# Dirichlet-Point Processes

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## Introduction

- Every minute:
  - 400h of video
  - 350 000 tweets
  - 500 000 comments
  - 4 200 000 searches

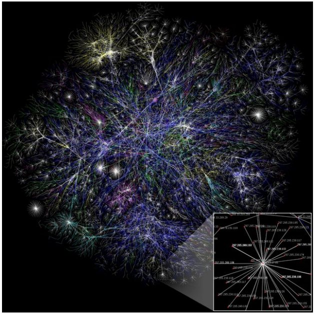


Figure 1: Snapshot of the internet (Wikipedia)

# Motivation

- Every minute:

  - 400h of video
  - 350 000 tweets
  - 500 000 comments
  - 4 200 000 searches

- How to make sense out of *that*?

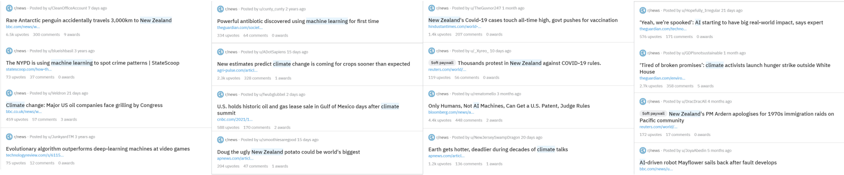



Figure 2: A typical stream from r/news

# Motivation

- Every minute:

  - 400h of video
  - 350 000 tweets
  - 500 000 comments
  - 4 200 000 searches

- How to make sense out of *that*?
  → Hidden semantic links


Figure 2: A typical stream from r/news – with topics

# Available information

- Main clues:
  - Textual information

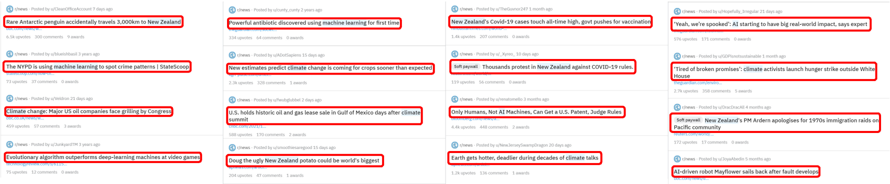



Figure 3: We can use textual information

# Available information

- Main clues:
  - Textual information
  - Temporal information

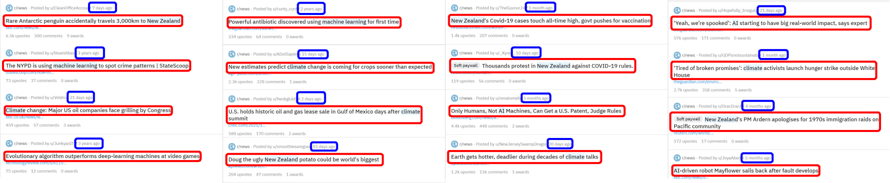


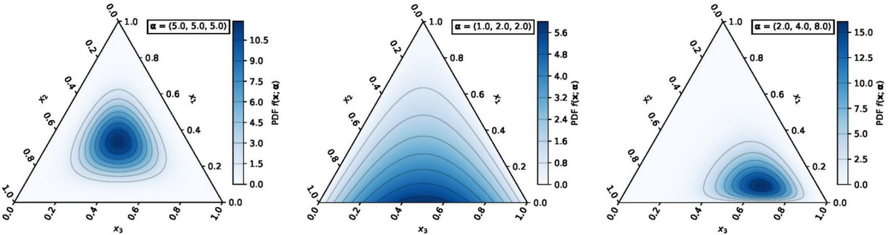

Figure 3: We can use textual information and temporal information

## Documents stream

- The data is therefore a documents stream

# Dirichlet process

- Dirichlet distribution: $\vec{X} \sim Dir(\alpha)$ s.t. $\sum_k X_k = 1$
- Often used as a prior distribution in Bayesian clustering
  - ◇ Typically $X_k$ is the probability to belong to cluster $k$



## Chinese restaurant process

- Chinese Restaurant Process:

$$CRP(C_i = c | C_1, C_2, ..., C_{i-1}, \alpha) = \begin{cases} \frac{N_c}{\alpha+N} \text{ if c = 1, ..., K} \\ \frac{\alpha}{\alpha+N} \text{ if c = K+1} \end{cases}$$

## Handling a stream of documents

- Chinese Restaurant Process:

$$CRP(C_i = c|C_1, C_2, ..., C_{i-1}, \alpha) = \begin{cases} \frac{N_c}{\alpha+N} \text{ if c = 1, ..., K} \\ \frac{\alpha}{\alpha+N} \text{ if c = K+1} \end{cases}$$

- Useful for sequential modeling (explicit posterior at each step, allows Gibbs sampling)

$$\underbrace{P(n^{th} obs = c|D, history)}_{Posterior} \propto \underbrace{P(D|n^{th} obs = c)}_{Likelihood} \times \underbrace{P(n^{th} obs = c|history)}_{CRP\ \ prior}$$

- Hypothesis: "rich-get-richer"

# Variants

- Variants of DP exist:
  - ◇ Uniform process [Wallach et al., 2010]
  - ◇ Pitman-Yor process [Pitman and Yor, 1997]
  - ◇ Hierarchical Dirichlet process [Teh et al., 2006]
  - ◇ Nested Dirichlet process [Rodríguez et al., 2008]

→ Most exhibit "rich-get-richer" property

→ All consider counts, none consider temporal dimension

## Modeling time as a continuous variable

- Time often "modeled" by sampling observations (DTM [Blei and Lafferty, 2006], RCRP [Ahmed and Xing, 2008, Diao and Jiang, 2014], DDCRP [Blei and Frazier, 2010] etc.)
    - ◇ Problems: how to slice data, which sampling function use, how to weight observations, etc.
- Modeling time explicitly: point processes

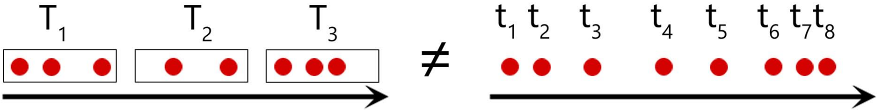



Figure 4: Data sampling/slicing is an approximation

## Poisson process

- Poisson processes are characterized by an **intensity** $\lambda$.
  - $\lambda \Delta t \overset{\Delta t \to 0}{=} P(\mathbb{N}(t+\Delta t) - \mathbb{N}(t) = 1)$
  - Instantaneous probability for *one* event

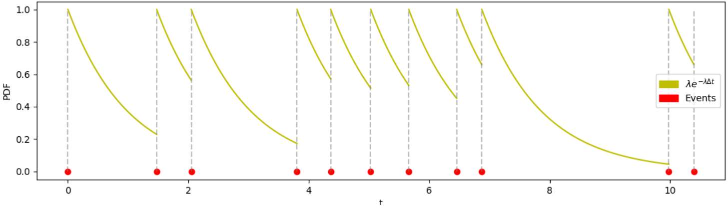



Figure 5: Could model radioactive decay events of atoms whose half-life is 1

## Non-homogeneous Poisson process

- $\lambda(t)$ is a function
- $\lambda(t)\Delta t \overset{\Delta t \to 0}{=} P(\mathbb{N}(t+\Delta t) - \mathbb{N}(t) = 1)$

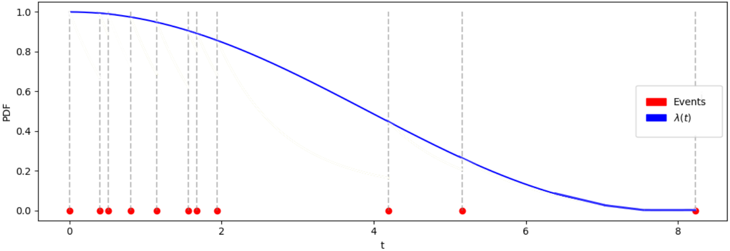



Figure 6: Could model cars arrival at gas station throughout a day

## Hawkes process

- Hawkes processes: $\lambda(t|\mathcal{H}_t)$ depends on past events $\mathcal{H}_t = \{t_i | t_i < t\}$

  $\rightarrow$ "Self-exciting process"

- Typically: $\lambda(t|\mathcal{H}_t) = \lambda_0 + \sum_{t_i \in \mathcal{H}_t} \phi(t - t_i)$

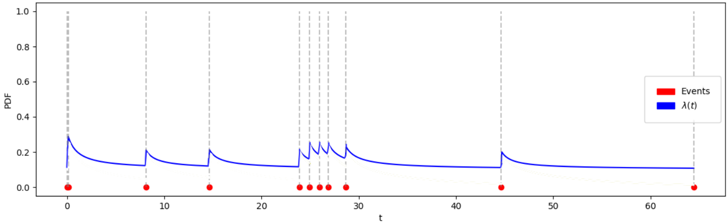



Figure 7: Could model online posting dynamics

## Inference

- Log-likelihood of a data stream $\mathcal{D} = \{t_0, ..., t_N\}$:

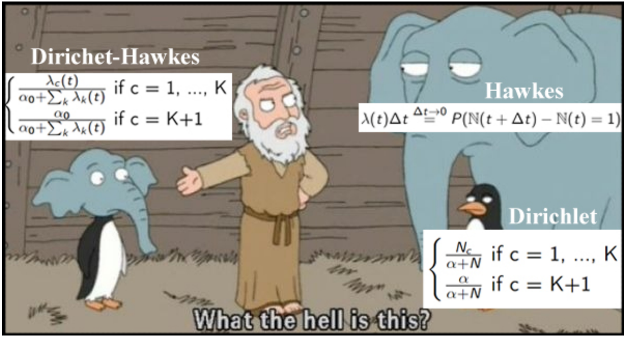
$$\ell(\lambda, \mathcal{D}) = -\int_{t_0}^{t_N} \lambda(t)dt + \sum_{t_i<t_N} \log \lambda(t_i) = \log \lambda(t_1) - \int_{t_0}^{t_1} \lambda(t)dt$$
$$+ \log \lambda(t_2) - \int_{t_1}^{t_2} \lambda(t)dt$$
$$+ ...$$
$$+ \log \lambda(t_N) - \int_{t_{N-1}}^{t_N} \lambda(t)dt$$

- Convex for certain shapes of $\lambda(t)$ (exp, ray, PL, Gaussian, ...).

## Dirichlet-Hawkes process

- [Du et al., 2015]: Dirichlet-Hawkes prior (Bayesian inference)

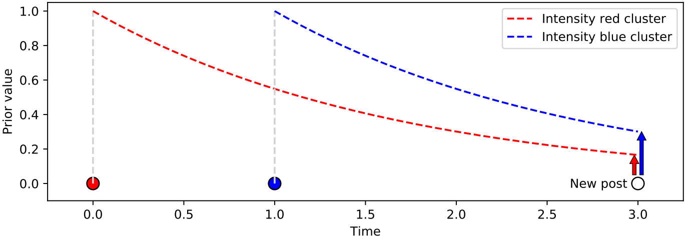
$$P(\text{cluster}|\text{text, time}, \mathcal{H}) \propto \underbrace{P(\text{text}|\text{cluster})}_{\substack{\text{Textual likelihood}\\ \text{(Dirichlet-Multinomial)}}} \times \underbrace{P(\text{cluster}|\text{time}, \mathcal{H})}_{\substack{\text{Temporal prior}\\ \text{(Dirichlet-Hawkes)}\\ \downarrow}}$$



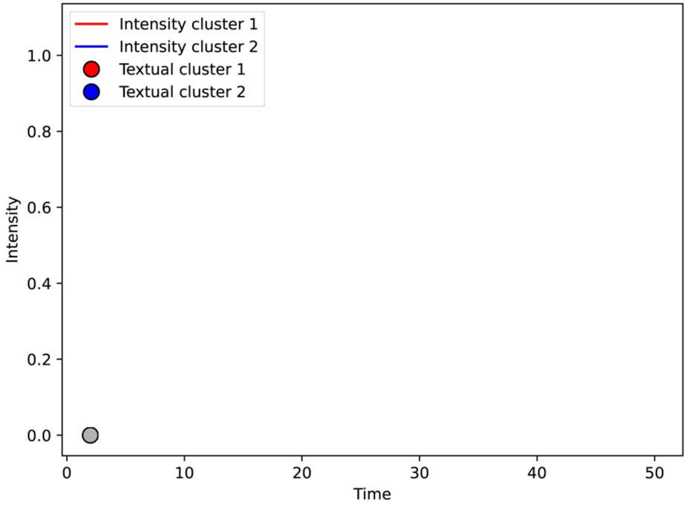
# Dirichlet-Hawkes process – Explicit

- $P(c|t,\mathcal{H})$: prior prob. of cluster $c$ at time $t$ given history $\mathcal{H}$
- $\lambda_c(t)$: Hawkes intensity of cluster $c$ at time $t$
- Dirichlet process with counts $N_c$ replaced by $\lambda_c(t)$

$$\underbrace{P(c|t,\mathcal{H})}_{\substack{\text{Temporal prior} \\ \text{(Dirichlet-Hawkes)} \\ \downarrow}} = \begin{cases} \frac{\lambda_c(t)}{\alpha_0+\sum_k \lambda_k(t)} & \text{if c = 1, ..., K} \\ \frac{\alpha_0}{\alpha_0+\sum_k \lambda_k(t)} & \text{if c = K+1} \end{cases}$$



# Inference (1 particle)



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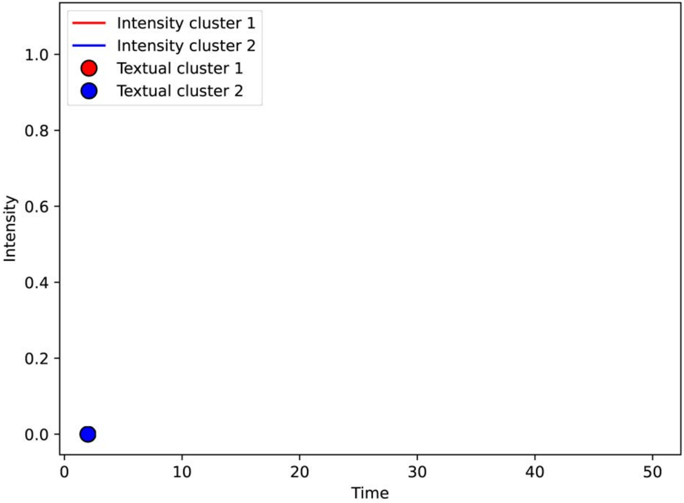

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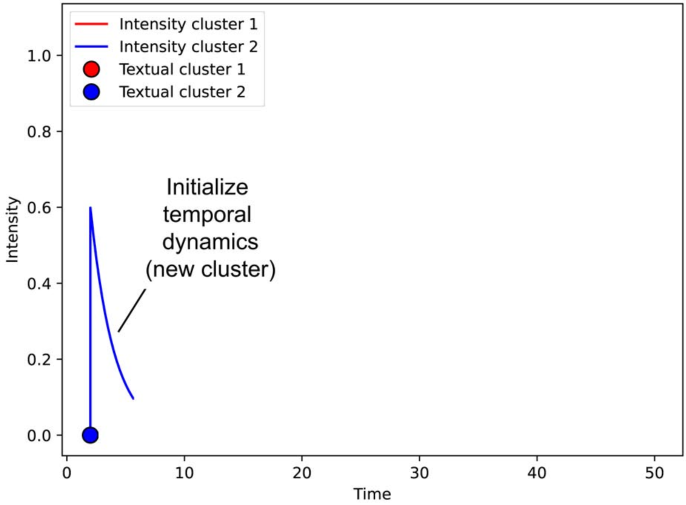

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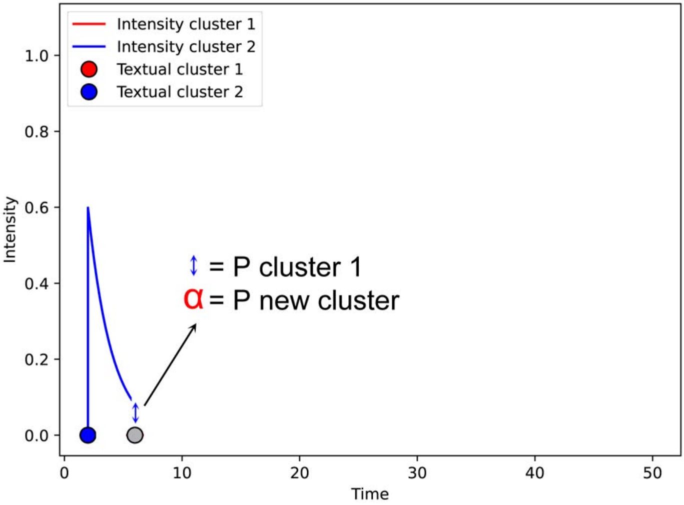

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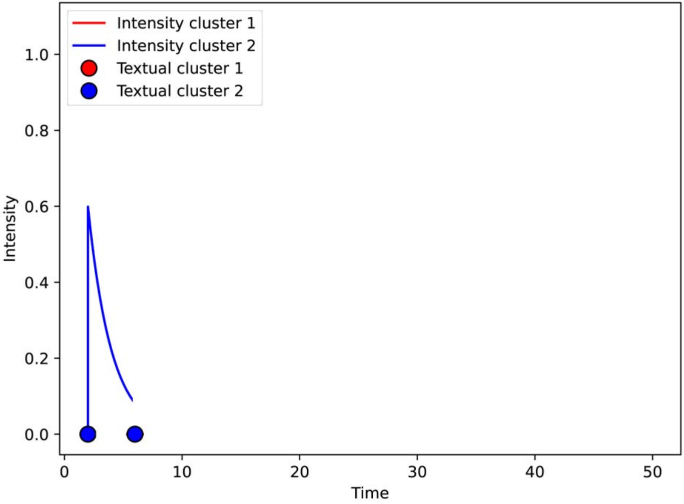

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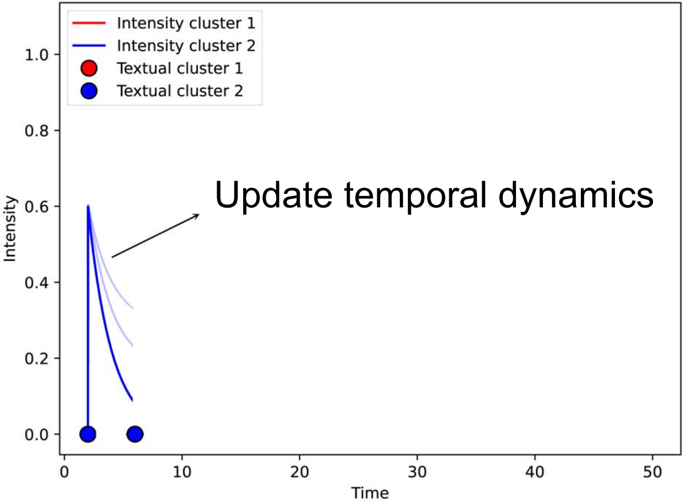

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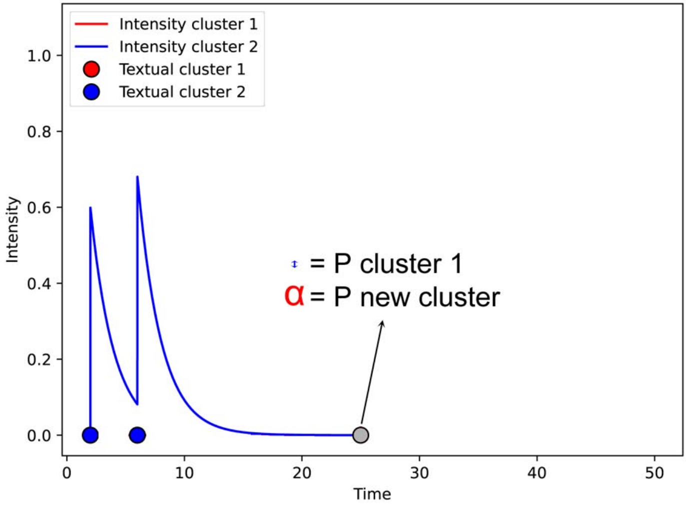

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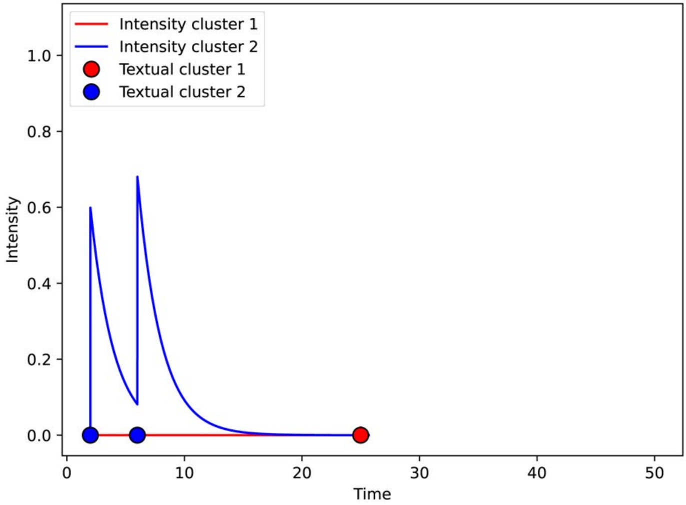

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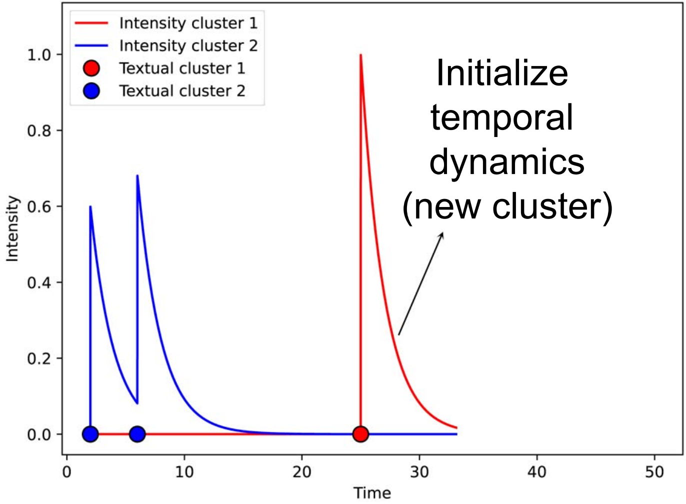

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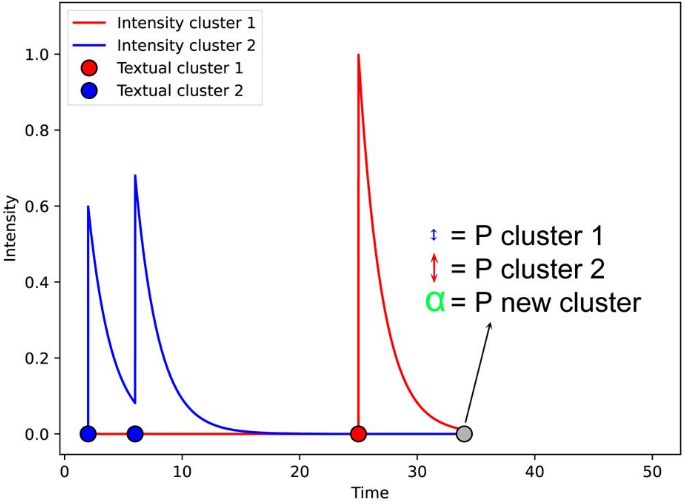

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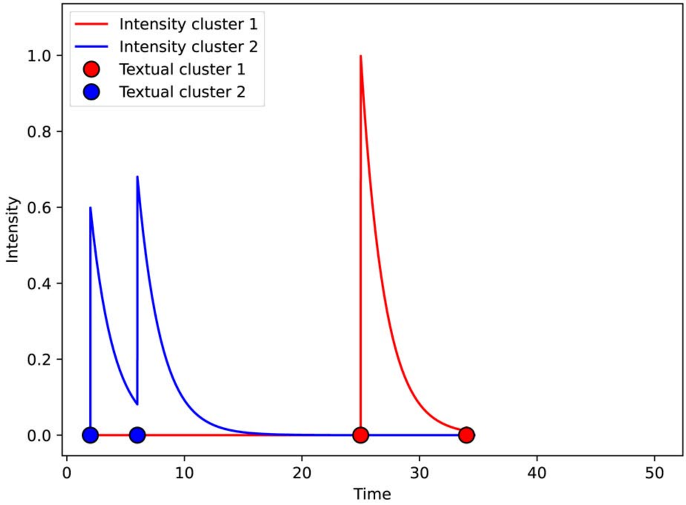

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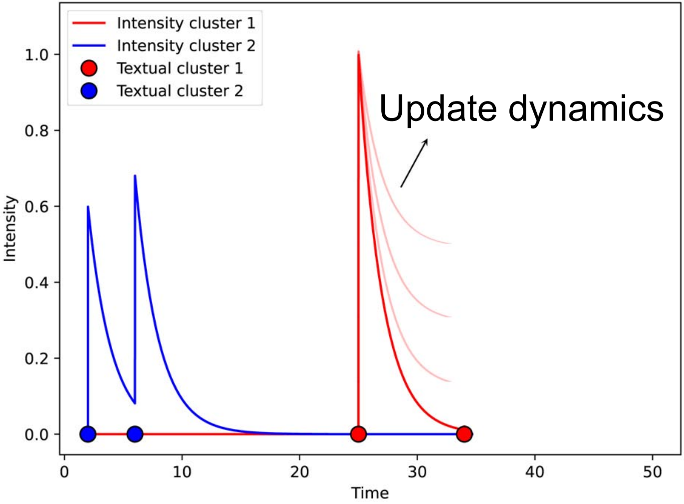
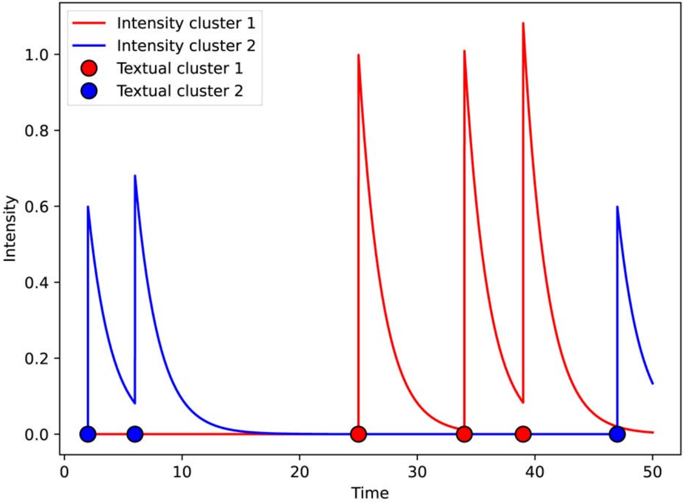
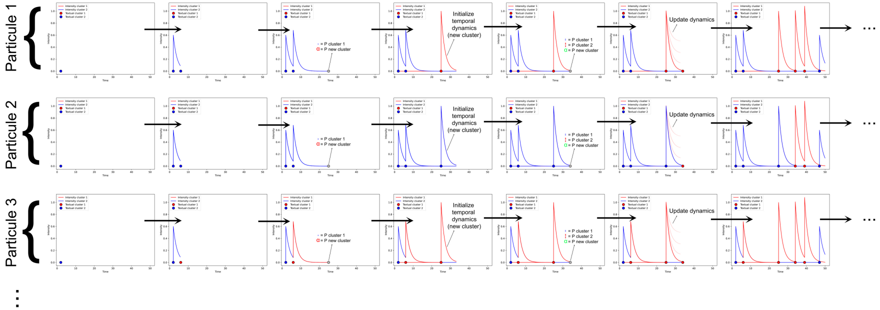


# Inference (1 particle)



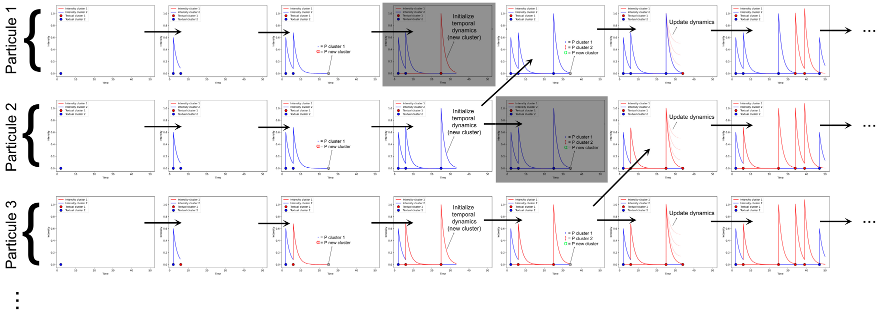
# Inference (all particles)

- Run simultaneously on several *particles*

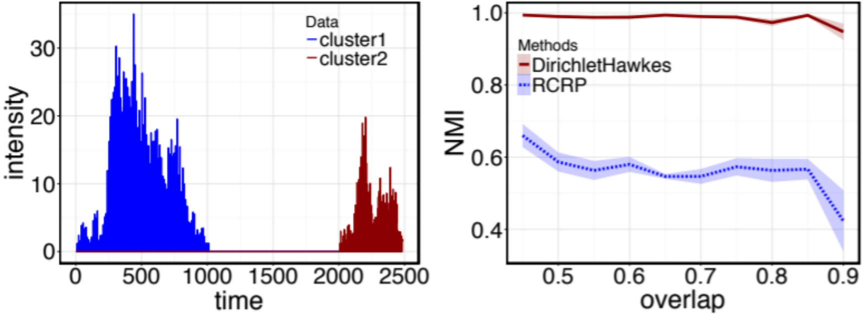


# Inference (all particles)

- Discard unlikely particles and replace them by more likely ones



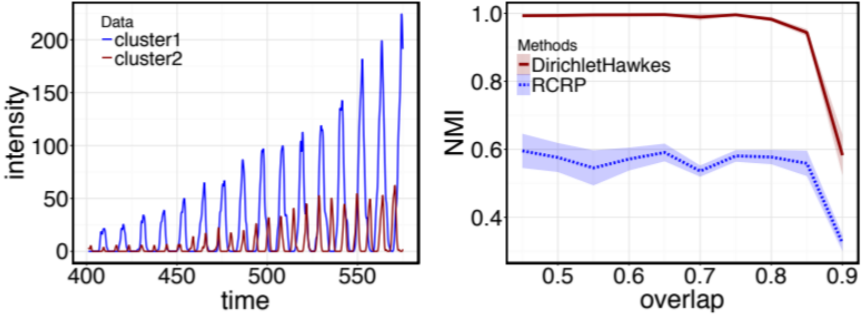
# Performances (well-separated)




(a) Temporally well-separated clusters.

Figure 10: [Du et al., 2015]

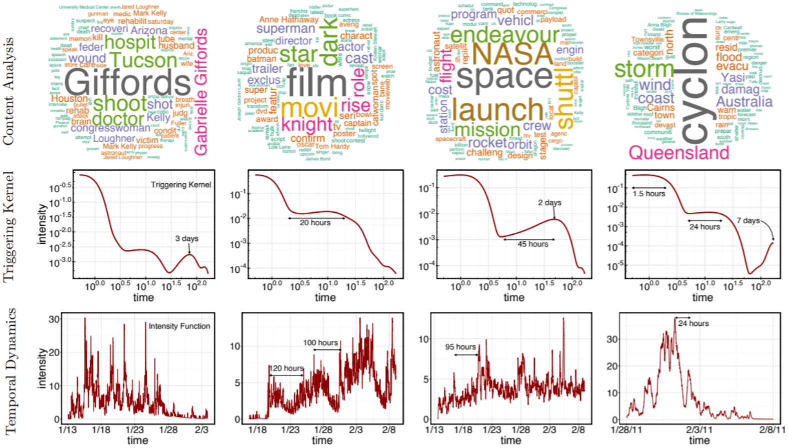
# Performances (“not” well-separated)



(b) Temporally interleaved clusters.

Figure 11: [Du et al., 2015]

# Output




(a) Tucson Shooting (b) Dark Knight Rise (c) Endeavour (d) Queensland Flooding

Figure 12: [Du et al., 2015]

## Variants

- Some variants based on Dirichlet-Hawkes process
  - ◇ Hierarchical (CRF) and Nested (nCRP) extensions of DHP
  - ◇ Not-vanishing DHP prior [Kapoor et al., 2018]

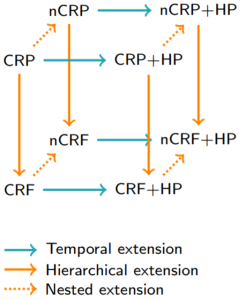



Figure 13: [Kapoor et al., 2018]

BUT!

## Dirichlet prior is a choice

- Dirichlet-based priors are an arbitrary choice
  - ◇ Other priors are as fit [Welling, 2006]
  - ◇ The choice of the prior matters [Wallach et al., 2009]
  - ◇ Few variations proposed [Wallach et al., 2010, Pitman and Yor, 1997]
- DP exhibits "rich-get-richer" property
  - ◇ Why linear dependence?
  - ◇ Why this assumption at all? [Wallach et al., 2010]

# Powered Dirichlet process

- Powered Chinese Restaurant Process:

$$PCRP(C_i = c|C_1, ..., C_{i-1}, \alpha, r) = \begin{cases} \frac{N_c^r}{\alpha+\sum_k N_k^r} & \text{if c = 1, ..., K} \\ \frac{\alpha}{\alpha+\sum_k N_k^r} & \text{if c = K+1} \end{cases}$$

  - ◇ r<0: "rich-get-poorer"
  - ◇ r=0: "rich-get-no-richer" (Uniform Process)
  - ◇ 0<r<1: "rich-get-less-richer"
  - ◇ r=1: "rich-get-richer" (Dirichlet Process)
  - ◇ r=$\frac{\log(N_k-\beta)}{\log(N_k)}$: "rich-get-richer" (Pitman-Yor Process)
  - ◇ r>1: "rich-get-more-richer"

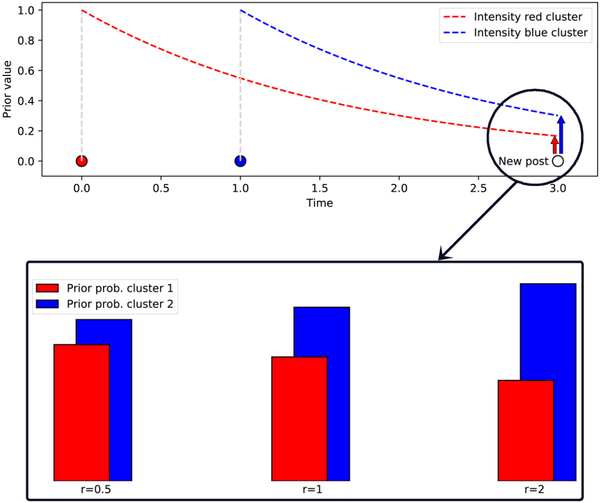
# PDP into DHP

- Powered Dirichlet-Hawkes Process [Poux-Médard et al., 2021]:

$$\underbrace{P(c|t,\mathcal{H},r)}_{\text{PDHP prior}} = \begin{cases} \frac{\lambda_c(t)^r}{\alpha_0+\sum_k \lambda_k(t)^r} & \text{if c = 1, ..., K} \\ \frac{\alpha_0}{\alpha_0+\sum_k \lambda_k(t)^r} & \text{if c = K+1} \end{cases}$$

- Generalization:
  - ◇ Uniform process: $r = 0$ (only textual information)
  - ◇ Dirichlet-Hawkes process: $r = 1$ (temporal and textual information)
  - ◇ Deterministic Hawkes process: $r \to \infty$ (only temporal information)

# Effect of $r$



1.0
0.8
0.6
0.4
0.2
0.0
Prior value
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Time
Intensity red cluster
Intensity blue cluster
New post
Prior prob. cluster 1
Prior prob. cluster 2
r=0.5
r=1
r=2

# Changes induced by PDHP

$$P(\text{cluster}|\text{text, time}) \propto \underbrace{P(\text{text}|\text{cluster})}_{\text{Textual likelihood}} \times \underbrace{P(\text{cluster}|\text{time, r, history})}_{\text{PDHP temporal prior}}$$

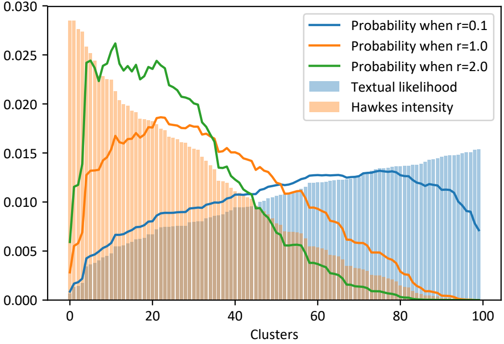
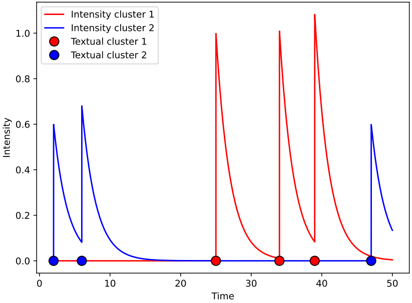


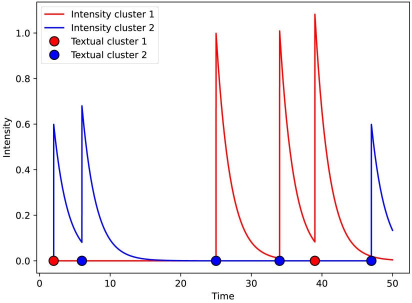

Figure 14: [Poux-Médard et al., 2021]

# Why is it relevant - Decorrelations

- Decorrelations:
  - Ex: influent journal publishing on a topic does not have same dynamics as less influent one on the same topic




$\longrightarrow$



# Results for various decorrelations

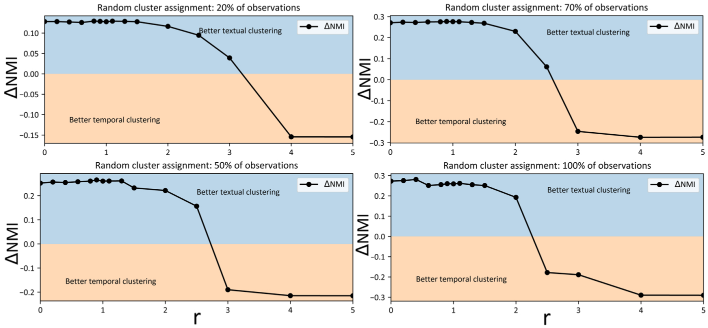



Figure 15: [Poux-Médard et al., 2021]

- PDHP retrieves either temporal or textual clusters
  - Small $r$: good textual clusters
  - Large $r$: good temporal clusters

# PDHP handles challenging situations

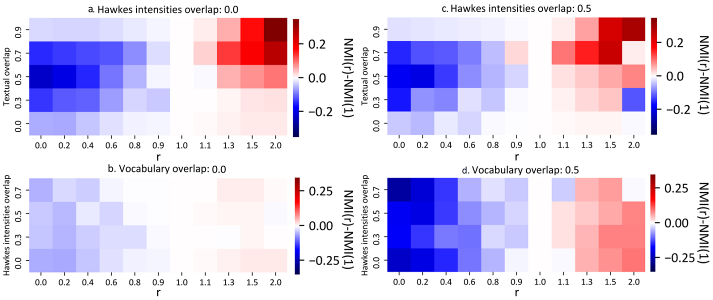



Figure 16: [Poux-Médard et al., 2021]

- PDHP adapts to various situations better than DHP (+0.3 NMI):
  - Large textual overlap
  - Large temporal overlap
  - No overlap

# Reddit r/news, r/TodayILearned, r/AskScience - Some metrics

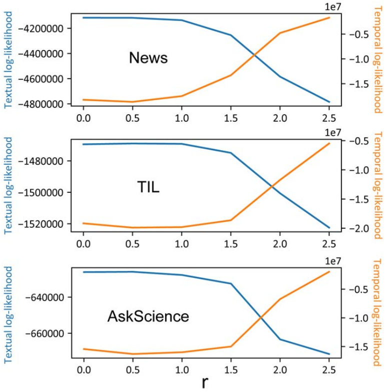



Figure 17: Textual and temporal likelihood vs $r$ [Poux-Médard et al., 2021]

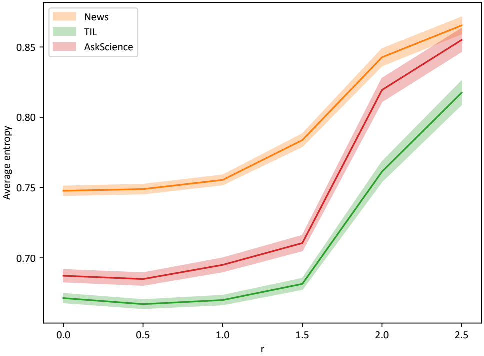
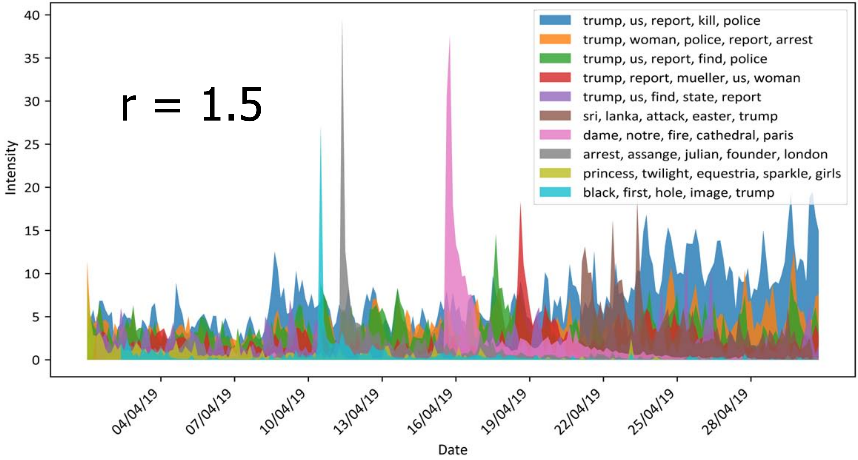



Figure 18: Entropy of textual clusters: sharper textual clusters for low $r$ [Poux-Médard et al., 2021]

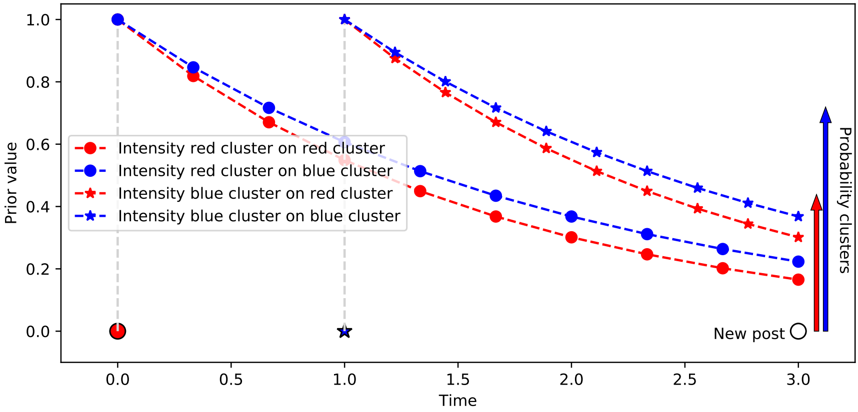
## Summary generation

- Powered Dirichlet-Hawkes prior: summary from data flows using temporal interactions

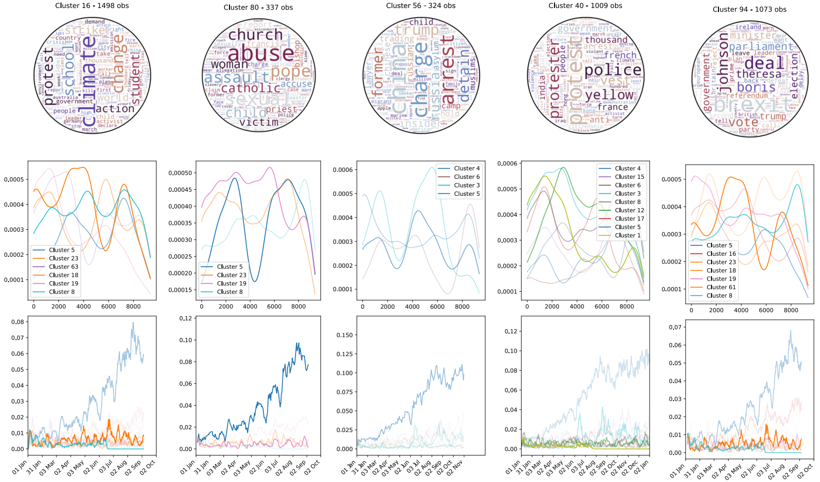


# Multivariate Powered Dirichlet-Hawkes process

- Extension: Multivariate Powered Dirichlet-Hawkes prior
  - How clusters influence each other



# Inferred clusters



# Cluster interaction network

- MPDHP prior: Cluster interaction network

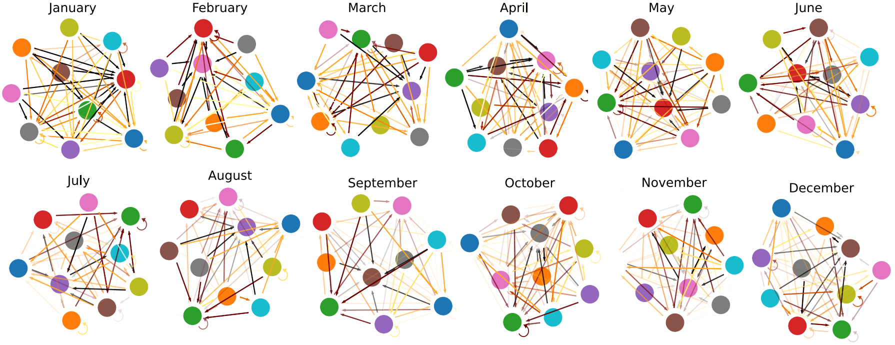



Figure 19: Topical interaction network over a year

# Perspective MPDHP – Summary generation

- Multivariate Powered Dirichlet-Hawkes prior: Cluster interaction network

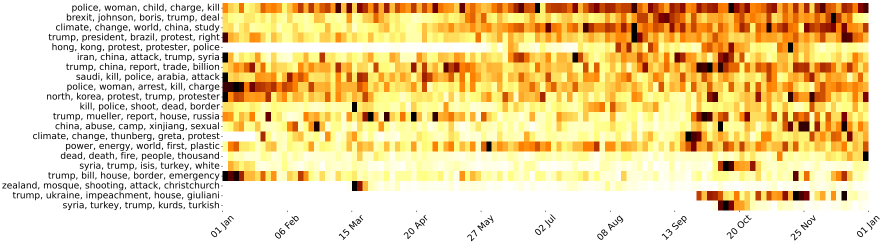



Figure 20: Inferred topics timeline

## Structure matters!

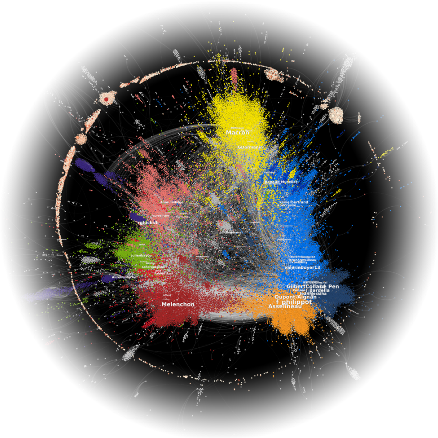



Figure 21: Sample of Twitter structure (Politoscope [Gaumont et al., 2018])

# Why (MP)DHP is incomplete

- (MP)DHP prior accounts for time but not structure
  - Infers aggregated dynamics
  - Misses the structural aspect: discussions are not the same among different groups

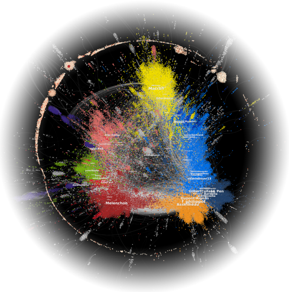


Figure 22: Sample of Twitter structure (Politoscope [Gaumont et al., 2018])

# Network inference – Literature

- Several works on network inference using survival analysis:
  - NetRate [Gomez-Rodriguez et al., 2011]
  - KernelCascade [Du et al., 2012]
  - MoNet [Wang et al., 2012]
  - InfoPath [Gomez-Rodriguez et al., 2013a]
  - TopicCascade [Du et al., 2013]
- They are all special cases of [Gomez-Rodriguez et al., 2013b]
  - Bridges the gap between network inference and point processes
  - Formulates each of previous models as a **counting point process**

## Temporal and structural prior

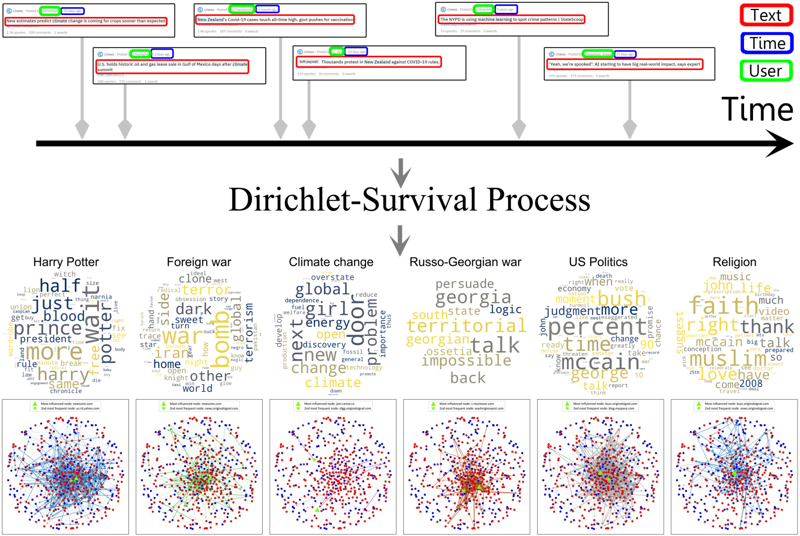
- Houston: **H**eterogeneous **O**nline **U**ser-**T**opic **N**etwork inference
- Prior on cluster membership $C_i$ of observation $i$ observed on node $u$ at time $t$ given history $\mathcal{H}$ and cluster-dependent networks $A$:

$$P(C_i = k|u, t, \mathcal{H}, A)$$

$$= \begin{cases} \frac{\lambda_0^{(k)} + \sum_{\mathcal{H}_{i,c}^{(k)}} H(t_i^c | t_j^c, \alpha_{u_j^c, u_i^c}^{(k)})}{\lambda_0^{(K+1)} + \sum_k^K \lambda_0^{(k)} + \sum_{\mathcal{H}_{i,c}^{(k)}} H(t_i^c | t_j^c, \alpha_{u_j^c, u_i^c}^{(k)})} & \text{if k = 1, ..., K} \\ \frac{\lambda_0^{(K+1)}}{\lambda_0^{(K+1)} + \sum_k^K \lambda_0^{(k)} + \sum_{\mathcal{H}_{i,c}^{(k)}} H(t_i^c | t_j^c, \alpha_{u_j^c, u_i^c}^{(k)})} & \text{if k = K+1} \end{cases}$$


$$= \begin{cases} \frac{\text{Strength of incoming edges of cluster/subnetwork k at time t}}{\text{Normalizing term}} & \text{if k = 1, ..., K} \\ \frac{\text{Probability of a new cluster/subnetwork k+1 at time t}}{\text{Normalizing term}} & \text{if k = K+1} \end{cases}$$

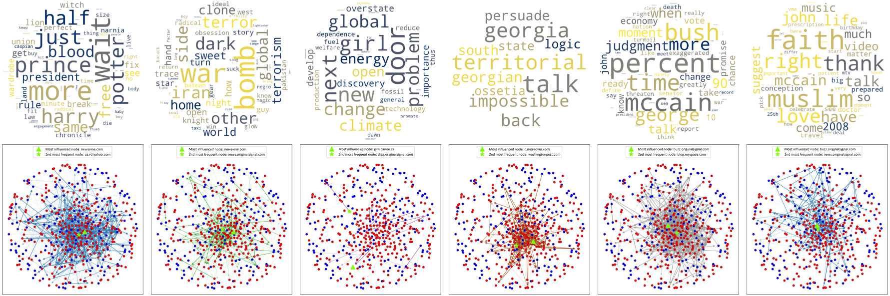
# Task


New estimates predict climate change is coming for crops sooner than expected
U.S. holds historic oil and gas lease sale in Gulf of Mexico days after climate summit
New Zealand's Covid-19 cases touch all-time high, govt pushes for vaccination
Thousands protest in New Zealand against COVID-19 rules.
The NYPD is using machine learning to spot crime patterns | StateScoop
'Yeah, we're spooked': AI starting to have big real-world impact, says expert
Text
Time
User
Time
Dirichlet-Survival Process
Harry Potter
Foreign war
Climate change
Russo-Georgian war
US Politics
Religion

# Results – Real world

- Memetracker data (2009)

Harry Potter | Foreign war | Climate change | Russo-Georgian war | US Politics | Religion



- Disclaimer: other works performing the same task, maybe better, without Dirichlet-Point processes [Barbieri et al., 2017]

## Conclusion

- Dirichlet and Hawkes process have an old and separate history
  - Only recently (2015) they have been brought together
  - Their reunion launched a new branch of inductive machine learning
- As many new perspectives as possible combinations

(**DP**, HDP, nHDP, **PDP**, IBP, PIBP, PnHDP, PPY, PnPY, PHPY, ...)

$\times$

(**Hawkes**, **Multi Hawkes**, **Survival**, Cox, Poisson, Determinantal, ...)

$=$

(**DHP**, HDHP, IBHP, **PDHP**, **MPDHP**, **Houston**, ...?)

Thanks for your attention!

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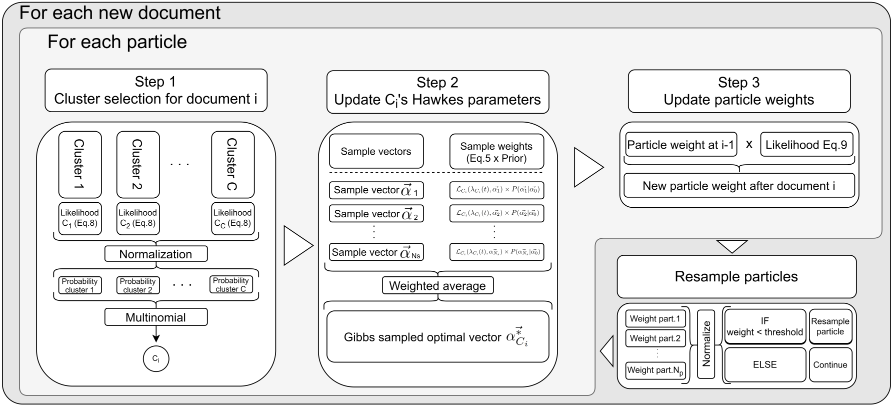
# Inference (summarized)



# PDP impact

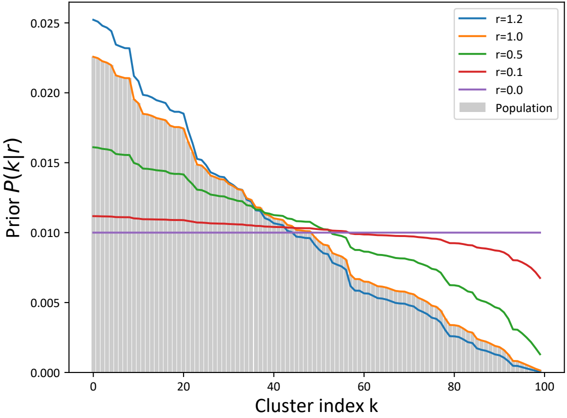
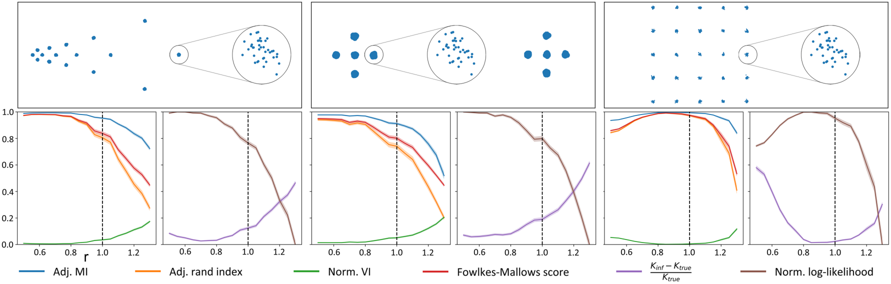



Figure 23: Prior probability for each of 100 clusters whose population is known (grey bars) w.r.t. $r$

# Results

- Use as prior for IGMM
- DP not always the best prior



# Why is it relevant - Overlaps

- Often, a piece of information is more informative than the other:
  - Twitter: short texts (few textual information) but informative cascade dynamics (helpful temporal information)
- Happens often because of overlaps:

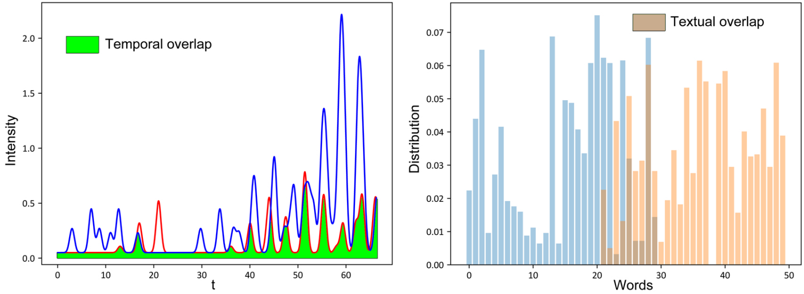



Figure 24: [Poux-Médard et al., 2021]

# Reddit r/news - Typical output

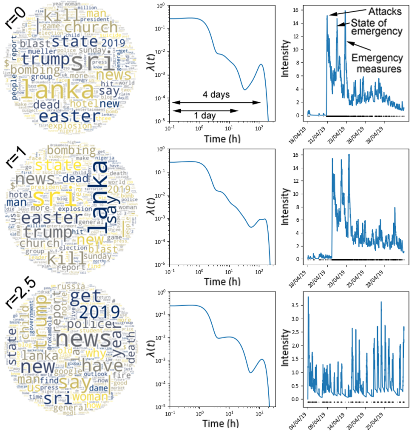
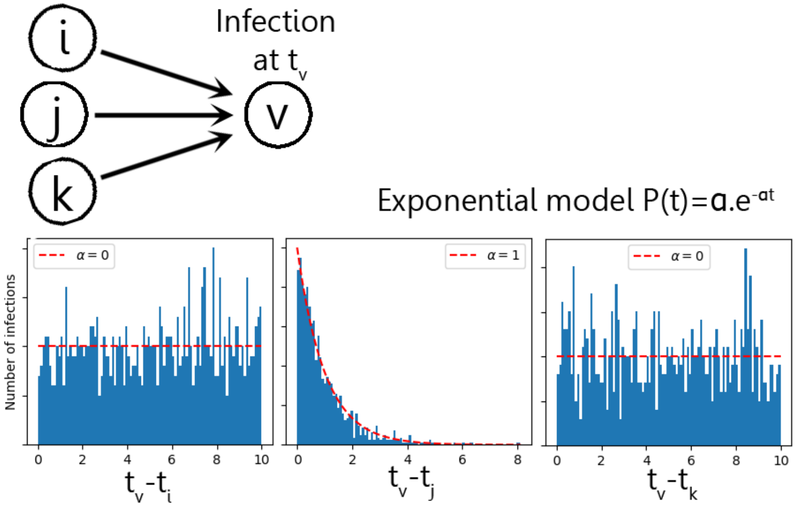



Figure 25: [Poux-Médard et al., 2021]

- Real world data: r/news
- Different clusters and dynamics for different $r$
  - Small $r$: similar vocabulary
  - Large $r$: specific dynamics

# Network inference



# Point process

- Network inference naturally embeds into point processes literature
  → We can derive a temporal *and* structural Bayesian prior

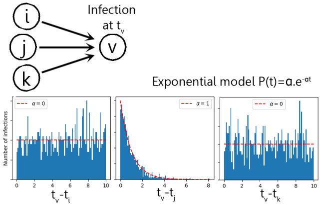



Figure 26: Survival process

Both are point processes
$<\approx>$

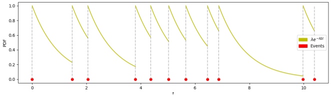
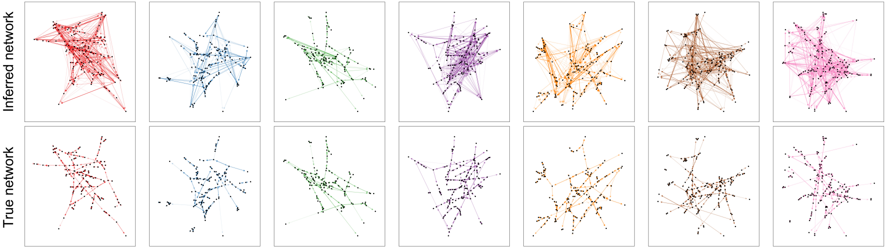



Figure 27: Hawkes process

# Results – Synthetic

- We simulate the spread of documents drawn from 5 topics, each with its own vocabulary and subnetwork



# Numerical results

| | | Houston | TC | DHP | NetRate |
|---|---|---|---|---|---|
| PL | NMI | **0.809** | 0.669 | 0.449 | - |
| | ARI | **0.688** | 0.330 | 0.063 | - |
| | AUC | **0.807** | 0.719 | - | 0.731 |
| | MAE | **0.267** | 0.338 | - | 0.460 |
| ER | NMI | **0.787** | 0.711 | 0.638 | - |
| | ARI | **0.631** | 0.488 | 0.411 | - |
| | AUC | **0.849** | 0.800 | - | 0.659 |
| | MAE | **0.229** | 0.278 | - | 0.481 |
| Blogs | NMI | **0.750** | 0.668 | 0.372 | - |
| | ARI | **0.609** | 0.365 | 0.023 | - |
| | AUC | 0.701 | 0.613 | - | **0.710** |
| | MAE | **0.374** | 0.444 | - | 0.499 |