August 26, 2016

Compressible problem

To describe single-phase flow through a porous medium, the continuity equations are used:

$$\frac{\partial(\rho\phi)}{\partial t} - \nabla \cdot \left(\frac{\rho \mathbf{K}}{\mu} (\nabla \mathbf{p} - \rho g \nabla z) \right) = q. \tag{1}$$

No rock compressibility is assumed $c_r = 0$, fluid compressibility is assumed as constant:

$$\rho(\mathbf{p}) = \rho_0 e^{c_f(\mathbf{p}-\mathbf{p}_0)}. \tag{2}$$

Well model

$$q = J(p_R - p_{bhp}),$$

where J is the productivity or injectivity index.

MRST solver

Using implicit discretization:

$$\frac{\phi\rho(\mathbf{p}^{n+1}) - \phi\rho(\mathbf{p}^{n})}{\Delta t^n} - \nabla \cdot (\rho(\mathbf{p}^{n+1})\frac{\mathbf{K}}{\mu}\nabla(\mathbf{p}^{n+1})) - q^n = 0. \tag{3}$$

$$q^n = W_j(p_r^n - p_{bhp}^n).$$

The latter system can be written in short vector form as:

$$\mathbf{F}(\mathbf{p}^{n+1}; \mathbf{p}^n) = 0, \tag{4}$$

Newton-Rhapson linearization method, the $(n+1)$-th iteration approximation is obtained from:

$$\frac{\partial \mathbf{F}(\mathbf{p}^n)}{\partial \mathbf{p}^n}\delta \mathbf{p}^n = -\mathbf{F}(\mathbf{p}^n), \qquad \delta \mathbf{p}^{n+1} = \mathbf{p}^{n+1} - \mathbf{p}^n,$$

where $\mathbf{J}(\mathbf{p}^n) = \frac{\partial \mathbf{F}(\mathbf{p}^n)}{\partial \mathbf{p}^n}$ is the Jacobian matrix, and $\mathbf{x} = \delta \mathbf{p}^{n+1}$ is the Newton update at iteration step $n+1$, $\mathbf{b} = \mathbf{F}(\mathbf{p}^n)$ is the function evaluated at the time n. The resulting system to solve is therefore:

$$\mathbf{J}\mathbf{x} = -\mathbf{b}.$$

NR Algorithm

while t $<$ totTime
  $t = t + dt$
  $step = step + 1$
  % Newton loop
  $resNorm = 1e99$
  $p0 = double(p_{ad})$ % Previous step pressure
  $nit = 0$

  while (resNorm $>$ tol) && (nit $<=$ maxits)
    % Newton update
      $\mathbf{J} = eq.jac\{1\}$ %Jacobian
      $res = eq.val$ %residual
      resNorm $=$ norm(res)
      $upd = -(\mathbf{J}/res)$ * %Newton update, the solution of this system is obtained with ICCG or DICCG
    % Update variables
      $p_{ad}.val + upd(pIx)$
      $bhp_{ad}.val + upd(bhpIx)$
      $qS_{ad}.val + upd(qsIx)$
      resNorm $=$ norm(res)
    $nit = nit + 1$
  end
end

System configuration

Size: 35 x35 grid cells.
Initial pressure 200 bar.
W1 = W2 = W3 = W4 = 100 bar.
W5 = 600 bars.
10 Snapshots, same conditions (first set of experiments).

Boundary conditions :

$$\frac{\partial P(y=1)}{\partial n} = \frac{\partial P(y=ny)}{\partial n} = \frac{\partial P(x=1)}{\partial n} = \frac{\partial P(x=nx)}{\partial n} = 0.$$

Snapshots (second set of experiments).

$\mathbf{z}_1$: W2 = W3 = W4 = 100 bars, W1 = 200 bars, W5 = 500 bars.

$\mathbf{z}_2$: W1 = W3 = W4 = 100 bars, W2 = 200 bars, W5 = 500 bars.

$\mathbf{z}_3$: W1 = W2 = W4 = 100 bars, W3 = 200 bars, W5 = 500 bars.

$\mathbf{z}_4$: W1 = W2 = W3 = 100 bars, W4 = 200 bars, W5 = 500 bars.

Results

Case 1

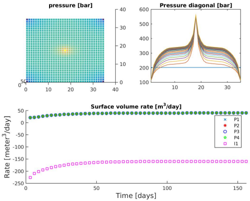


Figure: Solution of the compressible problem solved with the ICCG method.

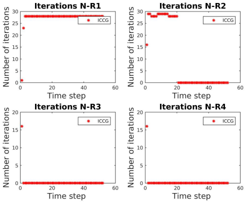


Figure: Iterations, ICCG

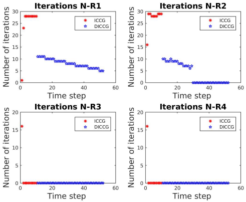


Figure: Iterations DICCG_{10}

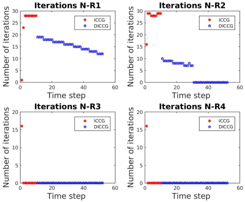


Figure: Iterations DICCG_{5POD}, 6-10

Results

Case 1

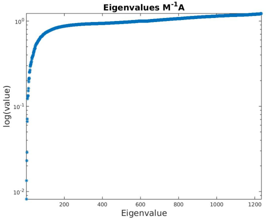


Figure: eigs, step 1 A

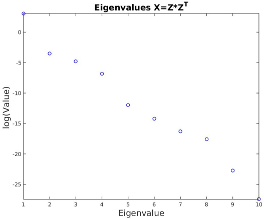


Figure: eigs POD, SVD

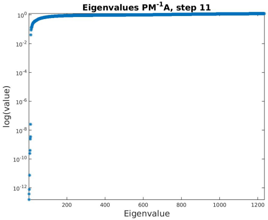


Figure: eigs, step 11 PA

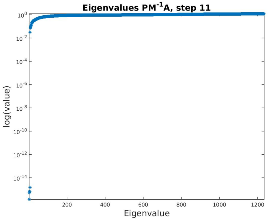
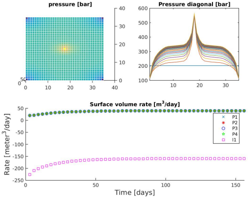


Figure: eigs, step 11 POD PA

Results

Case 1, $xi = x^{t-1}$



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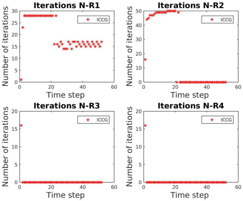


Figure: Iterations, ICCG

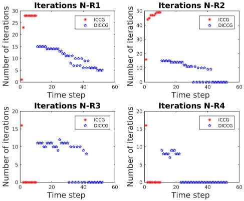


Figure: Iterations $DICCG_{10}$

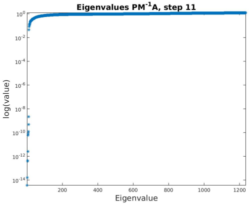


Figure: Iterations $DICCG_{5POD}$, 1-5

Results

Case 1, problems

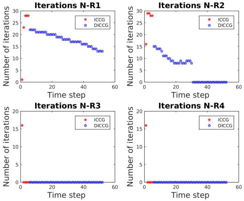


Figure: Iterations DICCG$_5$

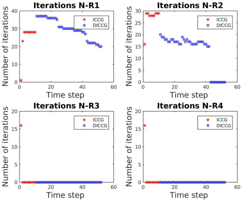
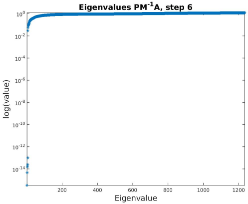


Figure: Iterations DICCG$_{5POD}$, 1-5



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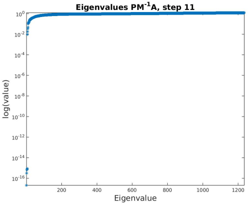
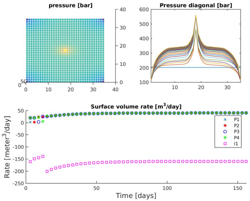


Figure: Iterations DICCG$_{5POD}$, 1-5

Results

Case 2



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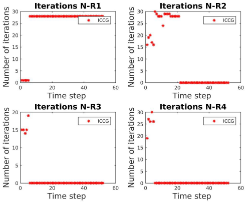


Figure: Iterations, ICCG

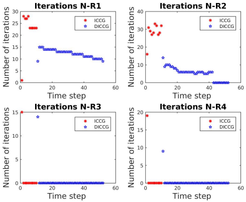


Figure: Iterations DICCG$_{10}$

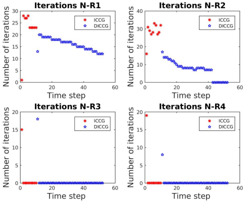


Figure: Iterations DICCG$_{5POD}$, 6-10

Results

Case 2

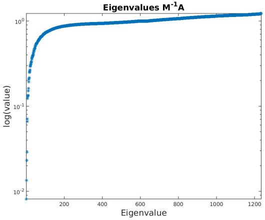


Figure: eigs, step 1 A

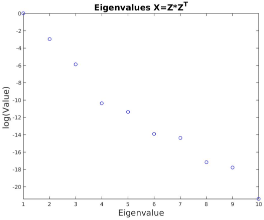


Figure: eigs POD, SVD

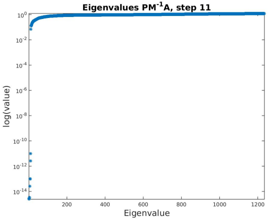


Figure: eigs, step 11 PA

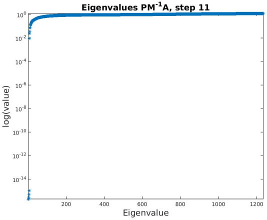
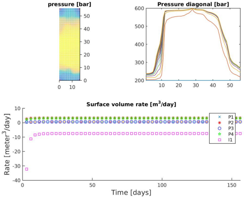


Figure: eigs, step 11 POD PA

Results

SPE 10, 16x56 grid cells, Case 1



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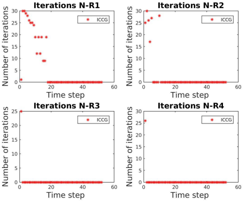


Figure: Iterations, ICCG

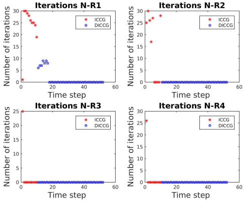


Figure: Iterations DICCG$_{10}$

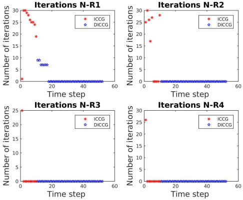


Figure: Iterations DICCG$_{5POD}$, 6-10

Results

SPE 10, 16x56 grid cells, Case 1

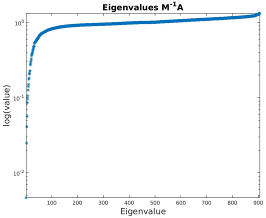


Figure: eigs, step 1 A

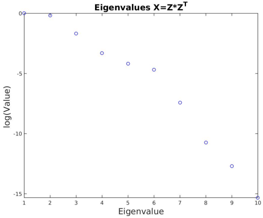


Figure: eigs POD, SVD

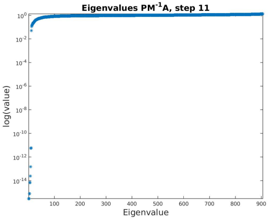


Figure: eigs, step 11 PA

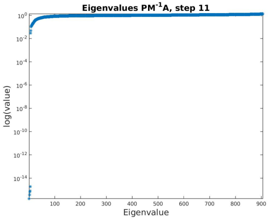
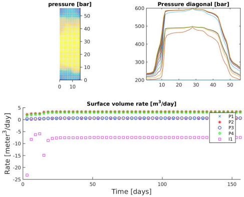


Figure: eigs, step 11 POD PA

Results

SPE 10, 16x56 grid cells, Case 2



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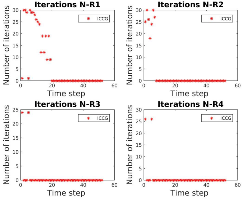


Figure: Iterations, ICCG

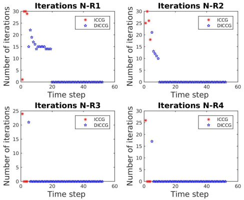


Figure: Iterations DICCG$_{10}$

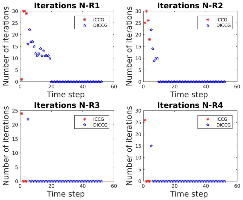


Figure: Iterations DICCG$_{3POD}$, 2, 3, 4

Results

SPE 10, 16x56 grid cells, Case 2

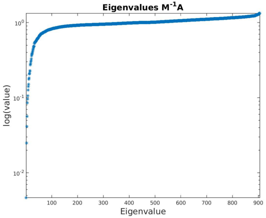


Figure: eigs, step 1 A

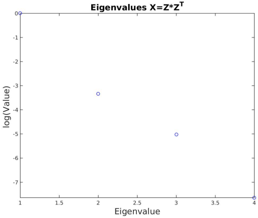


Figure: eigs POD, SVD

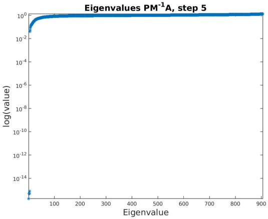


Figure: eigs, step 5 PA

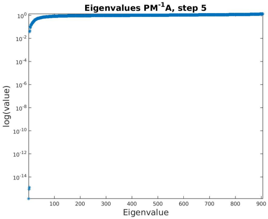


Figure: eigs, step 5 POD PA

Results

SPE 10, 60X220 grid cells, Case 1

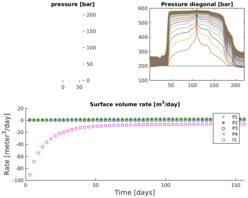


Figure: Solution of the compressible problem solved with the ICCG method.

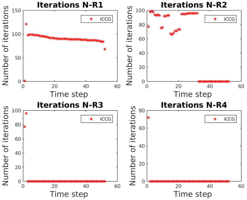


Figure: Iterations, ICCG

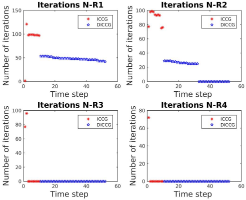


Figure: Iterations DICCG$_{10}$

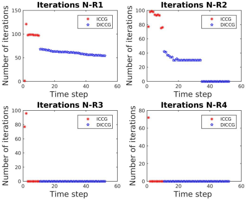


Figure: Iterations DICCG$_{5POD}$, 6-10

Results

SPE 10, 60X220 grid cells, Case 1

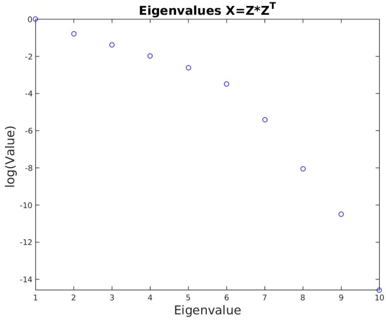
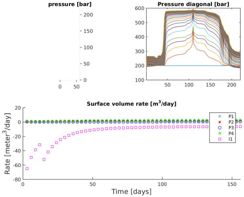


Figure: eigs POD, SVD

Results

SPE 10,60X220 grid cells, Case 2



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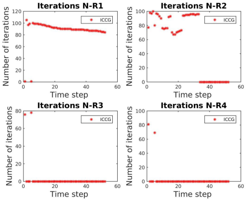

Figure: Iterations, ICCG

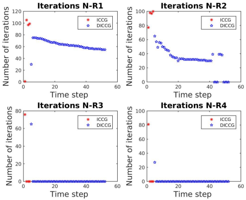

Figure: Iterations DICCG_{10}

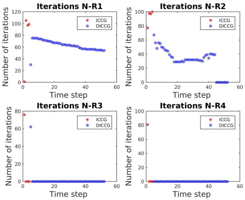

Figure: Iterations DICCG_{3POD}, 2, 3, 4

Results

SPE 10, 60X220 grid cells, Case 2

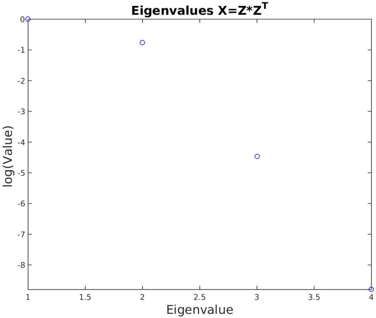

Figure: eigs POD, SVD

Paper

- JCAM Journal of Computational and Applied Mathematics
- ETNA Electronic Transactions on Numerical Analysid
- APNUM Applied Numerical Methods
- International Journal for Numerical Methods in Engineering