Cecilie, Lecture Friday 4 Oct 2019 ①

Blatt & Weisskopf (1952) pp. 386-389

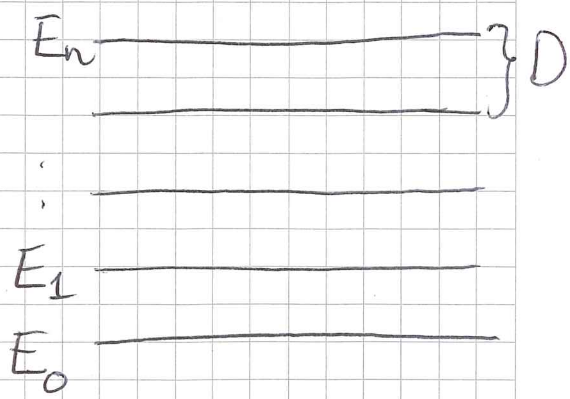
Highly excited levels

Assume equal spacing $D$:

Compound states that, below the neutron threshold, are approximately stationary and involve many nucleons

Have $N$ states, and the energy $E_n$ of state $n$ is given by

$$E_n = E_0 + nD$$




NOTE: the $N$ states ~~show~~ are considered to be of the same class, i.e. same spin & parity

$$E_1 = E_0 + D$$
$$E_2 = E_0 + 2D$$
$$\vdots$$
$$E_n = E_0 + nD$$

Linear combination of the N states, space dependence given by $\varphi_n$; gives the following wave function:

$$\Psi(t) = \sum_{n=1}^{N} a_n \varphi_n \exp\left(-\frac{iE_n t}{\hbar}\right)$$

Inserting $E_n = E_0 + nD$ :

$$\Rightarrow \Psi(t) = \sum_{n=1}^{N} a_n \varphi_n \exp\left(-\frac{i[E_0 + nD]\cdot t}{\hbar}\right)$$

$$= \exp\left(-\frac{iE_0 t}{\hbar}\right) \sum_{n=1}^{N} a_n \varphi_n \exp\left(-\frac{inDt}{\hbar}\right)$$

The wave function describes the same configuration at time $t$ as it does at time $t + \frac{2\pi\hbar}{D}$

$$\Rightarrow |\Psi[t + (2\pi\hbar/D)]|^2 = |\Psi(t)|^2$$

so that the period of motion is

$$\boxed{P = \frac{2\pi\hbar}{D}}$$

NOTE: holds only strictly for equally spaced levels

In reality, we work with an <u>average</u> level spacing $D$ (not necessarily equidistant)

The <u>key feature</u>: The period $P$ is <u>very large</u>

→ much larger than the period of a one-body motion in a nuclear potential well

→ this is due to the nuclear interaction involving now <u>many</u> nucleons, so that the time interval between the recurrences of the <u>same configuration</u> becomes very long

Now we consider decaying states, which in all essence are similar to the stationary ones

→ the decay is represented by a small perturbation to the stationary Hamiltonian

Let us look at the recurrence of a specific configuration:

- a decaying state is created by a particle a entering into the residual nucleus through the channel $\alpha$
- we ask: after what time will the particle a reappear at the nuclear surface, with the rest of the nucleus arranged in such a way that a can leave the nucleus in the same channel by which it came in

<u>However</u>, a will likely <u>not</u> leave the nucleus:

- the nuclear surface is equivalent to a strong and sudden change of potential
- the particle is very likely <u>reflected</u> back into the nucleus

$$\boxed{\underbrace{\text{Reflection}}_{\text{large}} + \underbrace{\text{Transmission}}_{\text{small}} = 1}$$

$\Rightarrow$ $\boxed{\text{Repetition of motion}}$

↳ the particle will try over and over again to escape

↳ <u>essential</u> for the existence of well-defined compound states:

$\Gamma \ll D$

⇒ Because of the reflection, the lifetime $\tau_\alpha$ is <u>longer</u> than $P$

Define the transmission coefficient toward the outside (escaping the nucleus):

$$T_\alpha = \frac{\#\ \text{successful escapes through ch.}\,\alpha}{\#\ \text{attempts to escape}}$$

⇒ This gives us

$$\tau_\alpha \sim \frac{P}{T_\alpha}$$

and from $\Gamma_\alpha \stackrel{def}{=} \frac{\hbar}{\tau_\alpha}$ :

$$\boxed{\Gamma_\alpha \sim \frac{T_\alpha}{P} \sim T_\alpha \cdot \frac{D}{2\pi}}$$

so, $\Gamma_\alpha \ll \frac{D}{2\pi}$ because $T_\alpha \ll 1$

Now, we remember the definition of the $\gamma$-strength function:

$$f = \frac{\langle \Gamma_\gamma(E_\gamma) \rangle}{E_\gamma^3 D}$$

and

$$T_\gamma = \frac{\langle \Gamma_\gamma(E_\gamma) \rangle}{D} \cdot 2\pi$$

$$\Rightarrow \boxed{T(E_\gamma) = f \cdot E_\gamma^3 \cdot 2\pi}$$