Dynamic fragmentation

G. Anciaux

Civil Enginering, Materials Science, EPFL

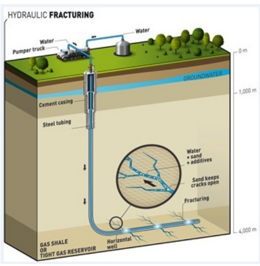
Dynamic fragmentation

A long history… many applications

- Mining industry, road excavation, fuel fragmentation (1930’s)
- 1940’s: seminal contribution of Mott (bomb shells)

Nevill-Francis Mott (Nobel prize picture)

- Applications in engineering (hydraulic fracturing, crash performance), medecine (kidney stone fragmentation), all the way to Space industry (orbital debris) and Astrophysics (asteroid impact, big bang),…



Hydraulic fracturing; Total E&P Denmark B.V

Energy-absorbing materials; 123rf.com

A conceptual image illustrating space debris orbiting Earth. (Image credit: johan63/iStock/Getty Images Plus)

Asteroid impact: artistic view

Experimental facts

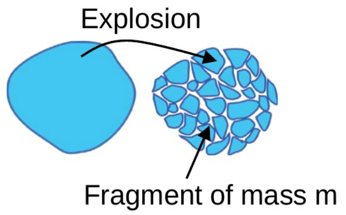
Cumulative distribution of masses

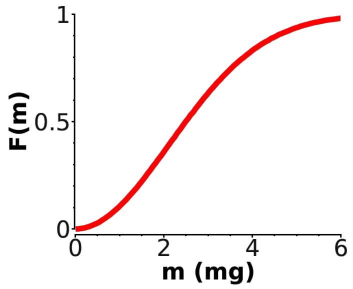
$$F(m) = 1 - e^{-(m/m_0)^\beta}$$

Probability of finding a fragment with mass $< m$

m_0 a characteristic (average) mass

P. Rosin, E. Rammler. *The Laws Governing the Fineness of Powdered Coal*. Journal of the Institute of Fuel. **7**,29-36. (1933)





$F(m) = 1 - e^{-(m/m_0)^\beta}$

- Exponential or power law cumulative distribution of fragment sizes
- Average fragment size decreases with higher loading energy
- Fragment velocities: inverse power law of the fragments' mass

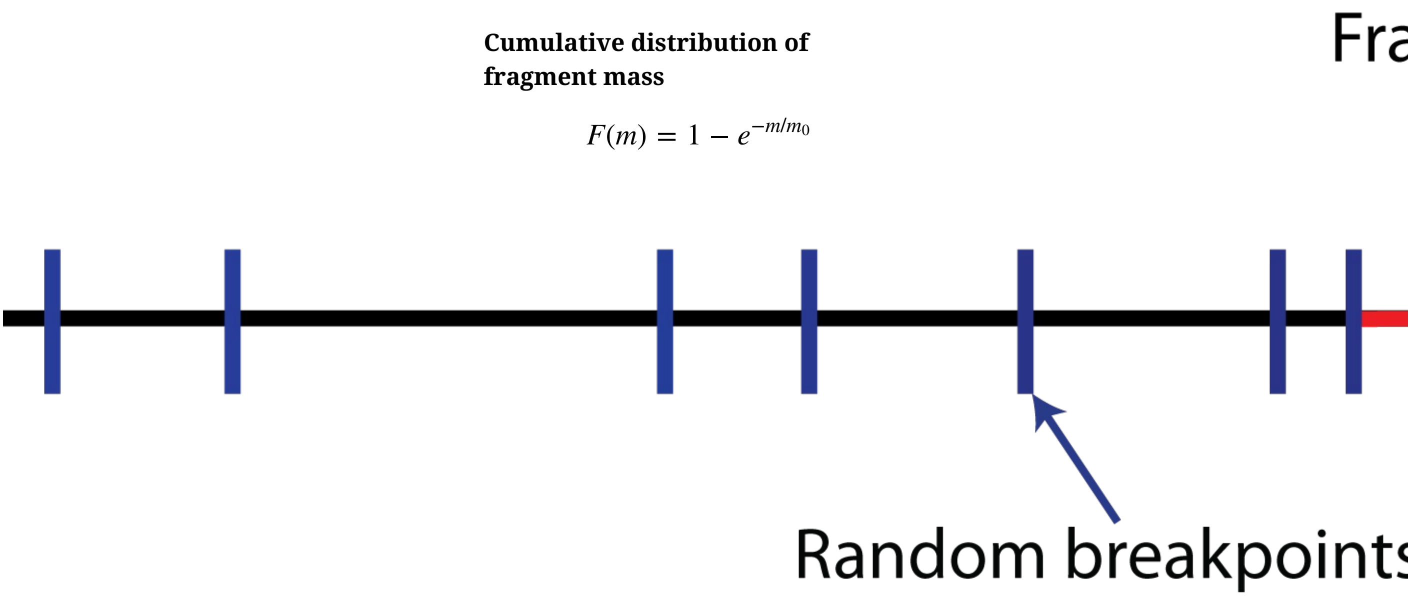
Analytical models: Early works

C. Lienau. *Random Fracture of a Brittle Solid*. Journal of the Franklin Institute. **221**(6),769-787. (1936)

assumption: no interactions between cracks

Cumulative distribution of fragment mass

$$F(m) = 1 - e^{-m/m_0}$$



Idea

- 1D line with randomly placed breakpoints
- Probability of finding k breakpoints within a given length l (Poisson distribution)

$$P(k,l) = \frac{\left(\frac{l}{l_0}\right)^k e^{-\frac{l}{l_0}}}{k!}$$

with:

- l_0: average spacing between breaks
- $P(0,l) = e^{-\frac{l}{l_0}}$: probability of finding 0 break in a length l
- $P(1,dl) = \frac{dl}{l_0}$: probability of finding 1 break within an infinitesimal length dl

Probability of finding 0 break in a length l **AND** 1 break within a length dl

$$\Longleftrightarrow$$

Probability of finding a fragment of size l within a precision dl

Therefore

$$f(l)dl = \frac{1}{l_0} e^{-\frac{l}{l_0}} dl$$

with $f(l)$ the fragment size distribution and $F(l) = \int f(l)dl$

$$F(l) = 1 - e^{-\frac{l}{l_0}}$$

Generalisation to a finite size line

$$F(l) = 1 - \left(1 - \frac{l}{L}\right)^{N_f - 1}$$

with

- N_f number of fragments
- converges to *Lienau* if $N_f \rightarrow \infty$

D. Grady. *Particle Size Statistics in Dynamic Fragmentation*. Journal of Applied Physics. **68**(12),6099-6105. (1990)

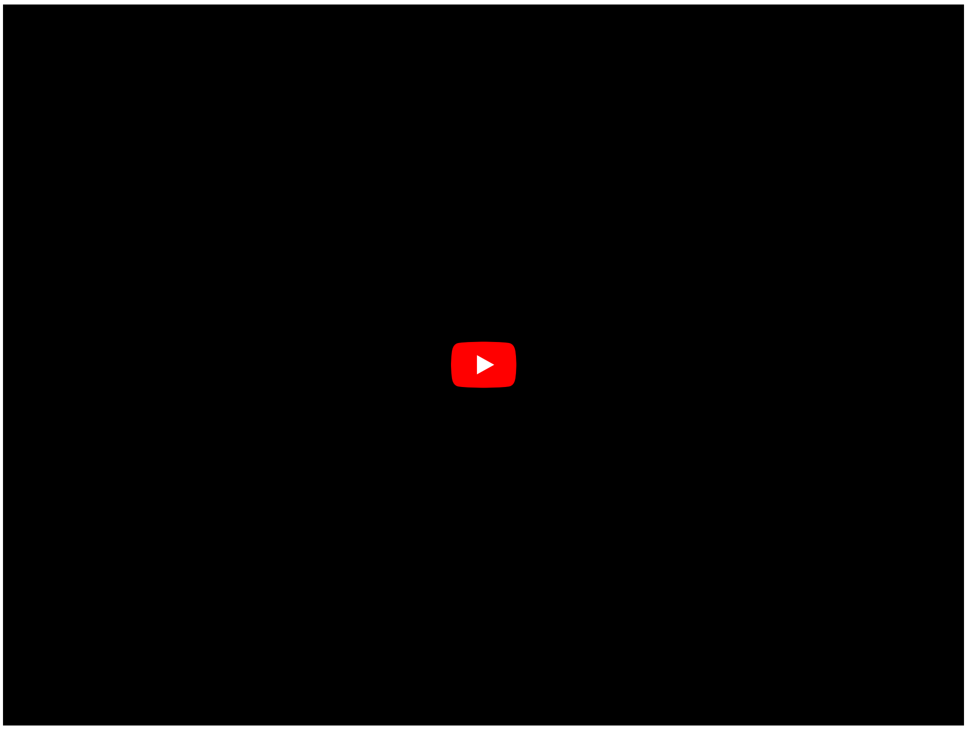
So

$$F(m) = 1 - e^{-(m/m_0)^\beta}$$

- How is the average m_0 varying ?
- Simplistic hypothesis (dynamics ?)

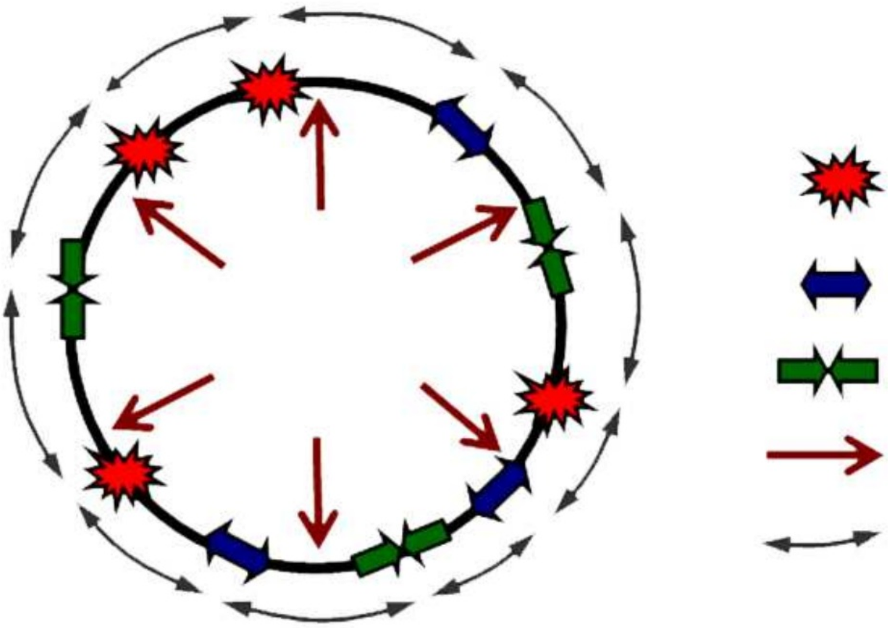
However … cracks interact via mechanical waves

Breaking spaghetti

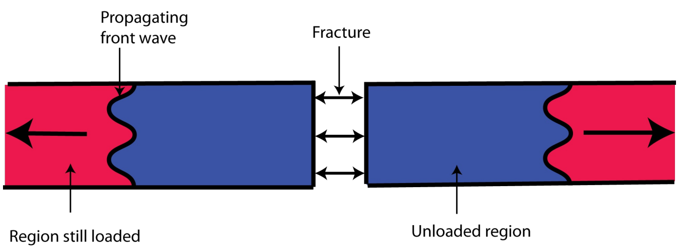


Mott's problem: expanding ring

- Expanding ring leading to a constant, longitudinal (1D) strain rate $\dot{\epsilon}$
- Perfectly plastic (yield Y)
- Random failure in both time and space
- Failure is instantaneous (immmediate stress drop)
- Waves are emited around fracture
  - Propagation distance: $x(t) = \sqrt{\frac{2Yt}{\rho\dot{\epsilon}}}$
- Elasto-plastic refinment (Lee, 1967)



Within a distance $2x(t)$ the stress drop protects from new crack nucleation



N. Mott, E. Linfoot. *A Theory of Fragmentation*.

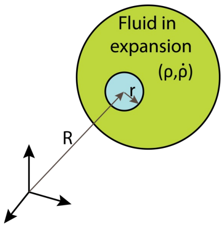
D. Grady, N. Mott. *Fragmentation of Rings and Shells: The Legacy of N. F. Mott*.

(Springer, 2006).

Grady and energy balance

Dynamic expansion of a fluid:

- Predict average fragment size
- Local kinetic energy = Failure energy



Grady's expanding fluid model. The large sphere represents the whole body. The small sphere of radius r limits a region of the fluid that will form a fragment.

$$s = \left(\frac{24G_c}{\rho\dot{\epsilon}^2}\right)^{1/3}$$

- s: characteristic size
- G_c: toughness
- ρ: volumetric mass
- $\dot{\epsilon}$: loading rate**

The average fragment size depends on

- The material properties
- The strain rate

D. Grady. *Local Inertial Effects in Dynamic Fragmentation*. Journal of Applied Physics. **53**(1),322-325. (1982)

S. Levy. *Exploring the Physics behind Dynamic Fragmentation through Parallel Simulations*.

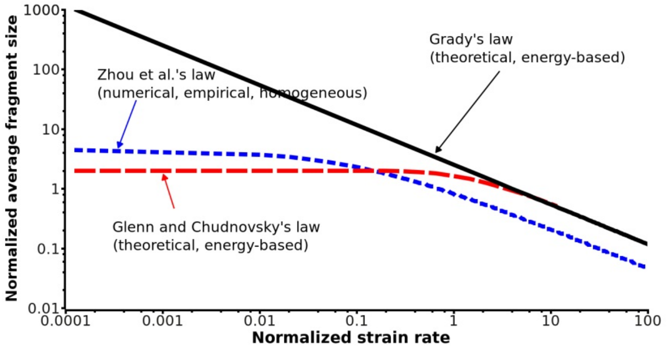
D. Turcotte. *Fractals and Fragmentation*. Journal of Geophysical Research: Solid Earth. **91**(B2),1921-1926. (1986)

But:

- No interaction between cracks (contact)
- No defects
- Inaccurate at low strain rates

In reality:

- Potential energy term missing in the energy balance
- Variation of material properties are important (defects)



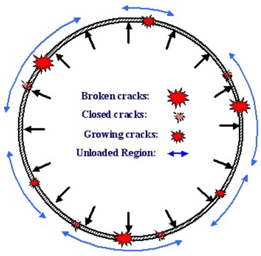
A. Chudnovsky, B. Kunin. *A Probabilistic Model of Brittle Crack Formation*. Journal of Applied Physics. **62**(10),4124-4129. (1987)

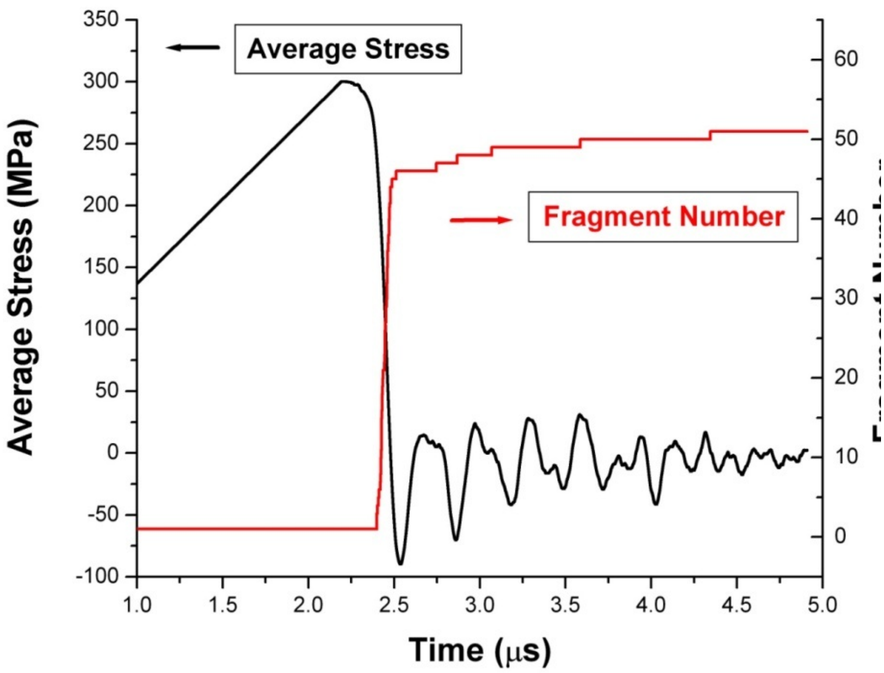
Mott's problem by numerics: [Zhou et al.]

- Distribution of defects matters
- Finite element with Cohesive element approach

F. Zhou, J.-F. Molinari, K. Ramesh. *Analysis of the Brittle Fragmentation of an Expanding Ring*. Computational Materials Science. **37**(1),74-85. (2006)

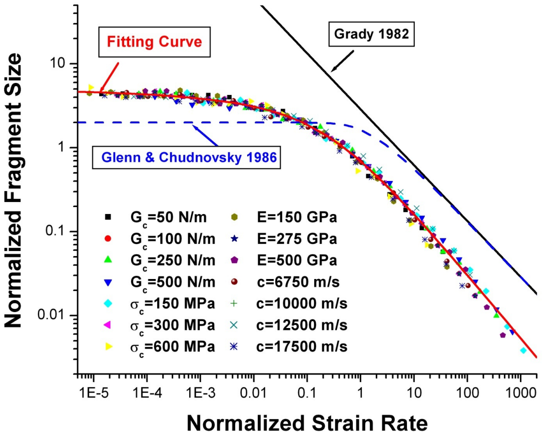
S. Levy, J.- . Molinari, I. Vicari, A. Davison. *Dynamic Fragmentation of a Ring: Predictable Fragment Mass Distributions*. Physical Review E. **82**(6),066105. (2010)





- Ceramic ring length: $L = 50$ mm
- Elastic parameters: $r = 2750$ Kg/m3, $E = 250$ GPa , $c = 10000$ m/s
- Fracture parameters: $\sigma_c = 300$ MPa, $\delta_c = 0.667$ mm, $G_c = 100$ N/m
- Defects distribution: *uniform or Weibul*

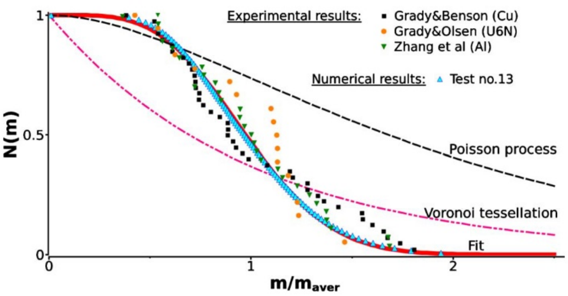
Average fragment size



- Smaller fragments than Grady (closer to experiments)

F. Zhou, J.-F. Molinari, K. Ramesh. *Analysis of the Brittle Fragmentation of an Expanding Ring*. Computational Materials Science. **37**(1),74-85. (2006)

Mass probability distribution



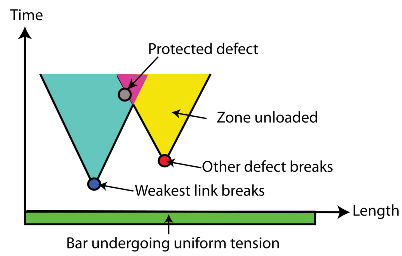
$$N(m) = (1 + (m/\mu)^{\beta})e^{-(m/\mu)^{\beta}}$$

- Numerics: $\beta \simeq 2.2$
- Grady and Kipp: $\beta = 1$
- Mott and Linfoot: $\beta = 1/2$

S. Levy, J.- . Molinari, I. Vicari, A. Davison. *Dynamic Fragmentation of a Ring: Predictable Fragment Mass Distributions*. Physical Review E. **82**(6),066105. (2010)

Secondary waves effect: Obscuration zone hypothesis?

- Defect protected if released by stress wave : Mott's assumption
- Simulations question validity of this assumption
  - when a defect "sees" a stress drop, it becomes unbreakable
  - for low strain rates (quasi-static): multiple wave passing
  - under-estimation of number of fragments
  - over-estimation of large



fragment sizes

C. Denoual, G. Barbier, F. Hild. *A Probabilistic Approach for Fragmentation of Brittle Materials under Dynamic Loading*. Comptes rendus de l'Acad\'emie des sciences. S\'erie IIb, M\'ecanique. **325**(12),685. (1997)

P. Forquin, F. Hild. *A Probabilistic Damage Model of the Dynamic Fragmentation Process in Brittle Materials*.

M. Chambart, S. Levy, J. Molinari. *How the Obscuration-Zone Hypothesis Affects Fragmentation: Illustration with the Cohesive-Element Method*. International Journal of Fracture. **171**(2),125-137. (2011)

Wrap-up

- Analytical models predict trends for various strain rates
  - Strain rates competing with wave propagations
  - Energy balance $E^{kin} \& E^{pot} \equiv Fracture\ energy$
  - Need material randomness (σ_c distributions) for low strain rates
  - Fragment distributions depend on material properties (brittle-ductile transition)
- Numerical approaches can bring
  - fragment interaction (contact)
  - complex geometries (in principle)
  - Relyable statistic: need lots of fragments and lots of elements per fragments....

How should we model fracture ?

Bibliography

P. Rosin, E. Rammler. *The Laws Governing the Fineness of Powdered Coal*. Journal of the Institute of Fuel. **7**,29-36. (1933)

C. Lienau. *Random Fracture of a Brittle Solid*. Journal of the Franklin Institute. **221**(6),769-787. (1936) 10.1016/S0016-0032(36)90526-4

N. Mott, E. Linfoot. *A Theory of Fragmentation*. 10.1007/978-3-540-27145-1_9

D. Grady. *Local Inertial Effects in Dynamic Fragmentation*. Journal of Applied Physics. **53**(1),322-325. (1982) 10.1063/1.329934

D. Turcotte. *Fractals and Fragmentation*. Journal of Geophysical Research: Solid Earth. **91**(B2),1921-1926. (1986) 10.1029/JB091iB02p01921

A. Chudnovsky, B. Kunin. *A Probabilistic Model of Brittle Crack Formation*. Journal of Applied Physics. **62**(10),4124-4129. (1987) 10.1063/1.339128

D. Grady. *Particle Size Statistics in Dynamic Fragmentation*. Journal of Applied Physics. **68**(12),6099-6105. (1990) 10.1063/1.347188

C. Denoual, G. Barbier, F. Hild. *A Probabilistic Approach for Fragmentation of Brittle Materials under Dynamic Loading*. Comptes rendus de l'Acad\'emie des sciences. S\'erie IIb, M\'ecanique. **325**(12),685. (1997) 10.1016/S1251-8069(97)82333-0

D. Grady, N. Mott. *Fragmentation of Rings and Shells: The Legacy of N. F. Mott*. (Springer, 2006).

F. Zhou, J.-F. Molinari, K. Ramesh. *Analysis of the Brittle Fragmentation of an Expanding Ring*. Computational Materials Science. **37**(1),74-85. (2006) 10.1016/j.commatsci.2005.12.017

P. Forquin, F. Hild. *A Probabilistic Damage Model of the Dynamic Fragmentation Process in Brittle Materials*. 10.1016/S0065-2156(10)44001-6

S. Levy, J.- . Molinari, I. Vicari, A. Davison. *Dynamic Fragmentation of a Ring: Predictable Fragment Mass Distributions*. Physical Review E. **82**(6),066105. (2010) 10.1103/PhysRevE.82.066105

S. Levy. *Exploring the Physics behind Dynamic Fragmentation through Parallel Simulations*. 10.5075/epfl-thesis-4898

M. Chambart, S. Levy, J. Molinari. *How the Obscuration-Zone Hypothesis Affects Fragmentation: Illustration with the Cohesive-Element Method*. International Journal of Fracture. **171**(2),125-137. (2011) 10.1007/s10704-011-9631-9