Let $\mathbf{AB} \in \mathbb{R}^{n\times n}$, and $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\alpha \in \mathbb{R}$.

| Operation | Operations | |
|---|---|---|
| | Full matrix | Sparse matrix (m non zero entries) |
| $\mathbf{x}^T\mathbf{y}$ | $n(*) + n - 1(+) = 2n - 1$ | $2n - 1$ |
| $\mathbf{x}(+/-)\mathbf{y}$ | $n$ | $n$ |
| $\alpha\mathbf{x}$ | $n$ | $n$ |
| $\mathbf{Ax}$ | $(n(*) + n - 1(+))n$ (r) $= 2n^2 - n$ | $(m(*) + m - 1(+))n$ (r) $= 2mn - n$ |
| $\mathbf{AB}$ | $[(n(*) + n - 1(+))n$ (r)$]n$ (c) $= 2n^3 - n^2$ | $[(m(*) + m - 1(+))n$ (r) $]m$ (c) $= 2m^2n - nm$ |
| $\mathbf{A} \in \mathbb{R}^{m\times n} \mathbf{B} \in \mathbb{R}^{n\times p}$ | | |
| $\mathbf{AB}$ | $mp(2n-1)$ | |
| $\mathbf{A} = \mathbf{LL}^T$ | $1/3n^3$ | |
| $\mathbf{Lx} = \mathbf{y}$ | $n^2$ | |
| $\mathbf{L}^T\mathbf{x} = \mathbf{y}$ | $n^2$ | |

For each matrix vector multiplication we have n multitplications and n-1 additions

| **Algorithm 3** CG method, solving $\mathbf{Ax} = \mathbf{b}$. | Operations | |
|---|---|---|
| | Full matrix | Sparse matrix |
| Give an initial guess $\mathbf{x}^0$.<br>Compute $\mathbf{r}^0 = \mathbf{b} - \mathbf{Ax}^0$ and set $\mathbf{p}^0 = \mathbf{r}^0$. | $2n^2$ | $2mn$ |
| **for** $k = 0, ...,$ until convergence | | |
| $\alpha^k = \frac{(\mathbf{r}^k, \mathbf{r}^k)}{(\mathbf{Ap}^k, \mathbf{p}^k)}$ | $2n^2 + 3n - 2$ | $2nm + 3n - 2$ |
| $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k\mathbf{p}^k$ | $2n$ | $2n$ |
| $\mathbf{r}^{k+1} = \mathbf{r}^k - \alpha^k\mathbf{Ap}^k$ | $2n^2 + n$ | $2nm + n$ |
| $\beta^k = \frac{(\mathbf{r}^{k+1}, \mathbf{r}^{k+1})}{(\mathbf{r}^k, \mathbf{r}^k)}$ | $4n - 1$ | $4n - 1$ |
| $\mathbf{p}^{k+1} = \mathbf{r}^{k+1} + \beta^k\mathbf{p}^k$ | $2n$ | $2n$ |
| **end** | | |
| Total each k | $4n^2 + 12n - 3$ | $4nm + 12n - 3$ |

| **Algorithm 4** ICCG method, solving $\mathbf{Ax} = \mathbf{b}$. | Operations | |
|---|---|---|
| Split preconditioner | Full matrix | Sparse matrix |
| Give an initial guess $\mathbf{x}^0$. | | |
| Compute $\mathbf{r}^0 = \mathbf{b} - \mathbf{Ax}^0$. | $2n^2$ | $2mn$ |
| Compute $\hat{\mathbf{r}}^0 = \mathbf{L}^{-1}\mathbf{r}^0$. | $n^2$ | $nm$ |
| Compute $\hat{\mathbf{p}}^0 = \mathbf{L}^{-T}\hat{\mathbf{r}}^0$. | $n^2$ | $nm$ |
| **for** $k = 0, ...,$ until convergence | | |
| $\alpha^k = \frac{(\mathbf{r}^k, \mathbf{r}^k)}{(\mathbf{Ap}^k, \mathbf{p}^k)}$ | $2n^2 + 3n - 2$ | $2nm + 3n - 2$ |
| $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k \mathbf{p}^k$ | $2n$ | $2n$ |
| $\mathbf{r}^{k+1} = \mathbf{r}^k - \alpha^k \mathbf{L}^{-1}\mathbf{Ap}^k$ | $3n^2 + n$ | $3nm + n$ |
| $\beta^k = \frac{(\mathbf{r}^{k+1}, \mathbf{r}^{k+1})}{(\mathbf{r}^k, \mathbf{r}^k)}$ | $4n - 1$ | $4n - 1$ |
| $\mathbf{p}^{k+1} = \mathbf{L}^{-T}\mathbf{r}^{k+1} + \beta^k \mathbf{p}^k$ | $n^2 + 2n$ | $nm + 2n$ |
| **end** | | |
| Total each k | $6n^2 + 12n - 3$ | $6nm + 12n - 3$ |


| **Algorithm 4** ICCG method, solving $\mathbf{Ax} = \mathbf{b}$. | Operations | |
|---|---|---|
| Split preconditioner | Full matrix | Sparse matrix |
| Give an initial guess $\mathbf{x}^0$. | | |
| Compute $\mathbf{r}^0 = \mathbf{b} - \mathbf{Ax}^0$. | $2n^2$ | $2mn$ |
| Compute $\hat{\mathbf{r}}^0 = \mathbf{L}^{-1}\mathbf{r}^0$. | $n^2$ | $nm$ |
| Compute $\hat{\mathbf{p}}^0 = \mathbf{L}^{-T}\hat{\mathbf{r}}^0$. | $n^2$ | $nm$ |
| **for** $k = 0, ...,$ until convergence | | |
| $\alpha^k = \frac{(\mathbf{r}^k, \mathbf{r}^k)}{(\mathbf{Ap}^k, \mathbf{p}^k)}$ | $2n^2 + 3n - 2$ | $2nm + 3n - 2$ |
| $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k \mathbf{p}^k$ | $2n$ | $2n$ |
| $\mathbf{r}^{k+1} = \mathbf{r}^k - \alpha^k \mathbf{L}^{-1}\mathbf{Ap}^k$ | $3n^2 + n$ | $3nm + n$ |
| $\beta^k = \frac{(\mathbf{r}^{k+1}, \mathbf{r}^{k+1})}{(\mathbf{r}^k, \mathbf{r}^k)}$ | $4n - 1$ | $4n - 1$ |
| $\mathbf{p}^{k+1} = \mathbf{L}^{-T}\mathbf{r}^{k+1} + \beta^k \mathbf{p}^k$ | $n^2 + 2n$ | $nm + 2n$ |
| **end** | | |
| Total each k | $6n^2 + 12n - 3$ | $6nm + 12n - 3$ |