HW 1: Self-Debugging System

Problem: http://www.cs.ou.edu/~qcheng/2603/hw1selfDebug.htm

Questions:

1. How long after receiving the emergency signal do all of the defective towers shut themselves down?
2. How do they know they're defective?
3. Why don't the non-defective towers shut down?

Attempt #1 (August 30, 2016 / 4:00 P.M.):

Firstly, if we assume that no towers are defective, then the first question is answered outright in that it would take no time for defective towers to shut down because there are not any. This does not answer the second or third question however.

Next, let's assume there is 1 tower that is defective in the system. That tower will, per the description of the towers messages and the emergency tower's broadcast:

1. Receive correct messages from all other 999 towers in the system
2. Send out a bad signal to all other 999 towers in the system
3. Receive the emergency broadcast from tower #1001

Using this information, we know that all towers that are not the defective tower realize (before tower #1001 sends out the emergency signal) that there is a defective tower in the system. Additionally, we know that since the tower that has the defect does not receive its own message, it does not know that there is a defective tower in the system *until* tower #1001 broadcasts the emergency signal.

So, we can say that as soon as tower #1001 sends out the emergency signal, the defective tower, having not received any defective transmissions from any other tower, will immediately know that it is defective and shut down.

Thus, by this logic, it would take 1 minute for all defective towers to shut down if there was only 1 defective tower in the system.

At this point I am still unsure how the system figures out which towers are defective if there are more than 1.

Attempt #2 (September 1, 2016 / 8:35 P.M.):

Again, as seen from my previous attempt, we know the answer to the 1st question if there are 0 defective towers and 1 defective tower in the system, but not more.

In regards to the second question, how the towers know they're defective, in the case of 1 tower being defective this tower will figure it out by realizing that in the minute before the emergency broadcast is sent it received no bad transmissions meaning that once the emergency signal is sent, there must be a defective tower in the system. Thus, it realizes that it is the defective tower and shuts down.

The non-defective towers do not shut down because they did receive 1 bad transmission during the 1-minute cycle before the emergency signal was sent out, thus they know that they are not the defective tower and keep running.

This logic does not hold, however, if more than 1 tower is defective.

If we assume that 2 towers are defective, we know that in the 1-minute cycle before the emergency broadcast, each of those towers will receive 1 bad transmission (due to not being able to receive their own transmission). After the emergency tower sends the emergency signal, which tells the whole system that there is/are a defective tower(s) in the system. However I'm still not sure exactly how the defective towers figure out that they themselves are defective at this point in time without receiving in the emergency broadcast the number of defective towers.

If they received how many towers in the system were defective through the emergency broadcast, it would be a simple case of math. i.e.

- Tower A is defective, receives 1 bad transmission out of 999 (from B)
- Tower B is defective, receives 1 bad transmission out of 999 (from A)
- Emergency signal goes out, saying that 2 of 998 towers are defective
- Tower A: 1 of 999 are defective, but 2 in system are defective from tower, so it must be a defective tower
- Tower B: *see Tower A*
- Both towers figure this out immediately after 1 minute and both shut down

As stated before, this is not a solution because the emergency broadcast does not transmit the number of defective towers in the system.

Attempt #3 (September 4, 2016 / 12:15 P.M.):

Taking Dr. Cheng's advice, I looked up Terry Tao's blog about the Blue Eyed Islander (https://terrytao.wordpress.com/2011/04/07/the-blue-eyed-islanders-puzzle-repost/)

In his blog, Mr. Tao describes a very similar problem to this involving a blue eyed traveler who travels to an island of tribal people who are not allowed to know their own eye color. If they figure out their eye color, they must commit ritual suicide the following day. They can all see each other's eye color, but cannot see their own.

After visiting the island, the traveler states "how unusual it is to see another blue-eyed person like myself in this region of the world".

The solution Mr. Tao provides is as follows:

- If there is 1 blue eyed islander, he will commit suicide the next day, realizing that no one else he has seen has blue eyes and thus the visitor must be referring to him.
    - This is the exact answer I concluded above, if you view the defective tower as the blue eyed islander, tower #1001 as the traveler, and the 1-minute cycle as the 1-day suicide
- If there are *n* number of blue eyed islanders, they will all be dead in *n* days. This is concluded by inferring that "each blue eyed person will reason as follows: 'If I am not blue-eyed, then there will only be *n*-1 blue eyed people on this island, and so they will all commit suicide *n*-1 days after the traveler's address.'" After this time passes, and no blue eyed people have committed suicide, each blue eyed person will realize that they themselves have blue eyes and will then commit suicide on the *n*th day.

Taking this similar problem and applying it to the broadcast tower problem, we see a perfect analogy, if we simply replace the variables with our own:

- Blue eyed traveler = tower #1001
- All islanders = all towers in the system
- All blue eyed islanders = all defective towers in system
- Suicide on the next day = 1-minute cycle of emergency broadcast

Thus we can take the solution above and adapt it to our own problem:

Assume there are *n* number of defective towers in the system. They all will know after the 1st signals are sent that there are $n - 1$ defective towers in the system. Then once the emergency tower sends out the signal that there is a problem in the system, they will wait for all other defective towers to shut down, thinking that they themselves are not

defective. After another minute has passed and the emergency broadcast is still being transmitted, the defective towers (let's pick one of them and call it Tower A), will still have received $n - 1$ bad transmissions. But again, Tower A will still not assume that it is a defective tower because it has no reason to (it doesn't receive its own transmission).

This cycle will continue until the $n - 1^{st}$ minute, after which, if Tower A is still getting the same readings as before (i.e. still receiving $n - 1$ bad transmissions), Tower A will realize that there are actually *n* defective towers, and that it is the *nth* tower. Once figuring this out, it (along with every other defective tower which is going through the same logical processing as Tower A) will shut down simultaneously on the *nth* minute.

So

Answers:

1. *n* minutes, where *n* is the number of defective towers
2. They know they are defective from realizing that if after $n - 1$ minutes they are still receiving bad transmissions, they must be a defective tower as well
3. The non-defective towers do not shut down because they are receiving *n* bad signals up until the *n* minute, at which point they will receive no bad signals (because all defective towers have shut down by this point).