MODEL QUESTION PAPER - 02

PART - A

1) Let p and q be primitive statements for which the conditional $p \to q$ is false. Determine the truth values of the following compound propositions.

(i) $p \wedge q$ (ii) $q \vee (\sim p)$

(i) $p \wedge q$

p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

(ii) $q \vee (\sim p)$

p	q	$\sim p$	$q \vee (\sim p)$
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

2) Consider the following propositions concerned with a triangle ABC.

p: ABC is isosceles q: ABC is equilateral r: ABC is equiangular.

Write down the following propositions in words.

(i) $p \wedge (\sim q)$ (ii) $p \to q$

(i) ABC is isosceles and not equilateral.

(ii) If ABC is isosceles then it is equilateral.

3) Write the formula for Mean, variance and standard deviation for Poisson distribution.

Mean $\Rightarrow \mu = m$.

variance $\Rightarrow V = m$

standard deviation $\Rightarrow SD = \sqrt{m}$.

4) Define Discrete Random variable and continuous Random variable.

Discrete Random variable: A random variable that takes finite or countable number of values for assigning sample point is called discrete random variable.

continuous random variable : A random variable that takes infinite number of values for assigning sample point is called continuous random variable.

5) Define Irreducible Markov chain.

A markov chain is said to be irreducible if its transition probability matrix is the regular stochastic matrix.

6) The transition matrix P of a markov chain is given by $P = \begin{bmatrix} 1/2 & 1/2 \\ 3/4 & 1/4 \end{bmatrix}$ then find i) $p_{21}^{(2)}$ ii) $p_{12}^{(2)}$

$$P^2 = \begin{bmatrix} 5/8 & 3/8 \\ 9/16 & 7/16 \end{bmatrix}$$

(i) $p_{21}^{(2)} = 9/16$ (ii) $p_{12}^{(2)} = 3/8$

7) Write the formula for rank correlation with repeated rank.

$$\rho = 1 - \frac{6}{n^3-n}\left[\Sigma d_i^2 + \frac{(m_1^3 - m_1)}{12} + \frac{(m_2^3 - m_2)}{12} + \frac{(m_3^3 - m_3)}{12} + \cdots\right]$$

8) If θ is the angle between lines of regression and $\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$ then Explain the significance when $r=0$ and $r=\pm 1$

→ when $r = 0 \Rightarrow \tan\theta = \infty$

$$\theta = \pi/2$$

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x}) \qquad x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$m_1 = r\frac{\sigma_y}{\sigma_x} \qquad m_2 = \frac{1}{r}\frac{\sigma_y}{\sigma_x}.$$

$$\tan\theta = \frac{\frac{1}{r}\frac{\sigma_y}{\sigma_x} - r\frac{\sigma_y}{\sigma_x}}{1 + \frac{1}{r}\frac{\sigma_y}{\sigma_x}\left(r\frac{\sigma_y}{\sigma_x}\right)}$$

$$\boxed{\tan\theta = \frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)}$$

//

9) Define the following

(i) Hypothesis : Hypothesis is decision making statement which is true or false.

(ii) Null Hypothesis : The hypothesis formulated for the purpose of its rejection is called NULL hypothesis.

(iii) Alternative Hypothesis : Any hypothesis which is not NULL is called alternative hypothesis.

10) Write the Test statistic formula for significance difference of two sample mean.

->
$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where $\bar{x}_1$ - mean of 1st sample
$\bar{x}_2$ - mean of 2nd sample
σ_1^2 - variance of 1st sample
σ_2^2 - variance of 2nd sample.
n_1 - size of 1st sample
n_2 - size of 2nd sample.

PART- B

1) a) Prove the following using the laws of logic :
$[Np \wedge (Nq \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$.

LHS

$[Np \wedge (Nq \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$

$[(Np \wedge Nq) \wedge r] \vee (q \wedge r) \vee (p \wedge r)$ (Associative)

$[N(p \vee q) \wedge r] \vee (q \wedge r) \vee (p \wedge r)$ (demorgan's)

$[N(p \vee q) \wedge r] \vee (q \vee p) \wedge r$ (distributive)

$[N(p \vee q) \vee (q \vee p)] \wedge r$ (distributive)

$[N(p \vee q) \vee (p \vee q)] \wedge r$ (commutative)

$\Rightarrow T \wedge r$ (Inverse)

$\Rightarrow r$ // (Identity)

5

1) b) Prove that the following argument is valid:

If a triangle has two equal sides, then it is isosceles.
If a triangle is isosceles, then it has two equal angles.
A certain triangle ABC does not have two equal angles.

∴ The triangle ABC does not have two equal sides.

→

S: be the set of all triangles

$P(x)$: x has two equal sides

$q(x)$: x is isosceles.

$r(x)$: x has two equal angles.

a: triangle ABC

$$\forall x \in S, \quad \begin{array}{l} P(x) \to q(x) \\ q(x) \to r(x) \\ \underline{\sim r(a)} \\ \sim P(a) \end{array}$$

$\forall S, [P(x) \to q(x)] \wedge [q(x) \to r(x)] \wedge \sim r(a)$

$P(a) \to q(a) \wedge q(a) \to r(a) \wedge \sim r(a)$ (Syllogism).

$P(a) \to r(a) \wedge \sim r(a)$

$\sim P(a)$ (modus tollens)

∴ It is valid.

1) c) Identify whether the following argument is valid:

$$\begin{array}{l} p \to q \\ r \to s \\ \underline{p \vee r} \\ \therefore q \vee s \end{array}$$

$\Rightarrow$ $(p \to q) \wedge (r \to s) \wedge (p \vee r)$

$(p \to q) \wedge (r \to s) \wedge (\sim(\sim p) \vee r)$

$(p \to q) \wedge (r \to s) \wedge (\sim p \to r)$ (negation of conditional)

$(p \to q) \wedge (\sim p \to r) \wedge (r \to s)$ (commutative)

$(p \to q) \wedge (\sim p \to s)$ (syllogism)

$(\sim q \to \sim p) \wedge (\sim p \to s)$ (contrapositive)

$(\sim q \to s)$ (negation of conditional)

$q \vee s$

∴ It is valid.

2) a) Prove the following $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$

p	q	r	$p \vee q$	$p \vee q \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$p \rightarrow r \wedge (q \rightarrow r)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	0	1	1	1	1	1	1
1	0	0	1	0	0	1	0
0	1	1	1	1	1	1	1
0	1	0	1	0	1	0	0
0	0	1	0	1	1	1	1
0	0	0	0	1	1	1	1

$\therefore [(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$

2) b) Prove that the following argument is not valid:

No engineering student is bad in studies.
Ram is not bad in studies.

$\therefore$ Ram is an engineering student.

s: be the set of all students.

$p(x)$: x is an engineering student

$q(x)$: x is bad in studies

a: Ram

$$\forall x \in s, \quad p(x) \rightarrow \sim q(x)$$
$$\frac{\sim q(a)}{p(a)}$$

$\forall s, [p(x) \rightarrow \sim q(x)] \wedge \sim q(a) \Rightarrow p(a)$

$\forall s, [p(a) \rightarrow \sim q(a)] \wedge \sim q(a)$

$\forall s, [\sim q(a) \rightarrow p(a)] \wedge \sim q(a)$ [converse]

$\forall s, \quad p(a)$ [modus pones]

$\therefore$ It is valid

2)c) check the validity of the following argument :

$(\sim p \vee \sim q) \rightarrow (r \wedge s)$

$r \rightarrow t$

$\sim t$

$\therefore p$

$[(\sim p \vee \sim q) \rightarrow (r \wedge s)] \wedge (r \rightarrow t) \wedge \sim t \Rightarrow p$

$[\sim p \vee \sim q] \rightarrow (r \wedge s) \wedge \sim r$ (modus tollens)

$[\sim (p \vee q) \rightarrow (r \wedge s)] \wedge (\sim r \vee \sim s)$

$[\sim (p \wedge q) \rightarrow (r \wedge s)] \wedge \sim (r \wedge s)$ (demorgan's)

$\sim [\sim (p \wedge q)]$

$p \wedge q$.

$\therefore$ It is not valid argument.

3)a) Find the constant K such that $f(x) = \begin{cases} Kx^2, & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$ is a p.d.f

also compute (i) $P(1 < x < 2)$ (ii) $P(x \leq 1)$

(iii) $P(x > 1)$ (iv) $p(x > 1)$, (v) $p(x > 2)$.

(i) $P(1 < x < 2) = \int_1^2 F(x)\, dx$

$F(x) = \begin{cases} \left(\frac{1}{9}\right) x^2 & 0 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$

$= \int_1^2 \frac{1}{9} x^2 dx$

$= \frac{1}{9} \left. \frac{x^3}{3} \right]_1^2$

$= \frac{1}{27} (8-1)$

$= \frac{7}{27}$.

(ii) $P(x \leq 1) = \int_{-\infty}^1 F(x) dx = \int_{-\infty}^0 \cancel{F(x)}^{0} dx + \int_0^1 F(x) dx$

$= \int_0^1 \frac{1}{9} x^2 dx$

$= \frac{1}{9} \left[\frac{x^3}{3} \right]_0^1$

$= \frac{1}{27} (1-0)$

$= \frac{1}{27}$.

(iii) $P(x>1) = \int_1^{\infty} f(x)\,dx$

$= \int_1^3 f(x)\,dx$

$= \int_1^3 \frac{x^2}{9}\,dx + \int_3^{\infty} \cancel{f(x)}^{0}\,dx$

$= \frac{1}{9} \frac{x^3}{3}\Big]_1^3$

$= \frac{1}{27}(27-1)$

$= \frac{26}{27}$ //

(iv) $P(x>2) = \int_2^{\infty} f(x)\,dx$

$= \int_2^3 f(x)\,dx$

$= \int_2^3 \frac{x^2}{9}\,dx + \int_3^{\infty} \cancel{f(x)}^{0}\,dx$

$= \frac{1}{9} \frac{x^3}{3}\Big]_2^3$

$= \frac{1}{27}(27-8)$

$= \frac{19}{27}$ //

3) b) obtain the mean, variance and standard deviation of the Binomial distribution.

$\mu = \sum_{x=0}^{n} x\, p(x)$

$= \sum_{x=0}^{n} x\, {}^nC_x\, p^x q^{n-x}$

$= 0 + {}^nC_1\, p^1 q^{n-1} + 2\, {}^nC_2\, p^2 q^{n-2} + \cdots + n\, {}^nC_n\, p^n q^0$

$= \frac{n!}{(n-1)!\,1!} p\, q^{n-1} + \cancel{2} \frac{n!}{(n-2)!\,\cancel{2!}} p^2 q^{n-2} + \cdots + \cancel{n} \frac{n!}{0!\,\cancel{n!}} p^n$

$= \frac{\cancel{(n-1)!}\,n}{\cancel{(n-1)!}} p\, q^{n-1} + \frac{\cancel{(n-2)!}(n-1)(n)}{\cancel{(n-2)!}} p^2 q^{n-2} + \cdots + np^n$

$= npq^{n-1} + (n-1)np^2q^{n-2} + \cdots + np^n$

$= np\,[q^{n-1} + (n-1)pq^{n-2} + \cdots + p^{n-1}]$

$= np\,[p+q]^{n-1}$

$= np\,[1]^{n-1}$

$\mu = np$

variance, $V = \Sigma x^2 p(x) - \mu^2$

$\because \mu = np$

$\sum_{x=0}^{n} x^2 p(x) = \sum_{x=0}^{n} (x^2 - x + x)\, p(x)$

$= \sum_{x=0}^{n} [x(x-1) + x]\, p(x)$

$= \sum_{x=0}^{n} x(x-1)\, p(x) + \sum_{x=0}^{n} x\, p(x)$

$= \sum_{x=0}^{n} x(x-1)\, p(x) + \mu$

at $x = 0, 1, 2, 3 \ldots n$

$= 0 + 0 + 2(1)\, {}^nC_2\, p^2 q^{n-2} + 3(2)\, {}^nC_3 \cdots + n(n-1)\, {}^nC_n\, p^n q^0 + \mu$

$$= 2\frac{n!}{(n-2)!\,2!}p^2q^{n-2} + 6\frac{n!}{(n-3)!\,3!}p^3q^{n-3} + \cdots + (n)(n-1)(1)p^n + \mu$$

$$= \frac{\cancel{(n-2)!}(n-1)(n)}{\cancel{(n-2)!}}p^2q^{n-2} + \frac{\cancel{(n-3)!}(n-2)(n-3)n}{\cancel{(n-3)!}}p^3q^{n-3} + \cdots + n(n-1)p^n + \mu$$

$$= n(n-1)p^2[q^{n-2} + pq^{n-3}(n-2) + \cdots + p^{n-2}] + \mu$$

$$= n(n-1)p^2[p+q]^{n-2} + \mu$$

$$= n(n-1)^2p^2(1)^{n-2} + \mu$$

$$v = \Sigma x^2 p(x) - \mu^2$$

$$= n(n-1)p^2 + \mu - \mu^2$$

$$= (n^2 - n)p^2 + np - n^2p^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$$= -np^2 + np$$

$$= np(1-p) = npq$$

$$v = np(1-p) = npq$$

$$S.D = \sqrt{v} = \sqrt{npq}\ //$$

3) c) Given that 2% of the fuses manufactured by a firm are defective, find by using poisson distribution, the probability that a box containing 200 fuses has (i) No defective fuses (ii) 3 or more defective fuses (iii) Atleast one defective fuse.

$n = 200$

$p = 2\% = 0.02$

$m = np = 200 \times 0.02$

$m = 4.$

(i) $P(x=0) = \dfrac{e^{-4}4^0}{0!}$

$= 0.0183$

(ii) $P(x \geq 3) = 1 - P(x<3)$

$= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= 1 - [0.0183 + 0.0733 + 0.1465]$

$= 1 - 0.2381$

$= 0.7619.$

(iii) $P(x \geq 1) = 1 - P(x=0)$

$= 1 - 0.0183$

$= 0.9817\ //$

4) a) The probability that an individual suffers a bad reaction from a certain injection is 0.001. Using poisson distribution, determine the probability that out of 2000 individuals (i) exactly 3 and (ii) More than 2 will suffer a bad reaction.

$n = 2000$

$P = 0.001$

$m = np = 2000 \times 0.001 = 2$

(i) $P(X=3) = \dfrac{e^{-2}\, 2^3}{3!}$

$= 0.1805$

(ii) $P(X>2) = 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - [0.1353 + 0.2707 + 0.2707]$

$= 0.3233$

(i) $P(X=3) = 0.1805$

(ii) $P(X>2) = 0.3233$

4) b) The probability that a news reader commits no mistake in reading the news is $1/e^3$. Find the probability that on a particular news broadcast he commits (i) only 2 mistakes (ii) more than 3 mistakes (iii) at most 3 mistakes.

$p(x=0) = \frac{1}{e^3}$ $x \to$ no of mistakes $p(x) = \dfrac{e^{-m} m^x}{x!}$

$= \dfrac{e^{-3}\, 3^x}{x!}$

Given $P(x=0) = \dfrac{1}{e^3}$

$= \dfrac{e^{-m} m^0}{0!} = \dfrac{1}{e^3}$

$e^{-m} = e^{-3}$

$\boxed{m=3}$

(i) $P(x=2) = \dfrac{e^{-3}\, 3^2}{2!} = \dfrac{0.0497 \times 9}{2} = 0.2236$.

(ii) $P(x>3) = 1 - P(x \leq 3)$

$= 1 - [P(0) + P(1) + P(2) + P(3)]$

$= 1 - (0.0497 + 0.1493 + 0.2240 + 0.2240)$

$= 1 - 0.647$

$P(x>3) = 0.353$

(iii) $P(x \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= \dfrac{e^{-3} 3^0}{0!} + \dfrac{e^{-3} 3^1}{1!} + \dfrac{e^{-3} 3^2}{2!} + \dfrac{e^{-3} 3^3}{3!}$,

$P(x \leq 3) = 0.647$

4) c) In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. It is given that $p(0<z<1.2263)=0.39$ and $p(0<z<1.4757)=0.43$.

→ Let x be the normal variat which represent marks percentage

x = marks.

Let z_1 be normal variat corresponding to $x=35 \Rightarrow z_1 = \frac{35-\mu}{\sigma}$

Let z_2 be normal variat corresponding to $x=60 \Rightarrow z_2 = \frac{60-\mu}{\sigma}$

$p(x<35) = 7\% = 0.07$

$p(z<z_1) = 0.07$

$0.5 + \phi(z_1) = 0.07$

$\phi(z_1) = 0.07 - 0.5$

$\phi(z_1) = -0.43$

$z_1 = -1.4757$.

$p(x<60) = 89\% = 0.89$

$p(z<z_2) = 0.89$

$0.5 + \phi(z_2) = 0.89$

$\phi(z_2) = 0.89 - 0.5$

$\phi(z_2) = 0.39$

$\phi(z_2) = 1.2263$

$\frac{35-\mu}{\sigma} = -1.4757$

$35-\mu = -1.4757\sigma$

$35 = (\mu - 1.4757)\sigma \rightarrow (1)$

$\frac{60-\mu}{\sigma} = 1.2263$

$60-\mu = 1.2263\sigma$

$60 = (\mu + 1.2263)\sigma \rightarrow (2)$.

$35 = (\mu - 1.4757)\sigma$

$60 = (\mu + 1.2263)\sigma$.

$\therefore \quad \mu = 48.65 \qquad \sigma = 9.2524$.

5) a) The joint probability distribution of two random variable X and Y are given below.

Y/X	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine i) E(X) and E(Y) ii) E(XY) iii) $\rho(X,Y)$.

x/Y	-4	2	7	F(x)
1	1/8	1/4	1/8	1/2
5	1/4	1/8	1/8	1/2
g(y)	3/8	3/8	2/8	1

x_i	1	5
$F(x_i)$	1/2	1/2

y_j	-4	2	7
$g(y_j)$	3/8	3/8	2/8

(i) $E(X) = \Sigma x_i F(x_i)$
$= 1(1/2) + 5(1/2)$
$= 3$

$E(Y) = \Sigma y_j\, g(y_j)$
$= (-4)(3/8) + 2(3/8) + 7(2/8)$
$= 1$

(ii) $E(XY) = \sum_i \sum_j x_i y_j\, f(x_i, y_j)$
$= 1(-4)(1/8) + 1(2)(1/4) + 1(7)(1/8) + 5(-4)(1/4) + 5(2)(1/8) + 5$
$(7)(1/8)$
$= 3/2 = 1.5.$

(iii) $\sigma_x^2 = E(x^2) - [E(x)]^2$
$= (1/2 + 25/2) - 9$
$\sigma^2_x = 4 \implies \sigma_x = 2$

$\sigma^2 y = E(y^2) - [E(y)]^2$
$= 16(3/8) + 12/8 + 49(2/8) - 1$
$\sigma^2 y = 18.75 \implies \sigma y = \sqrt{75/4}$

$$\rho(x,y) = \frac{cov(x\, y)}{\sigma_x \sigma_y}$$

$cov(x\, y) = [(XY) - E(x).E(Y)]$
$= 1.5 - 3(1)$
$= -1.5.$

$$\rho(x,y) = \frac{-1.5}{2(\sqrt{75/4})} = -\frac{\sqrt{3}}{25} = -0.1732$$

5) b) Prove that the markov chain where TPM is $\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible and find the the corresponding stationary probability vector.

$$P^2 = \begin{bmatrix} 1/2 & 1/6 & 1/3 \\ 1/4 & 7/12 & 1/6 \\ 1/4 & 1/3 & 5/12 \end{bmatrix}$$

$\therefore$ It is regular stochastic matrix

$\therefore$ It is irreducible.

let $v = (x\ y\ z)$ where $x+y+z=1$

$$vP = V$$

$$[x\ y\ z]\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = [x\ y\ z]$$

$\frac{y}{2} + \frac{z}{2} = x$ $\qquad$ $\frac{2x}{3} + \frac{z}{2} = y$ $\qquad$ $\frac{x}{3} + \frac{y}{2} = z$

$y+z = 2x$ $\qquad$ $2x + 3z = 6y$ $\qquad$ $2x+3y = 6z$

$y - 2x = z$

$4x + 3z = 6(2x - z)$

$4x + 3z = 12x - 6z$

$9z = 8x$

$z = 8x/9$

$x+y+z = 1$

$x + 10x/9 + 8x/9 = 1$

$9x + 10x + 8x = 9$

$27x = 9$

$x = \frac{9}{27} = \frac{1}{3}$ $\qquad$ $y = \frac{10}{9}\left(\frac{9}{27}\right) = \frac{10}{27}$

$z = \frac{8}{9}\left(\frac{1}{3}\right) = \frac{8}{27}$

$(x\ y\ z) = \left(1/3, \frac{10}{27}, \frac{8}{27}\right)$.

5) c) Three boys A, B, C are throwing ball to each other, A always throws to B and B always throws to C but C is just likely to throw the ball to B as to A, if C was a person to throw the ball find the probability that i) A has a ball ii) B has a ball iii) C has a ball.

$$P = \begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

$p^{(0)} = (0\ 0\ 1)$

$p^{(3)} = p^{(0)} \cdot p^{(3)}$

$$p^{(3)} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

$$p^{(3)} = (0\ 0\ 1)\begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{pmatrix} = (1/4\ \ 1/4\ \ 1/2)$$

after three throws a has ball = $1/4$

b has ball = $1/4$

c has ball = $1/2$.

6) a) suppose X and Y are independent random variables, X takes values. 2, 5, 7 with probability $1/2$, $1/4$, $1/4$ respectively. Y takes the values 3, 4, 5 with probability $1/3$, $1/3$, $1/3$.

(i) Find the joint probability distribution of X & Y.

(ii) show that cov (X, Y) is equal to zero.

x_i	2	5	7
$f(x_i)$	$1/2$	$1/4$	$1/4$

y_j	3	4	5
$g(y_j)$	$1/3$	$1/3$	$1/3$

$P_{11} = f(x_1)\,g(y_1)$

$= 1/2\,(1/3) = 1/6$

$P_{12} = 1/6$

$P_{13} = 1/6$

$P_{21} = 1/12$

$P_{22} = 1/12$

$P_{23} = 1/12$

$P_{31} = 1/12$

$P_{32} = 1/12$

$P_{33} = 1/12$

x/y	3	4	5	f(x)
2	$1/6$	$1/6$	$1/6$	$1/2$
5	$1/12$	$1/12$	$1/12$	$1/4$
7	$1/12$	$1/12$	$1/12$	$1/4$
g(y)	$1/3$	$1/3$	$1/3$	1

$E(X) = \sum_i x_i\, f(x_i)$

$= 2(1/2) + 5(1/4) + 7(1/4)$

$= 4$

$E(Y) = \sum y_j\, g(y_j)$

$= 3(1/3) + 4(1/3) + 5(1/3)$

$= 4$

$$E(x,y) = \sum_i \sum_j x_i y_j \, f(x_i \, y_j)$$

$$= 2(3)(1/6) + 2(4)(1/6) + 2(5)(1/6) + 5(3)(1/12) + 5(4)(1/12) + 5(5)(1/12) + 7(3)(1/12) + 7(4)(1/12) + 7(5)(1/12)$$

$$= 16$$

$$\operatorname{cov}(X\,Y) = E(XY) - E(X).E(Y)$$

$$= 16 - 16$$

$$= 0$$

6) b) show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is a regular stochastic matrix and find the corresponding unique fixed probability vector.

$$P^2 = \begin{bmatrix} 1 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \quad P^3 = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} \quad P^4 = \begin{bmatrix} 1 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ 1/8 & 3/8 & 1/2 \end{bmatrix}$$

$\therefore$ It is regular stochastic matrix

$$v = (x \; y \; z) \qquad x+y+z=1$$

$$vP = v$$

$$[x \; y \; z] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = [x \; y \; z]$$

$$\frac{z}{2} = x \qquad x = \frac{y}{2} \Rightarrow y = 2x$$

$$x + \frac{z}{2} = y$$

$$y = z \qquad z = 2x$$

$$x+y+z=1$$

$$x+2x+2x=1$$

$$5x=1$$

$$x = 1/5 \quad y = 2/5 \quad z = 2/5$$

$$v = (1/5 \;\; 2/5 \;\; 2/5)$$

6) c) A gamblers luck follows a pattern. If he wins a game, the probability of winning the next game is 0.6. However if he loses a game, the probability of losing the next game is 0.7. There is an even chance of gambler winning the first game. If so,

i) what is the probability of he winning the second game.

ii) what is the probability of he winning the third game.

→ $P(win|win) = 0.6$

$P(Lose|Lose) = 0.7$

$P(win) = P(Lose) = 0.5$

(i) $P(win|win) \times P(win) = 0.6 \times 0.5 = 0.3$

$P(win|Lose) \times P(Lose) = (1-0.7) \times 0.5 = 0.15$

Total probability $= 0.3 + 0.15$

$= 0.45$.

(ii) $P(win|win) \times P(win|win) \times P(win) = 0.6 \times 0.6 \times 0.5$

$= 0.18$

$P(win|Lose) \times P(Lose|win) \times P(win) = (1-0.7) \times (1-0.6) \times 0.5$

$= 0.06$

$P(win|win) \times P(win|Lose) \times P(Lose) = 0.6 \times (1-0.7) \times 0.5$

$= 0.09$

$P(win|Lose) \times P(Lose|Lose) \times P(Lose) = (1-0.7) \times 0.7 \times 0.5$

$= 0.105$.

Total probability $= 0.18 + 0.06 + 0.09 + 0.105$

$= 0.465$

∴ (i) 0.45

(ii) 0.465.

7) a) Find the coefficient of correlation for the following data:

x	55	56	58	59	60	60	62
y	35	38	39	38	44	43	45

x	y	$z = x-y$	$X = x-\bar{x}$	$Y = y-\bar{y}$	X^2	Y^2	XY
55	35	20	-3.57	-5.28	12.74	27.878	18.849
56	38	17	-2.57	-2.28	6.604	5.198	5.859
58	39	19	0.57	-1.28	0.324	1.638	-0.729
59	38	21	1.57	-2.28	2.464	5.198	-3.579
60	44	16	1.43	3.72	2.044	13.838	5.319
60	43	17	1.43	2.72	2.044	7.398	3.889
62	45	17	3.43	4.72	11.764	22.278	16.189
410	282				37.984	83.426	45.797

$$\bar{x} = \frac{\Sigma x}{n} = \frac{410}{7} = 58.57 \qquad \bar{y} = \frac{\Sigma y}{n} = \frac{282}{7} = 40.28$$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}} = \frac{45.797}{\sqrt{37.984}\sqrt{83.426}} = \frac{45.797}{6.323 \times 9.133}$$

$$= \frac{45.797}{57.752}$$

$$\boxed{r = 0.792}$$

7) b) Ten competitors in a beauty contest are ranked by two judges in the following order compute their rank correlation coefficient

I	1	6	5	3	10	2	4	9	7	8
II	6	4	9	8	1	2	3	10	5	7

$$\Sigma d_i^2 = (1-6)^2 + (6-4)^2 + (5-9)^2 + (3-8)^2 + (10-1)^2 + (2-2)^2 + (4-3)^2 + (9-10)^2 + (7-5)^2 + (8-7)^2$$

$$= 25 + 4 + 16 + 25 + 81 + 0 + 1 + 1 + 4 + 1$$

$$\Sigma d_i^2 = 158$$

$$\rho = 1 - \frac{6\,\Sigma d_i^2}{n^3 - n}$$

$$= 1 - \frac{6(158)}{10^3 - 10}$$

$$\boxed{\rho = 0.0424}$$

7) c) Fit a straight line $y = ax + b$ for the data

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

x_i	y_i	x_i^2	x_iy_i
1	1	1	1
3	2	9	6
4	4	16	16
6	4	36	24
8	5	64	40
9	7	81	63
11	8	121	88
14	9	196	126
56	40	524	364

(1) $\Rightarrow \Sigma y_i = a\Sigma x_i + nb$

$40 = 56a + 8b$

(2) $\Rightarrow \Sigma x_iy_i = a\Sigma x_i^2 + b\Sigma x_i$

$364 = 524a + 56b$

$\therefore a = 0.6364$

$b = 0.5455$

$\therefore \boxed{y = 0.6364x + 0.5455}$

8) a) Given $8x - 10y + 66 = 0$ and $4x - 18y = 214$ are the regression lines. Find the mean value of x and y and the coefficient of correlation also compute σ_y if $\sigma_x = 3$.

$8\bar{x} - 10\bar{y} = -66$

$4\bar{x} - 18\bar{y} = 214$

$\bar{x} = -32$

$\bar{y} = -19$

$x = \frac{1}{8}(10y - 66)$

$x = 1.25y - 8.25$

$y = \frac{1}{18}(4x - 214)$

$y = 0.222x - 11.888$

$(x - \bar{x}) = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$(y - \bar{y}) = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$

$r\frac{\sigma_x}{\sigma_y} = 1.25$

$r\frac{\sigma_y}{\sigma_x} = 0.22$

$r = \pm\sqrt{\text{coeff}(y) \times \text{coeff}(x)}$

$r = \pm\sqrt{1.25 \times 0.22}$

$r = \pm 0.5244$

$r\frac{\sigma_y}{\sigma_x} = 0.22$

$0.5244 \frac{\sigma_y}{3} = 0.22$

$0.5244\,\sigma_y = 0.66$

$\sigma_y = 1.2555$

8) b) Ten students got following percentage of marks in 2 subjects x and y compute their rank correlation coefficient.

x	78	36	98	25	75	82	90	62	65	39
y	84	51	91	60	68	62	86	58	53	47

x	y	R_x	R_y	$d_i = R_x - R_y$	d_i^2
78	84	4	3	1	1
36	51	9	9	0	0
98	91	1	1	0	0
25	60	10	6	4	16
75	68	5	4	1	1
82	62	3	5	-2	4
90	86	2	2	0	0
62	58	7	7	0	0
65	53	6	8	-2	4
39	47	8	10	-2	4
					30

$\Sigma d_i^2 = 30$

$$\rho = 1 - \frac{6\Sigma d_i^2}{n^3 - n}$$

$$= 1 - \frac{6(30)}{10^3 - 10}$$

$$\boxed{\rho = 0.8181}$$

8) c) Fit a least square geometric curve $y = ax^b$ for following data.

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

x_i	y_i	$X_i = \log x_i$	$Y_i = \log y_i$	$X_i Y_i$	X_i^2
1	0.5	0	-0.6931	0	0
2	2	0.6931	0.6931	0.4803	0.4803
3	4.5	1.0986	1.5040	1.6523	1.2069
4	8	1.3862	2.0794	2.8825	1.9215
5	12.5	1.6094	2.5257	4.0649	2.5901
15	27.5	4.7873	6.1091	9.08	6.1988

$\Sigma Y_i = nA + b\Sigma X_i$

$6.1091 = 5A + b4.7873$

$\Sigma X_i Y_i = A\Sigma X_i + b\Sigma X_i^2$

$9.08 = A4.7873 + b6.1988$

$A = -0.69338$, $B = 2.0002$

$a = e^A$

$a = e^{-0.6931}$

$a = 0.5$

$y = ax^b$

$$\boxed{y = 0.5\, x^{2.0002}}$$

9)a) A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. show that the die cannot be regarded as an unbiased one at 5% level of significance

$p = 2/6 = 1/3 \qquad q = 1 - 1/3 = 2/3 \qquad n = 9000$

$x = 3240 \qquad H_0$ = dice is unbiased.

$$Z = \frac{x - np}{\sqrt{npq}} = \frac{3240 - 9000(1/3)}{\sqrt{9000(1/3)(2/3)}} = 5.3666$$

at 5%

$Z_{0.05} = 1.96$

$\therefore Z_{cal} > Z_{0.05}$

$5.3666 > 1.96 = Z_{0.05}$

$\therefore$ It is rejected.

9)b) In an elementary school Examination, the mean rate of 32 boys was 72 with a standard deviation of 8, while the mean rate of 36 girls was 75 with S D of 6. Test hypothesis that the performance of girls is better than boys.

$n_1 = 32$

$\bar{x}_1 = 72$

$\sigma_1 = 8$

$n_2 = 36$

$\bar{x}_2 = 75$

$\sigma_2 = 6$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{72 - 75}{\sqrt{\frac{8^2}{32} + \frac{6^2}{36}}} = |-1.7321|$$

$z = 1.7321$

1% loss $\Rightarrow Z_{0.01} < -1.7321$

5% loss $\Rightarrow Z_{0.01} < -1.75$

$\therefore$ Hyp accepted.

9c) Fit a binomial distribution for the data

x	0	1	2	3	4
y	122	60	5	2	1

and test goodness of fit ($\chi^2_{0.05} = 7.815$ for 4 d.f)

$$P(x) = \frac{e^{-m} m^x}{x!} \qquad m = \frac{\Sigma fx}{\Sigma f} = \frac{0+60+10+6+4}{190}$$

$$m = \frac{80}{190}$$

$$m = 0.4210$$

x	O_i	E_i	$\frac{(O_i - E_i)^2}{E_i}$
0	122	124.71	0.0588
1	60	52.50	1.0714
2	5	11.05	3.3124
3	2	1.55	0.1306
4	1	0.163	4.2979
			8.8711

$F(0) = 124.71$

$$F(0) = 190 \; \frac{e^{-0.4210} (0.4210)^0}{0!}$$

$= 124.71$

$F(1) = 52.50$

$F(2) = 11.05$

$F(3) = 1.55$

$F(4) = 0.163$

$\chi^2_{cal} = 8.8711 > 7.815 = \chi^2$

$\therefore$ Hypothesis is rejected.

10) a)

$\rightarrow n = 200$

$x = 18$

$P_1 = 18/200 = 0.09$

$P_2 = 0.05$

$q = 0.95$

$$z = \frac{P_1 - P_2}{\sqrt{pq/n}}$$

$$= \frac{0.09 - 0.05}{\sqrt{\frac{0.05(0.95)}{200}}}$$

$z = 2.213$

p-value ≈ 0.0134

at 1%. $0.0134 > 0.01$ $\therefore$ rejected.

at 5% $0.0134 < 0.05$ $\therefore$ not rejected.

10) b)

$n_1 = 600$

$n_2 = 400$

$P_1 = 55\% = 0.55$

$P_2 = 48\% = 0.48$

H_0 : There is no significant in the opinion locality.

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

where

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$Z = \frac{0.55 - 0.48}{\sqrt{(0.522)(0.478)\left(\frac{1}{600}+\frac{1}{400}\right)}}$$

$$= \frac{600(0.55) + 400(0.48)}{600+400}$$

$P = 0.522$

$q = 1 - P = 0.478.$

$Z = 2.1710.$

at 1% loss $\quad Z_{cal} = 2.1710 < Z_{0.01} = 2.18$ (accepted)

at 5% loss $\quad Z_{cal} = 2.1710 > Z_{0.05} = 1.96$ (rejected)

10) c) $\quad n = 10 \quad D = n - 1 = 9$

x	$(x-\bar{x})^2$
63	23.04
63	23.04
66	3.24
67	0.64
68	0.04
69	1.44
70	4.84
70	4.84
71	10.24
71	10.24
678	81.6

$$\bar{x} = \frac{\Sigma x}{n} = \frac{678}{10} = 67.8.$$

$$s^2 = \frac{1}{n-1} \Sigma (x - \bar{x})^2$$

$$= \frac{1}{10-1} (81.6)$$

$$s^2 = 9.0667$$

$$s = 3.0111$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{67.8 - 66 (\sqrt{10})}{3.0111}$$

$$\boxed{t = 1.8904}$$

$$t_{cal} = 1.8904 < 2.262 = t_{0.05}$$

$\therefore$ H_0 is accepted.