Introduction to fracture mechanics (mostly LEFM)

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Outline

- Historical perspective
- Atomistic view of fracture and defects
- Griffith's theory
- Irwin, stress intensity factor
- Mode mixity
- Plastic-zone size estimates
- Cohesive zone model
- Dynamic fracture

T. Anderson. *Fracture Mechanics: Fundamentals and Applications, Fourth Edition*. (CRC Press, 2017).

L. Freund. *Dynamic Fracture Mechanics*. (Cambridge University Press, 1990).

F. Barras. *When Dynamic Cracks Meet Disorder: A Journey along the Fracture Process Zone*. EPFL (Lausanne), 2018.

Some rare structural failure events

 Hawaii, April 24, 1988, plane looses 1/3 of its roof due to stress fracture, at 6,000 meters altitude...

 Mineapolis, August 1, 2007, collapse of I-35W bridge

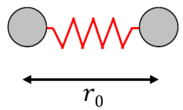
Accidents used to be not so rare...

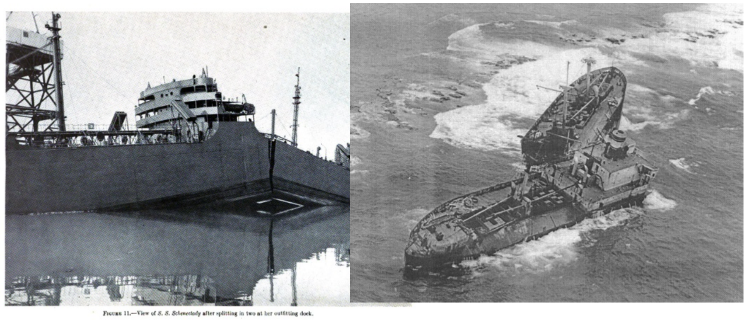
Historical perspective

- Hammurabi code (1750 BC): Darwinian design
- May 1453, siege of Constantinople: huge cast iron cannon explodes after firing three shots, killing 40 people
- Industrial revolution, modern era: accident skyrocket due to poor design (no stress analysis) and lack of materials understanding
  - 1919, explosion of molasse tank in Boston (40 dead, https://www.britannica.com/topic/Great-Molasses-Flood)
  - 1920's: 200 dead/year in the US due to rupture of axis of wheels in trains
- Consequence: engineers sometimes use security coefficients of 10 in structures
- 1st significant progress: Griffith's theory 1920 (but still "academic" = intellectual curiosity)
- Founding element : liberty ships story (Fracture Mechanics born after World War II)
  - German submarines sink cargo ships at 3X construction rate
  - Need drastic new design; call Henri Kaiser (construction engineer, built Hoover dam); Kaiser invents revolutionnary procedure: «all-welded hull», i.e. no rivets; success!!
  - Until 1943: one vessel broke in 2 while sailing between Siberia and Alaska
  - 2700 boats built in WWII, of which 400 sustained fracture under «low stress»
    - **10 broke in 2**

Atomic view of fracture

Fracture ≡ breaking of bonds





Reasons:

- Defects in weld (inexperienced welder)
- Fracture initiated at deck (where there is a stress concentration)
- Low toughness steel (especially at low temperature)
- And no rivets to block propagating crack

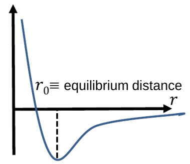
$\Rightarrow$ Irwin is called to start a lab at Naval Research Laboratory to study fracture mechanics

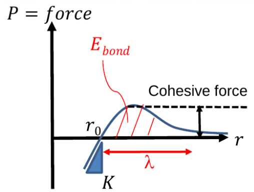
Birth of modern fracture mechanics:

- Defects and cracks are present in each structure.
- Can be detected with *Non-destructive Examination* (NDE)
- Engineer balances cost of reparation with risk of failure
- Concept of damage tolerance and fracture mechanics analysis (today backed up with numerical modeling)

Historical references

S. Antolovich, A. Saxena, W. Gerberich. *Fracture Mechanics - An Interpretive Technical History*. Mechanics Research Communications. **91**,46-86. (2018)



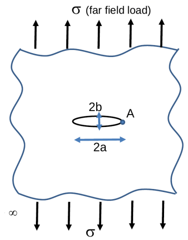


$$E_{bond} = \int_{r_0}^{\infty} P dr$$

- Approximation: $P(r) = P_c \sin(\frac{\pi r}{\lambda})$
- Small displacements: $P(r) = P_c \frac{\pi r}{\lambda}$
- Bond rigidity: $K = P_c \cdot (\pi/\lambda)$
- $\sigma_c = P_c/A$ and $K = EA/\lambda$
- $\sigma_c = \frac{E}{\pi}$

$$\gamma_s = \frac{1}{2}\int_0^{\lambda} \sigma_c \sin\left(\frac{\pi r}{\lambda}\right) dr = \sigma_c \frac{\lambda}{\pi} \equiv \text{surface energy} \equiv [J \cdot m^{-2}]$$

Defects: create stress concentrations



Elliptical hole in plate: curvature $\rho = b^2/a$

Charles Inglis, 1913

$$\sigma_A = \sigma\left(1 + \frac{2a}{b}\right) \Longleftrightarrow \sigma_A = \sigma\left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

Limit $a \gg b$

$$\sigma_A = 2\sigma\sqrt{\frac{a}{\rho}}$$

Problem, when $\rho \to 0, \sigma_A \to \infty$

i.e. in thin crack limit, all materials would break (leads to Griffith's theory)

But cracks cannot be infinitely thin, they plastify;

Atomically sharp crack: $\rho = \lambda$

$$\sigma_A \sim 2\sigma\sqrt{\frac{a}{\lambda}}$$

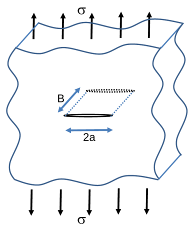
Fracture occurs if $\sigma_A = \sigma_c$

$$\sigma_{\text{failure}} = \sqrt{\frac{E\gamma_s}{4a}}$$

Rq: atomistically sharp crack cannot exist due to plasticity

Griffith's energy criteria (1920): Simple and powerful model

Plate under far field load:



Alan Arnold Griffith

Definitions

- $A = 2aB$: cracked surface
- dA: surface growth of crack
- dE_{pot}: potential energy variation (external and internal forces)
- dW_s: work necessary to create new surfaces

Critical point for propagation of a crack:

$$\frac{dE}{dA} = \frac{dE_{pot}}{dA} + \frac{dW_s}{dA} \leq 0$$

Released potential energy must be greater than energy necessary to create cracks

At critical point:

$$-\frac{dE_{pot}}{dA} = \frac{dW_s}{dA}$$

It can be shown (Inglis) that

$$E_{pot} = E^0_{pot} - \pi\frac{\sigma^2 a^2 B}{E} = E^0_{pot} - \frac{\pi\sigma^2 A^2}{4EB}$$

so that:

$$-\frac{dE_{pot}}{dA} = \frac{2\pi\sigma^2 A}{4EB}$$

Surface energy proportional to A

$$W_s = 2A\gamma_s$$

so that:

$$\frac{dW_s}{dA} = 2\gamma_s$$

$\Rightarrow$ Critical stress based on energetic criterion:

$$\sigma_f = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

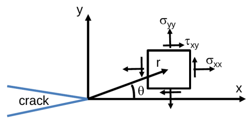
- Very close to Inglis ($\sqrt{\frac{E\gamma_s}{4a}}$)
- Works ok for brittle materials (not good for ductile materials)
- Does not require defining curvature ρ, unlike Inglis
- Profound model:

Small cracks heal, large cracks grow

Griffith's length: $a_c = \frac{2E\gamma_s}{\pi\sigma^2}$

From global to local analysis: stresses at crack tip

Rupture modes



∃ analytical solutions: Westgaard 1939, Irwin 1957, Sneddon 1946, Williams 1957

H. Westergaard. *Bearing Pressures and Cracks: Bearing Pressures Through a Slightly Waved Surface or Through a Nearly Flat Part of a Cylinder, and Related Problems of Cracks*. Journal of Applied Mechanics. **6**,A49-A53. (1939)

I. Sneddon, N. Mott. *The Distribution of Stress in the Neighbourhood of a Crack in an Elastic Solid*. Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences. **187**(1009),229-260. (1946)

G. Irwin. *Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate*. Journal of Applied Mechanics. **24**(3),361-364. (1957)

$$\sigma_{ij} = \left(\frac{K}{\sqrt{2\pi r}}\right) f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^{\frac{m}{2}} g^{(m)}_{ij}(\theta)$$

Limit $r \to 0$, leading term is in $\frac{1}{\sqrt{r}}$

Stress singularity at crack tip !

- K: Stress intensity factor

M. Williams. *On the Stress Distribution at the Base of a Stationary Crack*. Journal of Applied Mechanics. **24**(1),109-114. (1956)

3 rupture modes

Mode I (opening)

Mode II (in-plane shear)
Mode III (out-of-plane shear)

$f_{ij}^{I}(\theta), f_{ij}^{II}(\theta)$ and $f_{ij}^{III}(\theta)$ are known adimentional functions of θ, and are independent of the geometry

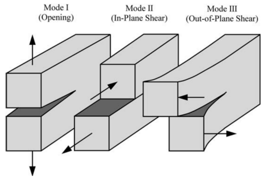
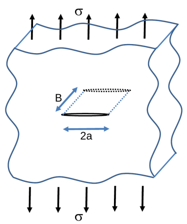


image from Anderson

Derivation of K: for through crack in an infinite plate under mode I opening



K depends on geometry and loading

- Known analytical derivation for through crack:

$$K_I = \sigma\sqrt{\pi a}$$

Other example, edge crack:

$$K_I = 1.12\sigma\sqrt{\pi a}$$

Analytical derivation difficult

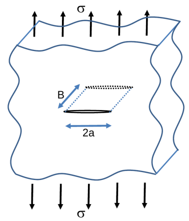
- Alternative: numerical evaluation

More analytical cases found in:

T. Anderson. *Fracture Mechanics: Fundamentals and Applications, Fourth Edition*. (CRC Press, 2017).

Equivalence between G and K: through crack example

Key points of LEFM: Griffith 1921, Westergaard 1939, Irwin 1957



Energy release rate:

$$G = -\frac{dE_{pot}}{dA} = \frac{\pi\sigma^2 a}{E}$$

Stress intensity factor:

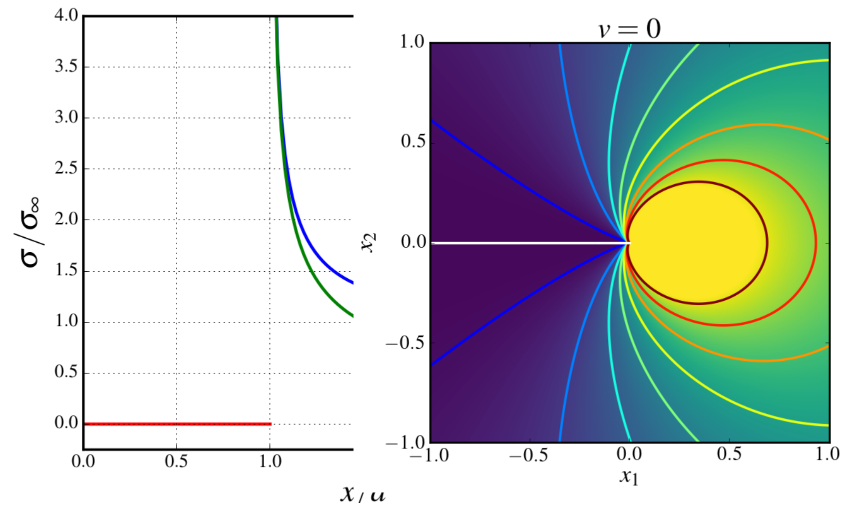
$$K_I = \sigma\sqrt{\pi a}$$

Gives:

$$G = \frac{K_I^2}{E}$$

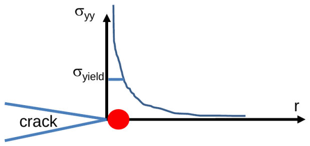
Mixed-mode (with Poisson effects):

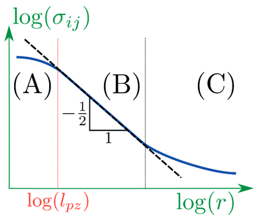
$$G = \frac{K_I^2}{E/(1-\nu^2)} + \frac{K_{II}^2}{E/(1-\nu^2)} + \frac{K_{III}^2}{2\mu}$$



- Stress intensity factor: $\sigma(r,\theta) \propto \frac{K}{\sqrt{2\pi r}} f(\theta)$
- Predict energy release by crack advance: $G = \frac{1-\nu^2}{E}(K_I^2 + K_{II}^2) + \frac{K_{III}^2}{2\mu}$
- Analyze the stability of crack in materials $G < G_c$

Crack tip plasticity: K dominated rupture





Stress singularity at crack tip, **but stresses cannot be infinite**

$$\Rightarrow \text{A plastic zone must exist}$$

Plastic/Process zone size can be estimated

- with the **Yield stress** σ_Y

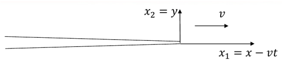
$$r = \frac{1}{\pi}\left(\frac{K}{\sigma_Y}\right)^2$$

Numerically:

- we will add a non-linear zone at crack tip to remove singularity
- cohesive zone model

Dynamic Fracture Mechanics: freund 1990, Kostrov and Das 1988

Moving crack (mode-III) in the x direction at velocity v



Elastodynamic wave equation (mode-III)

$$u_{z,xx} + u_{z,yy} = \frac{1}{c_s^2} u_{z,tt}$$

Apply the *Lorentz* transform: System of coordinates at tip

$$x_1 = \frac{x - vt}{\sqrt{1 - v^2/c_s/2}}, \quad x_2 = y, \quad x_3 = z, \quad t' = \frac{1 - vx/c^2}{\sqrt{1 - v^2/c_s{}^2}}$$

For a semi-infinite crack (elliptic PDE if $v \ll c_s$, no shocks), equation shape is preserved

$$\left(1 - \frac{v^2}{c_s^2}\right) u_{3,11} + u_{3,22} = \frac{1}{c_s^2} u_{3,t't'}$$

For a steady state propagating crack $\partial/\partial t' = 0$

$$\left(1 - \frac{v^2}{c_s^2}\right) u_{3,11} + u_{3,22} = 0$$

Dynamic Stress Intensity Factor:

$$\sigma_{yy} \simeq \frac{K_I^d}{\sqrt{2\pi r}}\Sigma(v,\theta) = \frac{1}{\left(1 - v^2/c_s^2\right)^4}\frac{K_I^s}{\sqrt{2\pi r}}\Sigma(v = 0, \theta)$$

K^s: static stress intensity factor

K^d: dynamic stress intensity factor
Contraction of space when approaching wave speed (larger stresses)

L. Freund. *Dynamic Fracture Mechanics*. (Cambridge University Press, 1990).

F. Barras. *When Dynamic Cracks Meet Disorder: A Journey along the Fracture Process Zone*. EPFL (Lausanne), 2018.

Asymptotic fields for dynamics crack: Mode I, Freund 1990

$$\sigma_{ij} = \frac{K_I(t)}{\sqrt{2\pi r}}\Sigma_{ij}^I(\theta, v) + \sigma_{ij}^{(1)} + o(1) \quad \text{as} \quad r \to 0. \qquad (4.3.10)$$

The functions $\Sigma_{ij}^I(\theta, v)$ that represent the angular variation of stress components for any value of instantaneous crack tip speed v are

$$\Sigma_{11}^I = \frac{1}{D}\left\{(1+\alpha_s^2)(1+2\alpha_d^2-\alpha_s^2)\frac{\cos\frac{1}{2}\theta_d}{\sqrt{\gamma_d}} - 4\alpha_s\alpha_d\frac{\cos\frac{1}{2}\theta_s}{\sqrt{\gamma_s}}\right\},$$

$$\Sigma_{12}^I = \frac{2\alpha_d(1+\alpha_s^2)}{D}\left\{\frac{\sin\frac{1}{2}\theta_d}{\sqrt{\gamma_d}} - \frac{\sin\frac{1}{2}\theta_s}{\sqrt{\gamma_s}}\right\},$$

$$\Sigma_{22}^I = -\frac{1}{D}\left\{(1+\alpha_s^2)^2\frac{\cos\frac{1}{2}\theta_d}{\sqrt{\gamma_d}} - 4\alpha_d\alpha_s\frac{\cos\frac{1}{2}\theta_s}{\sqrt{\gamma_s}}\right\}, \qquad (4.3.11)$$

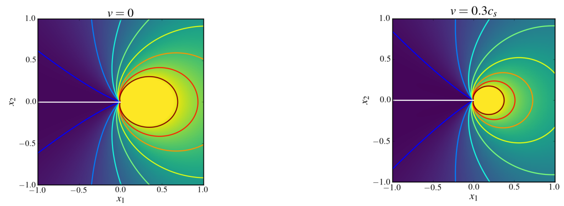
where

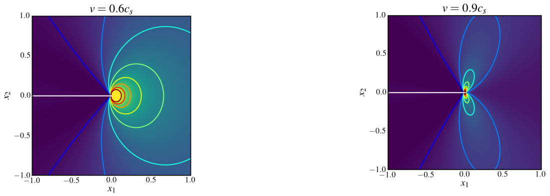
$$\gamma_d = \sqrt{1-(v\sin\theta/c_d)^2}, \quad \tan\theta_d = \alpha_d\tan\theta,$$
$$\gamma_s = \sqrt{1-(v\sin\theta/c_s)^2}, \quad \tan\theta_s = \alpha_s\tan\theta. \qquad (4.3.12)$$

L. Freund. *Dynamic Fracture Mechanics*. (Cambridge University Press, 1990).

Asymptotic fields for dynamic crack: Mode I, Freund 1990, Kostrov and Das 1988

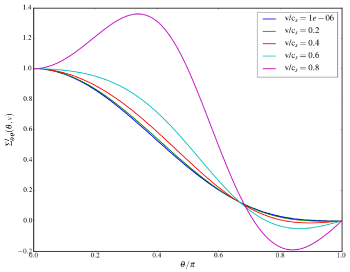
Hoop stress





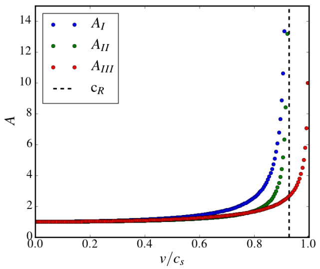
F. Barras. *When Dynamic Cracks Meet Disorder: A Journey along the Fracture Process Zone*. EPFL (Lausanne), 2018.

Asymptotic fields for dynamic crack: Maximum hoop stress



Dynamic energy release rate: Freund 1990, Kostrov and Das 1988

$$G(a, v) = \frac{1 - \nu^2}{E}\left[A_I(v)K_I^2 + A_{II}(v)K_{II}^2\right] + \frac{1}{2\mu}A_{III}(v)K_{III}^2$$



L. Freund. *Dynamic Fracture Mechanics*. (Cambridge University Press, 1990).

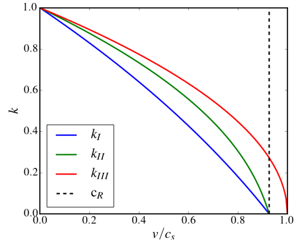
B. Kostrov, S. Das. *Principles of Earthquake Source Mechanics*. (Cambridge University Press, 1989).

K dependance on crack speed: Freund, 1990

Freund's assumption:

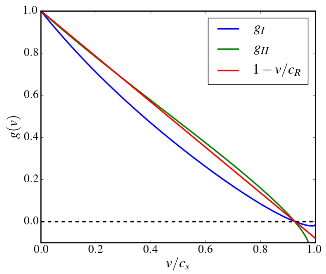
$$K(a, v) = k(v)K(a, v = 0)$$

$$k_I(v) \simeq \frac{1 - v/c_R}{\sqrt{1 - v/c_d}}$$
$$k_{II}(v) \simeq \frac{1 - v/c_R}{\sqrt{1 - v/c_s}}$$
$$k_{III}(v) \simeq \sqrt{1 - v/c_s}$$



Dynamic energy release rate: Freund, 1990

$$G(a, v) = \frac{1 - \nu^2}{E}A(v)k(v)^2K_I^2(a, v = 0) = G(a, v = 0)g(v)$$



Crack tip equation of motion: Freund, 1999

- Infinite homogeneous plate
- Uniform crack speed
- $G_{I,II} > 0$ when $v < c_R \Rightarrow$ Admissible crack speed

$$G \simeq G^{static}\left(1 - \frac{v}{c_R}\right)$$
for $v \leq c_R$

$G < 0, c_R < v < c_S$
$G \leq 0, v > c_S$
$G > 0, v = \sqrt{2}c_s$ (mode II)

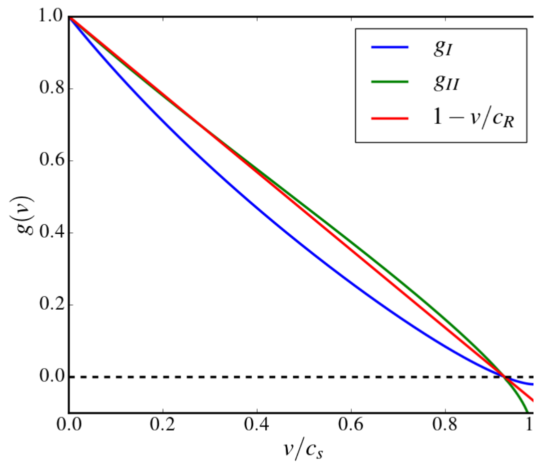


Table 2.1 – Summary of the admissible crack speeds predicted by LEFM.

	$0 < v_c < c_R$	$c_R \leq v_c < c_s$	$c_s < v_c < c_d$
Mode I	✓	✗	✗
Mode II	✓	✗	✓
Mode III	✓	✓	✗

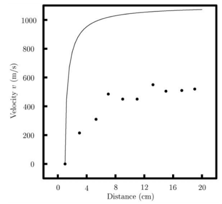
L. Freund. *Dynamic Fracture Mechanics*. (Cambridge University Press, 1990).

F. Barras. *When Dynamic Cracks Meet Disorder: A Journey along the Fracture Process Zone*. EPFL (Lausanne), 2018.

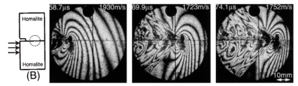
And is it working ?

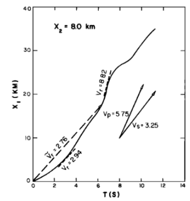
Kobayashi et al, 1974

Mode-I: no!



Mode-II: no!





A. Kobayashi, B. Wade, W. Bradley, S. Chiu. *Crack Branching in Homalite-100 Sheets*. Engineering Fracture Mechanics. **6**(1),81-92. (1974)

R. Archuleta. *A Faulting Model for the 1979 Imperial Valley Earthquake*. Journal of Geophysical Research: Solid Earth. **89**(B6),4559-4585. (1984)

A. Rosakis, O. Samudrala, D. Coker. *Cracks Faster than the Shear Wave Speed*. Science. **284**(5418),1337-1340. (1999)

A. Rosakis. *Intersonic Shear Cracks and Fault Ruptures*. Advances in Physics. **51**(4),1189-1257. (2002)

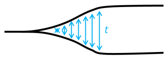
... but why ?

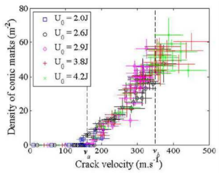
Scheibert et al. 2010

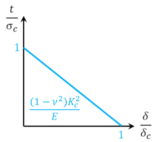
Mode-I: Interplay between crack and microstructure

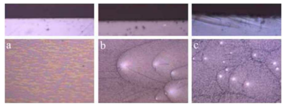
Burridge 1973, Andrews 1976

Mode-II: non-singular fracture theory









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D. Andrews. *Rupture Velocity of Plane Strain Shear Cracks*. Journal of Geophysical Research (1896-1977). **81**(32),5679-5687. (1976)

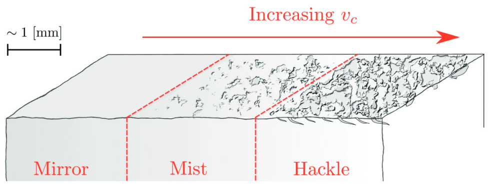
J. Scheibert, C. Guerra, F. Célarié, D. Dalmas, D. Bonamy. *Brittle-Quasibrittle Transition in Dynamic Fracture: An Energetic Signature*. Physical Review Letters. **104**(4),045501. (2010)

F. Barras, R. Carpaij, P. Geubelle, J.-F. Molinari. *Supershear Bursts in the Propagation of a Tensile Crack in Linear Elastic Material*. Physical Review E. **98**(6),063002. (2018)

Mitigated results

- slow crack propagation speeds: ok
- $v >$ a few tenths of cs: not ok
- underestimates the dissipated energy
- overestimates the crack propagation speed (already for $v > 0.65cR$)

Three dynamic phases



Postmortem appearance of the fracture surfaces

- Mirror: smooth surfaces.
- Mist: surface roughen (interplay between a crack front and microstructure)
- Hackle: microbranching instability

Relativistic contraction brings microstructure heterogeneity at play !!

$$\Rightarrow \text{Rupture front distorsion}$$

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