PRACTICAL – 13

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AIM: To fit logistic growth model using Yule's method, and estimate the trend values.

EXPERIMENT:

The following data represent the production of coffee at time *t*, where *t* is in years:

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Production (Kg.)	240.3	269.1	289.9	340.8	374.6	441.6	482.4	520.6	548.3	571.9	604.6	615.4	629.3	639.2
Time	15	16	17	18	19	20	21	22	23	24	25	26	27	
Production (Kg.)	650.2	661.9	670.8	692.8	710.3	731.6	750.7	771.9	792.2	802.6	812.7	814.9	820.8	

(a) Fit a Logistic growth model for this data using the yule's method.

(b) Estimate trend values of the time series using the above Logistic trend equation.

THEORY:

Logistic Curve:

i. A particular form of complex type of growth curves.

ii. Given by: $y = y_t = \frac{k}{1+\exp(a+bt)}$, $b>0$

iii. a, b, k are constants and yt is the value of the given time series, at time t.

Yule's Method:

i. We assume that the value of k is approximately known or obtained through any other method. Therefore there are two parameters a, and b, and two variables t and y_t.

ii. We use the principle of least square to estimate a and b. We have:
$a + bt = \log\left(\left(\frac{k}{y}\right) - 1\right)$ OR $v = a + bt$, => $v = \log\left(\left(\frac{k}{y}\right) - 1\right)$

iii. To find the value of k, we use method of three selected points. Hence the formula for k is given by: $k = \frac{y_2^2\,(y_1+y_3)-2y_1y_2y_3}{y_2^2-y_1y_3}$

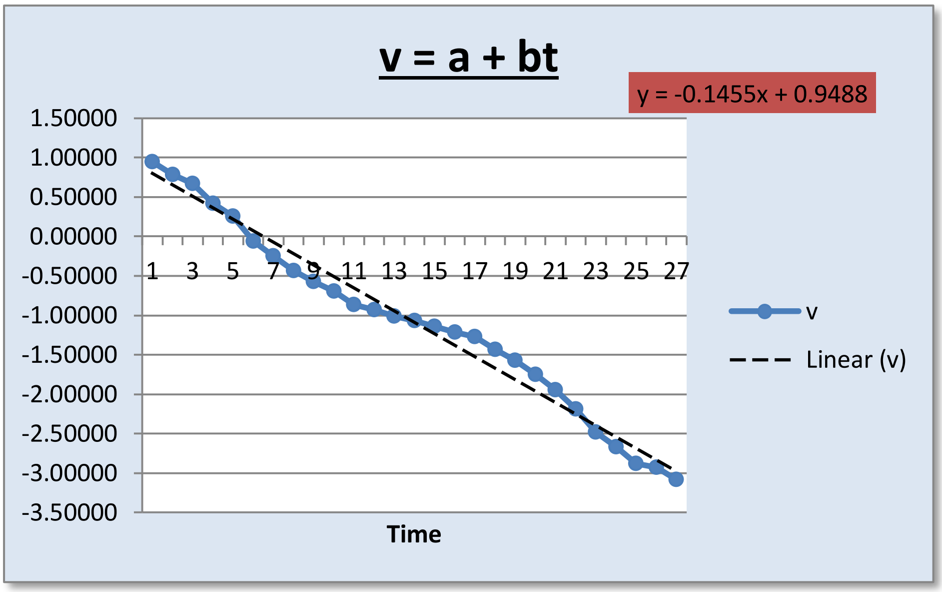
CALCULATIONS: (an excel file has been attached for reference to detailed calculations)

Table 13.1

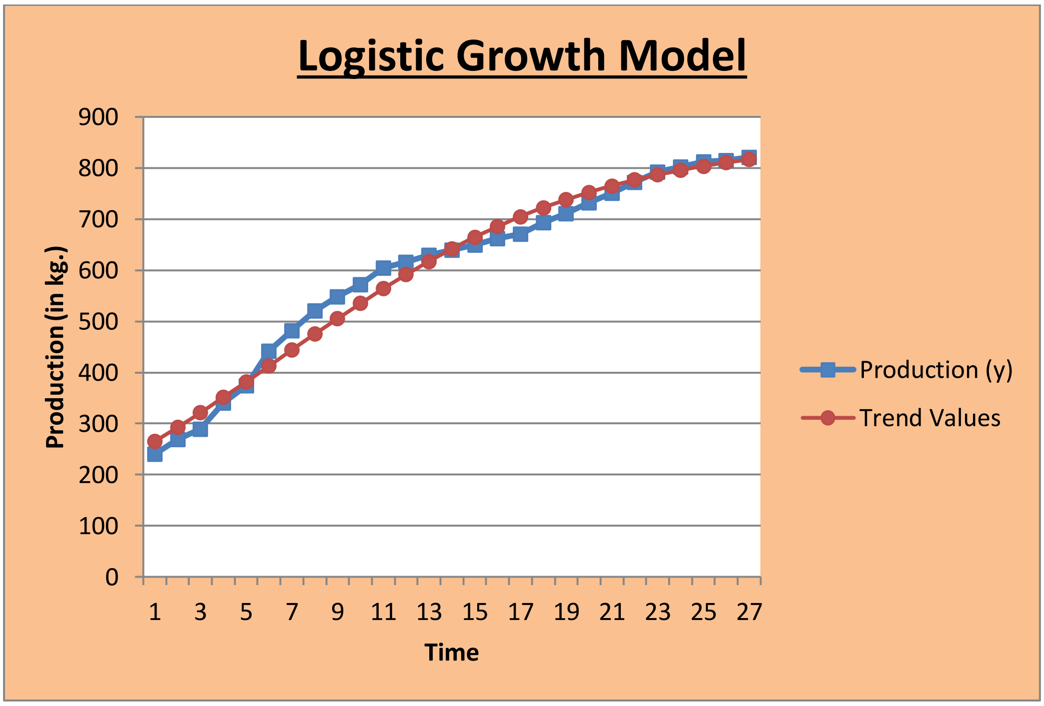
	Time (t)	Production (y)	v	Trend Values
t1=	1	240.3	0.94460	265.48876
	2	269.1	0.78368	292.88041
	3	289.9	0.67329	321.56518
	4	340.8	0.41771	351.31376
	5	374.6	0.25560	381.85647
	6	441.6	-0.05804	412.89199
	7	482.4	-0.24946	444.09874
	8	520.6	-0.43283	475.14800
	9	548.3	-0.57026	505.71760
	10	571.9	-0.69158	535.50515
	11	604.6	-0.86842	564.23942
	12	615.4	-0.92963	591.68931
	13	629.3	-1.01090	617.66956
t2=	14	639.2	-1.07070	642.04341
	15	650.2	-1.13927	664.72212
	16	661.9	-1.21497	685.66200
	17	670.8	-1.27470	704.85959
	18	692.8	-1.43178	722.34565
	19	710.3	-1.56849	738.17857
	20	731.6	-1.75343	752.43781
	21	750.7	-1.94260	765.21780
	22	771.9	-2.18989	776.62239
	23	792.2	-2.48368	786.76028
	24	802.6	-2.66791	795.74117
	25	812.7	-2.88050	803.67287
	26	814.9	-2.93265	810.65911
t3=	27	820.8	-3.08599	816.79798

k=	858.29791
a=	0.9488
b=	-0.1455

Graph 13.1



Graph 13.2



RESULT:

- Value of k was computed using the method of three selected points. Its value has been shown with Table 13.1. (k=858.29791).
- Value of v was computed using the value of k, and has been shown in Table 13.1.
- As v = a + bt represents a linear equation, hence a graph was plotted for v, and a linear trend-line was fitted to get the values of a and b. Values of a and b have been shown along with Table 13.1. (a = 0.9488, and b = -0.1455)
- We thus fit the values of k, a, and b so obtained in the equation of logistic curve, to compute trend values that have been shown in Table 13.1.
- Hence the logistic curve equation comes out to be: $y_t = \frac{858.29791}{1+\exp(0.9488-0.1455*t)}$