PRACTICAL => 5 (A)

AIM:- To compare exact and approximate prob. given by partial function expansion theorem.

PROBLEM:- If p_n is the probability of getting 2 consecutive heads in tossing a fair coin. Obtain the p.g.f P_s of p_n and use partial fraction expansion to obtain exact and approximate value of p_n.

THEORY:-

$P_n \Rightarrow$ Prob. that there are 2 consecutive heads at n^{th} trial.

$p_0 = 0 \quad , \quad p_1 = 0$

$p_2 = p.p = p^2$

$p_3 = THH = q\,pp = qp^2$

$p_4 = TTHH, HTHH, = q^2p^2 + qp^3$

$p_5 = TTTHH, HTTHH, THTHH = q^3p^2 + p^3q^2 + p^3q^2$

$= q(q^2p^2 + qp^3) + q^2p^3$

$= q\,p_4 + qp\,p_3$

In general, $\boxed{p_n = q\,p_{n-1} + qp\,p_{n-2}}$ — ① $\quad n \geq 3$

As. we know,

$$P(s) = \sum_{k=0}^{\infty} p_k s^k \Rightarrow \sum_{n=0}^{\infty} p_n s^n.$$

Multiply both side by s^n and sum it over all possible value

So

$$\sum_{n=3}^{\infty} p_n s^n = \sum_{n=3}^{\infty} q\,p_{n-1} s^n + \sum_{n=3}^{\infty} pq\,p_{n-2} s^n.$$

$$\Rightarrow \sum_{n=0}^{\infty} p_n s^n - (p_0 + p_1 s + p_2 s^2) = \sum_{n=0}^{\infty} qs\,p_{n-1} s^{n-1} - (p_0 + p_1 s) + pq s^2 \sum_{n=0}^{\infty} p_{n-2} s^{n-2} - p_0$$

$$\Rightarrow \sum_{n=0}^{\infty} p_n s^n - p_2 s^2 = qs\,p(s) + pqs^2\,p(s)$$

$$\Rightarrow p(s) - qs\,p(s) - pqs^2\,p(s) = p^2 s^2$$

$$\Rightarrow p(s)\left(1 - qs - pqs^2\right) = p^2 s^2$$

$$p(s) = \frac{p^2 s^2}{1 - qs - pqs^2}$$

Also we know that

$$p_n = A + \frac{U(s)}{V'(s)}$$

CALCULATION:-

Here $p = q = 1/2$

$$\text{So}, \; p(s) = \frac{s^2/4}{1 - s/2 - s^2/4}$$

By partial decomposition we get

$$\boxed{P_n = -1 + \frac{2(s-2)}{s^2+2s-4} \begin{matrix} \rightarrow U(s) \\ \rightarrow V'(s) \end{matrix}} \quad \text{— (ii)}$$

So, By solving $s^2 + 2s - 4 = 0$

$s_1 = -1 + \sqrt{5}$

$s_2 = -1 - \sqrt{5}$

From (ii) we get

$U(s) = 2(s-2)$

and $V'(s) = s^2 + 2s - 4$

Also $r = \frac{-U(s)}{V'(s)}$ $\quad r_1 = -\frac{U(s_1)}{V'(s_1)}$

So $r_1 = -\frac{U(s_1)}{V'(s_1)}$ $\quad r_2 = \frac{-U(s_2)}{V'(s_2)}$

$r_2 = -\frac{U(s_2)}{V'(s_2)}$

$\therefore$ Approximate $= \frac{r_1}{s_1^{n+1}}$

Error $= \frac{r_2}{s_2^{n+1}}$

Exact $p_n = \frac{r_1}{s_1^{n+1}} + \frac{r_2}{s_2^{n+1}}$

PRACTICA - 5(A)

- **Aim :-** To compare exact & approximate prob given by partial fraction expansion theorem.

- **Problem :-** If P_n is the probability of getting 2 consecutive heads in tossing a fair coin. Obtain the p.g.f P_s of P_n & use partial fraction expansion to obtain exact & approximate value of P_n.

- **Theory :-**

$P_n \Rightarrow$ Prob that there are 2 cons. heads at n^{th} trial.

$P_0 = 0 \ , \ P_1 = 0$

$P_2 = p.p = p^2$

$P_3 = THH = qp^2$

$P_4 = TTHH, HTHH = q^2p^2 + qp^3$

$P_5 = TTTHH, HTTHH, THTHH = q^3p^2 + p^3q^2 + p^3q^2$

So, $P_5 = q(q^2p^2 + qp^3) + q^2p^3$

$= qp_4 + qp_3$

So in general; $\boxed{P_n = q P_{n-1} + qp P_{n-2}}$ — (*)

As we know that ; $P(s) = \sum_{k=0}^{\infty} P_k s^k \Rightarrow \sum_{n=0}^{\infty} P_n s^n$

Mul both side by s^n & summing over all possible values of n in (*) eq^n

we get ; $\boxed{P(s) = \frac{p^2 s^2}{1 - qs - pqs^2}}$

Also ; we know that. $\boxed{r = \frac{-U(s)}{V'(s)}}$

- **Calculation :-**

Here, $p = q = \frac{1}{2}$

So, $P(s) = \frac{s^2/4}{1 - \frac{1}{2}s - \frac{1}{4}s^2}$

By partial decomposition we get,

$$\boxed{-1 + \frac{2s-4}{s^2+2s-4}} \quad \text{——} (*)(*)$$

So; By solving $s^2 + 2s - 4 = 0$

we get roots; $s_1 = -1 - \sqrt{5}$

$s_2 = -1 + \sqrt{5}$

Here from (*)(*) we get: $U(s) = 2s - 4 = 2(s-2)$

& $V(s) = s^2 + 2s - 4$

Also; $\gamma = \frac{-U(s)}{V'(s)}$

$$\boxed{\gamma = \frac{-2(s-2)}{2s+2}}$$

- Result :-

As can be seen from the table, the values of q_n obtained from partial fraction theorem & recursive are exactly same.

The approximate values are slightly different initially but as n ↑es, error becomes negligible.

PRACTICAL-5 (b)

• **Problem** :- If P_n is the prob. of getting 2 consecutive heads in tossing of a fair coin, obtain p.g.f $P(s)$ of P_n.

Use partial fraction expansion to obtain exact & approximate values of P_n if $p = 0.6$ & $q = 0.4$

• **Calculation** :-

$$P(s) = \frac{p^2 s^2}{1 - qs - pqs^2}$$

Here $p = 0.6$ & $q = 0.4$

$$\text{So, } P(s) = \frac{(0.6)^2 s^2}{1 - 0.4s - 0.6 \times 0.4 s^2}$$

$$P(s) = \frac{(0.36) s^2}{1 - 0.4s - 0.24 s^2}$$

By Partial decomposition;

$$\Rightarrow \frac{15s - \frac{75}{2}}{6\left(s + \frac{5}{6} + \frac{5\sqrt{7}}{6}\right)\left(s - \frac{5\sqrt{7}}{6} + \frac{5}{6}\right)}$$

$$\Rightarrow \boxed{\frac{30s - 75}{12s^2 + 20s - 50}}$$

So the roots are $\Rightarrow$
$$s_1 = -\frac{5}{6} + \frac{5\sqrt{7}}{6}$$
$$s_2 = \frac{-5}{6} - \frac{5\sqrt{7}}{6}$$

$$\boxed{U(s) = \frac{30s - 75}{12}}$$

$$V(s) = s^2 + \frac{20}{12}s - \frac{50}{12} = \boxed{s^2 + \frac{5s}{3} - \frac{25}{6}}$$

$$\text{So, } r = -\frac{U(s)}{V'(s)} = -\left[\frac{\frac{30s - 75}{12}}{2s + \frac{5}{3}}\right]$$

• **Result** :- As from the table, values of q_n from partial fraction th. & recursive are exactly same.
The approx. values are slightly different initially but as n ↑s, error becomes negligible

PART (C)

AIM:- To check if process is covariance stationary.

PROBLEM:- Simiulate a random walk of $Y(t) = Y(t-1) + e(t)$ where $y(0) = 0$, $e(t)$'s are iid RV with mean 0 and variance 1. Check if process is covariance stationary or not.

THEORY AND FORMULA:-

$$Y(0) = 0 \;,\; Y(1) = e(1) \;,\; Y(2) = e(1) + e(2) \; - - - - - \; Y(t) = e(1) + e(2) + - - - + e(t)$$

$$E[Y(t)] = 0 \qquad \{ E[e(t)] = 0 \}$$

$$Var(Y_t) = Var(e(1)) \; - - - = 1 + 1 + - - - + 1 = t$$

$$cov(Y_t, Y_s) = cov(e_1 + e_2 + e_3 + - - - e_t, \; e_1 + e_2 + - - - e_s)$$

$$= \sum_{i=1}^{t} \sum_{j=1}^{s} cov(e_i, e_j) + \sum_{i=1}^{} cov(e_i, e_j)$$

$$\Rightarrow 0 + \sum_{i=1}^{t} Var(e_i) \Rightarrow 0 + t \Rightarrow 0$$

Not a fun^n. of $|t-s|$ $\therefore$ so process is not covariance stationary

CALCULATION:-

E(X(t))	0.025107	0.040454	0.083471	0.137882	0.226674	0.178496	0.150678	0.193416	0.243789	0.31304
V(X(t))	0.989891	3.985868	5.298888	6.404874	7.355691	9.206643	9.514962	10.5975	11.68964	12.36041

COVARIANCE TABLE

	X(1)	X(2)	X(3)	X(4)	X(5)	X(6)	X(7)	X(8)	X(9)	X(10)
X(1)	0.997924									
X(2)	2.049828	3.991698								
X(3)	1.859239	4.165715	5.308877							
X(4)	1.942521	3.803289	5.374614	6.428919						
X(5)	1.938217	4.41548	6.326534	4.86942	7.336272					
X(6)	1.36194	4.598119	4.926872	4.316533	6.751434	9.25585				
X(7)	2.018541	3.108213	4.746456	6.126185	8.611725	8.216918	9.561501			
X(8)	1.904769	3.463189	5.731893	5.638496	8.200256	7.851811	7.877781	10.6073		
X(9)	1.555516	5.432542	5.453216	6.693936	5.873846	7.273252	9.545972	8.614498	11.67538	
X(10)	2.108264	4.819024	4.016697	5.701935	6.501295	8.377344	7.903502	10.18463	13.27151	12.42321