$f_{00}^{(1)} = 0.307$

$f_{00}^{(2)} = 0.384 \times 0.25 + 0.3076 \times 0.4$

$f_{00}^{(3)} = 0.384 \times 0.33 \times 0.25 + 0.3076 \times 0.333 \times 0.4 + 0.384 \times 0.416 \times 0.4$

$f_{00}^{(4)} = 0.384 \times (0.33)^2 \times 0.25 + 0.3076 \times (0.333)^2 \times 0.4 + 0.384 \times 0.416 \times 0.2667 \times 0.25$

$\vdots$

$f_{00}^{(n)} = 0.384 \times (0.33)^{n-2} \times 0.25 + 0.3076 \times (0.333)^{n-2} \times 0.4 + 0.384 \times 0.416 \times 0.2667 \times 0.25$

So, $\sum_{n=1}^{\infty} f_{00}^{(n)} = 1$

Hence State 0 is recurrent.

Since, all the states are in same class & recurrency is a class property then 1 & 2 are also recurrent.

- Result :-

The Obtained Markov Chain is irreducible because it consists only 1 class.

All 3 states in Markov Chain are recurrent.

PRACTICAL - 7

- **Aim** :- To Stimulate a Markov Chain from given state space & obtain TPM.
- **Problem** :- Stimulate a Markov Chain with state space $S = \{0, 1, 2\}$ & initial probability 1/3 each. Form TPM & find the nature of Markov chain.
- **Theory** :-

\# **Irreducible** ⇒ A markov chain is said to be irreducible if it consists of only 1 communicating class, i.e.; all states communicate with each other

\# **Reducible** ⇒ If a Markov Chain has more than 1 class, then it is called reducible markov chain.

\# **Recurrent** ⇒ State i is said to be recurrent if

$$\boxed{f_{ii} = 1}$$

i.e.; there is a probability that; upon entering state i the process will definitely return to state i.

Since a recurrent state definitely will be revisited after each visit, it will be visited infinitely often.

\# **Transient** ⇒ State i is said to be transient if

$$\boxed{f_{ii} < 1}$$

i.e.; upon entering state i, there is a positive probability that process never return to state i again.

In a finite-state Markov Chain, a transient state is visited only a finite no. of times.

- **Calculation** ⇒

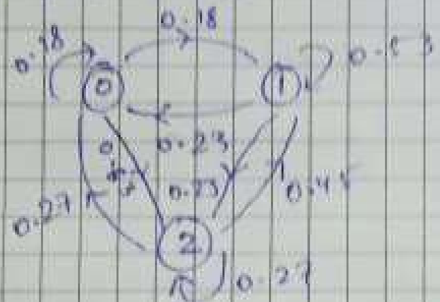
After generating random no. & calculating their frequency in the excel sheet we get,

$$n_{ij} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 2 & 2 & 7 \\ 5 & 11 & 5 \\ 5 & 8 & 5 \end{bmatrix} \end{array} \begin{array}{c} n_i \\ 11 \\ 21 \\ 18 \end{array}$$

Now divide each element by their respective row sum (n_i) to get T.P.M.

$$TPM = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.18 & 0.18 & 0.63 \\ 0.23 & 0.53 & 0.23 \\ 0.27 & 0.45 & 0.27 \end{bmatrix} \end{array}$$

Markov Chain $\Rightarrow$



(*) The markov chain consists only one class i.e. all states communicate with each other. So it is irreducible

$$f_{00}^{(1)} = 0.18$$

$$f_{00}^{(2)} = 0.18 \times 0.23 + 0.27 \times 0.63$$

$$f_{00}^{(3)} = 0.18 \times 0.53 \times 0.23 + 0.18 \times 0.23 \times 0.27 + 0.27 \times 0.27 \times 0.63$$

$$f_{00}^{(4)} = 0.18 \times (0.53)^2 \times 0.23 + 0.18 \times 0.23 \times 0.45 \times 0.23 + 0.18 \times 0.23 \times 0.27 \times 0.63 + 0.27 \times 0.27 \times 0.27 \times 0.63$$

$$f_{00}^{(5)} = 0.18 \times (0.53)^3 \times 0.23 + 0.18 \times (0.53)^2 \times 0.23 \times 0.45 \times 0.23 + 0.18 \times 0.23 \times 0.27 \times 0.27 \times 0.63 + (0.27)^4 \times 0.63$$

$\vdots$

So; $$f_{00}^{(n)} = 0.18 \times (0.53)^{n-2} \times 0.23 + 0.18 (0.53)^{n-4} \times 0.23 \times 0.45 \times 0.23 + 0.18 \times (0.53)^{n-4} \times 0.23 \times (0.27)^{n-4} \times 0.27 + 0.18 \times 0.23 \times (0.27)^{n-3} \times 0.63 + (0.27)^{n-1}\, 0.63$$

So; $$\sum_{n=1}^{\infty} f_{00}^{(n)} = 1$$

So we can say that state 0 is recurrent state

& since 1 & 2 belong to same class, so by class property of recurrence we can say that state $\{0, 1, 2\}$ recurrent state.

- Result $\Rightarrow$

The obtained TPM from excel sheet converted into Markov chain & is irreducible.

State $\{0, 1, 2\}$ is recurrent.

PRACTICAL - 8

- **Aim :-** Obtain Markov chain & find the nature of Markov Chain.

- **Problem :-** Obtain Markov chain from given TPM & find its nature.

$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & & & \\ 1 & & & \\ 2 & & & \end{array}$$

$$\begin{array}{c} 0 \\ 1 \\ 2 \end{array}\begin{bmatrix} 0.307692 & 0.384615 & 0.307692 \\ 0.25 & 0.33333 & 0.41667 \\ 0.4 & 0.2667 & 0.33333 \end{bmatrix}$$

- **Theory :-**

Irreducible ⇒ A markov chain is irreducible if it consists of only 1 communicating class ; i.e, all states communicate with each other.

Reducible ⇒ If a Markov Chain has more than 1 class then it is called reducible markov chain.

Recurrent ⇒ State i is said to be recurrent if

$$\boxed{f_{ii} = 1}$$

i.e.; there is a prob that, upon entering state i the process will definitely return to state i.

Since a recurrent state definitely will be revisited after each visit, it will be visited infinitely often.

Transient ⇒ State i is said to be transient if

$$\boxed{f_{ii} < 1}$$

i.e.; upon entering state i, there is a +ve prob that process never return to state i again.

In a finite - state markov chain, a transient state visited only a finite no. of times.

- **Calculation :-**