

$$F \sim R(0, \theta) \quad \tilde{\theta}_3 = \max x_i$$

Состоятельность:

$$\forall \theta \in \Theta \Leftrightarrow \tilde{\theta} \xrightarrow{P} \theta$$

$$P(|\max x_i - \theta| \geq \varepsilon) = P(\max x_i \geq \theta + \varepsilon) + P(\max x_i \leq \theta - \varepsilon) =$$

$$= 1 - \underbrace{P(\max x_i < \theta + \varepsilon)}_{(F(\theta + \varepsilon))^n = 1} + \underbrace{P(\max x_i < \theta - \varepsilon)}_{(F(\theta - \varepsilon))^n = (1 - \frac{\varepsilon}{\theta})^n \xrightarrow{n \rightarrow \infty} 0} + \underbrace{P(\max x_i = \theta - \varepsilon)}_0$$

$$\Rightarrow \tilde{\theta}_3 \xrightarrow{P} \theta \text{ состоятельность}$$

$$p = \begin{cases} e^{-\frac{x}{\theta}} \cdot \frac{1}{\theta}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

$$n=3 \quad \tilde{\theta}_1 = \bar{x}, \quad \tilde{\theta}_2 = x_{(2)}$$

$$M[\xi] = \int_{-\infty}^{\infty} e^{-\frac{x}{\theta}} \frac{x}{\theta} dx = \theta \int_0^{\infty} e^{-t} t dt =$$

$$= \left[ -e^{-t} t \Big|_0^{\infty} + \int_0^{\infty} e^{-t} dt \right] \theta = \theta$$

$$M[\xi^2] = \int_0^{\infty} e^{-\frac{x}{\theta}} \frac{x^2}{\theta} dx = \theta^2 \int_0^{\infty} e^{-t} t^2 dt =$$

$$= \theta^2 \left[ -e^{-t} t^2 \Big|_0^{\infty} + \int_0^{\infty} 2e^{-t} t dt \right] = 2\theta^2$$

$$a) M[\bar{x}] = \frac{1}{n} \cdot n M[\xi] = \theta$$

$\tilde{\theta}_1$  несмещенная



$$F(t) = \int_0^t p(x) dx$$

$$F(x) = \int_0^x p(t) dt = 1 - e^{-\frac{x}{\theta}}$$

$$\mathcal{L} = n p(x) C_{n-1}^{k-1} (1-F)^{n-k} F^{k-1} =$$

$$= 3 p(x) C_2^1 (e^{-\frac{x}{\theta}})' (1 - e^{-\frac{x}{\theta}}) =$$

$$= \frac{6}{\theta} (e^{-\frac{2x}{\theta}} - e^{-\frac{3x}{\theta}})$$

$$M[\tilde{\theta}_2] = 6 \int_0^{\infty} \frac{x}{\theta} (e^{-\frac{2x}{\theta}} - e^{-\frac{3x}{\theta}}) dx$$

$$\int_0^{\infty} x \frac{e^{-\frac{2x}{\theta}}}{\theta} dx = -\frac{1}{2} \int_0^{\infty} x de^{-\frac{2x}{\theta}} =$$

$$= -\frac{1}{2} [x e^{-\frac{2x}{\theta}} - \int_0^{\infty} e^{-\frac{2x}{\theta}} dx] = \frac{\theta}{4}$$

$$\int_0^{\infty} x \frac{e^{-\frac{3x}{\theta}}}{\theta} dx = -\frac{1}{3} [x e^{-\frac{3x}{\theta}} - \int_0^{\infty} e^{-\frac{3x}{\theta}} dx] = \frac{\theta}{9}$$

$$M[\tilde{\theta}_2] = 6 \left[ \frac{\theta}{4} - \frac{\theta}{9} \right] = \frac{5}{6} \theta$$

$$\tilde{\theta}_2 - \text{CMVY}, \quad \tilde{\theta}_2' = \frac{6}{5} X(2)$$

$$b) D[\tilde{\theta}_1] = D[\bar{X}] = \frac{1}{n^2} \cdot n D[\theta] = \frac{\theta^2}{3} = \frac{\theta^2}{3}$$

$$M[\tilde{\theta}_2'^2] = \frac{36}{25} \cdot 6 \cdot \int_0^{\infty} \frac{x^2}{\theta} (e^{-\frac{2x}{\theta}} - e^{-\frac{3x}{\theta}}) dx$$

$$\int_0^{\infty} \frac{x^2}{\theta} e^{-\frac{2x}{\theta}} dx = -\frac{1}{2} \int_0^{\infty} x^2 de^{-\frac{2x}{\theta}} =$$

$$= -\frac{1}{2} [x^2 e^{-\frac{2x}{\theta}} - \int_0^{\infty} 2x e^{-\frac{2x}{\theta}} dx] = \frac{\theta^2}{4}$$



$$\int_0^{\infty} \frac{x^2}{\theta} e^{-\frac{3x}{\theta}} dx = -\frac{1}{3} \int_0^{\infty} x^2 d e^{-\frac{3x}{\theta}} =$$

$$= -\frac{1}{3} \left[ x^2 e^{-\frac{3x}{\theta}} \Big|_0^{\infty} - 2 \int_0^{\infty} x e^{-\frac{3x}{\theta}} dx \right] = \frac{2\theta^2}{27}$$

$$M[\tilde{\theta}_1'^2] = \frac{36}{25} \cdot 6 \left( \frac{\theta^2}{4} - \frac{2\theta^2}{27} \right) = \frac{38}{25} \theta^2$$

$$D[\tilde{\theta}_1'^2] = \frac{13}{25} \theta^2$$

$$D[\tilde{\theta}_1'] < D[\tilde{\theta}_2'] \Rightarrow \tilde{\theta}_1 \text{ более эффективен.}$$

с) непрерывность модели:

1)  $p(x, \theta)$  - непрерывна, строго положителна при  $\theta > 0$

$$2) \frac{\partial}{\partial \theta} \int_0^{\infty} p(x, \theta) dx = 0$$

$$\int_0^{\infty} \frac{\partial}{\partial \theta} p(x, \theta) dx = \int_0^{\infty} \frac{\frac{x}{\theta^2} \cdot e^{-\frac{x}{\theta}} \cdot \theta - e^{-\frac{x}{\theta}}}{\theta^2} dx =$$

$$= \int_0^{\infty} \frac{x}{\theta^3} e^{-\frac{x}{\theta}} dx - \int_0^{\infty} \frac{1}{\theta^2} e^{-\frac{x}{\theta}} dx = 0$$

$$\Rightarrow \frac{\partial}{\partial \theta} \int_0^{\infty} p dx = \int_0^{\infty} \frac{\partial}{\partial \theta} p dx \quad \checkmark$$

$$3) I(\theta) = M \left[ \left( \frac{\partial \ln p(x, \theta)}{\partial \theta} \right)^2 \right] =$$

$$= M \left[ \left( \frac{\partial}{\partial \theta} \ln \frac{e^{-\frac{x}{\theta}}}{\theta} \right)^2 \right] = M \left[ \left( -\frac{x}{\theta^2} - \frac{1}{\theta} \right)^2 \right]$$

$$= M \left[ \frac{x^2}{\theta^4} + 2 \frac{x}{\theta^3} + \frac{1}{\theta^2} \right] =$$

$$= \int_0^{\infty} \left( \frac{x^2}{\theta^4} + 2 \frac{x}{\theta^3} + \frac{1}{\theta^2} \right) \frac{e^{-\frac{x}{\theta}}}{\theta} dx = \frac{1}{\theta^2}$$

$$I(\theta) \text{ непрерывна, } I > 0 \text{ при } \theta > 0 \quad \checkmark$$



модель рекуррентна

рекуррентность оценок:

агента носителя и допри  
на шлобыл какнакме из  $F$   
выплатено и для  $\tilde{\theta}_1$  и для  $\tilde{\theta}_2'$

оценки рекуррентны

$$D[\tilde{g}] \geq \frac{g^2(\theta)}{n \cdot I(\theta)}$$

$$\tilde{\theta}_1: \frac{\theta^2}{3} \geq \frac{\theta^2}{3}$$

$$\tilde{\theta}_2': \frac{13}{25} \theta^2 \geq \frac{\theta^2}{3}$$

$\tilde{\theta}_1$  - эффективная

$\tilde{\theta}_2'$  - дост. усл. эф. не вып.