

27. Prove {app-assoc} theorem

$$\begin{aligned}
 & (\text{append } [x_1 \dots x_n] (\text{append } ys \ zs)) = (\text{append } (\text{append } [x_1 \dots x_n] \ ys) \ zs) \\
 & (\text{append } [x_1 \dots x_n] (\text{append } ys \ zs)) \\
 & = (\text{append } (\text{cons } x_1 \ [x_2 \dots x_{n+1}] (\text{append } ys \ zs)) \quad \{\text{cons}\}) \\
 & = (\text{cons } x_1 (\text{append } [x_2 \dots x_{n+1}] (\text{append } ys \ zs)) \quad \{\text{app1}\}) \\
 & = (\text{cons } x_1 (\text{append } (\text{append } [x_2 \dots x_{n+1}] \ ys) \ zs)) \quad \{\text{inductive hyp}\}) \\
 & = (\text{append } (\text{cons } x_1 (\text{append } [x_2 \dots x_{n+1}] \ ys) \ zs)) \quad \{\text{app1}\}) \\
 & = (\text{append } (\text{append } (\text{cons } x_1 \ [x_2 \dots x_{n+1}] \ ys) \ zs)) \quad \{\text{app1}\}) \\
 & = (\text{append } (\text{append } [x_1 \dots x_{n+1}] \ ys) \ zs) \quad \{\text{cons}\})
 \end{aligned}$$

29. (defun rep (n x)

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  (if (zp n)
      Nil
      (cons x (rep (- n 1) x))))

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Prove {rep-len}

(len (rep n x)) = n

30.

(member-equal y nil) = nil {mem0}

(member-equal y (cons x xs)) = (equal y x) OR (member-equal y xs) {mem1}

Prove {member-equal}

(member-equal y (rep n x)) IMPLIES (member-equal y (cons x nil))

31. Prove {drop-all0}

$$(\text{len } (\text{nthcdr } (\text{len } xs) xs)) = 0 \quad P(n)$$

Base case:  $n = 0$

$$(\text{len } (\text{nthcdr } (\text{len } \text{nil}) \text{nil})) = 0$$

$$(\text{len } (\text{nthcdr } (0) \text{nil})) = 0 \quad \{\text{len0}\}$$

$$(\text{len } (\text{nil})) = 0 \quad \{\text{sfx0}\}$$

$$0 = 0 \quad \{\text{len0}\}$$

Inductive Hypothesis: For all  $n$ ,  $P(n)$

$$(\text{len } (\text{nthcdr } (\text{len } [x_1 \dots x_{n+1}]) [x_1 \dots x_{n+1}])) = 0$$

$$(\text{len } (\text{nthcdr } (\text{len } (\text{cons } x_1 [x_2 \dots x_{n+1}])) \text{cons } x_1 [x_2 \dots x_{n+1}])) \quad \{\text{cons 2x}\}$$

$$(\text{len } (\text{nthcdr } (+ 1 (\text{len } [x_2 \dots x_{n+1}])) \text{cons } x_1 [x_2 \dots x_{n+1}])) \quad \{\text{len1}\}$$

$$(\text{len } (\text{nthcdr } (+ (\text{len } [x_2 \dots x_{n+1}]) 1) \text{cons } x_1 [x_2 \dots x_{n+1}])) \quad \{+ \text{commutative}\}$$

$$(\text{len } (\text{nthcdr } (\text{len } [x_2 \dots x_{n+1}]) \text{cons } x_1 [x_2 \dots x_{n+1}])) \quad \{\text{sfx1, rest}\}$$

$$(\text{len } (\text{nthcdr } (\text{len } [x_2 \dots x_{n+1}])) [x_2 \dots x_{n+1}])) \quad \{\}$$

$$0 = 0 \quad \{\text{Inductive Hypothesis}\}$$