MODEL QUESTION PAPER - 1

PART - A

1) Let p: A circle is a conic

q: $\sqrt{5}$ is a real number.

Express the following compound propositions in words

(i) $p \wedge (\sim q)$ (ii) $q \vee (\sim p)$

Soln:

(i) A circle is a conic and $\sqrt{5}$ is not a real number.

(ii) $\sqrt{5}$ is a real number or a circle is not a conic.

2) Find the possible truth values of p, q, r in the following cases

(i) $p \rightarrow (q \vee r)$ is False (ii) $p \wedge (q \rightarrow r)$ is true.

Soln:

(i)

p	q	r	$q \vee r$	$p \rightarrow (q \vee r)$
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	0	0
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	0	1

$\therefore$ It is contigency.

(ii)

p	q	r	$q \rightarrow r$	$p \wedge (q \rightarrow r)$
1	1	1	1	1
1	1	0	0	0
1	0	1	1	1
1	0	0	1	1
0	1	1	1	0
0	1	0	0	0
0	0	1	1	0
0	0	0	1	0

$\therefore$ It is contigency.

3) Write the formula for Mean, variance and standard deviation for Binomial distribution.

Mean $\Rightarrow \mu = np$.

Variance $\Rightarrow V = npq$.

Standard Deviation $\Rightarrow S.D = \sqrt{V} = \sqrt{npq}$.

4) Define Probability Density Function and give an example.

It is a mathematical function that describes the probability distribution of a continuous random variable. It specifies the probability per unit of measurement of the variable.

Ex:- $F(x) = \frac{1}{(b-a)}$ if $a \le x \le b$

$F(x) = 0$ otherwise

Here $a = 1, b = 6$.

$F(x) = \frac{1}{5}$ if $1 \le x \le 6$

$F(x) = 0$ otherwise.

5) Define Markov chain and give one example.

A stochastic process which is such that the generation of the probability distribution depends only on the present state is called Markov process if state space of the process to discrete then it is called Markov chain.

Ex:- S = Sunny, C = cloudy, R = Rainy.

$$\begin{array}{c} \\ S \\ C \\ R \end{array}
\begin{array}{c} \begin{array}{ccc} S & C & R \end{array} \\ \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{array}$$

6) Write the formula for covarience and correlation for joint probability distribution.

Covarience :

$$\text{cov}(x, y) = \sum_i \sum_j x_i y_j \, f(x_i, y_j) - \mu_x \mu_y$$

Correlations

$$\rho(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

7) Write the normal equations for a straight line $y = a + bx$.

$$y = a + bx$$

$$\Sigma y_i = na + b \Sigma x_i$$

$$\Sigma x_i y_i = a \Sigma x_i + b \Sigma x_i^2$$

8) Given the regression lines $x = 19.13 - 0.87y$ and $y = 11.64 - 0.5x$ compute the coefficient of correlation.

→ Let regression lines passes through the point $(\bar{x}, \bar{y})$

$$\bar{x} = 19.13 - 0.87\bar{y}$$

$$\bar{y} = 11.64 - 0.5\bar{x}$$

$x = (-0.87y + 19.13)$ | $y = (-0.5x + 11.64)$

$(x - \bar{x}) = r \dfrac{\sigma_x}{\sigma_y}(y - \bar{y})$ | $(y - \bar{y}) = r \dfrac{\sigma_y}{\sigma_x}(x - \bar{x})$

$r \dfrac{\sigma_x}{\sigma_y} = -0.87$ | $r \dfrac{\sigma_y}{\sigma_x} = -0.5$

$$r = \pm\sqrt{-0.87 \times -0.5}$$

$$r = \pm\sqrt{0.435}$$

$$\boxed{r = \pm 0.6595}$$

//

9) Define the following (i) Type-I error (ii) Type-II error

(i) Type-I error: Rejecting the NULL hypothesis H_0 and accepting alternative hypothesis H_1 when actually H_0 is true is called Type-I error.

(ii) Type-II error: Accepting H_0 and Rejecting H_1 when actually H_0 is False is called type-II error.

10) Write the Test statistic formula for significance difference of two sample proportions.

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$ where $P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$

$P \rightarrow$ probability of success

$q \rightarrow$ probability of failures.

PART-B

1) (a) Negate and simplify each of the following:

(i) $\exists x, [p(x) \vee q(x)]$

(ii) $\forall x, [P(x) \wedge Nq(x)]$

(i) $\exists x, [p(x) \vee q(x)]$

$N[\exists x, \{p(x) \vee q(x)\}]$

$\forall x, N\{p(x) \vee q(x)\}$

$\forall x, Np(x) \wedge Nq(x)$.

(ii) $\forall x, [P(x) \wedge Nq(x)]$

$N[\forall x, \{p(x) \wedge Nq(x)\}]$

$\exists x, N\{p(x) \wedge Nq(x)\}$

$\exists x, NP(x) \vee q(x)$.

1) b) Prove that the following argument is valid :

All men are mortal

Sachin is a man

$\therefore$ sachin is mortal

→ $p(x)$: x is a mortal

A : sachin

$$\frac{\forall x \in S \quad p(x) \qquad a \in S}{p(a)}$$

$p(x)$ is true and $a \in S$ means $p(a)$ is true

$\therefore$ This is valid by universal specification.

1) c) Determine the validity of the following argument

$$\frac{\begin{array}{l} p \to (q \to r) \\ \sim q \to \sim p \\ p \end{array}}{\therefore p \to r}$$

$p \to (q \to r) \wedge (\sim q \to \sim p) \wedge p.$

$p \to (p \wedge q) \to r.$

$\therefore$ The given argument is not valid.

2) a) Construct the truth table for the following compound propositions.

(a) $(p \wedge q) \rightarrow (\sim r)$ (b) $q \wedge (\sim r \rightarrow p)$

(a)

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \rightarrow (\sim r)$
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	0	0	1
1	0	0	0	1	1
0	1	1	0	0	1
0	1	0	0	1	1
0	0	1	0	0	1
0	0	0	0	1	1

(b)

p	q	r	$\sim r$	$\sim r \rightarrow p$	$q \wedge (\sim r \rightarrow p)$
1	1	1	0	1	1
1	1	0	1	1	1
1	0	1	0	1	0
1	0	0	1	1	0
0	1	1	0	1	1
0	1	0	1	0	0
0	0	1	0	1	0
0	0	0	1	0	0

2) b) If I study, then I do not fail in the examination.
If I do not fail in the examination,
my father gifts a two wheeler to me.

$\therefore$ If I study then my father gifts a two wheeler to me.

p : I study

q : I do not fail in the exam

R : My father gifts a two wheeler to me

$p \to q$

$q \to r$

$r \to r$

2) c) Identify whether the following argument is valid :

$p \to q$

$q \to r$

$\therefore (p \vee q) \to r$

$(p \to q) \wedge (q \to r)$

$(\sim p \vee r) \wedge (\sim q \vee r)$

$(\sim p \wedge \sim q) \vee r$ [distributive]

$\sim (p \vee q) \vee r$

$(p \vee r) \to r$

$\therefore$ It is valid

3) a) The probability distribution of finite random variable X is

X	-3	-2	-1	0	1	2	3
P(X)	K	2K	3K	4K	3K	2K	K

Find k. Also find Mean Variance and standard deviation.

$\rightarrow$ $\Sigma p(x_i) \Rightarrow K + 2K + 3K + 4K + 3K + 2K + K = 1$

$16K = 1$

$K = 1/16$

x	-3	-2	-1	0	1	2	3
p(x)	1/16	2/16	3/16	4/16	3/16	2/16	1/16

* Mean $\mu = \Sigma x_i p(x_i)$

$= -3(1/16) + (-2)(2/16) - 1(3/16) + 0(4/16) + 1(3/16) + 0(4/16) +$

$1(3/16) + 2(2/16) + 3(1/16)$

$= -0.1875 - 0.25 - 0.1875 + 0.1875 + 0.25 + 0.1875$

$= 0$

* variance

$V = \sum x_i^2 p(x_i) - \mu^2$

$= (-3)^2(1/16) + (-2)^2(2/16) + (-1)^2(3/16) + 0^2(4/16) + 1^2(3/16) +$

$2^2(2/16) + 3^2(1/16)$

$V = 2.5$

* standard deviation

$SD = \sqrt{V} = \sqrt{2.5} = 1.5811$

3) b) obtain the mean, variance and standard deviation of the poisson distribution.

=>

$$p(x) = \frac{e^{-m} m^x}{x!}$$ is the poisson distribution.

then

x	0	1	2	3
$p(x)$	$\frac{e^{-m}m^0}{0!}$	$\frac{e^{-m}m^1}{1!}$	$\frac{e^{-m}m^2}{2!}$	$\frac{e^{-m}m^3}{3!}$

mean, variance and standard deviation of poisson distribution.

$$\mu = \sum_{x=0}^{\infty} x\, p(x)$$

$$= \sum_{x=0}^{\infty} x \frac{e^{-m} m^x}{x!}$$

$$= \sum_{x=0}^{\infty} \cancel{x} \frac{e^{-m} m^x}{\cancel{x}(x-1)!}$$

$$= e^{-m} \sum_{x=1}^{\infty} \frac{m^x}{(x-1)!}$$

$$= e^{-m} \left[\frac{m^1}{0!} + \frac{m^2}{1!} + \frac{m^3}{2!} + \cdots \right]$$

$= e^{-m} m [1 + m + \frac{m^2}{2!} + \cdots]$

$= e^{-m} m [e^m]$ [$\because$ by maclaurin's series]

$= e^{-m} m e^m$

$= m e^{-m+m}$

$= m [e^0]$

$= m [1]$

$\boxed{\mu = m}$

Variance [V]

$$V = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

$$= \sum_{x=0}^{\infty} (x^2 - x + x) p(x) - m^2$$

$$= \sum_{x=0}^{\infty} [x(x-1) + x] p(x) - m^2$$

$$= \sum_{x=0}^{\infty} x(x-1) p(x) + \sum_{x=0}^{\infty} x p(x) - m^2$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-m} m^x}{x!} + m - m^2$$

$$= \sum_{x=0}^{\infty} \cancel{x(x-1)} \frac{e^{-m} m^x}{\cancel{x(x-1)}(x-2)!} + m - m^2$$

$$= \sum_{x=2}^{\infty} \frac{e^{-m} m^x}{(x-2)!} + m - m^2$$

$$= e^{-m} \left[\frac{m^2}{0!} + \frac{m^3}{1!} + \frac{m^4}{2!} + \cdots \right] + m - m^2$$

$$= e^{-m} m^2 \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots \right] + m - m^2$$

$$= e^{-m} m^2 e^m + m - m^2$$

$$= m^2 e^{-m+m} + m - m^2$$

$$= m^2 e^0 + m - m^2$$

$$= \cancel{m^2} + m - \cancel{m^2}$$

$\boxed{V = m}$

$SD = \sqrt{V} = \sqrt{m} \Rightarrow \boxed{SD = \sqrt{m}}$.

3) c) The sales per day in a shop is exponentially distributed with the average sale amounting to Rs. 100 and net profit is 8 %. Find the probability that the net profit exceeds Rs. 30 on two consecutive days.

$m = 100$

$\frac{1}{\alpha} = 100 \Rightarrow \alpha = \frac{1}{100} = 0.01$ $\qquad f(x) = \alpha e^{-\alpha x}$

Let A be the amount for which profit is 8 %

$A \times 8\,\% = 30$

$A \times \frac{8}{100} = 30$

$A = \frac{30 \times 100}{8} = 375$

$P(x > 30) = 1 - P(x \leq 30)$

$= 1 - \text{Prob}(\text{sales} \leq 2{,}375)$

$= 1 - \int_0^{375} 0.01\ e^{-0.01x}$

$= 1 - 0.01 \left[\frac{e^{-0.01x}}{-0.01}\right]_0^{375}$

$= 1 + [e^{-0.01(375)} - e^{-0.01(0)}]$

$= 0.02351$

On two consecutive days $= 0.02351 \times 0.02351$

$= 0.00055 \simeq 0.0006$

$= 0.0006$

4) a) In 800 families with 5 children each how many families would be expected to have (i) 3 boys (ii) 5 girls (iii) either 2 or 5 boys (iv) atmost 2 girls by assuming probabilities of boys and girls to be equal.

$p = 0.5, \quad q = 0.5, \quad n = 5$

(i) $p(x) = {}^5C_x\ (0.5)^x\ (0.5)^{5-x}$

$F(x) = 800 \times p(x)$

where $x \to$ no of boys.

$P(x=3) = {}^5C_3 \, (0.5)^3 \, (0.5)^{5-3}$

$= 0.3125 \times 800$

$= 250$

(ii) $f(x=0) = {}^5C_0 \, (0.5)^0 \, (0.5)^{5-0} \times 800$

$= 25$

(iii) $f(x \geq 3) = p(3) + p(4) + p(5) \times 800$

$= 0.3125 + 0.15625 + 0.03125 \times 800$

$= 400$

(iv) $f(2) + f(3) = 800 \times 0.3125 + 0.3125$

$= 500$.

4) b) In a certain town the duration of shower has been exponentially distributed with mean 5 minutes what is the probability that shower will last for

(i) less than 10 minutes (ii) 10 minutes and more.

(iii) Between 10 and 12 minutes.

$\Rightarrow$ $\mu = 5$

$\frac{1}{\alpha} = 5$

$\alpha = \frac{1}{5}$

$$f(x) = \begin{cases} \frac{1}{5} e^{-x/5}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(i) $P(x \geq 10) = \int_{10}^{\infty} f(x)\,dx$

$= \int_{10}^{\infty} \frac{1}{5} e^{-x/5}\,dx$

$= \frac{1}{5}\left[\frac{e^{-x/5}}{-1/5}\right]_{10}^{\infty}$

$= -\left[e^{-\infty} - e^{-10/5}\right]$

$= -\left[e^{-2}\right] = 0.13533$

(ii) $p(x < 10) = \int_{-\infty}^{10} f(x)\,dx$

$= \int_{-\infty}^{0} \cancel{f(x)dx}^{\,0} + \int_{0}^{10} f(x)\,dx$

$$= \int_0^{10} f(x)dx$$

$$= \int_0^{10} \frac{1}{5} e^{-x/5} dx$$

$$= \frac{1}{5} \left[\frac{e^{-x/5}}{-1/5} \right]_0^{10}$$

$$= - [e^{-10/5} - e^{-0}]$$

$$= -(e^{-2} - 1)$$

$$= 0.86466$$

(iii) $P(10 < x < 12) = \int_{10}^{12} f(x) dx.$

$$= \int_{10}^{12} \frac{1}{5} e^{-x/5} dx.$$

$$= \frac{1}{5} \left[\frac{e^{-x/5}}{-1/5} \right]_{10}^{12}$$

$$= - [e^{-12/5} - e^{-10/5}]$$

$$= - [0.09071 - 0.13533]$$

$$= 0.04462 //$$

4) c) A manufacturer of air-mail envelopes knows from experience that the weight of the envelopes is normally distributed with mean 1.95gm and S.D 0.05gm. About how many envelops weighing.
(i) 2 gm or more (ii) 2.05 gm or more (iii) between 2 & 2.5 gm.
In a lot of 100 envelops (Given $A(1) = 0.3413$, $A(2) = 0.4772$)

(i) $z = \frac{(x - \mu)}{\sigma} = \frac{2 - 1.95}{0.05} = 1$

(ii) $z = \frac{(2.05 - 1.95)}{0.05} = 2$

(iii) $z_1 = \frac{(2 - 1.95)}{0.05} = 1$

$z_2 = \frac{(2.5 - 1.95)}{0.05} = 11$

(i) $P(z \geq 1) = 1 - P(z \leq 1)$

$= 1 - 0.8413$

$= 0.1587$

$= 0.1587 \times 100$

$P(z \geq 1) = 15.87$

(ii) $P(z \geq 2) = 1 - P(z \leq 2)$

$= 1 - 0.9772$

$= 0.0228$

$= 0.0228 \times 100$

$P(z \geq 2) = 2.28$.

(iii) $P(1 \leq z \leq 11) = 1 - 0.8413$

$= 0.1587$

$= 0.1587 \times 100$

$P(1 \leq z \leq 11) = 15.87$.

$(\because P(1 \leq z \leq 11) \approx P(z \leq 11) - P(z \leq 1) \approx 1 - 0.8413 = 0.1587)$

5) a) The joint probability distribution of two random variable X and Y are given below.

$Y \rightarrow$ / $X \downarrow$	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine i) $E(X)$ and $E(Y)$ (ii) $E(XY)$ (iii) $f(x, y)$.

X/Y	-2	-1	4	5	f(x)
1	0.1	0.2	0	0.3	0.6
2	0.2	0.1	0.1	0	0.4
g(y)	0.3	0.3	0.1	0.3	1

Marginal distribution table

x_i	1	2
$f(x_i)$	0.6	0.4

y_j	-2	-1	4	5
$g(y_j)$	0.3	0.3	0.1	0.3

(i) $E(X) = \Sigma_{x_i} f(x_i)$

$= 1(0.6) + 2(0.4)$

$= 0.6 + 0.8$

$E(X) = 1.4$

$E(Y) = \Sigma_{y_j}\, g(y_j)$

$= -2(0.3) - 1(0.3) + 4(0.1) + 5(0.3)$

$= 1$

$E(Y) = 1$

(ii) $E(XY) = \sum_i \sum_j x_i y_j f(x_i, y_j)$

$= 1(-2)(0.1) + 1(-1)(0.2) + 1(4)(0) + 1(5)(0.3) +$
$2(-2)(0.2) + 2(-1)(0.1) + 2(4)(0.1) + 2(5)(0)$

$= -0.2 - 0.2 + 0 + 1.5 - 0.8 - 0.2 + 0.8 + 0$

$E(XY) = 0.9$

(iii) $\rho(X, Y) = \dfrac{cov(X, Y)}{\sigma_x \sigma_y}$

$\sigma_x^2 = \Sigma x_i^2 f(x_i) - \mu_x^2$

$= 1^2(0.6) + 2^2(0.4) - (1.4)^2$

$= 0.6 + 1.6 - 1.96$

$= 2.2 - 1.96$

$\sigma_x^2 = 0.24 \Rightarrow \sigma_x = \sqrt{0.24} \qquad \sigma_x = 0.4898.$

$\sigma_y^2 = \Sigma y_j^2\, g(y_j) - \mu_y^2$

$= (-2)^2(0.3) + (-1)^2(0.3) + 4^2(0.1) + 5^2(0.3) - 1^2$

$= 1.2 + 0.3 + 1.6 + 7.5 - 1$

$= 10.6 - 1$

$\sigma_y^2 = 9.6 \Rightarrow \sigma_y = \sqrt{9.6} \qquad \sigma_y = 3.0983$

$$\mathrm{cov}(x,y) = \sum_i \sum_j x_i y_j \, f(x_i, y_j) - \mu_x \mu_y$$
$$= 0.9 - (1.4)(1)$$
$$= 0.9 - 1.4$$
$$\mathrm{cov}(x,y) = -0.5$$

$$\rho(x,y) = \frac{\mathrm{cov}(x,y)}{\sigma_x \sigma_y} = \frac{-0.5}{0.4898(3.0983)}$$

$$\boxed{\rho = -0.3294}$$ //

5) b) Every year a man trades his car for a new car if he has Maruthi, he trades it for an ambassador if he has an ambassador, he trades it fo santro. How ever if he has santro, he is just likely to trade it for a new santro as to trade for a Maruthi or an Ambassador In 2020 he bought his first car which was santro find the probability that he has i) 2022 santro ii) 2022 Maruthi.

$$P = \begin{array}{c} \\ S \\ M \\ A \end{array} \begin{array}{c} \begin{array}{ccc} S & M & A \end{array} \\ \begin{bmatrix} 2/3 & 1/6 & 1/6 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

since he bought his first car (santro) in 2020, we need to calculate the probability of having a santro after 2 years (2022).

(i) $P(S \text{ in } 2022) = P(S \text{ in } 2020) \times P(S|S)^2$
$$= 1 \times (2/3)^2$$
$$= 4/9$$
$$= 0.4444.$$

(ii) $P(M \text{ in } 2022) = P(S \text{ in } 2020) \times P(M|S) \times P(M|A)$
$$= 1 \times (1/6) \times 1$$
$$= 1/6$$
$$= 0.1667$$

$\therefore$ (i) = 0.4444

(ii) = 0.1667

5) c) Prove that the Markov chain where Transition Probability Matrix is

$$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

is irreducible and find the corresponding stationary probability vector.

$$P^2 = \begin{bmatrix} 0.5 & 0.28 & 0.22 \\ 0.2 & 0.66 & 0.14 \\ 0.6 & 0.12 & 0.28 \end{bmatrix}$$

∴ It is regular stochastic matrix.

∴ It is irreducible

Let $v = (x\ y\ z)$ where $x + y + z = 1$

$vP = V$

$$\cancel{(x\ y\ z)\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = [x\ y\ z]}$$

$$[x\ y\ z]\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix} = [x\ y\ z]$$

$0.6x + 0.1y + 0.6z = x \rightarrow ①$

$0.2x + 0.8y = y \rightarrow ②$

$0.2x + 0.1y + 0.4z = z \rightarrow ③$

$0.2x + 0.8y = y \rightarrow ②$

$0.2x = y - 0.8y$

$0.2x = 0.2y$

$\boxed{x = y}$

$0.2y + 0.1y + 0.4z = z \rightarrow ③$

$0.3y + 0.4z = z$

$0.3y = z - 0.4z$

$0.3y = 0.6z$

$z = \dfrac{0.3y}{0.6}$

$\boxed{z = 0.5y}$.

$0.6x + 0.1y + 0.6z = x \rightarrow ①$

$0.6x + 0.1x + 0.6z = x$

$0.7x + 0.6z = x$

$0.6z = x - 0.7x$

$0.6z = 0.3x$

$x = 2z$

$x = 2(0.5y)$

$\boxed{x = y}$

$x + y + z = 1$

$y + y + 0.5y = 1$

$2.5y = 1$

$y = 1/2.5$

$\boxed{y = 0.4} = \boxed{x = 0.4}$

$z = 0.5y$

$= 0.5(0.4)$

$\boxed{z = 0.2}$.

$[x \; y \; z] = [0.4, 0.4, 0.2]$

6) a) The joint probability distribution of two random variable X and Y are given below.

$Y \rightarrow$ / $X \downarrow$	1	3	6
1	1/9	1/6	1/18
3	1/6	1/4	1/12
6	1/18	1/12	1/36

Determine i) $E(X)$ and $E(Y)$ ii) $E(XY)$ iii) $\mathrm{cov}(X, Y)$.

$x \backslash y$	1	3	6	$f(x)$
1	1/9	1/6	1/18	1/3
3	1/6	1/4	1/12	1/2
6	1/18	1/12	1/36	1/6
$g(y)$	1/3	1/2	1/6	1

(i) $E(X) = \Sigma x_i f(x_i)$

$= 1(1/3) + 3(1/2) + 6(1/6)$

$= 1/3 + 3/2 + 1$

$E(X) = 17/6$

$E(Y) = \Sigma y_j g(y_j)$

$= 1(1/3) + 3(1/2) + 6(1/6)$

$= 1/3 + 3/2 + 1$

$E(Y) = 17/6$

(ii) $E(XY) = \Sigma_i \Sigma_j x_i y_j f(x_i y_j)$

$= 1(1)(1/9) + 1(3)(1/6) + 1(6)(1/18) + 3(1)(1/6) + 3(3)(1/4) +$
$3(6)(1/12) + 6(1)(1/18) + 6(3)(1/12) + 6(6)(1/36)$

$= 1/9 + 1/2 + 1/3 + 1/2 + 9/4 + 1 + 1/3 + 3/2 + 1$

$E(XY) = \frac{271}{36}$

(iii) $cov(XY) = E(XY) - E(x).E(Y)$.

$= \frac{271}{36} - (17/6)(17/6)$

$= \frac{271}{36} - \frac{289}{36}$.

$cov(XY) = -1/2$.

6) b) Determine the unique fixed probability vector of

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

$v(x\ y\ z)$ such that $x+y+z=1$

$$(x\ y\ z)\begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix} = (x\ y\ z)$$

$\frac{y}{6} = x$

$y = 6x$

$x + \frac{y}{2} + \frac{2z}{3} = y$

$\cancel{x} + \cancel{\frac{6x}{2}} + \cancel{\frac{2z}{3}} = \cancel{y}$

$x + \frac{2z}{3} = y - \frac{y}{2}$

$x + \frac{2z}{3} = \frac{y}{2}$

$x + \frac{2z}{3} = \frac{2z}{2}$

$x = z - \frac{2z}{3}$

$x = z/3$

$z = 3x$

$\frac{y}{3} + \frac{z}{3} = z$

$\frac{y}{3} = z - z/3$

$\frac{y}{3} = \frac{2z}{3}$

$y = 2z$.

$$x + y + z = 1$$
$$x + 6x + 3x = 1$$
$$10x = 1$$
$$x = 1/10$$
$$y = 6/10$$
$$z = 3/10$$
$$v = [\,1/10 \quad 6/10 \quad 3/10\,]$$

6) c) A software engineer goes to his work place every day by motor bike or by car. He never goes by bike on two consecutive days but if he goes by car on a day he is equally like to go by car or by bike on the next day. Find the transition matrix for the chain of the mode of transport he uses. If car is used on the first day of the week find the probability that i) bike is used ii) car is used after 4 days (or on the fifth day).

B → bike

C → car

$$P = \begin{array}{c|cc} & B & C \\ \hline B & 0 & 1 \\ C & 1/2 & 1/2 \end{array}$$

$$p^{(0)} = (0 \quad 1)$$

$$p^4 = p^{(0)} \cdot P^4$$

$$P^4 = \begin{bmatrix} 3/8 & 3/8 \\ 5/16 & 11/16 \end{bmatrix} \qquad p^4 = (0 \quad 1)\begin{bmatrix} 3/8 & 5/8 \\ 5/16 & 11/16 \end{bmatrix}$$

$$p^4 = [\,5/16 \quad 11/16\,]$$

After 4 days probability of using bike $= 5/16$

and car $= 11/16$ //

7) a) show that if θ is the angle between lines of regression, then

$$\tan\theta = \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$$

→ If θ is the angle between two lines $y_1 = m_1 x + c_1$

$y_2 = m_2 x + c_2$

then $\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$ where m_1 and m_2 represents slope

Regression line of 'y' on 'x' $\quad y - \bar{y} = r\dfrac{\sigma_y}{\sigma_x}(x - \bar{x}) \rightarrow$ ①

Regression line of 'x' on 'y' $\quad x - \bar{x} = r\dfrac{\sigma_x}{\sigma_y}(y - \bar{y}) \rightarrow$ ②

$$\Downarrow$$

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$$

$m_1 = r\dfrac{\sigma_y}{\sigma_x} \qquad m_2 = \dfrac{1}{r}\dfrac{\sigma_y}{\sigma_x}$

$$\tan\theta = \frac{\frac{1}{r}\frac{\sigma_y}{\sigma_x} - r\frac{\sigma_y}{\sigma_x}}{1 + \frac{1}{r}\frac{\sigma_y}{\sigma_x}\left(r\frac{\sigma_y}{\sigma_x}\right)}$$

$$\tan\theta = \frac{\frac{\sigma_y}{\sigma_x}\left(\frac{1}{r} - r\right)}{1 + \frac{\sigma_y^2}{\sigma_x^2}}$$

$$= \frac{\frac{\sigma_y}{\sigma_x}\left(\frac{1}{r} - r\right)}{\frac{\sigma_x^2 + \sigma_y^2}{\sigma_x^2}}$$

$$= \frac{\sigma_y}{\cancel{\sigma_x}}\left(\frac{\sigma_x^{\cancel{2}}}{\sigma_x^2 + \sigma_y^2}\right)\left(\frac{1-r^2}{r}\right)$$

$$\boxed{\tan\theta = \frac{\sigma_y \sigma_x}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)}$$

//

7) b) Ten competitors in a music contest are ranked by 3 judges A, B and C in the following order. Find the pair of judges have the nearest approach to common taste of music.

A	1	6	5	10	3	2	4	9	7	8
B	3	5	8	4	7	10	2	1	6	9
C	6	4	9	8	1	2	3	10	5	7

$D_1 = R_1 - R_2 \quad D_2 = R_1 - R_3 \quad D_3 = R_2 - R_3$

R_1	R_2	R_3	D_1	D_2	D_3	D_1^2	D_2^2	D_3^2
1	3	6	-2	-5	-3	4	25	9
6	5	4	1	2	1	1	4	1
5	8	9	-3	-4	-1	9	16	1
10	4	8	6	2	-4	36	4	16
3	7	1	-4	2	6	16	4	36
2	10	2	-8	0	8	64	0	64
4	2	3	2	1	-1	4	1	1
9	1	10	8	-1	-9	64	1	81
7	6	5	1	2	1	1	4	1
8	9	7	-1	1	2	1	1	4
						200	60	214

$$\rho = 1 - \frac{6 \Sigma d_i^2}{n^3 - n}$$

$$= 1 - \frac{6 \times 200}{10^3 - 10} = 1 - 1.2121$$

$$\boxed{\rho_1 = -0.2121}$$

$$\rho_2 = 1 - \frac{6 \times 60}{10^3 - 10} = 0.6363$$

$$\boxed{\rho_2 = 0.6363}$$

$$\rho_3 = 1 - \frac{6 \times 214}{10^3 - 10}$$

$$\boxed{\rho_3 = -0.2969}$$

7) c) calculate the coefficient of correlation and regression lines for the following data:

x	55	56	58	59	60	60	62
y	35	38	39	38	44	43	45

x	y	$z = x - y$	x^2	y^2	z^2
55	35	20	3025	1225	400
56	38	18	3136	1444	324
58	39	19	3364	1521	361
59	38	21	3486	1441	441
60	44	16	3600	1936	256
60	43	17	3600	1849	289
62	45	17	3844	2025	289
410	282	128	24050	1144	2360

$\bar{x} = 58.57 \quad \bar{y} = 40.28 \quad \bar{z} = 18.28$

$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = \frac{24050}{7} - 3430.4$

$\sigma_x^2 = 5.2693 \Rightarrow \sigma_x = 2.2954$

$\sigma_y^2 = 12.3787 \Rightarrow \sigma_y = 3.5183$

$\sigma_z^2 = 2.9844 \Rightarrow \sigma_z = 1.7275$

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_z^2}{2\sigma_x \sigma_y}$$

$r = 0.8078$

$y - \bar{y} = r \frac{\sigma_y}{\sigma_x}(x - \bar{x}) \qquad x - \bar{x} = r \frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$y = 1.3914x - 41.21 \qquad x = 0.5922y + 34.72$

$r = \sqrt{(1.3914)(0.5922)}$

$r = 0.9077$.

8) a) obtain the lines of regression and hence find the coefficient of correlation for the data.

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
1	8	-6	-7	36	49	42
3	6	-4	-9	16	81	36
4	10	-3	-5	9	25	15
2	8	-5	-7	25	49	35
5	12	-2	-3	4	9	6
8	16	1	1	1	1	1
9	16	2	1	4	1	2
10	10	3	-5	9	25	-15
13	32	6	17	36	289	102
15	32	8	17	64	289	136
70	150			204	818	360

$\bar{x} = \frac{\Sigma x}{n} = \frac{70}{10} = 7$ $\qquad$ $\bar{y} = \frac{\Sigma y}{n} = \frac{150}{10} = 15$.

Regression Y on X

$y - \bar{y} = \frac{\Sigma XY}{\Sigma X^2}(x - \bar{x})$

$y - 15 = \frac{360}{204}(x - 7)$

$y - 15 = 1.76(x - 7)$

$y - 15 = 1.76x - 12.3529$

$y = 1.76x - 12.3529 + 15$

$\boxed{y = 1.76x + 2.647}$

X on Y

$x - \bar{x} = \frac{\Sigma XY}{\Sigma Y^2}(y - \bar{y})$

$x - 7 = \frac{360}{818}(y - 15)$

$x - 7 = 0.44(y - 15)$

$x - 7 = 0.44y - 6.6014$

$x = 0.44y - 6.6014 + 7$

$\boxed{x = 0.44y + 0.3986}$

8) b) compute the rank correlation co-efficient for the data.

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

x	y	R_x	R_y	$d_i = R_x - R_y$	d_i^2
68	62	4	5	-1	1
64	55	6	7	-1	1
75	68	2.5	3.5	-1	1
50	45	9	10	-1	1
64	81	6	7	5	25
80	60	1	6	-5	25
75	68	2.5	3.5	-1	1
46	48	10	9	1	1
55	50	8	8	0	0
64	70	6	2	4	16
					72

$$\rho = 1 - \frac{6}{n^3-n}\left[\Sigma d^2 + \frac{(m_1^3-m_1)}{12} + \frac{(m_2^3-m_2)}{12} + \frac{(m_3^3-m_3)}{12}\right]$$

$m_1 = 2 \quad m_2 = 3 \quad m_3 = 2$

$$= 1 - \frac{6}{10^3-10}\left[72 + \frac{(8-2)}{12} + \frac{(27-3)}{12} + \frac{(8-2)}{12}\right]$$

$= 1 - 0.0060\,[72 + 0.5 + 2 + 0.5]$

$= 1 - 0.0060\,[75]$

$= 1 - 0.45$

$\boxed{\rho = 0.5500}$ //

2) c) Determine the equation of best fitting parabola $y = a + bx + cx^2$ for the following data:

x	0	1	2	3	4	5	6
y	14	18	23	29	36	40	46

$\Sigma y_i = na + b\Sigma x_i + c\Sigma x_i^2$

$\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2 + c\Sigma x_i^3$

$\Sigma x_i^2 y_i = a\Sigma x_i^2 + b\Sigma x_i^3 + c\Sigma x_i^4$

x_i	y_i	x_i^2	x_i^3	x_i^4	$x_i y_i$	$x_i^2 y_i$
0	14	0	0	0	0	0
1	18	1	1	1	18	18
2	23	4	8	16	46	92
3	29	9	27	81	87	261
4	36	16	64	256	144	576
5	40	25	125	625	200	1000
6	46	36	216	1296	276	1656
21	206	91	441	2275	771	3603

$$206 = 7a + b21 + c91 \rightarrow (1)$$

$$771 = a21 + b91 + c441 \rightarrow (2)$$

$$3603 = a91 + b441 + c2275 \rightarrow (3)$$

$A = 13.4523 \quad B = 4.9642 \quad C = 0.0833$

$$\boxed{y = 13.4523 + 4.9642x + 0.0833x^2}$$ //

9) a) In an examination given to students at a large number of different schools the mean grade was 74.5 and S.D grade was 8. At one particular school where 200 students took the examination the mean grade was 75.9. Discuss the significance of this result at both 5% and 1% level of significance.

$H_0 : \mu = 74.5$

$H_1 : \mu \neq 74.5$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$z = \frac{75.9 - 74.5}{\frac{8}{\sqrt{200}}}$$

$$\frac{8}{\sqrt{200}} = \frac{8}{14.142} \approx 0.5657$$

$$z = \frac{75.9 - 74.5}{0.5657} \approx 2.476$$

at 5% $z = \pm 1.96$

$\therefore$ failed.

at 1% $z = \pm 2.576$.

$2.476 > 1.96$

$2.476 < 2.576$.

9) b) In a city A, 20% of random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of random sample of 1600 school boys had the same defect. Is the difference between the proposing significant? Test at 5% significance level.

→ $P_1 = 0.20$

$P_2 = 0.185$

$n_1 = 900$

$n_2 = 1600$

$$SE = \sqrt{\frac{P_1(1-P_1)}{n_1} + \frac{P_2(1-P_2)}{n_2}}$$

$$SE = \sqrt{\frac{0.20(1-0.20)}{900} + \frac{0.185(1-0.185)}{1600}}$$

$$\Rightarrow \frac{0.20(0.80)}{900} = \frac{0.16}{900} \approx 0.0001778$$

$$\Rightarrow \frac{0.185(0.815)}{1600} = \frac{0.1502}{1600} \approx 0.0000939$$

$$Z = \frac{P_1 - P_2}{SE}$$

$$Z = \frac{0.20 - 0.185}{0.0165}$$

$$= \frac{0.015}{0.0165} \approx 0.909$$

$|0.909| < 1.96$.

∴ we accept the hypothesis.

9) c) Four coins are tossed 100 times and the following results were obtained.

No of Heads	0	1	2	3	4
Frequency	5	29	36	25	5

Fit a binomial distribution for the data and test the goodness of fit ($\chi^2_{0.05} = 9.49$ for 4 d.f.).

→ $P(X=0) = 1(1)(0.0625) = 0.0625 \times 100 = 6.25$

$P(X=1) = 4(0.5)(0.125) = 0.25 \times 100 = 25$

$P(X=2) = 6(0.25)(0.25) = 0.375 \times 100 = 37.5$

$P(X=3) = 4(0.125)(0.5) = 0.25 \times 100 = 25$

$P(x=4) = 1(0.0625)(1) = 0.0625 \times 100 = 6.25$

No of heads	freq	E_i	$\frac{(O_i - E_i)^2}{E_i}$
0	5	6.25	0.25
1	29	25	0.64
2	36	37.5	0.06
3	25	25	0
4	5	6.25	0.25
			1.2.

$$\chi^2_{cal} = 1.2 < 9.49 = \chi^2_{0.05}$$

$\therefore$ Hypothesis is accepted.

$\therefore$ It accepts binomial distribution.

10) a) A coin tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased at 1% level of significance.

$n = 1000$

$x = 540$

$p = 1/2 = 0.5$

$q = 1/2 = 0.5$

$Z_{0.05} = 1.96$

$$Z = \frac{x - np}{\sqrt{npq}}$$

$$= \frac{540 - 1000(0.5)}{\sqrt{1000(0.5)(0.5)}}$$

$$= \frac{40}{15.8113}$$

$$Z = 2.5298$$

$$2.5298 > 2.58 = Z_{0.01}$$

$\therefore$ Hypothesis is ~~rejected~~ accepted.

10) b) The mean and standard deviation of the maximum loads supported by 60 cables are 11.09 tons and 0.73 tons respectively. Find 99% confidence limits for the mean of the maximum loads of all cables produced by the company.

$\rightarrow$ $n = 60$

$\bar{x} = 11.09$

$\sigma = 0.73$

$$\bar{x} \pm 2.58\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$11.09 \pm 2.58\left(\frac{0.75}{\sqrt{60}}\right)$$

$$= +11.3331$$

$$= 10.8469.$$

$$\Rightarrow 10.85 \leq \mu \leq 11.33.$$

10) c) A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5, 3, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure? ($t_{0.05} = 2.201$ for 11 d.f.)

x	$(x-\bar{x})^2$
5	5.8564
2	0.3364
8	29.3764
-1	12.8164
3	0.1764
0	6.6564
6	11.6964
-2	20.9764
1	2.4964
5	5.8564
0	6.6564
4	2.0164
31	104.9168

$n = 12$

$D = n-1$

$D = 11$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{31}{12} = 2.58.$$

H_0: stimulus will not increase.

$$s^2 = \frac{1}{n-1}\Sigma(x-\bar{x})^2$$

$$= \frac{1}{11}(104.9168)$$

$$s^2 = 9.5379$$

$$s = 3.0883$$

$$t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{2.58-0}{\frac{3.0883}{\sqrt{12}}}$$

$$= \frac{2.58-0(\sqrt{12})}{3.0883}$$

$$t = 2.8939$$

$$t_{cal} = 2.89 > t_{0.05} = 2.201$$

∴ Hypothesis (H_0) is rejected.

∴ stimulus will increase the blood pressure.