PRACTICAL - 11

- **Aim** :- To measure the performance of model $(M/M/1):(\infty/FIFO)$.

- **Problem** :- The rate of arrival of customers at a public telephone booth follows Poisson dist with an avg. time of 10 min b/w one customer & the next. The duration of a phone call follow exp. dist. with mean time of 3 min.

(i) What is the prob. that a person arriving at the booth will have wait ?

(ii) What is the avg. length of non-empty queues that form from time to time ?

(iii) The Mahanagar Telephone Nigam Ltd. will install a II booth, when it is convinced that the customers would expect waiting for atleast 3 min. for their turn to make a call. By how much time should the flow of customers inc. in order to justify a II booth ?

(iv) Estimate fraction of a day that the phone will be in use.

(v) What is the prob. that it will take him more than 10 min. altogether to wait for phone & complete his call ?

- **Theory & formula** :-

Model $\{(M/M/1):(\infty/FIFO)\}$ $\Rightarrow$ This model deals with a queueing system having single service channel. Poisson input, Exponential service & there is no limit on the system capacity while the customers are served on a "first in, first out" basis.

Performance $\Rightarrow$

Avg. no. of customers in system or service $\Rightarrow$ $L_s = \dfrac{\lambda}{\mu - \lambda}$; $\lambda \Rightarrow$ Mean arrival, $\mu \Rightarrow$ Mean service

Avg. no. of customers in queue (length of the queue) $\Rightarrow$ $L_q = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

Avg. waiting time in system $\Rightarrow$ $W_s = \dfrac{1}{\mu-\lambda}$

Avg. waiting time in queue $\Rightarrow$ $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)}$

$\rho = \dfrac{\lambda}{\mu}$

Probability of no customer $\Rightarrow$ $P_0 = 1 - \rho = 1 - \dfrac{\lambda}{\mu}$

length of non-empty queue $\Rightarrow L_n = \frac{\mu}{\mu-\lambda}$

Probability of waiting time in queue $\geq n \Rightarrow$

$$P(\text{waiting time} \geq n) = \int_n^\infty \frac{\lambda}{\mu}(\mu-\lambda)\, e^{-(\mu-\lambda)t}\, dt$$

Prob. that an arrival will have to wait for more than N min. $\Rightarrow$

$$P[\text{waiting time} + \text{service time}] \geq N] = \rho^N$$

$$\rho^N = \int_N^\infty (\mu-\lambda)\, e^{-(\mu-\lambda)t}\, dt$$

- Calculation :-

$\lambda = \frac{1}{10} \times 60 = 6/hr.$ and $\mu = \frac{1}{3} \times 60 = 20/hr.$

(i) Prob. that person arrive at booth have to wait :-

$$P(w > 0) = 1 - P_0 = 1 - \left(1 - \frac{\lambda}{\mu}\right) = \frac{6}{20} = 0.3$$

(ii) Avg. length of non-empty queues :-

$$\frac{\mu}{\mu-\lambda} = \frac{20}{20-6} = \frac{20}{14} = 1.43$$

(iii) let λ' denotes the inc. arrival time rate.

So expected waiting time $\Rightarrow \frac{3}{60} = \frac{\lambda'}{\mu(\mu-\lambda')}$

$$\frac{3}{60} = \frac{\lambda'}{20(20-\lambda')} \Rightarrow \boxed{\lambda' = 10}$$

(iv) $\rho = \frac{\lambda}{\mu} = \frac{6}{20} = 0.3$

(v) $P(w \geq 10) = \int_{10}^\infty \lambda\left(1 - \frac{\lambda}{\mu}\right) e^{-(\mu-\lambda)t}\, dt = \int_{10}^\infty (0.30)(0.23)\, e^{-0.23t}\, dt$

$(\lambda = 0.10/\text{min.}$
$\mu = 0.33/\text{min.})$

$$= 0.069 \left[\frac{e^{-0.23t}}{-0.23}\right]_{10}^\infty$$

$$= 0.03$$

- **Result :-**

(i) Prob. that a person arriving at the booth will have to wait is 0.3

(ii) Avg. length of non empty queues is 1.43

(iii) Arrival rate should become 10 customers / hr to justify II booth.

(iv) Fraction of day that phone will be busy = 0.3.

(v) 3% of the arrivals on a avg. will have to wait for 10 min. or more before they can use the phone.

PRACTICAL-12

- **Aim** :- To measure the performance of model $(M/M/1):(\infty/FIFO)$.
- **Problem** :- At a public telephone booth in a post office arrivals are considered to be poisson with an avg. inter-arrival time of 12 min. The length of a phone call distributed exponentially with an avg. of 4 min.

(i) What is the prob. that a fresh arrival will not have to wait for the phone?

(ii) What is the prob. that an arrival will have to wait more than 10 min. before the phone is free?

(iii) What is the avg. length of queues that form from time to time?

- **Theory & formula** :-

\# $(M/M/1):(\infty/FIFO)$ ⇒ This model deals with a queuing system having single service channel. Poisson input, Exp. service & there is no limit on the system capacity while the customers are served on a "first in, first out" basis.

\# **Performance** ⇒ λ (Mean arrival), μ (Mean service), $\rho = \frac{\lambda}{\mu}$

Avg. no. of customers in service ⇒ $L_s = \frac{\lambda}{\mu-\lambda}$

Avg. no. of customers in queue ⇒ $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$

Avg. waiting time in system ⇒ $W_s = \frac{1}{\mu-\lambda}$

Avg. waiting time in queue ⇒ $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

Prob. of no customer ⇒ $P_0 = 1-\rho = 1-\frac{\lambda}{\mu}$

Length of non-empty queue ⇒ $L_n = \frac{\mu}{\mu-\lambda}$

Prob. of waiting time in queue $\geq n$ ⇒ $\int_n^{\infty} \frac{\lambda}{\mu}(\mu-\lambda)e^{-(\mu-\lambda)t}\,dt$

Prob. that arrival will have to wait for more than N min. ⇒ $\int_N^{\infty} (\mu-\lambda)e^{-(\mu-\lambda)t}\,dt$

- **Calculation** :-

$\lambda = \frac{1}{12} \times 60 = 5/hr$ & $\mu = \frac{1}{4} \times 60 = 15/hr$.

$$\rho = \frac{5}{15} = \frac{1}{3}$$

(i) $P_0 = 1 - \rho = 1 - \frac{1}{3}$

$= \frac{2}{3} = 0.66$

(ii) $P(w_q \geq 10) = \int_{10}^{\infty} \frac{\lambda}{\mu} (\mu - \lambda) e^{-(\mu-\lambda)t} dt$

$= \frac{\lambda}{\mu} e^{-(\frac{1}{4} - \frac{1}{12})10}$

$= \frac{1}{3} e^{-5/3}$

(iii) $L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{25}{15(15-5)} = \frac{\cancel{25}^{5}}{_{3}\cancel{15} \times \cancel{10}_{2}}$

$L_q = \frac{1}{6} = 0.16$

- Result :-

Prob. of arrival will not have to wait $= 0.66$

Prob. that an arrival will have to wait for more than 10 min. $= \frac{1}{3} e^{-5/3}$

Avg. length of queues $= 0.16$

PRACTICAL - 13

- **Aim** :- To measure the performance of model $(M/M/1):(\infty/FIFO)$.
- **Problem** :- At a certain petrol pump, customers arrive in a Poisson process with an average time of 5 min. b/w arrivals. The time interval b/w servers at the petrol pump follows an exp. dist. & mean time taken to service a unit is 2 min. Find :-

(i) Expected avg. queue length.
(ii) Avg. no. of customers in the system.
(iii) Avg. time a customer has to wait in the queue.
(iv) Avg. time a customer has to spend in the system.
(v) By how much time the flow of the customer be increased to justify the opening of another service point, where customer has to wait for 5 min. for the service?

- **Theory & formula** :-

$(M/M/1):(\infty/FIFO)$ ⇒ This model deals with a queuing system having single service channel. Poisson input, Exp. service & there is no limit on the system capacity, while the customers are served on a "first in, first out" basis.

Performance ⇒ λ (Mean Arrival), μ (Mean service) ; $\rho = \frac{\lambda}{\mu}$

Avg. no. of customers in service ⇒ $L_s = \frac{\lambda}{\mu-\lambda}$

Avg. no. of customers in queue ⇒ $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$

Avg. waiting time in system ⇒ $W_s = \frac{1}{\mu-\lambda}$

Avg. waiting time in queue ⇒ $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

Probability of no customer ⇒ $P_0 = 1-\rho = 1-\frac{\lambda}{\mu}$

length of non empty queue ⇒ $L_n = \frac{\mu}{\mu-\lambda}$

Prob. of waiting time in queue $\geq n$ ⇒ $\int_n^\infty \frac{\lambda}{\mu}(\mu-\lambda)\, e^{-(\mu-\lambda)t}\, dt$

- **Calculation** :-

$$\lambda = \frac{1}{5} \times 60 = 12/hr. \quad \& \quad \mu = \frac{1}{2} \times 60 = 30/hr$$

$$\rho = \frac{\lambda}{\mu} = \frac{12}{30} = \frac{2}{5}$$

(i) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{144}{30(30-12)} = \frac{\cancel{144}^{8}}{30 \times \cancel{18}}$

$L_q = \frac{4}{15} = 0.26$

(ii) $L_s = \frac{\lambda}{\mu-\lambda} = \frac{12}{(30-12)} = \frac{12}{18} = \frac{2}{3}$

$L_s = 0.66$

(iii) $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{12}{30(30-12)} = \frac{\cancel{12}^{2}}{{}_{15}\cancel{30} \times \cancel{18}\,3}$

$W_q = \frac{1}{45} = 0.0222$

(iv) $W_s = \frac{1}{\mu-\lambda} = \frac{1}{30-12} = \frac{1}{18}$

$W_s = 0.0555$

(v) let time increase $= \lambda'$

so, $\frac{5}{60} = \frac{1}{\mu-\lambda'}$

$\frac{5}{60} = \frac{1}{30-\lambda'} \Rightarrow 30-\lambda' = 12 \Rightarrow \lambda' = 18$

$\lambda' - \lambda = 18 - 12 = 6$

- **Result :-**

Average queue length = 0.26

Avg. no. of customers in system = 0.66

Avg. waiting time in the queue = 0.0222

Avg. waiting time in system = 0.0555

Customer arrival increase by 6/hr. to justify the opening of another service point.