Electrical Engineering
IIT Madras

EE 3101
Digital Signal Processing

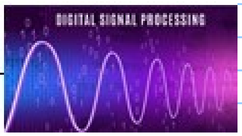
BS Electronic Systems programme

R. David Koilpillai

September – December 2024

Session # 14

October 29, 2024



EE3101 Digital Signal Processing

Session 14

Outline

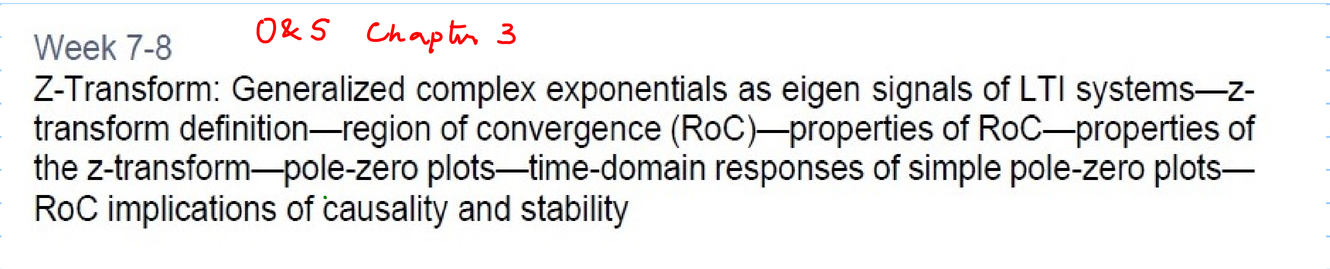
Last session

- DTFT properties
- DTFT examples

Today

- Introduction to Z Transform

O&S Chapter 3



Week 7-8

Z-Transform: Generalized complex exponentials as eigen signals of LTI systems—z-transform definition—region of convergence (RoC)—properties of RoC—properties of the z-transform—pole-zero plots—time-domain responses of simple pole-zero plots—RoC implications of causality and stability

✓ Sec 1 Z Transform

✓ Sec 2 Properties of ROC (Region of Convergence)

Sec 3 Inverse Z Transform (cover at end of week 8)

Sec 4 Z Transform Properties

Sec 5 Z Transform and LTI systems

Sec 6 Unilateral Z Transform (Omit)

Reading Assignment

O&S ch 3 The Z Transform

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$\frac{d}{d\omega} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\,(-jn)\, e^{-j\omega n} \qquad ①$$

$$j\frac{d}{d\omega} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} n\,x[n]\; e^{-j\omega n}$$

$$n\,x[n] \longleftrightarrow j\frac{d}{d\omega} X(e^{j\omega})$$

$$\frac{d^2}{d\omega^2} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n](-jn)(-jn)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} -n^2 x[n]\; e^{-j\omega n}$$

$$n^2 x[n] \longleftrightarrow -\frac{d^2}{d\omega^2} X(e^{j\omega})$$

Repeated application of Differentiation Theorem

$$\boxed{\begin{array}{ccc} x[n] & \longleftrightarrow & X(e^{j\omega}) \\ \underbrace{n\,x[n]}_{x_1[n]} & \longleftrightarrow & \underbrace{j\frac{d}{d\omega} X(e^{j\omega})}_{X_1(e^{j\omega})} \\ n\,x_1[n] & \longleftrightarrow & j\frac{d}{d\omega} X_1(e^{j\omega}) \end{array}}$$

Ex. Parseval's Theorem & Differentiation Theorem

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{jw})|^2 \, dw$$

$$\sum_{n=-\infty}^{\infty} |n\,x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X_1(e^{jw})|^2 \, dw = \frac{1}{2\pi}\int_{-\pi}^{\pi} \left|\frac{d}{dw} X(e^{jw})\right|^2 dw$$

$$\underbrace{n\,x[n]}_{x_1[n]} \longleftrightarrow \underbrace{j\frac{d}{dw} X(e^{jw})}_{X_1(e^{jw})}$$

#5 $u[n] \longleftrightarrow \frac{1}{1-e^{-jw}} + \pi \sum_{k=-\infty}^{\infty} \delta(w+2\pi k)$

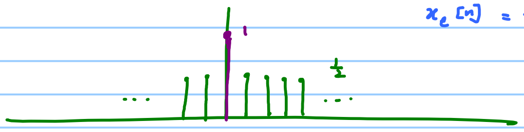
$x[n] = x_e[n] + x_o[n]$

$x_e[n] = \frac{1}{2} + \frac{1}{2}\delta[n] \longleftrightarrow \sum_{k=-\infty}^{\infty} \pi\,\delta(w+2\pi k) + \frac{1}{2}$

$X_o(e^{jw}) = \frac{1}{2}\left[\frac{1+e^{-jw}}{1-e^{-jw}}\right]$

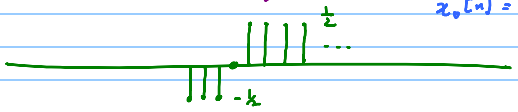
$u[n] \longleftrightarrow \pi \sum_{k=-\infty}^{\infty} \delta(w+2\pi k) + \frac{1}{1-e^{-jw}}$

$x_e[n] = u_e[n]$



$x_e[n] = \frac{1}{2} + \frac{1}{2}\delta[n]$

$x_o[n] = u_o[n]$



$x_o[n] = u[n] - \frac{1}{2} - \frac{1}{2}\delta[n]$

①

DTFT Transform Pair #7

$$x[n] = \frac{r^n \sin \omega_p (n+1)}{\sin \omega_p} u[n] \longleftrightarrow \frac{1}{1 - 2r \cos \omega_p e^{-j\omega} + r^2 e^{-j2\omega}} = X(e^{j\omega}) \quad \text{form } ①$$

Step 1 Factorization of $X(e^{j\omega})$

Step 2 Partial fraction expansion

Step 3 Obtain values of constants A, B

Step 4 Obtain $x[n]$

Ex 1. $x[n] = a^n \; u[n] \qquad 0 < a < 1$

$$X(e^{jw}) = \frac{1}{1-ae^{-jw}} \; \frac{1-ae^{jw}}{1-ae^{jw}} = \frac{1 - a\cos w - ja\sin w}{1 - 2a\cos w + a^2}$$

$$\begin{tikzcd} & X(e^{jw}) \arrow[dl] \arrow[dr] & \\ X_R(e^{jw}) & & X_I(e^{jw}) \end{tikzcd}$$

$$X_R(e^{jw}) = \frac{1 - a\cos w}{1 - 2a\cos w + a^2}$$

$$X_I(e^{jw}) = \frac{-a\sin w}{1 - 2a\cos w + a^2}$$

Ex 2

$$x[n] = \{\underset{\uparrow}{2}, 2, 3, 3, 4\}$$

$$y[n] = \{2, \underset{\uparrow}{4}, 9, 10, 13, 10, 8\}$$

Find $h[n]$ of the LTI system

Q1. Is $h[n]$ causal?

No.

Q2. Is $h[n]$ finite length? Yes

Q3. What is the length of $h[n]$

$$Y(e^{jw}) = H(e^{jw})\, X(e^{jw})$$

$$X(e^{jw}) = 2 + 2e^{-jw} + 3e^{-j2w} + 3e^{-j3w} + 4e^{-j4w}$$

$$Y(e^{jw}) = 2e^{jw} + 4 + 9e^{-jw} + 10e^{-j2w} + 13e^{-j3w} + 10e^{-j4w} + 8e^{-j5w}$$

$$\begin{array}{r} e^{jw} + 1 + 2e^{-j2w} \\ 2 + 2e^{-jw} + 3e^{-j2w} + 3e^{-j3w} + 4e^{-j4w} \overline{\smash{)}\, 2e^{jw} + 4 + 9e^{-jw} + 10e^{-j2w} + 13e^{-j3w} + 10e^{-j4w} + 8e^{-j5w}} \end{array}$$

$$H(e^{jw}) = e^{jw} + 1 + 2e^{-j2w}$$

$$h[n] = \{1, \underset{\uparrow}{1}, 2\}$$

Ex 3. $x[n] \longleftrightarrow X(e^{jw})$

Obtain the DTFT of the following signal in terms of $X(e^{jw})$

① $x_1[n] = x[1-n] + x[-1-n]$ $X_1(e^{jw})$ in terms of $X(e^{jw})$

$y_1[n] = x[n+1]$ $y_2[n] = x[n-1]$

$Y_1(e^{jw}) = e^{jw} X(e^{jw})$ $Y_2(e^{jw}) = e^{-jw} X(e^{jw})$

$x[n] \longleftrightarrow X(e^{jw})$

$x[-n] \longleftrightarrow X(e^{-jw})$

$x_1[n] = y_1[-n] + y_2[-n]$

$$X_1(e^{jw}) = Y_1(e^{jw})\Big|_{w \leftarrow -w} + Y_2(e^{jw})\Big|_{w \leftarrow -w}$$

$$= e^{-jw} X(e^{-jw}) + e^{jw} X(e^{-jw})$$

$$= X(e^{-jw}) \left[e^{-jw} + e^{jw}\right]$$

$$X_1(e^{jw}) = X(e^{-jw})\, 2\cos w$$

(ii) $x_2[n] = (n-1)^2\, x[n]$

$x[n] \longleftrightarrow X(e^{jw})$

$= (n^2 - 2n + 1)\; x[n]$

$n\, x[n] \longleftrightarrow j\frac{d}{dw} X(e^{jw})$

$n^2 x[n] \longleftrightarrow -\frac{d^2}{dw^2} X(e^{jw})$

$$X_2(e^{jw}) = -\frac{d^2}{dw^2} X(e^{jw}) - 2j\frac{d}{dw} X(e^{jw}) + X(e^{jw})$$

(iii) $x_3[n] = e^{j\frac{\pi}{2}n}\; x[n+2]$

$y_1[n] = x[n+2]$

$Y_1(e^{jw}) = e^{j2w} X(e^{jw})$

$$x_3[n] = e^{j\frac{\pi}{2}n} y_1[n] \longleftrightarrow Y_1\left(e^{j(w-\frac{\pi}{2})}\right) = e^{j2(w-\frac{\pi}{2})} X\left(e^{j(w-\frac{\pi}{2})}\right)$$

$$= e^{j2w} \cdot \underbrace{e^{-j\pi}}_{(-1)} X\left(e^{j(w-\frac{\pi}{2})}\right) = -e^{j2w} X\left(e^{j(w-\frac{\pi}{2})}\right)$$

Ex 4. Obtain the DTFT $x_1[n] = \underbrace{\overbrace{\cos w_0 n}^{w[n]}}\ \underbrace{u[n]}$

Using Multiplication Property

$$X_1(e^{jw}) = W(e^{jw}) \circledast U(e^{jw})$$

$$\left.\begin{aligned} \cos w_0 n &\longleftrightarrow \pi\left[\delta(w-w_0) + \delta(w+w_0)\right] = W(e^{jw}) \\ u[n] &\longleftrightarrow \pi\,\delta(w) + \frac{1}{1-e^{-jw}} \end{aligned}\right\} \quad -\pi \leq w < \pi$$

Convolving with a Dirac Delta $\Rightarrow$ freq shift

$$W(e^{jw}) * U(e^{jw}) = \left[\pi\left[\delta(w-w_0) + \delta(w+w_0)\right]\right] \circledast \left[\pi\,\delta(w) + \frac{1}{1-e^{-jw}}\right]$$

$$\boxed{\begin{aligned} \pi\,\delta(w-w_0) * \pi\,\delta(w) &= \pi^2\,\delta(w-w_0) \\ \pi\,\delta(w-w_0) * \frac{1}{1-e^{-jw}} &= \frac{\pi}{1-e^{-jw_0}} \end{aligned}}$$

$$= \pi^2\left(\delta(w-w_0) + \delta(w+w_0)\right) + \pi\underbrace{\left[\frac{1}{1-e^{-jw_0}} + \frac{1}{1-e^{jw_0}}\right]}_{=1}$$

$$\boxed{X_1(e^{jw}) = \pi + \pi^2\left(\delta(w-w_0 + \delta(w+w_0)\right) \quad -\pi \leq w < \pi}$$

(ii) $x_2[n] = \sin w_0 n \,.\, u[n]$

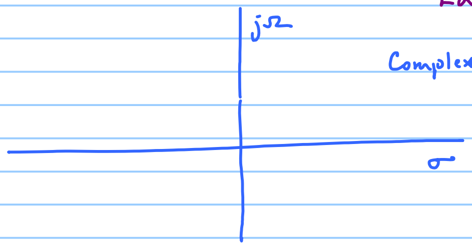
$$X_2(e^{jw}) = -\pi\cot\left(\frac{w_0}{2}\right) + \pi^2\left(\delta(w-w_0) - \delta(w+w_0)\right) \quad -\pi \leq w < \pi$$

Continuous time signals <
Fourier Transf
Laplace Transform

Z Transform

Laplace Transform $s = \sigma + j\Omega$

$\overset{\uparrow}{\text{Complex}}$



s covers entire complex plane

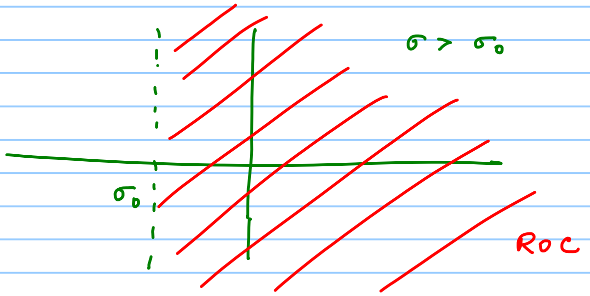
$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-\sigma t} e^{-j\Omega t} dt$$

Laplace Transform generalized form

Fourier Transform is a special case of LT

Region of convergence $\sigma > \sigma_0$ or $\sigma < \sigma_1$



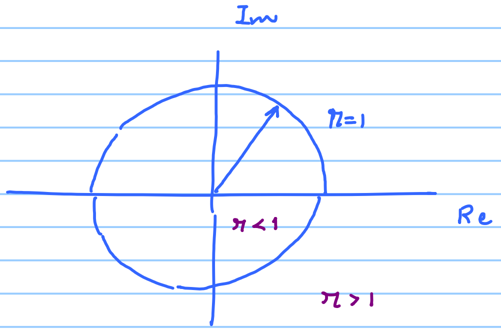
If ROC includes $j\Omega$ axis

$\Rightarrow$ FT exists

Z Transform is a generalization of the DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$



$$z = re^{j\omega}$$

$$z^{-n} = r^{-n} e^{-j\omega n}$$

Z Transform is the DT counterpart of the LT

DTFT $z = e^{j\omega} \Rightarrow |z| = 1$ or $r = 1$

General form $z = re^{j\omega}$

Defn $X(z) \triangleq \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} = \sum_{n=-\infty}^{\infty} \left[x[n]\, r^{-n} \right] e^{-j\omega n} \equiv$ DTFT of $x[n]\, r^{-n}$

$\{ r \in \mathbb{R} \mid r \geq 0 \}$

$$x[n] \xrightarrow{Z} X(z)$$

$$\xleftarrow[\text{Inv. ZT}]{}$$

ZT of $x[n] \equiv$ DTFT $(x[n] r^{-n})$

$\rightarrow$ Condition for existence (uniform convergence) $\Big\}$ Absolutely summable

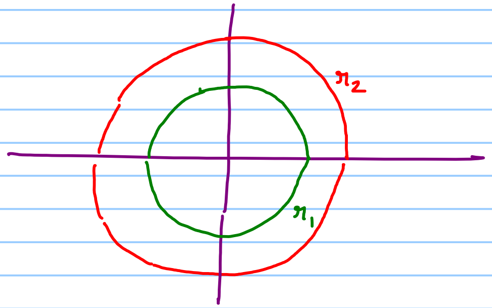
$$\sum_{n=-\infty}^{\infty} \left| x[n] r^{-n} \right| < \infty$$

ROC of ZT $\equiv$ range of values of r for which $x[n] r^{-n}$ is abs summable

$\boxed{\text{ROC depends on the value of } r}$

If ZT exists for one value of r ie, $z = re^{j\omega_0}$

then ZT exists for all points on circle with radius



ROC $|z| > r_1$

$|z| < r_1$

ROC

$|z| > r_2$

$|z| < r_1$

$r_1 < |z| < r_2$

$\Bigg\}$ ROC cannot include any poles

$H(x) = \frac{P(x)}{Q(x)}$ zeros of $P(x)$ = zeros of $H(x)$

Zeros of $Q(x)$ = poles of $H(x)$

For DTFT uniform conv. $|a|<1$

Ex $x[n] = a^n u[n]$

$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = \frac{1}{1-az^{-1}}$ $|az^{-1}| < 1$

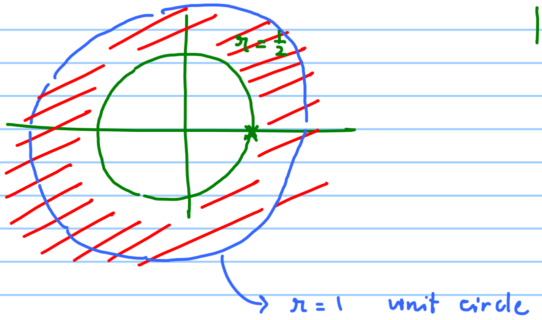
$|z| > |a|$

$X(z) = \frac{z}{z-a}$ zeros $z = 0$

pole $z = a$

$a = \frac{1}{2}$

DTFT exist

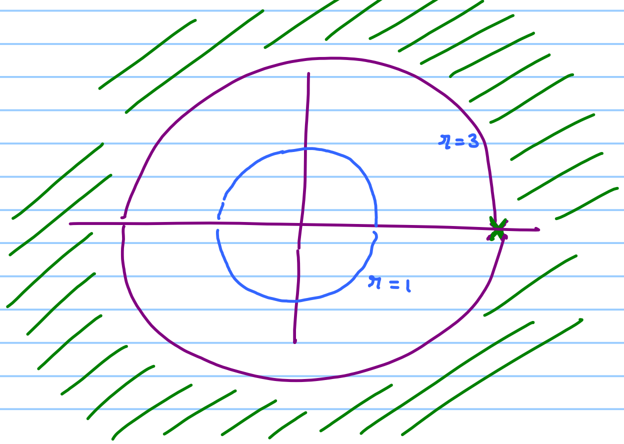


$a = 3$ RoC $|z| > 3$

ZT exists

RoC $|z| > 3$

DTFT does not exist



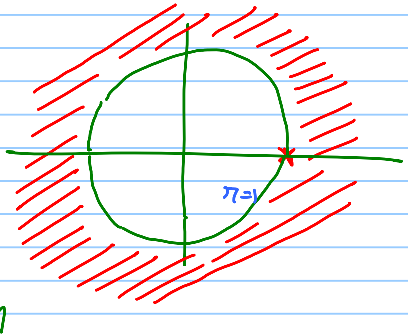
$x[n] = a^n \; u[n]$

$a = 1 \quad \Longrightarrow \quad x[n] = u[n]$

$X(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}}$ $\qquad |z^{-1}| < 1$

$|z| > 1$

DTFT does not converge uniformly



Ex

$x[n] = \{\underset{\uparrow}{1}, 2, 5, 7, 0, 1\}$

$X(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$

5th order polynomial $\Longrightarrow$ 5 zeros

pole = ? $\qquad z = 0 \quad \Longrightarrow$ 5 poles all @ $z = 0$

$x_1[n] = \{1, \underset{\uparrow}{2}, 5, 7, 0, 1\}$

$X_1(z) = \boxed{z} + 2 + 5z^{-1} + 7z^{-2} + z^{-4}$

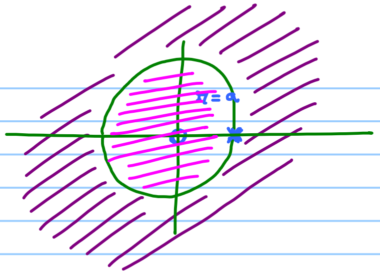
poles ? $\begin{cases} z = 0 \; ✓ \\ z = \infty \; ✓ \end{cases}$

causal seq.

Ex. $x[n] = a^n u[n]$

$$X(z) = \frac{1}{1-az^{-1}} \qquad |z| > a$$

$$= \frac{z}{z-a}$$



ROC should not inclu pole

Part 2

$$x[n] = -a^n u[-n-1]$$

$$\left\{ \cdots \; -\frac{1}{a^3} \;\; \overset{n=-2}{-\frac{1}{a^2}} \;\; \overset{n=-1}{-\frac{1}{a}} \;\; \underset{\uparrow}{0} \;\; 0 \;\; 0 \; \cdots \right\}$$

$$X(z) = -\sum_{n=-\infty}^{-1} a^n z^{-n}$$

$$= -\sum_{n=-\infty}^{-1} (az^{-1})^n \qquad m = -n$$

$$= -\sum_{m=1}^{\infty} (az^{-1})^{-m} = -\frac{\frac{1}{az^{-1}}}{1-\frac{1}{az^{-1}}} = \frac{1}{1-az^{-1}}$$

$$\left|\frac{1}{az^{-1}}\right| < 1$$

$$|az^{-1}| > 1$$

$$|z| < |a|$$

$$x[n] = -a^n u[-n-1] \longleftrightarrow X(z) = \frac{1}{1-az^{-1}} \qquad |z| < a.$$

Causal sequences
(right-sided sequences)
ROC $|z| > r_R$

(Left sided sequences)
ROC $|z| < r_L$

Result

A DT signal x[n] is uniquely represented by ZT & ROC

$\underline{\underline{X(z)}}$

Ex

$$x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u(-n-1)$$

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}}$$

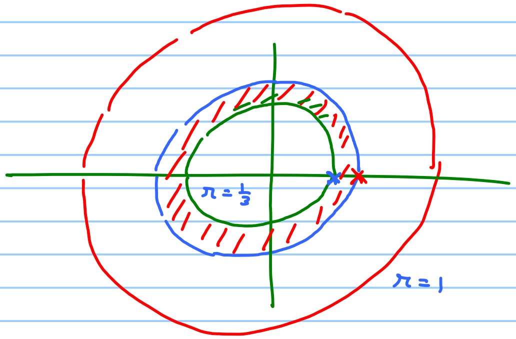
ROC_1 $\quad$ ROC_2

$|z| > \frac{1}{3}$ $\quad$ $|z| < \frac{1}{2}$

poles

@ $z = \frac{1}{3}$

$z = \frac{1}{2}$



$X(e^{jw})$ exists ? NO

→ ROC of X(z) is intersection of ROC_1 & ROC_2

Extension.

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{3}\right)^n u[-n-1]$$

RoC of X(z)

RoC$_1$ $|z| > \frac{1}{2}$ RoC$_2$ $|z| < \frac{1}{3}$ $\Rightarrow$ RoC$_1 \cap$ RoC$_2 = \{\phi\}$ null set

X(z) does not exist

RoC is a null set.

Finite Length Seq

$$x[n] = \begin{cases} a^n & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$X(z) = \sum_{n=0}^{N-1} a^n z^{-n} = \frac{1-(az^{-1})^N}{1-az^{-1}}$$

$|X(z)| < \infty$

ROC $\underline{\text{entire } Z \text{ plane}}$ except $z=0$

$$\le \sum_{n=0}^{N-1} |az^{-1}|^n < \infty$$

as long as az^{-1} is finite, $\Rightarrow |a| < \infty$ & $z \neq 0$

$$X(z) = \frac{1-(az^{-1})^N}{1-az^{-1}} = \frac{1}{z^{N-1}} \quad \frac{z^N - a^N}{\boxed{z-a}}$$

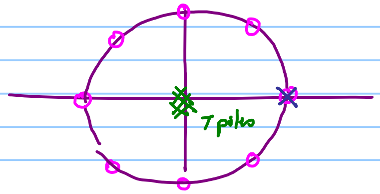
pole @ $z = a$

Closer look @ poles & zeros of $X(z)$

Zeros $z^N - a^N = 0 \Rightarrow z^N = a^N \Rightarrow z = a e^{j\frac{2\pi}{N}k}$ $K = 0, 1, \ldots N-1$

$$z^N = \left(a e^{j\frac{2\pi}{N}k}\right)^N = a^N$$

$N = 8$



Poles

$(N-1)$ poles @ $z=0$

pole-zero cancellation $\Rightarrow$ ROC is entire Z plane excluding $Z=0$

Ex

$$a^n u[n] \longleftrightarrow X(z) = \frac{1}{1-az^{-1}} = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$\frac{d}{dz} X(z) = \frac{d}{dz}\left(\sum_{n=0}^{\infty} a^n z^{-n}\right) = \sum_{n=0}^{\infty} a^n (-n) z^{-(n+1)}$$

$$= (-z^{-1}) \sum_{n=0}^{\infty} n a^n z^{-n}$$

$$-z \frac{d}{dz} X(z) = \sum_{n=0}^{\infty} n a^n z^{-n}$$

$$\boxed{n a^n u[n] \longleftrightarrow \frac{az^{-1}}{(1-az^{-1})^2}}$$

Properties of ROC

Assume $X(z)$ is a rational form $\frac{P(z)}{Q(z)}$

#1 ROC is a $\overbrace{\text{ring or a disc}}^{\text{annulus}}$ in Z plane centred @ origin

$$0 \le r_R < |z| < r_L \le \infty$$

#2 DTFT converges uniformly if ROC includes unit circle $\overset{\checkmark}{|z|=1}$ or $r = 1$

#3 ROC cannot include any poles

#4 If $x[n]$ is a finite duration seq

$$-\infty < N_1 \le n \le N_2 < \infty$$

Then ROC is the entire Z plane except possibly $z = 0$ or $z = \infty$

$$X(z) = \underbrace{x[-1]z}_{z=\infty \text{ pole}} + x[0] + \underbrace{x[1]z^{-1}}_{z=0 \text{ pole}}$$

#5 $x[n]$ is a right sided seq

$\Rightarrow$ seq is zero $n < N_1 < \infty$

ROC $|z| > r_R \leftarrow$ outermost pole of $X(z)$ (possibly exclude $z = \infty$)

#6 $x[n]$ is a left sided seq

$\Rightarrow$ seq is zero $n > N_2 > -\infty$

ROC $|z| < r_L \leftarrow$ innermost pole of $X(z)$ (possibly exclude $z = 0$)

#7 A two sided sequence is an ∞ duration seq that is neither left sided or right sided

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \underbrace{\sum_{n=0}^{\infty} x[n] z^{-n}}_{|z| > r_R} + \underbrace{\sum_{n=-\infty}^{-1} x[n] z^{-n}}_{|z| < r_L}$$

$$0 \leq r_R < |z| < r_L \leq \infty$$

#8 ROC must be a fully connected region
(no poles in ROC)

Table 3.1 SOME COMMON z-TRANSFORM PAIRS

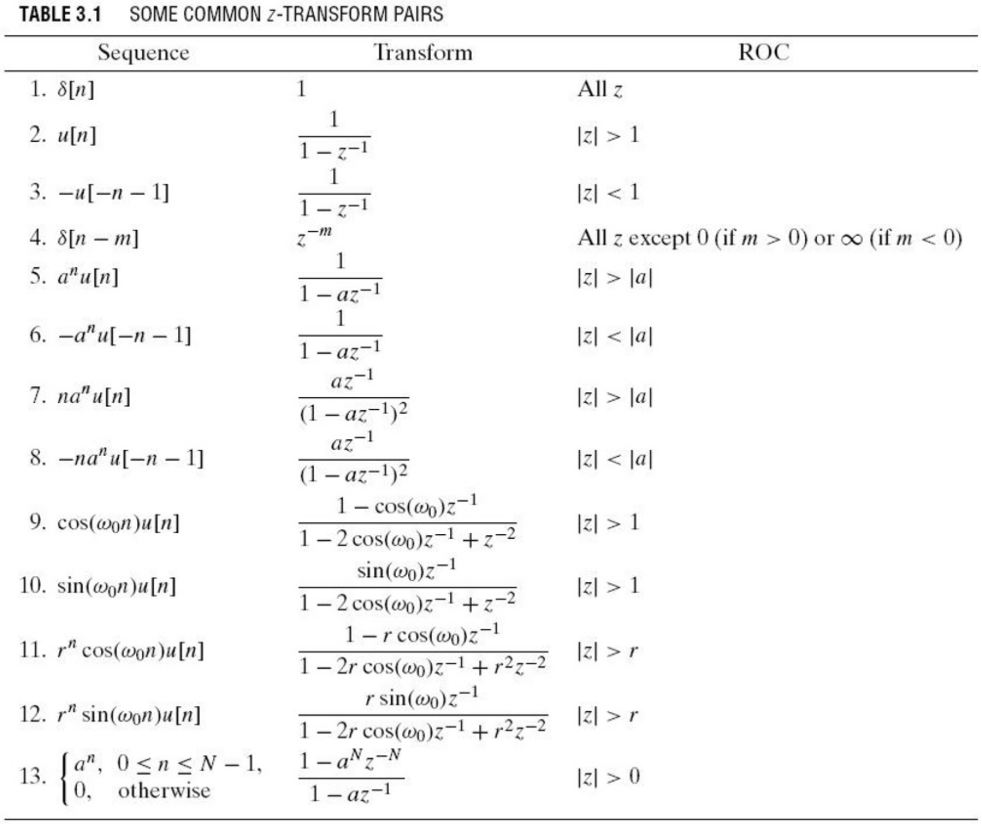


TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $\cos(\omega_0 n)u[n]$	$\dfrac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $\sin(\omega_0 n)u[n]$	$\dfrac{\sin(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $r^n\cos(\omega_0 n)u[n]$	$\dfrac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $r^n\sin(\omega_0 n)u[n]$	$\dfrac{r\sin(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{1-a^Nz^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

TABLE 3.2 SOME z-TRANSFORM PROPERTIES

Property Number	Section Reference	Sequence	Transform	ROC
		$x[n]$	$X(z)$	R_x
		$x_1[n]$	$X_1(z)$	R_{x_1}
		$x_2[n]$	$X_2(z)$	R_{x_2}
1	3.4.1	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
2	3.4.2	$x[n - n_0]$	$z^{-n_0}X(z)$	R_x, except for the possible addition or deletion of the origin or ∞
3	3.4.3	$z_0^n x[n]$	$X(z/z_0)$	$\|z_0\|R_x$
4	3.4.4	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x
5	3.4.5	$x^*[n]$	$X^*(z^*)$	R_x
6		$\mathcal{R}e\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Contains R_x
7		$\mathcal{I}m\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	Contains R_x
8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
9	3.4.7	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$