UNDERSATURATED OIL-GAS SIMULATION - IMPES TYPE SOLUTION

If the oil is under-saturated, there is no free gas in the pores. Thus, we really have one phase flow (in absence of water). However, if the amount of gas in solution varies, such as in the case of gas injection into an under-saturated reservoir, we need to keep track of the bubble point pressure, or saturation pressure, of the oil in order to account for the gas as well as in order to be able to compute the oil properties. Again, the oil density is given by:

$$\rho_o = \frac{\rho_{oS} + \rho_{gS} R_{so}}{B_o}$$

For an under-saturated oil, the oil pressure is per definition higher than the bubble point pressure:

$$P_o > P_{bp}$$

and there is no free gas in the reservoir:

$$S_o = 0.$$

Thus, the formation volume factor depends on both oil pressure and bubble point pressure (or saturation pressure):

$$B_o = f(P_o, P_{bp}).$$

The solution gas-oil ratio, on the other hand, is now a function of bubble point pressure only, and not on oil pressure:

$$R_{so} = f(P_{bp}).$$

Again, we will, for mass balance purposes, separate the oil density into two parts, one that remains liquid at the surface and one that becomes gas:

$$\rho_o = \frac{\rho_{oS} + \rho_{gS} R_{so}}{B_o} = \frac{\rho_{oS}}{B_o} + \frac{\rho_{gS} R_{so}}{B_o} = \rho_{oL} + \rho_{oG}$$

and the oil mass balance is written so that its continuity equation includes the liquid part only, while the gas mass balance includes the solution gas in the reservoir, and thus all free gas at the surface:

$$-\frac{\partial}{\partial x}(\rho_{oL} u_o) = \frac{\partial}{\partial t}(\phi \rho_{oL})$$

$$-\frac{\partial}{\partial x}(\rho_{oG} u_o) = \frac{\partial}{\partial t}[\phi(\rho_{oG})].$$

As in the case of saturated oil-gas systems, the solution gas will flow with the rest of the oil in the reservoir, at oil relative permeability, viscosity and pressure.

The Darcy equation for the single phase in the reservoir is thus the one for oil:

$$u_o = -\frac{k k_{ro}}{\mu_o} \frac{\partial P_o}{\partial x}$$

Although we have single phase flow, we will keep the relative permeability in the equations in case it is not equal to one. Substituting Darcy's equations and the liquid oil density and the solution gas density definitions into the

continuity equations, and including production/injection terms in the equations, we obtain the following two flow equations:
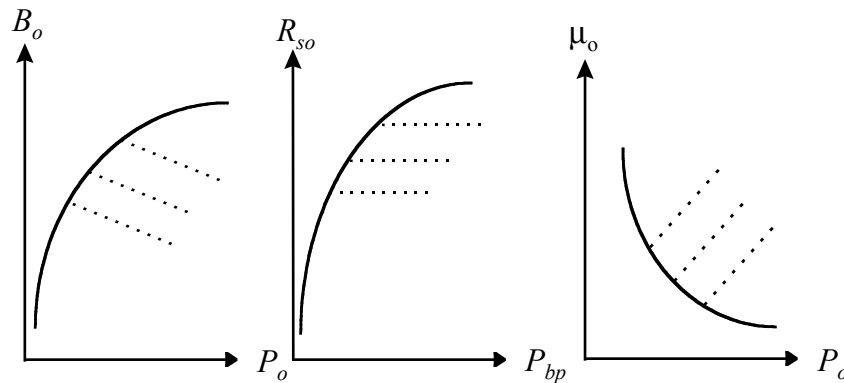
$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_o' = \frac{\partial}{\partial t}\left(\frac{\phi}{B_o}\right)$$

and

$$\frac{\partial}{\partial x}\left(R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right) - q_g' - R_{so}q_o' = \frac{\partial}{\partial t}\left(R_{so}\frac{\phi S_o}{B_o}\right),$$

A free gas rate term is included in the gas equation for gas injection purposes.

The *Black Oil* fluid properties for under-saturated oil are illustrated below:

In the figures, the solid lines represent saturated conditions, while the dotted lines represent under-saturated behavior, with the bubble point pressure being defined by the intersection of the dotted line and the saturated line. Thus, the bubble point pressure depends on the amount of gas present in the system. The more gas, the higher the bubble point pressure.

Discretization of flow equations

The discretization procedure for the flow terms of the oil-gas equations is very much similar to the one for saturated oil-gas equations.

Flow terms

The left side of the oil equation is identical to the one for a saturated system:

$$\frac{\partial}{\partial x}\left(\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)_i \approx T_{xo_{i+1/2}}(P_{o_{i+1}} - P_{o_i}) + T_{xo_{i-1/2}}(P_{o_{i-1}} - P_{o_i})$$

For the gas equation, we only have the solution gas terms, which may be approximated just as for the solution gas term of the saturated equation:

$$\frac{\partial}{\partial x}\left(R_{so}\frac{kk_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)_i \approx \left(R_{so}T_{xo}\right)_{i+1/2}(P_{o_{i+1}} - P_{o_i}) + \left(R_{so}T_{xo}\right)_{i-1/2}(P_{o_{i-1}} - P_{o_i})$$

Definitions of the terms in the equation are thus:

$$Tx_{o_{i+1/2}} = \frac{2\lambda_{o_{i+1/2}}}{\Delta x_i \left(\dfrac{\Delta x_{i+1}}{k_{i+1}} + \dfrac{\Delta x_i}{k_i} \right)}$$

$$Tx_{o_{i-1/2}} = \frac{2\lambda_{o_{i-1/2}}}{\Delta x_i \left(\dfrac{\Delta x_{i-1}}{k_{i-1}} + \dfrac{\Delta x_i}{k_i} \right)}$$

where

$$\lambda_o = \frac{k_{ro}}{\mu_o B_o}$$

and the upstream mobilities and solution-gas terms are selected as:

$$\lambda_{o_{i+1/2}} = \begin{cases} \lambda_{o_{i+1}} & if \ \ Po_{i+1} \geq Po_i \\ \lambda_{o_i} & if \ \ Po_{i+1} < Po_i \end{cases}$$

$$\lambda_{o_{i-1/2}} = \begin{cases} \lambda_{o_{i-1}} & if \ \ Po_{i-1} \geq Po_i \\ \lambda_{o_i} & if \ \ Po_{i-1} < Po_i \end{cases}$$

$$Rso_{i+1/2} = \begin{cases} Rso_{i+1} & if \ \ Po_{i+1} \geq Po_i \\ Rso_i & if \ \ Po_{i+1} < Po_i \end{cases}$$

$$Rso_{i-1/2} = \begin{cases} Rso_{i-1} & if \ \ Po_{i-1} \geq Po_i \\ Rso_i & if \ \ Po_{i-1} < Po_i \end{cases}$$

Oil storage term

We expand the right hand side of the oil equation as follows, keeping in mind that the formation volume factor is a function of oil pressure as well as bubble point pressure:

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B_o} \right) = \frac{1}{B_o}\frac{d\phi}{dP_o}\frac{\partial P_o}{\partial t} + \phi\frac{\partial(1/B_o)}{\partial P_o}\frac{\partial P_o}{\partial t} + \phi\frac{\partial(1/B_o)}{\partial P_{bp}}\frac{\partial P_{bp}}{\partial t}$$

where P_{bp} now is the variable bubble point pressure, a new unknown to be solved for together with oil pressure.

The two first terms above are identical to the two first terms of the saturated oil equation, with the exception that the derivative of the inverse formation volume factor is now a partial derivative, and that the the gas saturation is now zero. Thus, we can write the approximation of these two terms directly:

$$\left[\frac{1}{B_o}\frac{d\phi}{dP_o}\frac{\partial P_o}{\partial t} + \phi\frac{\partial(1/B_o)}{\partial P_o}\frac{\partial P_o}{\partial t} \right]_i \approx \frac{\phi_i}{\Delta t}\left[\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o} \right]_i \left(Po_i - Po_i^{\ t} \right)$$

For the last term, we use the standard approximation:

$$\left[\phi\frac{\partial(1/B_o)}{\partial P_{bp}}\frac{\partial P_{bp}}{\partial t} \right]_i \approx \frac{\phi_i}{\Delta t}\left[\frac{\partial(1/B_o)}{\partial P_{bp}} \right]_i \left(Pbp_i - Pbp_i^{\ t} \right)$$

Gas storage term

We expand the right hand side of the gas equation as follows:

$$\frac{\partial}{\partial t}\left(\frac{\phi R_{so}}{B_o}\right) = R_{so}\frac{\partial}{\partial t}\left(\frac{\phi}{B_o}\right) + \frac{\phi}{B_o}\frac{dR_{so}}{dP_{bp}}\frac{\partial P_{bp}}{\partial t}$$

Since the first term is equal to the right hand side of the oil equation, multiplied by the solution gas-oil ratio, we can write directly:

$$\left[R_{so}\frac{\partial}{\partial t}\left(\frac{\phi}{B_o}\right)\right]_i \approx \frac{(\phi R_{so})_i}{\Delta t}\left[\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o}\right]_i\left(P_{o_i} - P_{o_i}^{t}\right) + \frac{(\phi R_{so})_i}{\Delta t}\left[\frac{\partial(1/B_o)}{\partial P_{bp}}\right]_i\left(P_{bp_i} - P_{bp_i}^{t}\right)$$

The second term becomes:

$$\left[\frac{\phi}{B_o}\frac{dR_{so}}{dP_{bp}}\frac{\partial P_{bp}}{\partial t}\right]_i \approx \frac{\phi_i}{\Delta t B_{o_i}}\left[\frac{dR_{so}}{dP_{bp}}\right]_i\left(P_{bp_i} - P_{bp_i}^{t}\right)$$

Thus, the oil and gas equations in discretized form may be written, including the well terms:

$$Tx_{o_{i+1/2}}\left(P_{o_{i+1}} - P_{o_i}\right) + Tx_{o_{i-1/2}}\left(P_{o_{i-1}} - P_{o_i}\right) - q'_{oi}$$
$$= C_{poo_i}\left(P_{o_i} - P_{o_i}^{t}\right) + C_{bpq}\left(P_{bp_i} - P_{bp_i}^{t}\right), \qquad i = 1,N$$

$$(R_{so}Tx_o)_{i+1/2}\left(P_{o_{i+1}} - P_{o_i}\right) + (R_{so}Tx_o)_{i-1/2}\left(P_{o_{i-1}} - P_{o_i}\right) - (R_{so}q'_o)_i - q'_{gi}$$
$$= C_{pog_i}\left(P_{o_i} - P_{o_i}^{t}\right) + C_{bpg}\left(P_{bp_i} - P_{bp_i}^{t}\right), \qquad i = 1,N$$

where, for the oil equation:

$$C_{poq} = \frac{\phi_i}{\Delta t}\left(\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o}\right)_i$$

$$C_{poq} = \frac{\phi_i}{\Delta t}\left(\frac{\partial(1/B_o)}{\partial P_{bp}}\right)_i$$

and for the gas equation:

$$C_{pog} = \frac{(R_{so}\phi)_i}{\Delta t}\left(\frac{c_r}{B_o} + \frac{\partial(1/B_o)}{\partial P_o}\right)_i$$

and

$$C_{bpg} = \frac{\phi_i}{\Delta t}\left[R_{so}\frac{\partial(1/B_o)}{\partial P_{bp}} + \frac{1}{B_o}\frac{dR_{so}}{dP_{bp}}\right]_i$$

The derivative terms to be computed numerically for each time step based on the input table to the model, now are:

$$\left(\frac{\partial(1/B_o)}{\partial P_o}\right)_i, \left(\frac{\partial(1/B_o)}{\partial P_{bp}}\right)_i, \text{ and } \left(\frac{dR_{so}}{dP_{bp}}\right)_i$$

Boundary conditions

The boundary conditions for undersaturated oil-gas systems are somewhat simpler than those for saturated oil-gas systems since only one phase is produced in the reservoir. Normally, we inject gas in a grid block at constant surface rate or at constant bottom hole pressure, and produce oil (and solution gas) from a grid block at constant bottom hole pressure, or at constant surface oil rate. Again, we may sometimes want to specify constant reservoir voidage rate, where either the amount of injection of gas is to match a specified rate of oil production at reservoir conditions, so that average reservoir pressure remains constant, or the reservoir production rate is to match a specified gas injection rate.

Constant gas injection rate

Again, a gas rate term is already included in the gas equation. Thus, for a constant surface gas injection rate of Q_{gi} (negative) in a well in grid block i :

$$q'_{gi} = Q_{gi} / (A\Delta x_i) .$$

It is normally assumed that the injected gas immediately goes into solution with the oil in the injection gridblock. The bottomhole pressure in the well will therefore be a function of the oil mobility around the well, since there is not any free gas present. Thus, the bottom hole presure may be determined from the same expression for the well as for saturated oil-gas flow:

$$q'_{gi} = \frac{WC_i}{A\Delta x_i}\left(\frac{B_{oi}}{B_{gi}}\lambda_{oi} + \lambda_{gi}\right)(P_{o_i} - P_{bh_i}),$$

and since the gas mobility term is zero:

$$q'_{gi} = \frac{WC_i}{A\Delta x_i}\frac{B_{oi}}{B_{gi}}\lambda_{oi}(P_{o_i} - P_{bh_i}) .$$

The well constant is as before given by:

$$WC_i = \frac{2\pi k_i h}{ln(\frac{r_e}{r_w})},$$

and

$$r_e = \sqrt{\frac{\Delta y \Delta x_i}{\pi}} .$$

Injection at constant bottom hole pressure

Using the well equation above:

$$q'_{gi} = \frac{WC_i}{A\Delta x_i}\frac{B_{oi}}{B_{gi}}\lambda_{oi}(P_{o_i} - P_{bh_i}),$$

the terms must be included in the appropriate coefficients in the pressure solution. At the end of the time step, the above equation may be used to compute the actual gas injection rate for the step.

Constant oil production rate

For the oil equation, this condition is handled as for the saturated case. Thus, for a constant surface oil production rate of Q_{oi} (positive) in a well in grid block i :

$$q'_{oi} = Q_{oi} / (A \Delta x_i) .$$

Oil production will always be accompanied by solution gas production, and this is accounted for in the gas equation by the term $q'_{oi} Rs_{o_i}$.

At the end of a time step, after having solved the equations, the bottom hole production pressure for the production well may be calculated using the well equation for oil:

$$Q_{oi} = WC_i \lambda_{oi} (Po_i - Pbh_i) ,$$

As for oil-water systems, production wells in oil-gas systems are normally constrained by a minimum bottom hole pressure. If this is reached, the well should be converted to a constant bottom hole pressure well.

The gas-oil ratio, GOR, at the surface, is for undersaturated systems equal to the solution gas-oil ratio:

$$GOR_i = Rs_{o_i} .$$

Production at constant reservoir voidage rate

If the total production of fluids from a well in block i , at reservoir conditions, is to match the reservoir injection volume so that the reservoir pressure remains approximately constant, the following relationship must be obeyed:

$$Q_{oi} B_{oi} = -Q_{ginj} B_{ginj} = -Q_{ginj} \frac{B_{oinj}}{B_{ginj}} \lambda_{oi},$$

Thus,

$$q'_{oi} = \frac{1}{B_{oi}} \left(-Q_{ginj} B_{ginj} \lambda_{oi} \right) / (A \Delta x_i) ,$$

and the bottom hole pressure of the production well may be computed at the end of the time step from:

$$q'_{oi} = \frac{WC_i}{A \Delta x_i} \lambda_{oi} (Po_i - Pbh_i) .$$

Production at constant bottom hole pressure

For a production well in grid block i with constant bottom hole pressure, Pbh_i, we have an oil rate of:

$$Q_{oi} = WC_i \lambda_{oi} (Po_i - Pbh_i)$$

or,

$$q'_{oi} = \frac{WC_i}{A \Delta x_i} \lambda_{oi} (Po_i - Pbh_i) ,$$

and solution gas rate:

$$Rs_{o_i} q'_{oi} = Rs_{o_i} \frac{WC_i}{A \Delta x_i} \lambda_{oi} (Po_i - Pbh_i)$$

The terms appearing in the rate expressions will again have to be included in the appropriate matrix coefficients when solving for pressures. At the end of each time step, actual oil rate and GOR are computed.

Solution by IMPES method

The procedure for IMPES solution is similar to the previous cases. To be correct, the method should now be labeled IMPEBPP, for Implicit pressure, Explicit Bubble Point Pressure. We make the same type of assumptions in regard the coefficients in the equations:

$$Txo^t, Rso^t$$
$$Cpoo^t, Cpog^t .$$
$$Cbpo^t, Cbpg^t$$

Having made these approximations, the equations become:

$$Txo_{i+1/2}^t \left(Po_{i+1} - Po_i\right) + Txo_{i-1/2}^t \left(Po_{i-1} - Po_i\right) - q'_{oi}$$
$$= Cpoo_i^t \left(Po_i - Po_i^t\right) + Cbpo_i^t \left(Pb_{p_i} - Pb_{p_i}^t\right), \qquad i = 1, N$$

$$\left(R_{so}Txo\right)_{i+1/2}^t \left(Po_{i+1} - Po_i\right) + \left(R_{so}Txo\right)_{i-1/2}^t \left(Po_{i-1} - Po_i\right) - \left(R_{so}q'_o\right)_i - q'_{gi}$$
$$= Cpog_i^t \left(Po_i - Po_i^t\right) + Cbpg_i^t \left(Pb_{p_i} - Pb_{p_i}^t\right), \qquad i = 1, N$$

IMPES oil pressure solution

The oil pressure equation for the saturated oil-gas becomes:

$$\left[Txo_{i+1/2}^t + \alpha_i \left(R_{so}Txo\right)_{i+1/2}^t\right]\left(Po_{i+1} - Po_i\right) +$$
$$\left[Txo_{i-1/2}^t + \alpha_i \left(R_{so}Txo\right)_{i-1/2}^t\right]\left(Po_{i-1} - Po_i\right)$$
$$-q'_{oi} - \alpha_i \left(q'_g + R_{so}^t q'_{oi}\right)_i =$$
$$\left(Cpoo_i^t + \alpha_i Cpog_i^t\right)\left(Po_i - Po_i^t\right), \qquad i = 1, N$$

where

$$\alpha_i = -Cbpg_i^t / Cbpo_i^t .$$

We the rewrite the oil pressure equation as:

$$a_i Po_{i-1} + b_i Po_i + c_i Po_{i+1} = d_i, \qquad i = 1, N$$

where

$$a_i = Txo_{i-1/2}^t + \alpha_i \left(R_{so}Txo\right)_{i-1/2}^t$$

$$c_i = Txo_{i+1/2}^t + \alpha_i \left(R_{so}Txo\right)_{i+1/2}^t$$

$$b_i = -Txo_{i-1/2}^t - Txo_{i+1/2}^t - Cpoo_i^t - \alpha_i \left[\left(R_{so}Txo\right)_{i-1/2}^t + \left(R_{so}Txo\right)_{i+1/2}^t + Cpog_i^t\right]$$

$$d_i = -(Cpoo_i^t + \alpha_i Cpog_i^t)Po_i^t + q'_{oi} + \alpha_i \left(q'_g + R_{so}q'_o\right)_i$$

Modifications for boundary conditions

As before, all rate specified well conditions are included in the rate terms q'_{oi}, q'_{gi} and $Rs_{oi}q'_{oi}$, and are already appropriately included in the d_i term above.

For *injection of gas at bottom hole pressure specified well conditions*, the following expression apply:

$$q'_{gi} = \frac{WC_i}{A\Delta x_i} \frac{B_{oi}}{B_{gi}} \lambda_{oi}(Po_i - Pbh_i).$$

In a block with a well of this type, the following matrix coefficients are modified:

$$b_i = -(Txo^t_{i+1/2} + Txo^t_{i+1/2} + Cpo^t_i)$$

$$- \alpha_i \left[Txg^t_{i+1/2} + Txg^t_{i+1/2} + Cpog^t_i + \frac{WC_i}{A\Delta x_i} \frac{B^t_{oi}}{B^t_{gi}} \lambda^t_{oi} \right]$$

$$d_i = -(Cpoo^t_i + \alpha_i Cpog^t_i)Po^t_i - \frac{WC_i}{A\Delta x_i} \lambda^t_{oi} Pbh_i - \alpha_i \frac{WC_i}{A\Delta x_i} \frac{B^t_{oi}}{B^t_{gi}} \lambda^t_{oi} Pbh_i$$

For *production at bottom hole pressure specified well conditions*, we have the following expressions:

$$q'_{oi} = \frac{WC_i}{A\Delta x_i} \lambda_{oi}(Po_i - Pbh_i),$$

and

$$Rso_i q'_{oi} = \frac{WC_i}{A\Delta x_i} Rso_i \lambda_{oi}(Po_i - Pbh_i).$$

In a block with a well of this type, the following matrix coefficients are modified:

$$b_i = -(Txo^t_{i+1/2} + Txo^t_{i+1/2} + Cpo^t_i + \frac{WC_i}{A\Delta x_i} \lambda^t_{oi})$$

$$- \alpha_i \left[\left(R_{so}Txo\right)^t_{i+1/2} + \left(R_{so}Txo\right)^t_{i+1/2} + Cpog^t_i + \frac{WC_i}{A\Delta x_i} Rso_i \lambda^t_{oi} \right]$$

$$d_i = -(Cpoo^t_i + \alpha_i Cpog^t_i)Po^t_i - \frac{WC_i}{A\Delta x_i} \lambda^t_{oi} Pbh_i - \alpha_i \frac{WC_i}{A\Delta x_i} Rso^t_i \lambda^t_{oi} Pbh_i$$

Again, the pressure equation may now be solved for oil pressures by using Gaussian elimination.

IMPES bubble point pressure solution

Having obtained the oil pressures above, we need to solve for bubble point pressures using either the oil equation or the gas equation. In the following we will use the oil equation for this purpose:

$$Txo^t_{i+1/2}\left(Po_{i+1} - Po_i\right) + Txo^t_{i-1/2}\left(Po_{i-1} - Po_i\right) - q'_{oi}$$

$$= Cpoo^t_i\left(Po_i - Po^t_i\right) + Cbpo^t_i\left(Pbp_i - Pbp^t_i\right), \qquad i = 1, N$$

Since bubble point pressure only appears as an unknown in the last term on the right side of the oil equation, we may solve for it explicitly:

$$Pbp_i = Pbp^t_i + \frac{1}{Cbpo^t_i}\left[Txo^t_{i+1/2}\left(Po_{i+1} - Po_i\right) + Txo^t_{i-1/2}\left(Po_{i-1} - Po_i\right) - q'_{oi} - Cpoo^t_i\left(Po_i - Po^t_i\right)\right], \qquad i = 1, N$$

For grid blocks having pressure specified oil production wells, we make appropriate modifications, as discussed previously:

$$Pbp_i = Pbp_i^t + \frac{1}{Cbpo_i^t}\left[Txo_{i+1/2}^t\left(Po_{i+1} - Po_i\right) + Txo_{i-1/2}^t\left(Po_{i-1} - Po_i\right) - \frac{WC_i}{A\Delta x_i}\frac{B_{oi}}{B_{gi}}\lambda_{oi}^t\left(Po_i - Pbh_i^t\right) - Cpoo_i^t\left(Po_i - Po_i^t\right)\right]$$

$$i = 1, N$$

Having obtained oil pressures and water saturations for a given time step, well rates or bottom hole pressures may be computed, if needed, from, from the following expression for an injection well:

$$q'_{gi} = \frac{WC_i}{A\Delta x_i}\frac{B_{oi}}{B_{gi}}\lambda_{oi}(Po_i - Pbh_i),$$

and for a production well:

$$q'_{oi} = \frac{WC_i}{A\Delta x_i}\lambda_{oi}(Po_i - Pbh_i),$$

and

$$Rso_i q'_{oi} = \frac{WC_i}{A\Delta x_i}Rso_i\lambda_{oi}(Po_i - Pbh_i).$$

The surface gas-oil ratio is of course equal to the solution gas-oil ratio:

$$GOR_i = Rso_i.$$

Required adjustments in well rates and well pressures, if constrained by upper or lower limits, are made at the end of each time step, before all coefficients are updated before proceeding to the next time step.

Applicability of IMPES method

Applicability of the IMPES method for undersaturated oil-gas systems is fairly much as for the previously discussed systems. However, due to the fact that changes in bubble point pressure may be rapid, relatively small time step sizes may be required.

Textbook: Chapter 6, p. 57-62
                     Appendix B, p. 132-139
                     Appendix C, p. 140-144