NEURAL NETWORKS & DEEP LEARNING

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BINARY CLASSIFICATION

- $(x, y), x \in R^{n_x}, y \in \{0,1\}$
- $m: \#Training\ examples$
- $m_{test}: \#Test\ examples$
- $X = \begin{bmatrix} \vdots & \cdots & \vdots \\ x^{(1)} & \cdots & x^{(m)} \\ \vdots & \cdots & \vdots \end{bmatrix} \in (n_x, m)$
- $Y = \left[\, \mathrm{y}^{(1)} \ \ldots \ \mathrm{y}^{(\mathrm{m})} \right] \in (1, m)$

<u>Logistic regression</u>:

- $\hat{y} = P(y = 1|x) = \sigma(z + b); z = w^T \rightarrow \ \hat{Y} = \sigma(\Theta^{\mathrm{T}}\mathrm{X})$
- <u>Parameters</u>: $w \in R^{n_x}, b \in R \ \rightarrow \Theta = \begin{bmatrix} \theta_0 = b \\ \theta_1 \\ \vdots \\ \theta_{n_x} \end{bmatrix} \left.\vphantom{\begin{matrix} \theta_1 \\ \vdots \\ \theta_{n_x} \end{matrix}}\right\} \mathrm{w}$
- <u>PS</u>: $X_0 = 1, x \in R^{n_x} + 1$
- <u>Sigmoid function</u>: $\sigma(z) = \frac{1}{1+e^{-z}}$

<u>Loss error function</u>: $L(\hat{y}, y) = -ylog\hat{y} + (1 - y)\log(1 - \hat{y}) = \frac{1}{2}(\hat{y} - y)^2$

<u>Cost function</u>: $J(w, b) = \frac{1}{m}\sum_{i=1}^{m} L(\hat{y}^{(i)}, y^{(i)})$

GRADIENT DESCENT

Want to find w, b that minimize $J(w,b)$

Repeat {

$$w := w - \alpha \frac{dJ(w,b)}{dw}$$

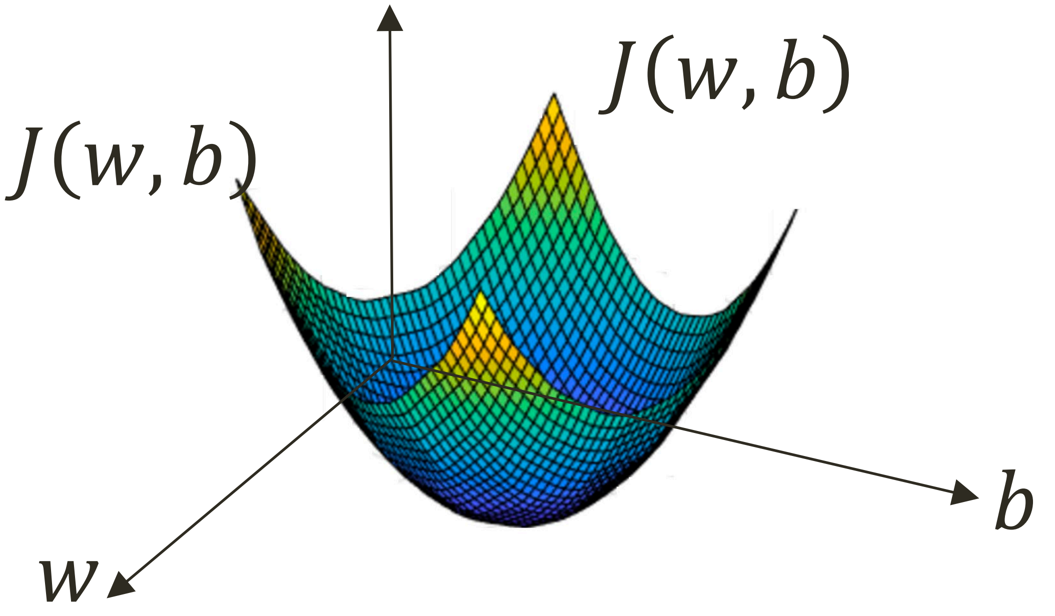
$$b := b - \alpha \frac{dJ(w,b)}{db}$$

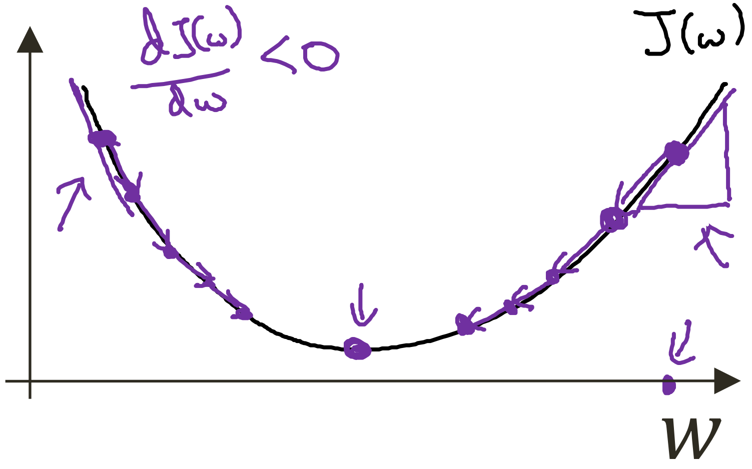
}

$\alpha = learning\ rate$

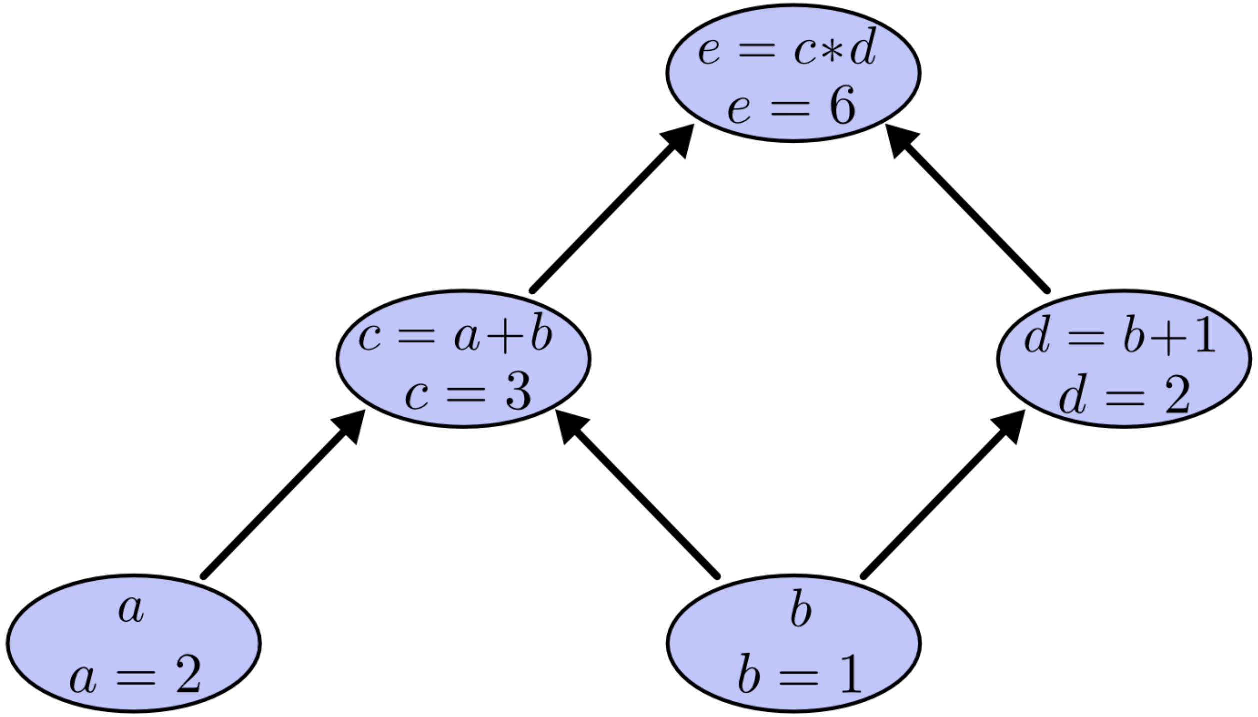
$dw = \frac{dJ(w,b)}{dw}$

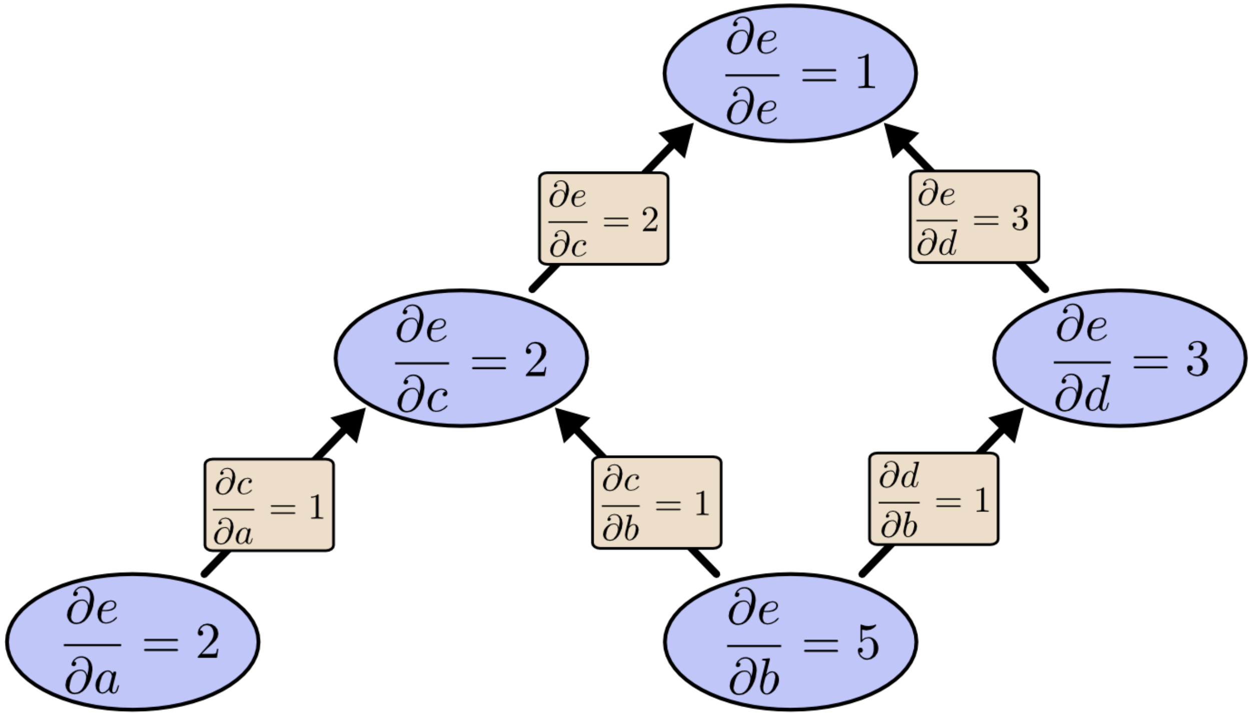
$db = \frac{dJ(w,b)}{db}$



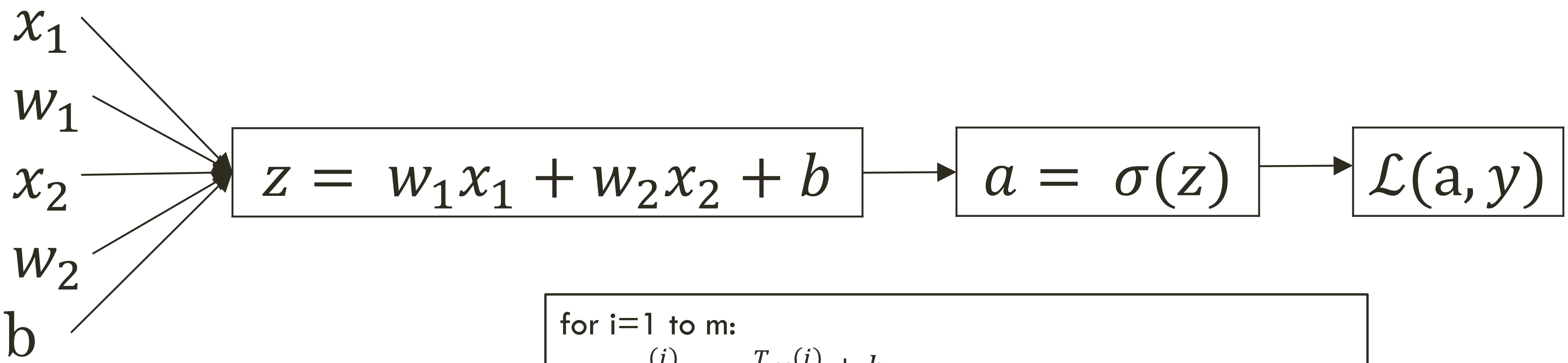


COMPUTATION GRAPH




$\frac{\partial e}{\partial e} = 1$
$\frac{\partial e}{\partial c} = 2$
$\frac{\partial e}{\partial d} = 3$
$\frac{\partial e}{\partial c} = 2$
$\frac{\partial e}{\partial d} = 3$
$\frac{\partial c}{\partial a} = 1$
$\frac{\partial c}{\partial b} = 1$
$\frac{\partial d}{\partial b} = 1$
$\frac{\partial e}{\partial a} = 2$
$\frac{\partial e}{\partial b} = 5$

LOGISTIC REGRESSION DERIVATIES



x_1

w_1

x_2

w_2

b

$z = \ w_1x_1 + w_2x_2 + b$

$a = \ \sigma(z)$

$\mathcal{L}(\mathrm{a}, y)$

$$dz = \frac{dL}{dz} = a - y$$

$$da = \frac{dL}{da} = -\frac{y}{a} + \frac{1-y}{1-a}$$

$$dw = x.dz$$

$$db = dz$$

```
for i=1 to m:
        $z^{(i)} = w^T x^{(i)} + b$
        $a^{(i)} = \sigma\left(z^{(i)}\right)$
        $J \mathrel{+}= -[y^{(i)} loga^{(i)} + \left(l1 - y^{(i)}\right)\log(1 - a^{(i)})$
        $dz^{(i)} = a^{(i)} - y^{(i)}$
        $dw_1 \mathrel{+}= x_1^{(i)} dz^{(i)}$
        $dw_2 \mathrel{+}= x_2^{(i)} dz^{(i)}$
        $db \mathrel{+}= dz^{(i)}$
$J /= m; dw_1 /= m; \mathrm{dw}_2 /= m; db /= m$
```

VECTORIZATION

Non vectorized: For loop → Slow

Vectorized: Tensor calculations → Fast

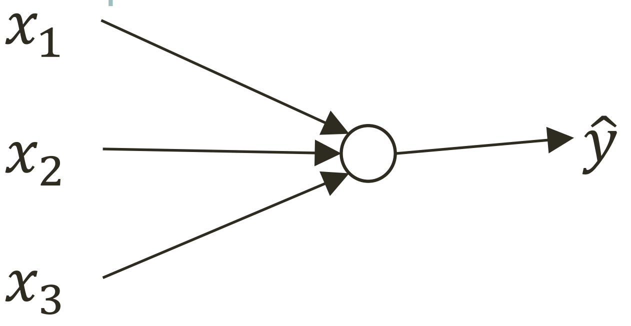
- Whenever possible, avoid explicit for-loops
- $u = Av \ \rightarrow u = np.dot(A, v)$
- $w_1, w_2 \dots \rightarrow one\ W$

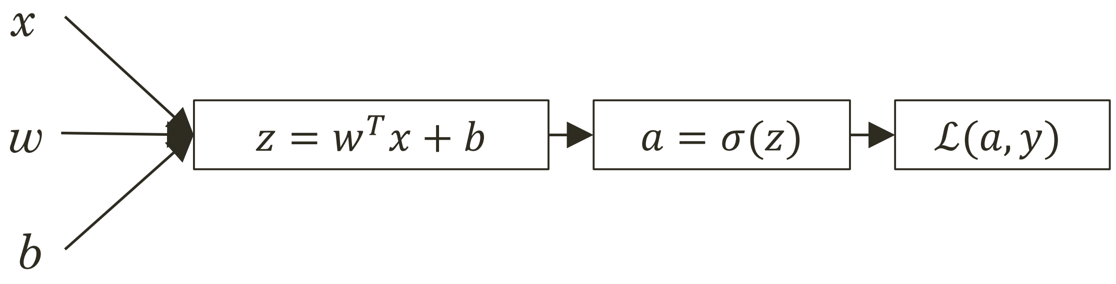
Vectorizing logistic regression:

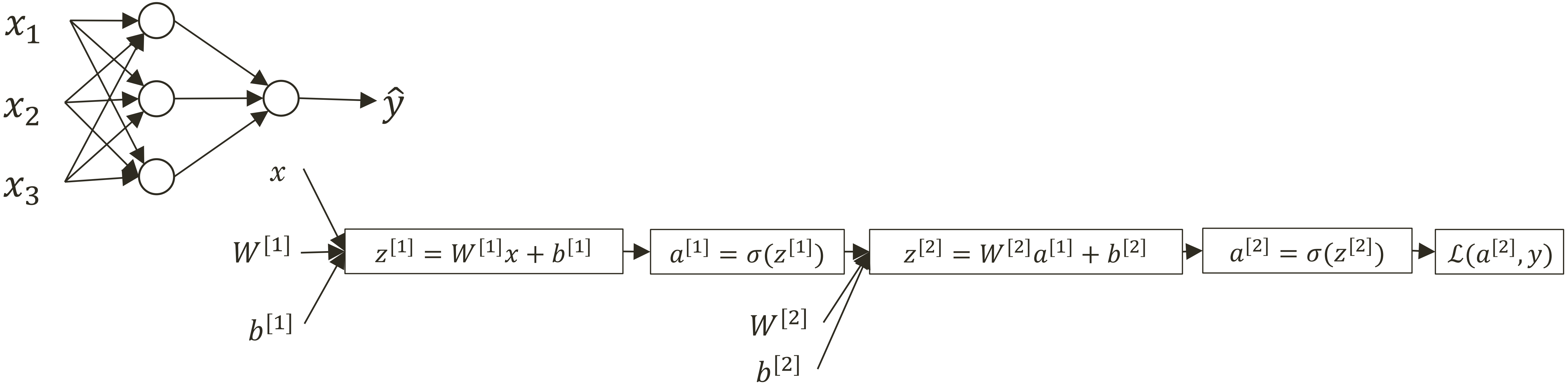
Python Broadcasting

- $Z = \left[z^{(1)} \dots z^{(n+1)}\right] = w^T X + [b \dots b] = \left[w^T x^{(1)} + b \dots w^T x^{(2)} + b\right] = \boldsymbol{np.dot(w.T, X) + b}$
- $A = \left[a^{(1)} \dots a^{(m)}\right] = \boldsymbol{\sigma(Z)}$
- $dZ^{[L]} = \boldsymbol{A}^{[L]} - \boldsymbol{Y} = \left[a^{(1)} - y^{(1)} \dots a^{(m)} - y^{(m)}\right]; dZ^{[l]} = w^{[l+1]T} dZ^{[l+1]} * g^{[l]'}\left(Z^{[l]}\right)$
- $db^{[l]} = \boldsymbol{\frac{1}{n} np.sum\left(dZ^{[l]}, axis = 1, keepdims = True\right)}, dw^{[l]} = \boldsymbol{\frac{1}{n} dZ^{[l]} A^{[l-1]T}}$
- $w := \boldsymbol{w - \alpha dw}; \quad b := \boldsymbol{b - \alpha db}$

NEURAL NETWORK







x_1
x_2
x_3
$\hat{y}$
x
$W^{[1]}$
$b^{[1]}$
$z^{[1]} = W^{[1]} x + b^{[1]}$
$a^{[1]} = \sigma(z^{[1]})$
$W^{[2]}$
$b^{[2]}$
$z^{[2]} = W^{[2]} a^{[1]} + b^{[2]}$
$a^{[2]} = \sigma(z^{[2]})$
$\mathcal{L}(a^{[2]}, y)$

NEURAL NETWORK REPRESENTATION

$$z^{[l]} = \begin{bmatrix} \vdots \\ z_i^{|l]} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ w_i^{[l]T} X + b_i^{[l]} \\ \vdots \end{bmatrix} = W^{[l]} x + b^{[l]}$$

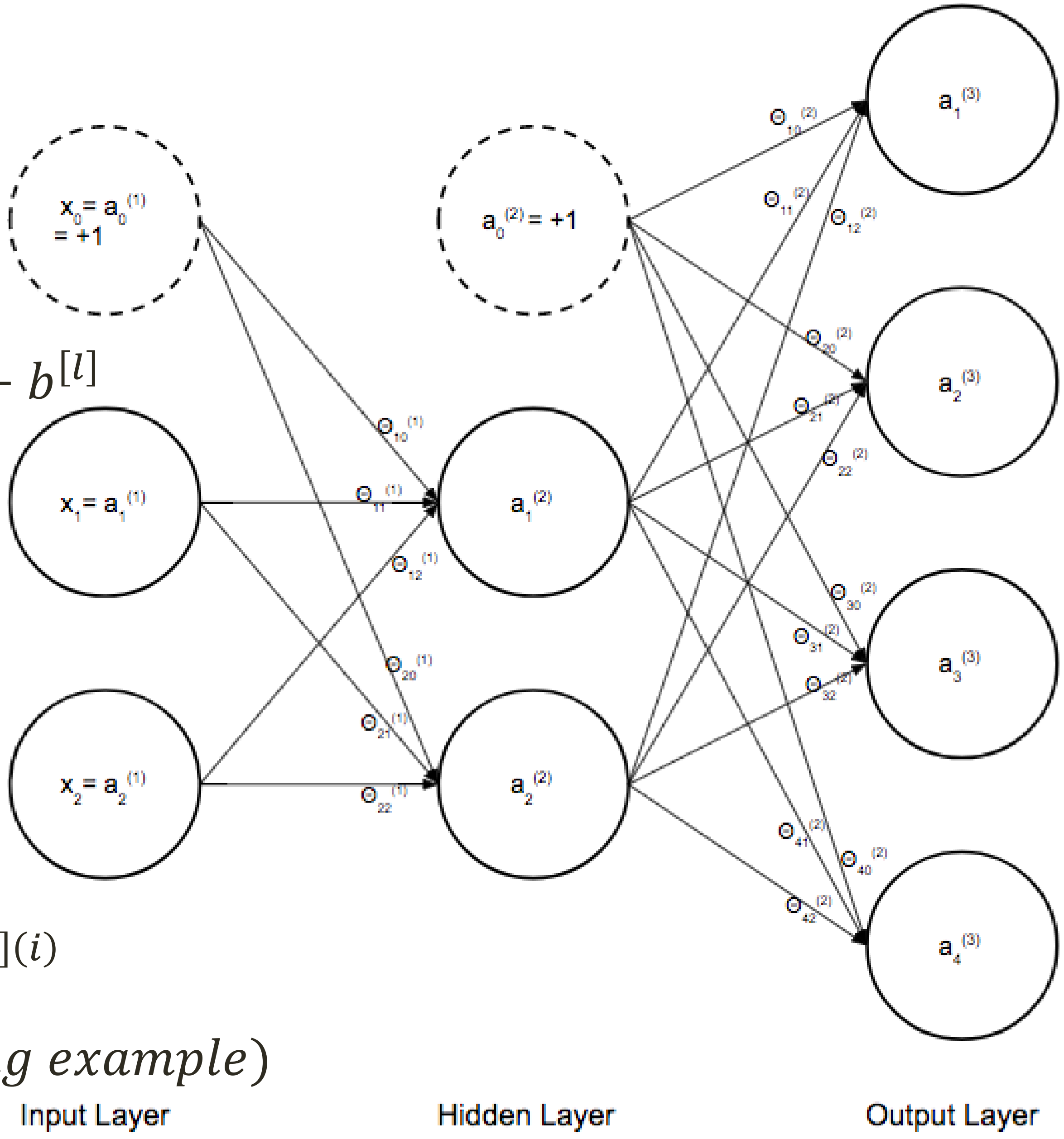
$$a^{[l]} = \begin{bmatrix} \vdots \\ a_i^{[l]} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ \sigma\left(z_i^{[l]}\right) \\ \vdots \end{bmatrix} = \sigma(z^{[l]})$$

$a_j^{[l](i)} = activation\ in\ node\ j\ in\ layer\ l$

$in\ example\ i$

➔ $for\ i = 1\ to\ m$: $Calculate\ z^{[l](i)}, a^{[l](i)}$

PS: $\mathrm{Z}^{[l]}, \mathrm{A}^{[l]} \in (\#hidden\ units, \#training\ example)$



MORE ON VECTORIZED IMPLEMENTATION

$$\underline{z^{[1](1)}} = W^{[1]}\underline{x^{(1)}} + \cancelto{0}{b^{[1]}}, \quad \underline{z^{[1](2)}} = W^{[1]}\underline{x^{(2)}} + \cancelto{0}{b^{[1]}}, \quad \underline{z^{[1](3)}} = W^{[1]}\underline{x^{(3)}} + \cancelto{0}{b^{[1]}}$$

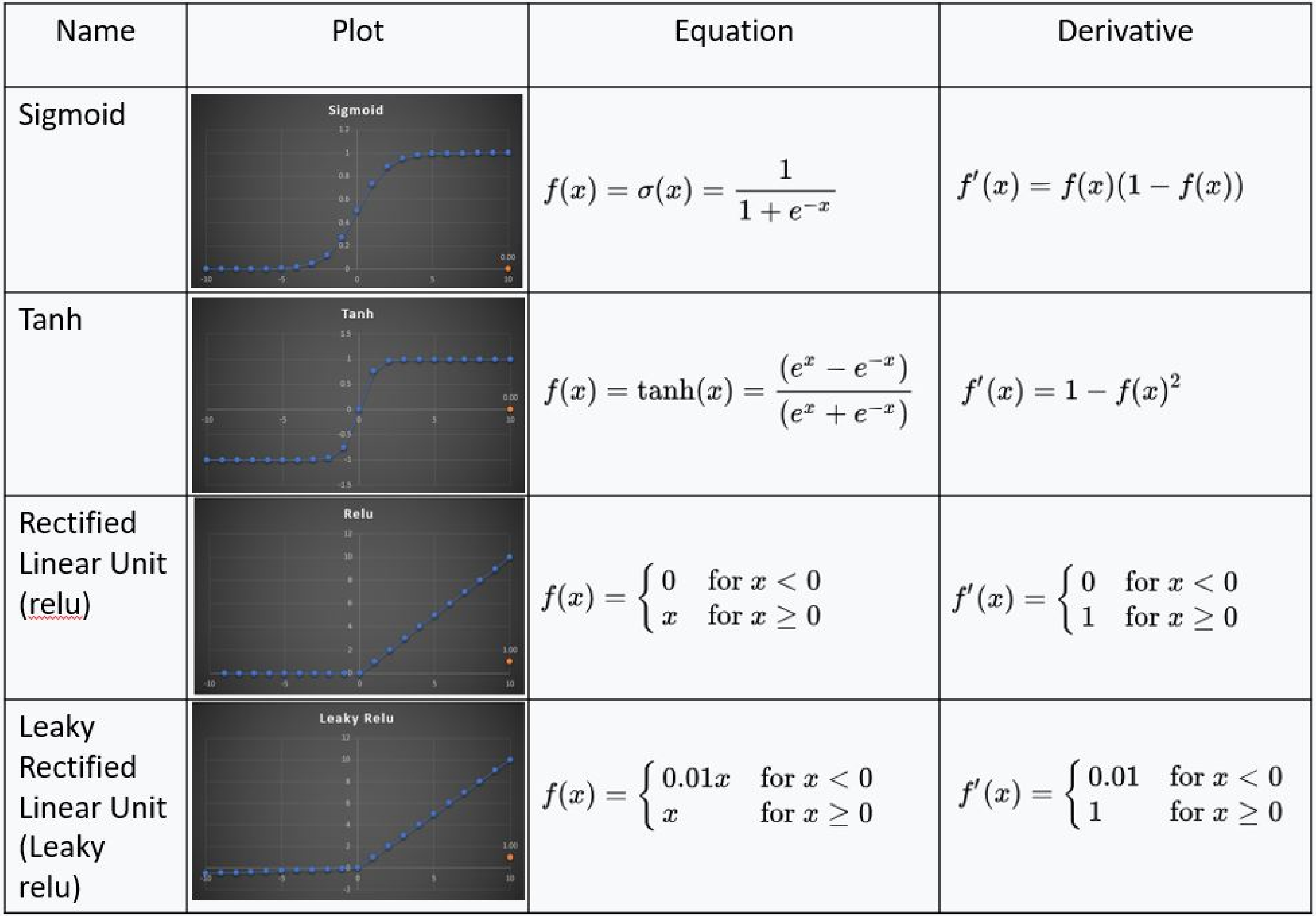
$$W^{[1]} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} \qquad W^{[1]}x^{(1)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(2)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad W^{[1]}x^{(3)} = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

$$W^{[1]} \underbrace{\begin{bmatrix} | & | & | & \\ x^{(1)} & x^{(2)} & x^{(3)} & \cdots \\ | & | & | & \end{bmatrix}}_{X} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} = \begin{bmatrix} | & | & | & \\ z^{[1](1)} & z^{[1](2)} & z^{[1](3)} & \cdots \\ | & | & | & \\ \uparrow & \uparrow & \uparrow & \\ +b^{[1]} & +b^{[1]} & +b^{[1]} & \end{bmatrix} = Z^{[1]}$$

$$Z^{[1]} = W^{[1]}X + b^{[1]}$$

$$W^{[1]}x^{(i)} = z^{[1](i)}$$

ACTIVATION FUNCTIONS



Name	Plot	Equation	Derivative
Sigmoid		$f(x) = \sigma(x) = \frac{1}{1+e^{-x}}$	$f'(x) = f(x)(1 - f(x))$
Tanh		$f(x) = \tanh(x) = \frac{(e^{x} - e^{-x})}{(e^{x} + e^{-x})}$	$f'(x) = 1 - f(x)^2$
Rectified Linear Unit (relu)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Leaky Rectified Linear Unit (Leaky relu)		$f(x) = \begin{cases} 0.01x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0.01 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$

SUMMARY:
FORWARD AND BACKWARD PROPAGATION

$$
\begin{aligned}
Z^{[1]} &= W^{[1]}X + b^{[1]} \\
A^{[1]} &= g^{[1]}(Z^{[1]}) \\
Z^{[2]} &= W^{[2]}A^{[1]} + b^{[2]} \\
A^{[2]} &= g^{[2]}(Z^{[2]}) \\
&\vdots \\
A^{[L]} &= g^{[L]}\left(Z^{[L]}\right) = \hat{Y}
\end{aligned}
$$

$$
\begin{aligned}
dZ^{[L]} &= A^{[L]} - Y \\
dW^{[L]} &= \frac{1}{m} dZ^{[L]} A^{[L]^T} \\
db^{[L]} &= \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[L]}, axis = 1, keepdims = True) \\
dZ^{[L-1]} &= dW^{[L]^T} dZ^{[L]} g'^{[L]}(Z^{[L-1]}) \\
&\vdots \\
dZ^{[1]} &= dW^{[L]^T} dZ^{[2]} g'^{[1]}(Z^{[1]}) \\
dW^{[1]} &= \frac{1}{m} dZ^{[1]} A^{[1]^T} \\
db^{[1]} &= \frac{1}{m} np.\mathrm{sum}(\mathrm{d}Z^{[1]}, axis = 1, keepdims = True)
\end{aligned}
$$

WEIGHT INITIALIZATION AND DIMENSIONALITY

Zero initialization → Activations will be the same

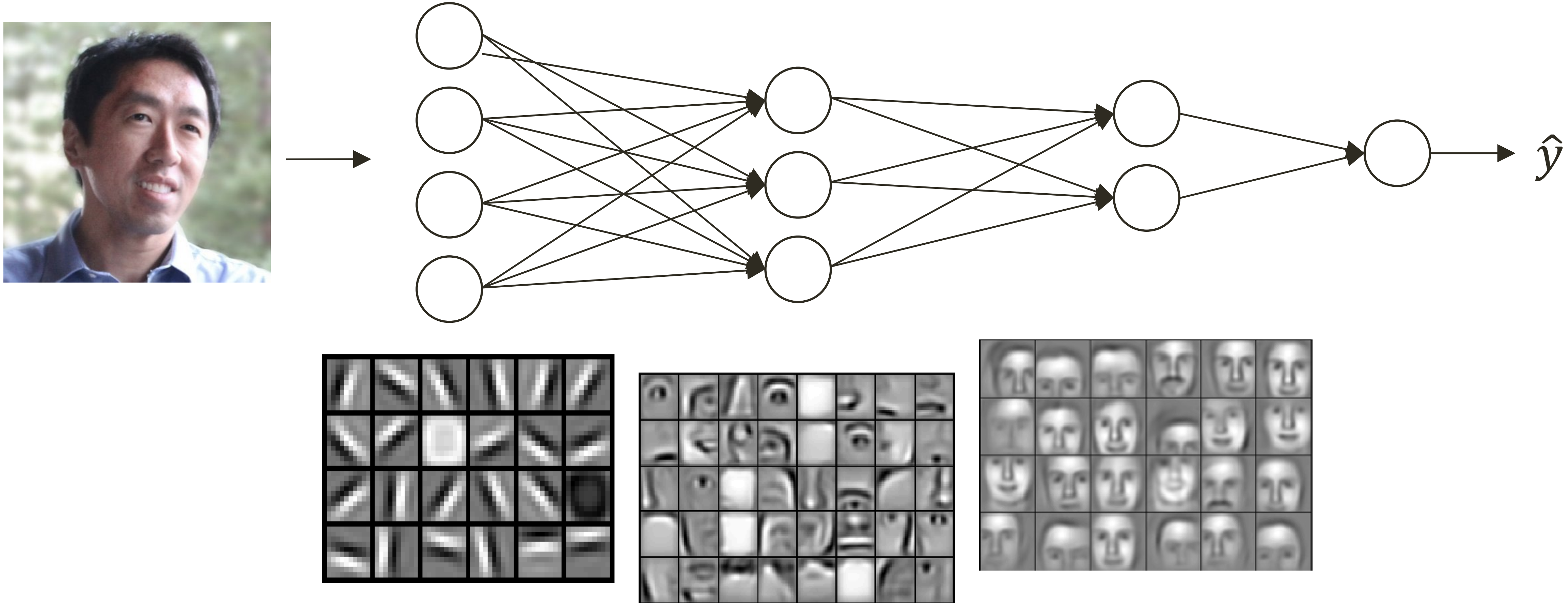
Random initialization → **Good**

PS: b can be initialized to zero

<u>Notations</u>:

- $n^{[l]} = \#units\ in\ layer\ l$
- $W^{[l]}, dW^{[l]} \in \left(n^{[l]}, n^{[l-1]}\right)$
- $A^{[l]}, Z^{[l]}, b^{[l]}, db^{[l]} \in \left(n^{[l]}, 1\right)$
- $A^{[l]}, dA^{[l]}, Z^{[l]}, dZ^{[l]} \in \left(n^{[l]}, m\right)$

DEEP REPRESENTATION

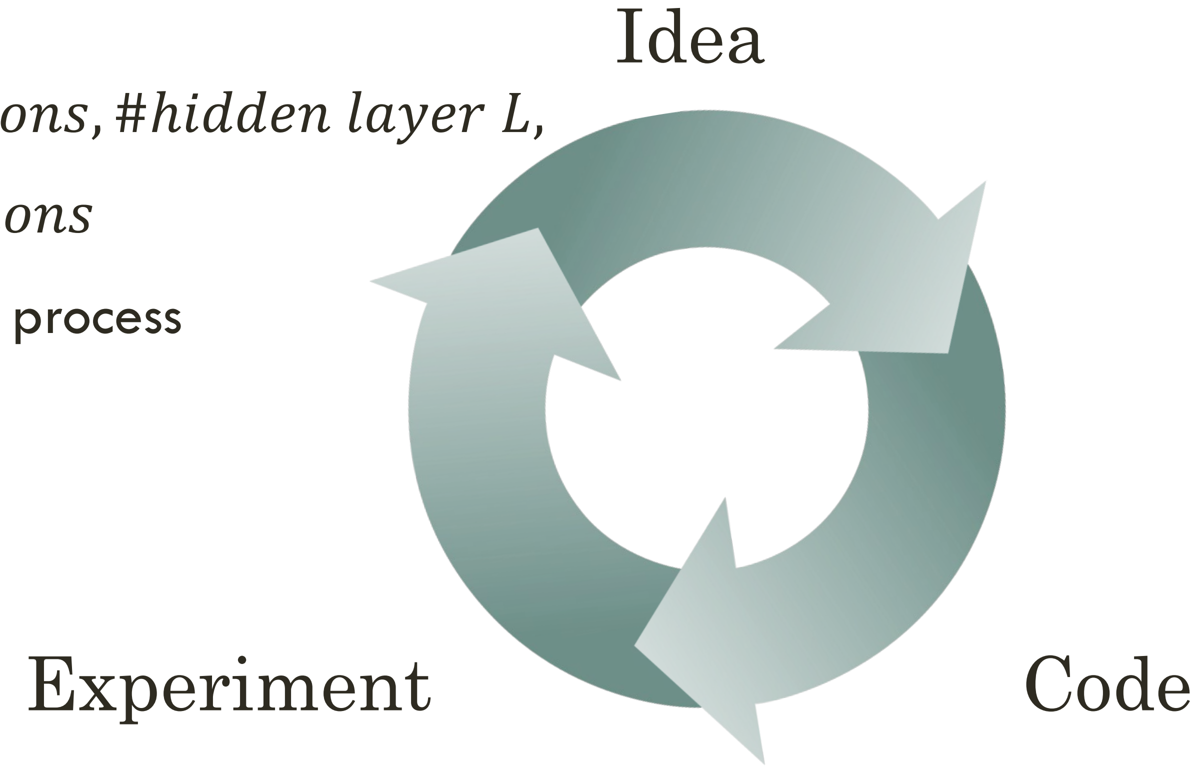


HYPERPARAMETERS

Parameters: W, b

Hyperparameters: $Learning\ rate\ \alpha, \#iterations, \#hidden\ layer\ L,$ $\#hidden\ units, choice\ of\ activation\ functions$

PS: Applying deep learning is a very empirical process



Idea

Code

Experiment