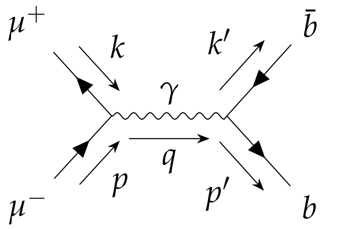
Project 1

TBD

February 17, 2022

1 Task 1

1.1 Feynman diagrams and scattering amplitudes



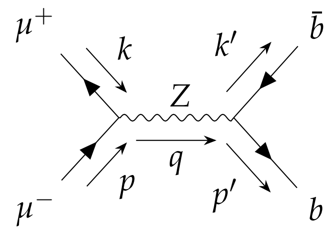
(a) Feynman diagram of $\mu^+\mu^- \to b\bar{b}$. The outermost arrows indicate momentum for the given particles, and the propagator has momentum $q = p - k$. Here the propagator is the photon field

Scattering amplitudes are given as follows. For diagram 1 we have a photon field as propagator, thus the vertices are given as $\frac{-ie\gamma^\rho}{3}$ for the bottom quarks, and $-ie\gamma^\rho$ for the muons. The propagator is given as

$$A_{\rho\nu,pro} = \frac{-ig_{\rho\nu}}{(p-k)^2 + i\epsilon}$$

Thus, our scattering amplitude is given as

$$\begin{aligned} i\mathcal{M}_1 &= \bar{v}_\mu(k)\frac{(-ie\gamma^\rho)}{3}u_\mu(p)\frac{-ig_{\rho\nu}}{(p-k)^2 + i\epsilon}\bar{u}_b(p')(-ie\gamma^\nu)v_b(k') \\ &= \frac{ie^2}{3(p-k)^2}\bar{v}_\mu(k)\gamma^\rho u_\mu(p)\bar{u}_b(p')\gamma_\rho v_b(k') \end{aligned} \tag{1}$$



(a) Feynman diagram of $\mu^+\mu^- \to b\bar{b}$. The outermost arrows indicate momentum for the given particles, and the propagator has momentum $q = p - k$. Here the propagator is the Z boson field

For diagram 2 we have a Z-boson propagator, thus the vertices are given as $\frac{ig}{\cos(\Theta_w)}\gamma^\rho(g_V^b - g_A^b\gamma^5)$ for the bottom quarks and $\frac{ig}{\cos(\Theta_w)}\gamma^\rho(g_V^\mu - g_A^\mu\gamma^5)$ for the muons. The propagator is given as

$$Z_{\rho\nu,prop} = \frac{-ig_{\rho\nu}}{(p-k)^2 - m_Z^2 + i\epsilon} \tag{2}$$

Thus our scattering amplitude is given as

$$\begin{aligned} i\mathcal{M}_2 &= \bar{v}_\mu(k)\frac{ig}{\cos(\Theta_w)}\gamma^\rho(g_V^b - g_A^b\gamma^5)u_\mu(p)\frac{-ig_{\rho\nu}}{(p-k)^2 - m_Z^2 + i\epsilon}\bar{u}_b(p')\frac{ig}{\cos(\Theta_w)}\gamma^\rho(g_V^b - g_A^b\gamma^5)v_b(k') \\ &= \frac{ig^2}{\cos^2(\Theta_W)[(p-k)^2 - m_Z^2]}\bar{v}_\mu(k)\gamma^\rho(g_V^b - g_A^b\gamma^5)u_\mu(p)\bar{u}_b(p')\gamma_\rho(g_V^\mu - g_A^\mu\gamma^5)v_b(k') \end{aligned} \tag{3}$$

1.2 Differential cross sections

To calculate the differential cross section we first need to find the squared scattering amplitude $|\mathcal{M}|^2 = \mathcal{M}\mathcal{M}^\dagger$ for the two diagrams and indeed also the cross term. In the first case with the photon propagator we have that

$$i\mathcal{M}_1 = \frac{ie^2}{3(p-k)^2}\bar{v}_\mu(k)\gamma^\rho u_\mu(p)\bar{u}_b(p')\gamma_\rho v_b(k') \tag{4}$$

and

$$i\mathcal{M}_1^\dagger = \frac{ie^2}{3(p-k)^2}\left[\bar{v}_\mu(k)\gamma^\rho u_\mu(p)\right]^\dagger\left[\bar{u}_b(p')\gamma_\rho v_b(k')\right]^\dagger \tag{5}$$

The squared amplitude can thus be written as

$$\begin{aligned} \mathcal{M}_1\mathcal{M}_1^\dagger &= \frac{e^4}{9q^4}\sum_{\text{all spin}}\left[\bar{v}_\mu(k)\gamma^\mu u_\mu(p)\right]\left[\bar{v}_\mu(k)\gamma^\nu u_\mu(p)\right]^\dagger\left[\bar{u}_b(p')\gamma_\mu v_b(k')\right]\left[\bar{u}_b(p')\gamma_\mu v_b(k')\right]^\dagger \\ &= \frac{16e^4}{3q^2}\left[2(p\cdot p')(k\cdot k') + 2(k\cdot p')(p\cdot k') + 2m_b^2(p\cdot k) + 2m_\mu^2(p'\cdot k') + 4m_b^2m_\mu^2\right] \end{aligned}$$

In the second case we have the Z boson propagator, and we have that

$$i\mathcal{M}_2 = \frac{ig^2}{\cos^2(\Theta_W)[(p-k)^2 - m_Z^2]}\bar{v}_\mu(k)\gamma^\rho(g_V^b - g_A^b\gamma^5)u_\mu(p)\bar{u}_b(p')\gamma_\rho(g_V^\mu - g_A^\mu\gamma^5)v_b(k')$$

$$\mathrm{iM}_2^\dagger = \frac{ig^2}{\cos^2(\Theta_W)[(p-k)^2 - m_Z^2]}\left[\bar{v}_\mu(k)\gamma^\rho(g_V^b - g_A^b\gamma^5)\right]^\dagger\left[u_\mu(p)\bar{u}_b(p')\gamma_\rho(g_V^\mu - g_A^\mu\gamma^5)v_b(k')\right]^\dagger.$$

The squared amplitude here is thus

$$\begin{aligned}\mathcal{M}_2\mathcal{M}_2^\dagger = \frac{4g^4}{\cos^4(\Theta_W)[(p-k)^2 - m_Z^2]^2}\Bigg[&AB\Bigg(2(p\cdot p')(k\cdot k + 2\,(\mathrm{p}\cdot\mathrm{k}')(\mathrm{p}\cdot\mathrm{k}')\Bigg)\\ &+ 2Bm_b^2(p\cdot k) + 2Am_\mu^2(p'\cdot k')\\ &+ 4m_b^2m_\mu^2AB\\ &- 8g_A^bg_V^bg_A^\mu g_V^\mu\Bigg((p\cdot k)(p'\cdot k') - (p\cdot k')(p'\cdot k)\Bigg)\Bigg]\end{aligned}$$

When squaring the total cross section we also get a cross term, i.e $\mathcal{M}_1\mathcal{M}_2^\dagger$. This is given as

$$\begin{aligned}\mathcal{M}_1\mathcal{M}_2^\dagger = \frac{8e^2g^2}{\cos^2(\theta_W)q^2(q^2 - m_Z^2)^2}\Bigg[&g_V^bg_A^b(k\cdot p')(p\cdot k') + g_V^\mu g_A^\mu(p\cdot p')(k\cdot k')\\ &+ g_V^bm_b^2(p\cdot k)g_V^\mu m_\mu^2(p'\cdot k')\\ &+ 2m_b^2m_\mu^2g_V^bg_V^\mu\\ &- 2g_A^bg_A^\mu\Bigg((k\cdot p')(p\cdot k') - (p\cdot p')(k\cdot k')\Bigg)\Bigg]\end{aligned}$$

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