Note Title 02-09-2024

Electrical Engineering
IIT Madras

EE 3101
Digital Signal Processing

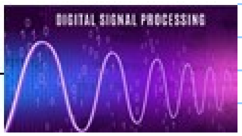
BS Electronic Systems programme

R. David Koilpillai

September – December 2024

Session # 13

October 25, 2024



EE3101 Digital Signal Processing

Session 12

EE3101
Session 12

Outline

Last session
- DTFT Symmetry Props
- DTFT Theorems
- DTFT Pairs

Week 5-6
Discrete-Time Fourier Transform (DTFT): Complex exponentials as eigensignals of LTI systems—DTFT definition—inversion formula—properties—relationship to continuous-time Fourier series (CTFS)

Week 7-8
Z-Transform: Generalized complex exponentials as eigen signals of LTI systems—z-transform definition—region of convergence (RoC)—properties of RoC—properties of the z-transform—pole-zero plots—time-domain responses of simple pole-zero plots—RoC implications of causality and stability

✓ O&S ch 2 Sec 7 Representation of Sequences by Fourier Transforms

✓ ch 2 Sec 8 Symmetry Properties of Fourier Transform

✓ ch 2 Sec 9 Fourier Transform Theorems

Properties & Applications

$\boxed{\text{Reading Assignment}}$

O&S ch 2 Sec 7, Sec 8 and Sec 9

Discrete Time Fourier Transform (DTFT)

DT seq $x[n] \xrightarrow[\text{DTFT}]{\mathcal{F}} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega \xleftarrow[\text{Inv DTFT}]{\mathcal{F}^{-1}} X(e^{j\omega})$

DTFT

TABLE 2.1 SYMMETRY PROPERTIES OF THE FOURIER TRANSFORM

Sequence $x[n]$	Fourier Transform $X(e^{j\omega})$
1. $x^*[n]$	$X^*(e^{-j\omega})$
2. $x^*[-n]$	$X^*(e^{j\omega})$ $\quad x^*[n] \longleftrightarrow X^*(e^{-j\omega})$
3. $\mathcal{R}e\{x[n]\}$	$X_e(e^{j\omega})$ (conjugate-symmetric part of $X(e^{j\omega})$)
4. $j\mathcal{I}m\{x[n]\}$	$X_o(e^{j\omega})$ (conjugate-antisymmetric part of $X(e^{j\omega})$)
5. $x_e[n]$ (conjugate-symmetric part of $x[n]$)	$X_R(e^{j\omega}) = \mathcal{R}e\{X(e^{j\omega})\}$
6. $x_o[n]$ (conjugate-antisymmetric part of $x[n]$)	$jX_I(e^{j\omega}) = j\mathcal{I}m\{X(e^{j\omega})\}$
The following properties apply only when $x[n]$ is real:	
7. Any real $x[n]$	$X(e^{j\omega}) = X^*(e^{-j\omega})$ (Fourier transform is conjugate symmetric)
8. Any real $x[n]$	$X_R(e^{j\omega}) = X_R(e^{-j\omega})$ (real part is even)
9. Any real $x[n]$	$X_I(e^{j\omega}) = -X_I(e^{-j\omega})$ (imaginary part is odd)
10. Any real $x[n]$	$\lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert$ (magnitude is even)
11. Any real $x[n]$	$\sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega})$ (phase is odd)
12. $x_e[n]$ (even part of $x[n]$)	$X_R(e^{j\omega})$
13. $x_o[n]$ (odd part of $x[n]$)	$jX_I(e^{j\omega})$

$$x[n] \xleftrightarrow{DTFT} X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jwn}$$

1. Conjugation $\quad X^*(e^{jw}) = \sum_{n=-\infty}^{\infty} x^*[n]\, e^{jwn}$

2. Replace $w \leftarrow -w$ $\quad X^*(e^{-jw}) = \sum_{n=-\infty}^{\infty} x^*[n]\, e^{-jwn}$

3. Replace $m = -n$ $\quad X^*(e^{jw}) = \sum_{m=-\infty}^{\infty} x^*[-m]\, e^{-jwm}$

Results

$$x[n] \longleftrightarrow X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jwn}$$

TABLE 2.2 FOURIER TRANSFORM THEOREMS

Sequence $x[n]$ $y[n]$	Fourier Transform $X(e^{j\omega})$ $Y(e^{j\omega})$	
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	
2. $x[n - n_d]$ (n_d an integer)	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$	Freq shift
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.	Time reversal
5. $nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$	Differentiation
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$	Convolution in Time
7. $x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	Multiplication in Time

periodic convolution
Integrating over one period
Convolution of periodic functions

Parseval's theorem:

8. $$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

9. $$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega}) d\omega$$

Ex 1. Sketch the DTFT of $x[n] = \sin \omega_0 n$ with $\omega_0 = \frac{2\pi}{5}$

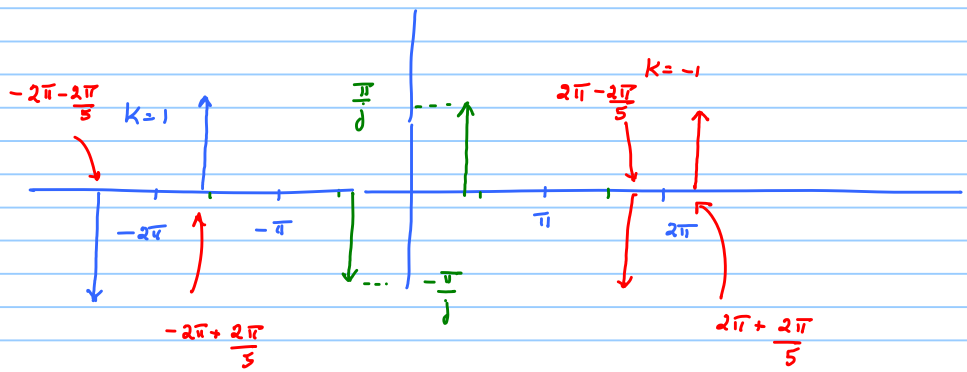
$$x[n] = \frac{1}{2j}\left[e^{j\omega_0 n} - e^{-j\omega_0 n}\right]$$

$$\frac{1}{2j} e^{j\omega_0 n} \longleftrightarrow \frac{1}{\cancel{2}j}\left[\cancel{2}\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 + 2\pi k)\right] = \frac{\pi}{j} \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 + 2\pi k)$$

$$\frac{1}{2j} e^{-j\omega_0 n} \longleftrightarrow$$

$$\sin \omega_0 n \longleftrightarrow \frac{\pi}{j} \sum_{k=-\infty}^{\infty} \left[\delta(\omega - \omega_0 + 2\pi k) - \delta(\omega + \omega_0 + 2\pi k)\right]$$

$\omega_0 = \frac{2\pi}{5}$



Ex 2

$x[n] \longleftrightarrow X(e^{jw})$

$x^*[n] \longleftrightarrow X^*(e^{-jw})$

Case 1

If $x[n]$ is real $\Rightarrow x[n] = x^*[n]$

$X(e^{jw}) = X^*(e^{-jw})$

DTFT is conjugate symmetric

Case 2

$x[n]$ is real and even

$x[n] = x^*[n] = x[-n]$

$X(e^{jw}) = X^*(e^{-jw}) = X(e^{-jw})$

$X(e^{jw})$ is real and even

even

Result

Complex number $c = a + jb$

If $c = c^*$ $\quad a + jb = a - jb$ $\quad b = 0 \Rightarrow c$ is real valued

If $c = -c^*$ $\quad a + jb = -a + jb \Rightarrow a = 0$

c is imaginary

Case 3

$x[n]$ is real and odd

$x[n] = x^*[n] = -x[-n] \implies X(e^{jw})$ is imaginary & odd

Case 4 $x[n]$ is imaginary

$x[n] = -x^*[n] \implies X(e^{jw})$ is conjugate symm

Case 5 $x[n]$ is imag & even

$x[n] = -x^*[n] = x[-n] \implies X(e^{jw})$ is imag & even

Verify

Case 6 $x[n]$ is imag & odd

$x[n] = -x^*[n] = -x[-n] \implies$ ______

Provide the answer with verification

Results from previous session

$x[n] \longleftrightarrow X(e^{j\omega})$

1. $(-1)^n x[n] = e^{j\pi n} x[n] \longleftrightarrow X(e^{j(\omega-\pi)})$

$X(e^{j\omega})$ is periodic with period $= 2\pi$

$X(e^{j(\omega-\pi+2\pi)}) = X(e^{j(\omega+\pi)})$

Frequency shifting

shifting to $\omega_0 = \pi$ (right shift) $e^{j\omega_0 n} x[n]$

is identical to $\omega_0 = -\pi$ (left shift) to freq $\omega_0 = -\pi$ $e^{-j\omega_0 n}$ $\omega_0 = \pi$

2. $e^{-j\frac{\pi}{2}n} x[n] \longleftrightarrow X(e^{j(\omega+\frac{\pi}{2})})$

$\{1, -j, -1, +j, 1, -j, -1, +j, \dots\}$

Frequency down conversion technique

- Signal centred around $\Omega_0 = 2\pi f_0$ and sampled @ $\Omega_s = 4\Omega_0$
- w/o multiplication

$T_s = \frac{1}{4f_0}$

Discrete Time

$\Omega_0 \longrightarrow \frac{\pi}{2}$ $\quad \omega_0 = \Omega_0 T_s = 2\pi f_0 T_s$

$= 2\pi f_0 \frac{1}{4f_0} = \frac{\pi}{2}$

$\Omega_s \longrightarrow 2\pi$

3 $n a^n u[n] \longleftrightarrow \dfrac{a\, e^{-j\omega}}{(1-\alpha\, e^{-j\omega})^2}$ Via differentiation theorem

4. $(n+1)\, a^n\, u[n+1] \longleftrightarrow \dfrac{1}{(1-a e^{-j\omega})^2}$

Note $u[n+1]$ starts @ $n=-1$

$(n+1)\, a^n\, u[n+1] = 0$ @ $n=-1$

Hence $(n+1)\, a^n\, u[n+1] \equiv \boxed{(n+1)\, a^n\, u[n]}$ Verify

5. Ideal filters
- LPF, HPF, BPF, BSF
- All specified via DTFT

6. Solving LCCDE using DTFT & obtaining $h[n]$

$a^n u[n] \longleftrightarrow \dfrac{1}{1-ae^{-j\omega}}$

$|a|<1$

$\longleftrightarrow \dfrac{1}{(1-ae^{-j\omega})^2}$

$\updownarrow$

obtained via first derivative

$\dfrac{d}{d\omega}\left(\dfrac{1}{1-ae^{-j\omega}}\right)$

$\longleftrightarrow \dfrac{1}{(1-ae^{-j\omega})^3} \longleftrightarrow$ obtained via second derivative

$\dfrac{d}{d\omega}\left(\dfrac{d}{d\omega}\dfrac{1}{1-ae^{-j\omega}}\right)$

Repeated application of Differentiation Theorem

$$x[n] \longleftrightarrow X(e^{j\omega})$$

$$\underbrace{n\,x[n]}_{x_1[n]} \longleftrightarrow \underbrace{j\frac{d}{d\omega}X(e^{j\omega})}_{X_1(e^{j\omega})}$$

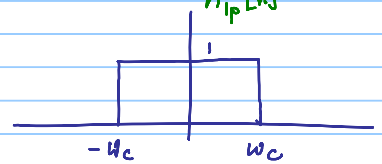
$$n\,x_1[n] \longleftrightarrow j\frac{d}{d\omega}X_1(e^{j\omega})$$

Property of Convolution

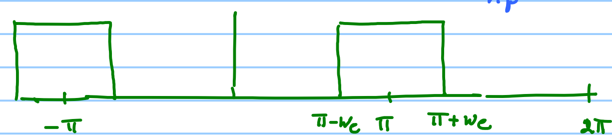
LPF
HPF
BPF
BSF

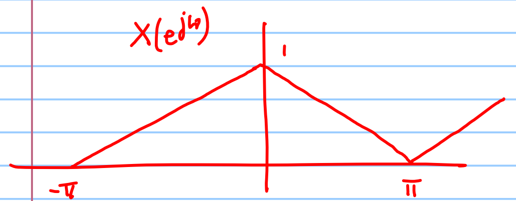
Ideal Filters

Ideal LPF

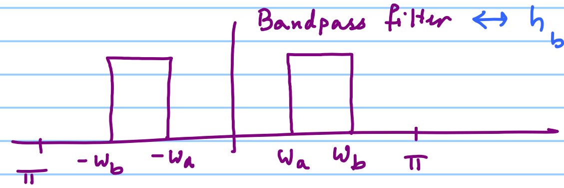


Ideal HPF $\longleftrightarrow$ $h_{hp}[n]$

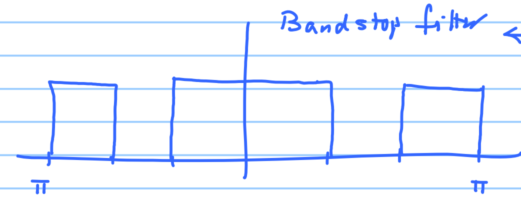




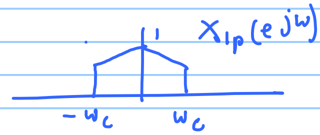
Bandpass filter $\longleftrightarrow$ $h_{bp}[n]$



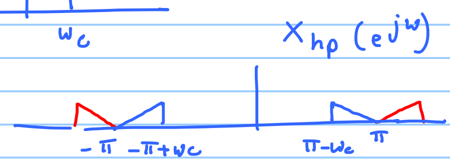
Bandstop filter $\longleftrightarrow$ $h_{bs}[n]$



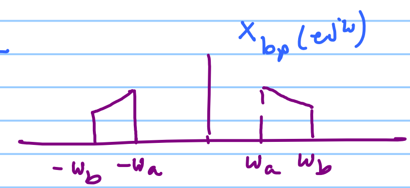
$x[n] * h_{lp}[n] \longleftrightarrow$



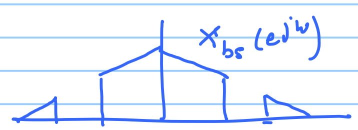
$x[n] * h_{hp}[n] \longleftrightarrow$



$x[n] * h_{bp}[n] \longleftrightarrow$



$x[n] * h_{bs}[n] \longleftrightarrow$



Ex 7b

LTI system represented by the LCCDE

$\longrightarrow$ $y[n] - \frac{1}{2} y[n-1] = x[n] - \frac{1}{4} x[n-1]$

Apply DTFT

$$Y(e^{j\omega})\left(1 - \frac{1}{2} e^{-j\omega}\right) = X(e^{j\omega})\left(1 - \frac{1}{4} e^{-j\omega}\right)$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - \frac{1}{4} e^{-j\omega}}{1 - \frac{1}{2} e^{-j\omega}}$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - \frac{1}{4}\left(\frac{1}{2}\right)^{n-1} u[n-1]$$

Task.

Sketch the circuit for LCCDE

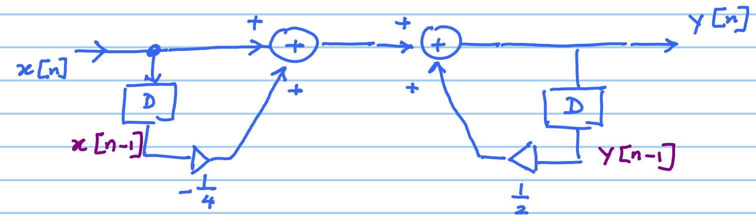


TABLE 2.3 FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n-n_0]$	$e^{-j\omega n_0}$
3. $1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4. $a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1 - ae^{-j\omega}}$
5. $u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6. $(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
7. $\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1 - 2r\cos\omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
8. $\frac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert < \omega_c, \\ 0, & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$ Ideal LPF
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
11. $\cos(\omega_0 n + \phi)$	$\sum_{k=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$

$$\frac{1}{2\pi} \longleftrightarrow \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k)$$

#11 $$\cos(\omega_0 n + \phi) = \frac{1}{2}\left[e^{j(\omega_0 n + \phi)} + e^{-j(\omega_0 n + \phi)}\right]$$

$$= \frac{e^{j\phi}}{2} \cdot e^{j\omega_0 n} + \frac{e^{-j\phi}}{2} \cdot e^{-j\omega_0 n}$$

Apply #10

$$\cos(\omega_0 n + \phi) \longleftrightarrow \pi e^{j\phi} \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_0 + 2\pi k)$$

$$+ \pi e^{-j\phi} \sum_{k=-\infty}^{\infty} \delta(\omega + \omega_0 + 2\pi k)$$

Ex. Parseval's Theorem & Differentiation Theorem

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{jw})|^2 \, dw$$

$$\sum_{n=-\infty}^{\infty} |n\, x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X_1(e^{jw})|^2 \, dw = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|\frac{d}{dw} X(e^{jw})\right|^2 dw$$

$$\underbrace{n\, x[n]}_{x_1[n]} \longleftrightarrow \underbrace{j \frac{d}{dw} X(e^{jw})}_{X_1(e^{jw})}$$

#5 $\quad u[n] \longleftrightarrow \dfrac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k)$

Note

$u[n]$ signal w. discontinuity

Not abs. summable

Not sq. summable

expect Dirac Delta if DTFT exists

Wrong approach

$$a^n u[n] \longleftrightarrow \frac{1}{1-ae^{-j\omega}} \qquad \boxed{|a|<1}$$

$$\lim_{a\to 1} a^n u[n] \longleftrightarrow \frac{1}{1-e^{-j\omega}}$$

Correct approach

$$x[n] = x_e[n] + x_o[n]$$

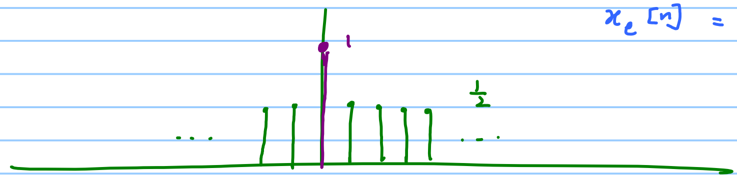
$x[n] = u[n]$

$x_e[n]$

$x_o[n]$

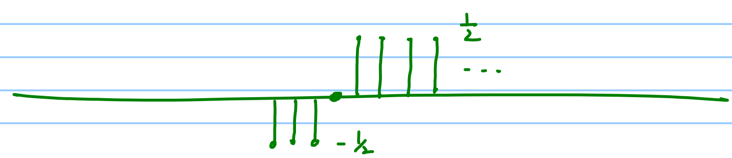
$x_e[n] = u_e[n]$

$x_e[n] = \frac{1}{2} + \frac{1}{2}\delta[n]$



$x_o[n] = u_o[n]$

$x_o[n] = u[n] - \frac{1}{2} - \frac{1}{2}\delta[n]$



$$x_e[n] = \frac{1}{2} + \frac{1}{2}\delta[n] \longleftrightarrow \frac{1}{\cancel{2}}\sum_{k=-\infty}^{\infty} \cancel{2}\pi\,\delta(\omega + 2\pi k) + \frac{1}{2}$$ ①

$$x_o[n] = u[n] - \frac{1}{2} - \frac{1}{2}\delta[n]$$

$$x_o[n] - x_o[n-1] = \boxed{u[n] - u[n-1]} - \frac{1}{2} - \frac{1}{2}\delta[n] - \left(u[n-1] - \frac{1}{2} - \frac{1}{2}\delta[n-1]\right)$$

$$\delta[n] + 0 - \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-1] = \frac{1}{2}\left[\delta[n] + \delta[n-1]\right]$$

$$x_o[n] - x_o[n-1] = \frac{1}{2}\left(\delta[n] + \delta[n-1]\right)$$

Taking DTFT

$$X_o(e^{j\omega}) - e^{-j\omega}X_o(e^{j\omega}) = \frac{1}{2}\left(1 + e^{-j\omega}\right)$$

$$X_o(e^{j\omega})\left[1 - e^{-j\omega}\right] = \frac{1}{2}\left(1 + e^{-j\omega}\right)$$

$$\boxed{X_o(e^{j\omega}) = \frac{1}{2}\left[\frac{1 + e^{-j\omega}}{1 - e^{-j\omega}}\right]}$$ ②

$$u[n] \longleftrightarrow \underbrace{\pi \sum_{k=-\infty}^{\infty} \delta(w + 2\pi k) + \frac{1}{2}}_{U_e(e^{jw}) = X_e(e^{jw})} \overbrace{+ \frac{1}{2} \frac{1 + e^{-jw}}{1 - e^{-jw}}}$$

$$u[n] \longleftrightarrow \pi \sum_{k=-\infty}^{\infty} \delta(w + 2\pi k) + \frac{1}{1 - e^{-jw}}$$

DTFT Transform Pair # 7

1

$$x[n] = \frac{r^n \sin w_p (n+1)}{\sin w_p} u[n] \longleftrightarrow \frac{1}{1 - 2r \cos w_p e^{-jw} + r^2 e^{-j2w}} = X(e^{jw}) \quad \text{form } 1$$

Step 1 Gen result from Algebra

$P(x)$ is a polynomial in x of order N with real coefficients

roots of $P(x)$ $\quad p_1, p_2 \ldots p_N$

p_i $\quad i = 1, N$ are real-valued, or occur as a complex conjugate pair

$$P(x) = \prod_{i=1}^{N_1} (x - p_i) \cdot \prod_{j=1}^{N_2} (x - p_j)(x - p_j^*) \qquad \boxed{N_1 + 2N_2 = N}$$

Step 2

$$(x - p_j)(x - p_j^*) = x^2 - 2\mathrm{Re}\{p_j\} x + |p_j|^2$$

Step 3

2

$$\Big(1 - \underbrace{r e^{jw_p}}_{p_j} \underbrace{e^{-jw}}_{x}\Big)\Big(1 - \underbrace{r e^{-jw_p}}_{p_j^*} \underbrace{e^{-jw}}_{x}\Big) = \left(1 - 2r \cos w_p e^{-jw} + r^2 e^{-j2w}\right)$$

$$X(e^{j\omega}) = \frac{1}{1 - 2r\cos\omega_p e^{-j\omega} + r^2 e^{-j2\omega}} = \frac{1}{(1 - re^{j\omega_p}e^{-j\omega})(1 - re^{-j\omega_p}e^{-j\omega})}$$

form 1 form 2

Step 4 Apply Partial Fraction Expansion to form 2

$$\frac{1}{(x-p_1)(x-p_2)} = \frac{A}{(x-p_1)} + \frac{B}{(x-p_2)} \quad \text{(PFE)}$$

$$X(e^{j\omega}) = \frac{A}{(1-re^{j\omega_p}e^{-j\omega})} + \frac{B}{(1-re^{-j\omega_p}e^{-j\omega})} \quad 3$$

Obtain A & B

Multiply 3 LHS & RHS by $(1-re^{j\omega_p}e^{-j\omega})$

Use $X(e^{j\omega})$ form 2

$$\frac{1}{(1-re^{-j\omega_p}e^{-j\omega})} = A + B\,\cancel{\frac{(1-re^{j\omega_p}e^{-j\omega})}{(1-re^{-j\omega_p}e^{-j\omega})}} = 0$$

$$(1-re^{j\omega_p}e^{-j\omega}) = 0$$

$$\Rightarrow e^{-j\omega} = \frac{1}{re^{j\omega_p}}$$

$$A = \frac{1}{1 - e^{-j2\omega_p}}$$

$$A = \frac{1}{1-e^{-j2w_p}} = \frac{1}{e^{-jw_p}\left(\underbrace{e^{jw_p} - e^{-jw_p}}_{2j\sin w_p}\right)} = \frac{e^{jw_p}}{2j\sin w_p}$$

IIIly

$$B = \frac{1}{1-e^{j2w_p}} = \frac{-e^{-jw_p}}{2j\sin w_p}$$

Form ③

$$X(e^{jw}) = \frac{A}{1-\underbrace{re^{jw_p}}_{a}e^{-jw}} + \frac{B}{1-\underbrace{re^{-jw_p}}_{b}e^{-jw}}$$

$$x[n] = A\,a^n u[n] + B\,b^n u[n] = A\,r^n e^{jw_p n} u[n] + B\,r^n e^{-jw_p n} u[n]$$

Substitute values A & B

$$= \frac{r^n}{\cancel{2j}\sin w_p}\left[\underbrace{e^{jw_p(n+1)} - e^{-jw_p(n+1)}}_{\cancel{2j}\sin w_p(n+1)}\right]u[n] = \frac{r^n \sin w_p(n+1)}{\sin w_p} = x[n]$$

Ex $x[n] = x_e[n] + x_o[n]$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} \left(x_e[n] + x_o[n] \right)^2$$

$$= \sum_{n=-\infty}^{\infty} x_e^2[n] + x_o^2[n] + \underbrace{\cancel{2 x_e[n] x_o[n]}}_{=0}$$

$x_e[n]\, x_o[n] = \text{even or odd ?}$

$x_e[-n]\, x_o[-n] = x_e[n] \left(- x_o[n] \right)$

$= - x_e[n]\, x_o[n]$

odd function

Ex

$$\sum_{n=-\infty}^{\infty} x_e[n] = x_e[0] + 2 \sum_{n=1}^{\infty} x_e[n]$$

$$\underbrace{\sum_{n=-\infty}^{\infty} x_o[n]}_{=0} = x_o[0] + \overbrace{\sum_{n=1}^{\infty} x_o[n] + x_o[-n]}^{=0}$$

$x_o[n] = - x_o[-n]$ odd function

@ $n=0$

$x_o[0] = - x_o[0]$

$\Rightarrow x_o[0] = 0$

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

odd × even ⟶ odd

even × odd ⟶ odd

even × even ⟶ even

odd × odd ⟶ even