1. (Quiz 1) Prove that

$$(x+1) \times (1+x) = x \times x + (1 + (1+1) \times x)$$

from the axioms in Figure 2.1. In each step, specify which axiom you use. You may not use any theorem.

Solution: Starting from the LHS:

$$
\begin{aligned}
& LHS \\
=& (x+1) \times (x+1) && \{+ \text{ commutative } \} \\
=& ((x+1) \times x) + ((x+1) \times 1) && \{\text{distributive law } \} \\
=& (x \times (x+1)) + ((x+1) \times 1) && \{\times \text{ commutative } \} \\
=& ((x \times x) + (x \times 1)) + ((x+1) \times 1) && \{\text{distributive law } \} \\
=& (x \times x) + ((x \times 1) + ((x+1) \times 1)) && \{+ \text{ associative } \} \\
=& (x \times x) + ((x \times 1) + (x+1)) && \{\times \text{ identity } \} \\
=& (x \times x) + (((x \times 1) + x) + 1) && \{+ \text{ associative } \} \\
=& (x \times x) + (1 + ((x \times 1) + x)) && \{+ \text{ commutative } \} \\
=& (x \times x) + (1 + ((x \times 1) + (x \times 1))) && \{\times \text{ identity } \} \\
=& (x \times x) + (1 + (x \times (1+1))) && \{ \text{ distributive law } \} \\
=& (x \times x) + (1 + ((1+1) \times x)) && \{ \times \text{ commutative } \}
\end{aligned}
$$

2. (Quiz 2) Prove that

$$x \vee (y \vee z) = (z \vee y) \vee x$$

from the axioms in Figure 2.3. In each step, specify which axiom you use. You may not use any theorem.

Solution: We start from the LHS:

$$\begin{aligned}
& LHS \\
=& x \vee (y \vee z) \\
=& x \vee (z \vee y) && \{ \vee \text{ commutative } \} \\
=& (z \vee y) \vee x && \{ \vee \text{ commutative } \}
\end{aligned}$$

3. Prove that

$$(x \wedge y) \vee y = y$$

from the axioms in Figure 2.3. In each step, specify which axiom you use. You may not use any theorem.

Solution: We start from the LHS:

$$\begin{aligned}
& LHS \\
= \;& (x \wedge y) \vee y \\
= \;& y \vee (x \wedge y) && \{ \vee \text{ commutative. } \} \\
= \;& (y \vee x) \wedge (y \vee y) && \{ \vee \text{ distributive.} \} \\
= \;& (y \vee x) \wedge (y) && \{ \vee \text{ idempotent.} \} \\
= \;& (x \vee y) \wedge y && \{\vee \text{ commutative}\} \\
= \;& \cdots && \{\text{Copy from Theorem 6 on} \\
& && \text{ page 20 and Theorem 5 on page 19}\} \\
= \;& RHS.
\end{aligned}$$

4. (Quiz 3) Prove that

$$(x \to y) \wedge (x \to z) = x \to (y \wedge z)$$

You may use the axioms of Boolean algebra (Figure 2.5), and the first ten theorems in Ex. 15.

Solution: We start from the LHS:

$$\begin{aligned}
&(x \to y) \wedge (x \to z) && \\
=&(\neg x \vee y) \wedge (\neg x \vee z) && \{\text{Implication (twice}\} \\
=&(\neg x) \vee (y \wedge z) && \{\ \vee \text{ distributive.}\} \\
=&x \to (y \wedge z) && \{\text{Implication}\}
\end{aligned}$$

5. Verify that

$$(x \to y) \wedge (x \to z) = x \to (y \wedge z)$$

using a truth table.

Solution:

x	y	z	$x \to y$	$x \to z$	$(x \to y) \wedge (x \to z)$	$y \wedge z$	$x \to (y \wedge z)$
f	f	f	t	t	t	f	t
f	f	t	t	t	t	f	t
f	t	f	t	t	t	f	t
f	t	t	t	t	t	t	t
t	f	f	f	f	f	f	f
t	f	t	f	t	f	f	f
t	t	f	t	f	f	f	f
t	t	t	t	t	t	t	t

The sixth column and the eight column are equal, thus the equation is verified.

6. (Quiz 4) Use the Karnaugh map to find a short Boolean formula for the true table:

x	y	z	$F(x,y,z)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Solution: First we draw the Karnaugh map: From the map, we have

z \ xy	00	01	11	10
0	1		1	1
1				

$$
\begin{aligned}
&\bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + xy\bar{z} \\
=&\bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + x\bar{y}\bar{z} + xy\bar{z} \\
=&(\bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z}) + (x\bar{y}\bar{z} + xy\bar{z}) \\
=&(\bar{x} + x)\bar{y}\bar{z} + x\bar{z}(\bar{y} + y) \\
=&\bar{y}\bar{z} + x\bar{z} \\
=&(\bar{y} + x)\bar{z}
\end{aligned}
$$

7. Draw a circuit diagram for the boolean formula

$$(a \to b) \land ((\neg b) \to c)$$

using AND, OR and NOT gates.

Solution: We need to convert the implication operator:

$$\begin{aligned} & (a \to b) \land ((\neg b) \to c) \\ =& ((\neg a) \lor b) \land ((\neg(\neg b)) \lor c) \\ =& ((\neg a) \lor b) \land (b \lor c) \end{aligned}$$

Now we can draw the circuit: