(9)

$6M + 8B \times 10 = 1$

$60M + 80B = 1 \rightarrow (1)$

$(26M + 48B) \times 2 = 1$

$52M + 96B = 1 \rightarrow (2)$

$\Rightarrow 780M + 1040B = 13 \rightarrow (3)$

(ii) $\times 15$

$\Rightarrow 780M + 1440B = 15 \quad (iv)$

iv - (iii)

$$\begin{array}{r} 780M + 1440B \\ -(780M + 1040B) \\ \hline 400B = 2 \end{array}$$

$\Rightarrow B = 1/200$

B in (1)

$60M + 80 \times 1/200 = 1$

$60M + 2/5 = 1 \rightarrow 60M = 3/5$

$M = 1/100$

$\therefore 15 \times 1/100 + 20 \times 1/200 = 15/100 + 20/200$

$= 0.15 + 0.1 = 0.25 = 1/4$

$\frac{1}{1/4} = 4$ days

(3) 80% in 20 days

A's = 80/20 = 4%

20% → A+B in 3 days

A+B = 20/3 ≈ 6.67%/day

4%/day → B's 6.67% − 4%

= 2.67% ≈ 8/3%

100% = 100/(8/3)

⇒ 100 × 3/8 = 37.5 days

(4) A → 1/24 ; B = 1/36, C = 1/72

4 × 1/24 = 1/6

B+C = 1/36 + 1/72 = (2+1)/72 = 3/72 = 1/24

4 days = 1/6 + 1/24 = 5/24

⇒ 24/5 = 4.8 = 4 + 1

16 days : 4 × 5/24 = 20/24

1 − 20/24 = 4/24 = 1/6 = 1 day

Types of numbers –

(1) Power cycles

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 6$ } 4

$2^5 = 2$ → 3 (2) take only unit place.
$2^6 = 4$
$2^7 = 8$
$2^8 = 6$

(2) $\frac{2^{71}}{10}$. unit place?

$71 \div 4 = 17$, remainder 3

$2^3 = 8$

(4) 2^{51}

$51 \div 2 = 25$, remainder 1

$2^1 = 2$

For 3:

$3^1 = 3$ $3^5 = 3$
$3^2 = 9$ $3^6 = 9$
$3^3 = 27$ $3^7 = 7$
$3^4 = 1$ $3^8 = 1$

(2) : 3^{86}

$86 \div 4 = 21$, remainder 2

$3^2 = 9$

3) $\frac{n(n+1)^2)}{2}$

Sum 34

Prod 8

$x + y = 34$

$x - y = 8$

$2x = 42$

$x = 21$

$20 + y = 34$

$y = 34 - 21$

$y = 13$ ⇒ $13 \times 21 = 273$

$14 \times 20 = 280$

product $(S+D)(S-D)/4$

$(34+2)(34-2)$

$\frac{21 \cancel{48} \times 36^{18}}{4 \cancel{2}}$

$P = 72$

Case 2:

$\text{Sum} = \sqrt{D^2 + 4P}$

$P = 72$

$D = 4$

$\Rightarrow \sqrt{(4)^2 + 4(72)}$

$\Rightarrow \sqrt{16 + 308}$

$= \sqrt{344}$

Case 3:

$D = \sqrt{S^2 - 4P}$

$P = 544$

$S = 49$

$\Rightarrow \sqrt{(49)^2 - 4(544)}$

$\sqrt{2401 - 42176}$

44

44

16   16+16   16   1936 $\sqrt{225}$

33   6

(8) 123!

$\frac{123^{24}}{8} + \frac{123^{4}}{25} + \frac{123}{125}$

$24 + 4 + 0 = 28$

(9) 137!

$\frac{137^{27}}{8} + \frac{137^{5}}{25} + \frac{137}{125} + \frac{137}{625}$ —

$27 + 5 + 1 + 0 = 33$

(8) Number system

⇒ divisibility rules (÷)

÷ by 2

unit 2, 4, 6, 8, 0

⇒ ÷ by 3

sum of digit is ÷ by 3

⇒ ÷ by 9

sum of digits is ÷ by 9

(a) 5746520

(2+8+6+7) − (5+3+4+0)

23 − 14 = 9

Not div by 11

(b) Div by 25

00 or 25

(c) Div by 7 or 13

diff of (sum of odd no) − (sum of even)

ex:- 4537792

4 (537 / 792

(792 + 4) − 537

796 − 537 = 259 ✓

for 7 div by 7

for 13 div by 13

796
537
259

(d) Sum of 1st n natural numbers $= \frac{n(n+1)}{2}$

$\frac{10 \times 11}{2} = \frac{55}{2}$ 5.5

(d) (sum of 1 to 110) − (sum of 1 to 90)

$\frac{n\,r(n+1)}{2} - n\frac{(n+1)}{2}$

$\frac{110 \times 111}{2} - \frac{90 \times 91}{2}$

6105 4095

$\frac{12210}{2} - \frac{8190}{2}$

$= 6105 - 4095$

$= 2010$

(e) $3401 \div 11$

$11 \overline{)3401}$ (307)

33

$\overline{10}$

$\overline{01}$

$\overline{11}$

$\overline{0}$

$\frac{P \times R \times T}{100} = SI$

$\frac{3900 \times 3}{100} = 117$ 3yy

Amt = P + SI

815 = P + 117

P = 815 − 117

= 698

$A = P\left(1 + \frac{\frac{R}{4}}{100}\right)^{4n}$

$A = 5000\left(1 + \frac{1.5}{100}\right)^{36}$

$5000\left(\frac{1000 + 1.5}{1000}\ \ \frac{1001.5}{1000}\right)^{24}$

$\Rightarrow 5000\left(\frac{1001.5}{1000}\right)^{24}$

$5000\left(\frac{1000 + 1.5}{1000}\right)^{3} \Rightarrow$

$\Rightarrow 5000\left(\frac{101.5}{100}\right)^{3}$

(6) $x, x+2, x+4$

$3x+6$

(7)

(8)

a

$\frac{a \times a \times a + b \times b \times b}{a \times a - a \times b + b \times b} \Rightarrow \frac{a^3+b^3}{a^2-ab+b^2}$

$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$\frac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2} = a+b$

$\frac{76}{233} = 100$

(9)

If $a+b+c=0$

$a+b=-c$

$b+c=-a$

$a+c=-b$

$(a+b)(b+c)(c+a)-2$

$\Rightarrow -c \cdot -b \cdot -a$

$\Rightarrow -abc$

(4) j Average & percentage

Sum = $\frac{n \times (n+1)}{2}$

$\frac{101 \times 101}{2}$

$\frac{5050}{2}$

$\frac{10100}{2}$

$= 5050$

$\frac{2}{10100}$

(5) Average of 18 players age = 23

Highest = 95 Lowest = ?

$\frac{?}{18} = 23$; $\frac{?}{18} = 25$

$\frac{414}{1494}$

Highest + lowest = 139

$= 139$

$\frac{45}{37}$ $= 39$

(6) $\frac{A+B+C}{3} = 45$ $= 135$

$\frac{A+B}{2} = 40$, 80

$\frac{B+C}{2} = 43$, 86

135
86
49

A = 49
B = 31
C = 55

$\frac{49 + B + 44}{3} = 45$

$93 + B = 135$

135
93
42

B

135
− 86
49

A + 86 = 135

B = 31

135
− 86
49

Boundary :-

If P% by x% then

Increased value = (100 + x)% of P value.

If A is R% [more] than B

A by $\frac{R}{R+100} \times 100$

A by $\frac{R}{100-R} \times 100$

⇒ Value decreased by x% & ↑ by y%

$-x + y - \frac{xy}{100}$

(Q) diff b/w 2 nos ⇒ $\frac{0.1x - 0.2y = 10}{90}$

$x - y = 90$

$\frac{10}{100}\% \; x = \frac{20}{100} y$

$0.1x - 0.2y = 10$

$x = 2y = \frac{0.1x}{0.2}$

$y = \frac{1}{2}x$

$x - \frac{1}{2}x = 90$

$\frac{2x - 1x}{2} = 90$

$1x = 180$

$x = 180$

$\frac{90\%x}{100} = \frac{20\%y}{100}$

$0.1x - 0.2y = 0$

$(1.07)^7$

107×1.07

(9) Present (P) = 1,00,000

R = 7%

n = 4 years. from now what?

$$P \times \left(1 + \frac{R}{100}\right)^{n}$$

$$1{,}00{,}000 \times \left(1 + \frac{7}{100}\right)^{4}$$

$$(1.07)^4$$

1.07 × 1.07 = 1.1449

1.1449 × 1.07 = 1.14490 + 1.1449 = 1

$$100000 \times 1.3107960$$

(10) 2 plots ⇒ 500000

gain 15% on one plot

loss 15% on another

$$= +x - y - \frac{xy}{100}$$

$$15 - 15 - \frac{225}{100}$$

$$\Rightarrow -2.25\%$$

$$-x + y - \frac{xy}{100}$$

$$-15 + 15 - \frac{225}{100} \Rightarrow -2.25\%$$

Simple Interest (S.I)

SI

(1) Amount = P + interest

∴ SI = $\frac{P \times T \times R}{100}$ ⇒ $\frac{P \times T \times R}{100}$

(3) R ⇒ yearly due usually

(Q) P = 6500 ₹

R = 8%

T = $2\frac{1}{2}$ years = 2 + 0.5 yrs = 2.5 yrs

SI = $\frac{P \times R \times T}{100}$ ⇒ $\frac{6500 \times 8 \times 4T}{100}$

⇒ 130 × 10 ⇒ 1300

Amt = P + SI

⇒ 6500 + 1300

= 7800

(Q) P = 30000

R = 12%

Amt = 33000

R = 15%

$SI = \frac{P \times R \times T}{100}$ $A = P + SI$

$3000 = P +$

$\frac{P \cdot 12\%}{T \times S}$ 2 3000 ₹ → $Amt = P + SI$

$\frac{P \times 15 \times T}{100} = \frac{P \times 12 \times T}{100} = 300$

$\frac{P \times 3 \times T}{100} = 300$

$SI = \frac{P \times R \times T}{100}$ ⇒ $\frac{10,000 \times 12}{100} = 1200$

(Q) $S = 12500 \xrightarrow{4y} 15,500$

$SI = 2$

$SI = \frac{P \times R \times T}{100}$ $Amt = P + SI$

$15500 = 12500 + SI$

3000

$3000 = \frac{12500 \times R \times 4}{100}$

$\frac{6}{3000} = R = 6\%$
500

Compound Test

diff = $P\left(4 \times \frac{R}{100}\right)^n = P\left(\frac{R}{100}\right)^2$

$n = 2$

diff => $P\left(\frac{R}{100}\right)^2 = \frac{SI \times R}{200}$

SI = diff $\times \left(\frac{100}{5}\right)^2$

=) diff for 3 yrs.

diff = $P\left(\frac{R}{100}\right)^2 \times \left(\frac{R}{100} + 3\right)$

Profit & Loss

gain = (SP) - (CP)

loss = (CP) - SP)

gain % = $\frac{\text{gain} \times 100}{CP}$

selling price (SP) = $\frac{(100 + \text{gain} \times CP)}{100}$

=) $\left[\frac{100 - \text{loss}}{100} \times CP\right]$

(8) loss = $\left[\frac{\text{common loss \& gain \%}}{10}\right]^2 - \left(\frac{2c}{10}\right)^2$

(6) 200 - 350

300 - 405

$x = 8$

CP = 350

$\frac{8}{2800}$

405

$\frac{8}{3400}$

(d) CP = 900

SP = 1080

gain = 180

$\frac{180}{900} \times 100$

= 20%

(e) A → 10 h, B → 15 hr, C → 30 hr

LCM, 60

$\frac{1}{10} + \frac{1}{15} - \frac{1}{30} \rightarrow \frac{6+5-2}{60}$

2	10, 15, 30
2	5, 6, 15
3	5, 3, 15
5	5, 1, 5
	1, 1, 1

$\frac{2c}{3} = 6.6$ hr

(c) Pipe A → 3 hrs

$\frac{1}{3} + \frac{1}{45}$ → $\frac{3+2}{90}$

→ $\frac{1}{18}$ → 18 hrs

(d) First t m

Second: (t + 10) m

$\frac{1}{t} = \frac{1}{(t+10)}$ → $\frac{1}{12} = \frac{t+10+t}{t(t+10)}$ → $\frac{1}{2}$

$\frac{2t+10}{t^2+10t} = \frac{1}{12}$

$24t + 120 = t^2 + 10t$

$-t^2 + 24t - 10t + 120$

$-t^2 + 14t + 120$

$t^2 - 14t - 120$

$t^2 + 6t - 20t - 120$

$t(t+6) - 20(t+6) = 0$

$(t-20)(t+6)$

$t = 20$

(3) $\frac{\frac{a+b}{b}}{\frac{a-b}{b}} = \frac{\frac{c+d}{d}}{\frac{c-d}{d}}$ $= \frac{a+b}{a-b}$ $= \frac{c+d}{c-d}$

if $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-b}$

(4) $\frac{5}{6}$ (or) $5:6$

$= \frac{40}{48} = 5/6$

$54 : 64$

(5) $17 : 18$ & $10 : 11$

Fraction ?

$\frac{17}{18}$ $\frac{10}{11}$ $\Rightarrow 187 > 180$

$\frac{17}{10}$ is greater

(6) 3rd proportional to 18 & 54

$a : b : c$

$a : b :: b : c$

$18 : 54 :: 54 : c$

$\frac{a}{b} = \frac{b}{c}$

$ac = b^2$

$b^2 = ac$

$(54)^2 = 18c$

$$\frac{54 \times 54}{18} = c$$

$$c = 162$$

$$3^{10} \text{ Ans}$$

$$= 2$$

8) Ratio 3 : 8

$$\frac{3k+5}{8k+5} = \frac{9}{54}$$

$$154 + 25k = 162 + 16k$$

$$16(k+25) = 4(16k+10)$$

$$154 - 16k = 10 - 25$$

$$-k = -15$$

$$k = 15$$

$$\frac{3k}{8k} = \frac{3 \times 15}{8 \times 15} = \frac{45}{120}$$

$$= \frac{3}{8}$$

(4) A & B light capacity work in 15 day

B alone in 20 day

A alone ?

$$A + B \rightarrow \frac{1}{15}$$

$$B: \frac{1}{20}$$

$$\frac{1}{15} - \frac{1}{20} = \frac{1}{60}$$ ∴ 60 days

(a) 14 km/hr $S = \frac{D}{T}$

20 km/hr $S = \frac{D}{T}$

$20x - 140$

distance $= x$

$\frac{x}{10} - \frac{x}{15} = 2$

$\frac{3x - 2x}{30} = 2$

for $x = 60$ dist

$S = 60/hr$

for $x = 1$ $T = \frac{1}{10} = \frac{60}{T}$ $T = 6hr$

at 10 AM

+ 6 hrs

$\frac{60}{5} = 12$

(b) $S = 60 km/hr$

$T = 9 min = \frac{9}{3600} hr$

$5/3 \times 9$

$= 150$

$S = \frac{D}{T}$ $S = \frac{75}{T}$

$\frac{S_1}{S_2} = \frac{T_2}{T_1} \Rightarrow \frac{5}{3} \times 60 \times \frac{1}{75}$

Probability

(4). 15 boys

10 girls

prob of 2 boy & 1 girl selected

selection of boy × selection girl

same space out of which selection is to be made

$$= \frac{{}^{10}C_1 \times {}^{15}C_2}{{}^{25}C_3} = \frac{15 \times 5 \times 9}{25 \times 8 \times 23} = \frac{21}{46}$$

$$\frac{15 \times 7}{5}$$

$$nCr = \frac{n!}{r!(n-r)!} = {}^{15}C_2 = \frac{15!}{2!(15-2)!} = \frac{15!}{2!(13)!}$$

$$\frac{15!}{2 \times 13!} = \frac{15 \times 14 \times 13!}{2 \times 13!} = \frac{15 \times 14}{2} = 105$$

$$2 \quad 105$$

$${}^{10}C_1 = \frac{10!}{(10-1)!} = \frac{10!}{9!} = \frac{10}{1} = 10$$

$$\Rightarrow \frac{10 \times 105}{2300} = \frac{1050}{2300}$$

$${}^{25}C_3 = \frac{25!}{3!(25-3)!}$$

$$= \frac{25!}{6 \times (22)!} = \frac{25 \times 24 \times 23}{6}$$

$$\frac{21}{46}$$

$$= 2300$$

(Q): 2 cards from 52 cards.

Prob that 1 is spades & one is heart is?

$$\frac{{}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_2} = \frac{13 \times 13}{26 \times 51} = \frac{13}{102}$$

$${}^{13}C_1 = \frac{13!}{12!} = 13$$

$$\frac{52!}{50}$$

$${}^{52}C_2 = \frac{52!}{2(50!)} = \frac{52 \times 51 \times 50}{2}$$

$$\frac{52 \times 51}{2}$$

$$52 \times 51 \times 25$$

234

$$\frac{13 \times 13}{1326} = \frac{13}{102}$$

$$\frac{13 \times 13}{26 \times 51} \quad \frac{13}{102}$$

$$\frac{13 \times 13}{1326}$$

102

52
51
25,5+1,2
25,15,2
2652
2652.0

26

$$\frac{26}{51} \quad \frac{46}{51}$$

10 30+2,6
20,30+4,6
20,34,6

1326

2346

(P) A bag contain 6 white & 4 black balls, 2 balls are drawn at random find the prop that they are the of the same colour

$$\frac{6C_2 + 4C_2}{10C_2}$$

w b
w b
w b
w b
w
w

10

45

$$10C_2 = \frac{10!}{2 \times 8!} = \frac{10!}{2 \times (10-2)!} \quad \frac{10 \times 9 \times 8!}{2 \times 8!} \Rightarrow \frac{90}{2} = 45$$

$6C_2$ = 2 cases : (i) both white
(ii) both black

(i) $6C_2 = \frac{6!}{2} \quad 6 \times 5 \quad \frac{30}{3} = 15.$

(ii) $4C_2 = \frac{4!}{2} = 6 \Rightarrow \frac{4!}{2 \times (4-2)!} \quad \frac{2}{2}$

$15 + 6 = 21 = \frac{7}{21} \quad \frac{1}{15} \quad \frac{3}{9} \quad \frac{4 \times 3 \times 2!}{2! \times 2!} = 6$

$= \frac{1}{15}$ 11

(Q) A bag contain 5 white, 6 red, 7 black balls, 3 balls drawn randomly, the prop all are red are?

$\frac{17 \times 9}{153}$

$$18C_2 \Rightarrow \frac{18!}{2!(18-2)!} = \frac{18 \times 17 \times 16!}{2 \times 1} = 153$$

(b) all three are red?

$^{18}C_3 = \frac{18!}{3!(18-3)!} = \frac{18 \times 17 \times 16 \times 15!}{3! \times 15!}$

$= \frac{18 \times 17 \times 16}{3 \times 2 \times 1} = \frac{272 \times 3}{6} = \frac{816}{6}$

$^6C_3 = \frac{6!}{3!(6-3)!} = \frac{6 \times 5 \times 4 \times 3}{3 \times 2 \times 1} = 20$

$\frac{20}{816} = \frac{10}{408} = \frac{5}{204}$

(b) A box has 3 blue, 6 green, 2 yellow marbles if 2 are drawn random, what is the prob the 1st is green

4 red

$\frac{11}{^{15}C_2}$

$^{15}C_2 = \frac{15!}{2!(15-2)!} = \frac{15 \times 14 \times 13!}{2 \times 13!} = \frac{15 \times 14}{2} = 105$

using complementary method.

P(at least one green) = 1 − P(no green)

non green marbles = 3 + 4 + 2 = 9

choose 2 from 9:

$^9C_2 = \frac{9!}{2!(9-2)!} = \frac{9 \times 8 \times 7!}{2 \times 1 \times 7!} = 36$

P(no green) $= \frac{36}{105} = \frac{12}{35}$

1 is green $= 1 - \frac{12}{35} = \frac{35-12}{35} = \frac{23}{35}$

(Q) either Blue or yellow

$$= \frac{{}^3C_2 + {}^2C_2 + {}^5C_2}{{}^{15}C_2}$$

B B, Y Y, (B Y)

$3 \times 2 = {}^3C_2$

$${}^5C_2 = \frac{5!}{2!\,3!}$$

$${}^3C_2 = \frac{3!}{2(3-2)!} = 3$$

$${}^2C_2 = \frac{2!}{2} = 1$$

blue / yellow: ${}^3C_1$, ${}^2C_1$

$3 \times 2 = 6$

$\Rightarrow 6 + 1 + 3 = 10$

$$\frac{10}{{}^{15}C_2} \Rightarrow \frac{10}{105} = \frac{2}{21}$$

$$\frac{15 \times 14 \times 13!}{2! \times 13!} = 105$$

(Q) 3B, 4R, 6G, 2Y

$4 + 6 + 2 \Rightarrow 12$, ${}^{12}C_4$

$${}^5C_2 = \frac{5!}{2 \times (3!)}$$

$$\frac{{}^{15}C_4 + {}^{15}C_6 + {}^{15}C_2}{{}^{15}C_4}$$

$$\Rightarrow \frac{{}^{12}C_4}{{}^{15}C_4} = \frac{12!}{4!\,(8!)} = 12 \times 11 \times 10 \times 9$$

$${}^{15}C_4 = \frac{15 \times 14 \times 13 \times 12 \times 11!}{4!\,(11!)}$$

$$\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2}$$

$30 \times 4 = 330$

$$\frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$$

$15 \times 7 \times 13 = 105 \times 13 = 1365$

(Q) 10B

$10B \rightarrow 10!$

$(A+B)$ — AB, BA: $2!$

$1 + 8 \Rightarrow 9! \times 2!$

$$\frac{9! \times 2!}{10!} = \frac{1}{5}$$

(Q) prob that a leap year has 53 sunday & 52 monday.

$366 \div 7 = 52$ weeks $+ 2$ extra day

53 sunday & 52 monday $= \frac{2}{7}$

(Q) ANALOGY

P I B Q D C L Z

A B C D E F G H I J K L M N O
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

P Q R S T U V W X Y Z
16 17 18 19 20 21 22 23 24 25 26

1, 14, 1, 12, 15, 7, 25

15, 5, 1, 5, 9, 9, 1

16, 9, 2, 17, 24, 12, 26