Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
A		2222354

The Power Curve of Cyclists Based on the Differential Equation Model

Summary

Cycling road time trial is a common sports event, and based on the characteristics of the event itself, it not only has high requirements on the overall physical quality of the athletes, but also the strategy during the competition. In order to provide better guiding suggestions for cyclists, this article starts from the power curve of cyclists to show a scientific guide for athletes.

Before starting the experiment, we **fitted and regression** analyzed a large amount of power data to simulate the rider's output power and thresholds over different time periods. We use **differential equation models** to predict the power values that athletes should use at the corresponding position during the race.

First, to simplify the model, we set the same equipment for all athletes.

Second, we selected three roadmaps with their own styles: the **2021 Olympic Time Trial course in Tokyo, Japan**; the **2021 UCI World Championship time trial course in Flanders, Belgium** and our own designed roadmap. Based on data on athletes with two different power curves riding on the track, we found that due to the strength of the time trial specialists in **aerobic endurance**, they consistently performed well on multiple tracks.

Then we collected 15 days of weather data for Qinghai Lake. Based on this, we found that athletes can adjust the riding posture and position in the team according to the corresponding wind direction and wind intensity, thereby reducing the impact of changes in **weather conditions** on the **power curve**.

At the same time, we also completed the sensitivity analysis of the model to explore the robustness of the model for certain parameters.

Fourth, we expanded the model to multiple riders on the same team to explore how to distribute the output power of the riders in the same stage. The results showed that in general, the stable runners lead the ride, ensuring that the climbers and sprinters can maintain good power and in the climbing stage led by the climbers and other runners.

Keywords: Cyclist; Power Curve; Differential Equation Model; Sensitivity Analysis

Contents

1 Introduction

1.1 Problem Background

As our life becoming more and more abundant, different ideas of bicycle road race emerged, so there are so many types of races, and a single cyclist in different length of time will produce different levels of power output, a power curve [1] can display the maximum power of the cyclist can produce in a fixed period of time, and the more power produced, the less time a cyclist has to maintain that power. And as the race goes on, the athlete loses more and more strength, so the power output of the athlete in the past is important.

Chance of winning may also vary in various types of competitions, such as the criterium [2], a team time trial, and an individual time trial. Cyclists can participate in different races according to their power curves, and they can also adjust their power curves according to different race types.

(a) An Individual Time Trial [3]

(b) A Team Time Trial [4]

1.2 Restatement of the Problem

According to the given information, we are required to design a mathematical model that can determine the relation between the rider's position and power applied. Besides, we should also consider the limit on the total power and the power curve. And the model should include the following:

1) Use the model to define the power curve of two different types of riders. One is for a time trial specialist, and the other is for a rider of different type. And the model should also consider the different personal conditions of different genders.

2) Apply the model to different types of time trial courses. One is for the **2021 Olympic Time Trial course in Tokyo, Japan**, one is for the **2021 UCI World Championship time trial course in Flanders, Belgium,** and the other is the course designed by ourselves.

3) Consider the influence of weather conditions, such as the direction and strength of wind, find the sensitivity of the results for small differences caused by the weather and environment.

4) Find the sensitivity of the results for rider deviations from the target power profile.

5) Analysis how to extend the model to apply to a team time trial of six riders per team for the optimal power use, and the time of the team is determined by the fourth rider crossing the finish line.

6) Two pages of rider's race guidance, which is easy to understand for the one who have no knowledge in mathematics.

1.3 Our Work

To address this, where the rider is on the track and how much power he should exert at that position is the focus of our attention.

Task 1: Prediction Class **Differential Equation Model**.

We decided to build a predictive model to guide the riders to adjust the output power reasonably according to factors such as geographical location and their own conditions during the race.

According to physics and knowledge about **differential equation models**, the expression of distance and time, the expression of power and time is established, and the unknown variables are gradually eliminated, so as to achieve the purpose of establishing the relationship between position and power.

Task 2: Power-Time Curve.

Analyze the relationship between power and time to explore the range and amplitude of power fluctuations when riding. We used the large amount of data collected by the network to fit and regression analyze it to determine the power time curve.

Task 3: Make long-term measurements of fixed routes to study the effects of changes in weather fluctuations.

The model established earlier was applied to collect the wind direction and wind intensity data of Qinghai Lake for a period of 15 days, and the degree of impact of the model under this factor was predicted.

Task 4: In order to ensure that athletes' power remains within the team's expected range of power even if they deviate from the target, we explore the maximum range that athletes can deviate from in key parts of the track.

2 Assumptions and Explanations

The balance between power supply and consumption needs to be ensured by making assumptions about the resistance encountered during riding and determining the relationship between power consumption and speed in order to overcome the resistance.

- **Assumption 1: All the routes in road cycling have the same terrain.**

  Explanation: In order to ensure that the rider's power consumption is not affected by changes in the terrain during the ride.

- **Assumption 2: A cyclist's performance affected by psychological or mental conditions**

  Explanation: If a cyclist's performance is affected by psychological and mental conditions, there is no guarantee that all the power stored in his body can be used for the race

3 Notations

Important notations used in this paper are listed below:

Symbol	Definition	Unit
$f(t)$	Momentum exerted by the rider	N
F	The biggest momentum	N
k	Momentum limiting coefficient	/
t	Time	s
τ^{-1}	The ratio factor of in vivo and in vitro resistance to velocity	/
ω	The power provided per unit time	/
E_0	Total power stored in the body	kj
r	The resistance in vivo and in vitro	N
m	Unit mass	kg
$\dot{E}(t)$	The rate of change of the power stored in the body	kj
$s(t)$	Expression of distance associated with time	km

Table 3.1: Notations

4 Differential equation model

4.1 Model's Description

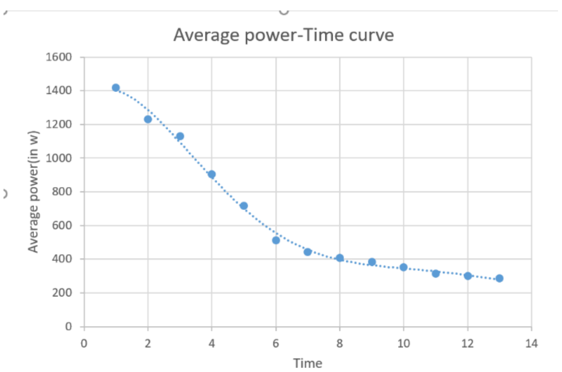
According to the relation between power and speed $P = F \times v$, we introduce time parameter t to match the relation between power and speed with the position of the rider, then the power that the rider should take at each position can be obtained.

From the power data collected from the network, we divided the time into four intervals: 10s,5min,2h, and more than 2h. At the same time, by **fitting and regression analysis** the power data, we find the power and time curve.

Time\male	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5	Ave
1 s	1111	1218	1393	1623	1749	1418.8
5 s	995	1091	1202	1344	1529	1232.2
10s	913	991	1113	1240	1385	1128.4
30 s	707	991	831	947	1040	903.2
1 min	580	766	677	744	820	717.4
5 min	432	617	472	503	531	511
10 min	399	450	435	455	481	444
20 min	369	387	403	426	453	407.6

30 min	347	361	384	406	427	385
60 min	310	329	350	368	398	351
120 min	282	296	312	330	355	315
180 min	266	281	297	315	338	299.4
240 min	252	268	284	298	325	285.4

Table 4.1: The data of the power curve [5]



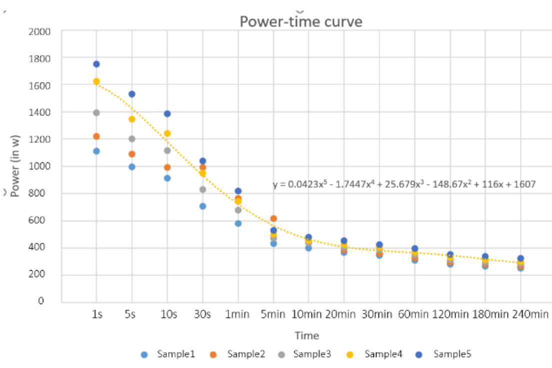


Table 4.2: Average power curve

Table 4.3: Power curve

4.2 Analysis of physical principles

Through the physical calculation of speed and distance, thus related to the relationship of the position of the rider on the track and the power.

Athletes overcome resistance to achieve or maintain a certain speed in the process of riding, it needs to consume a certain amount of power to complete. The power consumed comes from two sources. One is the power produced by the respiratory and circulatory systems through the metabolism of oxygen. The second is by eating and drinking before the race.

In the process of riding, due to the limitations of their own physiological conditions (total stored power is limited), internal and external resistance and other factors, the speed will change as time goes on. In the process of riding, the greater the momentum provided by the rider's body, the less resistance from inside and outside the body, and the speed also increases. The total power stored in the human body will continue to be consumed as time goes on, so it can be seen that in the process of riding, the momentum provided by the rider will continue to decrease, while the internal and external resistance increases, and the speed gradually decreases.

1. The internal and external resistance is directly proportional to the speed, and the proportional coefficient is τ^{-1}, the maximum momentum is F, and the initial speed is 0.

2. The power that provided per unit time by the respiratory and circulatory system through oxygen metabolism is a constant ω, and the total power stored in the body is Eo.

3. In order to ensure that the model is applicable to any type of athlete and eliminate some factors that vary from person to person, especially the weight factor of the driver, we make a model for the unit mass of the driver, assuming that the mass of the following types is m = 1kg.

4.3 Operation

From the condition we can know:

$$\frac{f(t)'}{f(t)} = -\frac{1}{k}$$

Integrate both sides of this equation with respect to the independent variable t:

$$f(t) = c_1 e^{-\frac{t}{K}} (c_1 \text{ is an arbitrary constant})$$

Because of $f(0) = F$, solve for the constant c_1:

$$f(t) = Fe^{-\frac{t}{K}}$$

According to Newton's Second Theorem of Motion [6]: $F = ma = m\frac{dv}{dt}$,

$$m\frac{dv}{dt} = f - r$$

$\because r = \frac{1}{\tau}V$， $f(t) = Fe^{-\frac{t}{K}}$

$$m\frac{dv}{dt} = Fe^{-\frac{t}{k}} - \frac{v}{\tau}$$

Integrate both sides with respect to t:

$$mv(t) = c_2 e^{-\frac{t}{\tau}} + \frac{Fk\tau}{k-\tau} e^{-\frac{t}{k}}$$

Substitute into v(0) = 0:

$$v(t) = \frac{Fk\tau}{(k-\tau)m}\left(e^{-\frac{t}{k}} - e^{-\frac{t}{\tau}}\right)$$

Because of $v = \frac{ds}{dt}$,

$$s(t) = \frac{FK\tau}{(K-\tau)m}\left(-ke^{-\frac{t}{R}} + \tau e^{\frac{-t}{\tau}}\right) + c_3$$

Substitute into $S(0) = 0$.

$$s(t) = \frac{Fk\tau}{(k-\tau)m}\left(-\tau + k - ke^{-\frac{t}{k}} + \tau e^{\frac{-t}{\tau}}\right)$$

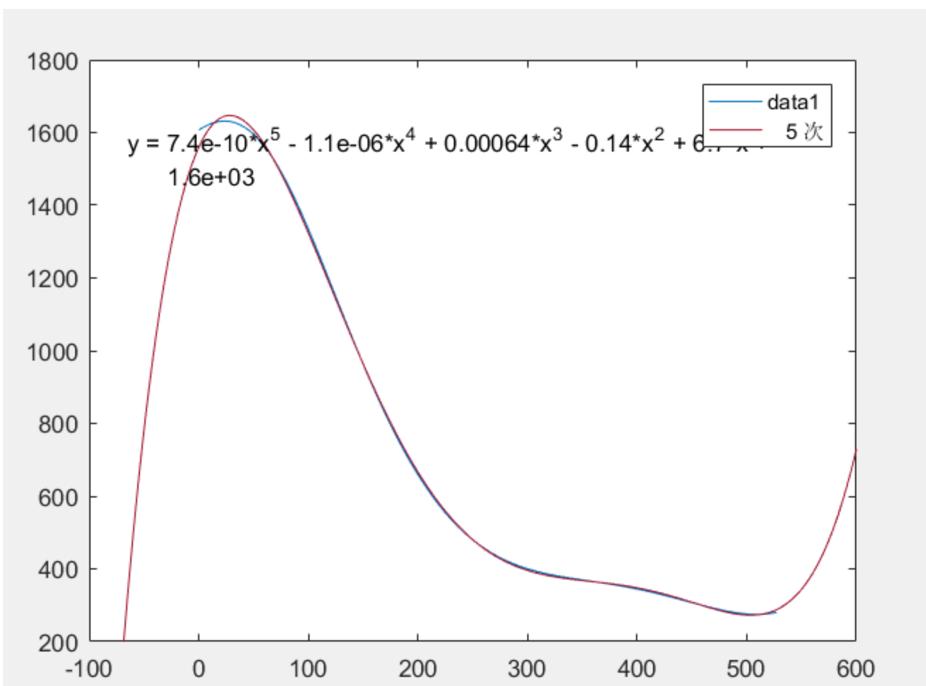


Table 4.4: Power-Distance curve

4.4 Curve of Recovery time and Threshold exceeded time

Since the rider can choose to go over the limit on the power curve for a short time, but the rider needs additional time to recover at lower power levels, we will set up a graph of the time over the limit versus the recovery time curve.

5 Two types of the power curve

5.1 Preparation

In order to accurately define the power curve of two types of cyclist, and one of them is for the time trial, another under the consideration of different sexes, we collected a lot data about the power of professional athletes in each time interval number, according to the conversion relation of power and power, then the reasonable and real power curve is obtained.

5.2 The power curve of a time trial specialist

The strength of aerobic endurance is an important factor in determining whether a time trial athlete can achieve excellent results. The aerobic endurance of time trial athletes is reflected in maintaining a high heart rate for a long time. There is training in this area of professional training: the second stage of the aerobic endurance zone, which is 55%~75% FTP intensity, is maintained in the middle range of the aerobic range training. The fifth stage of VO2 intensity indicates that riders should not train more than 120% FTP and ensure that lactic acid does not exceed the level of anaerobic threshold, so that interval training can help increase VO2 max.

For time trial athletes, mastering the rhythm of the game and controlling the consumption of power are the prerequisites for ensuring excellent results. Acceleration is fast at the start of the race, but not more than the aerobic range, and the second half is further accelerated, slightly higher than the speed of the anaerobic threshold, and ensures that after riding for three kilometers, the output power reaches a stable state.

5.3 The power curve of female athlete

Women in hypoxic environments, due to the less efficient redistribution of blood flow to active tissues and/or lower hemoglobin carrying capacity than male athletes, rely more on partial oxygen extraction, resulting in fewer oxygen extraction reserves than men, manifested by an increased rate of decreased muscle oxygen saturation at an early stage of constant power testing and maintained at low levels.

Time\female	Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
5 s	629	783	856	932	1017
10s	568	711	791	865	917
30 s	435	515	592	630	684
1 min	363	422	460	523	544
5 min	280	299	321	341	370
10 min	245	273	290	311	336
20 min	234	251	273	294	305
30 min	214	236	259	273	292
60 min	201	215	238	253	266

Table 5.1: The data about power curve of female athlete [7]

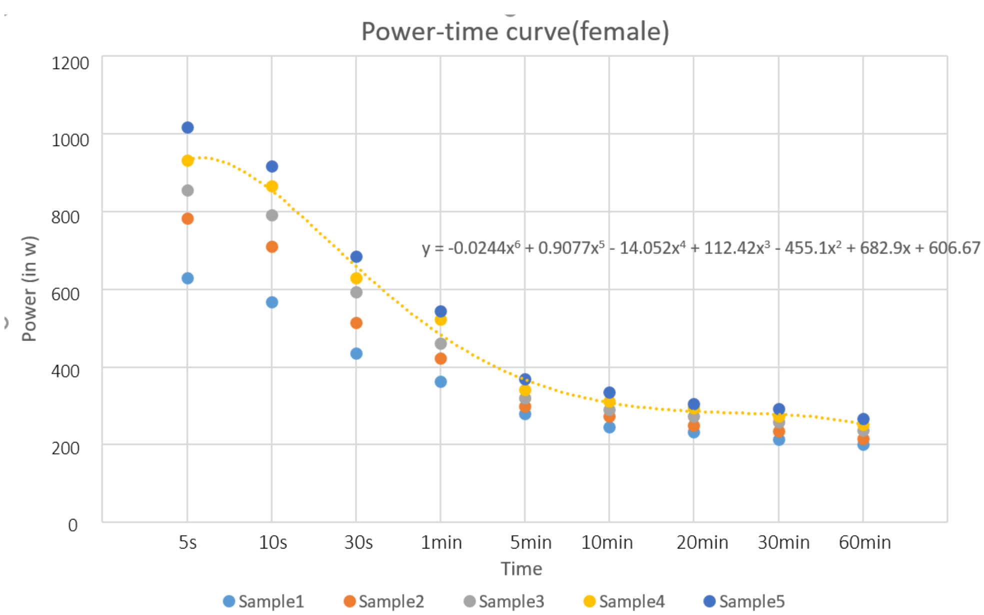


Table 5.2: The power-time curve of female

6 The Application of our model

We plan to select three types of time trial routes to ensure that the power curves are adaptable and visible. Their route characteristics are listed below. All the data about time trials are collected from the race authorities and are authentic.

6.1 2021 Olympic Time Trial course in Tokyo, Japan

The men's race will take place over a 22.1kilometer course with two laps, while the women's race will take place over a single lap. Road cycling's home base is in Fuji International Circuit at the foot of Mount Fuji, southwest of Tokyo. Individual time trials start and finish on the track, but riders need to travel through the nearby countryside.

Table 6.1: Route slope map of the course [8]

6.2 2021 UCI World Championship time trial course in Flanders, Belgium

The individual time trial started in Knokke-Heist, a small city near the Dutch border in the far north of Belgium, and finished in Bruges, a major city in the country's north.

Unlike the complex, winding course with lots of ups and downs and climbs in the majors, the individual time trial course at The World Championships has few technical difficulties and is basically flat. The only possible source of uncertainty and difficulty is the wind on the coastline.

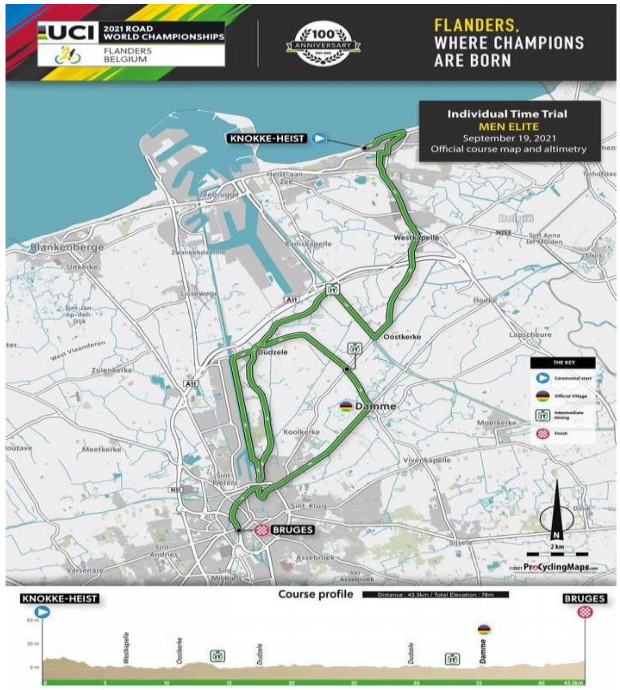


Table 6.2: Slope map of UCI World Championship route

6.3 A course designed by ourselves

This course has four sharp turns, the first turn is downhill, so it will be easier, while t he second, third and fourth sharp turns are uphill, so we need to use more power to pass, but if we use too much power there, it would betoo late to turn or even wrestling, so we s hould control the strength when turning. Apart from these four sharp turns, the rest of the road is quite smooth, so the difficulty of this circuit is the four sharp turns.

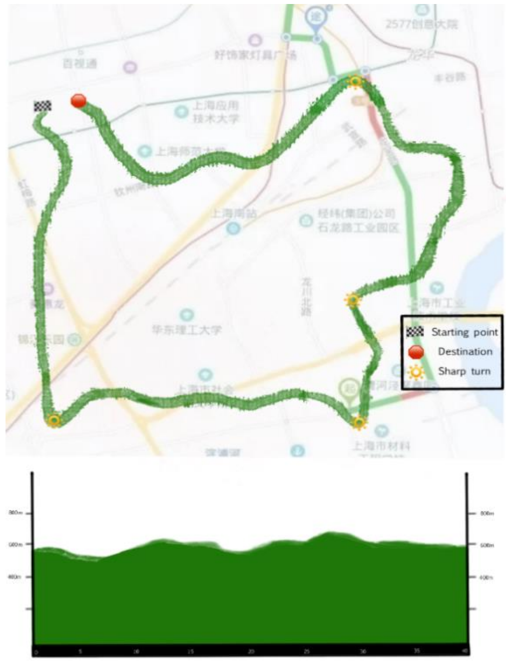


Table 6.3: A course designed by ourselves

7 Weather sensitivity analysis

7.1 The effects of weather conditions on the human body

According to the research papers on meteorological conditions, when the nerve receptors of the human body receive the stimulation from the environment due to weather changes, they will cause a series of extremely complex physiological reactions and physiological adjustment in the body, thus adjusting the balance between the human body and the environment.

7.2 The influence of weather conditions on the performance of riders

The best race temperature conditions for road cycling are 15 to 20 degrees Celsius and humidity is 50 to 60 percent. When the temperature and humidity are in the ideal environment, the human body feels comfortable, the rider will play more stable. But when the temperature and humidity exceed their appropriate range, for example, if the temperature and humidity are too high, the human body may be difficult to evaporate sweat, easy to produce heat stroke.

If the temperature and humidity are too low, the body's water will evaporate a lot, and the evaporative heat dissipation takes away a lot of heat from the body, so the body is not enough to support the race. To prevent this, riders will change into clothes and pants suitable for the current temperature and humidity before the race.

As a speed movement, how to overcome air resistance is of great significance for improving speed. According to the fluid dynamics and the characteristics of the bicycle itself, more than 95% of the movement of the bicycle comes from the air resistance, and maintaining the correct riding posture can greatly reduce the wind resistance, so that the rider and the bicycle become "streamlined."

Table 7.1: A chart of air flow trends during cycling [9]

For example, in the seventh stage of the 2021 Tour de France, the riders' association from Vierzon to La Creusot,, the most prominent weather feature of this stage is the large altitude climb gap. The stage is 228 kilometers long, and the accumulated altitude climb is 3000m. There is a huge difference between the starting point and the ending point of the wind, temperature and humidity.

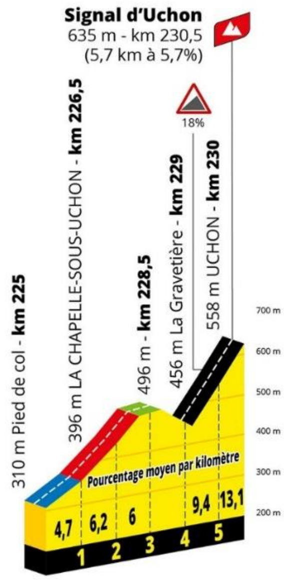


Table 7.2: Elevation plan of race route [10]

On the other hand, different weather will have a certain impact on the sight of riders, riders will choose appropriate equipment according to different weather. For example, athletes can change the color of cycling glasses lenses according to sunny, rainy and cloudy conditions.

At the same time, according to the real-time wind strength, that is, the size of the wind speed, adjust the transmission ratio on the bicycle, in order to maximize the speed in the race. The gear ratio of a bicycle is the ratio of the number of teeth of the bicycle sprocket to the flywheel.

The front gear of the bicycle is the active gear (the number of teeth is a), and the rear gear is the passive gear (its number of teeth is b), then the front and rear gear ratios are g = c/f. On the bicycle, the large and small gears are connected by chains, so the linear velocity of the two is the same. When the rear wheel and the pinion rotate on the same axis, the angular velocity between the two wheels is the same. With the rider's momentum F^' brought to the bike with each pedal, the rider transmits the momentum through the crank into the tension of the chain F^', which is transmitted through the chain to the flywheel, thus driving the rear wheel forward.

The formula for power in **the chain drive system**:$P = \frac{vF}{1000}$ (when the output power is unchanged, the tension is inversely proportional to the chain speed)

Table 7.3: Road bicycle of the Cercelo [11]

When riding, the rider adapts to the changes in the competition environment to determine the appropriate transmission ratio, and transmits the power of the pedal to the rear wheel through the transmission ratio to obtain the best power conversion benefit. For example, when climbing a hill, the rider will choose to reduce the number of teeth of the sprocket, increase the gear ratio, and thus reduce the output of force.

From the above information, we need to discuss the effect on power P when there is a small change in parameter r.

To measure the sensitivity of the result to the parameters using the relative amount of change, we record the sensitivity of P to r as $S(P,r)$, defined as $S(P,r) = \frac{\frac{\Delta P}{P}}{\frac{\Delta r}{r}} \approx \frac{dP}{dr} * \frac{r}{P}$.

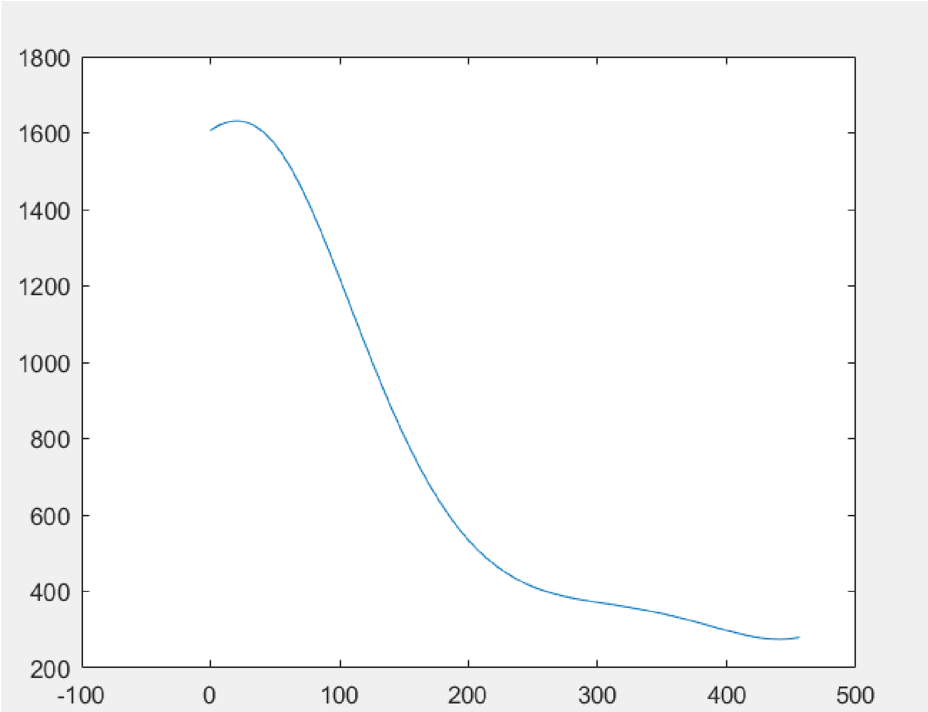


Table 7.4: The weather sensitivity analysis

Data\Date	3.15	3.16	3.17	3.18	3.19
The directin of the wind	Southeast	East	West	Southwest	West
The level of the wind	Level 2	Level 2	Level 3	Level 2	Level 2
Data\Date	3.2	3.21	3.22	3.23	3.24
The directin of the wind	West	West	West	West	Southwest
The level of the wind	Level 3	Level 3	Level 3	Level 2	Level 1
Data\Date	3.25	3.26	3.27	3.28	3.29
The directin of the wind	West	West	West	West	West
The level of the wind	Level 2	Level 3	Level 3	Level 3	Level 2

Table 7.5: The weather data of Qinghai Lake [12]

8 The Sensitivity of Cyclists to Target Deviation

Based on the information we have gathered, there may be several factors for athletes to deviate from the target power distribution:

- Before the game, the power stored in the body is absorbed through diet, etc.

- Power fluctuations caused by their own physiological conditions, for example, during the race, a car crash occurs, and the human body and the body receive certain damage.

- During the race, the environment and weather changed suddenly.

9 Further Extension

9.1 Rules of a contest

There are six drivers in a team. According to the rules of the game, each team sent 4 athletes to participate in, between each team two to three minutes apart from the start, in the process of the game, the athletes arrange in formation according to the wind direction, the strength of the wind and so on, each take turns to lead about 200m, exchange positions to lead riding.

9.2 Optimal allocation of power use

According to the model, the leader should be expert of the time trial at the beginning of the race, to guarantee the smooth start and the stability of team cycling, make a rotation in the midway, according to the priorities of uphill slope and curve, real-time schedule climbing hand or other contestants pace, in the sprint phase, we should pay attention to their body power consumption level, arrange the person to prepare for the sprint, Also pay attention to the turnover of other teams.

10 A Rider's r Race Guide for Directeur Sportif

Individual time trial requires an aerodynamic rider to ride alone for a certain distance. The finish time of the athlete shall be regarded as the competition result. Tempo and resistance play a role in the race, as do pre-race resupply and training. By reading this guide to the race, you can have a better plan and arrangement for the cyclists, so as to achieve better results.

✧ Resistance:

According to our research, resistance during the riding time plays a part in race performance, so aerodynamics plays a crucial role in timing testing. The rider's own body is responsible for most of the drag, and a more flexible position allows the rider to ride faster with little effort.

Therefore, minimize the front windward area, flatten your back, and tuck your arms and elbows in. Make sure you don't set it too low to maintain your posture.

✧ Rhythm of riding:

The model showed that, after starting, leaving the cushion and sprinting within 10 seconds, the athlete would reach a fairly high speed. "Then return to the seat cushion and strike a timing pose. It's easy to get adrenaline going on race day, so avoid starting too fast. According to our model, when cyclists ride at full speed for a short time within 1min, the cycling time increases and the resistance mode they choose gradually increases, which is beneficial for athletes to create the best performance. So try to keep your sprints within a minute.

One method that is tried and trusted is called Negative Split. This involves the athlete riding more conservatively in the first half of the race and putting in more effort in the second half. This can be done digitally using a power meter or heart rate monitor to reach the athlete's functional threshold. Alternatively, you can choose to pace for the time trial purely by feeling. This is achieved by starting at a moderate pace and gradually increasing the athlete's effort so that by the time the athlete reaches the final quarter of the race, the athlete's strength is at its maximum. There is a feeling of wanting to stop and a need for mental fortitude to reach the finish line.

And for the sharp turns, it is recommended to ride along the right side of the road for some distance before entering the turn and then accelerate the turn to ensure the speed and reduce the risk.

✧ Supply before the race:

Many cyclists gorge on pasta in the days leading up to a big race. While consuming carbohydrates in this way ensures an athlete's glycogen stores are adequate, moderation is the best strategy when it comes to time trials.

For shorter races, such as 16km or 40km time trials, a carbohydrate-rich meal such as pasta or rice before the race should suffice.

On race morning, focus again on carbs. A bowl of oats and a banana is ideal. On your way to the time trial, have a cup of coffee 45 minutes early to refresh yourself, and be sure to drink plenty of fluids before starting the race.

✧ Training before the race:

Time trials are about strength and resistance. Although more aerodynamics can help athletes achieve the latter, there is still a need to raise the functional threshold to achieve the best performance, which means training for this ability.

When training, use timing bikes and posture to minimize the impact of drag. At the same time, try to keep the upper body still, which will help the athlete improve aerodynamics and save power.

Having a strong core will make this job easier, so make sure your athletes' outdoor workouts include plenty of lunges, planks and sit-ups, and don't forget to stretch regularly. Stretching the gluteus maximus, hamstrings and lower back gives the athlete more flexibility to better maintain this demanding but crucial low riding position.

References

[1] Power Curve: The most powerful visual performance a driver can maintain for a specific length of time.

[2] Criterium: A bicycle race on an enclosed track. The length can be specified by a fixed number of turns or the maximum number of turns in a predetermined time period.

[3] Source: https://tse1-mm.cn.bing.net/th/id/

[4] Source: https://tse1-mm.cn.bing.net/th/id/R-C

[5] Source: https://zhuanlan.zhihu.com/p/448008831

[6] Newton's second law of Motion: It states that the acceleration of a body is directly proportional to the net force on the body and inversely proportional to its mass.

[7] Source: https://zhuanlan.zhihu.com/p/448008831

[8] Source: https://www.biketo.com/racing/47976.html?all=1

[9] Source: https://zhuanlan.zhihu.com/p/71297258

[10] Source: https://www.wildto.com/news/51472.html

[11] Source: https://www.cervelo.com/en/

[12] Source: https://zhidao.baidu.com/question/1829353116726535242.html

Appendices

```
x=[0:0.01:14]

y=0.0423*x.^5-1.7447*x.^4+25.679*x.^3-148.67*x.^2+110*x+1535;

plot(x,y),title('Weather sensitivity analysis')

grid on

axis([0 14 0 1800]);

set(gca,'ytick',[0 200 400 600 800 1000 1200 1400 1600 1800])



t=[0:0.01:14];

s=-(1000.206)*(-1.0205+40-40*exp(t/40)+1.0205*exp((-t)/1.0205))/(40-1.0205);

y=0.0423*t.^5-1.7447*t.^4+25.679*t.^3-148.67*t.^2+116*t+1607;

plot(s,y)
```