TIME SERIES ANALYSIS
PRACTICAL – 8

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AIM: To Estimate trend using Grompertz curve by method of partial sums and comment on fitting of Grompertz curve in comparison with given data.

EXPERIMENT:

The following data gives the amount of savings and loan association in the US from 1945 to 1971:

YEAR	AMOUNT	YEAR	AMOUNT	YEAR	AMOUNT
1945	7.40	1954	27.3	1963	91.3
1946	8.50	1955	32.1	1964	101.9
1947	9.80	1956	37.1	1965	110.4
1948	11.00	1957	41.9	1966	114
1949	12.50	1958	48	1967	124.5
1950	14.00	1959	54.6	1968	131.6
1951	16.10	1960	62.1	1969	135.5
1952	19.20	1961	70.9	1970	146.4
1953	22.30	1962	80.2	1971	174.5

Estimate trend using Gompertz curve by method of Partial Sums.
Comment on fitting of Gompertz curve in comparison with given data.
Forecast amount for next five years.

THEORY:

METHOD OF PARTIAL SUMS:

Equation for modified exponential curve: $y_t = a + bc^t$ (1)

The given time series data is split up into 3 equal parts, each containing n consecutive values of y_t corresponding to t=1,2,…,n ; t=n+1,n+2,…,2n ; t=2n+1,2n+2,…,3n. Let S1, S2 and S3 represent the partial sums of the 3 parts respectively, such that,

- $S_1 = \sum_{t=1}^{n} yt$
- $S_2 = \sum_{t=n+1}^{2n} yt$
- $S_3 = \sum_{t=2n+1}^{3n} yt$

Substituting for y_t from equation (1), we get the values of a, b and c.

- $c = \left(\frac{S3-S2}{S2-S1}\right)^{1/n}$
- $b = \frac{(c-1)((S2-S1)^3)}{c((S3-2S2+S1)^\wedge 2)}$
- $a = \frac{1}{n}\left[\frac{S1S3-(S2^\wedge 2)}{S3-2S2+S1}\right]$

GROMPERTZ CURVE:

The Grompertz curve describes a trend in which the growth increments of the logarithms are declining by a constant percentage. Thus the natural values of the trend would show a declining ratio of increase, but the ratio does not decrease by either a constant amount or a constant percentage.

Equation: $y_t = a + b^{c\^t}$
Taking log both sides,
$\log y_t = \log a + c^t * \log b$

Let, $\log y_t = Y_t$, $\log a = A$, $\log b = B$
Therefore, we get
$Y_t = A + Bc^t$

The above equation is comparable to the equation of modified exponential curve.

CALCULATIONS: (An excel sheet has also been attached)

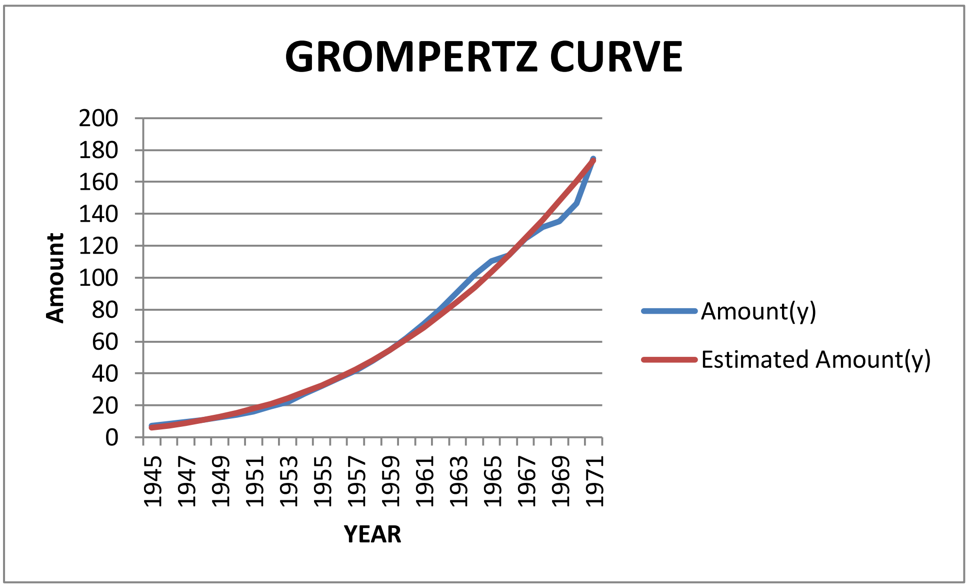
Table 8.1

t	YEAR	Amount(y)	Y_t	Yt(estimated)	Estimated Amount(y)
1	1945	7.4	2.00148	1.813663938	6.132876798
2	1946	8.5	2.140066	2.010003181	7.46334109
3	1947	9.8	2.282382	2.199228218	9.018050848
4	1948	11	2.397895	2.381596828	10.82217022
5	1949	12.5	2.525729	2.557357447	12.90167886
6	1950	14	2.639057	2.726749511	15.28312855
7	1951	16.1	2.778819	2.89000378	17.99337762
8	1952	19.2	2.95491	3.047342653	21.05930811
9	1953	22.3	3.104587	3.198980469	24.50753127
10	1954	27.3	3.306887	3.345123802	28.36408699
11	1955	32.1	3.468856	3.485971741	32.65414307
12	1956	37.1	3.613617	3.62171616	37.40170007
13	1957	41.9	3.735286	3.752541981	42.62930727
14	1958	48	3.871201	3.878627426	48.35779485
15	1959	54.6	4.000034	4.000144259	54.6060269
16	1960	62.1	4.128746	4.117258021	61.39067932
17	1961	70.9	4.26127	4.230128253	68.72604592
18	1962	80.2	4.384524	4.338908716	76.62387531
19	1963	91.3	4.514151	4.443747601	85.09324037
20	1964	101.9	4.623992	4.544787727	94.14044157
21	1965	110.4	4.70411	4.642166739	103.7689444
22	1966	114	4.736198	4.736017295	113.9793504
23	1967	124.5	4.824306	4.826467247	124.7694017
24	1968	131.6	4.879767	4.913639812	136.134016
25	1969	135.5	4.908972	4.997653745	148.0653521
26	1970	146.4	4.986343	5.078623495	160.5529019
27	1971	174.5	5.161925	5.156659367	173.5836068
28	1972			5.231867668	187.1419964
29	1973			5.304350851	201.2103446
30	1974			5.374207661	215.7688425
31	1975			5.441533262	230.7957826
32	1976			5.506419369	246.2677526

S1=	22.82493
S2=	34.77042
S3=	43.33976
a=	1383.363
b=	0.003616
c=	0.963766
B=	-5.62233
A=	7.232273

R^2=	0.996155895

Graph 8.1



RESULT:

- Trend values using Grompertz Curve (method of partial sums) have been calculated and shown in Table 8.1.
- The trend values have been plotted along with the given values in Graph 8.1.
- The amount for the next 5 years has been forecasted and shown in Table 8.1.
- Correlation coefficient between the given amount and the estimated amount is: R^2 = 0.996155895.

CONCLUSION:

- Estimated values are increasing exponentially.
- The R^2 value calculated is almost equal to 1 (0.996155895). This indicates that the values estimated are almost equal to the given values.