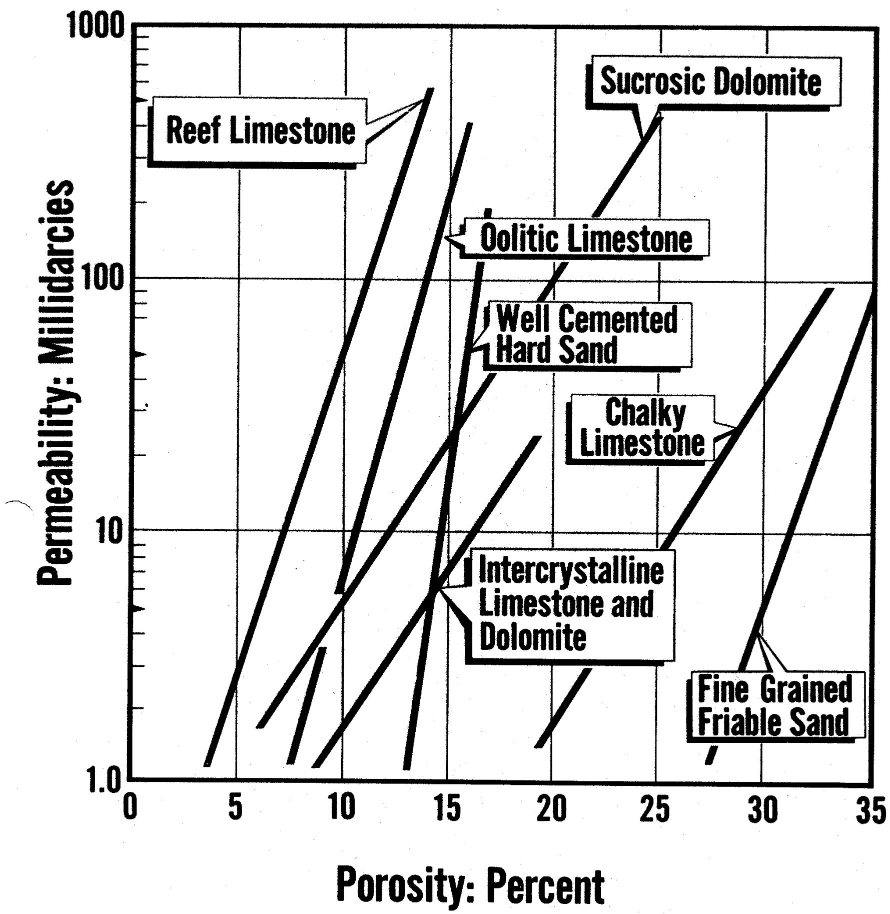
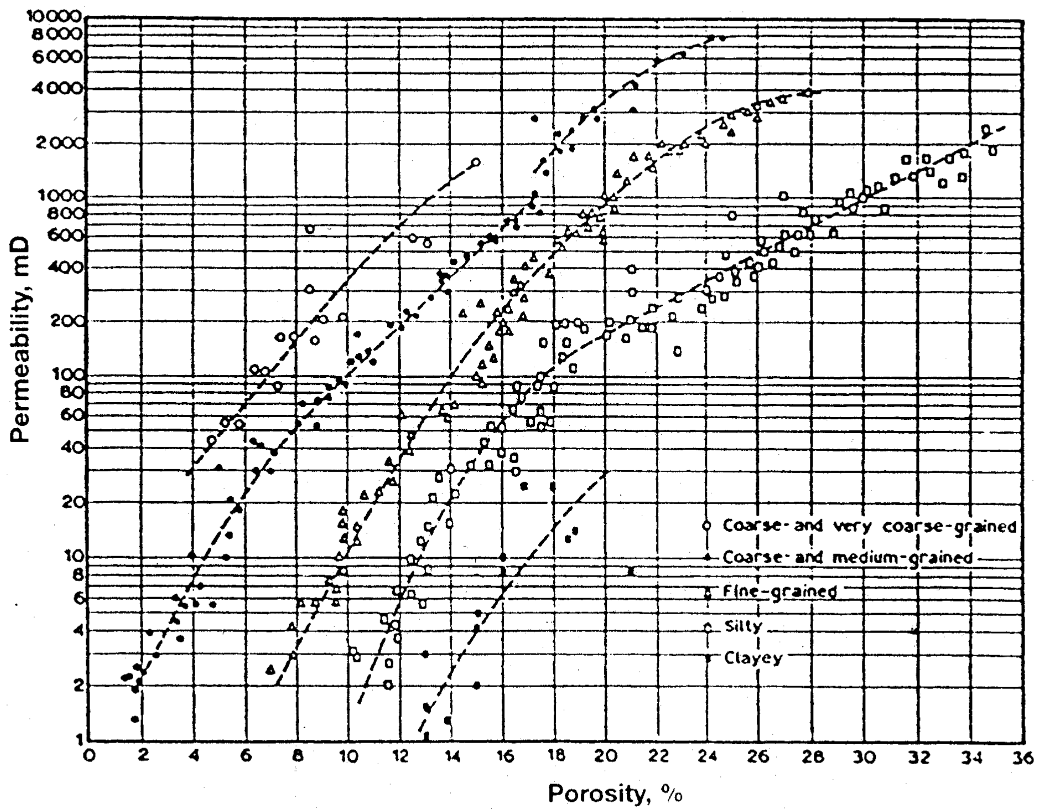
Porosity – Permeability Relationships



Permeability and porosity trends for various rock types [CoreLab,1983]

Porosity – Permeability Relationships



Influence of grain size on the relationship between porosity and permeability [Tiab & Donaldson, 1996]

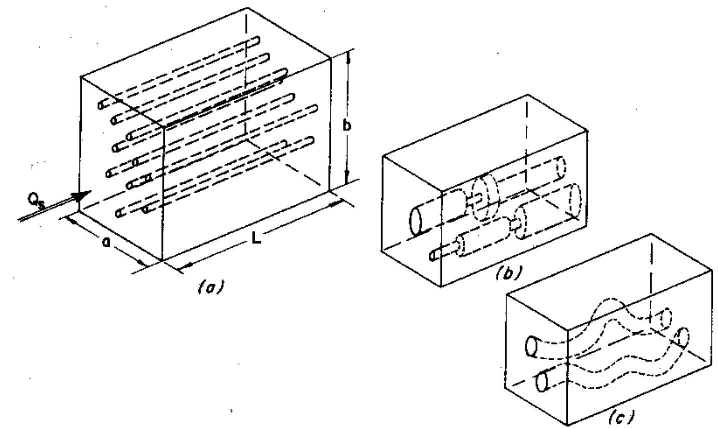
Porosity – Permeability Relationships

- **Darcy's Law (1856)** – empirical observations of flow to obtain permeability
- **Slichter (1899)** – theoretical analysis of fluid flow in packed uniform spheres
- **Kozeny (1927),Carmen (1939)** – capillary tube model

Porosity – Permeability Relationships

Capillary Tube Model

Define porosity → $\phi = n_t \pi r^2 \sqrt{\tau}$



Where r is radius of the capillary tube,

n_t is number of tubes/ unit area

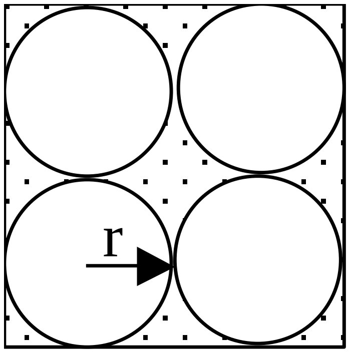
Define permeability → $k = \dfrac{\pi n_t r^4}{8\sqrt{\tau}}$

Porosity-permeability relationship → $k = \dfrac{\phi r^2}{8\tau}$

Porosity – Permeability Relationships

Example

For cubic packing shown, find ϕ and k.



Number of tubes per unit area: 4 tubes/$(4r)^2$

Porosity → $$\phi = \frac{1}{4r^2} * \pi r^2 = \frac{\pi}{4}$$

Tortuosity → $$\tau = \left(\frac{L_a}{L}\right)^2 = 1$$

Permeability → $$k = \frac{\phi r^2}{8\tau} = \frac{\pi}{4} * \frac{r^2}{8(1)} = \frac{\pi r^2}{32}$$

Porosity – Permeability Relationships

Define specific surface area

S_{pv} – specific surface area per unit pore volume

S_{pv} = 2/r (for cylindrical pore shape)

S_{bv}- …unit bulk volume

S_{gv}- …unit grain volume

$$\tau = \left(\frac{L_a}{L}\right)^2$$

$$S_{bv} = \phi * S_{pv}$$

$$S_{gv} = \left(\frac{\phi}{1-\phi}\right) S_{pv}$$

Carmen – Kozeny Equation

$$k = \frac{\phi r^2}{8\tau}$$

S_{pv} = 2/r

$$k = \frac{\phi}{k_z S_{pv}^2}$$

Where

Kz, Kozeny constant-shape factor to account for variability in shape and length

Porosity – Permeability Relationships

Carmen – Kozeny Equation

$$k = \frac{\phi r^2}{8\tau} \xleftarrow{} S_{pv} = 2/r$$

$$k = \frac{\phi}{k_z S_{pv}^2}$$

Where

Kz, Kozeny constant-shape factor to account for variability in shape and length

Carmen – Kozeny Equation

$$k_z = k_o * \tau$$

Tortuosity, τ

$$\tau = \left(\frac{L_a}{L} \right)^2$$

k_o is a shape factor

= 2 for circular

= 1.78 for square

Porosity – Permeability Relationships

Example: spherical particles with diameter, d_p

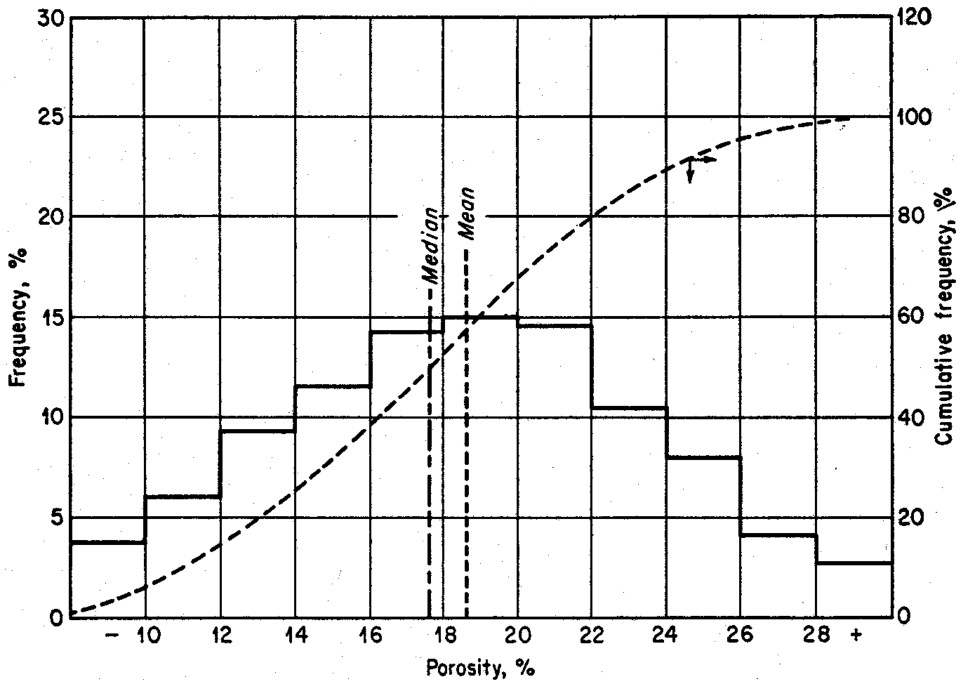
$$k = \frac{\phi}{k_z S_{pv}^2}$$

$$\Big\downarrow \; ??$$

$$k = \frac{\phi^3 d_p^2}{72(1-\phi)^2 \tau}$$

Distribution of Rock Properties

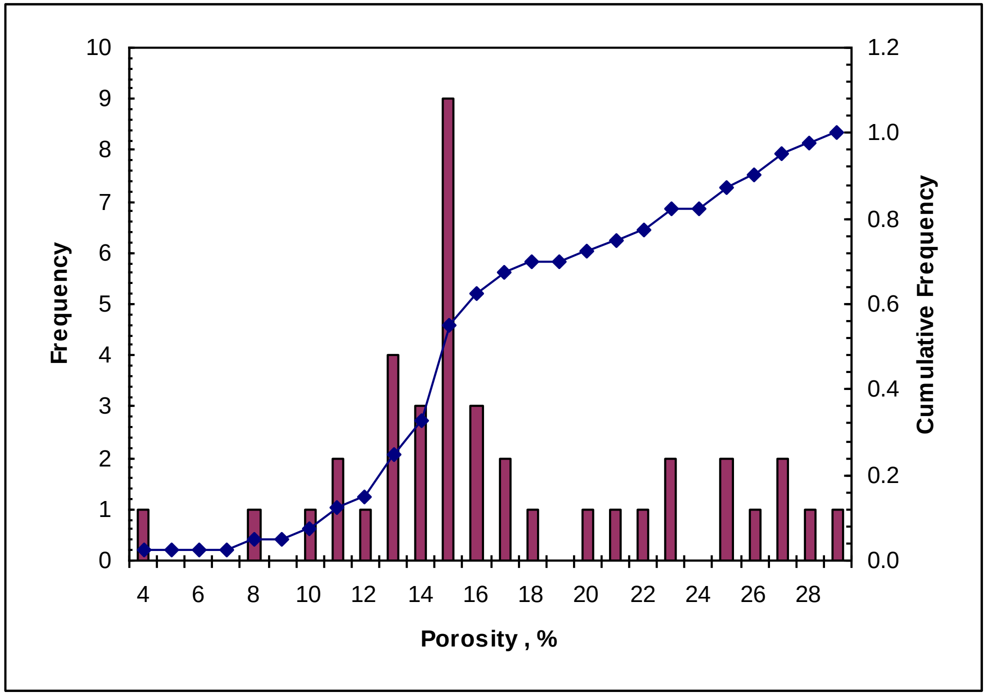
Porosity Distribution



Expected porosity histogram
[Amyx,et at., 1960]

Distribution of Rock Properties

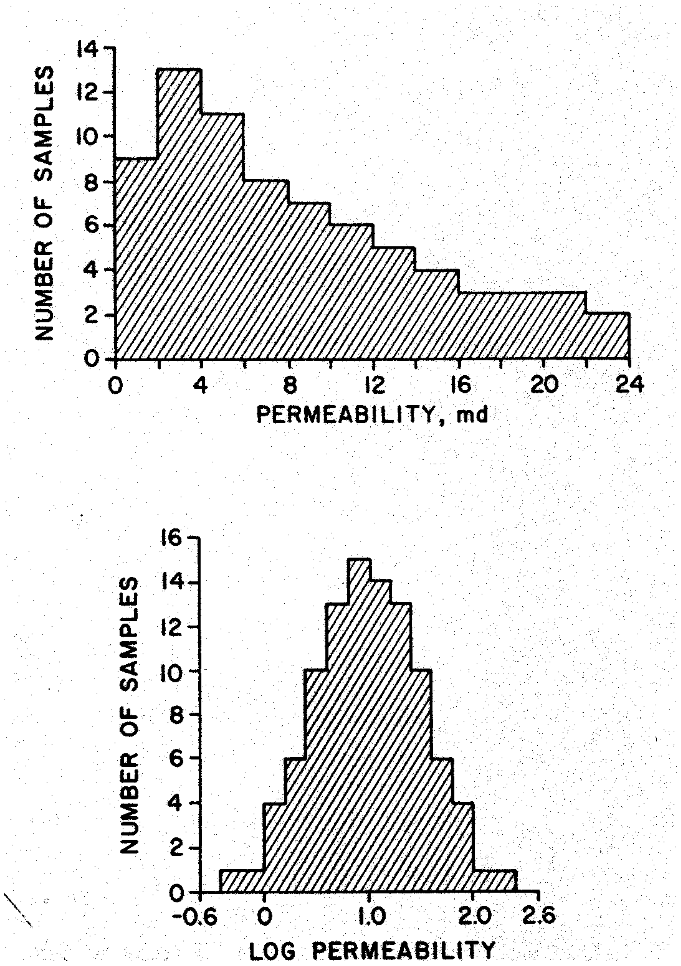
Porosity Distribution



Actual porosity histogram
[NBU42W-29, North Burbank Field]

Distribution of Rock Properties

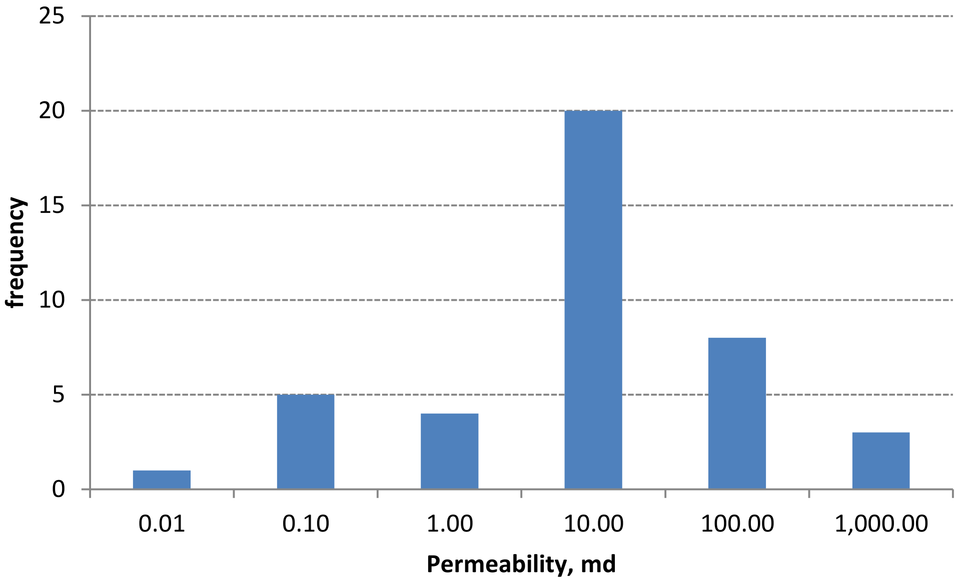
Permeability Distribution



Expected Skewed normal and log normal histograms for permeability [Craig,1971]

Distribution of Rock Properties

Permeability Distribution



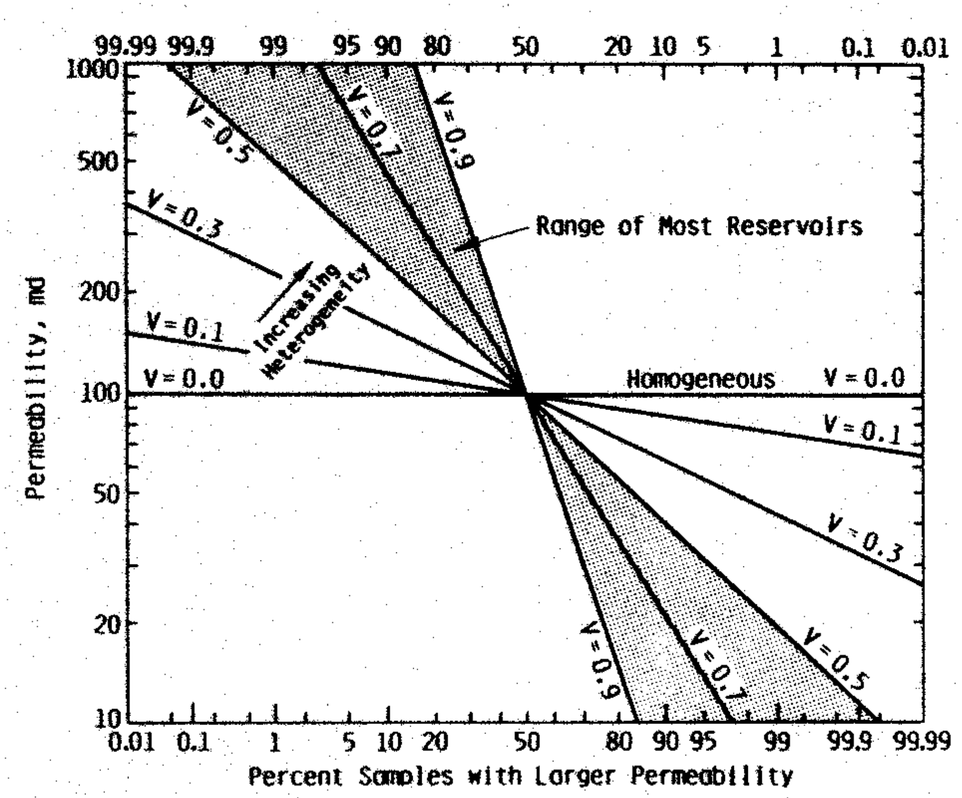
Actual permeability histogram
[NBU42W-29, North Burbank Field]

Distribution of Rock Properties

Permeability Variation

Dykstra-Parsons Coefficient

$$V = \frac{k_{50} - k_{84.1}}{k_{50}}$$

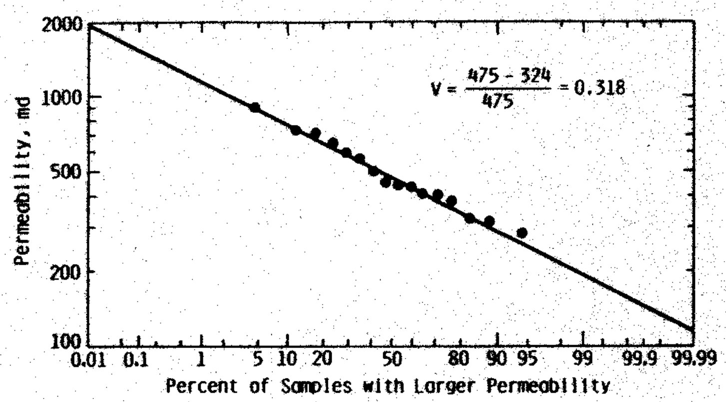


Characterization of reservoir heterogeneity by permeability variation
[Willhite, 1986]

Distribution of Rock Properties

Permeability Variation

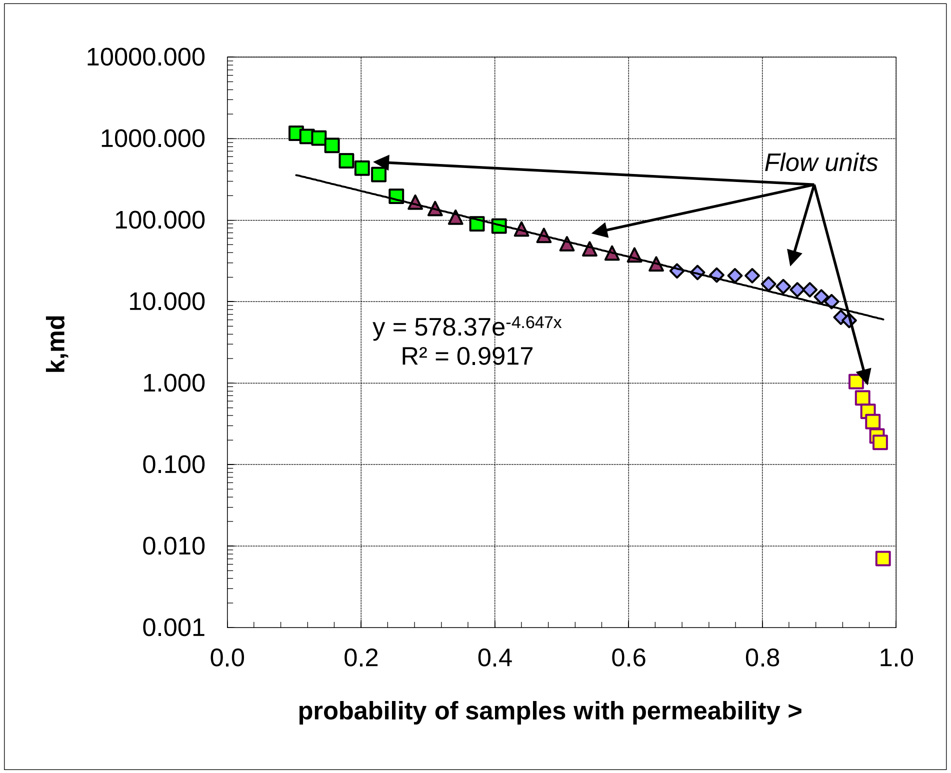
Permeability to Air (md)	Number of Samples With Larger Permeability	Cumulative Frequency Distribution (% > k_i)
1,212	0	0
900	1	5.8
730	2	11.8
714	3	17.7
650	4	23.5
591	5	29.4
565	6	35.3
500	7	41.2
450	8	47.1
440	9	52.9
430	10	58.8
420	11	64.7
407	12	70.6
381	13	76.5
324	14	82.4
315	15	88.2
283	16	94.1



Example of log normal permeability distribution [Willhite, 1986]

Distribution of Rock Properties

Permeability Variation



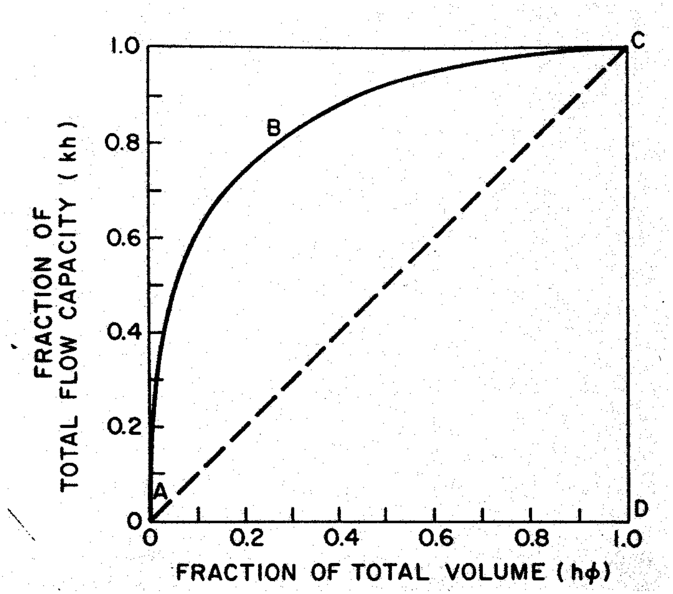
Actual Dykstra-Parsons probability plot
[NBU42W-29, North Burbank Field]

Distribution of Rock Properties

Permeability Variation

Lorenz Coefficient

$$L_k = \frac{\text{Area ABCA}}{\text{Area ADCA}}$$



Flow capacity vs storage capacity distribution
[Craig, 1971]

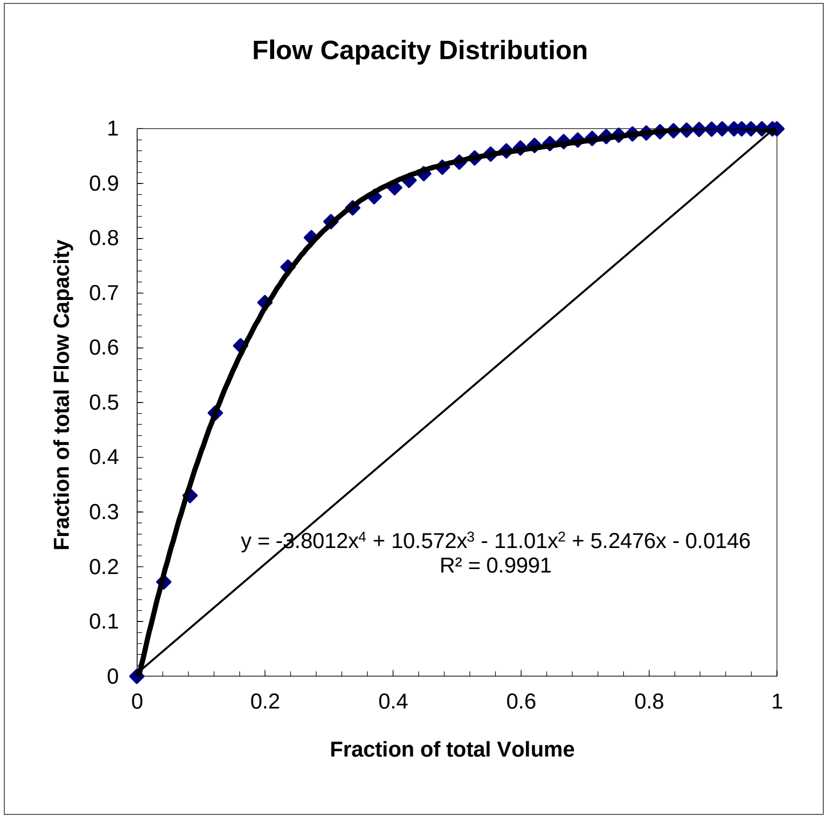
Distribution of Rock Properties

Permeability Variation

Lorenz Coefficient

$$L_k = \frac{Area\ ABCA}{Area\ ADCA}$$

$$= 0.643$$

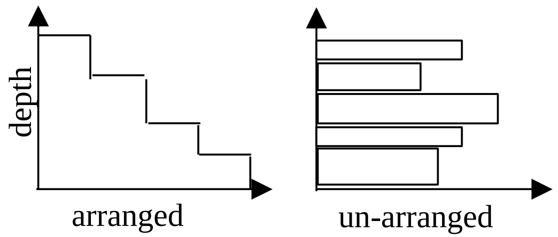


Actual Lorenz plot
[NBU42W-29, North Burbank Field]

Distribution of Rock Properties

Drawback of statistical approaches

- Sequential ordering of data



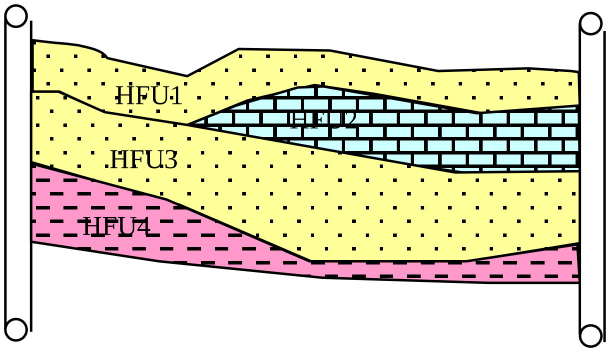
Schematic of statistical approach of arranging data in comparison to true reservoir data, which is not ordered.

- reliance only on permeability variations for estimating flow in layers. Does not account for:
  - phase mobility, pressure gradient, Swirr and the k/ϕ ratio

Distribution of Rock Properties

Hydraulic Flow Unit

- unique units with similar petrophysical properties that affect flow.
  - Hydraulic quality of a rock is controlled by pore geometry
  - It is the distinction of rock units with similar pore attributes, which leads to the separation of units into similar hydraulic units.
  - not equivalent to a geologic unit. The definition of geologic units or facies are not necessarily the same as the definition of a flow unit.



Schematic illustrating the concept of flow units.

Distribution of Rock Properties

- Start with CK equation

$$\sqrt{\frac{k}{\phi}} = \left(\frac{\phi}{1-\phi}\right)\left(\frac{1}{\sqrt{k_o \tau}\, S_{gv}}\right)$$

- Take the log

$$\log(RQI) = \log(\phi_r) + \log(FZI)$$

where the Reservoir quality index (RQI) is given by,

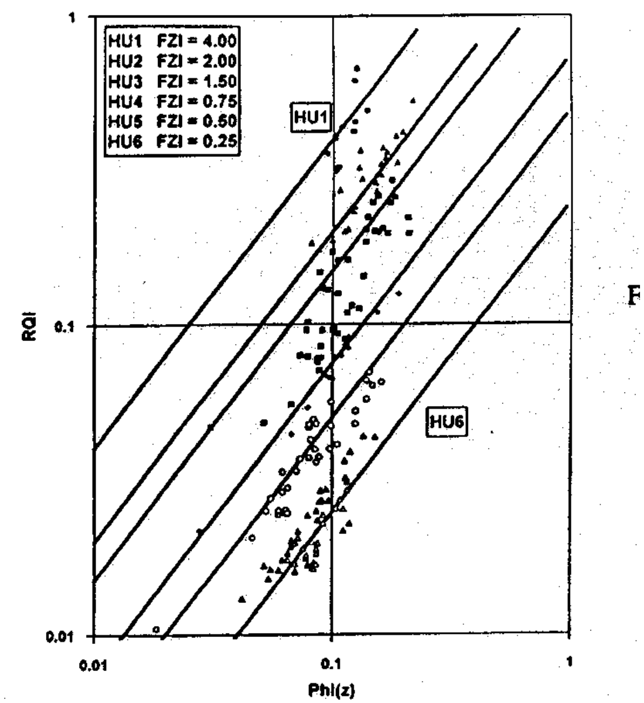
$$RQI\{\mu m\} = 0.0314\sqrt{\frac{k\{md\}}{\phi}}$$

the Flow Zone Indicator (FZI) is,

$$FZI = \frac{1}{S_{gv}\sqrt{k_z}}$$

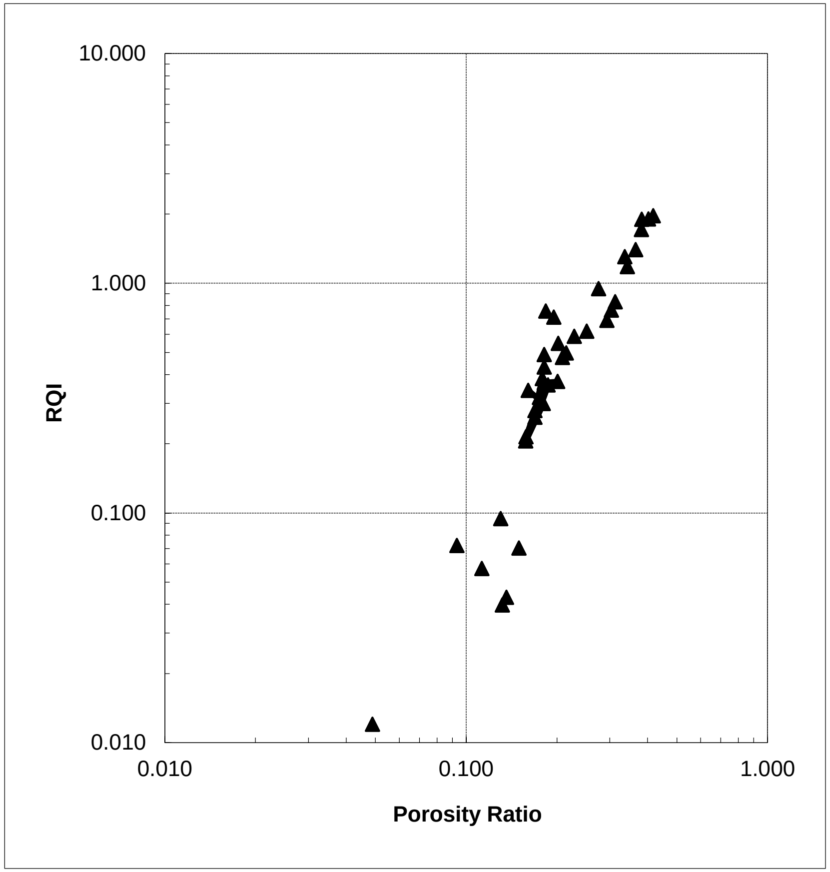
and the pore-to-grain volume ratio is expressed as

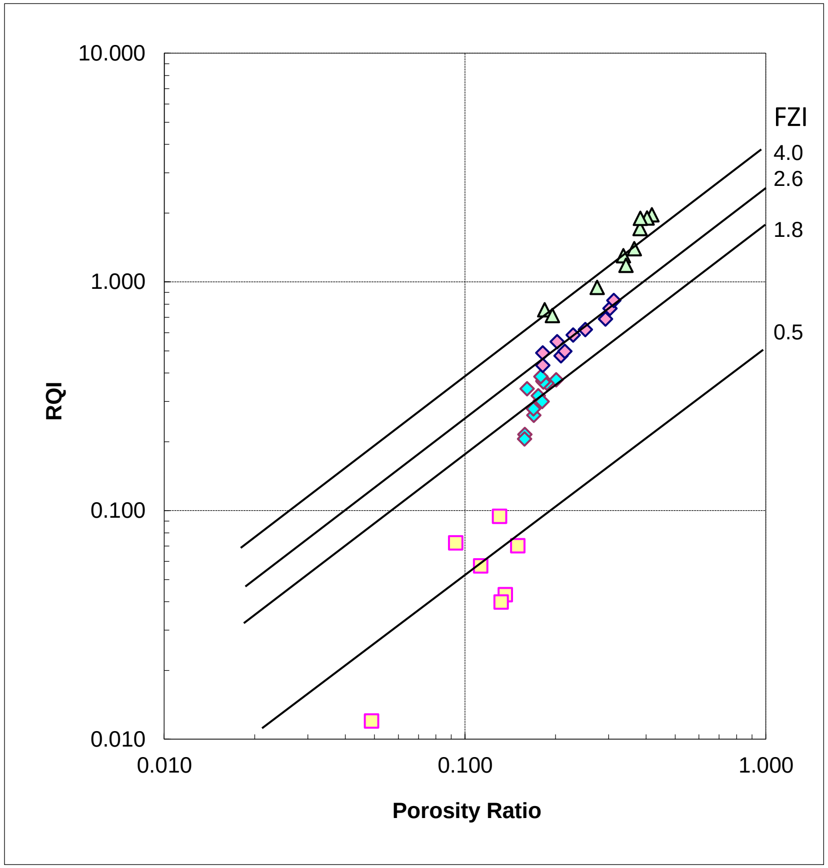
$$\phi_r = \frac{\phi}{1-\phi}$$



Plot of RQI vs ϕ_r for East Texas Well [Amaefule, et al.,1993]

Distribution of Rock Properties





HFU
[NBU42W-29, North Burbank Field]

Distribution of Rock Properties