Partial Differentiation Problems

1. **Compute**
   Given:

$$Z = e^{x} t\cos t, \quad y = t\sin t$$

   Z = ext cos t, y = t sin t

   Find:

$$\frac{dZ}{dt} \text{ at } t = \frac{\pi}{2}$$

   dtdZ at t = 2π

2. **Prove:**
   If

$$u = \tan^{-1}\left(\frac{x}{y}\right)$$

   u = tan−1 (yx)

   then show that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$

   x∂x∂u + y∂y∂u = 0.

3. **Prove:**
   If

$$u = f\left(\frac{x}{y}\right)$$

   u = f (yx)

   then show that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$

   x∂x∂u + y∂y∂u = 0.

4. **Prove:**
   If

$$u = \log\frac{x^2 + y^2}{x + y}$$

   u = log x + yx2 + y2

   then show that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1.$$

   x∂x∂u + y∂y∂u = 1.

5. **Find Maxima/Minima:**
   Show that the function:

$$f(x,y,z) = (x + y + z)^3 - 3(x + y + z) - 24xyz + a^3$$

f(x, y, z) = (x + y + z)3 − 3(x + y + z) − 24xyz + a3

has a maximum at $(1, 1, 1)$(1, 1, 1).

6. **Verify the Equation:**
If

$$f(x, y, z - 2x) = 0$$

f(x, y, z − 2x) = 0

then show that under suitable conditions, the equation

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 2x$$

x∂x∂z − y∂y∂z = 2x

holds. Determine these conditions.

7. **Prove:**
If

$$z = f\left(\frac{ny - mz}{mx - lz}\right)$$

z = f (mx − lzny − mz)

then prove that:

$$(nx - lz)\frac{\partial z}{\partial x} + (ny - mz)\frac{\partial z}{\partial y} = 0.$$

(nx − lz)∂x∂z + (ny − mz)∂y∂z = 0.

8. **Prove Mixed Partial Derivatives:**
If

$$u = x^y$$

u = xy

then show that:

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}.$$

∂y∂x∂2u = ∂x∂y∂2u.

9. **Prove Second-Order Partial Differentiation:**
If

$$Z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$$

Z = x2 tan−1 (xy) − y2 tan−1 (yx)

then prove that:

$$\frac{\partial^2 Z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}.$$

∂y∂x∂2Z = x2 + y2x2 − y2.

10. **State and Prove Euler's Theorem:**

   - State Euler’s theorem on homogeneous functions.
   - Provide a proof with an example.

11. **Prove:**
If

$$u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$$

u = sin−1 yx + tan−1 xy

then show that:

$$\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$

∂x∂u + y∂y∂u = 0.

12. **Find Maxima/Minima:**
Find the maximum or minimum values of the function:

$$f(x,y) = x^2y^2(1 - x - y).$$

f(x, y) = x2y2(1 − x − y).