

Two-phase flow

Capillary pressure

Relative permeabilities

Theory of Buckley Leverett

Waves associated with conservation laws

Only part discussed during lectures is relevant

$$\frac{\partial \mathbf{G}}{\partial t} + \frac{\partial u \mathbf{F}}{\partial x} = 0$$

Hans Bruining

Contents

- Explain how one can solve the oil displacement problem by water (1D) numerically and compare to the Buckley Leverett theory; ignore gravity
- Run the spreadsheet both for a high oil-water viscosity ratio, one in which the viscosities are about equal, and one for a low oil-water viscosity ratio
- Explain that the tangent to the fractional flow function from the point (S_{wc} , 0) makes it possible to determine the shock saturation.

Capillary pressure

Water wet

Water-wet (oil is non-wetting)

water

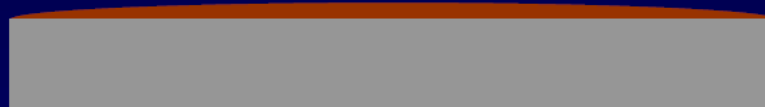


$$\theta = 0^\circ$$

Oil wet

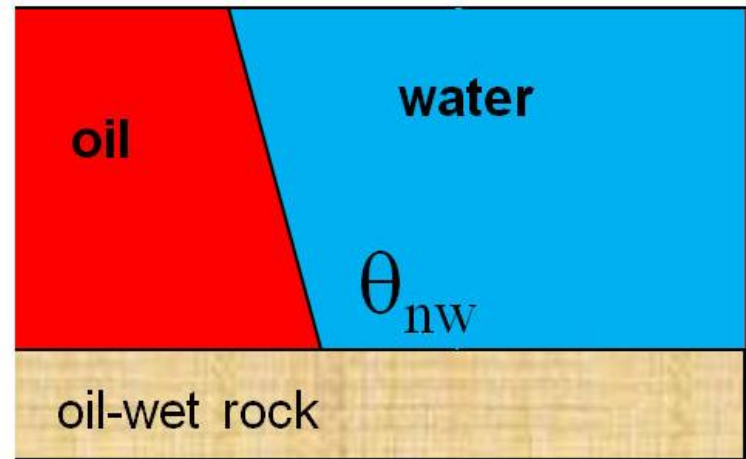
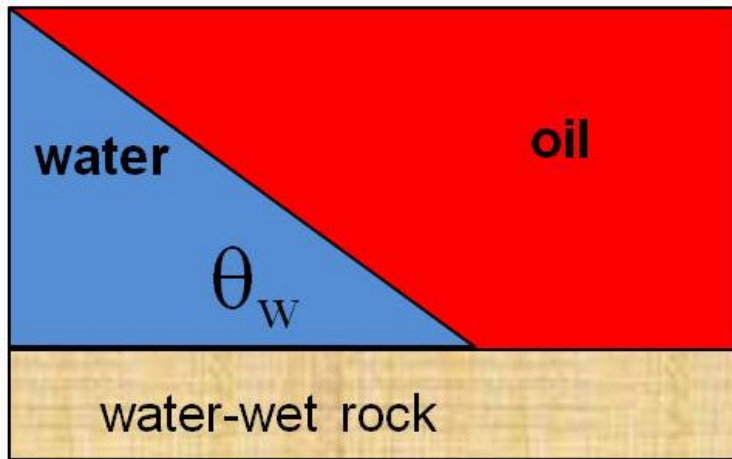
Oil-wet (oil is spreading)

water



$$\theta = 180^\circ$$

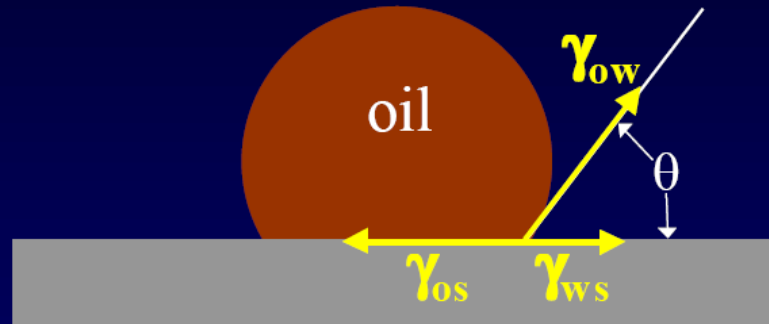
Contact angles for water-wet and oil-wet rock



Partial wetting

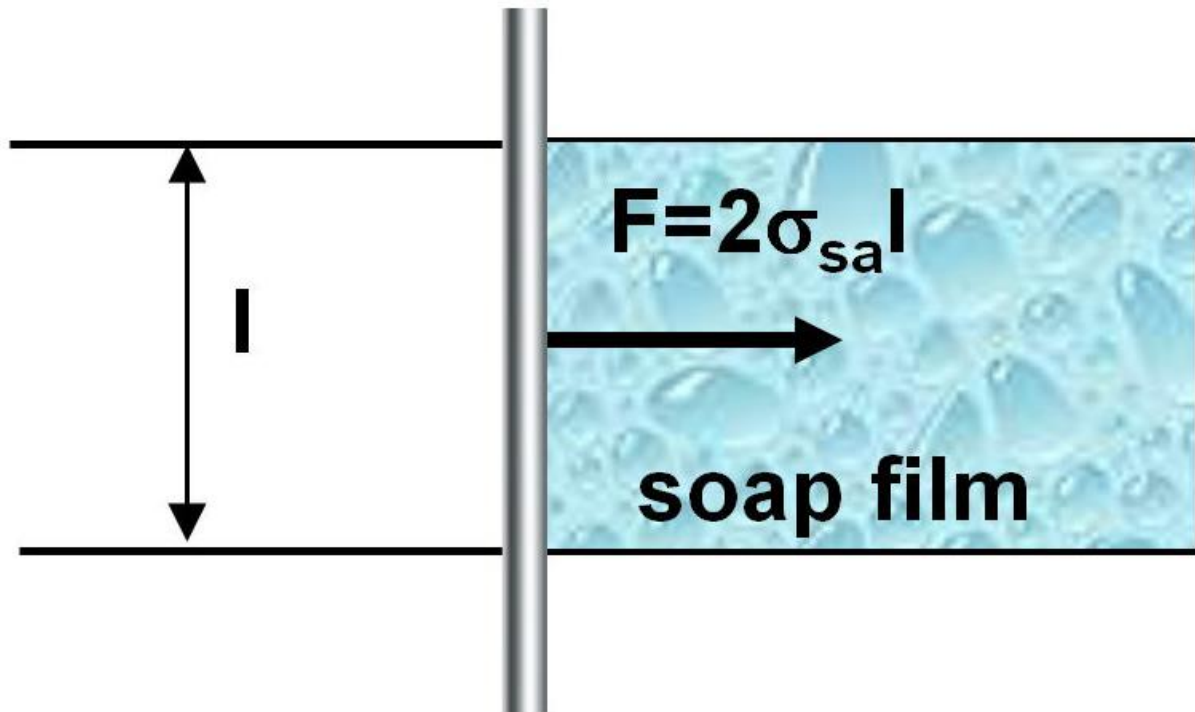
Partial wetting: $0^\circ < \theta < 180^\circ$

water



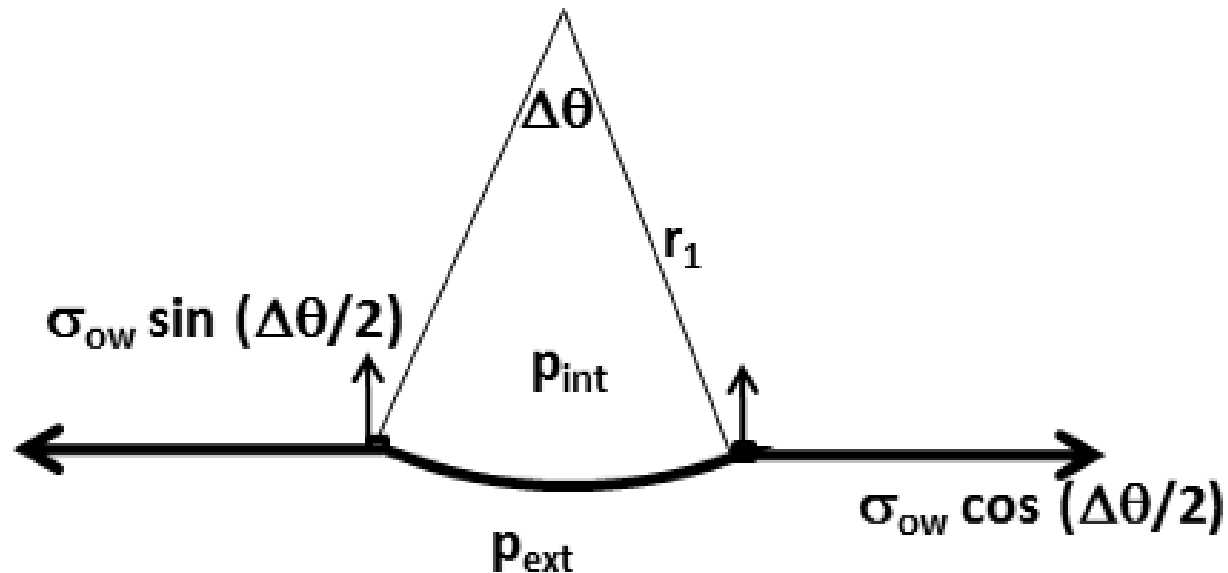
$$\gamma_{os} = \gamma_{ws} + \gamma_{ow} \cos \theta$$

Interfacial tension

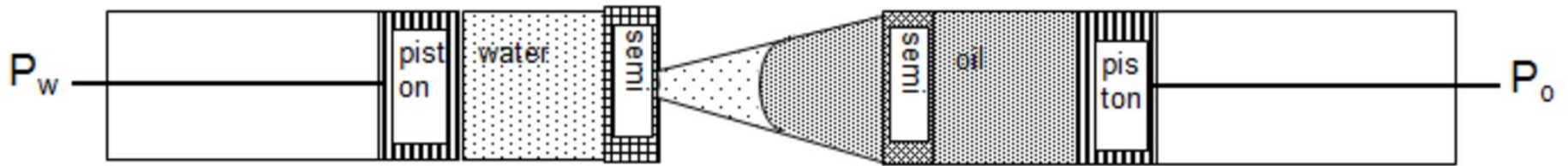


Laplace's formula

$$2\sigma_{gw} \sin(\Delta\theta/2) = (p_{int} - p_{ext})r_1\Delta\theta$$



Capillary pressure



Capillary pressure

$$\begin{aligned}(P_g - P_w)dV &= \sigma_{gs}dA_{gs} + \sigma_{ws}dA_{ws} + \sigma_{gw}dA_{gw} = \\ &= (\sigma_{gs} - \sigma_{ws})dA_{gs} + \sigma_{gw}dA_{gw} = \\ &= \sigma_{gw} \cos \theta dA_{gs} + \sigma_{gw}dA_{gw}\end{aligned}$$

$$P_c = \sigma_{gw} \cos \theta \frac{dA_{gs}}{dV} + \sigma_{gw} \frac{dA_{gw}}{dV}$$

Capillary bundle model

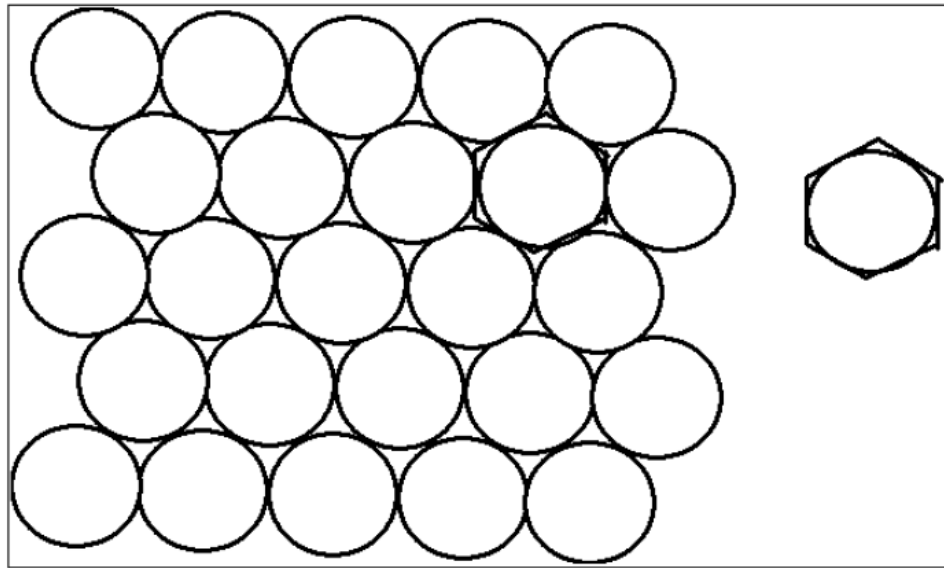


Figure 7: Cross-section through the capillary bundle model.

$$k = \varphi r^2/8$$

On the right we show a hexagonal unit cell with area A . We only need to consider the unit cell. We assume that all cylinders have a radius r . Using Poiseuille flow we now that the total flow rate Q in a unit cell i.e. through a single capillary tube is given by

$$Q = -\frac{\pi r^4}{8\mu} \frac{\Delta\phi}{\Delta L}$$

In this equation we use μ to denote the viscosity, ϕ to denote the potential $\phi = (P + \rho g z)$, L the length of the tubes. Furthermore we use that the hexagonal unit cell has an area A . The cross-section of the tube is given by πr^2 . Therefore the porosity $\varphi = \frac{\pi r^2}{A}$. The Darcy velocity $u = \frac{Q}{A}$.

$$u = -\frac{\pi r^4}{8\mu A} \frac{\Delta\phi}{\Delta L} = -\frac{\varphi r^2}{8\mu} \frac{\Delta\phi}{\Delta L}$$

Leverett

Apparently there exists some relation between the characteristic radius r of the pore size and $\sqrt{\frac{k}{\varphi}}$. This relation was used by Leverett to assume with Laplaces formula (see Equation 4) that $P_c \sim \frac{\sigma}{r} \cos \Theta \sim \sigma \cos \Theta \sqrt{\frac{\varphi}{k}}$. where σ is the interfacial tension and Θ is the contact angle. The term $\sigma \cos \Theta \sqrt{\frac{\varphi}{k}}$ has units [Pa] and the remainder is a dimensionless function. It has been the merit of Leverett [14] to set up this theory, which is relatively simple and sufficiently accurate for engineering applications. Leverett was the first to define a dimensionless capillary pressure function which is now known under the name Leverett J function and can be used to compare pressure data to one another.

$$P_c = \sigma \cos(\Theta) \sqrt{\left(\frac{\phi}{k}\right)} J(S_w). \quad (7)$$

$$P_c = \sigma_{ow} \gamma \sqrt{\frac{\varphi}{k}} \left(\frac{\frac{1}{2} - S_{wc}}{1 - S_{wc}} \right)^{\frac{1}{\lambda}} \left(\frac{S_w - S_{wc}}{1 - S_{wc}} \right)^{\frac{-1}{\lambda}}$$

Assignment

- Take reasonable values for S_{wc} . Use $\gamma = 1/2$
- Draw the capillary pressure curve (log-scale) versus the water saturation for five values of $0.1 < \lambda < 10$

Capillary pressure curves

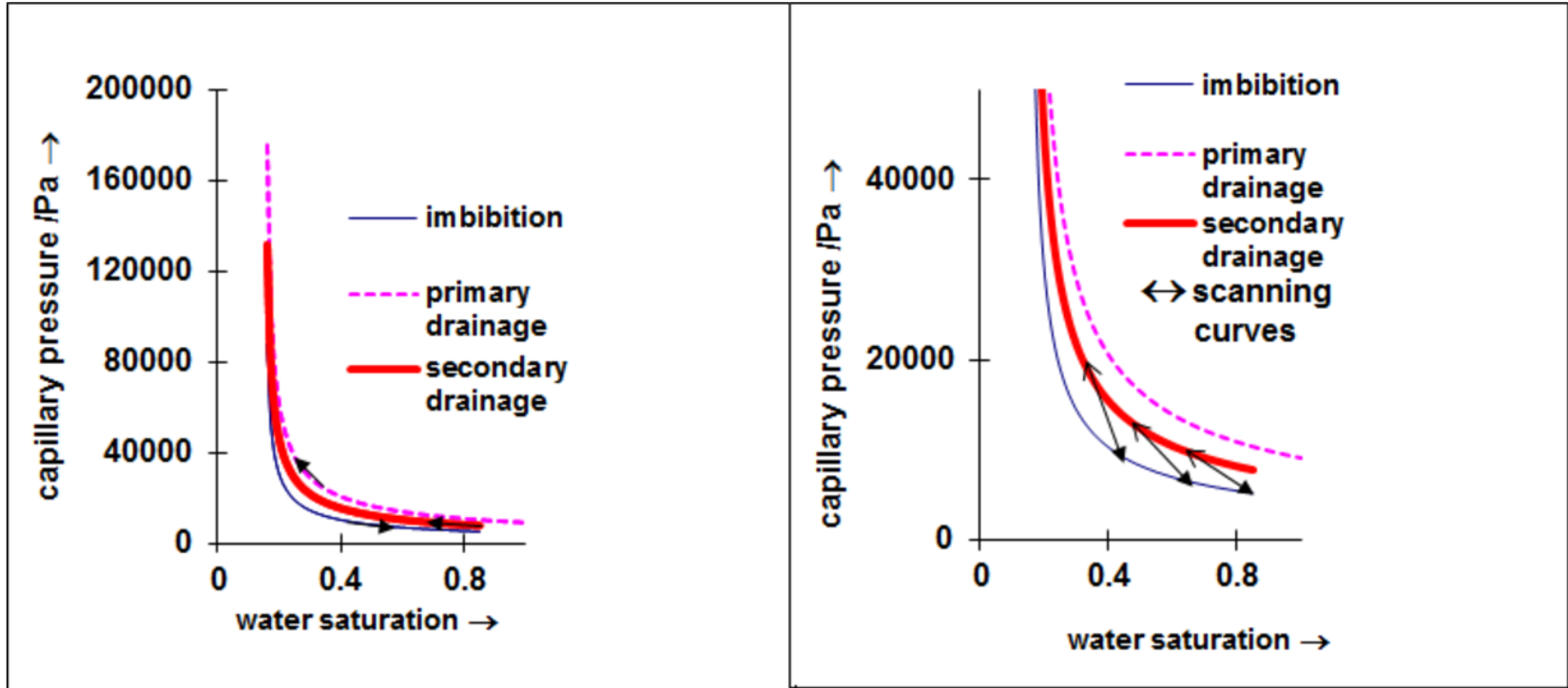


Figure 10: Primary drainage, imbibition and secondary drainage capillary pressure curves. The arrows in the left indicate the direction of saturation. In the right Figure the arrows indicate the imbibition scanning curves.

Assignments surface chemistry

Use internet to find answers to questions that are not treated in the lecture notes

problems from colloid and surface chemistry, J. Theodoor Overbeek, deceased Feb. 2007, A self-study guide

1. What pressure in the atmosphere would be necessary to blow water at 25°C out of a sintered glass filter with a uniform pore diameter of $0.10\text{ }\mu\text{m}$. Answ: 28.4 atmospheres.
2. Two vertical cylindrical glass rods of radius 1 cm and with well polished flat ends are separated by a $1\text{ }\mu\text{m}$ thick layer of water between the two ends. If the water meniscus is semicircular all around the cylindrical circumference, calculate the length the lower cylinder may have before it is detached by gravity from the upper one. Assume the glass density to be 2500 kg/m^3 .

Assignments ctd

3. The Kelvin equation relates the vapor pressure p_r for a curved surface to the vapor pressure p_o for the flat surface the molar volume of the liquid v_L and the pressure difference between the pressure for the curved surface P_2 and for the flat surface P_1 . The surface tension is σ . The equation reads

$$RT \ln \frac{p_r}{p_o} = v_L (P_2 - P_1). \quad (10)$$

The Young-Laplace equation reads

$$P_c = P_2 - P_1 = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right). \quad (11)$$

If the vapor pressure of a rain drop is larger than the pressure on a flat surface, what do you think then of the vapor pressure in a water-wet porous medium? Answ: is lower.

Assignments ctd

4. Most rocks are completely water-wet. Explain why most rocks are covered with a water layer. For which conditions would oil spread on water and when would it form lenses. If oil spreads on water, what is the consequence for residual oil.
5. The isoelectric point is defined as the pH at which the zeta potential is zero (surface charge is zero). At higher pH the surface becomes negatively charged. The i.e.p. of hydrated SiO_2 is 2, and of CaCO_3 is 9. Explain why lime stone reservoirs are often oil-wet. What is the consequence for fractured reservoirs?

Assignments ctd

7. Explain that the capillary pressure P_c in porous media can be represented as

$$P_c = \gamma \sigma \cos \theta \sqrt{\frac{\varphi}{k}} \frac{1/2 - S_{wc}}{1 - S_{wc}}^{1/\lambda} \frac{S_w - S_{wc}}{1 - S_{wc}}^{-1/\lambda}, \quad (12)$$

where φ is the porosity, θ the contact angle, γ is a number between 0.3 and 0.7, k is the permeability, S_{wc} is the connate water saturation, and λ is the sorting factor.

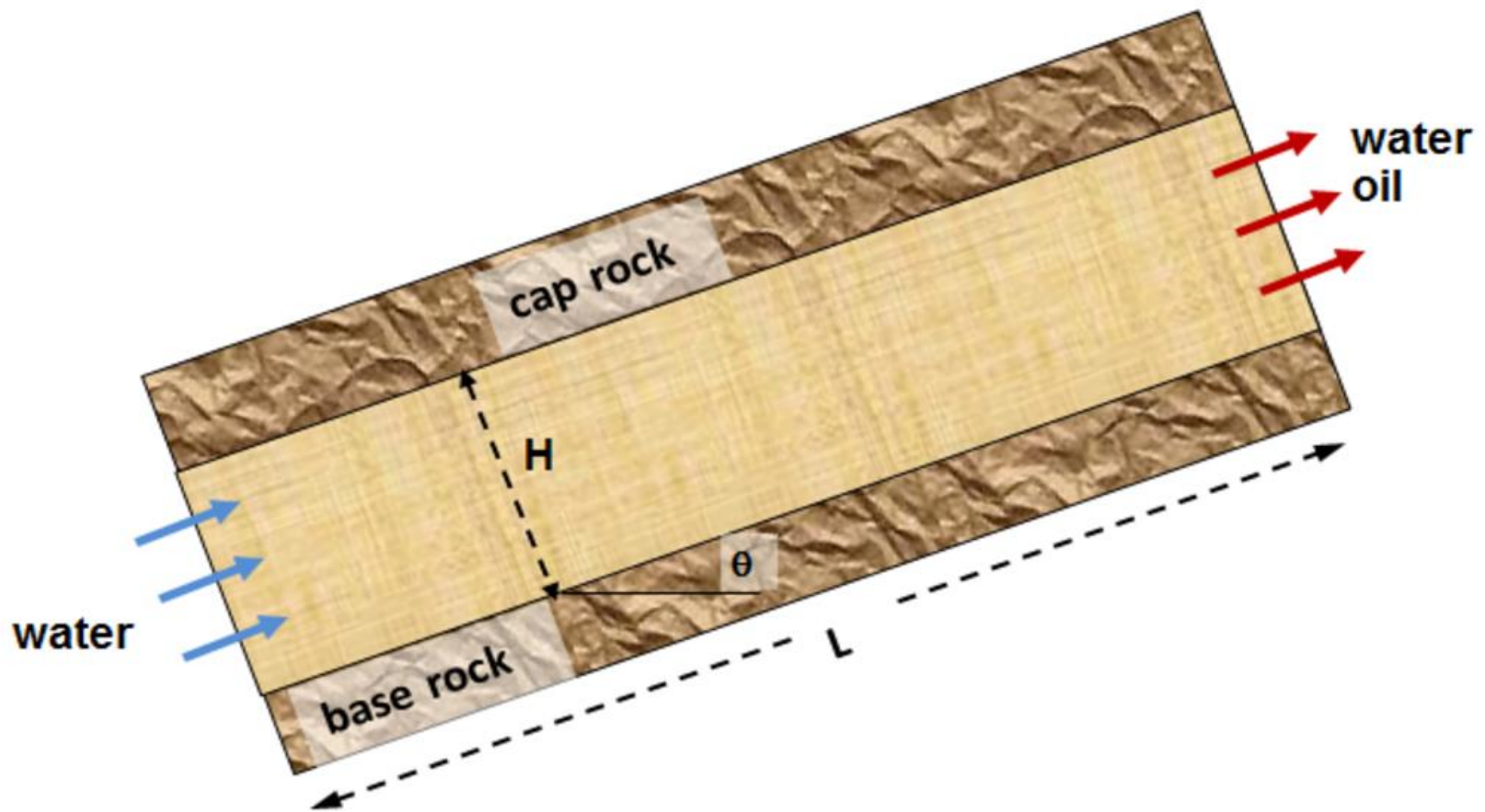
8. Two small sand blocks of equal volume and porosities but with different permeabilities are put in close contact. There is equilibrium. The average saturation $S_w = 0.5$. Use some reasonable numbers in Eq. 12 and determine the saturation in both sand blocks. $\lambda = 0.5$ and $\gamma = 0.5$.
9. Determine the saturation profile in a formation that consists of 20 stacked layers with random permeabilities, log-normally distributed with a Dykstra Parsons' coefficient of $V_{DP} = 0.7$. There is capillary gravity equilibrium. The fluids are oil and water. Use some reasonable values and place the $P_c = 0$ plane at the bottom of the reservoir. Smooth the random field by an averaging window which takes the average of four neighboring permeabilities.

Two-phase flow

Oil displacement by water in layer

Buckley-Leverett theory

Schematic layer



Assumptions for BL theory I

1. We consider a layer of length L , constant thickness $H \ll L$ bounded from above and below respectively by impermeable cap- and base rock. The layer can be under a dip ϑ . Injection and production occurs via a line drive configuration or via edge water drive such that two- dimensional flow in a vertical cross-section occurs. The dip (layer) direction is indicated by "x" and the cross-dip (X-dip) direction is indicated by "y" .
2. The fluids and rock are considered incompressible
3. The permeability and the porosity in the reservoir are considered constant. There exists a water saturation below which water cannot flow in the porous medium. This saturation is called the connate water saturation S_{wc} . The initial water saturation in the oil reservoir is equal to this connate water saturation. There exists also an oil saturation called the residual oil saturation S_{or} below which oil cannot flow.

Assumptions for BL theory II

4. Both the water and oil phase flow independently of each other and behave according to a generalized form of Darcy's law.
5. In the dip direction we disregard (as much as possible) the capillary forces and we take only the viscous and gravity forces into consideration
6. The pressure gradient only depends only on the x - coordinate (in the dip direction) and is thus constant in the X -dip direction. Moreover an equilibrium saturation has been attained balancing gravity and capillary forces in the X -dip direction subject to a given capillary pressure at the boundary between the reservoir and the base rock. Conditions are such that capillary forces dominate over gravity forces in the X -dip direction. Therefore the saturation is approximately constant in the X -dip direction. This is also called completely dispersed flow. This reduces the physical problem to a problem in $1 - D$.

Notions in porous media flow

- Porosity ϕ : Void fraction
- Saturation S_{α} : Volume fraction of void occupied by Liquid $\alpha=w,o,g$
- Irreducible oil saturation S_{or} : Saturation at and below which oil is not able to flow if subjected to a pressure gradient
- Connate saturation S_{wc} : Saturation at and below which water is not able to flow if subjected to a pressure gradient

Notions Ctd. Darcy's law

- p =pressure [Pa]
- Total flow rate= Q [m^3/s] (total discharge)
- Darcy velocity $u=Q/A$, where A is surface area ($u=q$ =specific discharge)
- Permeability k (1-Darcy = $0.987\text{e-}12 \text{ m}^2$)
- Viscosity: μ [Pa s] ($1\text{cP}=1\text{x}10^{-3} \text{ Pa s}$)
- Darcy's Law:

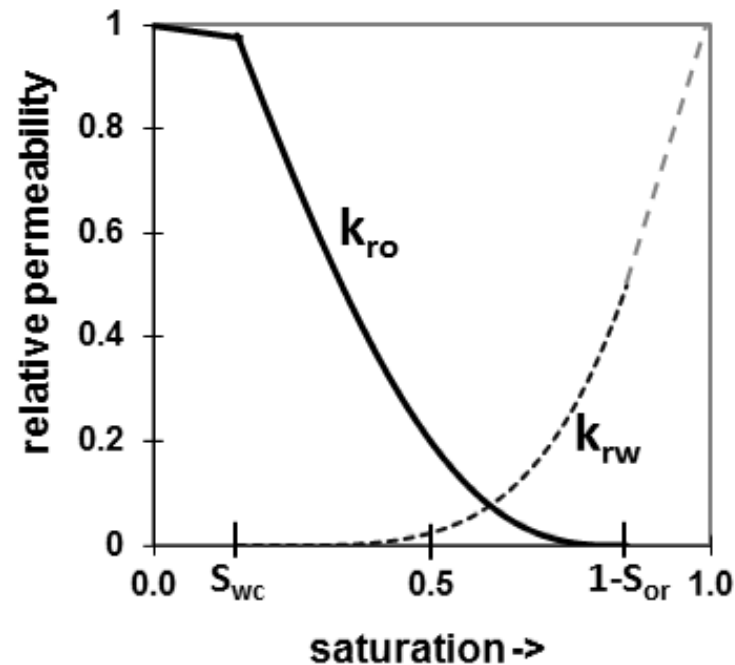
$$u = -\frac{k}{\mu} \nabla p$$

Darcy's law for two phase flow

Oil flows through a porous medium that consists of rock and water with a permeability k_o , whereas water flows through a porous medium that consists of oil and rock with a permeability k_w . Within this concept one expects that the permeability is only a function of the saturation (and not of the viscosity contrast between the fluids) i.e. $k_o = k_o(S_o)$ en $k_w = k_w(S_w)$. Therefore we can write Darcy's law for the oil and water phase as follows: (z is opposite to the direction of the force of gravity)

$$\begin{aligned}\mathbf{u}_o &= -\frac{k_o}{\mu_o}(\text{grad}P_o + \rho_o g \mathbf{e}_z), \\ \mathbf{u}_w &= -\frac{k_w}{\mu_w}(\text{grad}P_w + \rho_w g \mathbf{e}_z),\end{aligned}\tag{1}$$

Relative permeabilities for water wet systems



Note that the end point permeability for water is much lower than for oil

Brooks-Corey rel-perms

There are a number of semi-empirical relations that one can use when (more the rule than the exception) experimental data of relative permeabilities are lacking. We suggest to use the Brooks-Corey relations.

$$\begin{aligned}k_{rw} &= k'_{rw} S_{we}^{\frac{2+3\lambda}{\lambda}} \\k_{ro} &= k'_{ro} (1 - S_{we})^2 (1 - S_{we}^{\frac{2+\lambda}{\lambda}}) \\S_{we} &= \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}, k'_{rw} = \frac{1}{2}, k'_{ro} = 1\end{aligned}\tag{2}$$

Capillary pressure

$$\frac{(1 - k_{rwp} * S_{we}(1 - s_{or})^{(2/\lambda + 3)})}{s_{or} * (x - 1) + 1}$$

$$p_{cb} = \gamma * \sigma_{ow} * \sqrt{\phi / \mu} * ((0.5 - s_{wc}) / (1 - s_{wc}))^{(1/\lambda)}$$
$$p_{cba} = p_{cb} * s_{co}^{(1/\lambda_{dim})} * ((s_{co} - s_{wc}) / (1 - s_{wc}))^{(-1/\lambda)}$$

$$k_{ro}(s) = k_{rop} * (1 - S_{we}(x)) * (1 - S_{we}(x)) * (1 - S_{we}(x)^{(2/\lambda + 1)})$$

$$d(u, t) * \text{test}(u) - f_w(u) * \text{test}(u_x) + 1/Pe * u_x * \text{test}(u_x)$$

Power-law rel-perms

Sometimes for illustration purposes the (erroneous) oversimplified relations are used

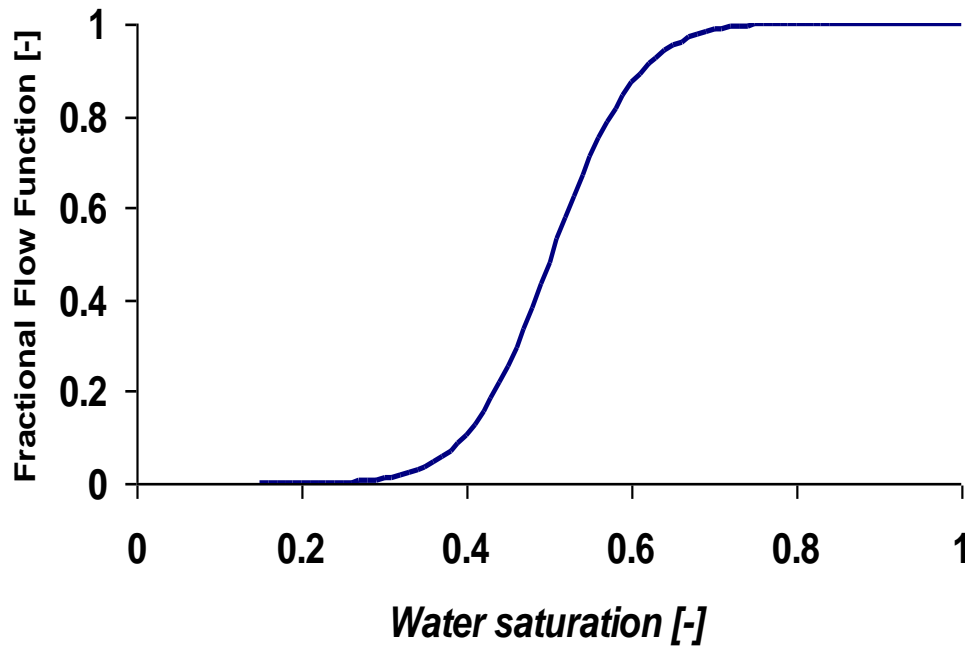
$$\begin{aligned}k_{rw} &= k'_{rw} S_{we}^{n_w}, \\k_{ro} &= k'_{ro} S_{oe}^{n_o},\end{aligned}\tag{3}$$

with $S_{we} = (S_w - S_{wc})/(1 - S_{wc} - S_{or})$ and $S_{oe} = (S_o - S_{or})/(1 - S_{wc} - S_{or})$ and where k'_{rw} and k'_{ro} ($(k'_{rw} \neq k'_{ro})$) are the end-point permeabilities respectively for water and oil. The exponents n_w and n_o can assume values between one and seven.

Water fractional flow function $f_w(S_w)$

- Fraction of total flow carried by water

fractional flow versus saturation



$$u_w = -\frac{kk_{rw}}{\mu_w} \nabla p$$

$$u_o = -\frac{kk_{ro}}{\mu_o} \nabla p$$

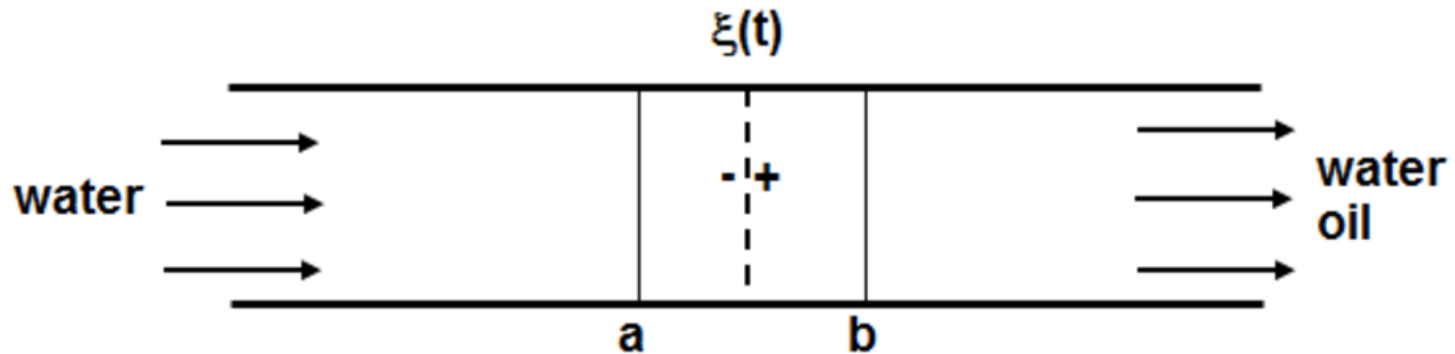
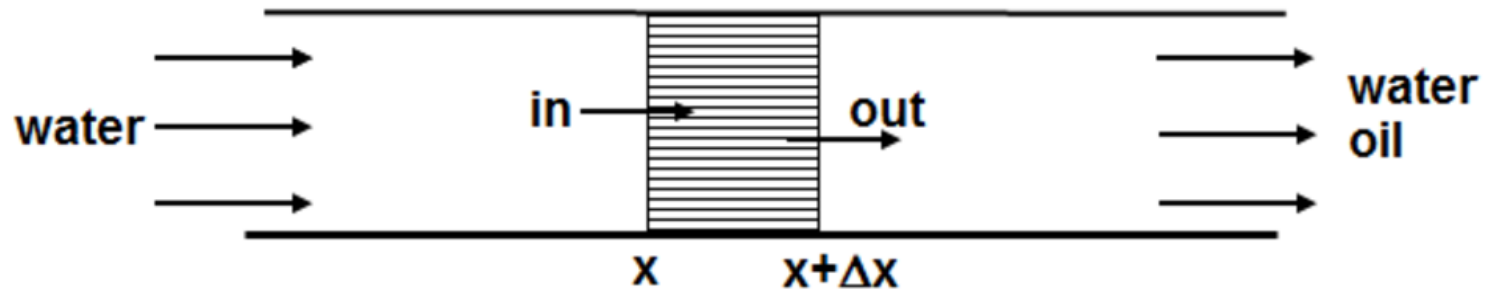
$$u = u_w + u_o$$

$$u_w = \frac{\frac{kk_{rw}}{\mu_w}}{\frac{kk_{rw}}{\mu_w} + \frac{kk_{ro}}{\mu_o}} u := u f_w$$

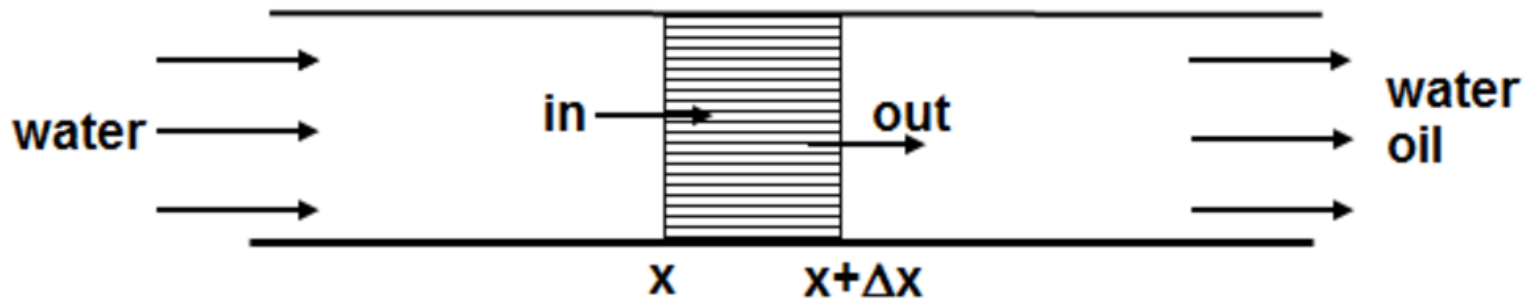
Assignment

- Take reasonable values for S_{wc} .
- Draw the relative permeabilities versus the water saturation for five values of $0.1 < \lambda < 10$
- Draw the fractional flow functions versus the water saturation for five values of $0.1 < \lambda < 10$

Mass balance equation



Mass balance outside discontinuities

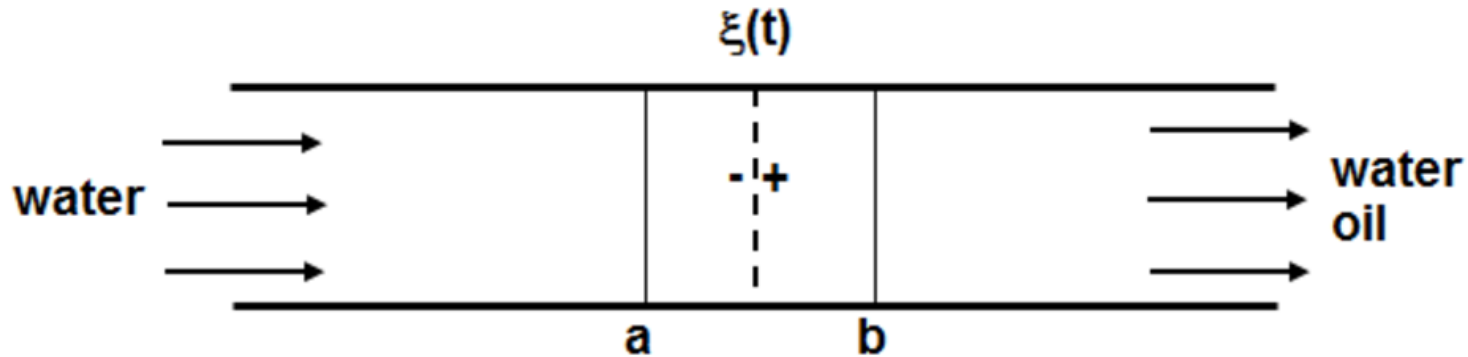


$$\frac{d}{dt}(\varphi S_{\alpha} A \Delta x) = A(u_{\alpha, in} - u_{\alpha, out}). \quad (1)$$

where S_{α} denotes the average saturation in the block Δx . In the limit $\Delta x \rightarrow 0$ we can rework this equation to a partial differential equation:

$$\varphi \frac{\partial S_{\alpha}}{\partial t} + \frac{\partial u_{\alpha}}{\partial x} = 0. \quad (2)$$

Mass balance at saturation discontinuity



We shall now derive the volume balance at the discontinuities. The position where the discontinuity occurs (see Figure) is denoted by $\xi(t)$.

We can derive an equation for the velocity with which the discontinuity progresses. The derivation is due to Lax. We write in one dimension over the interval (a, b) , such that $a < \xi(t) < b$:

$$\varphi \frac{d}{dt} \int_a^b S_\alpha dx = u_\alpha(a) - u_\alpha(b). \quad (3)$$

We split the integral on the left in a part in which we integrate from a to $\xi(t)$ and in a part in which we integrate from $\xi(t)$ to b :

$$\varphi \frac{d}{dt} \left(\int_a^{\xi(t)} S_\alpha dx + \int_{\xi(t)}^b S_\alpha dx \right) = u_\alpha(a) - u_\alpha(b). \quad (4)$$

Leibnitz's rule

To continue we apply Leibnitz's rule which state how to differentiate integrals.

$$\begin{aligned}\frac{d}{d\lambda} \left(\int_{u(\lambda)}^{v(\lambda)} f(x, \lambda) dx \right) &= \int_{u(\lambda)}^{v(\lambda)} \frac{\partial}{\partial \lambda} f(x, \lambda) dx \\ &+ f(v, \lambda) \frac{\partial v}{\partial \lambda} - f(u, \lambda) \frac{\partial u}{\partial \lambda}.\end{aligned}\tag{5}$$

We use the superscript $-$ to indicate a point which is just upstream of $\xi(t)$ and the superscript $+$ to indicate a point just downstream of $\xi(t)$. Note, a and b are constant in equation. We find with the introduced notation:

$$\begin{aligned}\varphi(S_{\alpha}^{-} - S_{\alpha}^{+}) \frac{\partial \xi}{\partial t} + \int_a^{\xi^{-}(t)} \varphi \frac{\partial S_{\alpha}}{\partial t} dx + \int_{\xi^{+}(t)}^b \varphi \frac{\partial S_{\alpha}}{\partial t} dx \\ = u_{\alpha}(a) - u_{\alpha}(b).\end{aligned}\tag{6}$$

Final part in derivation of shock condition

In the domains $(a, \xi^-(t))$ and $(\xi^+(t), b)$ is the saturation differentiable (only not at $\xi(t)$) such that the equation $\varphi \frac{\partial S_\alpha}{\partial t} + \frac{\partial u_\alpha}{\partial x} = 0$ holds. Therefore we substitute $\partial u_\alpha / \partial x$ for $\varphi \partial S_\alpha / \partial t$ and we obtain:

$$\varphi(S_\alpha^- - S_\alpha^+) \frac{\partial \xi}{\partial t} - \int_a^{\xi^-(t)} \frac{\partial u_\alpha}{\partial x} dx - \int_{\xi^+(t)}^b \frac{\partial u_\alpha}{\partial x} dx = u_\alpha(a) - u_\alpha(b).$$

The integrals are easy to evaluate and we find:

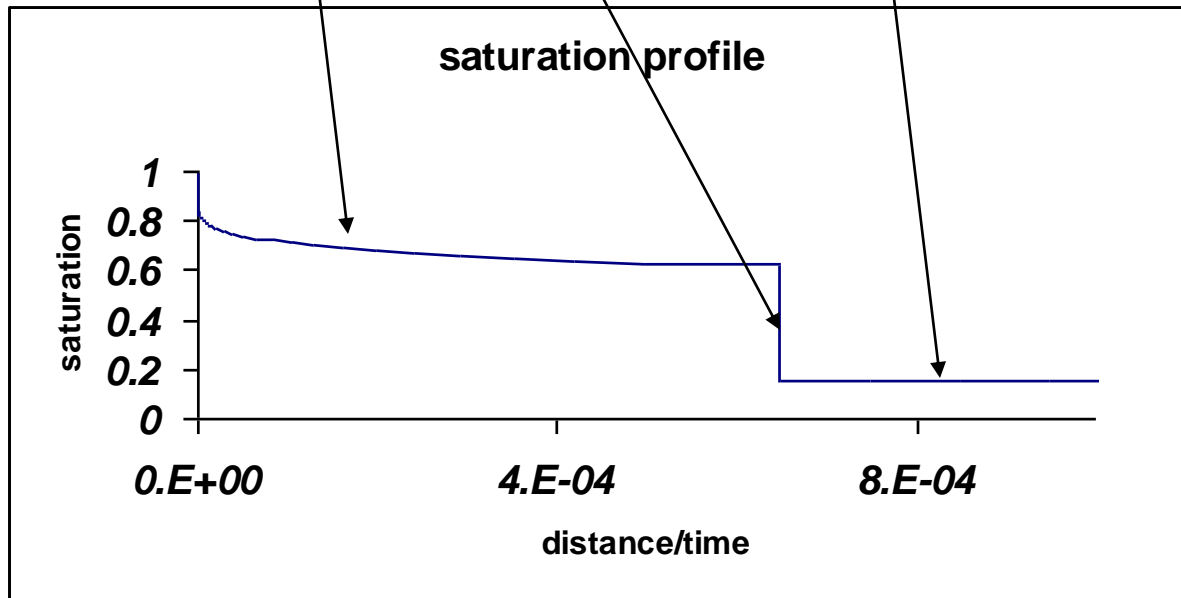
$$\begin{aligned} \varphi(S_\alpha^- - S_\alpha^+) \frac{\partial \xi}{\partial t} - u_\alpha|_a^{\xi^-(t)} - u_\alpha|_{\xi^+(t)}^b &= u_\alpha(a) - u_\alpha(b). \\ \varphi(S_\alpha^- - S_\alpha^+) \frac{\partial \xi}{\partial t} + u_\alpha(a) - u_\alpha^- + u_\alpha^+ &= u_\alpha(a) - u_\alpha(b) \end{aligned}$$

and we obtain

$$\varphi(S_\alpha^- - S_\alpha^+) \frac{\partial \xi}{\partial t} = u_\alpha^- - u_\alpha^+. \quad (7)$$

Solution of model equations consists of three parts

- Rarefaction, Shock, Constant state



Solution procedure I

The solution of partial differential equation is always based on rewriting the equation in terms of an ordinary differential equation. In this case we can achieve this through a similarity transformation that is a convenient combination of x and t such that the equation and boundary condition can all be expressed in this new variable. In this case we use $\eta = x/t$. We start with the material balance equation.

$$\frac{\partial S_w}{\partial t} + \frac{u}{\phi} \frac{\partial f_w}{\partial x} = 0.$$

We use $\frac{\partial \eta}{\partial t} = -\frac{x}{t^2}$ and $\frac{\partial \eta}{\partial x} = \frac{1}{t}$. Substitution in the equation with $x/t = \eta$ leads to an ordinary differential equation. We also use $f_w = f_w(S_w)$ i.e. $\frac{\partial f_w}{\partial \eta} = \frac{df_w}{dS_w} \frac{\partial S_w}{\partial \eta}$,

$$-\eta \frac{dS_w}{d\eta} + \frac{u}{\phi} \frac{df_w}{dS_w} \frac{dS_w}{d\eta} = 0.$$

Solution procedure II

$$-\eta \frac{dS_w}{d\eta} + \frac{u}{\phi} \frac{df_w}{dS_w} \frac{dS_w}{d\eta} = 0.$$

We assume, for simplicity, that a horizontal reservoir filled with oil at connate water saturation $S_w = S_{wc}$ in which we inject water. This initial condition can be expressed in terms of η i.e. $S_w(\eta \rightarrow \infty) = S_{wc}$. Water injection means that for $x = 0$ and thus for $\eta = 0$ we have the saturation $S_w = 1 - S_{or}$.

We observe that there are two solutions namely the **constant state** $\frac{dS_w}{d\eta} = 0$ i.e.

$$S_w = \text{constant},$$

and the **rarefaction wave**:

$$\eta = \frac{u}{\phi} \frac{df_w}{dS_w}. \quad (11)$$

Solution procedure III

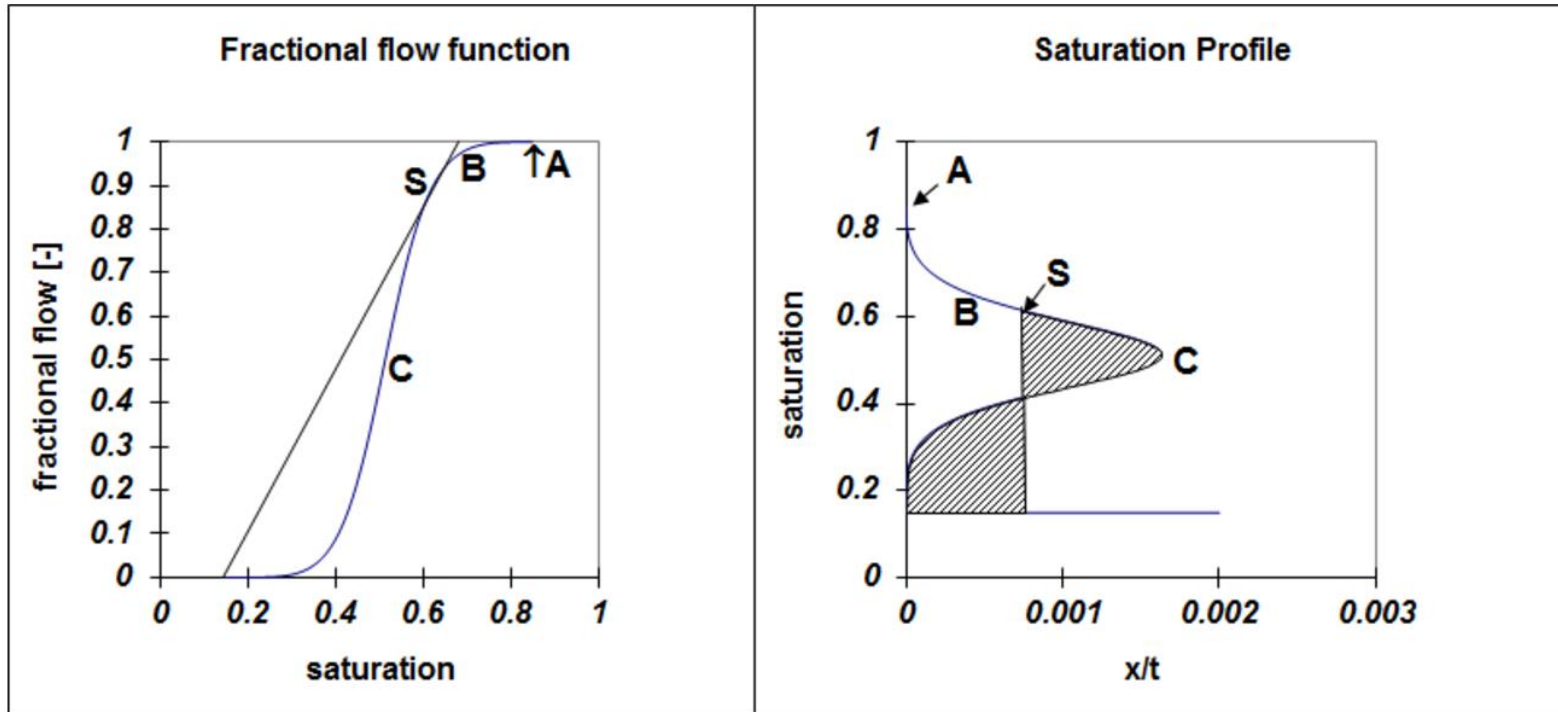


Figure 14: Construction of Saturation profile from fractional flow function. The part from A to S is found from $x/t = u/\varphi \frac{df_w}{dS_w}$ and is called rarefaction part. The shock saturation is found from tangent line to the fractional flow function starting at $(S_{wc}, 0)$. Alternatively the (hatched) areas right and left (below) the shock line should be equal.

Solution procedure IV

The capillary forces are required to avoid a multi-valued solution (physical nonsense) and thus to obtain shock like behavior instead. It is a good example to show that mathematics and physical models have to be considered together. The problem with the solution shows that something is wrong with the physical model i.e. it is not allowed to disregard capillary forces. We had anticipated that a shock like part needs to be included in the solution. Therefore we gave a derivation for the velocity of the shock:

$$v_s = \frac{\partial \xi}{\partial t} = \frac{u}{\phi} \frac{f_\alpha^- - f_\alpha^+}{(S_\alpha^- - S_\alpha^+)}, \quad (12)$$

where we have substituted the definition of the fractional flow function. We use the notation v_s to denote the velocity of the shock.

Summarizing we can say that the solution can be constructed from three solution parts i.e. the constant state solution, the rarefaction wave solution 11 and the shock solution 12.

Solution procedure V

$\frac{f_w^-}{S_w^- - S_{wc}}$ must be equal to the velocity $\frac{x}{t} = \eta \propto \frac{df_w}{dS_w}$ with which the saturation point S moves. Thus we equate η in Eq. (30) to v_s in Eq. (31). We divide on both sides by $\frac{u}{\varphi}$ and we obtain (see Fig. 14):

$$\frac{f_\alpha^- - f_\alpha^+}{(S_\alpha^- - S_\alpha^+)} = \left(\frac{df_w}{dS_w}\right)_{shock}. \quad (32)$$

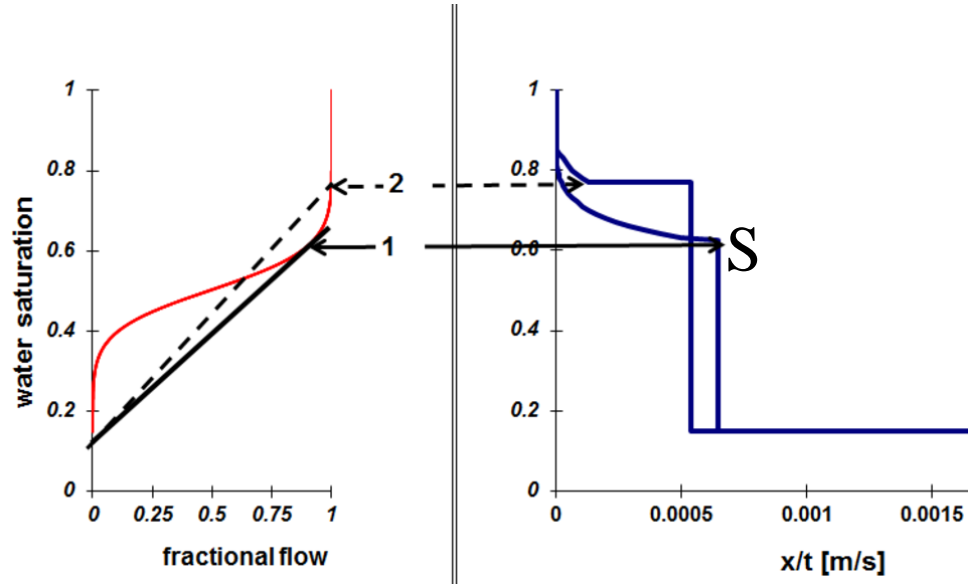
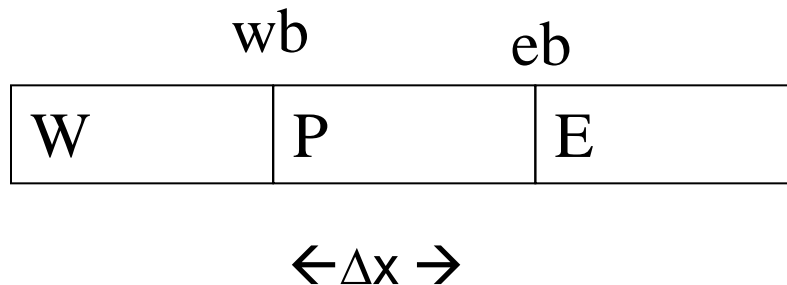


Figure 15: Tangent construction (1) according to Welge giving the right solution. The intersection construction (2) satisfies all requirements except for the entropy condition. Such a solution does not follow as a limit when the capillary forces tend to zero.

numerical

- $\varphi \partial_t S_w + \partial_x u f_w = 0$
- $S_w(t + \Delta t) = S_w(t) + u (f_{wW} - f_{wP}) \Delta t / (\varphi \Delta x)$



- *Upstream weighting*

Numerical solution

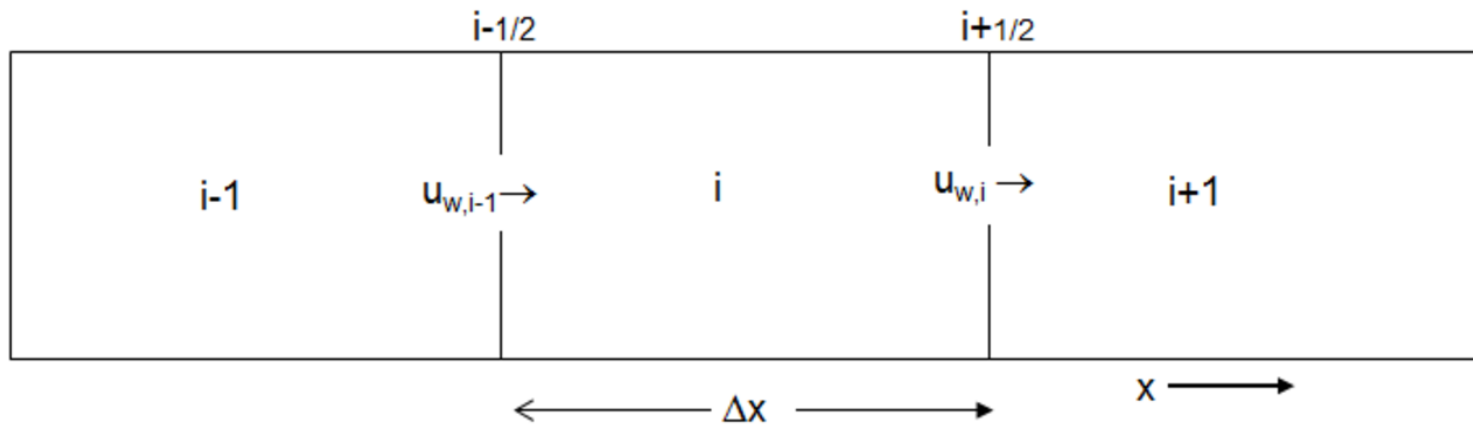


Figure 17: Labelling convention of flow into cell and out of cell. The properties u_w^{i-1} and u_w^i are determined by the center of the cell from which the flow is coming.

Rearrange expression for water flux

$$u_w = -\lambda_w \left(\frac{\partial P_w}{\partial x} + \rho_w g \sin \theta \right)$$

$$u_o = -\lambda_o \left(\frac{\partial P_o}{\partial x} + \rho_o g \sin \theta \right)$$

where λ_α , P_α and ρ_α are the mobility, the pressure and the density of phase α respectively, θ is the dip angle, and g is the acceleration due to gravity Hence we obtain

$$u = u_w + u_o = u_w - \lambda_o \left(\frac{\partial P_o}{\partial x} + \rho_o g \sin \theta \right) =$$
$$u_w - \frac{\lambda_o}{\lambda_w} \lambda_w \left(\frac{\partial P_w}{\partial x} + \rho_w g \sin \theta \right) - \lambda_o \left(\frac{\partial P_c}{\partial x} + (\rho_o - \rho_w) g \sin \theta \right)$$

Water flux = convective part + diffusive part

We can now resubstitute Darcy's equation to get rid of the pressure term. Rearranging the terms we can write explicitly for the Darcy's velocity of water

$$\begin{aligned} u_w &= \frac{\lambda_w}{\lambda_w + \lambda_o} u - \frac{\lambda_w \lambda_o}{\lambda_w + \lambda_o} (\rho_w - \rho_o) g \sin \theta + \frac{\lambda_w \lambda_o}{\lambda_w + \lambda_o} \frac{\partial P_c}{\partial x} := \\ &= u_{fw}(S_w) + \frac{\lambda_w \lambda_o}{\lambda_w + \lambda_o} \frac{\partial P_c}{\partial x} \end{aligned}$$

Input file

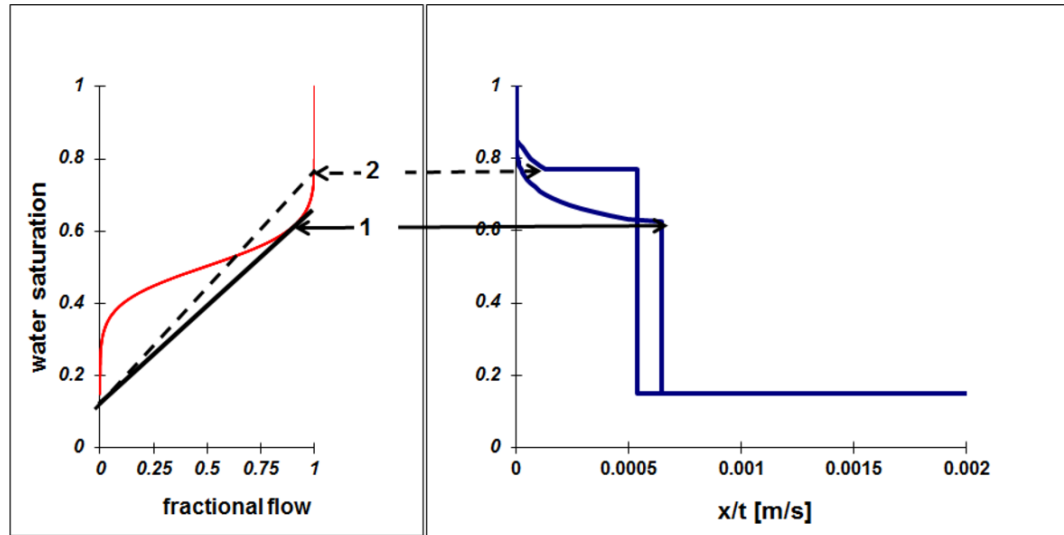
	A	B	C
1	start	1	[-]
2	utot	0.00001	[m/s]
3	dx	1	[m]
4	swc	0.15	[-]
5	visw	0.001	[Pa s]
6	viso	0.003	[Pa s]
7	phi	0.3	[-]
8	prefac	=utot*dt/phi/dx	[-]
9			
10	dt	=0.01*dx*phi/utot	[s]
11			

Program to simulate BL flow

	B	C	D
12			
13	dist	=0.5*dx	=dx+C13
14	krw	=(sw-sw _c)^2	=(sw-sw _c)^2
15	kro	=(1-sw)^2	=(1-sw)^2
16	fw	=krw/visw/(krw/visw+kro/viso)	=krw/visw/(krw/visw+kro/viso)
17	dsw	=prefac*(1-fw)	=prefac*(C16-fw)
18	dist	=C13	=D13
19	sw	=IF(start=0,sw _c ,sw+dsw)	=IF(start=0,sw _c ,sw+dsw)

Copy column “D” as far right as one wants to have results

Solution procedure VI



This is, however, not the only solution that can be constructed. Consider the solutions connected by line (2) in the figure. Here we have drawn the intersection line from point $(S_{wc}, 0)$ with the $f_w - S_w$ curve. If we consider only the upper intersection point then we find the following solution. Above the highest intersection point of the line from $(S_{wc}, 0)$ with the fractional flow curve we use the rarefaction wave solution $\frac{x}{t} = \frac{u}{\phi} \frac{df_w}{dS_w}$. Thus we construct the solution upstream of the shock. The intersection dotted line represents the shock in the same way as the tangent drawn line in the left part of the figure. In the right part of the figure we observe that the velocity of the shock is not equal to the velocity of the rarefaction wave. Between the shock and the rarefaction wave we have a constant state solution. It is clear that the solution drawn in the right part of the figure satisfies all the conditions which we require from the solution.

Solution procedure VII

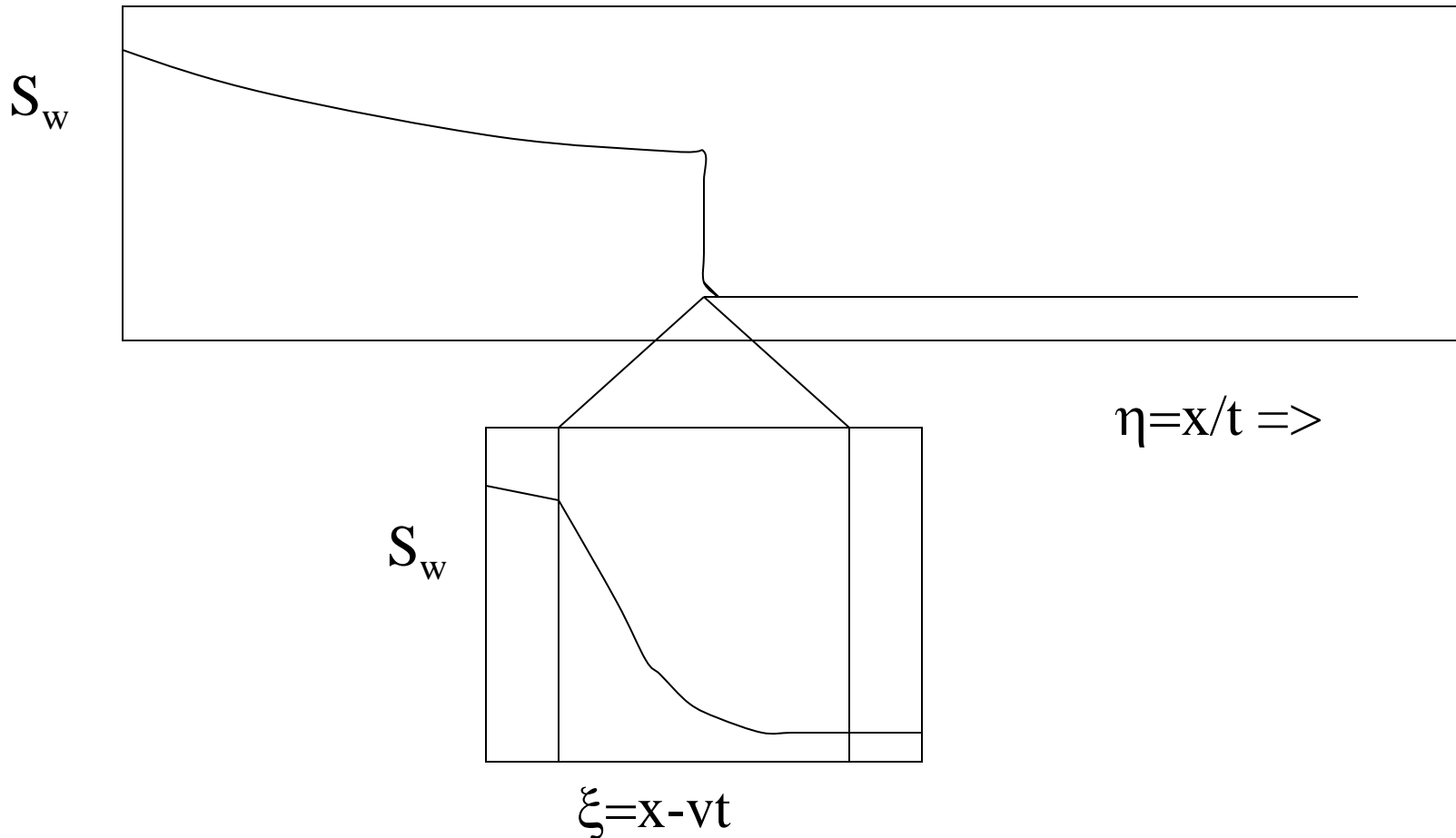
It turns out that in Equation

$$\phi \frac{\partial S_\alpha}{\partial t} + \frac{\partial u_\alpha}{\partial x} = \lim_{D \rightarrow 0} D \frac{\partial^2 S_\alpha}{\partial x^2}$$

i.e. as the diffusion constant tends to zero this does not imply that we can disregard the right term. The capillary diffusion term can be small but is positive. It is possible to show (the local entropy production term is always positive) that in the limit of a zero diffusion constant the only admissible solution is the one picked out by the tangent shock line. It is, however, outside the scope of these lectures.

The name entropy condition can be given the physical meaning by observing that according to (non-equilibrium) thermodynamics a positive entropy production term implies a positive diffusion constant. The entropy condition is known in Petroleum Engineering under the name construction of Welge. Welge (a genius) intuitively picked out the right solution without fully understanding the problem. Therefore the left plot of the figure is right and the right plot of the figure is incorrect.

Method of matched asymptotic expansions



Calculation of production curves

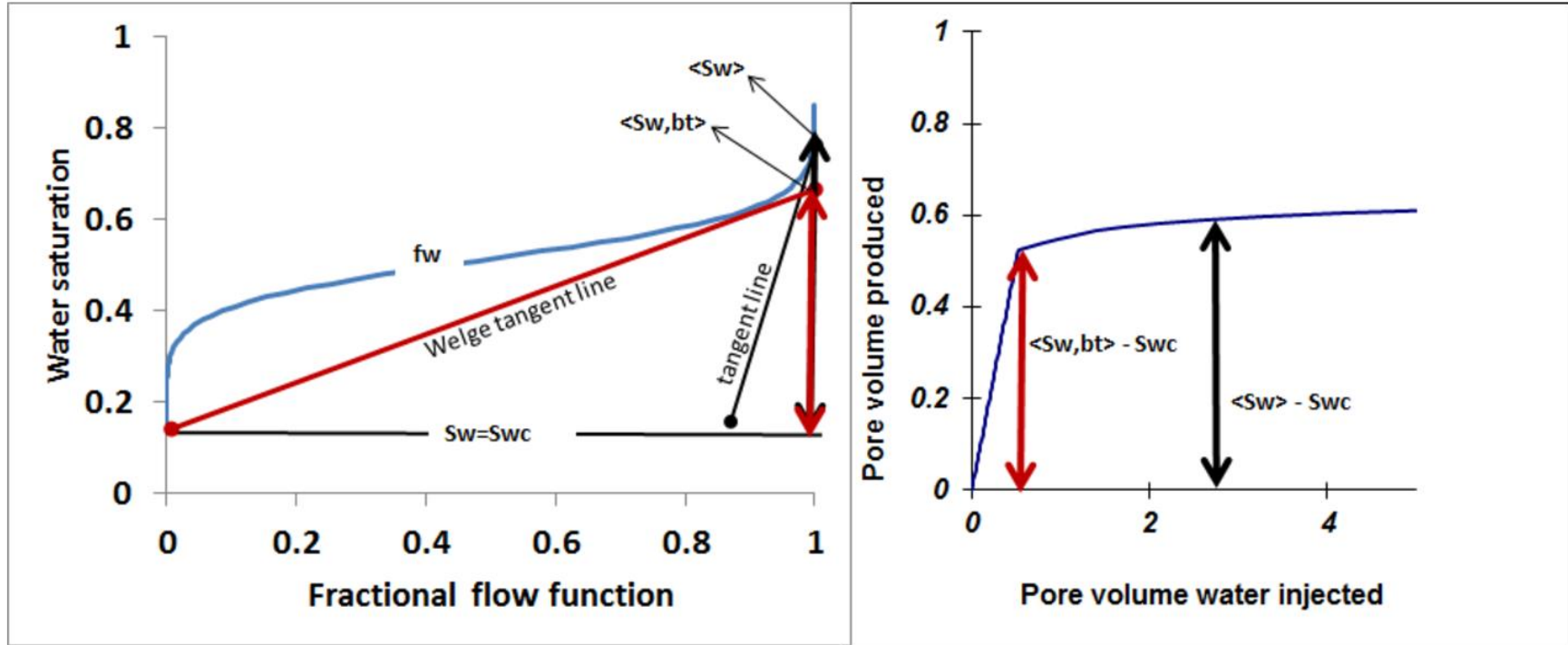


Figure 16: Extrapolation of line tangent to fractional flow curve at the saturation at the production point $S_w = S_{w,end}$ towards the line $f_w = 1$ gives the average saturation $\langle S_w \rangle$ in the reservoir. The tangent line originating from the connate water saturation gives the average saturation at breakthrough $\langle S_{w,bt} \rangle$. The pore volume of oil produced $N_{pd} = \langle S_w \rangle - S_{wc}$. The pore volume of oil produced at breakthrough is $N_{pd,bt} = \langle S_{w,bt} \rangle - S_{wc}$. The pore volume of water injected is $W_{id} = dS_w/df_w$.

Conclusions

- Dispersion/diffusion is over-estimated in simulators
- Therefore we like a comparison with zero diffusion
- In the limit of zero diffusion, conservation law type of equations give rise to a combination of rarefactions, shocks and constant states
- The shock region can be described as a traveling wave. The traveling wave has only solutions for some conditions, due to the fact that the diffusion-coefficient is positive. These conditions can be termed entropy conditions