Continuum damage models *Phase-field method* (Variational approach)

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Outline

- Introduction
- Continuum approach: local versus non local continuum damage model
- Phase-field approach
- Dynamic crack branching

Continuum damage mechanics

Definition of a continuous damage variable

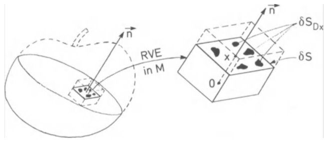
with:

- δS RVE section area
- δS_d area intersecting micro-cracks and/or micro-cavities
- isotropic damage

$$d = \max_{\text{RVE planes}} \frac{\delta S_d}{\delta S}$$

Then:

- $d = 0 \Rightarrow$ Undamaged RVE
- $d = 1 \Rightarrow$ Fully broken RVE in 2 parts



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Continuum damage mechanics

Helmholtz free energy

$$\psi(\epsilon, D) = \frac{1}{2\rho}[(1-D)\epsilon : C : \epsilon]$$

Constituvive law

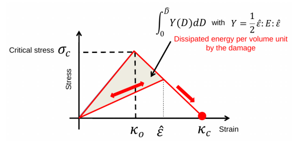
$$\sigma = \rho\frac{\partial\psi}{\partial\epsilon} = (1-D)C : \epsilon$$

Volume energy ensity

$$\bar{Y} = \rho\frac{\partial\psi}{\partial D} = -\frac{1}{2}\epsilon : C : \epsilon$$

Strain energy density release rate

$$Y(D) = -\bar{Y}(D) = \frac{1}{2}\epsilon : C : \epsilon$$



Dissipated energy

$$E^{\text{dissipated}} = \int Y(D)dD$$

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Damage evolution

- A set of constraints (comparable to yielding) are necessary:

Positive dissipation constraint

$$Y(D)\dot{D} \geq 0$$

Increasing damage constraint

$$\dot{D} \geq 0$$

Threshold constraint (example)

$$F(Y, d) = Y - Y_d - Sd \leq 0$$

- with Y_d and S material parameters to be fitted to obtain the right dissipation

These constraints allow to compute the damage evolution (similar to plastic flow evolution)

Example: if the constraint is violated ($F > 0$), projecting on the constraint surface leads to:

$$F(Y, d^{n+1}) = F(Y, d^n) + \frac{\partial F}{\partial d}\Delta d = 0$$

which bring the evolution of d:

$$d^{n+1} = \frac{-Y + Y_d}{S}$$

For the constraint the $d \leq 1$ we can change it to:

$$d^{n+1} = \min(\frac{-Y + Y_d}{S}, 1)$$

Remark: *F can also be defined be means of an equivalent strain $\epsilon^e q$ instead of Y, leading to other formulations*

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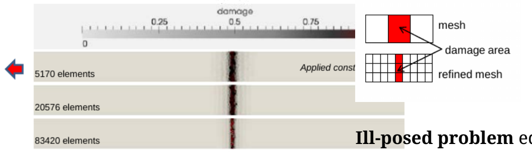
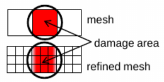
Local/Non-Local Continuum damage mechanics



Ill-posed problem equilibrium balance becomes elliptic in dynamics

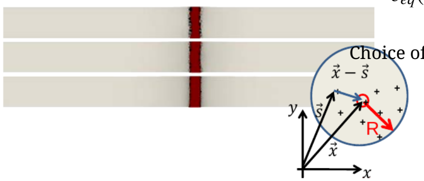


- Several methods to avoid localization

For instance: Gradient, Delay, **Integral type**

$$\epsilon_{eq}^{nl}(\mathbf{x}) = \int_{\Omega} \alpha(\mathbf{x},\mathbf{s})\epsilon_{eq}(\mathbf{s})ds$$

Choice of R ?



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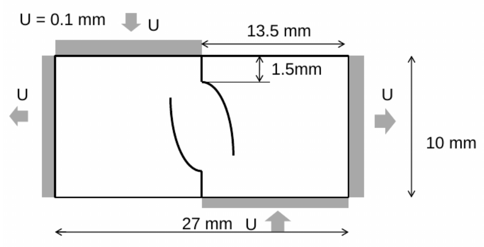
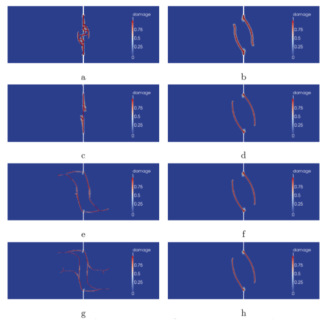
Mesh convergence: Nooru Mohamed test



Mesh convergence: Nooru-Mohamed test



Mesh 1

Element size: 0.12mm
Number of elements: 17,934

Mesh 2

Element size: 0.1mm
Number of elements: 25,647

Mesh 3

Element size: 0.05mm
Number of elements: 104,089

Mesh 4

Element size: 0.02mm

Variational approach to fracture: Phase-field

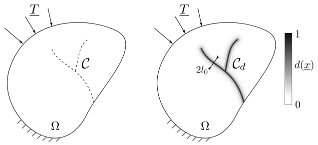
Idea

- The crack discontinuous topology is regularized by a **continuous** phase-field function
  - This function is the damage $d \in [0, 1]$ of the material
  - Introduction of a regularization length scale l_0

Variational approach to fracture: Phase-field

Better suited for

- complex crack paths: dynamic crack branching, instabilities
- crack propagation in heterogeneous media
- multiphysics coupling



With a discontinuous representation

- Γ: the crack path
- G_c: the critical energy release rate
- ψ: the *Helmholtz* free energy
- $\Psi_0 = \rho\psi$: the energy density

The total energy becomes

$$E(u,\Gamma) = \underbrace{\int_{\Omega\backslash\Gamma} \Psi_0(\epsilon) d\Omega}_{\text{elastic energy}} + \underbrace{G_c \int_\Gamma ds}_{\text{dissipated energy}}$$

Regularization leads to

- d: damage phase field (strong link with **damage gradient models**)
- l_0: caracteristic regularization length
- k: residual stiffness at full failure

$$E(u,d) = \int_\Omega [(1-d)^2 + k]\Psi_0(\epsilon) d\Omega + \frac{G_c}{2l_0}\int_\Omega \left(d^2 + l_0^2 ||\nabla d||^2\right) d\Omega$$

Remarks:

The information of the crack path is **now contained in the phase field** d

Dissipated energy density $\frac{G_c}{2l_0}\left(d^2 + l_0^2||\nabla d||^2\right)$

The problem becomes

find u,d minimizers of E(u, d) with the constraint $\Delta d > 0$

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Equilibrium: Euler-Lagrange equations

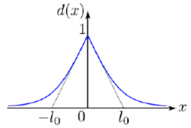
Constitutive law

$$\sigma = [(1-d)^2 + k]\frac{\partial \Psi_0}{\partial \epsilon} = [(1-d)^2 + k] C : \epsilon$$

Damage equation

$$0 = 2(1-d)\Psi_0 + \frac{G_c}{l_0}\left(d + l_0^2 \Delta d\right)$$

- This equation leads to spreading of damage



Resolution algorithm: alternate minimization

1. At fixed u, solve for d (constrainted optimization for irreversibility)
2. At fixed d, solve for u: elastodynamics problem with degraded stiffness

$\Rightarrow$ regularization of the crack surface with a phase-field

Compression/traction separation

$$E(u,d) = \int_\Omega \left\{[(1-d)^2 + k]\Psi_0^+ + \Psi_0^-\right\} d\Omega + \frac{G_c}{2l_0}\int_\Omega \left(d^2 + l_0^2||\nabla d||^2\right) d\Omega$$

with the separation of **compression/traction** strains:

$$\Psi_0^\pm = \frac{1}{2}\lambda\langle tr(\epsilon)\rangle_\pm^2 + \mu tr(\epsilon_\pm^2)$$

which then brings:

$$\sigma = [(1-d)^2 + k]\frac{\partial \Psi_0^+}{\partial \epsilon} + \frac{\partial \Psi_0^-}{\partial \epsilon}$$
$$0 = 2(1-d)\Psi_0^+ + \frac{G_c}{l}\left(d + l_0^2 \Delta d\right)$$

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Irreversibility of damage using history

$$0 = 2(1-d)\mathcal{H}(x) + \frac{G_c}{l}\left(d + l_0^2 \Delta d\right)$$
$$\mathcal{H} = \max_t \Psi_0^+(\epsilon, t)$$

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Variational approach to fracture:

Phase-field

Choice of dissipation function

$$\frac{G_c}{2l_0}\left(d^2 + l_0^2||\nabla d||^2\right)$$

most widely used model in phase-field literature

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$$\frac{3G_c}{8l_0}\left(d + l_0^2||\nabla d||^2\right)$$

Faster decay of d

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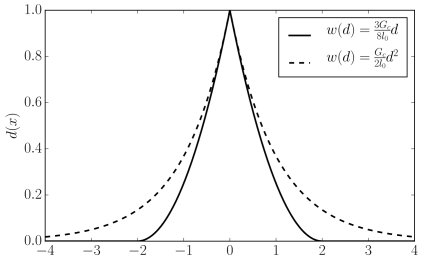

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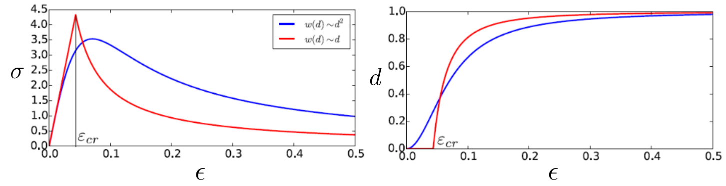
Comparison of dissipation models

- Un-physical spreading of the phase field

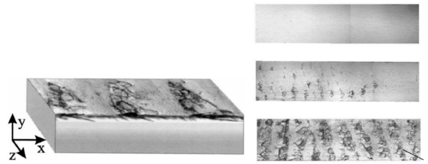


Remark: presence/absence of a purely elastic phase

Introduction: some experimental facts on dynamic fracture

Prediction of a limit crack velocity: c_R (mode I), c_S (mode III) never attained in experiments, rarely exceed $0.4-0.6c_R$ explained by **crack tip instabilities** [Fineberg et al.]:

- microbranching ($\sim 0.4c_R$): small ($1-100\mu m$ in PMMA) short-lived micro-cracks, highly localized in z direction:

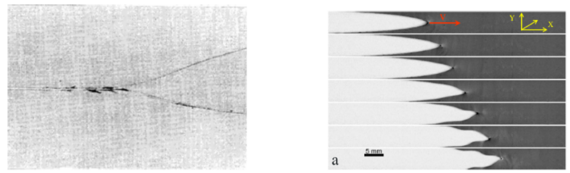


- mirror, mist, hackle patterns

Introduction: some experimental facts on dynamic fracture

Prediction of a limit crack velocity: c_R (mode I), c_S (mode III) never attained in experiments, rarely exceed $0.4-0.6c_R$ explained by **crack tip instabilities** [Fineberg et al.]:

- increase of microbranch width ⇒ macroscopic branching



- microbranching can be suppressed in thin smaples or strongly anisotropic materials ⇒ oscillatory instability at 0.9 c_R

Variational approach to fracture: Phase-field

elastic strain energy density:

$$\Psi(\epsilon, d) = (1-d)^2\Psi_0^+ + \Psi_0^-$$

fracture energy density:

$$\frac{3G_c}{8l_0}\left(d + l_0^2||\nabla d||^2\right)$$

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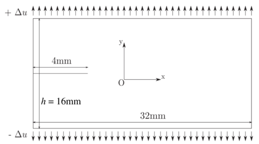
Crack branching: Pre-strained plate



Prestrained PMMA plate, fixed boundaries *[Zhou, 1996]*

$E = 3.09 GPa$

$\nu = 0.35$
$\rho = 1180 kg/m^3$
$G_c = 300 J/m^2$
c_R = 906 m/s$

- $\Gamma = \frac{dE}{da} = 2E(\Delta U)^2/h$
  $\Rightarrow$ crack should accelerate to c_R
- transition from straight propagation to branched patterns
- apparent toughness increases with loading/crack velocity

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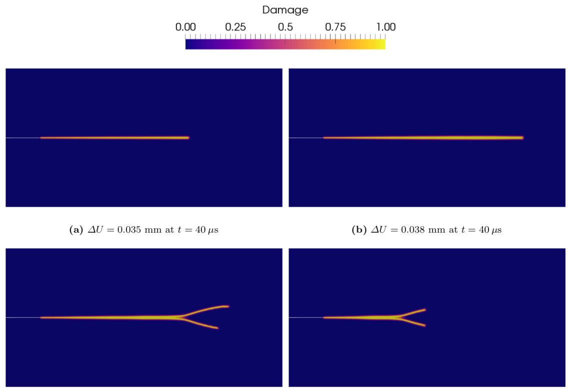
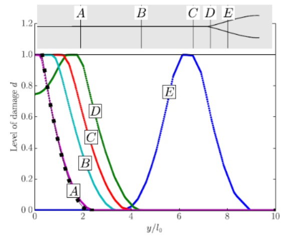
Variational approach to fracture: Dynamic crack branching



(a) $\Delta U = 0.035$ mm at $t = 40\,\mu s$
(b) $\Delta U = 0.038$ mm at $t = 40\,\mu s$
(c) $\Delta U = 0.040$ mm at $t = 40\,\mu s$
$\Delta U = 0.045$ mm at $t = 20\,\mu s$

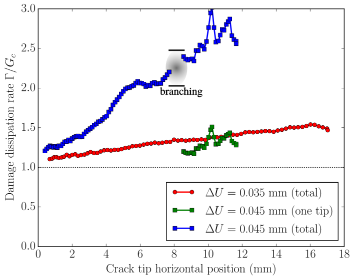
Variational approach to fracture: Damage zone thickening



- progressive thickening of the damaged band before branching
- branching viewed as a progressive transition from a widening crack to two crack tips screening each other
- branching angle seems to depend on geometry

Variational approach to fracture: Apparent fracture energy

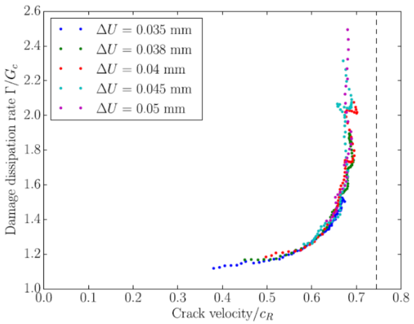
Damage dissipation rate $\Gamma = \frac{dE_{frac}}{da}$ interpreted as the apparent fracture energy



suggests a critical value of $\Gamma \simeq 2G_c$ associated to branching

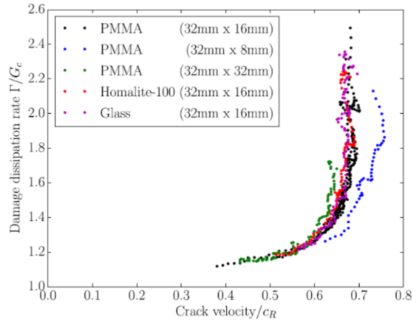
Variational approach to fracture: Velocity toughening

during propagation and before macroscopic branching

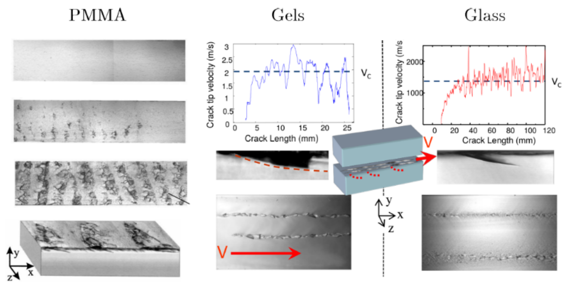


existance of a well-defined $\Gamma(v)$ relationship associated to a velocity-toughening mechanism

Variational approach to fracture: Velocity toughening



But branching is a 3D instability: Fineberg et al.

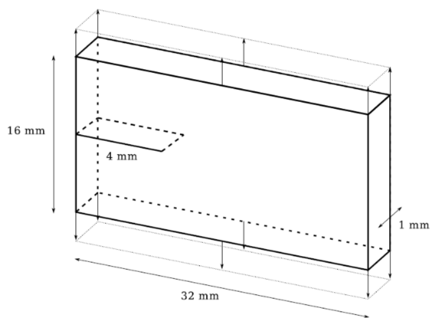


- increasing branch length with loading/velocity
- z-localization: much more localized for Gels/Glass than for PMMA
- x-periodicity: $10 - 100\mu m$ in PMMA, from *nm* to *mm* in glass
- microbranching **suppressed for thin samples**

Variational approach to fracture: From 2D to 3D

Same setup, same material parameters, now: $L \times H \times W$ plate

starting with $W = 1mm$, $l_0 = 0.004mm$



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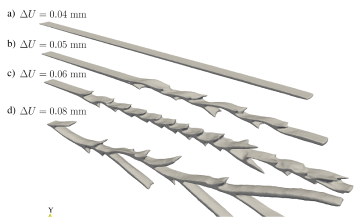
Variational approach to fracture: Effect of loading



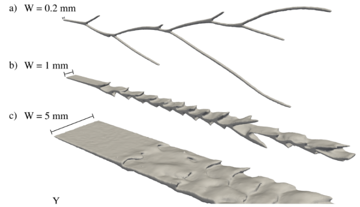
- single straight crack → microscopic branches → macroscopic branches

- nice quasi-periodic regime at intermediate loading:

- less z-invariance at smaller loading consistent with experiments

Variational approach to fracture: Effect of thickness

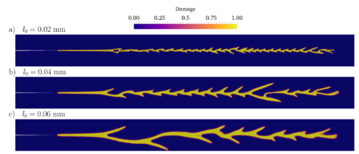
same loading $\Delta U = 0.06mm$



- microbranching clearly suppressed for small width
- increasing localization with increasing width
- no periodicity for larget width:

Variational approach to fracture: Influence of l_0

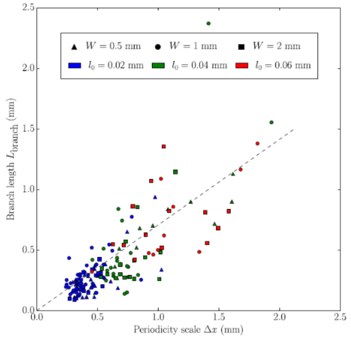
$\Delta U = 0.06mm$, $W = 1mm$



- strong efect on Δx
- initiation occurs at roughly the same time
- total dissipated energy almost identical ($\pm 2\%$)
- no microbranching when $l_0 \simeq 0.1mm$
- other plate width suggest $W_{crit} \simeq 10 l_0$ for this loading

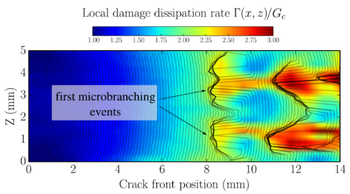
Variational approach to fracture: Emergent geometrical features

Correlation between microbranch length L_{branch} and Δx



Variational approach to fracture: Transition to branching

$W = 5mm$: localized microbranching events



- first events when $\Gamma \geq 2G_c$ **locally**
- velocity overshoot ahead of the first event initiates the second one
- after that, complex dynamics....

Conclusions

- Variational approach to dynamic fracture shows great prospects (comes at a cost)
- Mesh independency
- Reproduces many experimental features
- Many many open questions... (in dynamics)

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