PRACTICAL – 12

Submitted By: Ridam Singhal(5040)

AIM: To fit logistic growth model using Hotelling's method, and estimate the trend values.

EXPERIMENT:

The following data represent the size of a population at time t, where t is in years:

Time	Population size
2007	956
2008	1225
2009	2879
2010	4994
2011	11525
2012	16190
2013	22573
2014	30677
2015	38517
2016	44003
2017	49034
2018	56244

(a) Fit a Logistic growth model for this data using the Hostelling's method.
(b) Estimate trend values of the time series using the above Logistic trend equation.

THEORY:

Logistic Curve:

i. A particular form of complex type of growth curves.
ii. Given by: $y = y_t = \frac{k}{1+\exp(a+bt)}$, b>0
iii. a, b, k are constants and yt is the value of the given time series, at time t.

Hotelling's Method:

i. An elegant and indigenous method to fit a logistic curve.
ii. We have, $\frac{1}{y_t} . \frac{\Delta y_t}{\Delta t} = -b + \frac{b}{k} y_t$ OR U = A + By, where $U = \frac{1}{y_t} . \frac{\Delta y_t}{\Delta t}$, A = -b, and B = $\frac{b}{k}$
iii. A and B, and this b and k can be obtained by principle of least square.
iv. a is obtained by assuming that the curve passes through mean of Y and mean of t.

CALCULATIONS: (an excel file has been attached for reference to detailed calulations)

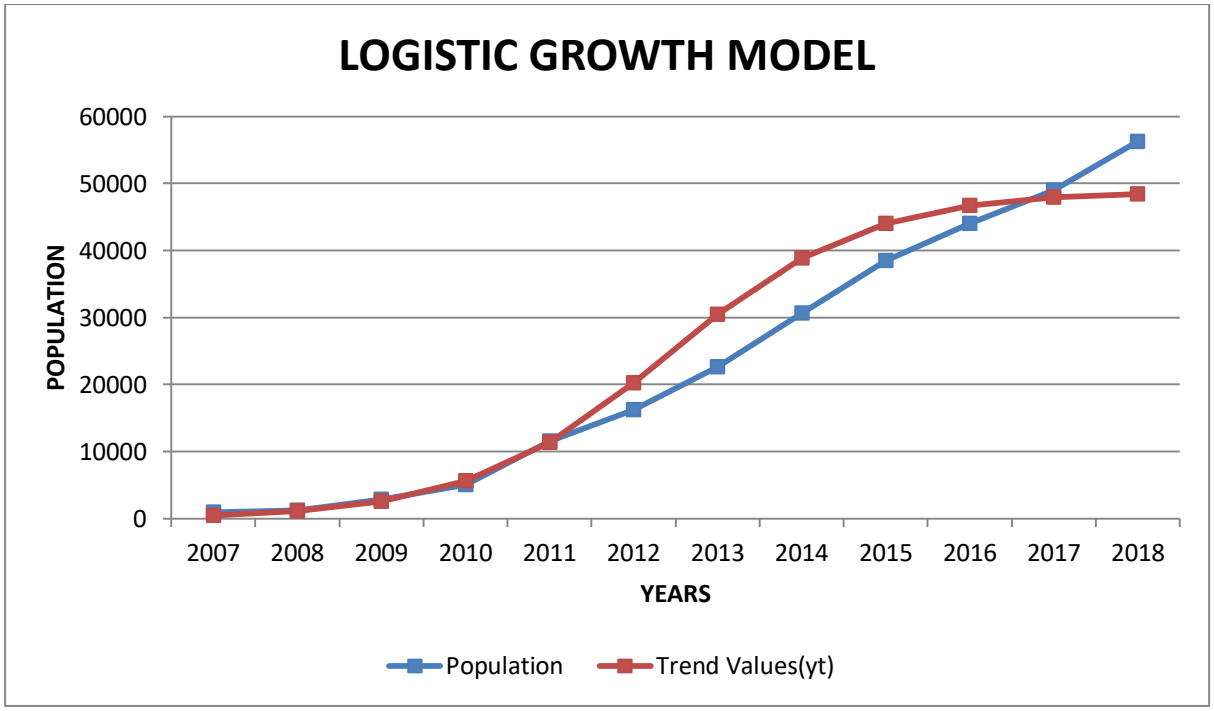
Table-12.1

t	Time	Population	U	Trend Values(y_t)
1	2007	956	0.281380753	482.8642551
2	2008	1225	1.350204082	1117.151764
3	2009	2879	0.73463008	2540.880184
4	2010	4994	1.307769323	5567.487788
5	2011	11525	0.404772234	11318.45652
6	2012	16190	0.394255713	20233.90909
7	2013	22573	0.35901298	30472.71123
8	2014	30677	0.255566059	38860.79856
9	2015	38517	0.142430615	44030.79496
10	2016	44003	0.114333114	46679.84261
11	2017	49034	0.147040829	47909.36233
12	2018	56244		48453.759

B	A
-1.74351E-05	0.851998238

b=	-0.851998238		k=	48866.81053		a=	5.459186018

Graph-12.1



RESULT:

- Values for the constants a, b, and k have been computed and shown under Table 12.1, along with the values of A and B.
- Hence the logistic curve equation comes out to be: $y_t = \frac{48866.81053}{1+\exp(5.459186-0.851998*t)}$
- Trend values have been computed and shown in Table 12.1.
- Logistic growth model is shown in Graph 12.1.