PRACTICAL-4(A)

- **Aim** :- Check whether the process is Covariance stationary or not.

- **Problem** :- Let $X_t = A_1 + A_2 t$ where $A_i \sim U(a, b)$ such that $E(a_i) = 0$ & $V(a_i = 1)$ for all $i = 1, 2, \ldots$ Verify if the process is cov. stationary?

- **Theory** :-

A second order process or a Covariance stationary process (X_t); $[X(t), t \in T]$ is stationary if its mean f^n is independent of time & $C(s, t)$ is f^n of time difference $|t - s|$.

If for arb. $t_1, t_2, \ldots, t_n$ the joint dist^n r.v. $[X(t_1), X(t_2), \ldots, X(t_n)]$ and $[X(t_1+h), X(t_2+h), \ldots, X(t_n+h)]$ are same for all $h > 0$ the it is said to be strictly stationary.

- **Calculation** :-

$$X_t = A_1 + A_2 t$$

$$\& \ A_i \sim U(a, b)$$

so Mean $= \dfrac{a+b}{2}$

Also, $E(a_i) = 0$

so $\dfrac{a+b}{2} = 0$

so $\boxed{a = -b}$

Variance $= \dfrac{(a-b)^2}{12}$

Also, $V(a_i) = 1$

so, $\dfrac{(a-b)^2}{12} = 1 \Rightarrow (a-b)^2 = 12$

$a = -b$

so, $4b^2 = 12$

$b^2 = 3 \Rightarrow \boxed{b = \pm\sqrt{3}}$

$X(t) = A_1 + A_2 t \Rightarrow V(t) = 1 + t^2$

$X(s) = A_1 + A_2 s$

So, $Cov(s,t) = Cov[X(s), X(t)]$

$= E[X(s), X(t)] - \underbrace{E(X,s)\ E[X(t)]}_{0}$

$= E[X(s), X(t)]$

$= E[(A_1 + A_2 t)(A_1 + A_2 s)]$

$= E[A_1^2 + A_1 A_2 s + A_1 A_2 t + A_2^2 ts]$

A_1 & A_2 are independent & $V(a_i = 1)$

So, $\boxed{Cov(s,t) = 1 + st}$

- Result :-

As can be seen from the covariance table, $V(t)$ keeps increasing as the value of t increases.
So variance is not independent of the value of t.

So, the Process is not Covariance stationary.

PRACTICAL - 4 (b)

- **Problem :-** Consider the process;

$X(t) = A\cos\omega t + B\sin\omega t$, where A & B are selected such that mean $= 0$ & var. $= 1$, ω is a +ve const. Check whether the process is cov. stationary or not.

- **Calculation :-**

$$X(t) = A\cos\omega t + B\sin\omega t$$

$$\text{mean} = 0$$

$$X(s) = A\cos\omega s + B\sin\omega s$$

$$\begin{aligned}
Cov(s,t) &= Cov[X(s), X(t)] \\
&= E[X(s), X(t)] - \underbrace{E[X(s)]\ E[X(t)]}_{0} \\
&= E[(A\cos\omega s + B\sin\omega s)(A\cos\omega t + B\sin\omega t)] \\
&= E[A^2\cos\omega t + \cos\omega s + \underbrace{AB\cos\omega s\sin\omega t}_{0} + \\
&\qquad \underbrace{BA\sin\omega s\cos\omega t}_{0} + B^2\sin\omega s\sin\omega t] \\
&= \cos\omega t\cos\omega s + \sin\omega t\sin\omega s \\
&= \cos(\omega t - \omega s)
\end{aligned}$$

So, $Cov(s,t) = \boxed{\cos(|t-s|\omega)}$

- **Result :-**

As can be seen from the cov. table, $V(t)$ is almost equal to 1. So it is independent of the time t.

Also, the cov. fx^n is fx^n of $|(s-t)|$.

So, the process is covariance stationary.