

$$x(t) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \quad (1)$$

$$\rightarrow t=0 \quad p_0, \dot{p}_0, \ddot{p}_0 = v_0, a_0 \quad (4)$$

$$t \Rightarrow t_f \quad p_f, \dot{p}_f, \ddot{p}_f = v_f, a_f \quad (5)$$

$$\dot{x}(t) = 5a_5 t^4 + 4a_4 t^3 + 3a_3 t^2 + 2a_2 t + a_1 \quad (2)$$

$$\ddot{x}(t) = 20a_5 t^3 + 12a_4 t^2 + 6a_3 t + 2a_2 \quad (3)$$

$$x(0) = a_0, \quad x(t_f) = a_5 t_f^5 + a_4 t_f^4 + a_3 t_f^3 + a_2 t_f^2 + a_1 t_f + a_0$$

$$\dot{x}(0) = a_1, \quad \dot{x}(t_f) = 5a_5 t_f^4 + 4a_4 t_f^3 + 3a_3 t_f^2 + 2a_2 t_f + a_1$$

$$\ddot{x}(0) = 2a_2, \quad \ddot{x}(t_f) = 20a_5 t_f^3 + 12a_4 t_f^2 + 6a_3 t_f + 2a_2$$

$$p_0 = a_0, \quad \dot{p}_0 = a_1, \quad \ddot{p}_0 = 2a_2$$

$$\begin{aligned} a_0 &= p_0 \\ a_1 &= \dot{p}_0 \\ a_2 &= \ddot{p}_0 / 2 \end{aligned}$$

$$x(t_f) = p_f, \quad \dot{x}(t_f) = \dot{p}_f, \quad \ddot{x}(t_f) = \ddot{p}_f$$

$$p_f = a_5 t_f^5 + a_4 t_f^4 + a_3 t_f^3 + \frac{\ddot{p}_0}{2} t_f^2 + \dot{p}_0 t_f + p_0$$

$$\dot{p}_f = 5a_5 t_f^4 + 4a_4 t_f^3 + 3a_3 t_f^2 + \ddot{p}_0 t_f + \dot{p}_0$$

$$\ddot{p}_f = 20a_5 t_f^3 + 12a_4 t_f^2 + 6a_3 t_f + \ddot{p}_0$$

$$\begin{bmatrix} x(0) \\ x(t_f) \\ \dot{x}(0) \\ \dot{x}(t_f) \\ \ddot{x}(0) \\ \ddot{x}(t_f) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ t_f^5 & t_f^4 & t_f^3 & t_f^2 & t_f & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 5t_f^4 & 4t_f^3 & 3t_f^2 & 2t_f & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 20t_f^3 & 12t_f^2 & 6t_f & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$\begin{bmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = A^{-1} \begin{bmatrix} x(0) \\ \dot{x}(0) \\ \ddot{x}(0) \\ \ddot{x}(t_f) \\ \ddot{x}(0) \\ \ddot{x}(t_f) \end{bmatrix} = A^{-1} \begin{bmatrix} p_0 \\ p_f \\ V_0 \\ V_f \\ a_0 \\ a_f \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{-6}{t_f^5} & \frac{6}{t_f^5} & \frac{-3}{t_f^4} & \frac{-3}{t_f^4} & \frac{-1}{2t_f^3} & \frac{1}{2t_f^3} \\ \frac{15}{t_f^4} & \frac{-15}{t_f^4} & \frac{0}{t_f^3} & \frac{7}{t_f^3} & \frac{3}{2t_f^2} & \frac{-1}{t_f^2} \\ \frac{-10}{t_f^3} & \frac{10}{t_f^3} & \frac{-6}{t_f^2} & \frac{-4}{t_f^2} & \frac{-3}{2t_f} & \frac{1}{2t_f} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{a}_5 = \frac{-6}{t_f^5} p_0 + \frac{6}{t_f^5} p_f - \frac{3}{t_f^4} V_0 - \frac{3}{t_f^4} V_f - \frac{1}{2t_f^3} a_0 - \frac{1}{2t_f^3} a_f$$

$$\bar{a}_4 = \frac{15}{t_f^4} p_0 - \frac{15}{t_f^4} p_f + \frac{0}{t_f^3} V_0 + \frac{7}{t_f^3} V_f + \frac{3}{2t_f^2} a_0 - \frac{1}{2t_f^2} a_f$$

$$\bar{a}_3 = \frac{-10}{t_f^3} p_0 + \frac{10}{t_f^3} p_f - \frac{6}{t_f^2} V_0 - \frac{4}{t_f^2} V_f - \frac{3}{2t_f} a_0 - \frac{1}{2t_f} a_f$$

$$\bar{a}_2 = a_0$$

$$\bar{a}_1 = V_0$$

$$\bar{a}_0 = p_0$$