AST3220, spring 2020: Project 1

Moral speech

This project consists of a set of tasks, some analytical, some numerical. You should structure your answers as a report with an introduction, methods, results, discussion and conclusion. It is important that you explain how you think, just writing down a bunch of equations with no explanations will not give you a maximum score. I recommend that you write the report using LaTeX. Posting handwritten lecture notes and solutions to problems is a privilege that belongs to the lecturer alone.

Your figures should have a clear layout with proper axis labels and units, and with a caption explaining what the figure shows. The figures should be referenced in the main text. You are also required to hand in your source code in a form that can be easily compiled. If you use python, use python 3 as this makes testing your codes easier for us.

VERY IMPORTANT: Use your candidate number, and nothing else, to identify yourself in the report. In previous years we have seen several examples of students handing in reports with their full name and/or e-mail address. The evaluation process is supposed to be anonymous. Therefore, if we find your full name in the report, we will deduct 5 points from your score.

Exploring possible expansion histories

Goals of this project:

1. Derive equation (1) (5 points)

2. Use it to explore possible expansion histories in the ΛCDM case, in particular show by examples that the lines and labels in the figure included with the project are reasonable (10 points)

3. Find a simple empirical test for the "No Big Bang" cosmologies in terms of the redshift which can be used to rule them out (15 points)

4. Explore possibilites when $w < -1$ (10 points)

5. You will need to write a routine for evaluating the luminosity distance numerically for a given model, and in your report you should test it

against two cases (of your own choice) where $d_\mathrm{L}(z)$ can be found analytically (5 points)

6. Use supernova data to constrain $\Omega_{\mathrm{m}0}$ and $\Omega_{\Lambda 0}$. You should include a plot showing the 95 % confidence region in the $\Omega_{\mathrm{m}0}$-$\Omega_{\Lambda 0}$-plane (20 points)
7. Assuming $\Omega_{\mathrm{m}0} = 0.3$, use supernova data to constrain $\Omega_{\mathrm{w}0}$ and w. You should include a plot showing the 95 % confidence region in the $\Omega_{\mathrm{w}0}$-w-plane (15 points)
8. Can you find any interesting expansion histories (i.e., something different from the standard case of a Big Bang followed by eternal expansion) within the allowed region of parameter values in the two cases? (10 points)

In addition to the two plots mentioned above, you can include any other plots that you think will be helpful to the reader and help you make your point.

The last 10 points are awarded for a well-written and structured report.

You should use the value $h = 0.7$ for the dimensionless Hubble constant wherever you need it.

Background information

We have found some analytical solutions of the first Friedmann equation, but in most cases this is not possible. However, it is possible to explore the qualitative properties of the solutions in a simple, graphical way. With matter, spatial curvature, and a component with equation of state parameter w (the case with a cosmological constant corresponds to $w = -1$, $\Omega_{w0} = \Omega_{\Lambda 0}$) the first Friedmann equation is

$$\frac{1}{H_0^2}\left(\frac{\dot{a}}{a}\right)^2 = \Omega_{\mathrm{m}0}\left(\frac{a_0}{a}\right)^3 + \Omega_{\mathrm{k}0}\left(\frac{a_0}{a}\right)^2 + \Omega_{w0}\left(\frac{a_0}{a}\right)^{3(1+w)}.$$

By introducing new variables $x = a/a_0$ and $\tau = H_0 t$, this equation can be written in dimensionless form:

$$\left(\frac{dx}{d\tau}\right)^2 + \left[-\left(\frac{\Omega_{\mathrm{m}0}}{x} + \frac{\Omega_{w0}}{x^{1+3w}}\right)\right] = \Omega_{\mathrm{k}0} = \text{constant}. \qquad (1)$$

Note that the first term on the left-hand side is always ≥ 0, just like a kinetic energy. So this equation has the same form as the equation for conservation of energy of a particle moving in one dimension in a potential,

$$T + U = E = \text{constant},$$

This gives us a quick way of checking the qualitative nature of the solution of the Friedmann equations for given values of the density parameters. Plot the 'potential energy' U as a function of $x = a/a_0$, and in the same plot draw the line $E(x) = 1 - \Omega_{\mathrm{m}0} - \Omega_{w0} =$ constant. The allowed values of x (that is, a) are those for which U lies below or touches E. The scale factor cannot have values fow which U lies above E, because that would mean that the 'kinetic energy' would be negative.

Along with this text you will receive a table of measured luminosity distances with associated errors. The table is in the format (redshift, luminosity distance, error). The distances and the errors are given in units of Gpc (1 Gpc = 10^9 pc). You will now use these measurements to find empirical constraints on cosmological parameters. Let us, as an example, look at the case when we want to constrain $\Omega_{\mathrm{m}0}$ and $\Omega_{\Lambda 0}$. Let us call the expression for the luminosity distance based on these two parameters for our model. Given values for $\Omega_{\mathrm{m}0}$ and $\Omega_{\Lambda 0}$, we want to know the probability of the model, given the data, $P(\text{model}|\text{data})$. There is no ready recipe for calculating this probability, but a result known as Bayes' theorem says that

$$P(\text{model}|\text{data}) = \frac{P(\text{data}|\text{model})P(\text{model})}{P(\text{data})} \tag{2}$$

The second factor in the numerator is the probability we would assign to the model before obtaining the data, and it is called the *prior*. The factor in the denominator is known as the *evidence*. We will, as is quite common, consider both of these factors to be constants, and we then have the result

$$P(\text{model}|\text{data}) \propto P(\text{data}|\text{model}). \tag{3}$$

The probability on the right-hand side is known as the *likelihood*, and the point is that it is possible to work out how to calculate it. For example, we will assume that the observations are drawn from a Gaussian distribution. This means that we assume that if we measure the luminosity distance to the ith redshift z_i to be d_{L}^i with measurement error σ_i, then the probability distribution for the true luminosity distance $d_{\rm L}(z_i)$ is

$$P(d_{\rm L}(z_i)) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left[-\frac{(d_{\rm L}(z_i) - d_L^i)^2}{2\sigma_i^2}\right].$$

If we also assume that the measurements are uncorrelated, it can be shown that the likelihood is given by

$$P(\text{data}|\text{model}) = \frac{1}{(2\pi \prod_{i=1}^N \sigma_i^2)^{1/2}} \exp\left[-\frac{1}{2}\sum_{i=1}^N \frac{(d_L(z_i;\vec{p}) - d_L^i)^2}{\sigma_i^2}\right],$$

where $d_L(z_i, \vec{p})$ is the model prediction for the luminosity distance to redshift z_i for given parameter vector $\vec{p}$, and N is the number of observations. In our case, $\vec{p} = (\Omega_{\rm m0}, \Omega_{\Lambda0})$. To find the most probable values of $\Omega_{\rm m0}$ and $\Omega_{\Lambda0}$, we want to maximize the likelihood as a function of these two parameters, and this is equivalent to minimizing the quantity

$$\chi^2(\vec{p}) = \sum_{i=1}^N \frac{(d_L(z_i;\vec{p}) - d_L^i)^2}{\sigma_i^2}.$$

We cannot be sure that the most probable value is the *true* value. All the data allow us to find is the most probable value, and the range in which the true value probably lies. In the case you consider here, it can be shown that there is a 95 % probability that the true values of $\Omega_{\rm m0}$ and $\Omega_{\lambda0}$ if found in the region which satisfies

$$\chi^2(\vec{p}) - \chi^2_{\rm min} < 6.17,$$

where $\chi^2_{\rm min}$ is the minimum value of χ^2.