Electrical Engineering
IIT Madras

EE 3101
Digital Signal Processing

BS Electronic Systems programme

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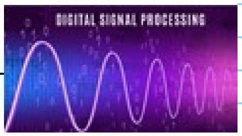
September – December 2024

Session # 21

November 26, 2024



26/11/24 Note Title

EE 3101 Digital Signal Processing

EE3101 Session 21

03-01-2018

Session 21

Outline

Last session

- Allpass systems, properties
- Relationship between Mag & Phase
- Higher order TF
- Minimum Phase Systems

Today

Week 9-10 O&S chapter 5

Frequency Domain Analysis of LTI Systems: Frequency response of systems with rational transfer function—definitions of magnitude and phase response—frequency response of single complex zero/pole—frequency response of simple configurations (second order resonator, notch filter, averaging filter, comb filter, allpass systems)—

phase response—definition of principal phase—zero-phase response—conditions that have to be met for a filter to have generalized linear phase—four types of linear phase FIR filters—on the zero locations of a linear phase FIR filter—constrained zeros at z = 1 and at z = -1 and their implications on choice of filters Type I through Type IV (with focus on Type I)

O&S chapter 7 Filter Design Techniques.

Week 11-12 (DFT & FFT)

Discrete Fourier Transform (DFT): Definition of the DFT and inverse DFT—relationship to discrete-time Fourier series—matrix representation—DFT as the samples of the DTFT and the implied periodicity of the time-domain signal—circular convolution and its relationship with linear convolution—effect of zero padding—introduction to the Fast Fourier Transform (FFT) algorithm—decimation-in-time

Reading Assignment

O&S ch 5 Transform Analysis of LTI systems

O&S ch 8 Discrete Fourier Transform

O&S Ch 5 Transform Analysis of LTI System

✓ 5.1 Freq response of LTI systems $h[n] \longleftrightarrow H(e^{j\omega})$

✓ 5.2 Systems characterized by LCCDE

✓ 5.3 Freq response of Rational Transfer Function

✓ 5.4 Relationship between magnitude and phase

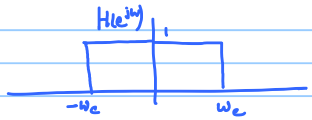
✓ 5.5 Allpass System

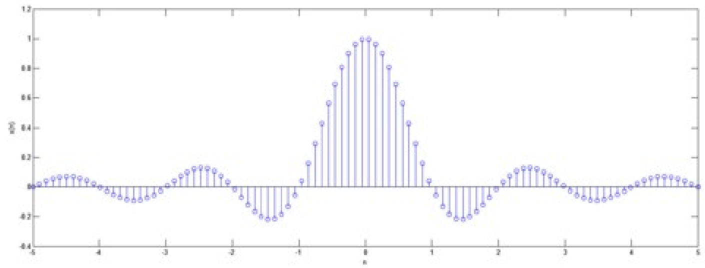
✓ 5.6 Min. phase system

5.7 Linear phase $\Leftarrow$

Ideal LPF

$$H(e^{jw}) = \begin{cases} 1 & |w| < w_c \\ 0 & \text{otherwise} \end{cases} \quad \longleftrightarrow \quad h[n] = \frac{\sin w_c n}{\pi n}$$





$H(e^{jw})$ = real-valued

$\arg H(e^{jw}) = 0 \quad \forall w \Rightarrow$ zero phase function

even symmetry $h[n] = h[-n]$

non-causal

infinite duration

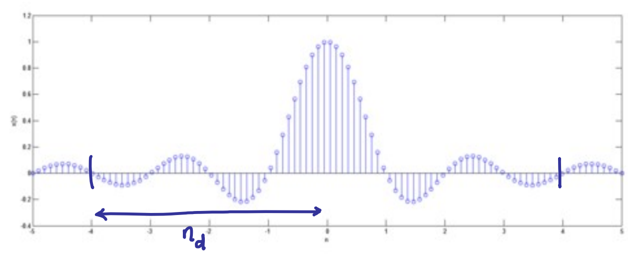
Zero-phase TF

→ non-causal

→ symmetry < even / odd

→ finite or inf duration

* LTI systems with linear phase are realizable



Truncation + shift (Windowing)

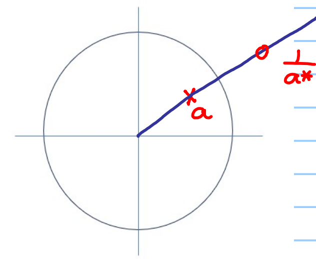
$$h_{prac}[n] = \frac{\sin \cdot w_c (n-n_d)}{\pi (n-n_d)} \left[u[n] - u[n-(2n_d+1)]\right]$$

Linear phase $\quad -wn_d$

Group Delay $\quad n_d$ samples

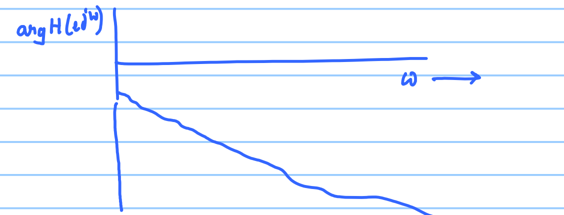
Allpass Systems

$$\boxed{H_{ap}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}} = \frac{z^{-1}(1 - a^* z)}{(1 - az^{-1})}$$



$$H_{ap}(e^{jw}) = e^{-jw} \frac{(1 - a^* e^{jw})}{(1 - a e^{-jw})} \implies |H_{ap}(e^{jw})| = 1 \quad \forall w$$

Group delay $\tau(w) = \dfrac{1 - r^2}{1 + r^2 - 2r\cos(w - \theta)}$



Causal, stable allpass $\Rightarrow$ all poles inside the unit circle $r < 1$

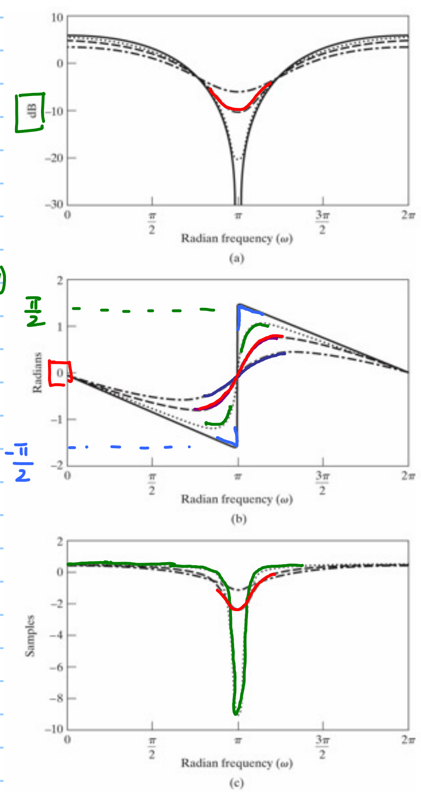
$\tau(w) > 0 \ \forall w \quad \Rightarrow$ arg $H_{ap}(e^{jw})$ is a monotone decreasing function zero on the unit circle

$|H(e^{j\omega})|$

Arg $H(e^{j\omega})$

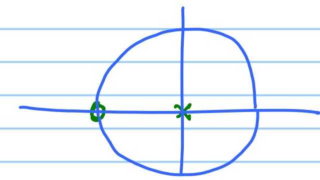
$r = 1$
$r = 0.9$
$r = 0.7$
$r = 0.5$

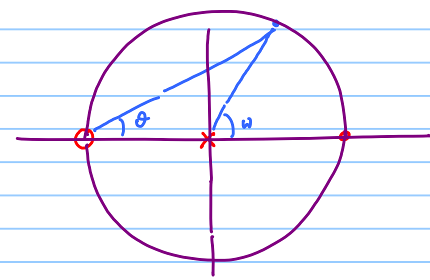
GD

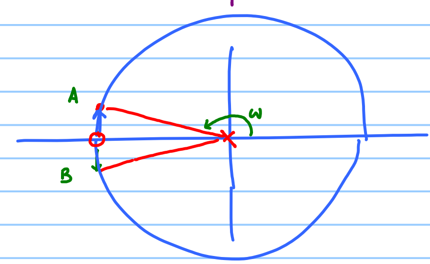


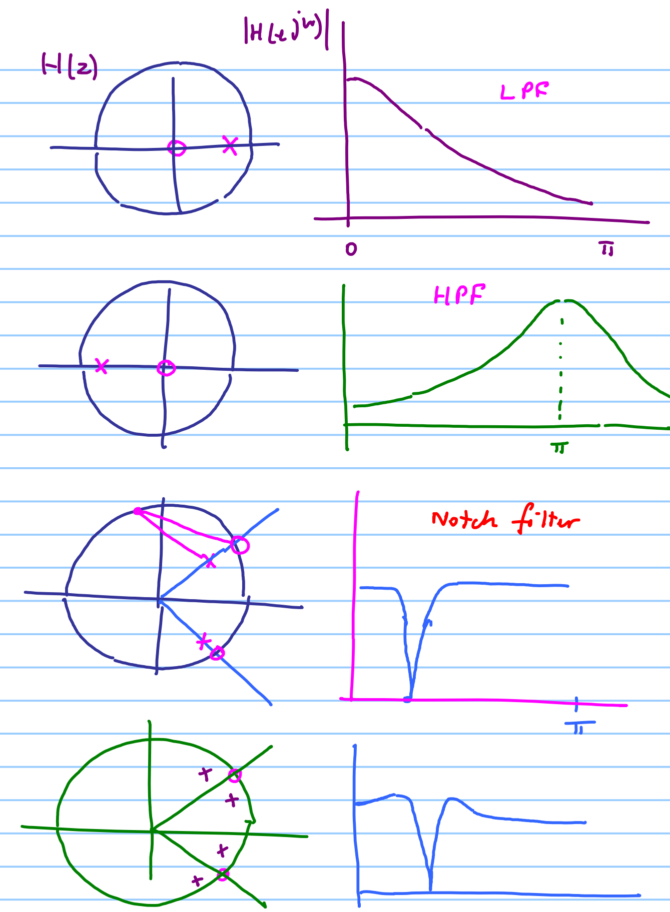
$H(z) = (1 + z^{-1})$ — zero @ $z = -1$

$\boxed{c_0 = -1}$

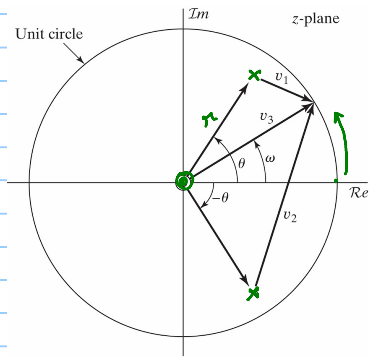








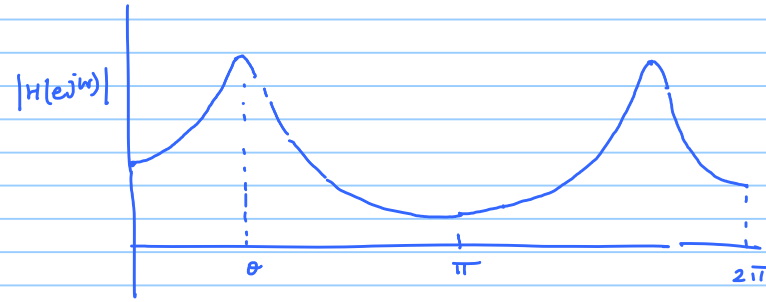
O&S Example 5.6



$$H(z) = \frac{1}{(1 - a z^{-1})(1 - a^* z^{-1})}$$

$$a = r e^{j\theta}$$

$$|H(e^{jw})| = \frac{|v_3|^2}{|v_1||v_2|} = \frac{1}{|v_1||v_2|}$$



$$\text{Arg } H(e^{jw}) = 2w - \angle v_1 - \angle v_2$$

@ $w = 0$ $\quad$ $\arg H(e^{jw}) \sim 0$

If $h[n]$ real-valued $\quad$ $|H(e^{jw})|$ even funcn. of w

$\arg H(e^{jw})$ odd function of w

Magnitude Squared Response

$$|H(e^{j\omega})|^2 = H(e^{j\omega})\, H^*(e^{j\omega})$$

$$\underbrace{h[n] * h^*[-n]}_{\text{autocorrelation}} \longleftrightarrow |H(e^{j\omega})|^2$$

$$\left.\begin{array}{l} h[n] \longleftrightarrow H(e^{j\omega}) \\ h^*[-n] \longleftrightarrow H^*(e^{j\omega}) \end{array}\right\} \quad h[n] * h^*[-n] \longleftrightarrow H(z)\, H^*\!\left(\frac{1}{z^*}\right)$$

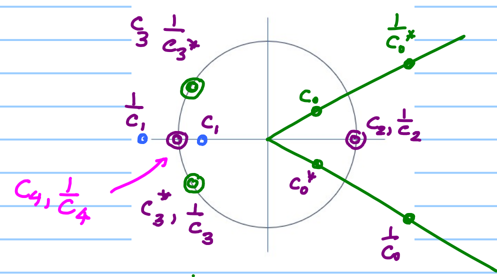
From property of allpass functions

$$\frac{z^{-1}(1-a^*z)}{(1-az^{-1})} \Rightarrow \text{Magnitude response of } \left.(1-az^{-1})\right|_{z=e^{j\omega}} \equiv \text{Mag. response of } \left.(z^{-1}-a^*)\right|_{z=e^{j\omega}}$$

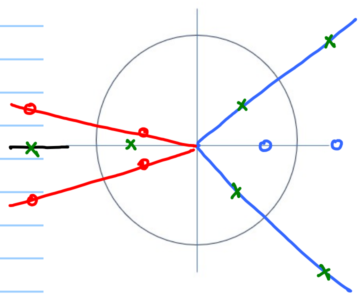
or $z^{-1}(1-a^*z)$

Observations about Magnitude Squared function

- If c_0 is a zero/pole, $\frac{1}{c_0^*}$ is also a zero/pole
- If $h[n]$ is real valued, then c_0, c_0^*, $\frac{1}{c_0^*}, \frac{1}{c_0}$
- If zero/pole on real axis, $c_1, \frac{1}{c_1}$ are pole/zero
- If zero/pole on unit circle, but not on real axis $c_3, \frac{1}{c_3^*}$
- If zero/pole @ $z=\pm 1$, double zero/pole $c_4, 1/c_4$



Example $H(z)\,H^*\left(\frac{1}{z^*}\right)$



$H(z)$ 3 poles, 3 zeros

$H(z)\,H^*\left(\frac{1}{z}\right) \sim$ 6 poles, 6 zeros

possible values for TF $H(z)$ $2^6 = 64$ with same $|H(e^{j\omega})|$

possible TF with real coefficients $2^4 = 16$

possible TF $\underbrace{\text{causal, stable}}_{\text{all poles inside } |z|=1}$ $2^3 = 8$

possible TF causal, stable, real coeff $2^2 = 4$

Min phase Property

$H(z) = z^{-1} - z_0^*$ $\quad |z_0| < 1$ $\quad$ zero @ $z = \frac{1}{z_0^*}$ outside unit circle

$= \frac{(1 - z_0 z^{-1})}{(1 - z_0 z^{-1})} \cdot (z^{-1} - z_0^*) = \underbrace{(1 - z_0 z^{-1})}_{H'(z)} \underbrace{\frac{z^{-1} - z_0^*}{1 - z_0 z^{-1}}}_{AP,\ \text{stable, causal}}$ $\quad$ pole @ $z = z_0$

Note $|H(e^{jw})| = |H'(e^{jw})|$

$H(z) = H'(z)\, H_{ap}(z)$ $\leftarrow$ causal stable AP

$|H(e^{jw})| = |H'(e^{jw})|$

unwrapped phase

$\arg H(e^{jw}) = \arg H'(e^{jw}) + \underbrace{\arg H_{ap}(e^{jw})}_{\leq 0}$ $\quad \longrightarrow \quad GD\{H(e^{jw})\} > GD\{\underbrace{H'(e^{jw})}_{\text{min } \phi \text{ system}}\}$

$\arg H(e^{jw}) \leq \arg H'(e^{jw})$ $\quad \begin{cases} = 0 & @\ w = 0 \\ < 0 & w \neq 0 \end{cases}$

phase lag $H(e^{jw}) = -\arg H(e^{jw})$

Phase lag $H(e^{jw}) \geq$ phase lag $H'(e^{jw})$

TF with poles & zeros inside unit circle (causal, stable) $\Big\}$ min ϕ ~~lag~~ system

min ϕ system

Properties Min ϕ systems

(Min ϕ lag systems)

* Causal, stable $\longrightarrow$ all poles & zeros inside unit circle
* Min ϕ lag compared to any other TF with same mag response
* Min GD "
* Max partial energy "

Ex.

$$H(z) = \left(\frac{a_0}{b_0}\right) \frac{\prod_{k=1}^{M} (1 - c_k z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})}$$

zeros @ $z = c_k$ $k = 1, \dots M$

poles @ $z = d_k$ $k = 1, \dots N$

$h[n] \longleftrightarrow H(z)$

$h^*[n] \longleftrightarrow H^*(z^*)$

$h^*[-n] \longleftrightarrow H^*\left(\frac{1}{z^*}\right)$

$$h^*[n] \longleftrightarrow H^*(z^*) = \left(\frac{a_0^*}{b_0^*}\right) \frac{\prod_{k=1}^{M} (1 - c_k^* z^{-1})}{\prod_{k=1}^{N} (1 - d_k^* z^{-1})}$$

zeros @ $z = c_k^*$

poles @ $z = d_k^*$

$$h^*[-n] \longleftrightarrow H^*\left(\frac{1}{z^*}\right) = \left(\frac{a_0^*}{b_0^*}\right) \frac{\prod_{k=1}^{M} (1 - c_k^* z)}{\prod_{k=1}^{N} (1 - d_k^* z)}$$

zeros @ $z = \frac{1}{c_k^*}$

poles @ $z = \frac{1}{d_k^*}$

If $H(z)$ is LTI, causal & stable, which of the following systems are causal, stable

(a) $G(z) = H(z)\, H^*(z^*)$ causal, stability

$z = d_k$ $z = d_k^*$

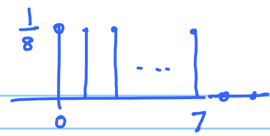
(b) $G(z) = H(z^{-1})$ poles $z = \frac{1}{d_k}$ causality, unstable

noncausal, stable $|z| < \min\left\{\frac{1}{d_k}\right\}$ $k = 1, \dots N$

(c) $G(z) = H(-z)$ poles @ $z = -d_k$ causal, stable

$g[n] = (-1)^n h[n]$

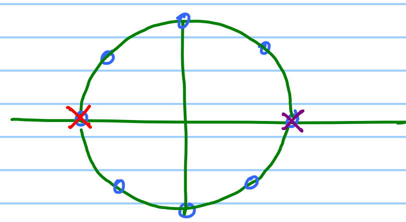
Ex MA filter $N = 8$

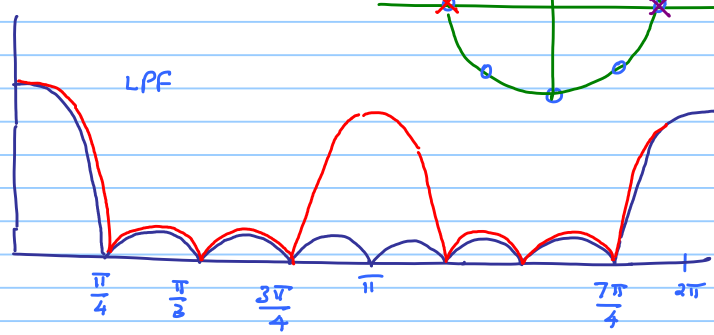


$$h[n] = \frac{1}{8}\left[u[n] - u[n-8] \right]$$

$$H(e^{j\omega}) = \frac{1}{8}\sum_{n=0}^{7} e^{-j\omega n} = \frac{1}{8}\,\frac{1-e^{-j8\omega}}{1-e^{-j\omega}} = \frac{1}{8}\, e^{-j\omega\frac{7}{2}}\,\frac{\sin 4\omega}{\sin\frac{\omega}{2}}$$

$$H(z) = \frac{1-z^{-8}}{1-z^{-1}}$$





$|H(e^{j\omega})|$ $\qquad$ $\arg H(e^{j\omega})$

Relationship between Mag. Resp and the Phase Resp

* $H(e^{j\omega}) = |H(e^{j\omega})| \, e^{j \arg H(e^{j\omega})}$

* Magnitude response does not provide information about phase resp., and vice versa

* If mag. response known, & # poles and # zeros are known, only finite # choices for the phase resp

* Additional constraints
  - → causal, stable
  - → real coefficients
  - → Min ϕ lag

Statement

Min ϕ lag system causal & stable <
- all poles inside unit circle
- all zeros inside unit circle

- → Min ϕ lag
- → Min GD
- → Max Partial Energy

LTI system causal & stable $\quad H(z) \quad H_{min}(z) \quad H_{max}(z)$

mixed $\quad$ all zeros inside unit circle $\quad$ all zeros outside

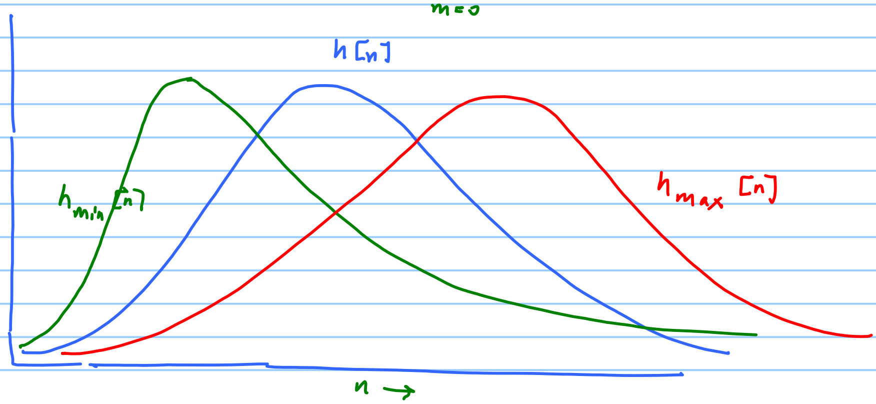
$$|H(e^{j\omega})| = |H_{min}(e^{j\omega})| = |H_{max}(e^{j\omega})| \quad \forall\, \omega$$

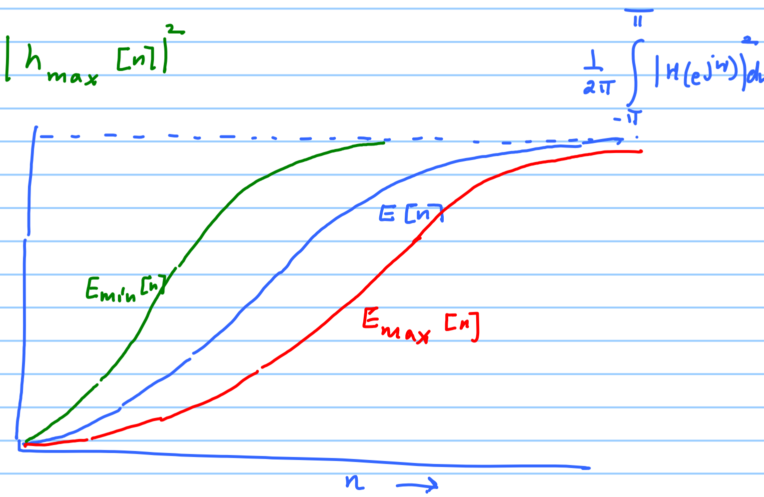
Parseval's theorem

$$\sum_{n=0}^{\infty} |h[n]|^2 = \sum_{n=0}^{\infty} |h_{min}[n]|^2 = \sum_{n=0}^{\infty} |h_{max}[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |H(e^{j\omega})|^2 d\omega$$

Partial Energy $\quad E[n] = \sum_{m=0}^{n} |h[m]|^2$

$$E_{min}[n] = \sum_{m=0}^{n} |h_{min}[n]|^2 \qquad E_{max}[n] = \sum_{m=0}^{n} |h_{max}[n]|^2$$





Partial Energy

$$E_{min}[n] \geq E[n] \geq E_{max}[n]$$

$\downarrow$

Min GD

Min ϕ lag

Result

Any ration TF $H(z)$ (causal & stable) can be expressed as

$$H(z) = H_{min}(z)\, H_{ap}(z)$$

Ex1

$$H(z) = \frac{(1-c_0 z^{-1})(1-c_0^* z^{-1})}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})} \qquad c_0 = \frac{3}{2}e^{j\frac{\pi}{4}}$$

✓ Causal & stable

$$(1-c_0 z^{-1})(1-c_0^* z^{-1}) = (-c_0)\left(z^{-1}-\frac{1}{c_0}\right)(-c_0^*)\left(z^{-1}-\frac{1}{c_0^*}\right) = |c_0|^2\left(z^{-1}-\frac{1}{c_0}\right)\left(z^{-1}-\frac{1}{c_0^*}\right)$$

zeros @ $z = \frac{1}{c_0}, \frac{1}{c_0^*}$

$$|c_0|^2 \left(z^{-1} - \frac{1}{c_0}\right)\left(z^{-1} - \frac{1}{c_0^*}\right) = |c_0|^2 \underbrace{\left(1 - \frac{1}{c_0} z^{-1}\right)\left(1 - \frac{1}{c_0^*} z^{-1}\right)}_{\text{zeros inside unit circle}} \underbrace{\frac{\left(z^{-1} - \frac{1}{c_0}\right)\left(z^{-1} - \frac{1}{c_0^*}\right)}{\left(1 - \frac{1}{c_0} z^{-1}\right)\left(1 - \frac{1}{c_0^*} z^{-1}\right)}}_{H_{ap}(z)}$$

zeros @ $= c_0, c_0^*$

poles @ $= \frac{1}{c_0^*}, \frac{1}{c_0}$

$$H(z) = \frac{\left(1 - c_0 z^{-1}\right)\left(1 - c_0^* z^{-1}\right)}{\left(1 - \frac{1}{3} z^{-1}\right)\left(1 - \frac{1}{4} z^{-1}\right)} = |c_0|^2 \, H_{min}(z) \, H_{ap}(z)$$

$$H_{min}(z) = \frac{\left(1 - \frac{1}{c_0} z^{-1}\right)\left(1 - \frac{1}{c_0^*} z^{-1}\right)}{\left(1 - \frac{1}{3} z^{-1}\right)\left(1 - \frac{1}{4} z^{-1}\right)}$$

Ex $H(z) = \left(1 - c_1 z^{-1}\right)\left(1 - c_1^* z^{-1}\right)\left(1 - c_2 z^{-1}\right)\left(1 - c_2^* z^{-1}\right)$

$c_1 = 0.9 e^{j0.6\pi}$ $\quad$ $c_2 = 1.25 e^{j0.8\pi}$

$H(z) = H_{min}(z) \, H_{ap}(z)$ $\quad$ $H_{min}(z) =$

$$H_{ap}(z) = |c_2|^2 \frac{\left(z^{-1} - \frac{1}{c_2}\right)\left(z^{-1} - \frac{1}{c_2^*}\right)}{\left(1 - \frac{1}{c_2} z^{-1}\right)\left(1 - \frac{1}{c_2^*} z^{-1}\right)}$$

Linear Phase

Zero phase $\rightarrow$ example Ideal LPF $h[n]$

$h[n]$ real, causal

$$h[n] \longleftrightarrow H(e^{j\omega})$$
$$h[-n] \longleftrightarrow H^*(e^{j\omega})$$

$$h_e[n] \longleftrightarrow \underbrace{H_R(e^{j\omega})}_{}$$ Real valued

$$h_o[n] \longleftrightarrow j\,\underbrace{H_I(e^{j\omega})}_{}$$ Real-valued

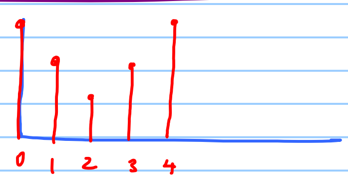
$$j = e^{j\frac{\pi}{2}}$$

$$\boxed{h_e[n-N_0] \longleftrightarrow e^{-j\omega N_0} H_R(e^{j\omega})}$$

$\arg H_e(e^{j\omega}) = -\omega N_0$ Linear phase

To make even seq $h_e[n]$ causal

FIR Linear ϕ TF



order $= M =$ even

even symmetry

$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} + h[4]e^{-j4\omega}$$

$$= e^{-j2\omega}\left[h[2] + h[1]\left[e^{j\omega} + e^{-j\omega}\right] + h[0]\left[e^{j2\omega} + e^{-j2\omega}\right]\right]$$

$$= e^{-j2\omega}\left[h[2] + 2h[1]\cos\omega + 2h[0]\cos 2\omega\right]$$

Type I Lin ϕ FIR

(Length is odd) Order = M $\quad H(z) = \sum_{k=0}^{M} h[k] z^{-k}$

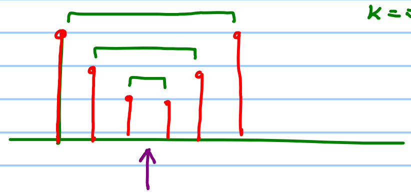
M is even

FIR $\quad h[n] = \begin{cases} h[M-n] & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$

$$H(e^{j\omega}) = \sum_{k=0}^{M} h[k] e^{-j\omega k} = e^{-j\omega \frac{M}{2}} \sum_{k=0}^{\frac{M}{2}} a[k] \cos \omega k$$

$a[0] = h\left[\frac{M}{2}\right]$

$a[k] = 2h\left[\frac{M}{2} - k\right]$



Type II Lin ϕ FIR

M = odd

even symmetry

$h[n] = h[M-n]$

$$H(e^{j\omega}) = e^{-j\omega \frac{M}{2}} \sum_{k=1}^{\frac{M+1}{2}} b[k] \cos\left(\omega\left(k - \tfrac{1}{2}\right)\right)$$

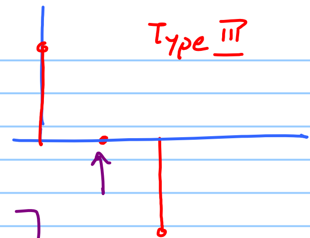
$b[k] = 2h\left[\frac{M+1}{2} - k\right] \quad k = 1, 2, \ldots \frac{M+1}{2}$

$$\omega = \pi \Rightarrow H(e^{j\pi}) = e^{-j\pi \frac{M}{2}} \sum_{k=1}^{\frac{M+1}{2}} b[k] \cos\left(\pi\left(k - \tfrac{1}{2}\right)\right) = 0$$

$$\boxed{H(e^{j\omega}) = 0 \quad \omega = \pi}$$

Type III Lin ϕ

M even $\quad h[n] = -h[M-n]$



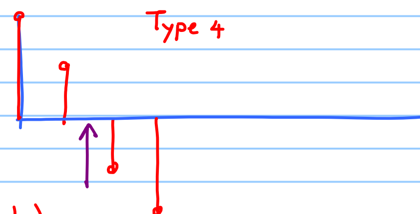
$$H(e^{j\omega}) = j\, e^{-j\omega\frac{M}{2}} \left[\sum_{k=1}^{\frac{M}{2}} c[k] \sin \omega k \right]$$

$$c[k] = 2h\left[\frac{M}{2}-k\right] \qquad k = 1, 2, \ldots \frac{M}{2}$$

$$\boxed{\begin{aligned} H(e^{j0}) &= 0 \\ H(e^{j\pi}) &= 0 \end{aligned}} \Rightarrow \begin{aligned} &\text{LPF} \times \\ &\text{HPF} \times \end{aligned}$$

Type IV Lin ϕ

M odd $\quad h[n] = -h[M-n]$



$$H(e^{j\omega}) = j\, e^{-j\omega\frac{M}{2}} \sum_{k=1}^{\frac{M+1}{2}} d[k] \sin \omega\left(k-\tfrac{1}{2}\right)$$

$$d[k] = 2h\left[\frac{M+1}{2}-k\right] \qquad k = 1 \ldots \frac{M+1}{2}$$

$$\boxed{H(e^{j0}) = 0} \qquad \text{LPF} \times$$

nc = no constraint

Table

Symm	M	Type	$H(e^{j\omega})$	@ $\omega = 0$	@ $\omega = \pi$	Applic.	
even	even	I	$e^{-j\omega\frac{M}{2}} \sum_{k=0}^{\frac{M}{2}} a[k] \cos \omega k$	nc	nc	Any filter	
even	odd	II	$e^{-j\omega\frac{M}{2}} \sum_{k=1}^{\frac{M+1}{2}} b[k] \cos \omega(k-\frac{1}{2})$	nc	0	except HPF	
odd	even	III	$j e^{-j\omega\frac{M}{2}} \sum_{k=1}^{\frac{M}{2}} c[k] \sin \omega k$	0	0	HPF ✗ LPF ✗	Differentiator Hilbert Transformer
odd	odd	IV	$j e^{-j\omega\frac{M}{2}} \sum_{k=1}^{\frac{M+1}{2}} d[k] \sin \omega(k-\frac{1}{2})$	0	nc	HPF ✓ LPF ✗	"

FIR lin ϕ with real coefficient

$$h[n] = \pm h[M-n]$$

$$H(z) = h[0] + h[1]z^{-1} + \cdots \qquad + h[1]z^{-(M-1)} + h[0]z^{-M}$$

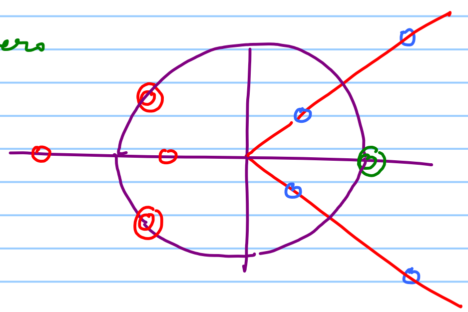
$$= z^{-M}\left(h[0]z^{M} + h[1]z^{M-1} + \quad \cdots \quad + h[1]z + h[0]\right)$$

$$\boxed{H(z) = z^{-M} H(z^{-1})}$$

$$H(z) = (1 - z_0 z^{-1})(1 - z_0 z)$$

$$H(z^{-1}) = (1 - z_0 z)(1 - z_0 z^{-1})$$

If z_0 is a zero of $H(z)$ FIR lin ϕ TF, then $\frac{1}{z_0}$ is also a zero



$h[n]$ is real-valued

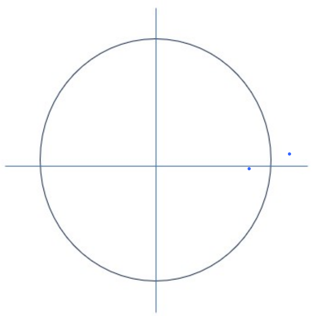
$$\left.\begin{array}{l}\text{If } z_0 \text{ is a zero,} \\ z_0^* \text{ is a zero}\end{array}\right\} \quad \begin{array}{l}\frac{1}{z_0} \text{ is a zero} \\ \frac{1}{z_0^*} \text{ is a zero}\end{array}$$

O & S Ch 7 Filter Design

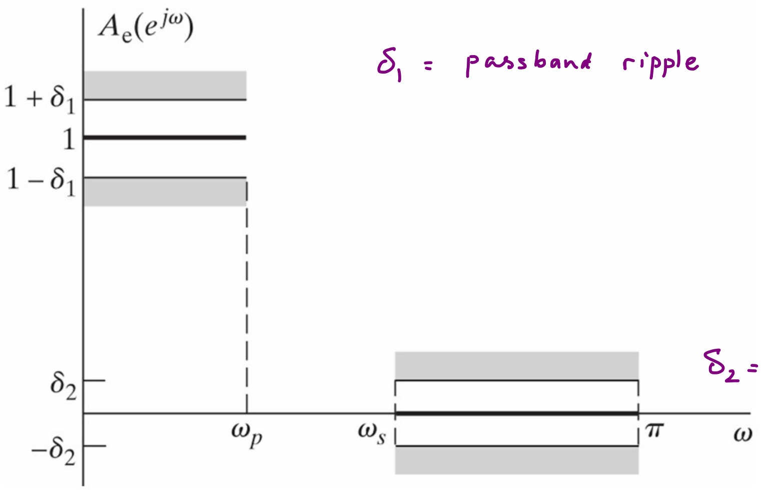
Filters - very import building block

Filter → Freq selective → some frequencies amplified
→ " attenuated

Filter ⟷ LTI ⟷ h[n] ⟷ H(z) → IIR
→ FIR → lin ϕ
→ Min ϕ

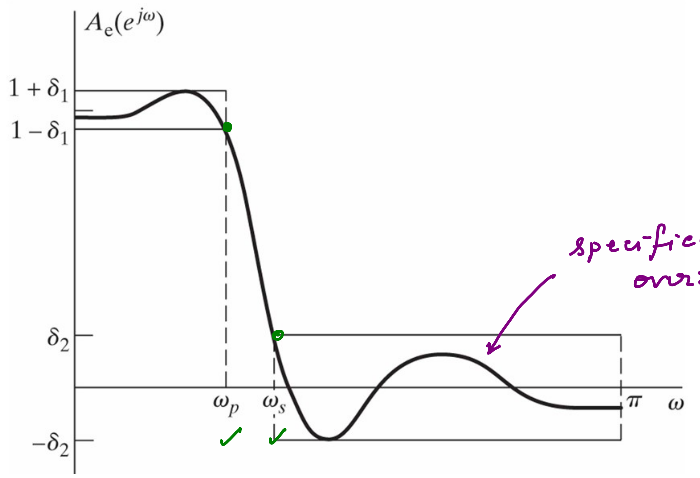


Filter Specifications (LPF)



δ_1 = passband ripple

δ_2 = stopband Attenuation



specifications oversatisfied

Passband $[0, w_p]$

Stopband $[w_s, \pi]$

Transition band $[w_p, w_s]$

LPF

- w_p passband edge (radians)
- w_s stopband edge (radians)
- δ_1
- δ_2

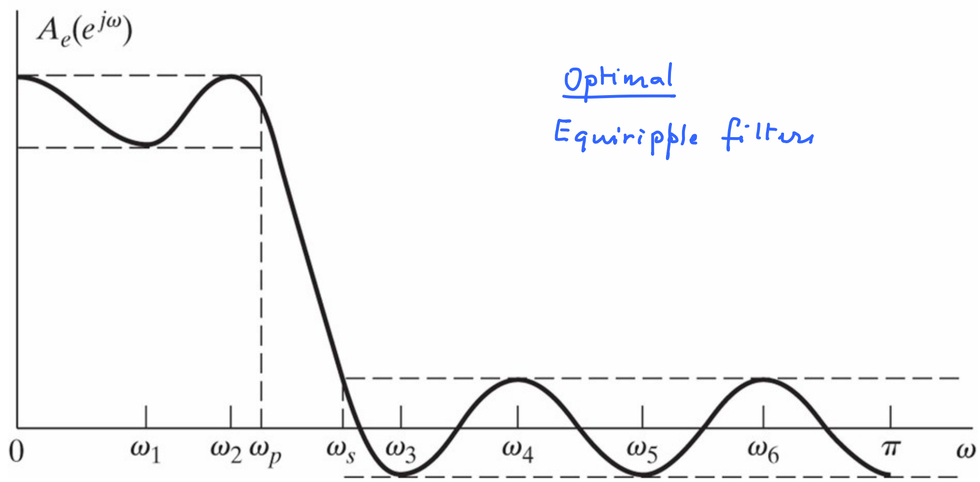
Attenuation $= -20\log_{10} \delta_2$ (dB)

Filter meets all specifications

FIR filter order = M

length M+1

Is the filter optimal in terms of length of filter?



Parks - McClellan Algorithm

ω_p, ω_s $\quad \Delta\omega = \omega_s - \omega_p$

Transition band

δ_1

δ_2

M

Estimation of Filter Order

PM Filter ≡ Equiripple

$$M = \text{filter order} = \frac{-10\log_{10}\delta_1\delta_2 - 13}{2.324\ \Delta\omega}$$

$M \uparrow$ if $\delta_1 \downarrow$ or $\delta_2 \downarrow$

$M \uparrow$ if $\Delta\omega \downarrow$

Example

$\delta_1 = 0.01$

$A_s = 60\text{ dB} = \delta_2 = 0.001$

$\Delta\omega = 0.2\pi$

$\omega_p = 0.4\pi$

$\omega_s = 0.6\pi$

$M \simeq 26$

Case 2

$A_s = 80\text{ dB}$ $\delta_2 = 0.0001$

$M \simeq 52$

Practical Example

LPF Passband [0, 2000 Hz] $\delta_1 = 0.01$

Stopband [3000 Hz] $\delta_2 = 0.001$

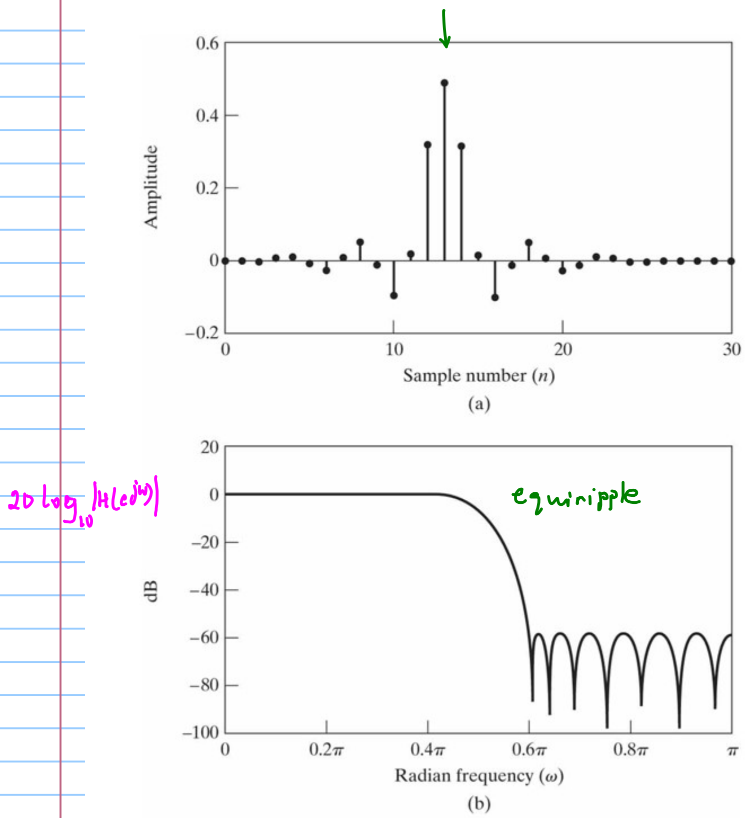
Sampling freq 10000 Hz

Anti Aliasing filter 5000 Hz

$$\omega_p = \frac{2\pi \; 2000}{10000} = 0.4\pi$$

$$\omega_s = \frac{2\pi \; 3000}{10000} = 0.6\pi$$

point of symmetry



Parks - McClellan (PM) Design Example

$\omega_p = 0.4\pi$

$\omega_s = 0.6\pi$

$K = 10$

$M = 26$ (Type I FIR filter)

$\delta_1 = K\delta_2$

$\delta_1 = 0.009$

Filter Order estimation

$$M = \frac{-10\log_{10}\delta_1\delta_2 - 13}{2.324\,\Delta\omega}$$

$\delta_1 = 0.009$

$\delta_2 = 0.0009$

$\Delta\omega = 0.2\pi$

$M \sim 25.96 = 26.$

Verify via design that filter satisfies

1. the specifications for δ_1 and δ_2
2. Equiripple response

Discrete Fourier Transform

* Representation of DT seq $x[n]$ → DTFT $X(e^{j\omega})$; → ZT $X(z)$

$x[n]$ finite $\longleftrightarrow X(e^{j\omega})$

DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

Time: discrete (in n), aperiodic

Freq: continuous (in ω), periodic

Discrete Time Fourier Series

If $x[n]$ is periodic ⇒ period $= N$

Represent via DT Fourier Series

$e^{j\frac{2\pi}{N}k}$ $k = 0, 1 \ldots$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} C_k e^{j\frac{2\pi}{N}kn}$$

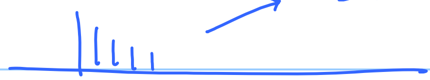
Fourier Coefficients

$$C_k = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}nk}$$

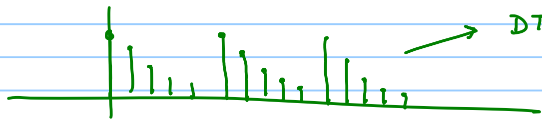
Time: discrete, periodic

Freq: $\boxed{\text{discrete}}$, periodic

N distinct values



DTFT $X(e^{j\omega})$ continuous



DTFS discrete

Discrete Fourier Transform (DFT)

efficient implem

Fast Fourier Transform (FFT)