Brunel Network

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Model The Leaky integrate-and-fire (IF) neuron model is described as an electrical model of a neuron with a resistance R and capacity C:

$$\tau \frac{dV}{dt}(t) = -V(t) + RI(t) \text{ with } \tau = RC \tag{1}$$

where the network contribution $RI(t)$ from a neuron j to a neuron i is described as :

$$RI(t)_i = \tau \sum_j J_{ij} \sum_{t'} \delta(t - t'_j - D) \tag{2}$$

where J_{ij} is the postsynaptic potential amplitude, $\delta(t)$ the dirac function, t'_j the spiking time of pre-synaptic neuron j at time t' and D the transmission delay. For simplicity we represent $V(t)$ as V_t.

Analytical solution We consider that the neuron is able to steeply rise its potential when there is an input of current, thus discarding the exponential charge of the capacitance. The discharge follows an exponential decay with the time constant τ. We compute two different stepping methods from the analytical solution:

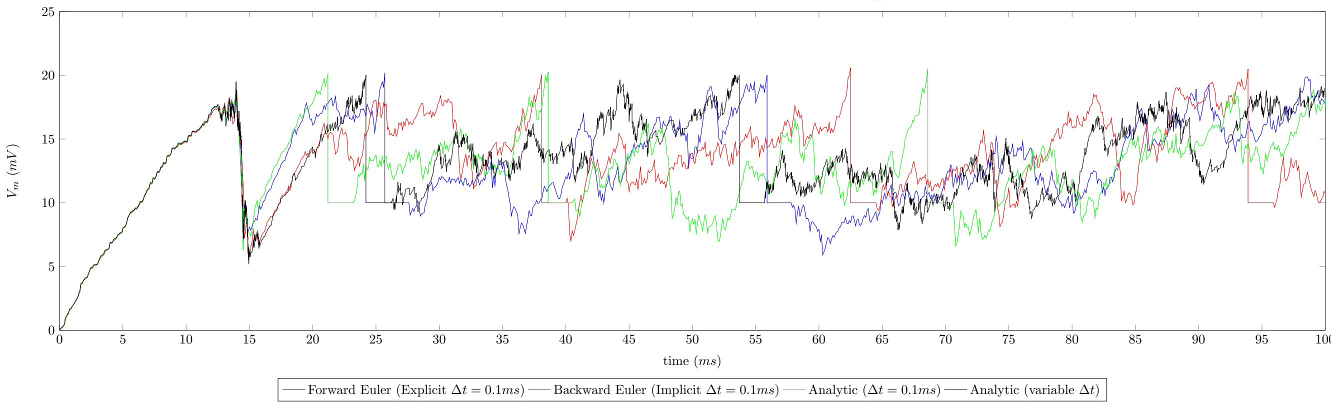
- **Two step algorithm**:
  1. Calculate decay from previous step: $V_t \leftarrow V_{t-\Delta t} exp(\frac{-dt}{\tau})$
  2. Add voltage change of network current RI between $t - \Delta t$ and t: $V_t \leftarrow V_t + RI_{(t-\Delta t,t]}/\tau$
- **fixed step** interpolation assumes Δt to be constant throughout the execution. The **Variable step** method advances the neuron with the smallest t on the network, with a Δt computed as the minimum of the two following values:
  - the time difference to the next incoming spike;
  - $t^\star + D$ where $t^\star$ is the time of the neuron with the second smallest t, i.e. the largest step that can be taken from neuron at t so that it won't miss a spike from neuron at time $t^\star$, if it spikes.

Euler methods For efficiency purposes, we implemented two approximated fixed-step Euler methods:

- **Explicit Forward Euler:** $\tau \frac{dV_t}{dt} = -V_{t-\Delta t} + RI_{(t-\Delta t,t]} \Leftrightarrow V_t = V_{t-\Delta t} + \frac{dV_{t-\Delta t}}{dt}\Delta t \Leftrightarrow V_t = \frac{-V_{t-\Delta t} + RI_{(t-\Delta t,t]}}{\tau}\Delta t$
- **Implicit Backward Euler:** $\tau \frac{dV_t}{dt} = -V_t + RI_{(t-\Delta t,t]} \Leftrightarrow \frac{V_t - V_{t-\Delta t}}{\Delta t} = -V_t + RI_{(t-\Delta t,t]} \Leftrightarrow V_t = \frac{RI_{(t-\Delta t,t]} + \tau * V_{t-\Delta t}}{\Delta t + \tau}$

with $\frac{dV_t}{dt}$ and $RI_{(t-\Delta t,t]}$ computed from equations 1 and 2 respectively[1].

Results We present the potential over time of neuron 0 for the aforementioned methods on a $100ms$ simulation of a Brunel network of 10000 neurons with random seed 0 and default neuron parameters:



[1] $RI_{(t-\Delta t,t]}$ is divided by Δt so that it is expressed in voltage per time-unit instead of per time-step.