Note Title 02-09-2024

Electrical Engineering
IIT Madras

EE 3101
Digital Signal Processing

BS Electronic Systems programme

R. David Koilpillai

September – December 2024

EE 3101 Digital Signal Processing

- Welcome
- Course Overview
- Introduction

Faculty : Prof. R. David Koilpillai
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Instructor . Mr. Tony Varkey
tony@study.iitm.ac.in

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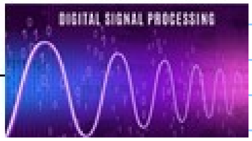
Introduction

- A warm welcome to EE3101 DSP Course !!
- Builds on EE2101 Signals & Systems course
- Concepts of Signals, Systems and their representation
- In Discrete Time …
- DSP translates Signals & Systems into the domain of computers
- Digital representation of Signals
  - Implementation, Transmission, Storage, Retrieval …
  - Compression & Reconstruction
  - Encryption and Decryption
- Many interesting applications
- One of them is the Cellphone …



2G digitized voice
data

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1989 - Motorola Micro tac

1992 Nokia 101

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1996 - Motorola StarTAC

1998-2000

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2002 – Blackberry 5810

2004 – Motorola Razor

Ericsson Family

2006 – Blackberry Pearl

2007 Apple iPhone

2012 iPhone 5

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2 G

Single standard Communications device with some applications

evolution

4 G

Applications device with Communications functionality (Multiple DSP applications)

Cellphone Cameras

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- An exciting application of DSP …

Pro camera system

48MP Main | Ultra Wide | Telephoto

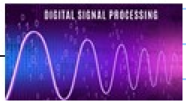
Super-high-resolution photos (24MP and 48MP)

Next-generation portraits with Focus and Depth Control

Up to

10x

optical zoom range



Touch Screen

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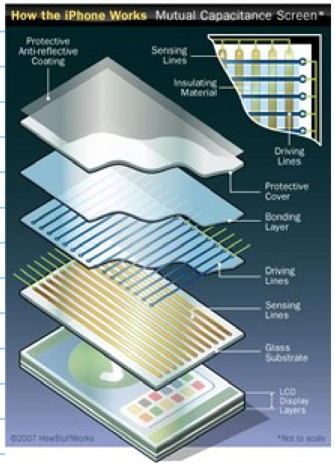
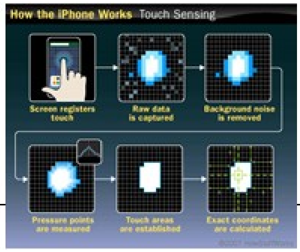


- Multi-touch, Capacitive method
- Layer of capacitive material
- Capacitors in coordinate system
- Sense changes at each point on grid.
- Works only if you touch it with fingertip
- Not stylus or non-conductive gloves



Wireless Channel

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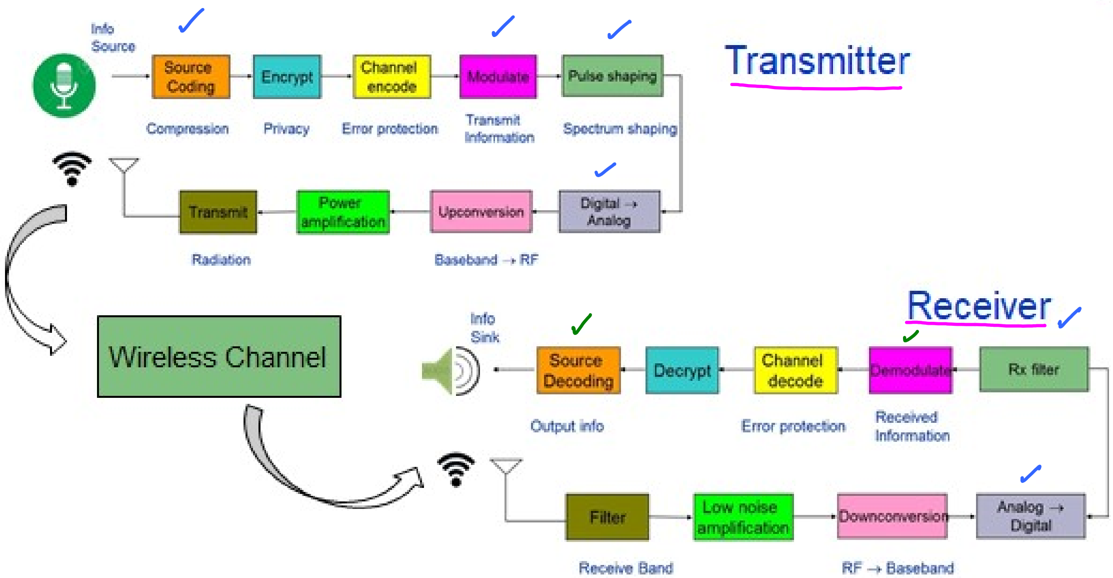
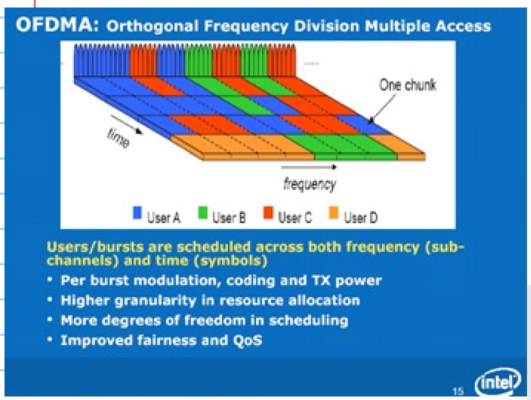


DSP involved in multiple parts of Receiver

① Synchronization { freq, time }

② Equalization



(OFDM)

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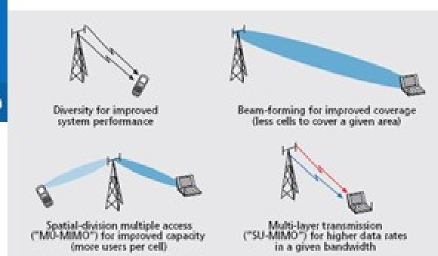
Ref: A. Muchumbe (Intel) – Mobile WiMAX Presentation



Ref: Dahlman, IEEE Comm Mag, Apr 2009

Key Aspects of 4G systems

- OFDMA
- Smart Antennas

DSP in a Cellphone

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- In Transmitter
- In Receiver
- Camera
  - Motion deblurring, Special effects, ...
- Display
- Video
- Music
- Applications
  - Google Lens
  - Adaptive equalizer (Bass boost, ...)
  - Acoustic effects (Concert Hall ...)
-
-
-

DSP involved in OFDM modulation

DSP involved in Smart Antenna algorithms

Motivation for Studying DSP EE3101

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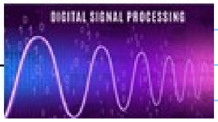
- ✓ A practical course
- ✓ Many applications
- Essential in communications
- Foundation for Machine Learning / Deep Learning / Neural Networks
- DSP applied in all branches of Engineering, Medicine, ...
- DSP applications in Finance, Operations Research ...



David Koilpillai Profile

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Education

B.Tech, IIT Madras, MS, PhD Caltech, USA

Work Experience

IIT Madras (2002 – present)

- Qualcomm Institute Chair Professor (Mar 2016 – present)
- Professor, Electrical Engineering Department (Jun 2002 – present)
- Head, EE Department (Aug 2019 – Aug 2022)
- Dean (Planning) (Oct 2011 – Oct 2017)

CEWiT – Chief Scientist (Jan 2007 – Jul 2007)

Co-Chair, IIT Hyderabad Task Force (Jun 2008 – Dec 2009)

Ericsson Inc, USA (1994-2002)

- Director, Advanced Technologies, Research and Patents

General Electric Corporate R& D Center, NY, USA (1990 – 1994)

Professional

- Areas of expertise: Cellular - 4G and 5G, DSP for wireless and optical communications
- 32 Issued US patents, 10 Canadian patents, 19 WIPO/European patents, 1 Indian Patent (+ 2 in process)
- Publications: 80 Conference and Journal publications
- Ericsson Inventor of Year Award 1999
- Fellow, Indian National Academy of Engineering
- Srimathi Marti Annapurna Gurunath Award for Excellence in Teaching 2014
  - Best Teacher Award of IIT Madras
- IITM Alumni Association Award for Distinguished Service – August 2017

Digital Signal Processing

Course ID: EE3101
Course Credits: 4+1
Course Type: Diploma
Pre-requisites: **EE2101 - Signals and Systems**

What you'll learn

To teach the fundamentals of Digital Signal Processing

Course structure & Assessments

- 5 credit course,
- weekly online assignments,
- 2 in-person invigilated quizzes,
- 1 in-person invigilated end term exam.

Evaluation
Quiz 1
Quiz 2
End Semester Exam
Weekly Assignments

Lec 1
2

Week 1-2

Introduction to sampling - Review of Signals and Systems: Basic operations on signals - Discrete time complex exponentials and other basic signals – system properties (linearity, time-invariance, memory, causality, BIBO stability) -- impulse response and convolution

Week 3-4

Sampling: Impulse train sampling—relationship between impulse trained sampled continuous-time signal spectrum and the DTFT of its discrete-time counterpart—scaling of the frequency axis—relationship between true frequency and digital frequency—reconstruction through sinc interpolation—aliasing—effect of sampling at a discontinuous point—relationship between analog and digital sinc—effects of oversampling—discrete-time processing of continuous-time signals

Week 5-6

Discrete-Time Fourier Transform (DTFT): Complex exponentials as eigensignals of LTI systems—DTFT definition—inversion formula—properties—relationship to continuous-time Fourier series (CTFS)

DSP Books

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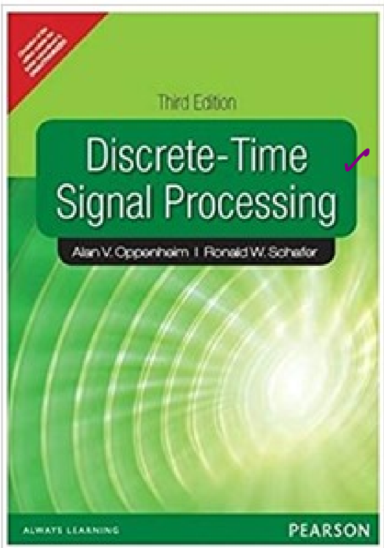
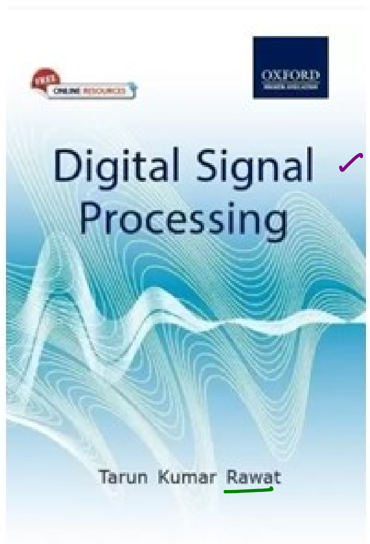
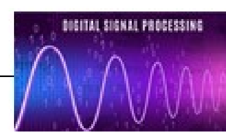
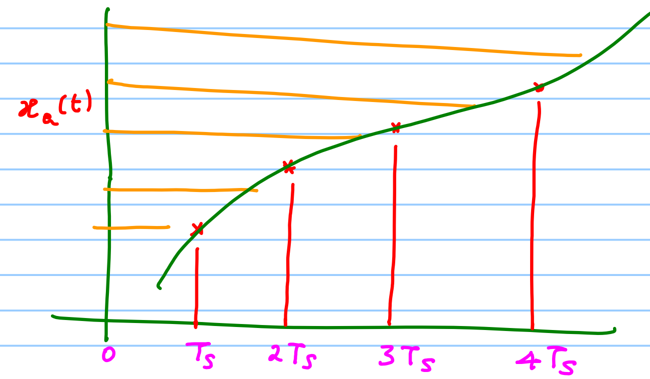






In DSP



Finite precision

Discrete Time, Amplitude ∞ precision $\rangle$ DT signal

Discrete Time, Discrete in Amplitude, Quantized $\rangle$ Digital Signal

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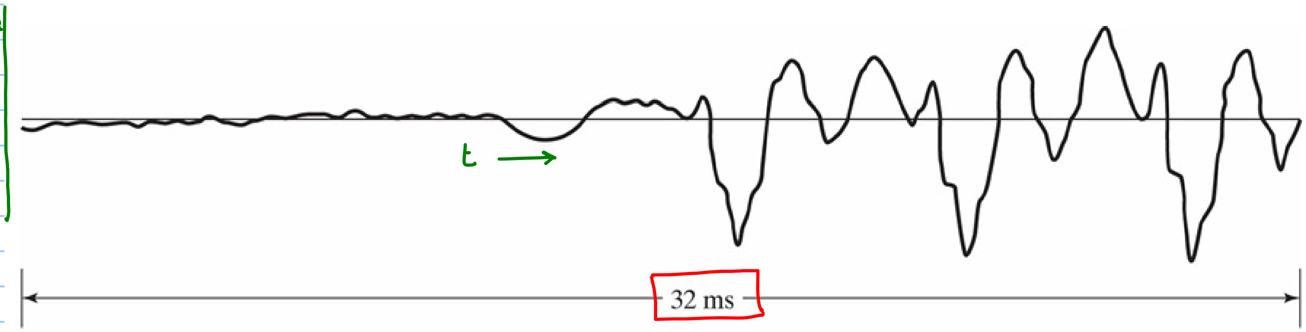
Week 1 …

Speech signal $s(t)$ time independent information bearing

Amplitude $s(t)$



(a)

Signal $s(t)$ continuous $t \in \mathbb{R}$ continuous variable

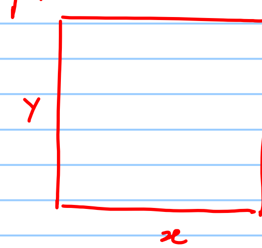
$s(t)$ one-dim signal

colour photograph

$I_r(x,y)$

$I_g(x,y)$

$I_b(x,y)$



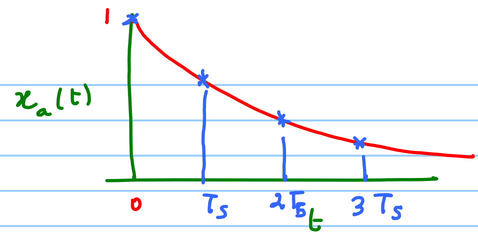
video $I_r(x, y, t)$

$I_g(x, y, t)$

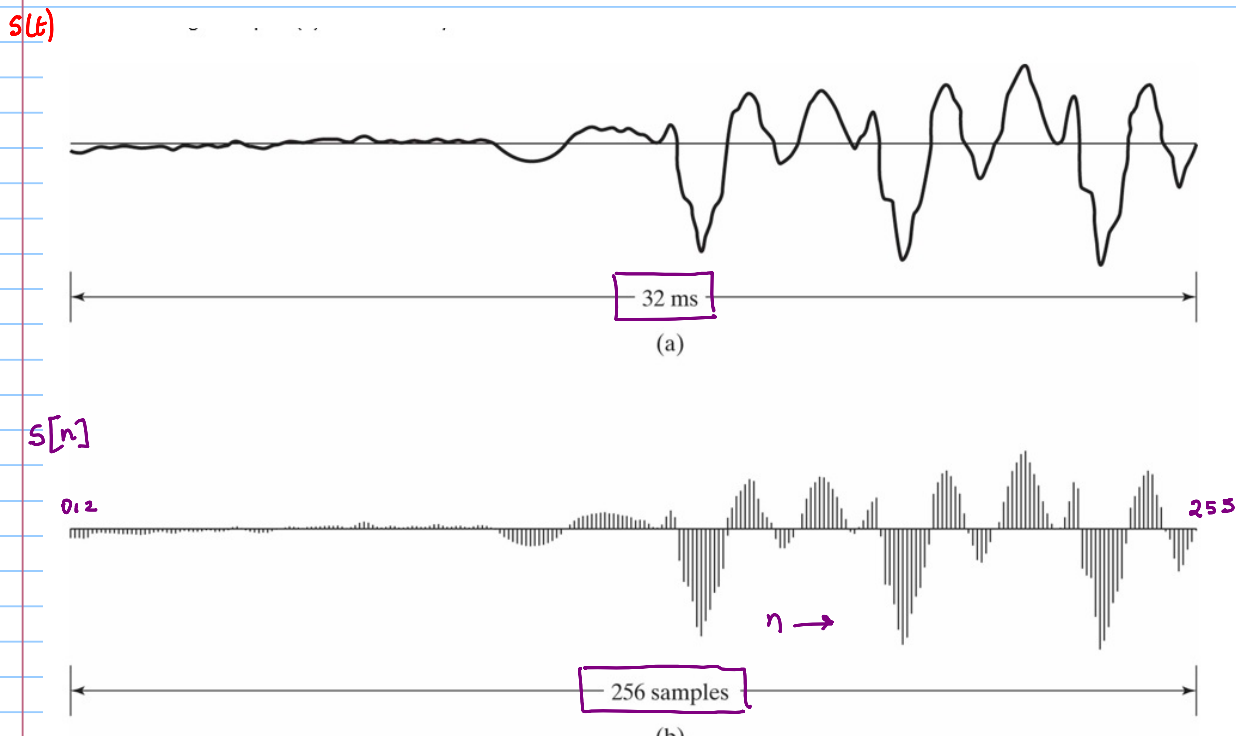
$I_b(x, y, t)$

Eg.

$x_a(t) = e^{-t}\, u(t)$



$x_a(t)\Big|_{t = nT_s} \quad n = 0, \pm 1, \pm 2, \ldots$



$32 \times 8 = 256$ samples

Nyquist sampling Theorem

$$f_s = \frac{1}{T_s} \geq 8 \text{ KHz}$$

8000 samples / sec

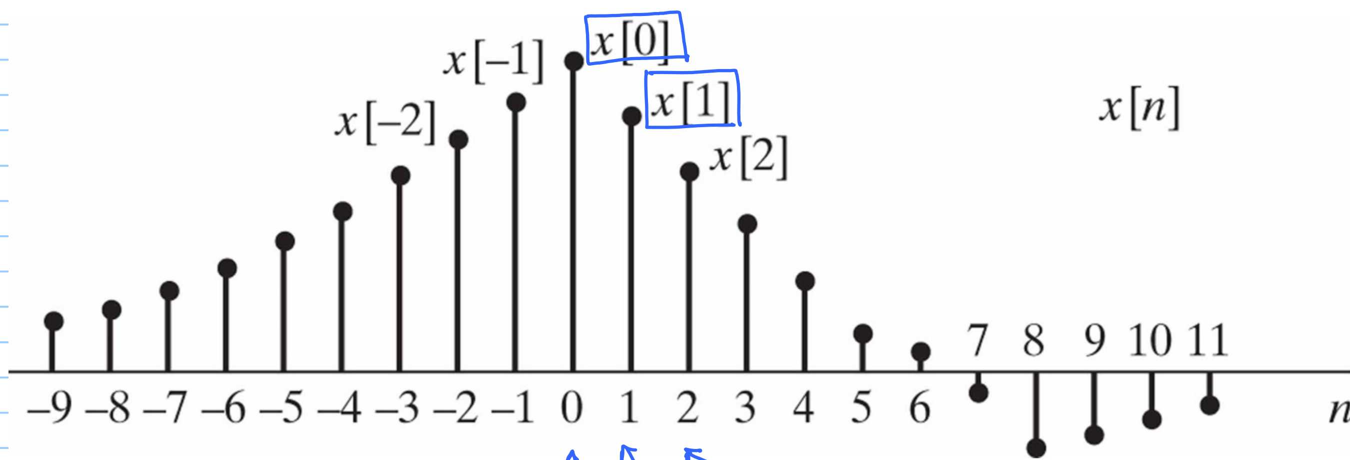
8 samples / msec

Compact Disc

oversatisfies Nyquist

Sampling @ 44.1 KHz

44,100 samples



$x_a(0)$ $x_a(T_s)$ $x_a(2T_s)$

... $x[-2]$ $x[-1]$ $x[0]$ $x[1]$ $x[2]$ ...

$\{x[n]\} = \{ \quad x[-1] \quad x[0] \quad x[1] \quad x[2] \quad \ldots \quad \}$

DT signals – a sequence of numbers

$$x[n] = \{\ \dots\quad 0.9 \quad -0.2 \quad \underset{\uparrow}{\overset{x[0]}{2.2}} \quad 1.1 \quad -3.7 \quad \dots\ \}$$

$x[-1]$ $x[0]$ $x[1]$ $x[2]$

index 0

$\{x[n]\}$

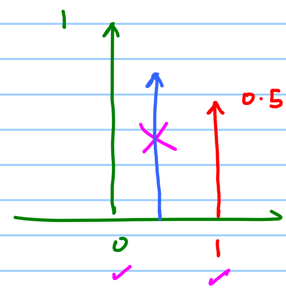
square brackets

$x[n] \in \mathbb{C}$

Special case is the subset $\mathbb{R}$

defined only for $n \in \mathbb{Z}$ set of integers

$x[n]$ undefined if $n \notin \mathbb{Z}$



$$CT \xrightarrow[\text{Sampling}]{\text{uniform}} DT$$

Continuous Time

Discrete Time

$$x_a(t)\Big|_{t=nT_s} = x[n]$$

T_s = sampling period (sec)

$\frac{1}{T_s}$ = sampling freq (Hz)

$$CT \xrightarrow{\text{uniform sampling}} DT$$

Loss of information?

$\longleftarrow$ No Loss of information

Nyquist Sampling Theorem $\Rightarrow$ if satisfied, there is no loss of information in the DT signal

Basic Signals in CT

1. Unit Step $u(t)$
2. Dirac Delta $\delta(t)$
3. exponential Ae^{-at}
4. Causal seq $Ae^{-at}u(t)$
5. Sinusoids $Ae^{j\omega t}$ or $B\sin(\omega t+\phi)$

   $B\cos(\omega t+\phi)$

DT Signals

① Temperature measured hourly

$x[1]$ $x[2]$ . . .

1AM 2AM

An example of a signal that is inherently Discrete-Time.

$\boxed{\text{Next Session}}$

Basic DT Signals

Operations performed on DT Signals

DT systems

I/O relations in DT system.

$\boxed{\text{Reading Assignment}}$

Oppenheim & Schafer : Sec 2.0, 2.1

Rawat : Sec 1.1 – 1.3

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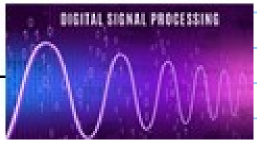

EE 3101 Digital Signal Processing

EE3101
Lec 2

Note Title

Outline

- Recap of last session
  - CT vs DT
  - DT vs Digital

Faculty : Prof. R. David Koilpillai
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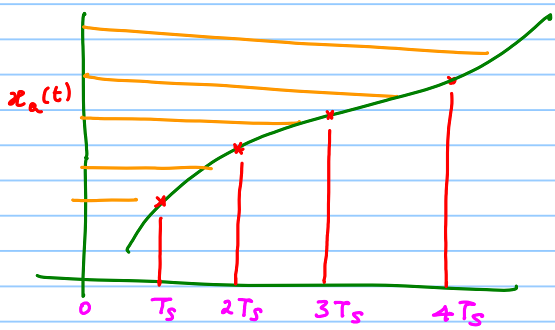
Week 1-2
Introduction to sampling - Review of Signals and Systems: Basic operations on signals - Discrete time complex exponentials and other basic signals – system properties (linearity, time-invariance, memory, causality, BIBO stability) -- impulse response and convolution

$\boxed{\text{Reading Assignment}}$

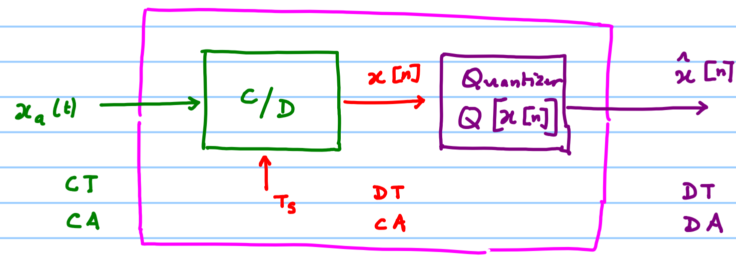
Oppenheim & Schafer : Sec 2.0, 2.1
Rawat : Sec 1.1 – 1.3

Discrete Time Signal Processing vs Digital Signal Processing

C - Continuous
D - Discrete
T - Time
A - Amplitude



Finite precision



A/D converter

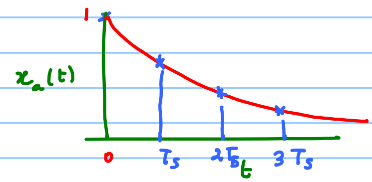
Discrete Time
Amplitude ∞ precision
→ DT signal

Discrete Time
Discrete in Amplitude
Quantized
→ Digital Signal

	$x_a(t)$	$x[n]$	$\hat{x}[n]$
Amplitude	C	C	D
Time	C	D	D

Uniform Sampling

$x[n] \triangleq x_a(t)\Big|_{t = nT_s} \qquad n = 0, \pm 1, \pm 2, \ldots$



T_s = sampling period (sec)

$\frac{1}{T_s}$ = sampling freq (Hz)

$t \in \mathbb{R}$ continuous variable

$x[n] \in \mathbb{C}$

special case is the subset $\mathbb{R}$

$x[n]$
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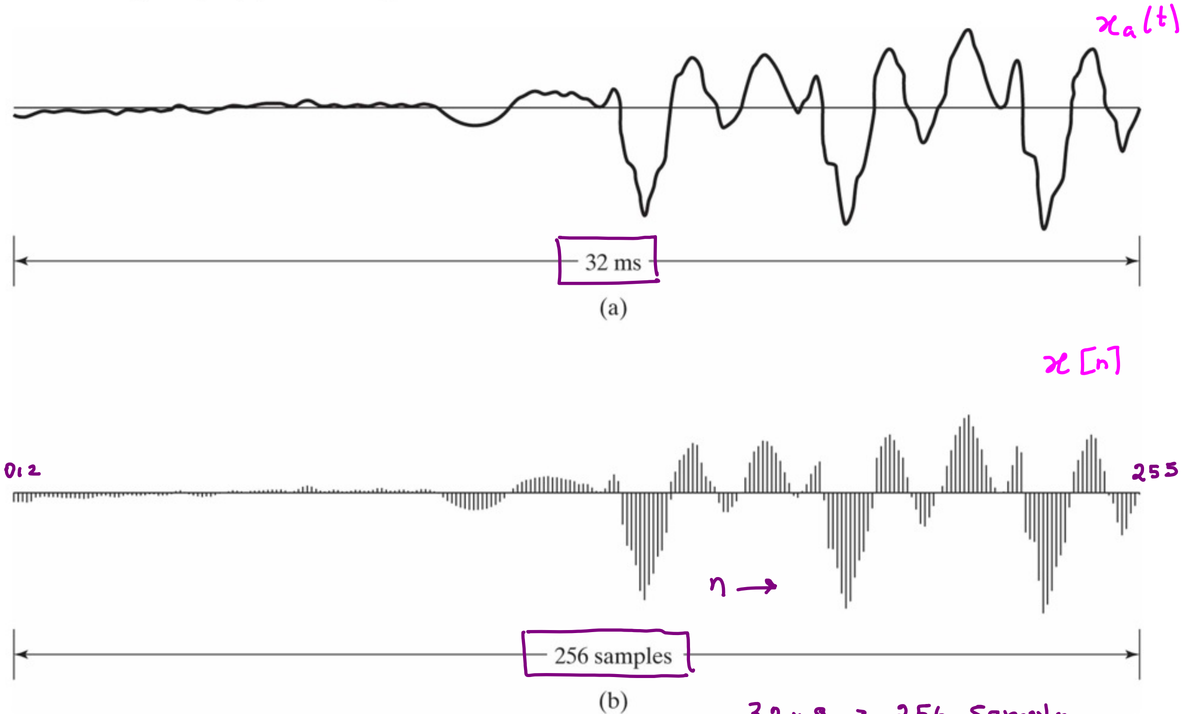


$32 \times 8 = 256$ samples

Nyquist sampling Theorem

$$f_s = \frac{1}{T_s} \geq 8 \text{ KHz}$$

8000 samples / sec

8 samples / msec

DT signals

8000 samples/sec

Each sample ∞ precision

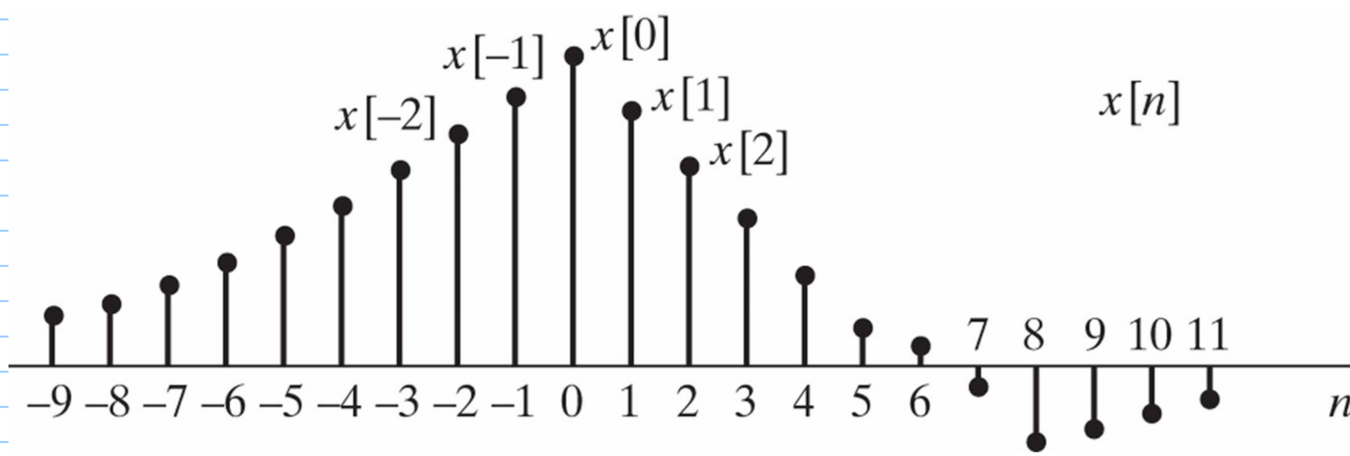
$\Rightarrow$ No quantization

Digital signal

8000 samples/sec

Suppose each sample quantized to 8 bits

$\hat{x}[n]$ Digital signal $\longrightarrow$ $8000 \times 8 = \underline{64{,}000}$ bits/sec



$\{x[n]\} = \{ \quad x[-1] \quad x[0] \quad x[1] \quad x[2] \quad \ldots \quad \}$

index $n=0$

$\{1, 2, 3, 4, 5\}$

$x[-2]$ $x[-1]$ $x[0]$ $x[1]$ $x[2]$

$x_a(t)\Big|_{t = nT_s} = x[n]$

T_s = sampling period (sec)

$\frac{1}{T_s}$ = sampling freq (Hz)

$CT \xrightarrow[\text{sampling}]{\text{uniform}} DT$ Loss of information?

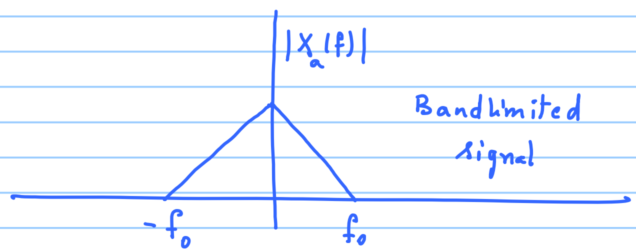
$\longleftarrow$

if No Loss of information

Nyquist Sampling Theorem $\Rightarrow$ if satisfied, There is no loss of information in the DT signal

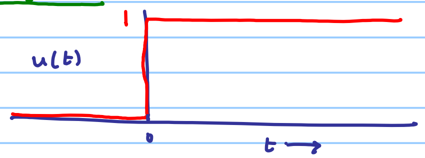
Bandlimited signal

There is a class of signals that are inherently discrete time signals * *

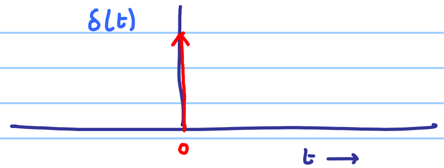


Basic CT Signals

unit step $u(t)$

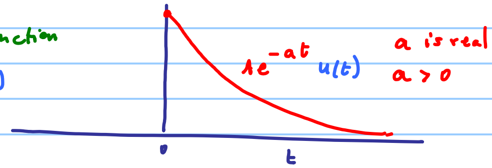


Dirac Delta $\delta(t)$



Exponential function

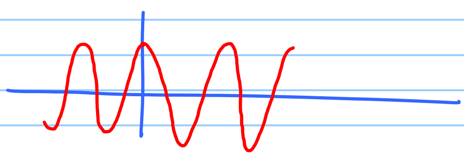
$Ae^{-at}\,u(t)$



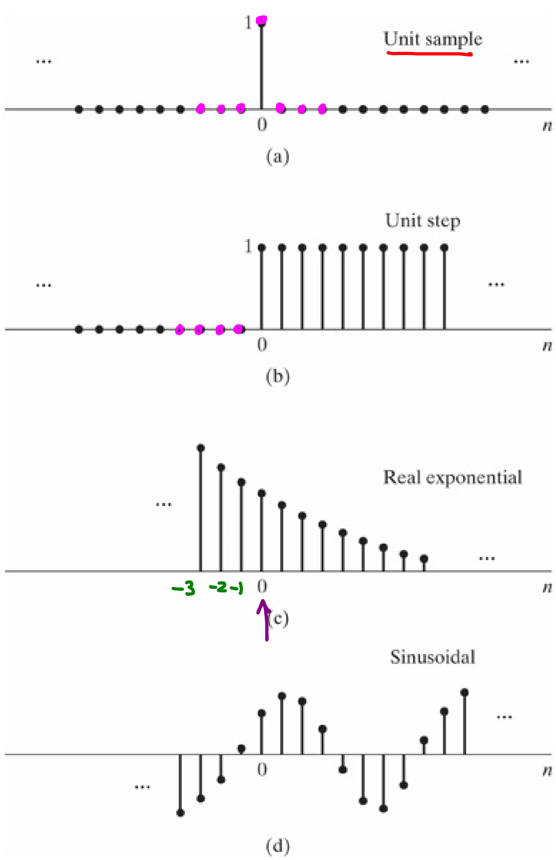
Sinusoidal $Ae^{j\omega t}$

$B\cos(\omega t + \phi)$

$C\sin(\omega t + \phi)$



Basic DT signals



unit sample

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

unit step

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Exponential

$$x[n] = A\alpha^n$$

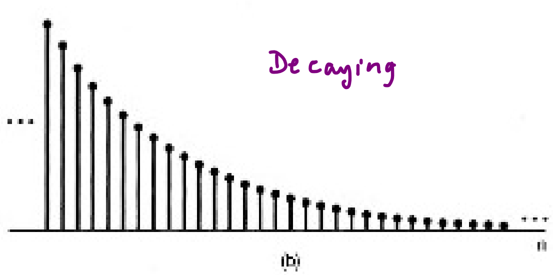
(Decaying)

A, α are real numbers

A positive

$0 < \alpha < 1$

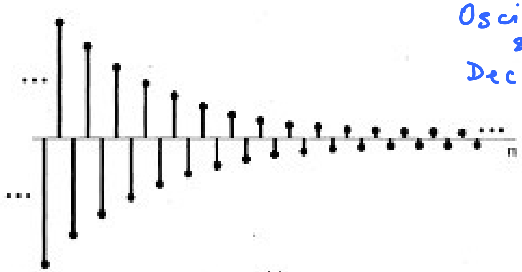
Exponential signals



$A\alpha^n$ $\quad 0 < \alpha < 1$

$A\alpha^n$ $\quad \alpha > 1$

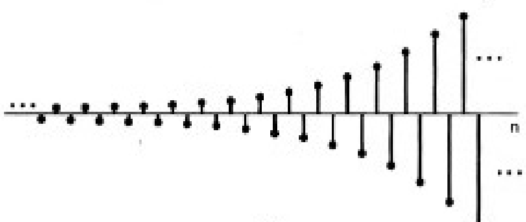
Exponential Signals (contd)



(c)

Oscillating & Decaying

$-1 < \alpha < 0$

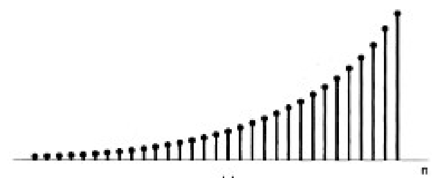


(d)

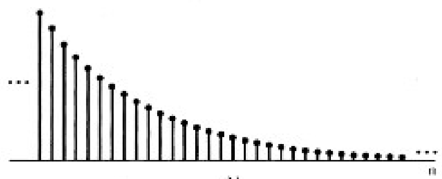
Oscillating & Growing

$-1 < \alpha$

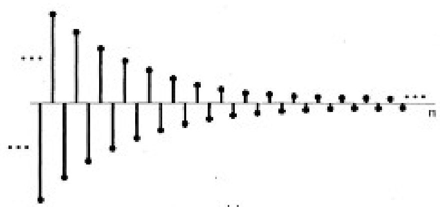
Family of Exponential Signals


(a)

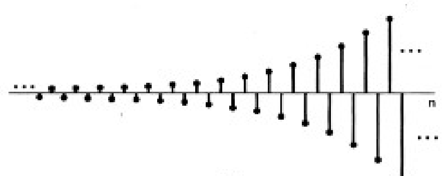
growing


(b)

decaying

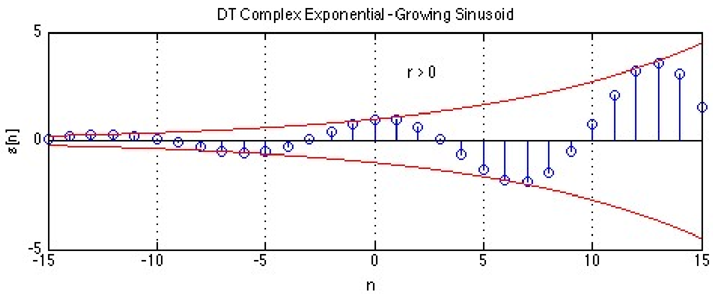

(c)

decaying & oscillating

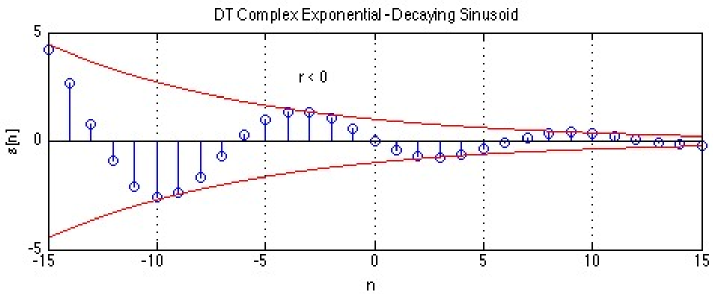

(d)

growing & oscillating

Sinusoidal signals

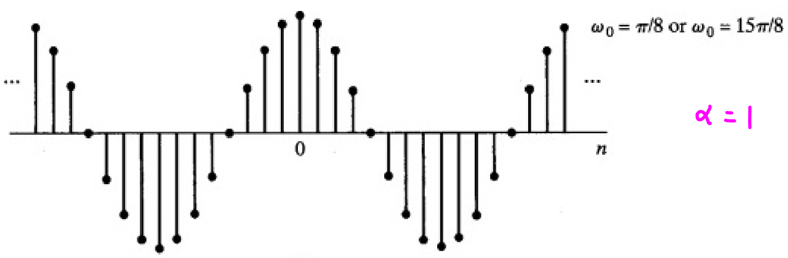


exp. growing sinusoid



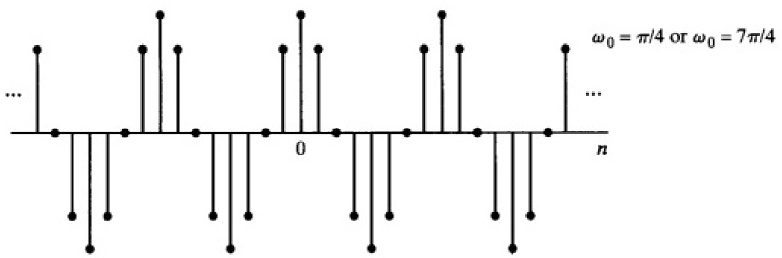
exp. decaying sinusoid

DT Sinusoidal signals



(b)

$\alpha = 1$



Continuous time sinusoid $A\cos(\Omega_0 t + \theta)$

$$x[n] = A\cos(\omega_0 n + \phi)$$

ω_0 DT frequency

dimensionless

$\boxed{\omega_0 n = \text{rads}}$

$\boxed{\Omega_0 T_s = \omega_0}$

Ω_0 rads/sec

T_s sec

$\boxed{\omega_0 \text{ rads}}$

$$x[n] = A\alpha^n e^{j(\omega_0 n + \phi)}$$

$$= A\alpha^n\left[\underbrace{\cos(\omega_0 n + \phi)} + j\,\underbrace{\sin(\omega_0 n + \phi)}\right]$$

$j = \sqrt{-1}$

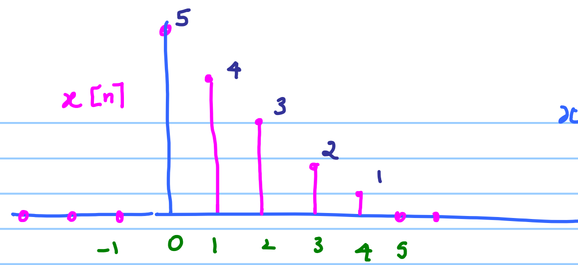
Basic Operations

$x[n]$

$y[n] = x[n - n_0]$ $\quad n_0$ integer

$n_0 > 0$ delay $\quad -\infty < n < \infty$

$n_0 < 0$ advance



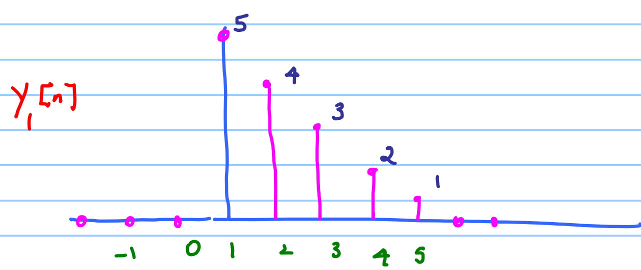
$x[n] = \{0, 0, \ldots 0, 5, 4, 3, 2, 1, 0, 0, 0 \ldots\}$

n	-2	-1	0	1	2	3	4	5	6	7
$x[n]$	0	0	5	4	3	2	1	0	0	0
$y_1[n] = x[n-1]$		0	0	5	4	3	2	1	0	0
$y_2[n] = x[n+1]$	0	5	4	3	2	1	0	0	0	0

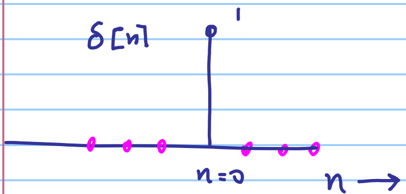
$x[n] = \{0, 0, 0, 5, 4, 3, 2, 1, 0, 0, 0\}$

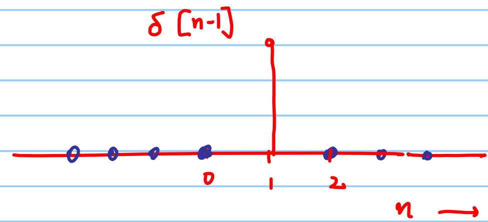
$y_1[n] = \{0, 0, 0, 5, 4, 3, 2, 1, 0, 0, 0\}$

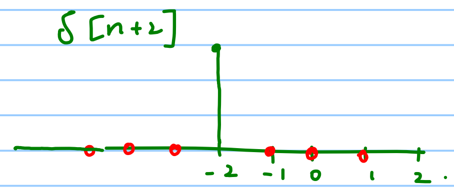
$y_2[n] = \{0, 0, 0, 5, 4, 3, 2, 1, 0, 0, 0\}$



unit sample





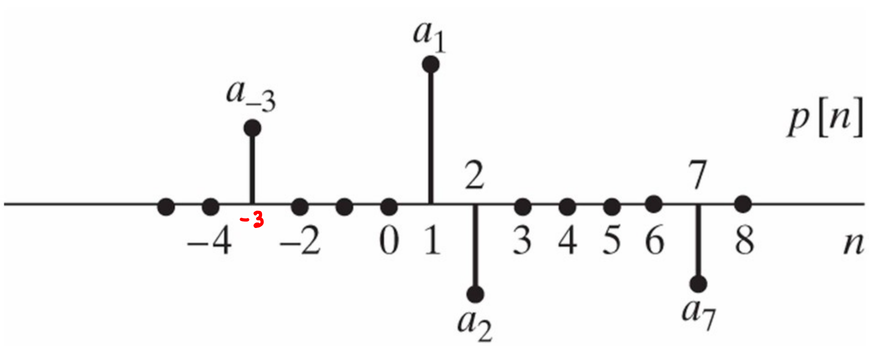


Example

$$\begin{array}{cccccccccccc} n \rightarrow & & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ x[n] & = & \{1, & 0, & 2, & 0, & 3, & 0, & 2, & 0, & 0, & 1\} \\ & & \uparrow & & & & & & & & & \end{array}$$

$$x[n] = \delta[n] + 2\,\delta[n-2] + 3\,\delta[n-4] + 2\,\delta[n-6] + 1\,\delta[n-9]$$

Example



$x[n] = a_{-3}\,\delta[n+3] + a_1\,\delta[n-1] + a_2\,\delta[n-2] + a_7\,\delta[n-7]$

Generalization

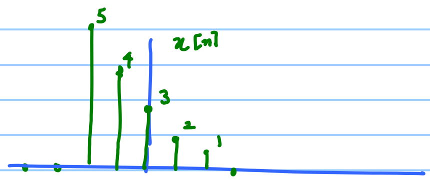
$x[n] = \Big\{ \quad \ldots \; x[-2] \;\; x[-1] \;\; x[0], \;\; x[1], \; x[2] \;\; \ldots$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] = \ldots + x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1] + \ldots$$

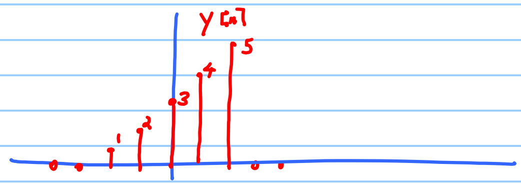
Sum   scaling   shifting

Time Reversal

$$\boxed{y[n] = x[-n]}$$

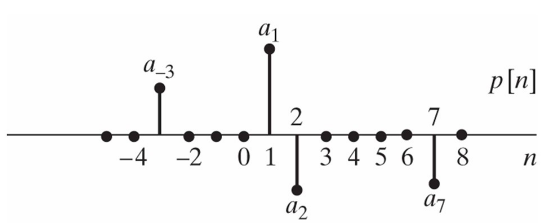


$x[n] = \{0, 0, 0, 5, 4, 3, 2, 1, 0, 0, 0 \cdots\}$



$y[n] = \{0, 0, 0\ 0\ 1\ 2\ 3, 4, 5, 0, 0 \cdots\}$

Example Time-Reversal



$p[n] = \{0, 0, a_{-3}, 0\ 0\ 0\ a_1\ a_2\ 0\ 0\ 0\ 0\ a_7\ 0\ 0\ 0\}$

$y[n] = p[-n] = \{0\ a_7\ 0\ 0\ 0\ 0\ a_2\ a_1\ 0\ 0\ 0\ a_{-3}\ 0\ 0 \cdots$

Basic Operations on DT sequences

DT sequences $x_1[n]$, and $x_2[n]$

Sequence / signal addition $y_1[n] = x_1[n] + x_2[n] \quad \forall n$

$y_1[0] = x_1[0] + x_2[0]$

$y_1[1] = x_1[1] + x_2[1]$

$\vdots$

Scalar addition $y_2[n] = \alpha + x_2[n] \quad \forall n$

$y_2[0] = \alpha + x_2[0]$

$y_2[1] = \alpha + x_2[1]$

$\vdots$

Sequence multiplication $y_3[n] = x_1[n] \cdot x_2[n] \quad \forall n$

$y_3[0] = x_1[0] \cdot x_2[0]$

$\vdots$

Scalar multiplication $y_4[n] = \alpha\, x_1[n] \quad \forall n.$

Example

$$x_1[n] = \{1.5, \underset{\uparrow}{2}, 3.4, -5, 10\}$$

$$x_2[n] = \{2.2, \underset{\uparrow}{3}, 2, 4.2, 8\}$$

$$y_1[n] = x_1[n] + x_2[n]$$

$$y_2[n] = x_1[n]\, x_2[n]$$

$$y_3[n] = \frac{3}{2}\, x_2[n]$$

$$y_4[n] = \alpha + x_1[n] \qquad \alpha = 2$$

Definitions

$\{x[n]\}$ → finite length
→ infinite length

Finite length $x[n]$ is non-zero in $N_1 \le n \le N_2$ $\begin{cases} N_1 > -\infty \\ N_2 < \infty \end{cases}$

unit step → finite No.
→ infinite length Yes.

unit sampl → finite Yes

Length of a finite length seq $\boxed{\text{Length} = N_2 - N_1 + 1}$

Example

$N_2 = 3$ $N_1 = -2$ $\Longrightarrow$ Length $3 - (-2) + 1 = 6$

$(N_2 - N_1 + 1)$

n	-2	-1	0	1	2	3	
	N_1					N_2	Non zero

Length $= 6$

Example

$x[n]$ is non-zero in the range $2 \le n \le 5$

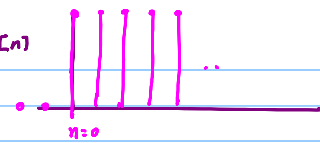
Length $N = 4$

Causal signal

$x[n] = 0 \quad n < 0$

unit step $u[n]$

Causal ✓



Right sided seq

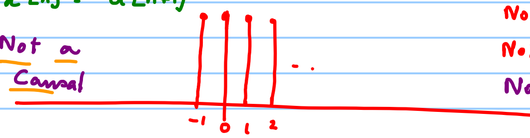
$x[n] = 0 \quad n < N_1$

N_1 + or −

can be ∞ length seq

$x[n] = u[n+1]$

Not a Causal



right-sided seq

Not causal

Not anticausality

Non causal

Anticausal signal

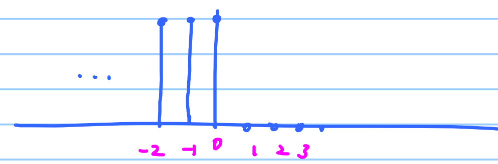
$x[n] = 0 \quad$ for $n \geq 0$

can be ∞ length

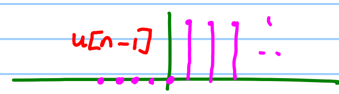
$u[n]$

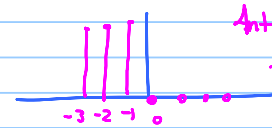
$x[n] = u[-n]$

is not an anticausal signal



$u[n-1]$





Anticausal signal

$\begin{cases} x[n] = u[n-1] \\ y[n] = x[-n] = u[-n-1] \end{cases}$

Left sided seq

$x[n] = 0$ for $n > N_2 \qquad N_2 \begin{cases} + \\ - \end{cases}$

$u[-n]$ is a left-sided, not anticausal

Anticausal seq is a Left sided seq

$u[-n-1]$ is a left-sided, anticausal

DT Sinusoids Properties

$x[n] = A\sin(\omega_0 n + \phi)$ or $A\cos(\omega_0 n + \phi)$

$x(t) = A\sin(\Omega_0 t + \theta)$

$\Omega_0 \quad 0 \longrightarrow \infty$

low freq   freq

Freq $\omega_1 = \omega_0 + 2\pi k$ shifted by a multiple of 2π

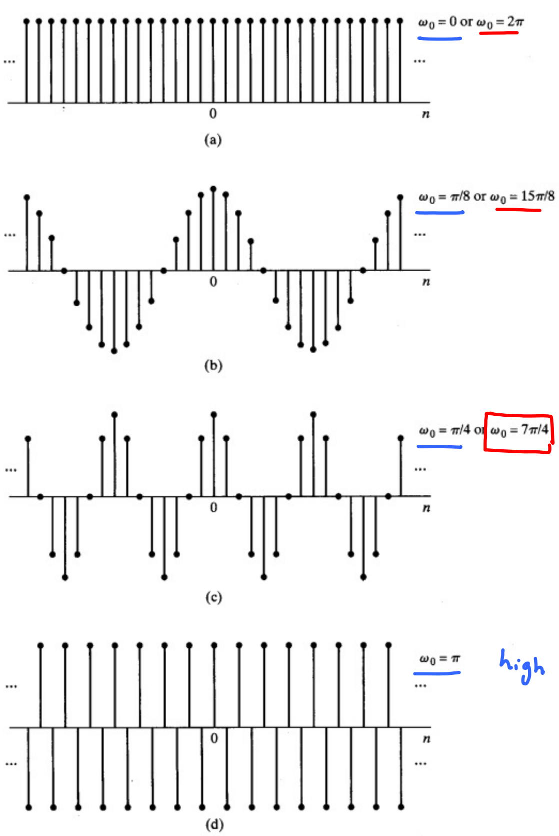
$$A\cos(\omega_1 n + \phi) = A\cos\left((\omega_0 + 2\pi k)n + \phi\right)$$

$$= A\cos(\omega_0 n + 2\pi k n + \phi) = A\cos(\omega_0 n + \phi)$$

$\boxed{\text{Periodicity in frequency for DT Sinusoids}}$

Illustration

DT sinusoids

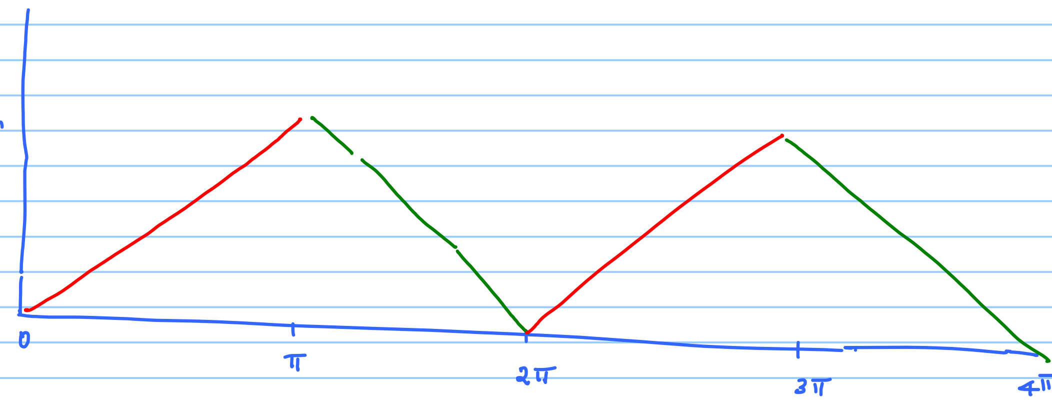


Low freq

$A\cos(\omega_0 n + \phi)$

> $\omega = \pi$

high freq



$A\cos(\omega_0 n + \phi)$

$\omega_0 = \pi$

Observation #1 ***

DT sinusoids are periodic in frequency

Period $= 2\pi$ or any multiple of 2π

Time behaviour

CT $\rightarrow$ all sinusoids are period $T = \frac{2\pi}{\Omega_0}$

DT $\quad A\cos(\omega_0 n + \phi)$

If Periodic with period $= N \quad \Rightarrow \quad \underbrace{A\cos(\omega_0(n + kN) + \phi)}_{x[n+kN] = x[n]} = A\cos(\omega_0 n + \phi)$

$\omega_0 kN = $ multiple of 2π

$k=1 \quad \omega_0 N = $ multiple of 2π

$\omega_0 N = 2\pi m \quad$ m is integer

A DT sinusoid is periodic iff $\boxed{\frac{2\pi}{\omega_0} = \frac{N}{m}}$ Rational function

N, m are integers

Examples

Condition for periodicity

$$\boxed{\frac{2\pi}{\omega_0} = \frac{N}{m}}$$

N = period

m = any Integer

(i) $x_1[n] = \cos \frac{\pi}{4} n$ $\qquad \omega_0 = \frac{\pi}{4}$

$$\frac{2\pi}{\frac{\pi}{4}} = \frac{8}{1} = \frac{N}{m}$$

$m = 1$

Then $N = 8$

Periodic function with period = 8

(2) $x_2[n] = \cos \frac{3\pi}{8} n$

$\omega_0 = \frac{3\pi}{8}$

$$\frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{3\pi}{8}} = \frac{16}{3} = \frac{N}{m}$$

$m = 3$

Then $N = 16$

Periodic with period = 16

$\frac{3\pi}{8} = 0.375\pi \longrightarrow$ periodic

$\frac{\pi}{4} = 0.25\pi \sim 0.7854 \longrightarrow$ periodic

(3) $\boxed{x_3[n] = \cos 0.785 n}$

Not periodic

$$\frac{2\pi}{0.785} = \frac{N}{m}$$

No value of m for which N is integer

Fourier representation

Discrete Time signal with period $= N$

$\omega_0 N = 2\pi m$

$$\frac{2\pi}{\omega_0} = \frac{N}{m} \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{N} m$$

$$\omega_0 = \frac{2\pi}{N} m \qquad \underbrace{m = 0, 1, 2, 3, \dots N-1}$$

# of periodic sequences with period $= N$
(sinusoisal)

$\Rightarrow$ N sequences