Electrical Engineering
IIT Madras

EE 3101
Digital Signal Processing

BS Electronic Systems programme

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Session # 16

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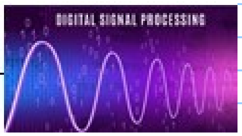


Session 16

Outline

Last session
- Z Transform properties

Today
- Pole - Zero plots
- RoC
- Causality
- Stability
- Inverse Z Transform

Week 7-8 O&S Chapter 3

Z-Transform: Generalized complex exponentials as eigen signals of LTI systems—z-transform definition—region of convergence (RoC)—properties of RoC—properties of the z-transform—pole-zero plots—time-domain responses of simple pole-zero plots—RoC implications of causality and stability

✓ Sec 1 Z Transform
✓ Sec 2 Properties of RoC (Region of Convergence)
Sec 3 Inverse Z Transform (Cover at end of week 8)
✓ Sec 4 Z Transform Properties
Sec 5 Z Transform and LTI systems
× Sec 6 Unilateral Z Transform (Omit)

Reading Assignment

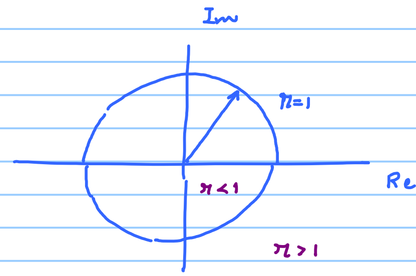
O&S ch 3 The Z Transform

Z Transform is a generalization of the DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

General form $z = r e^{j\omega}$



Defn $X(z) \triangleq \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} = \sum_{n=-\infty}^{\infty} \left[x[n]\, r^{-n}\right] e^{-j\omega n}$

$\equiv$ DTFT of $x[n]\, r^{-n}$

$\{r \in \mathbb{R} \mid r \geq 0\}$

$x[n] \xrightarrow{Z} X(z)$ ROC

$x[n] \xleftarrow{\text{Inv. } ZT} X(z)$

DTFT $z = e^{j\omega}$ A special case of Z Transform

$\Rightarrow |z| = 1$ or $r = 1$

ZT can exist even if DTFT does not exist

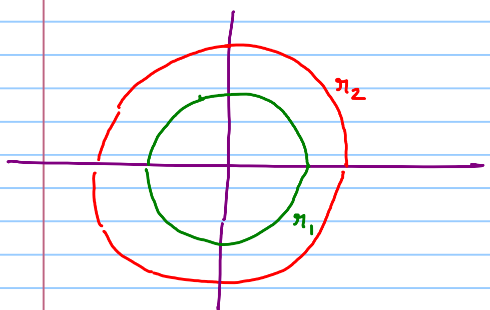
ZT of $x[n] \equiv$ DTFT $\left(x[n]\, r^{-n}\right)$

ROC $\Rightarrow$ where ZT exists

$\Rightarrow$ range of values of r for which $x[n]\, r^{-n}$ is abs summable

$\boxed{\text{ROC depends on the value of } r}$

If ZT exists for one value of r ie, $z = re^{j\omega_0}$ then ZT exists for all points on circle with radius



ROC $|z| > r_1$ $\longrightarrow$ right-sided (subset : Causal seq)

ROC cannot include any poles

$|z| < r_1$ $\longrightarrow$ left-sided

ROC

$|z| > r_2$

$|z| < r_1$

$r_1 < |z| < r_2$ $\longrightarrow$ Two-sided

Properties of ROC

Assume $X(z)$ is a rational form $\frac{P(z)}{Q(z)}$

#1 ROC is a ring or a disc in Z plane centred @ origin

annulus

right-seq — Two-sided

$$0 \leq r_R < |z| < r_L \leq \infty$$

Left-sided

$$\boxed{0 \leq r_R < |z| < r_L \leq \infty}$$

Right sided sequence r_R is radius of outermost pole

Left sided sequence r_L is radius of innermost pole

ROC of finite length sequences $\rightarrow$ entire Z plane except possibly $z=0$, and/or $z=\infty$

Table 3.1 SOME COMMON z-TRANSFORM PAIRS

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Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6. $-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7. $na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8. $-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9. $\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10. $\sin(\omega_0 n)u[n]$	$\frac{\sin(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11. $r^n\cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
12. $r^n\sin(\omega_0 n)u[n]$	$\frac{r\sin(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$

ROC

① $X(z) = 1$ entire Z plane

② $X(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1-z^{-1}}$ $\lvert z\rvert > 1$

④ $m = 2$ $\{0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ \dots\}$

$X(z) = z^{-2}$ $\begin{cases} \text{excl. } z = 0 \\ \text{entire } z \text{ plane} \end{cases}$

⑬ $x[n] = a^n$ $0 \le n \le N-1$

$X(z) = \frac{1-a^N z^N}{1-az^{-1}}$ pole-zero cancellation

ROC entire Z except $z=0$

$\lvert z\rvert > 0$

7

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$n\,x[n] \longleftrightarrow -z \frac{d}{dz} X(z)$$

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z| > a$$

$$n\,a^n u[n] \longleftrightarrow -z \frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right) = \frac{a z^{-1}}{(1-az^{-1})^2} \qquad |z| > a$$

Ex $H(z) = \dfrac{K(1+4z^{-1})(1+2.5z^{-1})}{(1+3z^{-1})(1-1.5z^{-1})(1-0.9z^{-1})}$

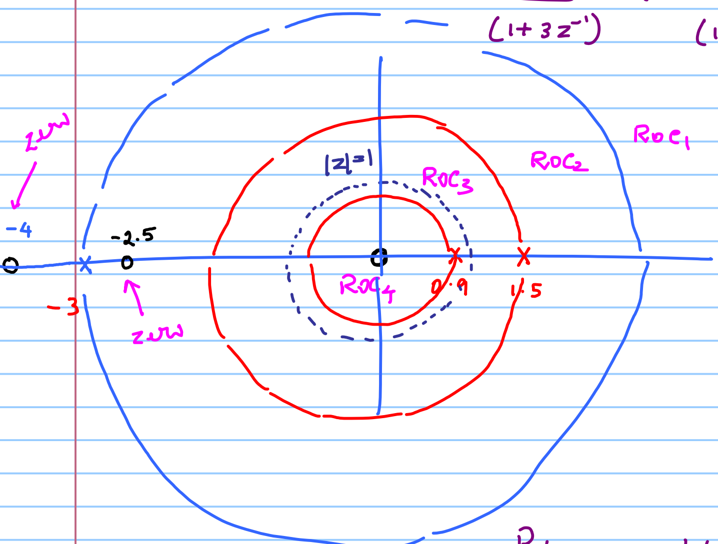
zeros $z = -4, \quad z = -2.5, \quad z = 0$

poles $z = -3, \quad z = 1.5, \quad z = 0.9$

Partial Fraction Expansion

$$= \frac{A_1}{(1+3z^{-1})} + \frac{A_2}{(1-1.5z^{-1})} + \frac{A_3}{(1-0.9z^{-1})}$$

Assume A_1, A_2, A_3 are computed



Pole - zero plot

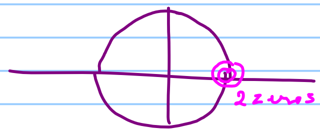
Obtain the ROC for which the DTFT exists & sequence $x[n]$

$ROC_3 \quad 0.9 < |z| < 1.5 \longrightarrow$ includes unit circle $\Rightarrow$ DTFT exists

$$h[n] = A_3(0.9)^n u[n] - A_2(1.5)^n u[-n-1] - A_1(-3)^n u[-n-1]$$

Pole-zero plot

zeros @ $z=-4, \quad z = -2.5$



Examples

$H_1(z) = (1+4z^{-1})$ zero @ $z=-4$ ✓

$= 1 + \frac{4}{z}$

$= \frac{z+4}{z}$ zero @ $z=-4$

pole @ $z=0$

$H_2(z) = \frac{1}{1+3z^{-1}} = \frac{z}{z+3}$ pole @ $z=-3$

zero @ $z=0$

poles = # zeros in the finite Z plane

$H_3(z) = \frac{1+4z^{-1}}{1+3z^{-1}} = \frac{z+4}{\cancel{z}} \cdot \frac{\cancel{z}}{z+3}$ $\longrightarrow$ pole @ $z=-3$

zero @ $z=-4$

In general

$$H(z) = \frac{\prod_{k=1}^{M}(1-c_k z^{-1})}{\prod_{m=1}^{N}(1-d_m z^{-1})}$$

Let $N > M$

zeros @ $z = c_k$ $k=1,\dots M$, $(N-M)$ zeros @ $z=0$

poles @ $z = d_m$ $m=1,\dots N$

TABLE 3.2 SOME z-TRANSFORM PROPERTIES

	Property Number	Section Reference	Sequence	Transform	ROC
			$x[n]$ ⟷	$X(z)$	R_x
			$x_1[n]$ ⟷	$X_1(z)$	R_{x_1} or R_1
			$x_2[n]$ ⟷	$X_2(z)$	R_{x_2} or R_2
Linearity	1	3.4.1	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	Contains $\boxed{R_{x_1} \cap R_{x_2}}$
Shift	2	3.4.2	$x[n - n_0]$	$z^{-n_0}X(z)$	R_x, except for the possible addition or deletion of the origin or ∞
Multip. by expon. seq	3	3.4.3	$z_0^n x[n]$	$\boxed{X(z/z_0)}$	$\|z_0\|R_x$ $\|z_0\| r_R < \|z\| < \|z_0\| r_L$
Differentiation	4	3.4.4	$nx[n]$	$-z\dfrac{dX(z)}{dz}$	R_x
	5	3.4.5	$x^*[n]$	$X^*(z^*)$	R_x
	6		$\mathcal{R}e\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Contains R_x
	7		$\mathcal{I}m\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	Contains R_x
	8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
	9	3.4.7	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
			$R_{xx}[n]$	$X\left(\frac{1}{z}\right) X(z)$	$\max\left(r_R, \frac{1}{r_L}\right) < \|z\| < \min\left(r_L, \frac{1}{r_R}\right)$

$$aX(z) = \sum_{n=-\infty}^{\infty} a x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (x_1[n] + x_2[n]) z^{-n}$$

$$= X_1(z) + X_2(z)$$

Linearity

PFE ⟷ Linearity Property

$$\frac{A_1}{1-a_1 z^{-1}} + \frac{A_2}{1-a_2 z^{-1}} + \frac{A_3}{1-a_3 z^{-1}}$$

$$a_1^n u[n] + a_2^n u[n] + a_3^n u[n]$$

ROC_1 ROC_2 ROC_3

$ROC = ROC_1 \cap ROC_2 \cap ROC_3$

Ex $x[n] = r_0^n \sin w_0 n \; u[n]$

$$= \frac{1}{2j}\left[r_0^n e^{jw_0 n} - r_0^n e^{-jw_0 n}\right] u[n]$$

Property of Linearity

2 Time-Shift Property

$x[n] \longleftrightarrow X(z) \quad ROC = R_x$

$x[n-n_0] \longleftrightarrow z^{-n_0} X(z) \quad ROC = R_x$

- If n_0 is positive, $z=0$ excl. from ROC
- If n_0 is negative, $z=\infty$ excl. from ROC

$$X(z) = \frac{a + b z^{-1}}{1-\alpha z^{-1}}$$ Obtain $x[n]$ using properties of ZT

$$\frac{a + b z^{-1}}{1-\alpha z^{-1}} = \frac{a}{1-\alpha z^{-1}} + b\, z^{-1} \frac{1}{1-\alpha z^{-1}}$$ (Linearity)

shift

$$x[n] = a \cdot \alpha^{n} u[n] + b\, \alpha^{n-1} u[n-1] \qquad ROC \ |z| > \alpha$$

A method of computing inverse Z Transform

#3 Multiplication by exponential seq

$x[n] \longleftrightarrow X(z)$ $\quad r_R < |z| < r_L$

$z_0^n \, x[n] \longleftrightarrow X\left(\frac{z}{z_0}\right)$ $\quad |z_0| r_R < |z| < |z_0| r_L$

$e^{j\omega_0 n} \, x[n] \longleftrightarrow X(z e^{-j\omega_0})$ $\quad \boxed{R_x}$

$r_0^n \, x[n] \longleftrightarrow X\left(\frac{z}{r_0}\right)$ $\quad r_0 r_R < |z| < r_0 r_L$

$$X\left(\frac{z}{z_0}\right) = X\left(\frac{z}{r_0 e^{j\omega_0}}\right) = X\left(\frac{z}{r_0} e^{-j\omega_0}\right)$$ combination of scaling of RoC & freq. shift

RoC $\quad r_R < \left|\frac{z}{z_0}\right| < r_L$

$|z_0| r_R < |z| < |z_0| r_L$

$\boxed{\text{scale the RoC}}$

$|z_0| > 1$ expanding

< 1 shrinking of RoC

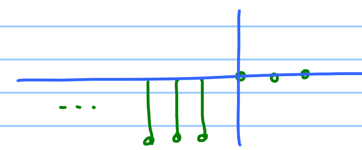
Ex

$x[n] = u[n] \longleftrightarrow X(z) = \frac{1}{1-z^{-1}} \quad |z|>1$

$x_1[n] = x[n] = -u[-n-1] \longleftrightarrow X(z) = \frac{1}{1-z^{-1}} \quad |z|<1 \quad R_{x_1}$

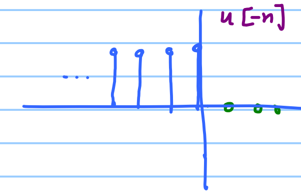
$x_1[n]$

$u[-n-1]$



$x_2[n]$

$u[-n]$



$x_2[n] = x[n] = u[-n]$

$\longleftrightarrow X(z) = -\frac{1}{1-z^{-1}} \cdot z^{-1} = -\frac{z^{-1}}{1-z^{-1}} \quad R_{x_1}$

$x_2[n] = -x_1[n-1]$

$X(z) = 1 + z + z^2 + \cdots$

?

$\underbrace{0 <}\, |z| < 1$

or $|z|<1$ ✓

Conjugation

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

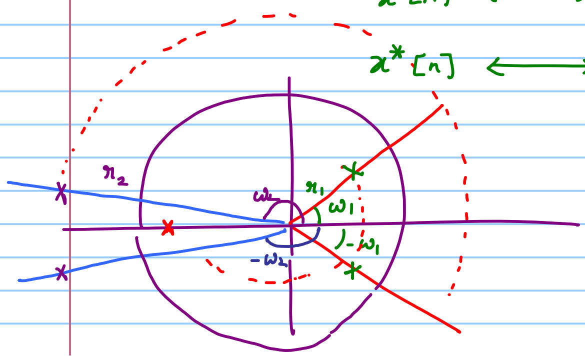
$$X^*(z) = \left(\sum_{n=-\infty}^{\infty} x[n]\, z^{-n}\right)^* = \sum_{n=-\infty}^{\infty} x^*[n]\, (z^{-n})^*$$

$$= \sum_{n=-\infty}^{\infty} x^*[n]\, (z^*)^{-n}$$

$$X^*(z^*) = \sum_{n=-\infty}^{\infty} x^*[n]\, z^{-n}$$

$x[n] \longleftrightarrow X(z) \qquad R_x$

$x^*[n] \longleftrightarrow X^*(z^*) \qquad ? \qquad R_x$



$$(z^{-n})^* = (z^*)^{-n}$$

$$(z^{-n})^* = \left(\frac{1}{z^n}\right)^* = \left(\frac{1}{z^*}\right)^n = (z^*)^{-n}$$

$z = r e^{j\omega} \qquad z^* = r e^{-j\omega}$

$z^{-n} = r^{-n} e^{-j\omega n} \qquad (z^*)^{-n} = r^{-n} e^{j\omega n}$

$(z^{-n})^* = r^{-n} e^{j\omega n}$

$(z^{-n})^* = (z^*)^{-n}$

$$X(z) = \frac{\prod_k (1 - c_k z^{-1})}{\prod_m (1 - d_m z^{-1})} \qquad \text{poles @ } z = d_m \quad m = 1, \dots M$$

$$X^*(z^*) = \frac{\prod_k (1 - c_k^* z^{-1})}{\prod_m (1 - d_m^* z^{-1})} \qquad \text{poles @ } z = d_m^* \quad m = 1, \dots M$$

Table   7 cases   Write down ROC

Specify whether ROC represents a seq. that is ① causal ② BIBO stable

① $\boxed{|z| > r \qquad 0 < r < 1}$

- Causal
- BIBO stable

② $|z| > r \qquad r > 1$

- Causal
- Not BIBO Stable

③ $|z| < r \qquad r > 1$

- Anticausal
- stable

④ Anticausal

Not BIBO stable

⑤ $r_1 < |z| < r_2$

Two-sided

BIBO Stable

⑥, ⑦ Two sided

BIBO unstable

Example

$x[n]$ is $\underbrace{\text{absolutely summable signal}}$ DTFT exists with rational Z Transform $X(z)$ with a pole @ $z = \frac{1}{2}$

(a) Can $x[n]$ be finite duration?

NO

$X(z)$ has a pole @ $z = \frac{1}{2}$

RoC $\neq$ entire Z plane $\Rightarrow$ not a finite duration seq.

(b) Can $x[n]$ be a left-sided sequence?

NO

$|z| < \frac{1}{2}$

Given condition RoC includes unit circle

(c) Can $x[n]$ be a right-sided sequence?

Yes

$|z| > \frac{1}{2}$

(d) Can $x[n]$ be two-sided?

Possibly Yes.

$\frac{1}{2} < |z| < r_L$

Exercises

① $x[n] = \left(\frac{1}{2}\right)^{n+1} u[n+1]$ ST ROC $\frac{1}{2} < |z| < \infty$

② Given $X(z) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{-n+1} z^{n}$ Obtain ROC $|z| < \frac{1}{2}$

③ $x[n] = -\left(\frac{1}{2}\right)^{n} u[-n-1] + 2^{n} u[-n-1]$ ST ROC $|z| < \frac{1}{2}$

Property #5 Conjugation $x[n] \longleftrightarrow X(z)$ R_x

$x^*[n] \longleftrightarrow X^*(z^*)$ R_x

Property #6

$Re\{x[n]\} = \frac{1}{2}[x[n] + x^*[n]] \longleftrightarrow \frac{1}{2}[X(z) + X^*(z^*)]$ R_x

R_x R_x

Verify #7 $Im\{x[n]\} \longleftrightarrow \frac{1}{2j}[X(z) - X^*(z^*)]$ R_x

Time reversal

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

Replace $m = -n$

$$X(z) = \sum_{m=-\infty}^{\infty} x[-m] z^{m}$$

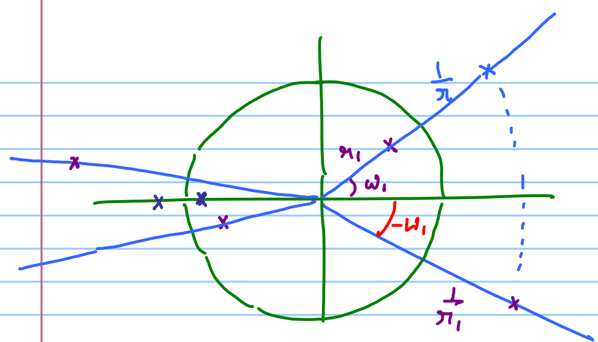
$$X\left(\frac{1}{z}\right) = \sum_{m=-\infty}^{\infty} x[-m] z^{-m} \Rightarrow x[-n] \longleftrightarrow X\left(\frac{1}{z}\right)$$

$$X(z) = \frac{\prod_k (1-c_k z^{-1})}{\prod_m (1-d_m z^{-1})}$$

poles @ $z = d_m$ $m = 1, \cdots N$

$$X\left(\frac{1}{z}\right) = \frac{\prod_k (1-c_k z)}{\prod_m (1-d_m z)}$$

poles $z = \frac{1}{d_m}$ $m = 1, \cdots N$



$d_1 = r_1 e^{j\omega_1}$

$\frac{1}{d_1} = \frac{1}{r_1} e^{-j\omega_1}$

$\frac{1}{d_1^*} = \frac{1}{r_1} e^{j\omega_1}$

ROC of $X(z)$ $\quad r_R < |z| < r_L$

ROC of $X\left(\frac{1}{z}\right)$ $\quad r_R < \left|\frac{1}{z}\right| < r_L$

$\frac{1}{r_L} < |z| < \frac{1}{r_R}$

$\left\{ \begin{array}{l} \left|\frac{1}{z}\right| < r_L \\ |z| > \frac{1}{r_L} \end{array} \right.$

$r_R < \left|\frac{1}{z}\right|$

$|z| < \frac{1}{r_R}$

Property #8

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad R_x \qquad r_R < |z| < r_L$$

$$X^*(z^*) = \sum_{n=-\infty}^{\infty} x^*[n] z^{-n} \qquad R_x$$

$m = -n$

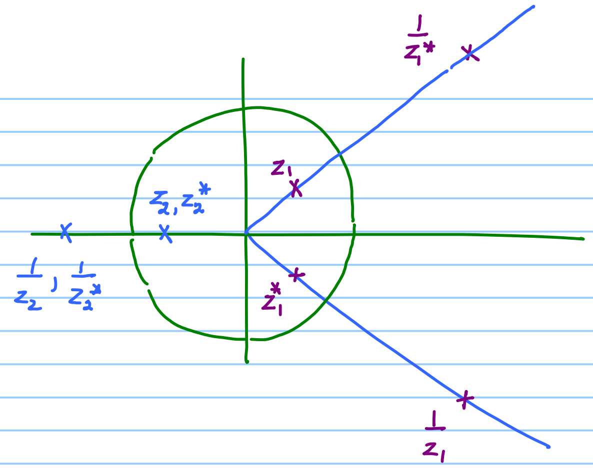
$$X^*(z^*) = \sum_{m=-\infty}^{\infty} x^*[-m] z^{m}$$

$$X^*\left(\frac{1}{z^*}\right) = \sum_{m=-\infty}^{\infty} x^*[-m] z^{-m} \Rightarrow x^*[-m] \longleftrightarrow X^*\left(\frac{1}{z^*}\right)$$

$$X(z) = \frac{\prod_k (1 - c_k z^{-1})}{\prod_m (1 - d_m z^{-1})}$$

$$X^*\left(\frac{1}{z^*}\right) = \frac{\prod_k (1 - c_k^* z)}{\prod_m (1 - d_m^* z)}$$

poles @ $z = \frac{1}{d_m^*}$ $\quad m = 1, \ldots M$



$r_R < |z| < r_L$

$x^*[-n] \longleftrightarrow X^*\left(\frac{1}{z^*}\right) \qquad \frac{1}{r_L} < |z| < \frac{1}{r_R}$

$x[n] \longleftrightarrow X(z) \qquad r_R < |z| < r_L$

$x^*[n] \longleftrightarrow X^*(z^*) \qquad r_R < |z| < r_L$

$x[-n] \longleftrightarrow X\left(\frac{1}{z}\right) \qquad \frac{1}{r_L} < |z| < \frac{1}{r_R}$

$x^*[-n] \longleftrightarrow X^*\left(\frac{1}{z^*}\right) \qquad \frac{1}{r_L} < |z| < \frac{1}{r_R}$

#9 Convolution Property of ZT

$x_1[n] \longleftrightarrow X_1(z) \qquad R_{x_1}$

$x_2[n] \longleftrightarrow X_2(z) \qquad R_{x_2}$

$$x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k] \longleftrightarrow \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k] \right) z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x_1[k] \sum_{n=-\infty}^{\infty} x_2[n-k] \underbrace{z^{-n} z^{k}}_{z^{-(n-k)}} z^{-k}$$

$$= \underbrace{\sum_{k=-\infty}^{\infty} x_1[k]\, z^{-k}}_{X_1(z)} \; \underbrace{\sum_{n=-\infty}^{\infty} x_2[n-k]\, z^{-(n-k)}}_{X_2(z)}$$

$$x_1[n] * x_2[n] \longleftrightarrow \underbrace{X_1(z)\, X_2(z)}$$

ROC $\quad R_{x_1} \cap R_{x_2}$

(take into account pole-zero cancellation)

Autocorrelation $x[n]$

$$r_{xx}[n] = \sum_{k=-\infty}^{\infty} x[k]\, x[k+n]$$

(Difference wrt convolution
No time reversal)

$$r_{xx}[n] \longleftrightarrow R_{xx}(z) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k]\, x[n+k] \right) z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x[k] \sum_{n=-\infty}^{\infty} x[n+k]\, \underbrace{z^{-n}\, z^{-k}}_{z^{-(n+k)}}\, z^{+k}$$

$$= \underbrace{\sum_{k=-\infty}^{\infty} x[k]\, z^{k}}_{X\left(\frac{1}{z}\right)} \underbrace{\sum_{n=-\infty}^{\infty} x[n+k]\, z^{-(n+k)}}_{X(z)}$$

$$R_{xx}(z) = X\left(\frac{1}{z}\right) X(z)$$

$\frac{1}{r_L} < |z| < \frac{1}{r_R}$ $r_R < |z| < r_L$

$\max\left(r_R, \frac{1}{r_L}\right) < |z| < \min\left(r_L, \frac{1}{r_R}\right)$

$|z| > r_R$
$|z| > \frac{1}{r_L}$
$|z| > \max\left(r_R, \frac{1}{r_L}\right)$

$|z| < \frac{1}{r_R}$
$|z| < r_L$
$|z| < \min\left(r_L, \frac{1}{r_R}\right)$

Initial Value Theorem

If $x[n]$ is causal $\left(\begin{array}{l} x[n] = 0 \text{ for } n < 0 \\ x[0] \neq 0 \end{array} \right)$

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n} = x[0] + x[1] z^{-1} + x[2] z^{-2} + \cdots$$

$$\lim_{z \to \infty} X(z) = x[0] + 0 + 0 + 0 \cdots$$

$$\frac{1}{z} \to 0$$

$$\boxed{\lim_{z \to \infty} X(z) = x[0]}$$

Causal seq

example

$$a^n u[n] \longleftrightarrow X(z) = 1 + a z^{-1} + a^2 z^{-2} + \cdots$$

$$= \frac{1}{1 - a z^{-1}}$$

$$x[0] = \lim_{z \to \infty} \frac{1}{1 - a z^{-1}} = \frac{1}{1 - 0} = 1$$