

## PRACTICAL – 14

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**AIM:** To plot a correlogram for the given moving average process.

### **EXPERIMENT:**

Plot the correlogram of Moving Average Process for the following.

$$Y_t = \frac{1}{2}\epsilon_t + \frac{3}{4}\epsilon_{t-1} - \frac{3}{8}\epsilon_{t-2} - \frac{15}{16}\epsilon_{t-3} - \frac{15}{32}\epsilon_{t-4} - \frac{29}{64}\epsilon_{t-5} - \frac{29}{128}\epsilon_{t-6}.$$

### **THEORY:**

Correlogram: A correlogram is a visual way to show autocorrelation in the data that changes over time (i.e. time series data).

Correlogram of Moving Average: For a moving average of extent  $m$ , with weights  $(a_1, a_2, \dots, a_m)$  of random components  $(\epsilon_i; i=1,2,\dots)$ , the generated series is given by:

$$y_i = a_1\epsilon_{i+1} + a_2\epsilon_{i+2} + \dots + a_{k+1}\epsilon_{i+k+1} + \dots + a_m\epsilon_{i+m}$$

$$y_{i+k} = a_1\epsilon_{i+k+1} + a_2\epsilon_{i+k+2} + \dots + a_{m-k}\epsilon_{i+m-k} + \dots + a_m\epsilon_{i+k+m}$$

Where  $\epsilon_i$ 's are iid  $N(0, \sigma^2)$ . Thus,

$$E(y_i) = 0 = E(y_{i+k})$$

$$\text{And } \text{Var}(y_i) = E(y_i^2) = (a_1^2 + a_2^2 + \dots + a_m^2)\sigma^2 = \sigma^2 \sum_{j=1}^m a_j^2 \quad \forall i = 1, 2, \dots$$

$$\text{Similarly, } \text{Var}(y_{i+k}) = E(y_{i+k}^2) = \sigma^2 \sum_{j=1}^m a_j^2$$

$$E(y_i y_{i+k}) = (a_1 a_{k+1} + a_2 a_{k+2} + \dots + a_{m-k} a_m) \sigma^2 = \sigma^2 \sum_{j=1}^{m-k} a_j a_{j+k}, \quad k < m$$

$$\left. \begin{aligned} r_k &= \frac{E(y_i y_{i+k})}{\sqrt{\text{Var}(y_i) \text{Var}(y_{i+k})}} \\ &= \frac{\sum_{j=1}^{m-k} a_j a_{j+k}}{\sum_{j=1}^m a_j^2}, \text{ if } k < m \\ r_k &= 0, \text{ if } k \geq m \end{aligned} \right\}$$

## **CALCULATIONS:**

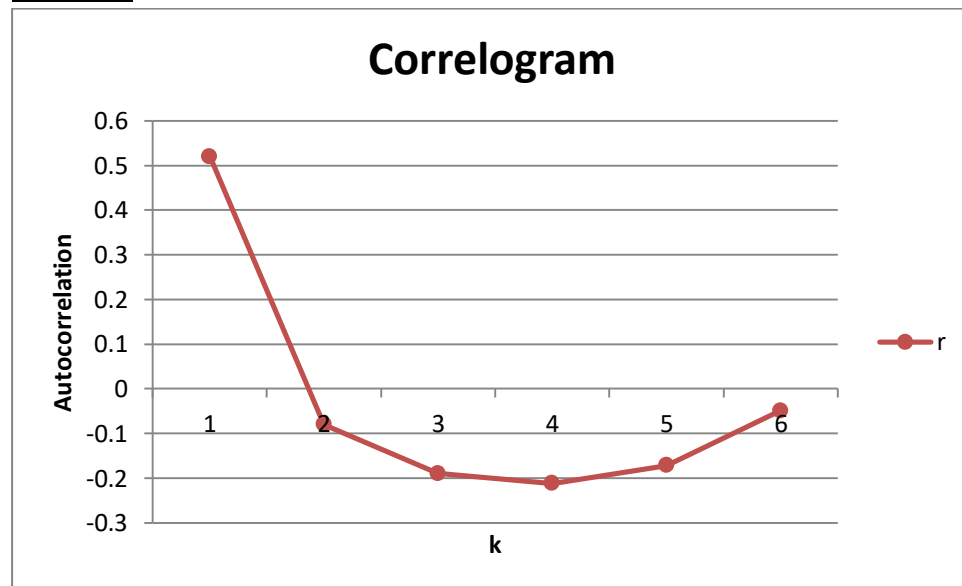
Table 14.1

m	a	a2	k					
			1	2	3	4	5	6
1	0.5	0.25						
2	0.75	0.5625	0.375					
3	-0.375	0.140625	-0.28125	-0.1875				
4	-0.9375	0.878906	0.351563	-0.70313	-0.46875			
	-							
5	0.46875	0.219727	0.439453	0.175781	-0.35156	-0.23438		
	-							
6	0.45313	0.205322	0.212402	0.424805	0.169922	-0.33984	0.22656	
	-							
7	0.22656	0.051331	0.102661	0.106201	0.212402	0.084961	0.16992	0.11328
SUM	-							
	1.21094	2.308411	1.199829	-0.18384	-0.43799	-0.48926	0.39648	0.11328

Table 14.2

k	1	2	3	4	5	6
r	0.519764	0.07964	0.18974	0.21195	0.17176	0.04907

Graph 14.1



## **RESULT:**

- The value of autocorrelation ( $r_k$ ) for the order k have been shown in Table 14.2
- Graph 14.1 shows the correlogram plotted for the moving average.