(Z_{i-1}-Z_i)+\overset{(\nu)}{T}{}^{n+1}_{x_{i+1/2}}\gamma^n_{i+1/2}(Z_{i+1}-Z_i)]-G_{w_i}(\frac{1}{B\mu})^{\overset{(\nu)}{n+1}}_i p^n_{wf_i}-\frac{V_{b_i}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_i p^n_i.\end{aligned}\tag{8.43}$$

The unknowns in Eq. 8.43 are the pressures of blocks $i-1$, i, and $i+1$ at time level $n+1$ and current iteration $\nu+1$ ($\overset{(\nu+1)}{p}{}^{n+1}_{i-1}$, $\overset{(\nu+1)}{p}{}^{n+1}_{i}$, and $\overset{(\nu+1)}{p}{}^{n+1}_{i+1}$).

The general flow equation for interior block n in multidimensional flow using simple iteration on transmissibilities can be expressed as

$$\begin{aligned}&\sum_{l\in\psi_n}\overset{(\nu)}{T}{}^{n+1}_{l,n}\overset{(\nu+1)}{p}{}^{n+1}_{l}-[\sum_{l\in\psi_n}\overset{(\nu)}{T}{}^{n+1}_{l,n}+\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n+G_{w_n}(\frac{1}{B\mu})^{\overset{(\nu)}{n+1}}_n]\overset{(\nu+1)}{p}{}^{n+1}_n\\&=\sum_{l\in\psi_n}\overset{(\nu)}{T}{}^{n+1}_{l,n}\gamma^n_{l,n}(Z_l-Z_n)-G_{w_n}(\frac{1}{B\mu})^{\overset{(\nu)}{n+1}}_n p^n_{wf_n}-\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n p^n_n.\end{aligned}\tag{8.44}$$

8.5.5 Newton's Iteration (Fully Implicit) Method

In the fully implicit method, transmissibilities, well production rates, and fictitious well rates if present are dated at the current time level (time level n+1). Gravities are dated at the old time level as mentioned in Chapter 7. By dating nonlinear terms and unknown pressures at the current time level and the current iteration and using the old iteration level in calculating $(\frac{\phi}{B_g})'_i$, Eq. 8.15b becomes

$$\begin{aligned}&\overset{(\nu+1)}{T}{}^{n+1}_{x_{i-1/2}}[(\overset{(\nu+1)}{p}{}^{n+1}_{i-1}-\overset{(\nu+1)}{p}{}^{n+1}_{i})-\gamma^n_{i-1/2}(Z_{i-1}-Z_i)]+\overset{(\nu+1)}{T}{}^{n+1}_{x_{i+1/2}}[(\overset{(\nu+1)}{p}{}^{n+1}_{i+1}-\overset{(\nu+1)}{p}{}^{n+1}_{i})-\gamma^n_{i+1/2}(Z_{i+1}-Z_i)]\\&+\overset{(\nu+1)}{q}{}^{n+1}_{sc_i}=\frac{V_{b_i}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_i[\overset{(\nu+1)}{p}{}^{n+1}_{i}-p^n_i].\end{aligned}\tag{8.45}$$

The first, second, and third terms on the LHS of Eq. 8.45 can be approximated using the fully implicit method as

$$\begin{aligned}&\overset{(\nu+1)}{T}{}^{n+1}_{x_{i\mp1/2}}[(\overset{(\nu+1)}{p}{}^{n+1}_{i\mp1}-\overset{(\nu+1)}{p}{}^{n+1}_{i})-\gamma^n_{i\mp1/2}(Z_{i\mp1}-Z_i)]\cong\overset{(\nu)}{T}{}^{n+1}_{x_{i\mp1/2}}[(\overset{(\nu)}{p}{}^{n+1}_{i\mp1}-\overset{(\nu)}{p}{}^{n+1}_{i})-\gamma^n_{i\mp1/2}(Z_{i\mp1}-Z_i)]\\&+[(\overset{(\nu)}{p}{}^{n+1}_{i\mp1}-\overset{(\nu)}{p}{}^{n+1}_{i})-\gamma^n_{i\mp1/2}(Z_{i\mp1}-Z_i)][\frac{\partial T_{x_{i\mp1/2}}}{\partial p_i}\bigg|^{\overset{(\nu)}{n+1}}(\overset{(\nu+1)}{p}{}^{n+1}_{i}-\overset{(\nu)}{p}{}^{n+1}_{i})+\frac{\partial T_{x_{i\mp1/2}}}{\partial p_{i\mp1}}\bigg|^{\overset{(\nu)}{n+1}}(\overset{(\nu+1)}{p}{}^{n+1}_{i\mp1}-\overset{(\nu)}{p}{}^{n+1}_{i\mp1})]\\&+\overset{(\nu)}{T}{}^{n+1}_{x_{i\mp1/2}}[(\overset{(\nu+1)}{p}{}^{n+1}_{i\mp1}-\overset{(\nu)}{p}{}^{n+1}_{i\mp1})-(\overset{(\nu+1)}{p}{}^{n+1}_{i}-\overset{(\nu)}{p}{}^{n+1}_{i})],\end{aligned}\tag{8.46}$$

$$q_{sc_i}^{\overset{(\nu+1)}{n+1}} \cong q_{sc_i}^{\overset{(\nu)}{n+1}} + \left.\frac{dq_{sc_i}}{dp_i}\right|^{\overset{(\nu)}{n+1}} (p_i^{\overset{(\nu+1)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) \tag{8.32}$$

The RHS of Eq. 8.45 can be rewritten as

$$\frac{V_{b_i}}{\alpha_c \Delta t}\left(\frac{\phi}{B_g}\right)_i'[p_i^{\overset{(\nu+1)}{n+1}} - p_i^n] = \frac{V_{b_i}}{\alpha_c \Delta t}\left(\frac{\phi}{B_g}\right)_i'[(p_i^{\overset{(\nu+1)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) + (p_i^{\overset{(\nu)}{n+1}} - p_i^n)]. \tag{8.47}$$

Substitution of Eqs. 8.32, 8.46, and 8.47 into Eq. 8.45 and collecting terms yields the final form for the fully implicit flow equation for interior gridblock i,

$$\begin{aligned}
&\{T_{x_{i-1/2}}^{\overset{(\nu)}{n+1}} + [(p_{i-1}^{\overset{(\nu)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) - \gamma_{i-1/2}^n (Z_{i-1} - Z_i)] \left.\frac{\partial T_{x_{i-1/2}}}{\partial p_{i-1}}\right|^{\overset{(\nu)}{n+1}} \}\delta p_{i-1}^{\overset{(\nu+1)}{n+1}} \\
&-\{T_{x_{i-1/2}}^{\overset{(\nu)}{n+1}} - [(p_{i-1}^{\overset{(\nu)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) - \gamma_{i-1/2}^n (Z_{i-1} - Z_i)] \left.\frac{\partial T_{x_{i-1/2}}}{\partial p_i}\right|^{\overset{(\nu)}{n+1}} + T_{x_{i+1/2}}^{\overset{(\nu)}{n+1}} \\
&-[(p_{i+1}^{\overset{(\nu)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) - \gamma_{i+1/2}^n (Z_{i+1} - Z_i)] \left.\frac{\partial T_{x_{i+1/2}}}{\partial p_i}\right|^{\overset{(\nu)}{n+1}} - \left.\frac{dq_{sc_i}}{dp_i}\right|^{\overset{(\nu)}{n+1}} + \frac{V_{b_i}}{\alpha_c \Delta t}\left(\frac{\phi}{B_g}\right)_i'\}\delta p_i^{\overset{(\nu+1)}{n+1}} \\
&+\{T_{x_{i+1/2}}^{\overset{(\nu)}{n+1}} + [(p_{i+1}^{\overset{(\nu)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) - \gamma_{i+1/2}^n (Z_{i+1} - Z_i)] \left.\frac{\partial T_{x_{i+1/2}}}{\partial p_{i+1}}\right|^{\overset{(\nu)}{n+1}} \}\delta p_{i+1}^{\overset{(\nu+1)}{n+1}} \\
&= -\{T_{x_{i-1/2}}^{\overset{(\nu)}{n+1}} [(p_{i-1}^{\overset{(\nu)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) - \gamma_{i-1/2}^n (Z_{i-1} - Z_i)] + T_{x_{i+1/2}}^{\overset{(\nu)}{n+1}} [(p_{i+1}^{\overset{(\nu)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}) - \gamma_{i+1/2}^n (Z_{i+1} - Z_i)] \\
&+q_{sc_i}^{\overset{(\nu)}{n+1}} - \frac{V_{b_i}}{\alpha_c \Delta t}\left(\frac{\phi}{B_g}\right)_i'(p_i^{\overset{(\nu)}{n+1}} - p_i^n)\}.
\end{aligned} \tag{8.48}$$

The unknowns in Eq. 8.48, which reflects the fully implicit treatment of nonlinearities in the flow equation for interior block i, are the pressure changes over an iteration in blocks $i-1$, i, and $i+1$ [($p_{i-1}^{\overset{(\nu+1)}{n+1}} - p_{i-1}^{\overset{(\nu)}{n+1}}$), ($p_i^{\overset{(\nu+1)}{n+1}} - p_i^{\overset{(\nu)}{n+1}}$), and ($p_{i+1}^{\overset{(\nu+1)}{n+1}} - p_{i+1}^{\overset{(\nu)}{n+1}}$)]. Note that for the first iteration ($\nu = 0$), $p_i^{\overset{(0)}{n+1}} = p_i^n$ for $i = 1, 2, 3, \ldots n_x$ and the first-order derivatives are evaluated at old time level n.

The fully implicit method general equation for block n has the form

$$\sum_{l\in\psi_n}\{\overset{(\nu)}{T_{l,n}^{n+1}}+[(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})-\gamma_{l,n}^n(Z_l-Z_n)]\frac{\partial T_{l,n}}{\partial p_l}\bigg|^{\overset{(\nu)}{n+1}}+\sum_{m\in\xi_n}\frac{\partial q_{sc_{m,n}}}{\partial p_l}\bigg|^{\overset{(\nu)}{n+1}}\}\delta\overset{(\nu+1)}{p_l^{n+1}}$$

$$-\{\sum_{l\in\psi_n}\Big(\overset{(\nu)}{T_{l,n}^{n+1}}-[(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})-\gamma_{l,n}^n(Z_l-Z_n)]\frac{\partial T_{l,n}}{\partial p_n}\bigg|^{\overset{(\nu)}{n+1}}\Big)-\sum_{l\in\xi_n}\frac{\partial q_{sc_{l,n}}}{\partial p_n}\bigg|^{\overset{(\nu)}{n+1}} \tag{8.49a}$$

$$-\frac{dq_{sc_n}}{dp_n}\bigg|^{\overset{(\nu)}{n+1}}+\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})_n'\}\delta\overset{(\nu+1)}{p_n^{n+1}}=-\{\sum_{l\in\psi_n}\overset{(\nu)}{T_{l,n}^{n+1}}[(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})-\gamma_{l,n}^n(Z_l-Z_n)]$$

$$+\sum_{l\in\xi_n}\overset{(\nu)}{q_{sc_{l,n}}^{n+1}}+\overset{(\nu)}{q_{sc_n}^{n+1}}-\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})_n'(\overset{(\nu)}{p_n^{n+1}}-p_n^n)\}.$$

Note that the summation term $\sum_{m\in\xi_n}\frac{\partial q_{sc_{m,n}}}{\partial p_l}\bigg|^{\overset{(\nu)}{n+1}}$ in Eq. 8.49a contributes a maximum of one term for neighboring block l if and only if block n is a boundary block and block l falls next to reservoir boundary m. In addition, $\frac{\partial q_{sc_{m,n}}}{\partial p_l}$ and $\frac{\partial q_{sc_{m,n}}}{\partial p_n}$ are obtained from the flow rate equation of the fictitious well, which depends on the prevailing boundary condition. Note also that Eq. 8.49a does not produce a symmetric matrix because of the term

$$[(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})-\gamma_{l,n}^n(Z_l-Z_n)]\frac{\partial T_{l,n}}{\partial p_n}\bigg|^{\overset{(\nu)}{n+1}}$$

Coats, Ramesh, and Winestock (1977) derived the fully implicit equations for their steam model without conservative expansions of the accumulation terms. Although their equations do not conserve the material balance during iterations, they preserve it at convergence. Their method of obtaining the fully implicit iterative equation is applied here for the compressible fluid described by the implicit form of Eq. 7.12. This equation is written in a residual from at time level $n+1$, i.e., all terms are placed on one side of the equation and the other side is zero. Each term at time level $n+1$ in the resulting equation is approximated by its value at the current iteration level $\nu+1$, which in turn can be approximated by its value at the last iteration level ν, plus a linear combination of the unknowns arising from partial differentiation with respect to all unknown pressures. The unknown quantities in the resulting equation are the changes over an iteration of all the unknown pressures in the original equation. The resulting fully implicit iterative equation for block n is

$$\sum_{l\in\psi_n}\{\overset{(\nu)}{T_{l,n}^{n+1}}+[(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})-\gamma_{l,n}^n(Z_l-Z_n)]\frac{\partial T_{l,n}}{\partial p_l}\bigg|^{(\nu)}_{n+1}+\sum_{m\in\xi_n}\frac{\partial q_{sc_{m,n}}}{\partial p_l}\bigg|^{(\nu)}_{n+1}\}\delta \overset{(\nu+1)}{p_l^{n+1}}$$

$$-\{\sum_{l\in\psi_n}\left(\overset{(\nu)}{T_{l,n}^{n+1}}-[(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})-\gamma_{l,n}^n(Z_l-Z_n)]\frac{\partial T_{l,n}}{\partial p_n}\bigg|^{(\nu)}_{n+1}\right)-\sum_{l\in\xi_n}\frac{\partial q_{sc_{l,n}}}{\partial p_n}\bigg|^{(\nu)}_{n+1} \tag{8.49b}$$

$$-\frac{dq_{sc_n}}{dp_n}\bigg|^{(\nu)}_{n+1}+\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n\bigg|^{(\nu)}_{n+1}\}\delta \overset{(\nu+1)}{p_n^{n+1}}=-\{\sum_{l\in\psi_n}\overset{(\nu)}{T_{l,n}^{n+1}}[(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})-\gamma_{l,n}^n(Z_l-Z_n)]$$

$$+\sum_{l\in\xi_n}\overset{(\nu)}{q_{sc_{l,n}}^{n+1}}+\overset{(\nu)}{q_{sc_n}^{n+1}}-\frac{V_{b_n}}{\alpha_c\Delta t}[(\frac{\phi}{B_g})_n^{\overset{(\nu)}{n+1}}-(\frac{\phi}{B_g})_n^n]\}.$$

Eq. 8.49b is similar to Eq. 8.49a with two exceptions that are related to the accumulation term. First, while Eq. 8.49a preserves material balance at every current iteration, Eq. 8.49b preserves material balance only at convergence. Second, the term $(\frac{\phi}{B_g})'_n$ in Eq. 8.49a represents the chord slope that results from a conservative expansion, whereas the term $(\frac{\phi}{B_g})'_n\bigg|^{(\nu)}_{n+1}$ in Eq. 8.49b represents the slope of $(\frac{\phi}{B_g})_n$, both terms being evaluated at last iteration level ν. Third, the last term on the RHS of Eq. 8.49a, $\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n(\overset{(\nu)}{p_n^{n+1}}-p_n^n)$, is replaced with $\frac{V_{b_n}}{\alpha_c\Delta t}[(\frac{\phi}{B_g})_n^{\overset{(\nu)}{n+1}}-(\frac{\phi}{B_g})_n^n]$ in Eq. 8.49b. For single-phase flow, where the accumulation term is a function of pressure only, these two terms are equal because both represent the accumulation term evaluated at the last iteration.

The next set of examples demonstrates the mechanics of implementing the explicit method, simple iteration method, and fully implicit method of linearization in solving the equations for single-well simulation. It should be noted that the simple iteration and fully implicit methods produce close results because, contrary to the explicit method, the transmissibility in both methods is updated every iteration. All methods in this problem show the same convergence property for a time step of one month because, over the pressure range 1515 to 4015 psia, the product μB is approximately a straight line having a small slope (-4.5×10^{-6} cp-RB/scf-psi).

Example 8.1 Consider the reservoir described in Example 7.13, where a 6-in vertical well is drilled on 20-acre spacing in a natural gas reservoir. The reservoir is described by four gridblocks in the radial direction as shown in Figure 8–2. The reservoir is horizontal and has 30-ft net thickness and homogeneous and isotropic *rock properties with $k = 15$ md and $\phi = 0.13$. Initially, reservoir pressure is 4015 psia. Table 8–1 presents gas FVF and viscosity dependence on pressure. The external reservoir boundaries are sealed to fluid flow. Let the well produce with a FBHP of 1515 psia. Find the pressure distribution in the reservoir after one month (30.42 days) using a single time step. Solve the problem using the implicit formulation with the explicit transmissibility method of linearization and present the simulation results up to six months.*

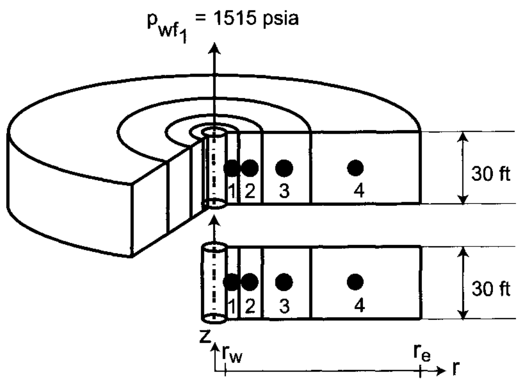


Figure 8–2 Discretized 1D reservoir in Example 8.1.

Solution

Gridblock locations, bulk volumes, and geometric factors in the radial direction are calculated in exactly the same way as in Example 7.12. The results are presented in Table 8–2.

For single-well simulation in a horizontal reservoir (Z_n = constant) with no-flow boundaries ($\sum_{l \in \xi_n} q_{sc_{l,n}}^{n+1} = 0$), the implicit flow equation with explicit transmissibility is obtained from Eq. 8.41. For gridblock n with a well operating under a specified FBHP,

$$\sum_{l \in \psi_n} T_{l,n}^{n} p_l^{\overset{(\nu+1)}{n+1}} - [\sum_{l \in \psi_n} T_{l,n}^{n} + \frac{V_{b_n}}{\alpha_c \Delta t} (\frac{\phi}{B_g})'_n + G_{w_n} (\frac{1}{B\mu})_n^n] p_n^{\overset{(\nu+1)}{n+1}} = -G_{w_n} (\frac{1}{B\mu})_n^n p_{wf_n}^n - \frac{V_{b_n}}{\alpha_c \Delta t} (\frac{\phi}{B_g})'_n p_n^n . \tag{8.50a}$$

For gridblock n without a well,

Table 8–1 Gas FVF and Viscosity in Example 8.1

Pressure (psia)	GFVF (RB/scf)	Gas Viscosity (cp)
215.00	0.016654	0.0126
415.00	0.008141	0.0129
615.00	0.005371	0.0132
815.00	0.003956	0.0135
1015.00	0.003114	0.0138
1215.00	0.002544	0.0143
1415.00	0.002149	0.0147
1615.00	0.001857	0.0152
1815.00	0.001630	0.0156
2015.00	0.001459	0.0161
2215.00	0.001318	0.0167
2415.00	0.001201	0.0173
2615.00	0.001109	0.0180
2815.00	0.001032	0.0186
3015.00	0.000972	0.0192
3215.00	0.000922	0.0198
3415.00	0.000878	0.0204
3615.00	0.000840	0.0211
3815.00	0.000808	0.0217
4015.00	0.000779	0.0223

Table 8–2 Gridblock Locations, Bulk Volumes, and Geometric Factors

n	i	r_i (ft)	$G_{r_{i+1/2}}$ (RB-cp/D-psi)	V_{b_n} (ft^3)
1	1	0.5611	1.6655557	340.59522
2	2	3.8014	1.6655557	15631.859
3	3	25.7532	1.6655557	717435.23
4	4	174.4683	1.6655557	25402604

$$\sum_{l\in\psi_n} T_{l,n}^n \overset{(v+1)}{p_l^{n+1}} - [\sum_{l\in\psi_n} T_{l,n}^n + \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n] \overset{(v+1)}{p_n^{n+1}} = -\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n p_n^n . \quad \textbf{(8.50b)}$$

The gas in this reservoir flows toward the well in gridblock 1. Therefore, gridblock 4 is upstream to gridblock 3, gridblock 3 is upstream to gridblock 2, and gridblock 2 is upstream to gridblock 1. In solving this problem, we use upstream weighting (Section 8.4.1.1) of the pressure dependent terms in transmissibility.

First time step calculations ($n = 0$, $t_{n+1} = 30.42$ days, and $\Delta t = 30.42$ days)

Assign $p_1^n = p_2^n = p_3^n = p_4^n = p_{in} = 4015$ psia.

For the first iteration ($v = 0$), assume $\overset{(v)}{p_n^{n+1}} = p_n^n = 4015$ psia for $n = 1, 2, 3, 4$. In addition, we estimate $(\frac{\phi}{B_g})'_n$ between p_n^n and $p_n^n - \varepsilon$ where $\varepsilon = 1$ psi. Table 8–3 presents the estimated values of the FVF and viscosity using linear interpolation within table entries, chord slope $(\frac{\phi}{B_g})'_n$, and $\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n$ for all gridblocks at the first iteration. Note that at $p = 4014$ psia, $B_g = 0.00077914$ RB/scf and $\mu_g = 0.0222970$ cp. For example, for gridblock 1,

$$(\frac{\phi}{B_g})'_1 = \frac{(\frac{\phi}{B})_1^{\overset{(v)}{n+1}} - (\frac{\phi}{B})_1^n}{\overset{(v)}{p_1^{n+1}} - p_1^n} = \frac{(\frac{0.13}{0.00077914}) - (\frac{0.13}{0.000779})}{4014 - 4015} = 0.0310567$$

$$\frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 = \frac{340.59522 \times 0.0310567}{5.614583 \times 30.42} = 0.0619323$$

$$T_{r_{2,1}}^n \Big|_2 = T_{r_{1,2}}^n \Big|_2 = G_{r_{1+1/2}} (\frac{1}{\mu B})_2^n = 1.6655557 \times (\frac{1}{0.0223000 \times 0.00077900}) = 95877.5281$$

for upstream weighting of transmissibility.

In addition, for the production well in wellblock 1, G_{w_1} is calculated using Eq. 6.10a, yielding

$$G_{w_1} = \frac{2 \times \pi \times 0.001127 \times 15 \times 30}{\log_e (0.5611 / 0.25)} = 3.941572$$

$$G_{w_1} (\frac{1}{\mu B})_1^n = 3.941572 \times (\frac{1}{0.0223000 \times 0.00077900}) = 226896.16 \text{ scf/D-psi.}$$

Therefore, $T^n_{r_{1,2}}\Big|_2 = T^n_{r_{2,3}}\Big|_3 = T^n_{r_{3,4}}\Big|_4 = 95877.5281$ scf/D-psi. Note also that $T^n_{r_{l,n}} = T^n_{r_{n,l}}$.

For gridblock 1, $n = 1$ and $\psi_1 = \{2\}$. Therefore, Eq. 8.50a becomes

$$-[T^n_{2,1}\Big|_2 + \frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 + G_{w_1}(\frac{1}{B\mu})^n_1]p_1^{\overset{(\nu+1)}{n+1}} + T^n_{2,1}\Big|_2\, p_2^{\overset{(\nu+1)}{n+1}}$$
$$= -G_{w_1}(\frac{1}{B\mu})^n_1 p^n_{wf_1} - \frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 p^n_1. \quad \textbf{(8.51)}$$

Substitution of the values in this equation gives

$$-[95877.5281 + 0.0619323 + 226896.16]p_1^{\overset{(\nu+1)}{n+1}} + 95877.5281 p_2^{\overset{(\nu+1)}{n+1}}$$
$$= -226896.16 \times 1515 - 0.0619323 \times 4015$$

or after simplification,

$$-322773.749 p_1^{\overset{(\nu+1)}{n+1}} + 95877.5281 p_2^{\overset{(\nu+1)}{n+1}} = -343747929. \quad \textbf{(8.52)}$$

For gridblock 2, $n = 2$ and $\psi_2 = \{1,3\}$. Therefore, Eq. 8.50b becomes

$$T^n_{1,2}\Big|_2\, p_1^{\overset{(\nu+1)}{n+1}} - [T^n_{1,2}\Big|_2 + T^n_{3,2}\Big|_3 + \frac{V_{b_2}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_2]p_2^{\overset{(\nu+1)}{n+1}} + T^n_{3,2}\Big|_3\, p_3^{\overset{(\nu+1)}{n+1}} = -\frac{V_{b_2}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_2 p^n_2. \quad \textbf{(8.53)}$$

Substitution of the values in this equation gives

$$95877.5281 p_1^{\overset{(\nu+1)}{n+1}} - [95877.5281 + 95877.5281 + 2.84243]p_2^{\overset{(\nu+1)}{n+1}}$$
$$+95877.5281 p_3^{\overset{(\nu+1)}{n+1}} = -2.84243 \times 4015$$

or after simplification,

$$95877.5281 p_1^{\overset{(\nu+1)}{n+1}} - 191757.899 p_2^{\overset{(\nu+1)}{n+1}} + 95877.5281 p_3^{\overset{(\nu+1)}{n+1}} = -11412.3496 \quad \textbf{(8.54)}$$

For gridblock 3, $n = 3$ and $\psi_3 = \{2,4\}$. Therefore, Eq. 8.50b becomes

$$T^n_{2,3}\Big|_3\, p_2^{\overset{(\nu+1)}{n+1}} - [T^n_{2,3}\Big|_3 + T^n_{4,3}\Big|_4 + \frac{V_{b_3}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_3]p_3^{\overset{(\nu+1)}{n+1}} + T^n_{4,3}\Big|_4\, p_4^{\overset{(\nu+1)}{n+1}} = -\frac{V_{b_3}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_3 p^n_3. \quad \textbf{(8.55)}$$

Substitution of the values in this equation gives

$$95877.5281 p_2^{\overset{(\nu+1)}{n+1}} - [95877.5281 + 95877.5281 + 130.455]p_3^{\overset{(\nu+1)}{n+1}} + 95877.5281 p_4^{\overset{(\nu+1)}{n+1}}$$
$$= -130.455 \times 4015$$

or after simplification,

Table 8–3 Estimated Gridblock FVF, Viscosity, and Chord Slope at Old Iteration $\nu = 0$

Block n	$\overset{(0)}{p_n^{n+1}}$ (psia)	B_g (RB/scf)	μ_g (cp)	$(\phi / B_g)'_n$	$\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n$
1	4015	0.00077900	0.0223000	0.0310567	0.0619323
2	4015	0.00077900	0.0223000	0.0310567	2.84243
3	4015	0.00077900	0.0223000	0.0310567	130.455
4	4015	0.00077900	0.0223000	0.0310567	4619.10

$$95877.5281 \overset{(\nu+1)}{p_2^{n+1}} - 191885.511 \overset{(\nu+1)}{p_3^{n+1}} + 95877.5281 \overset{(\nu+1)}{p_4^{n+1}} = -523777.862 \tag{8.56}$$

For gridblock 4, $n = 4$ and $\psi_4 = \{3\}$. Therefore, Eq. 8.50b becomes

$$T_{3,4}^n \Big|_4 \overset{(\nu+1)}{p_3^{n+1}} - [T_{3,4}^n \Big|_4 + \frac{V_{b_4}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_4] \overset{(\nu+1)}{p_4^{n+1}} = -\frac{V_{b_4}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_4 p_4^n. \tag{8.57}$$

Substitution of the values in this equation gives

$$95877.5281 \overset{(\nu+1)}{p_3^{n+1}} - [95877.5281 + 4619.10] \overset{(\nu+1)}{p_4^{n+1}} = -4619.10 \times 4015$$

or after simplification,

$$95877.5281 \overset{(\nu+1)}{p_3^{n+1}} - 100496.6251 \overset{(\nu+1)}{p_4^{n+1}} = -18545676.2 \tag{8.58}$$

The results of solving Eqs. 8.52, 8.54, 8.56, and 8.58 for the unknown pressures are $\overset{(1)}{p_1^{n+1}} = 1559.88$ psia, $\overset{(1)}{p_2^{n+1}} = 1666.08$ psia, $\overset{(1)}{p_3^{n+1}} = 1772.22$ psia, and $\overset{(1)}{p_4^{n+1}} = 1875.30$ psia.

For the second iteration ($\nu = 1$), we use $\overset{(1)}{p_n^{n+1}}$ to estimate the values of FVF in order to estimate chord slope $(\frac{\phi}{B_g})'_n$ and $\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n$ for gridblock *n*. Table 8–4 lists these values. For example, for gridblock 1,

$$(\frac{\phi}{B_g})'_1 = \frac{(\frac{\phi}{B})_1^{\overset{(\nu)}{n+1}} - (\frac{\phi}{B})_1^n}{\overset{(\nu)}{p_1^{n+1}} - p_1^n} = \frac{(\frac{0.13}{0.0019375}) - (\frac{0.13}{0.000779000})}{1559.88 - 4015} = 0.0406428$$

Table 8-4 Estimated Gridblock FVF and Chord Slope at Old Iteration $\nu = 1$

Block n	$\overset{(1)}{p_n^{n+1}}$ (psia)	$\overset{(1)}{B_{g_n}^{n+1}}$ (RB/scf)	$(\phi / B_g)'_n$	$\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n$
1	1559.88	0.0019375	0.0406428	0.0810486
2	1666.08	0.0017990	0.0402820	3.68676
3	1772.22	0.0016786	0.0398760	167.501
4	1875.30	0.0015784	0.0395013	5875.07

$$\frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 = \frac{340.59522 \times 0.0406428}{5.614583 \times 30.42} = 0.0810486$$

Note that for the explicit transmissibility treatment,

$$T^n_{r_{1,2}}\Big|_2 = T^n_{r_{2,3}}\Big|_3 = T^n_{r_{3,4}}\Big|_4 = 95877.5281 \text{ scf/D-psi and } G_{w_1}(\frac{1}{\mu B})^n_1 = 226896.16 \text{ scf/D-psi}$$

for all iterations.

For gridblock 1, $n = 1$. Substitution of the values in Eq. 8.51 gives

$$-[95877.5281 + 0.0810486 + 226896.16]\overset{(\nu+1)}{p_1^{n+1}} + 95877.5281\overset{(\nu+1)}{p_2^{n+1}}$$
$$= -226896.16 \times 1515 - 0.0810486 \times 4015$$

or after simplification,

$$-322773.768\overset{(\nu+1)}{p_1^{n+1}} + 95877.5281\overset{(\nu+1)}{p_2^{n+1}} = -343748006. \quad \textbf{(8.59)}$$

For gridblock 2, $n = 2$. Substitution of the values in Eq. 8.53 gives

$$95877.5281\overset{(\nu+1)}{p_1^{n+1}} - [95877.5281 + 95877.5281 + 3.68676]\overset{(\nu+1)}{p_2^{n+1}}$$
$$+95877.5281\overset{(\nu+1)}{p_3^{n+1}} = -3.68676 \times 4015$$

or after simplification,

$$95877.5281\overset{(\nu+1)}{p_1^{n+1}} - 191758.743\overset{(\nu+1)}{p_2^{n+1}} + 95877.5281\overset{(\nu+1)}{p_3^{n+1}} = -14802.3438 \quad \textbf{(8.60)}$$

For gridblock 3, $n = 3$. Substitution of the values in Eq. 8.55 gives

$$95877.5281 \overset{(\nu+1)}{p_2^{n+1}} - [95877.5281 + 95877.5281 + 167.501] \overset{(\nu+1)}{p_3^{n+1}}$$
$$+95877.5281 \overset{(\nu+1)}{p_4^{n+1}} = -167.501 \times 4015$$

or after simplification,

$$95877.5281 \overset{(\nu+1)}{p_2^{n+1}} - 191922.557 \overset{(\nu+1)}{p_3^{n+1}} + 95877.5281 \overset{(\nu+1)}{p_4^{n+1}} = -672516.495 \tag{8.61}$$

For gridblock 4, $n = 4$. Substitution of the values in Eq. 8.57 gives

$$95877.5281 \overset{(\nu+1)}{p_3^{n+1}} - [95877.5281 + 5875.07] \overset{(\nu+1)}{p_4^{n+1}} = -5875.07 \times 4015$$

or after simplification,

$$95877.5281 \overset{(\nu+1)}{p_3^{n+1}} - 101752.599 \overset{(\nu+1)}{p_4^{n+1}} = -23588411.0 \tag{8.62}$$

The results of solving Eqs. 8.59, 8.60, 8.61, and 8.62 for the unknown pressures are $\overset{(2)}{p_1^{n+1}} = 1569.96$ psia, $\overset{(2)}{p_2^{n+1}} = 1700.03$ psia, $\overset{(2)}{p_3^{n+1}} = 1830.00$ psia, and $\overset{(2)}{p_4^{n+1}} = 1956.16$ psia. Iterations continue until the convergence criterion is satisfied. Table 8–5 shows the successive iterations for the first time step. Note that it took four iterations to converge. The convergence criterion was set as given by Eq. 7.179; i.e.,

$$\max_{1 \le n \le N} \left| \frac{\overset{(\nu+1)}{p_n^{n+1}} - \overset{(\nu)}{p_n^{n+1}}}{\overset{(\nu)}{p_n^{n+1}}} \right| \le 0.001. \tag{8.63}$$

Table 8–5 Pressure Solution at $t_{n+1} = 30.42$ Days for Successive Iterations

$\nu+1$	$\overset{(\nu+1)}{p_1^{n+1}}$ **(psia)**	$\overset{(\nu+1)}{p_2^{n+1}}$ **(psia)**	$\overset{(\nu+1)}{p_3^{n+1}}$ **(psia)**	$\overset{(\nu+1)}{p_4^{n+1}}$ **(psia)**
0	4015.00	4015.00	4015.00	4015.00
1	1559.88	1666.08	1772.22	1875.30
2	1569.96	1700.03	1830.00	1956.16
3	1569.64	1698.94	1828.15	1953.57
4	1569.65	1698.98	1828.23	1953.68

Table 8–6 Converged Pressure Solution and Gas Production at Various Times

$n+1$	Time (day)	v	p_1^{n+1} (psia)	p_2^{n+1} (psia)	p_3^{n+1} (psia)	p_4^{n+1} (psia)	q_{gsc}^{n+1} (MMscf/D)	Cumulative production (MMMscf)
1	30.42	4	1569.65	1698.98	1828.23	1953.68	–12.4003	–0.377217
2	60.84	3	1531.85	1569.07	1603.85	1636.31	–2.28961	–0.446867
3	91.26	3	1519.81	1530.96	1541.87	1552.37	–0.639629	–0.466324
4	121.68	2	1516.45	1519.88	1523.27	1526.58	–0.191978	–0.472164
5	152.10	2	1515.44	1516.49	1517.53	1518.55	–0.058311	–0.473938
6	182.52	2	1515.13	1515.45	1515.77	1516.09	–0.017769	–0.474478

After reaching convergence, the time is incremented by $\Delta t = 30.42$ days and the above procedure is repeated. Table 8–6 shows the converged solutions at various times up to six months of simulation time.

Example 8.2 Consider the problem described in Example 8.1. Apply the simple iteration on transmissibility method to find the pressure distribution in the reservoir after one month (30.42 days) using a single time step. Present the simulation results up to six months.

Solution

Table 8–2 reports the gridblock locations, bulk volumes, and geometric factors in the radial direction. For single-well simulation in a horizontal reservoir (Z_n = constant) with no-flow boundaries ($\sum_{l\in\xi_n} q_{sc_{l,n}}^{n+1} = 0$), the implicit flow equation with simple iteration on transmissibility is obtained from Eq. 8.44.

For gridblock n with a well operating under a specified FBHP,

$$\sum_{l\in\psi_n} \overset{(\nu)}{T_{l,n}^{n+1}} \overset{(\nu+1)}{p_l^{n+1}} - [\sum_{l\in\psi_n} \overset{(\nu)}{T_{l,n}^{n+1}} + \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n + G_{w_n} \overset{(\nu)}{(\frac{1}{B\mu})_n^{n+1}}] \overset{(\nu+1)}{p_n^{n+1}}$$
$$= -G_{w_n} \overset{(\nu)}{(\frac{1}{B\mu})_n^{n+1}} p_{wf_n}^n - \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n p_n^n . \quad \textbf{(8.64a)}$$

For gridblock n without a well,

$$\sum_{l\in\psi_n} \overset{(\nu)}{T_{l,n}^{n+1}} \overset{(\nu+1)}{p_l^{n+1}} - [\sum_{l\in\psi_n} \overset{(\nu)}{T_{l,n}^{n+1}} + \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n] \overset{(\nu+1)}{p_n^{n+1}} = -\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n p_n^n . \quad \textbf{(8.64b)}$$

As mentioned in Example 8.1, the gas in this reservoir flows toward the well in gridblock 1, gridblock 4 is upstream to gridblock 3, gridblock 3 is upstream to gridblock 2, and gridblock 2 is upstream to gridblock 1. In solving this problem, we use upstream weighting (Section 8.4.1.1) of the pressure dependent terms in transmissibility.

First time step calculations ($n = 0$, $t_{n+1} = 30.42$ days, and $\Delta t = 30.42$ days)

For the first iteration ($v = 0$), assume $\overset{(v)}{p_n^{n+1}} = p_n^n = 4015$ psia for $n = 1, 2, 3, 4$.

Therefore, $G_{w_1} (\frac{1}{\mu B})_1^{\overset{(0)}{n+1}} = G_{w_1} (\frac{1}{\mu B})_1^n = 226896.16$ scf/D-psi and $\overset{(0)}{T_{r_{1,n}}^{n+1}} = T_{r_{1,n}}^n$, or

more explicitly, $\overset{(0)}{T_{r_{1,2}}^{n+1}}\Big|_2 = \overset{(0)}{T_{r_{2,3}}^{n+1}}\Big|_3 = \overset{(0)}{T_{r_{3,4}}^{n+1}}\Big|_4 = 95877.5281$ scf/D-psi. Consequently,

the equations for gridblocks 1, 2, 3, and 4 are given by Eqs. 8.52, 8.54, 8.56, and 8.58, respectively, and the unknown pressures are $\overset{(1)}{p_1^{n+1}} = 1559.88$ psia,

$\overset{(1)}{p_2^{n+1}} = 1666.08$ psia, $\overset{(1)}{p_3^{n+1}} = 1772.22$ psia, and $\overset{(1)}{p_4^{n+1}} = 1875.30$ psia.

For the second iteration ($v = 1$), we use $\overset{(1)}{p_n^{n+1}}$ to estimate the values of FVF, gas

viscosity, and chord slope $(\frac{\phi}{B_g})'_n$ and then calculate $\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n$ for gridblock *n*.

Table 8–7 lists these values in addition to the upstream value of interblock transmissibility ($\overset{(v)}{T_{r_{n,n+1}}^{n+1}}$). For example, for gridblock 1,

$$(\frac{\phi}{B_g})'_1 = \frac{(\frac{\phi}{B})_1^{\overset{(v)}{n+1}} - (\frac{\phi}{B})_1^n}{\overset{(v)}{p_1^{n+1}} - p_1^n} = \frac{(\frac{0.13}{0.0019375}) - (\frac{0.13}{0.000779000})}{1559.88 - 4015} = 0.0406428$$

$$\frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 = \frac{340.59522 \times 0.0406428}{5.614583 \times 30.42} = 0.0810486$$

$$\overset{(v)}{T_{r_{1,2}}^{n+1}}\Big|_2 = \overset{(v)}{T_{r_{2,1}}^{n+1}}\Big|_2 = G_{r_{1+1/2}} (\frac{1}{\mu B})_2^{\overset{(v)}{n+1}} = 1.6655557 \times (\frac{1}{0.0153022 \times 0.0017990}) = 60502.0907$$

for upstream weighting of transmissibility. In addition, for the production well in wellblock 1,

$$G_{w_1} (\frac{1}{\mu B})_1^{\overset{(v)}{n+1}} = 3.941572 \times (\frac{1}{0.01506220 \times 0.00193748}) = 135065.6$$

For gridblock 1, $n = 1$ and $\psi_1 = \{2\}$. Therefore, 8.64a becomes

Table 8–7 Estimated Gridblock FVF and Chord slope at Old Iteration $\nu = 1$

Block n	$\overset{(1)}{p}{}_n^{n+1}$ (psia)	$\overset{(1)}{B}{}_{g_n}^{n+1}$ (RB/scf)	$\overset{(1)}{\mu}{}_{g_n}^{n+1}$ (cp)	$\overset{(\nu)}{T}{}_{r_{n,n+1}}^{n+1}\Big\vert_{n+1}$	$(\phi / B_g)'_n$	$\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n$
1	1559.88	0.0019375	0.0150622	60502.0907	0.0406428	0.0810486
2	1666.08	0.0017990	0.0153022	63956.9105	0.0402820	3.68676
3	1772.22	0.0016786	0.0155144	66993.0320	0.0398760	167.501
4	1875.30	0.0015784	0.0157508	—	0.0395013	5875.07

$$-[\overset{(\nu)}{T}{}_{2,1}^{n+1}\Big|_2 + \frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 + G_{w_1}(\frac{1}{B\mu})_1^{\overset{(\nu)}{n+1}}]\overset{(\nu+1)}{p}{}_1^{n+1} + \overset{(\nu)}{T}{}_{2,1}^{n+1}\Big|_2 \overset{(\nu+1)}{p}{}_2^{n+1}$$
$$= -G_{w_1}(\frac{1}{B\mu})_1^{\overset{(\nu)}{n+1}} p_{wf_1}^n - \frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 p_1^n. \tag{8.65}$$

Substitution of the values in this equation gives

$$-[60502.0907 + 0.0810486 + 135065.6]\overset{(\nu+1)}{p}{}_1^{n+1} + 60502.0907\overset{(\nu+1)}{p}{}_2^{n+1}$$
$$= -135065.6 \times 1515 - 0.0810486 \times 4015$$

or after simplification,

$$-195567.739\overset{(\nu+1)}{p}{}_1^{n+1} + 60502.0907\overset{(\nu+1)}{p}{}_2^{n+1} = -204624660. \tag{8.66}$$

For gridblock 2, $n = 2$ and $\psi_2 = \{1,3\}$. Therefore, Eq. 8.64b becomes

$$\overset{(\nu)}{T}{}_{1,2}^{n+1}\Big|_2 \overset{(\nu+1)}{p}{}_1^{n+1} - [\overset{(\nu)}{T}{}_{1,2}^{n+1}\Big|_2 + \overset{(\nu)}{T}{}_{3,2}^{n+1}\Big|_3 + \frac{V_{b_2}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_2]\overset{(\nu+1)}{p}{}_2^{n+1} + \overset{(\nu)}{T}{}_{3,2}^{n+1}\Big|_3 \overset{(\nu+1)}{p}{}_3^{n+1} = -\frac{V_{b_2}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_2 p_2^n. \tag{8.67}$$

Substitution of the values in this equation gives

$$60502.0907\overset{(\nu+1)}{p}{}_1^{n+1} - [60502.0907 + 63956.9105 + 3.68676]\overset{(\nu+1)}{p}{}_2^{n+1}$$
$$+63956.9105\overset{(\nu+1)}{p}{}_3^{n+1} = -3.68676 \times 4015$$

or after simplification,

$$60502.0907\overset{(\nu+1)}{p}{}_1^{n+1} - 124462.688\overset{(\nu+1)}{p}{}_2^{n+1} + 63956.9105\overset{(\nu+1)}{p}{}_3^{n+1} = -14802.3438 \tag{8.68}$$

For gridblock 3, $n = 3$ and $\psi_3 = \{2,4\}$. Therefore, Eq. 8.64b becomes

$$\overset{(\nu)}{T_{2,3}^{n+1}}\Big|_3 \overset{(\nu+1)}{p_2^{n+1}} - [\overset{(\nu)}{T_{2,3}^{n+1}}\Big|_3 + \overset{(\nu)}{T_{4,3}^{n+1}}\Big|_4 + \frac{V_{b_3}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_3]\overset{(\nu+1)}{p_3^{n+1}} + \overset{(\nu)}{T_{4,3}^{n+1}}\Big|_4 \overset{(\nu+1)}{p_4^{n+1}} = -\frac{V_{b_3}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_3 p_3^n. \quad \textbf{(8.69)}$$

Substitution of the values in this equation gives

$$63956.9105\overset{(\nu+1)}{p_2^{n+1}} - [63956.9105 + 66993.0320 + 167.501]\overset{(\nu+1)}{p_3^{n+1}}$$
$$+66993.0320\overset{(\nu+1)}{p_4^{n+1}} = -167.501 \times 4015$$

or after simplification,

$$63956.9105\overset{(\nu+1)}{p_2^{n+1}} - 131117.443\overset{(\nu+1)}{p_3^{n+1}} + 66993.0320\overset{(\nu+1)}{p_4^{n+1}} = -672516.495 \quad \textbf{(8.70)}$$

For gridblock 4, $n = 4$ and $\psi_4 = \{3\}$. Therefore, Eq. 8.64b becomes

$$\overset{(\nu)}{T_{3,4}^{n+1}}\Big|_4 \overset{(\nu+1)}{p_3^{n+1}} - [\overset{(\nu)}{T_{3,4}^{n+1}}\Big|_4 + \frac{V_{b_4}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_4]\overset{(\nu+1)}{p_4^{n+1}} = -\frac{V_{b_4}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_4 p_4^n. \quad \textbf{(8.71)}$$

Substitution of the values in this equation gives

$$66993.0320\overset{(\nu+1)}{p_3^{n+1}} - [66993.0320 + 5875.07]\overset{(\nu+1)}{p_4^{n+1}} = -5875.07 \times 4015$$

or after simplification,

$$66993.0320\overset{(\nu+1)}{p_3^{n+1}} - 72868.1032\overset{(\nu+1)}{p_4^{n+1}} = -23588411.0 \quad \textbf{(8.72)}$$

Table 8–8 Pressure Solution at $t_{n+1} = 30.42$ **Days for Successive Iterations**

$\nu+1$	$\overset{(\nu+1)}{p_1^{n+1}}$ (psia)	$\overset{(\nu+1)}{p_2^{n+1}}$ (psia)	$\overset{(\nu+1)}{p_3^{n+1}}$ (psia)	$\overset{(\nu+1)}{p_4^{n+1}}$ (psia)
0	4015.00	4015.00	4015.00	4015.00
1	1559.88	1666.08	1772.22	1875.30
2	1599.52	1788.20	1966.57	2131.72
3	1597.28	1773.65	1937.34	2087.32
4	1597.54	1775.64	1941.60	2094.01
5	1597.51	1775.38	1941.02	2093.08

Table 8–9 Converged Pressure Solution and Gas Production at Various Times

$n+1$	Time (day)	ν	p_1^{n+1} (psia)	p_2^{n+1} (psia)	p_3^{n+1} (psia)	p_4^{n+1} (psia)	q_{gsc}^{n+1} (MMscf/D)	Cumulative production (MMMscf)
1	30.42	5	1597.51	1775.38	1941.02	2093.08	−11.3980	−0.346727
2	60.84	3	1537.18	1588.10	1637.63	1685.01	−2.95585	−0.436644
3	91.26	3	1521.54	1536.87	1552.07	1566.82	−0.863641	−0.462916
4	121.68	2	1517.03	1521.84	1526.63	1531.31	−0.268151	−0.471073
5	152.10	2	1515.62	1517.10	1518.58	1520.02	−0.082278	−0.473576
6	182.52	2	1515.19	1515.64	1516.09	1516.54	−0.025150	−0.474341

The results of solving Eqs. 8.66, 8.68, 8.70, and 8.72 for the unknown pressures are $\overset{(1)}{p_1^{n+1}} = 1599.52$ psia, $\overset{(1)}{p_2^{n+1}} = 1788.20$ psia, $\overset{(1)}{p_3^{n+1}} = 1966.57$ psia, and $\overset{(1)}{p_4^{n+1}} = 2131.72$ psia.

Iterations continue until the convergence criterion is satisfied. Table 8–8 shows the successive iterations for the first time step. Note that it took five iterations to converge. The convergence criterion was set as given by Eq. 8.63. After reaching convergence, the time is incremented by $\Delta t = 30.42$ days and the above procedure is repeated. Table 8–9 shows the converged solutions at various times up to six months of simulation time.

Example 8.3 Consider the problem described in Example 8.1. Apply Newton's iteration method to find the pressure distribution in the reservoir after one month (30.42 days) using a single time step, and present the simulation results up to six months.

Solution

Table 8–2 reports the gridblock locations, bulk volumes, and geometric factors in the radial direction. For single-well simulation in a horizontal reservoir (Z_n = constant) with no-flow boundaries ($\sum_{l \in \xi_n} q_{sc_{l,n}}^{n+1} = 0$), the implicit flow equation with implicit transmissibility is obtained from Eq. 8.49a.

For gridblock n with a well operating under a specified FBHP,

$$\sum_{l\in\psi_n}\{\overset{(\nu)}{T_{l,n}^{n+1}}+(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})\frac{\partial T_{l,n}}{\partial p_l}\bigg|^{\overset{(\nu)}{n+1}}\}\delta \overset{(\nu+1)}{p_l^{n+1}}$$
$$-\{\sum_{l\in\psi_n}[\overset{(\nu)}{T_{l,n}^{n+1}}-(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})\frac{\partial T_{l,n}}{\partial p_n}\bigg|^{\overset{(\nu)}{n+1}}]-\frac{dq_{sc_n}}{dp_n}\bigg|^{\overset{(\nu)}{n+1}}+\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n\}\delta\overset{(\nu+1)}{p_n^{n+1}} \tag{8.73a}$$
$$=-\{\sum_{l\in\psi_n}\overset{(\nu)}{T_{l,n}^{n+1}}(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})+\overset{(\nu)}{q_{sc_n}^{n+1}}-\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n(\overset{(\nu)}{p_n^{n+1}}-p_n^n)\}.$$

For gridblock n without a well,

$$\sum_{l\in\psi_n}\{\overset{(\nu)}{T_{l,n}^{n+1}}+(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})\frac{\partial T_{l,n}}{\partial p_l}\bigg|^{\overset{(\nu)}{n+1}}\}\delta\overset{(\nu+1)}{p_l^{n+1}}-\{\sum_{l\in\psi_n}[\overset{(\nu)}{T_{l,n}^{n+1}}-(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})\frac{\partial T_{l,n}}{\partial p_n}\bigg|^{\overset{(\nu)}{n+1}}]$$
$$+\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n\}\delta\overset{(\nu+1)}{p_n^{n+1}}=-\{\sum_{l\in\psi_n}\overset{(\nu)}{T_{l,n}^{n+1}}(\overset{(\nu)}{p_l^{n+1}}-\overset{(\nu)}{p_n^{n+1}})-\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n(\overset{(\nu)}{p_n^{n+1}}-p_n^n)\}. \tag{8.73b}$$

As mentioned in Example 8.1, gridblock 4 is upstream to gridblock 3, gridblock 3 is upstream to gridblock 2, and gridblock 2 is upstream to gridblock 1. Upstream weighting of the pressure dependent terms in transmissibility is used.

First time step calculations ($n=0$, $t_{n+1}=30.42$ days, and $\Delta t=30.42$ days)

For the first iteration ($\nu=0$), assume $\overset{(0)}{p_n^{n+1}}=p_n^n=4015$ psia for $n=1,2,3,4$.

Consequently, $T_{n,n+1}\Big|_n^{\overset{(0)}{n+1}}=95877.5281$ for all gridblocks,

$(\overset{(0)}{p_l^{n+1}}-\overset{(0)}{p_n^{n+1}})\frac{\partial T_{l,n}}{\partial p_l}\bigg|^{\overset{(0)}{n+1}}=0$ for all values of l and n, and $\frac{V_{b_n}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_n$ is obtained as shown in Table 8–3.

For wellblock 1, $\frac{d}{dp}(\frac{1}{\mu B})\bigg|_1^{\overset{(0)}{n+1}}=2.970747$,

$$\overset{(0)}{q_{sc_1}^{n+1}}=-G_{w_1}(\frac{1}{\mu B})_1^{\overset{(0)}{n+1}}(\overset{(0)}{p_1^{n+1}}-p_{wf_1})$$

$$=-3.941572\times(\frac{1}{0.0223000\times0.0007790})\times(4015-1515)=-567240397,$$

$$\frac{\mathrm{d}q_{sc_1}}{\mathrm{d}p_1}\Bigg|_{1}^{\overset{(0)}{n+1}} = -G_{w_1}[(\frac{1}{\mu B})_1^{\overset{(0)}{n+1}} + \frac{\mathrm{d}}{\mathrm{d}p}(\frac{1}{\mu B})\Bigg|_{1}^{\overset{(0)}{n+1}} (p_1^{\overset{(0)}{n+1}} - p_{wf_1})]$$

$$= -3.941572 \times [(\frac{1}{0.0223000 \times 0.0007790}) + 2.970747 \times (4015 - 1515)] = -256169.692.$$

In addition, the flow equation for gridblock n with a well (Eq. 8.73a) reduces to

$$\sum_{l \in \psi_n} T_{l,n}^{\overset{(0)}{n+1}} \delta p_l^{\overset{(1)}{n+1}} - \{\sum_{l \in \psi_n} T_{l,n}^{\overset{(0)}{n+1}} - \frac{\mathrm{d}q_{sc_n}}{\mathrm{d}p_n}\Bigg|^{\overset{(0)}{n+1}} + \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n\} \delta p_n^{\overset{(1)}{n+1}} = -q_{sc_n}^{\overset{(0)}{n+1}} \tag{8.74a}$$

and that for gridblock n without a well (Eq. 8.73b) reduces to

$$\sum_{l \in \psi_n} T_{l,n}^{\overset{(0)}{n+1}} \delta p_l^{\overset{(1)}{n+1}} - \{\sum_{l \in \psi_n} T_{l,n}^{\overset{(0)}{n+1}} + \frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n\} \delta p_n^{\overset{(1)}{n+1}} = 0. \tag{8.74b}$$

For gridblock 1, $n = 1$ and $\psi_1 = \{2\}$. Substitution of the relevant values in Eq. 8.74a yields

$$-\{95877.5281 - (-256169.692) + 0.06193233\} \times \delta p_1^{\overset{(1)}{n+1}} + 95877.5281 \times \delta p_2^{\overset{(1)}{n+1}}$$
$$= -(-567240397)$$

or

$$-352047.281 \times \delta p_1^{\overset{(1)}{n+1}} + 95877.5281 \times \delta p_2^{\overset{(1)}{n+1}} = 567240397 \tag{8.75}$$

For gridblock 2, $n = 2$ and $\psi_2 = \{1, 3\}$. Substitution of the relevant values in Eq. 8.74b results in

$$95877.5281 \times \delta p_1^{\overset{(1)}{n+1}} + 95877.5281 \times \delta p_3^{\overset{(1)}{n+1}}$$
$$-\{95877.5281 + 95877.5281 + 2.842428\} \times \delta p_2^{\overset{(1)}{n+1}} = 0$$

or

$$95877.5281 \times \delta p_1^{\overset{(1)}{n+1}} - 191757.899 \times \delta p_2^{\overset{(1)}{n+1}} + 95877.5281 \times \delta p_3^{\overset{(1)}{n+1}} = 0 \tag{8.76}$$

For gridblock 3, $n = 3$ and $\psi_3 = \{2, 4\}$. Substitution of the relevant values in Eq. 8.74b results in

$$95877.5281 \times \delta p_2^{\overset{(1)}{n+1}} + 95877.5281 \times \delta p_4^{\overset{(1)}{n+1}}$$
$$-\{95877.5281 + 95877.5281 + 130.4553\} \times \delta p_3^{\overset{(1)}{n+1}} = 0$$

or

$$95877.5281 \times \delta \overset{(1)}{p_2^{n+1}} - 191885.511 \times \delta \overset{(1)}{p_3^{n+1}} + 95877.5281 \times \delta \overset{(1)}{p_4^{n+1}} = 0 \quad \textbf{(8.77)}$$

For gridblock 4, $n = 4$ and $\psi_4 = \{3\}$. Substitution of the relevant values in Eq. 8.74b results in

$$95877.5281 \times \delta \overset{(1)}{p_3^{n+1}} - \{95877.5281 + 4619.097\} \times \delta \overset{(1)}{p_4^{n+1}} = 0$$

or

$$95877.5281 \times \delta \overset{(1)}{p_3^{n+1}} - 100496.626 \times \delta \overset{(1)}{p_4^{n+1}} = 0 \quad \textbf{(8.78)}$$

The results of solving Eqs. 8.75 through 8.78 for the pressure change over the first iteration are $\delta \overset{(1)}{p_1^{n+1}} = -2179.03$, $\delta \overset{(1)}{p_2^{n+1}} = -2084.77$, $\delta \overset{(1)}{p_3^{n+1}} = -1990.57$ and $\delta \overset{(1)}{p_4^{n+1}} = -1899.08$. Therefore, $\overset{(1)}{p_1^{n+1}} = 1835.97$ psia, $\overset{(1)}{p_2^{n+1}} = 1930.23$ psia, $\overset{(1)}{p_3^{n+1}} = 2024.43$ psia, and $\overset{(1)}{p_4^{n+1}} = 2115.92$ psia.

For the second iteration ($\nu = 1$), we use $\overset{(1)}{p_n^{n+1}}$ to estimate the values of FVF, gas viscosity, $(\frac{\phi}{B_g})'_n$, $\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n$, transmissibility and its derivative with respect to block pressure. Table 8–10 lists these values.

For example, for gridblock 1,

$$(\frac{\phi}{B_g})'_1 = \frac{(\frac{\phi}{B})_1^{\overset{(\nu)}{n+1}} - (\frac{\phi}{B})_1^n}{\overset{(\nu)}{p_1^{n+1}} - p_1^n} = \frac{(\frac{0.13}{0.00161207}) - (\frac{0.13}{0.000779})}{1835.97 - 4015} = 0.03957679,$$

$$\frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 = \frac{340.59522 \times 0.03957679}{5.614583 \times 30.42} = 0.07892278,$$

$$\overset{(\nu)}{T_{r_{1,2}}^{n+1}} = T_{1,2}\Big|_2^{\overset{(\nu)}{n+1}} = G_{r_{1+1/2}}(\frac{1}{\mu B})_2^{\overset{(\nu)}{n+1}} = 1.6655557 \times (\frac{1}{0.01588807 \times 0.00153148}) = 68450.4979,$$

$$\frac{\partial T_{1,2}}{\partial p_1}\Big|_2^{\overset{(\nu)}{n+1}} = 0, \text{ and } \frac{\partial T_{1,2}}{\partial p_2}\Big|_2^{\overset{(\nu)}{n+1}} = G_{r_{1+1/2}} \frac{\mathrm{d}}{\mathrm{d}p}(\frac{1}{\mu B})\Big|_2^{\overset{(\nu)}{n+1}} = 1.6655557 \times 16.47741 = 27.444044$$

for upstream weighting of transmissibility. In addition, for the production well in wellblock 1,

Table 8–10 Estimated Gridblock Functions at Old Iteration $\nu = 1$

n	$\overset{(1)}{p_n^{n+1}}$ (psia)	$\overset{(1)}{B_{g_n}^{n+1}}$ (RB/scf)	$\overset{(1)}{\mu_{g_n}^{n+1}}$ (cp)	$(\phi / B_g)'_n$	$\frac{V_{b_n}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_n$	$\frac{d}{dp}(\frac{1}{\mu B})\Big\vert_n^{\overset{(\nu)}{n+1}}$	$\frac{\partial T_{n,n+1}}{\partial p_n}\Big\vert_n^{\overset{(\nu)}{n+1}}$	$T_{n,n+1}\Big\vert_n^{\overset{(\nu)}{n+1}}$
1	1835.97	0.00161207	0.01565241	0.03957679	0.07892278	14.68929	24.465831	66007.6163
2	1930.23	0.00153148	0.01588807	0.03933064	3.599688	16.47741	27.444044	68450.4979
3	2024.43	0.00145235	0.01612828	0.03886858	163.2694	12.78223	21.289516	71104.7736
4	2115.92	0.00138785	0.01640276	0.03855058	5733.667	14.28023	23.784518	73164.3131

$$\overset{(\nu)}{q_{sc_1}^{n+1}} = -G_{w_1} (\frac{1}{\mu B})_1^{\overset{(\nu)}{n+1}} (\overset{(\nu)}{p_1^{n+1}} - p_{wf_1})$$

$$= -3.941572 \times (\frac{1}{0.01565241 \times 0.00161207}) \times (1835.97 - 1515) = -50137330$$

$$\frac{dq_{sc_1}}{dp_1}\Bigg\vert^{\overset{(\nu)}{n+1}} = -G_{w_1} [(\frac{1}{\mu B})_1^{\overset{(\nu)}{n+1}} + \frac{d}{dp}(\frac{1}{\mu B})\Bigg\vert_1^{\overset{(\nu)}{n+1}} (\overset{(\nu)}{p_1^{n+1}} - p_{wf_1})]$$

or

$$\frac{dq_{sc_1}}{dp_1}\Bigg\vert^{\overset{(\nu)}{n+1}} = -3.941572 \times [(\frac{1}{0.01565241 \times 0.00161207}) + 14.68929 \times (1835.97 - 1515)]$$

$= -174971.4.$

For gridblock 1, $n = 1$ and $\psi_1 = \{2\}$. Therefore, 8.73a becomes

$$\begin{aligned} &-[T_{1,2}\Big\vert_2^{\overset{(\nu)}{n+1}} - (\overset{(\nu)}{p_2^{n+1}} - \overset{(\nu)}{p_1^{n+1}}) \frac{\partial T_{1,2}}{\partial p_1}\Bigg\vert_2^{\overset{(\nu)}{n+1}} - \frac{dq_{sc_1}}{dp_1}\Bigg\vert^{\overset{(\nu)}{n+1}} + \frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1] \delta \overset{(\nu+1)}{p_1^{n+1}} \\ &+[T_{1,2}\Big\vert_2^{\overset{(\nu)}{n+1}} + (\overset{(\nu)}{p_2^{n+1}} - \overset{(\nu)}{p_1^{n+1}}) \frac{\partial T_{1,2}}{\partial p_2}\Bigg\vert_2^{\overset{(\nu)}{n+1}}] \delta \overset{(\nu+1)}{p_2^{n+1}} \\ &= -\{T_{1,2}\Big\vert_2^{\overset{(\nu)}{n+1}} (\overset{(\nu)}{p_2^{n+1}} - \overset{(\nu)}{p_1^{n+1}}) + \overset{(\nu)}{q_{sc_1}^{n+1}} - \frac{V_{b_1}}{\alpha_c \Delta t}(\frac{\phi}{B_g})'_1 (\overset{(\nu)}{p_1^{n+1}} - p_1^n)\}. \end{aligned} \tag{8.79}$$

Substitution of the values in Eq. 8.79 gives

$$-[68450.4979 - (1930.23 - 1835.97) \times 0 - (-174791.4) + 0.07892278] \delta \overset{(\nu+1)}{p_1^{n+1}}$$

$$+[68450.4979 + (1930.23 - 1835.97) \times 27.444044] \delta \overset{(\nu+1)}{p_2^{n+1}}$$

$$= -\{68450.4979 \times (1930.23 - 1835.97) + (-50137330) - 0.07892278 \times (1835.97 - 4015)\}.$$

After simplification, the equation becomes

$$-243242.024\times\delta\overset{(\nu+1)}{p_1^{n+1}}+71037.4371\times\delta\overset{(\nu+1)}{p_2^{n+1}}=43684856.7 \tag{8.80}$$

For gridblock 2, $n=2$ and $\psi_2=\{1,3\}$. Therefore, Eq. 8.73b becomes

$$\begin{aligned}
&[\left.\overset{(\nu)}{T_{1,2}}\right|_2^{n+1}+(\overset{(\nu)}{p_1^{n+1}}-\overset{(\nu)}{p_2^{n+1}})\left.\frac{\partial T_{1,2}}{\partial p_1}\right|_2^{\overset{(\nu)}{n+1}}]\delta\overset{(\nu+1)}{p_1^{n+1}}\\
&-[\left.\overset{(\nu)}{T_{1,2}}\right|_2^{n+1}-(\overset{(\nu)}{p_1^{n+1}}-\overset{(\nu)}{p_2^{n+1}})\left.\frac{\partial T_{1,2}}{\partial p_2}\right|_2^{\overset{(\nu)}{n+1}}+\left.\overset{(\nu)}{T_{3,2}}\right|_3^{n+1}-(\overset{(\nu)}{p_3^{n+1}}-\overset{(\nu)}{p_2^{n+1}})\left.\frac{\partial T_{3,2}}{\partial p_2}\right|_3^{\overset{(\nu)}{n+1}}\\
&+\frac{V_{b_2}}{\alpha_c\Delta t}(\frac{\phi}{B_g})_2']\delta\overset{(\nu+1)}{p_2^{n+1}}+[\left.\overset{(\nu)}{T_{3,2}}\right|_3^{n+1}+(\overset{(\nu)}{p_3^{n+1}}-\overset{(\nu)}{p_2^{n+1}})\left.\frac{\partial T_{3,2}}{\partial p_3}\right|_3^{\overset{(\nu)}{n+1}}]\delta\overset{(\nu+1)}{p_3^{n+1}}\\
&=-\{[\left.\overset{(\nu)}{T_{1,2}}\right|_2^{n+1}(\overset{(\nu)}{p_1^{n+1}}-\overset{(\nu)}{p_2^{n+1}})+\left.\overset{(\nu)}{T_{3,2}}\right|_3^{n+1}(\overset{(\nu)}{p_3^{n+1}}-\overset{(\nu)}{p_2^{n+1}})]-\frac{V_{b_2}}{\alpha_c\Delta t}(\frac{\phi}{B_g})_2'(\overset{(\nu)}{p_2^{n+1}}-p_2^n)\}.
\end{aligned} \tag{8.81}$$

In the above equation,

$$\overset{(\nu)}{T_{r_{3,2}}^{n+1}}=\left.\overset{(\nu)}{T_{3,2}}\right|_3^{n+1}=G_{r_{2+1/2}}(\frac{1}{\mu B})_3^{\overset{(\nu)}{n+1}}=1.6655557\times(\frac{1}{0.01612828\times0.00145235})=71104.7736,$$

$$\left.\frac{\partial T_{3,2}}{\partial p_2}\right|_3^{\overset{(\nu)}{n+1}}=0,\ \text{and}\ \left.\frac{\partial T_{3,2}}{\partial p_3}\right|_3^{\overset{(\nu)}{n+1}}=G_{r_{2+1/2}}\left.\frac{\mathrm{d}}{\mathrm{d}p}(\frac{1}{\mu B})\right|_3^{\overset{(\nu)}{n+1}}=1.6655557\times12.78223=21.289516.$$

Substitution of these values in Eq. 8.81 gives

$$\begin{aligned}
&[68450.4979+(1835.97-1930.23)\times0]\delta\overset{(\nu+1)}{p_1^{n+1}}-[68450.4979-(1835.97-1930.23)\\
&\times27.444044+71104.7736-(2024.43-1930.23)\times0+3.599688]\delta\overset{(\nu+1)}{p_2^{n+1}}\\
&+[71104.7736+(2024.43-1930.23)\times21.289516]\delta\overset{(\nu+1)}{p_3^{n+1}}\\
&=-\{[68450.4979\times(1835.97-1930.23)+71104.7736\times(2024.43-1930.23)]\\
&-3.599688\times(1930.23-4015)\}
\end{aligned}$$

or after simplification,

$$68450.4979\times\delta\overset{(\nu+1)}{p_1^{n+1}}-142145.810\times\delta\overset{(\nu+1)}{p_2^{n+1}}+73110.2577\times\delta\overset{(\nu+1)}{p_3^{n+1}}=-253308.066 \tag{8.82}$$

For gridblock 3, $n=3$ and $\psi_3=\{2,4\}$. Therefore, Eq. 8.73b becomes

$$
\begin{aligned}
&[T_{2,3}\Big|_3^{\overset{(\nu)}{n+1}} + (\overset{(\nu)}{p}{}_2^{n+1} - \overset{(\nu)}{p}{}_3^{n+1})\frac{\partial T_{2,3}}{\partial p_2}\Big|_3^{\overset{(\nu)}{n+1}}]\delta \overset{(\nu+1)}{p}{}_2^{n+1} \\
&-[T_{2,3}\Big|_3^{\overset{(\nu)}{n+1}} - (\overset{(\nu)}{p}{}_2^{n+1} - \overset{(\nu)}{p}{}_3^{n+1})\frac{\partial T_{2,3}}{\partial p_3}\Big|_3^{\overset{(\nu)}{n+1}} + T_{4,3}\Big|_4^{\overset{(\nu)}{n+1}} - (\overset{(\nu)}{p}{}_4^{n+1} - \overset{(\nu)}{p}{}_3^{n+1})\frac{\partial T_{4,3}}{\partial p_3}\Big|_4^{\overset{(\nu)}{n+1}} \\
&+\frac{V_{b_3}}{\alpha_c \Delta t}\left(\frac{\phi}{B_g}\right)_3']\delta \overset{(\nu+1)}{p}{}_3^{n+1} + [T_{4,3}\Big|_4^{\overset{(\nu)}{n+1}} + (\overset{(\nu)}{p}{}_4^{n+1} - \overset{(\nu)}{p}{}_3^{n+1})\frac{\partial T_{4,3}}{\partial p_4}\Big|_4^{\overset{(\nu)}{n+1}}]\delta \overset{(\nu+1)}{p}{}_4^{n+1} \\
&= -\{[T_{2,3}\Big|_3^{\overset{(\nu)}{n+1}}(\overset{(\nu)}{p}{}_2^{n+1} - \overset{(\nu)}{p}{}_3^{n+1}) + T_{4,3}\Big|_4^{\overset{(\nu)}{n+1}}(\overset{(\nu)}{p}{}_4^{n+1} - \overset{(\nu)}{p}{}_3^{n+1})] - \frac{V_{b_3}}{\alpha_c \Delta t}\left(\frac{\phi}{B_g}\right)_3'(\overset{(\nu)}{p}{}_3^{n+1} - p_3^n)\}
\end{aligned}
\tag{8.83}
$$

where

$$\overset{(\nu)}{T}{}_{r_{4,3}}^{n+1} = T_{4,3}\Big|_4^{\overset{(\nu)}{n+1}} = G_{r_{3+1/2}}\left(\frac{1}{\mu B}\right)_4^{\overset{(\nu)}{n+1}} = 1.6655557 \times \left(\frac{1}{0.01640276 \times 0.00138785}\right) = 73164.3131,$$

$$\frac{\partial T_{4,3}}{\partial p_3}\Big|_4^{\overset{(\nu)}{n+1}} = 0, \text{ and } \frac{\partial T_{4,3}}{\partial p_4}\Big|_4^{\overset{(\nu)}{n+1}} = G_{r_{3+1/2}}\frac{\mathrm{d}}{\mathrm{d}p}\left(\frac{1}{\mu B}\right)\Big|_4^{\overset{(\nu)}{n+1}} = 1.6655557 \times 14.28023 = 23.784518.$$

Substitution of these values in Eq. 8.83 gives

$$
\begin{aligned}
&[71104.7736 + (1930.23 - 2024.43) \times 0]\delta \overset{(\nu+1)}{p}{}_2^{n+1} - [71104.7736 - (1930.23 - 2024.43) \\
&\times 21.289516 + 73164.3131 - (2115.92 - 2024.43) \times 0 + 163.2694]\delta \overset{(\nu+1)}{p}{}_3^{n+1} \\
&+[73164.3131 + (2115.92 - 2024.43) \times 23.784518]\delta \overset{(\nu+1)}{p}{}_4^{n+1} \\
&= -\{[71104.7736 \times (1930.23 - 2024.43) + 73164.3131 \times (2115.92 - 2024.43)] \\
&-163.2694 \times (2024.43 - 4015)\}.
\end{aligned}
$$

After simplification, the equation becomes

$$71104.7736 \times \delta \overset{(\nu+1)}{p}{}_2^{n+1} - 146437.840 \times \delta \overset{(\nu+1)}{p}{}_3^{n+1} + 75340.4074 \times \delta \overset{(\nu+1)}{p}{}_4^{n+1} = -320846.394 \tag{8.84}$$

For gridblock 4, $n = 4$ and $\psi_4 = \{3\}$. Therefore, Eq. 8.73b becomes

$$[\overset{(v)}{T_{3,4}}\Big|_4^{n+1}+(\overset{(v)}{p_3^{n+1}}-\overset{(v)}{p_4^{n+1}})\frac{\partial T_{3,4}}{\partial p_3}\Bigg|_4^{\overset{(v)}{n+1}}]\delta \overset{(v+1)}{p_3^{n+1}}$$

$$-[\overset{(v)}{T_{3,4}}\Big|_4^{n+1}-(\overset{(v)}{p_3^{n+1}}-\overset{(v)}{p_4^{n+1}})\frac{\partial T_{3,4}}{\partial p_4}\Bigg|_4^{\overset{(v)}{n+1}}+\frac{V_{b_4}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_4]\delta \overset{(v+1)}{p_4^{n+1}} \tag{8.85}$$

$$=-\{[\overset{(v)}{T_{3,4}}\Big|_4^{n+1}(\overset{(v)}{p_3^{n+1}}-\overset{(v)}{p_4^{n+1}})]-\frac{V_{b_4}}{\alpha_c\Delta t}(\frac{\phi}{B_g})'_4(\overset{(v)}{p_4^{n+1}}-p_4^n)\}.$$

Substitution of the values in Eq. 8.85 gives

$$[73164.3131+(2024.43-2115.92)\times 0]\delta \overset{(v+1)}{p_3^{n+1}}$$

$$-[73164.3131-(2024.43-2115.92)\times 23.784518+5733.667]\delta \overset{(v+1)}{p_4^{n+1}}$$

$$=-\{[73164.3131\times(2024.43-2115.92)]-5733.667\times(2115.92-4015)\}.$$

After simplification, the equation becomes

$$73164.3131\times\delta \overset{(v+1)}{p_3^{n+1}}-81074.0745\times\delta \overset{(v+1)}{p_4^{n+1}}=-4194735.68 \tag{8.86}$$

The results of solving Eqs. 8.80, 8.82, 8.84, and 8.86 for the pressure change over the second iteration are $\delta \overset{(2)}{p_1^{n+1}}=-221.97$, $\delta \overset{(2)}{p_2^{n+1}}=-145.08$, $\delta \overset{(2)}{p_3^{n+1}}=-77.72$ and $\delta \overset{(2)}{p_4^{n+1}}=-18.40$. Therefore, $\overset{(2)}{p_1^{n+1}}=1614.00$ psia, $\overset{(2)}{p_2^{n+1}}=1785.15$ psia, $\overset{(2)}{p_3^{n+1}}=1946.71$ psia, and $\overset{(2)}{p_4^{n+1}}=2097.52$ psia. Iterations continue until the convergence criterion is satisfied. Table 8–11 shows the successive iterations for the first time step. As can be seen, it took four iterations to converge. The convergence criterion was set as given by Eq. 8.63. After reaching convergence, the time is incremented by $\Delta t = 30.42$ days and the above procedure is repeated. Table 8–12 shows the converged solutions at various times up to six months of simulation time.

8.6 Summary

The flow equation for an incompressible fluid (Eq. 8.1) is linear. The flow equation for a slightly compressible fluid has very weak nonlinearity caused by the product μB that appears in the interblock flow terms, fictitious well flow rate, and well production rate. This product can be assumed constant without introducing noticeable errors; hence, the flow equation for a slightly compressible fluid becomes linear (Eq. 8.9). The flow equation for a compressible fluid has weak nonlinearity, but it needs to be linearized. Linearization involves treatment in both space and time of the transmissibilities, well production rate, fic-

Table 8–11 Pressure Solution at $t_{n+1} = 30.42$ Days for Successive Iterations

$\nu+1$	$p_1^{n+1\,(\nu+1)}$ (psia)	$p_2^{n+1\,(\nu+1)}$ (psia)	$p_3^{n+1\,(\nu+1)}$ (psia)	$p_4^{n+1\,(\nu+1)}$ (psia)
0	4015.00	4015.00	4015.00	4015.00
1	1835.97	1930.23	2024.43	2115.92
2	1614.00	1785.15	1946.71	2097.52
3	1597.65	1775.45	1941.04	2093.09
4	1597.51	1775.42	1941.09	2093.20

Table 8–12 Converged Pressure Solution and Gas Production at Various Times

$n+1$	Time (day)	ν	p_1^{n+1} (psia)	p_2^{n+1} (psia)	p_3^{n+1} (psia)	p_4^{n+1} (psia)	q_{gsc}^{n+1} (MMscf/D)	Cumulative production (MMMscf)
1	30.42	4	1597.51	1775.42	1941.09	2093.20	–11.3984	–0.346740
2	60.84	3	1537.18	1588.11	1637.66	1685.05	–2.95637	–0.436673
3	91.26	3	1521.54	1536.88	1552.08	1566.84	–0.863862	–0.462951
4	121.68	2	1517.04	1521.84	1526.63	1531.32	–0.268285	–0.471113
5	152.10	2	1515.63	1517.10	1518.58	1520.03	–0.082326	–0.473617
6	182.52	2	1515.19	1515.64	1516.10	1516.54	–0.025165	–0.474382

titious well flow rate, and coefficient of pressure in the accumulation term. Linearization of transmissibility in space uses any of the methods discussed in Section 8.4.1.1. In general, linearization in time of any nonlinear term can be accomplished using the explicit method, simple-iteration method, explicit transmissibility method, simple-iteration on transmissibility method, or fully implicit method. Section 8.4.1.2 presented linearization of transmissibility, Section 8.4.2 presented linearization of the well production rate, Section 8.4.3 presented linearization of fictitious well rates, and Section 8.4.4 presented linearization of the coefficient of pressure change in the accumulation term. The linearized flow equation is obtained by substituting the linearized terms in the flow equation. The linearized 1D flow equation for a compressible fluid is expressed as Eq. 8.34 for the explicit method, Eq. 8.37 for the simple-iteration method, Eq. 8.40 for the explicit transmissibility method, Eq. 8.43 for the simple iteration on transmissibility method, and Eq. 8.48 for the fully implicit method. The corresponding linearized flow equations for a compressible fluid in multiple dimensions are given by Eqs. 8.35, 8.38, 8.41, 8.44, and 8.49, respectively.

8.7 Exercises

8-1 Define the linearity of Eq. 8.1 by examining the various terms in the equation.

8-2 Define the linearity of Eq. 8.9 by examining the various terms in the equation.

8-3 Explain why Eq. 8.5 can be looked at as a nonlinear equation.

8-4 Explain why Eq. 8.11 is a nonlinear equation.

8-5 Examine Eq. 8.30, used for the linearization of the well production rate, and point out the differences between the explicit method (Eq. 8.30a) and the explicit transmissibility method (Eq. 8.30b).

8-6 Examine Eq. 8.31, used for the linearization of the well production rate, and point out the differences between the simple-iteration method (Eq. 8.31a) and the simple iteration on transmissibility method (Eq. 8.31b).

8-7 Consider the one-dimensional, inclined reservoir shown in Figure 8–3. The reservoir is volumetric and homogeneous. The reservoir contains a production well located in gridblock 2. At the time of discovery ($t = 0$), fluids were in hydrodynamic equilibrium and the pressure of gridblock 2 was 3,000 psia. All gridblocks have $\Delta x = 400$ ft, $w = 200$ ft, $h = 80$ ft, $k = 222$ md, and $\phi = 0.20$. The well in gridblock 2 produces fluid at a rate of 10^6 scf/D. Table 8–1 gives the gas FVF and viscosity. Gas density at standard conditions is 0.05343 lbm/ft^3. Estimate the initial pressure distribution in the reservoir. Find the well FBHP and pressure distribution in the system at fifty and one hundred days. Use the implicit formulation with the explicit transmissibility method.

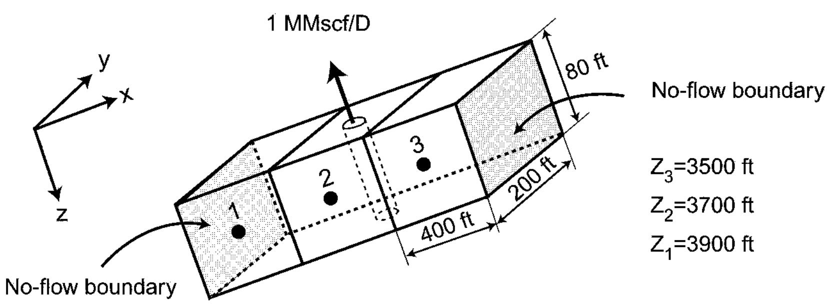


Figure 8–3 Discretized 1D reservoir in Exercise 8-7.

8-8 Consider the 1D flow problem described in Exercise 8-7. Find the pressure distribution in the reservoir at fifty and one hundred days. Use the implicit formulation with the simple iteration on transmissibility method.

8-9 Consider the 1D flow problem described in Exercise 8-7. Find the pressure distribution in the reservoir at fifty and one hundred days. Use the implicit formulation with the fully implicit method.

8-10 A vertical well is drilled on 16-acre spacing in a natural gas reservoir. The reservoir is described by four gridpoints in the radial direction as shown in Figure 8–4. The reservoir is horizontal and has 20-ft net thickness and homogeneous and isotropic rock properties with $k = 10$ md and $\phi = 0.13$. Initially, reservoir pressure is 3015 psia. Table 8–1 presents the gas FVF and viscosity dependence on pressure. The external reservoir boundaries are sealed to fluid flow. Well diameter is 6 in. The well produces under a constant FBHP of 2015 psia. Find the pressure distribution in the reservoir every month (30.42 days) for two months. Take time steps of 30.42 days. Use the implicit formulation with the explicit transmissibility method.

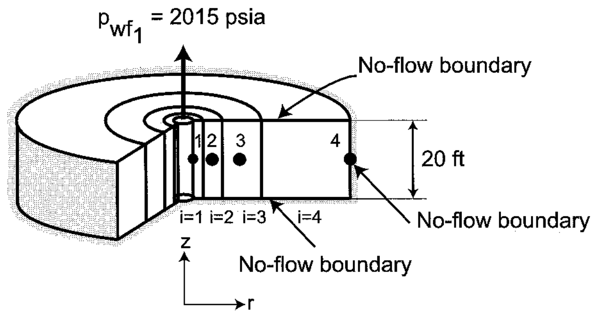


Figure 8–4 Discretized reservoir in Exercise 8-10.

8-11 Consider the single-well simulation problem presented in Exercise 8-10. Find the pressure distribution in the reservoir at one and two months. Use the implicit formulation with the simple iteration on transmissibility method.

8-12 Consider the single-well simulation problem presented in Exercise 8-10. Find the pressure distribution in the reservoir at one and two months. Use the implicit formulation with the fully implicit method.

8-13 Consider the 2D single-phase flow of natural gas taking place in the horizontal, homogeneous reservoir shown in Figure 8–5. The external reservoir boundaries are sealed to fluid flow. Gridblock properties are $\Delta x = \Delta y = 1000$ ft, $h = 25$ ft, and $k_x = k_y = 20$ md and $\phi = 0.12$. Initially, reservoir pressure is 4015 psia. Table 8–1 presents the gas FVF and viscosity dependence on pressure. The well in gridblock 1 produces gas at a rate of 10^6 scf/D. Well diameter is 6 in. Find the pressure distribution in the reservoir and the FBHP of the well every month (30.42 days) for two months. Check the material balance every time step. Use the implicit formulation with the explicit transmissibility method. Observe symmetry and take time steps of 30.42 days.

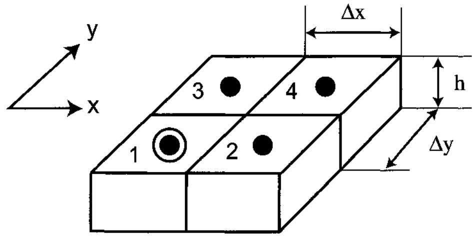


Figure 8–5 Discretized 2D reservoir in Exercise 8-13.

8-14 Consider the 2D flow problem described in Exercise 8-13. Find the pressure distribution in the reservoir and the FBHP of the well at one and two months. Check the material balance every time step. Use the implicit formulation with the simple iteration on transmissibility method.

8-15 Consider the 2D flow problem described in Exercise 8-13. Find the pressure distribution in the reservoir and the FBHP of the well at one and two months. Check the material balance every time step. Use the implicit formulation with the fully implicit method.

8-16 Derive Eq. 8.49b, that represents the fully implicit equation without conservative expansion of accumulation term for compressible fluid, using the method of Coats, Ramesh, and Winestock (1977) as outlined in the text.