Solving Coupled ODEs

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1 Introduction

In order to solve Simultaneous differential equations using Runge Kutta method we proceed in a manner similar to the one used for solving single differential equation.

We need to apply each step of the algorithm to all the ODEs before moving ahead with the next step.

Here, RK method of order 4 is used.

k1 must be calculated for all the differential equations before k2 is calculated. The same is true for k2, k3, and k4. For a System of ODEs, k1,k2,k3,k4 are vectors.

In the second part of the problem, initial values of x,y,z are Jointly Gaussian with mean (1,1,1) and co-variance of identity matrix

$$f(x1, x2, x3) = \frac{1}{\sqrt{(2\pi)^3 det(C)}} exp(-1/2(X - M)C^-1(X - M)^T)$$

Here, x1=x x2=y x3=z

C: covariance matrix X: variable vector M: Mean vector

Substituting the values of Mean and Covariance matrices, we obtain the following pdf

$$f(x1, x2, x3) = \frac{1}{\sqrt{(2\pi)^3}} exp(-1/2(x - 1)^2 + (y - 1)^2 + (z - 1)^2)$$

Sampling of Multivariate (given) Gaussian probability density function:

1. Find standard normal random variables (N(0,1)) from uniformly distributed random variables (U(0,1)) by using Box Muller Transform.

2. The standard normal random variables obtained above are transformed to the required Jointly Gaussian random variables by using the following transformation.

$$Z = cX + m$$

where, Z: vector of required pdf
X: vector formed by taking random variables from step 1
c: Lower triangular matrix obtained from C (Covariance Matrix)
m: Vector of means of X

c is generally obtained from C by using Cholesky decomposition:

$$C = cc^T$$