PRACTICAL - 9

- **Aim** :- To calculate the probability of poisson process.
- **Problem** :- Let $N(t)$ be the no. of computer failures in the interval 0 to t. We suppose that $N(t)$ is Poisson process with mean rate of 1/week. Calculate the prob. that;

(i) System operate with no failure for 2 weeks.

(ii) System will have exactly 2 failures during a given week knowing that it operate without failure in previous 2 weeks.

(iii) 2 weeks elapse before the III failure occur.

- **Theory** :-

The counting process $[X(t); t \geq 0]$ is a Poisson Process having rate λ where $\lambda > 0$ if;

(i) $X(0) = 0$

(ii) Process have independend increment

(iii) The no. of events in any interval of length t is a Poisson dist. with mean $\boxed{\lambda t}$.

$$P[X(t+s) - X(s) = n] = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

(*) The Poisson Process has a stationary & independent increment.

(*) Interarrival times ⇒ The intervals between successive occurances (called interarrival times) of a poisson process with mean λt are identically & independently distributed r.v. which follows the

negative exponential law with mean $1/\lambda$

(*) So, the Poisson Process has independent exponentially distributed interarrival times & gamma distributed waiting times.

2 weeks: $t \longrightarrow t+2$

- Calculation :-

(i) No failure for 2 consecutive weeks $\Rightarrow$

$$P[N(t+2) - N(t) = 0]$$

$$P[N(2) - N(0) = 0] \Rightarrow P[N(2) = 0]$$

(being stationary)

So; $\Rightarrow \dfrac{e^{-2}(2)^0}{0!}$ $(\lambda = 2)$

$$= \boxed{e^{-2}}$$

(ii) exactly 2 failures during a given week $\Rightarrow$

So here, $\lambda = 1$

So; $\Rightarrow P[N(1) = 2] \Rightarrow \dfrac{e^{-1}(1)^2}{2!} = \dfrac{e^{-1}}{2}$

$$\Rightarrow \boxed{\frac{1}{2e}}$$

(iii) 2 weeks elapse before the III failure $\Rightarrow$

Poisson process follows exp. dist.

So; $N(t) = \lambda e^{-\lambda t}$

Prob. that 2 weeks elapse before III failure is $\Rightarrow P[T > 2]$

T is time until III failure.

~~$P[T>2] = (1 - [\lambda e^{-\lambda t}])$~~

Here $\lambda = 1$ & $t = 2$

So. $P[T > 2] = \int_2^{\infty} \lambda e^{-\lambda t}\, dt = \int_2^{\infty} e^{-t}\, dt \Rightarrow \left[-e^{-t}\right]_2^{\infty}$

$$\Rightarrow \boxed{e^{-2}}$$

- Result :- (i) No failure for 2 weeks $= e^{-2}$

(ii) Prob. of 2 failures knowing that it operate w/o failure in prev. 2 weeks $= \dfrac{1}{2e}$

(iii) 2 weeks elapsed before III failure $= e^{-2}$

PRACTICAL - 10

- **Aim :-** To calculate the probability of Poisson Process.
- **Problem :-** City buses arrive at a certain street corner b/w 5 a.m. & 11 p.m. according to poisson process with rate 4/hr. let T_1 be the waiting time (in min) until the I bus (after 5 a.m.) & M be the total no. of buses arrive at this corner b/w 5 a.m. & 5:15 a.m.

Calculate the prob $[T_1 \in (a, b) / M = 1]$ when interval $[a, b]$ is included b/w 0 to 15

- **Theory :-**

The counting process $[X(t) ; t \geq 0]$ is a poisson process having rate λ where $\lambda > 0$ if ;

(i) $X(0) = 0$

(ii) Process have independent increment.

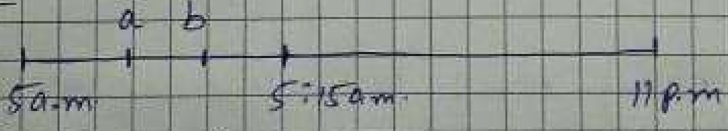
(iii) The no. of events in any interval of length t is a poisson dist. with λt.

(*) The Poisson Process has a stationary & independent increment.

(*) Interarrival Times :- The intervals b/w successive occurances (called interarrival times) of a poisson process with mean λt are identically & independently distributed r.v. which follows the ;

negative exponential law with mean $1/\lambda$.

- **Calculation :-**



$N(t) \Rightarrow$ no. of buses arrive at the corner during $[0, t]$.

$$P[T_1 \in [a, b] / M = 1] = P[N(a) - N(0) = 0, N(b) - N(a) = 1, N(15) - N(b) = 0]$$

$$= P[N(a) - N(0) = 0] \cdot P[N(b) - N(a) = 1] \cdot P[N(15) - N(b) = 0]$$

[Becoz Poisson Process stationary & ind. increments]

$= e^{-\lambda a} \cdot e^{-\lambda(b-a)} \cdot \lambda(b-a) \times e^{-\lambda(15-b)}$

$= e^{-\lambda(a+b-a+15-b)} \cdot \lambda(b-a)$

$= e^{-\lambda(15)} \cdot \lambda(b-a)$

$\{N(t) : t \geq 0\}$ — poisson process, $\lambda = \frac{1}{15}$

So, $\Rightarrow P[T_1 \in [a,b] \mid M = 1] = e^{-1} \cdot \frac{1}{15}(b-a)$

$= \boxed{\frac{b-a}{15e}}$

- **Result :-**

$P[T_1 \in [a,b] \mid M = 1]$ when interval $[a,b]$ is included b/w 0 to 15 is $\Rightarrow \boxed{\frac{b-a}{15e}}$