

Let  $\mathbf{A}\mathbf{B} \in \mathbb{R}^{n \times n}$ , and  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ ,  $\alpha \in \mathbb{R}$ .

Operation	Operations	
	Full matrix	Sparse matrix (m non zero entries)
$\mathbf{x}^T \mathbf{y}$	$n(*) + n - 1(+) = 2n - 1$	$2n - 1$
$\mathbf{x}(+/-)\mathbf{y}$	$n$	$n$
$\alpha \mathbf{x}$	$n$	$n$
$\mathbf{A}\mathbf{x}$	$(n(*) + n - 1(+))n(\text{r}) = 2n^2 - n$	$(m(*) + m - 1(+))n(\text{r}) = 2mn - n$
$\mathbf{A}\mathbf{B}$	$[(n(*) + n - 1(+))n(\text{r})]n(\text{c}) = 2n^3 - n^2$	$[(m(*) + m - 1(+))n(\text{r})]m(\text{c}) = 2m^2n - nm$
$\mathbf{A} \in \mathbb{R}^{m \times n} \mathbf{B} \in \mathbb{R}^{n \times p}$		
$\mathbf{A}\mathbf{B}$	$mp(2n - 1)$	
$\mathbf{A} = \mathbf{L}\mathbf{L}^T$	$1/3n^3$	
$\mathbf{L}\mathbf{x} = \mathbf{y}$	$n^2$	
$\mathbf{L}^T \mathbf{x} = \mathbf{y}$	$n^2$	

Algorithm 1 CG method, solving $\mathbf{A}\mathbf{x} = \mathbf{b}$ .	Operations	
	Full matrix	Sparse matrix
Give an initial guess $\mathbf{x}^0$ . Compute $\mathbf{r}^0 = \mathbf{b} - \mathbf{A}\mathbf{x}^0$ and set $\mathbf{p}^0 = \mathbf{r}^0$ .	$2n^2$	$2mn$
<b>for</b> $k = 0, \dots$ , until convergence $\alpha^k = \frac{(\mathbf{r}^k, \mathbf{r}^k)}{(\mathbf{A}\mathbf{p}^k, \mathbf{p}^k)}$ $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k \mathbf{p}^k$ $\mathbf{r}^{k+1} = \mathbf{r}^k - \alpha^k \mathbf{A}\mathbf{p}^k$ $\beta^k = \frac{(\mathbf{r}^{k+1}, \mathbf{r}^{k+1})}{(\mathbf{r}^k, \mathbf{r}^k)}$ $\mathbf{p}^{k+1} = \mathbf{r}^{k+1} + \beta^k \mathbf{p}^k$ <b>end</b>	$2n^2 + 3n - 2$ $2n$ $2n^2 + n$ $4n - 1$ $2n$	$2nm + 3n - 2$ $2n$ $2nm + n$ $4n - 1$ $2n$
Total each k	$4n^2 + 12n - 3$	$4nm + 12n - 3$

Algorithm 2 Incomplete Cholesky factorization IC(0)	Operations	
	Full matrix	Sparse matrix
<b>for</b> $j = 2, \dots, n$ 1) Eliminate $a_{ij}$ from $\mathbf{R}_j$ Divide $\mathbf{R}_i$ by $a_{ii}$ Multiply $\mathbf{R}_i$ by $a_{ij}$ Subtraction $\mathbf{R}_j - \mathbf{R}_i$ Total number of operations is	$n - i$ $(n - i + 1)(n - i)$ $(n - i + 1)(n - i)$ $\sum_{i=1}^{n-1} (n - i)(2n - 2i + 3)$	$m - i$ $(m - i + 1)(n - i)$ $(m - i + 1)(n - i)$ $\sum_{i=1}^{n-1} (n - 1)(2n - 2i + 3)$

Algorithm 2 ICCG method, solving $\mathbf{A}\mathbf{x} = \mathbf{b}$ .	Operations	
Split preconditioner	Full matrix	Sparse matrix
Give an initial guess $\mathbf{x}^0$ . Compute $\mathbf{r}^0 = \mathbf{b} - \mathbf{A}\mathbf{x}^0$ . Compute $\hat{\mathbf{r}}^0 = \mathbf{L}^{-1}\mathbf{r}^0$ . Compute $\hat{\mathbf{p}}^0 = \mathbf{L}^{-T}\hat{\mathbf{r}}^0$ .	$2n^2$ $n^2$ $n^2$	$2mn$ $nm$ $nm$
<b>for</b> $k = 0, \dots$ , until convergence $\alpha^k = \frac{(\mathbf{r}^k, \mathbf{r}^k)}{(\mathbf{A}\mathbf{p}^k, \mathbf{p}^k)}$ $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k \mathbf{p}^k$ $\mathbf{r}^{k+1} = \mathbf{r}^k - \alpha^k \mathbf{L}^{-1} \mathbf{A}\mathbf{p}^k$ $\beta^k = \frac{(\mathbf{r}^{k+1}, \mathbf{r}^{k+1})}{(\mathbf{r}^k, \mathbf{r}^k)}$ $\mathbf{p}^{k+1} = \mathbf{L}^{-T} \mathbf{r}^{k+1} + \beta^k \mathbf{p}^k$ <b>end</b>	$2n^2 + 3n - 2$ $2n$ $3n^2 + n$ $4n - 1$ $n^2 + 2n$	$2nm + 3n - 2$ $2n$ $3nm + n$ $4n - 1$ $nm + 2n$
Total each k	$6n^2 + 12n - 3$	$6nm + 12n - 3$

Let  $\mathbf{A} \in \mathbb{R}^{n \times n}$ , with  $m$  non zero entries per row.

Algorithm 3 DICCG method, solving $\mathbf{Ax} = \mathbf{b}$ .	Operations	
Split preconditioner	Full matrix	Sparse matrix
Let $\mathbf{Z} \in \mathbb{R}^{n \times p}$ $\mathbf{E} = \mathbf{Z}^T \mathbf{A} \mathbf{Z}$ $\mathbf{Qx} = \mathbf{Z} \mathbf{E}^{-1} \mathbf{Z}^T \mathbf{x}$ $\mathbf{Px} = (\mathbf{I} - \mathbf{A} \mathbf{Q}) \mathbf{x}$	$2np(2n - 1)$ $n^2 + 3np - n + 2p^2 - 2p$ $3n^2 + 3np + 2p^2 - 2p - n$	$2np(2m - 1)$ $n^2 + 3np - n + 2p^2 - 2p$ $2nm + n^2 + 3np + 2p^2 - 2p - n$

Algorithm 3 DICCG method, solving $\mathbf{Ax} = \mathbf{b}$ .	Operations	
Split preconditioner	Full matrix	Sparse matrix
Give an initial guess $\mathbf{x}^0$ . Compute $\mathbf{r}^0 = \mathbf{b} - \mathbf{Ax}^0$ . Compute $\hat{\mathbf{r}}^0 = \mathbf{Pr}^0$ . Compute $\mathbf{y}^0 = \mathbf{L}^{-1} \hat{\mathbf{r}}^0$ . Compute $\hat{\mathbf{p}}^0 = \mathbf{L}^{-T} \mathbf{y}^0$ .	$2n^2$ $3n^2 + 3np + 2p^2 - 2p - n$ $n^2$ $n^2$	$2mn$ $2nm + n^2 + 3np + 2p^2 - 2p - n$ $nm$ $nm$
<b>for</b> $k = 0, \dots$ , until convergence $\hat{\mathbf{w}}^0 = \mathbf{PAp}^k$ . $\alpha^k = \frac{(\hat{\mathbf{r}}^k, \mathbf{y}^k)}{(\mathbf{p}^k, \hat{\mathbf{w}}^k)}$ $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha^k \mathbf{p}^k$ $\mathbf{r}^{k+1} = \mathbf{r}^k - \alpha^k \mathbf{L}^{-1} \mathbf{Ap}^k$ $\beta^k = \frac{(\mathbf{r}^{k+1}, \mathbf{r}^{k+1})}{(\mathbf{r}^k, \mathbf{r}^k)}$ $\mathbf{p}^{k+1} = \mathbf{L}^{-T} \mathbf{r}^{k+1} + \beta^k \mathbf{p}^k$ <b>end</b> Total each k	$5n^2 + 3np + 2p^2 - 2p - 2n$ $2n^2 + 3n - 2$ $2n$ $3n^2 + n$ $4n - 1$ $n^2 + 2n$	$4nm + n^2 + 3np + 2p^2 - 2p - 2n$ $2nm + 3n - 2$ $2n$ $3nm + n$ $4n - 1$ $nm + 2n$
$\mathbf{x}_{it} = \mathbf{Qb} + \mathbf{P}^T \mathbf{x}^{k+1}$	$11n^2 + 3np + 2p^2 - 2p + 10n - 3$ $4n^2 + 6np + 4p^2 - 4p - 2n$	$10nm + n^2 + 3np + 2p^2 - 2p + 10n - 3$ $2nm + 2n^2 + 6np + 4p^2 - 4p - 2n$

Operation	Operations	
	Full matrix	Sparse matrix (m non zero entries)
$\mathbf{A} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$	$13n^3$	$11n^3 + 2mn^2$
$\mathbf{x}(+/-)\mathbf{y}$	$n$	$n$
$\alpha \mathbf{x}$	$n$	$n$
$\mathbf{Ax}$	$(n(*) + n - 1(+))n \text{ (r)} = 2n^2 - n$	$(m(*) + m - 1(+))n \text{ (r)} = 2mn - n$
$\mathbf{AB}$	$[(n(*) + n - 1(+))n \text{ (r)}]n \text{ (c)} = 2n^3 - n^2$	$[(m(*) + m - 1(+))n \text{ (r)}]m \text{ (c)} = 2m^2n - nm$
$\mathbf{A} \in \mathbb{R}^{m \times n} \mathbf{B} \in \mathbb{R}^{n \times p}$		
$\mathbf{AB}$	$mp(2n - 1)$	
$\mathbf{A} = \mathbf{LL}^T$	$1/3n^3$	
$\mathbf{Lx} = \mathbf{y}$	$n^2$	
$\mathbf{L}^T \mathbf{x} = \mathbf{y}$	$n^2$	