Delft University of Technology

Faculty of Electrical Engineering, Mathematics and Computer Science

Mathematical Methods For Physics: Finite–Element Analysis (wi4243-III)
2016/2017 Take Home Exams, First Series

All exercises are taken from the book *Numerical Methods in Scientific Computing*

1. Consider the minimal surface problem on the domain Ω with boundary $\Gamma = \Gamma_1 \cup \Gamma_2$

$$\min_{u\in\Sigma} \int_\Omega (1 + u_x^2 + u_y^2)^{\frac{1}{2}} \, d\Omega, \tag{1}$$

$$\Sigma := \{u : \overline{\Omega} \to \mathbb{R} \mid u = g_1 \text{ on } \Gamma_1\}, \tag{2}$$

   where $\overline{\Omega} := \Omega \cup \Gamma$.

   Derive the Euler-Lagrange equation on Ω and the natural boundary condition on Γ_2.

2. Let $\Omega \subset \mathbf{R}^2$ be a region with boundary $\partial\Omega$. On Ω we consider the PDE for the pressure p:

$$-\mathrm{div}\left(\frac{h^3}{12\mu} \,\mathrm{grad}\, p + \frac{h}{2}\mathbf{u}\right) \;=\; 0, \tag{3}$$

   where $h > 0$ is a function of the spatial coordinates, $\mu > 0$ (viscosity) a given constant, $\mathbf{u}$ a given constant velocity vector. The boundary condition for this PDE is:

$$p = 0 \qquad \text{on } \partial\Omega. \tag{4}$$

   Derive the minimization problem corresponding to Equation (3), with boundary condition (4). Motivate why you are allowed to derive the minimization problem using the method that you are currently using. Your answer should not contain any partial derivates of order higher than one.

To be submitted before December 2, 2016, 16:00, EWI, HB03.310