# PRACTICAL - 6 (B).

- **Problem :-** Find mean & variance of compound probability distⁿ $S_N$ where $S_N = X_1 + X_2 + \ldots + X_N$ ; where $N \sim$ Poisson (5) & $X \sim$ Binomial (5, 0.6). Also verify the result.

- **Calculation :-**

$$N \sim \text{Poisson}(5)$$
$$X \sim \text{Binomial}(5, 0.6)$$

So; $E(N) = 5$ | $E(X_i) = 5 \times 0.6 = 3$

& $V(N) = 5$ | & $V(X) = npq = 5 \times 0.6 \times 0.4 = 1.2$

$$E[S_N] = E(X) \cdot E(N)$$
$$E[S_N] = 3 \times 5 = 15$$

So, $\boxed{E[S_N] = 15}$

$$V[S_N] = [E(X_i)]^2 \cdot V(N) + V(X)\, E(N)$$
$$= (3)^2 \cdot 5 + 1.2(5)$$
$$= 45 + 6 = 51$$

So, $\boxed{V[S_N] = 51}$

- **Result :-**

Compound probability distribution has been obtained in the excel sheet.

Mean & variance in excel sheet ⇒ $E[S_N] = 16.4$
$V[S_N] = 48.16327$

Theoretical Mean & Variance ⇒ $E[S_N] = 15$
$V[S_N] = 51$

So, the obtained Mean & Variance are very close to their respective theoretical values, thus verifying our result.

# PRACTICAL - 6 (A)

- **Aim** :- To obtain compound probability distribution, its mean & variance.

- **Problem** :- Find mean & variance of compound probability distⁿ $S_N$ where $S_N = X_1 + X_2 + \cdots + X_N$; where $N \sim$ poisson(6) & $X \sim$ Bernoulli (0.45). Also verify the result.

- **Theory** :-

  let $X_i$; $i = 1, 2, \ldots$ be the iid r.v. with probability

  $P[X_i = k] = p_k$ & p.g.f of series is $P(s) = \sum p_k s^k$

  for $i = 1, 2, \ldots$

  let $S_N = X_1 + X_2 + \cdots + X_N$ where $N$ is a r.v. independent of $X_i$.
  let distⁿ of $N$ be given by ;⇒ $\boxed{P[N = n] = g_n}$ & p.g.f of $N$ is

  $\boxed{G(s) = \Sigma g_n s^n}$, then p.g.f. of $S_N$ is given by compound function, i.e., $\boxed{H(s) = G[P(s)]}$

$$\begin{cases} E[S_N] = E(N) \cdot E(X_i) \\ V[S_N] = [E(X_i)]^2 \cdot V(N) + V(X)\, E(N) \end{cases}$$

- **Calculation** :-

  $N \sim$ Poisson (6)
  $X_i \sim$ Bernoulli (0.45)

| | |
|---|---|
| So; $E(N) = 6$ | $E(X_i) = 0.45$ |
| & $V(N) = 6$ | $V(X) = pq = 0.45(0.55)$ |

$$E[S_N] = E(N) \cdot E(X_i)$$

$$\text{So, } E[S_N] = 6(0.45) = \boxed{2.7}$$

$$V[S_N] = [E(X_i)]^2 \cdot V(N) + V(X)\, E(N)$$

$$\text{So, } V[S_N] = (0.45)^2 (6) + (0.45)(0.55)\, 6$$

$V(S_N) = 1.215 + 1.485$

$\boxed{V(S_N) = 2.7}$

- Result :-

Compounded probability distribution has been obtained in the excel sheet.

The obtained Mean & variance are very close to their respective theoretical values, thus ~~very~~ verifying our result.