# Review for Final Exam

1. Let $S$ be the surface parameterized by $\vec{r}(u,v) = \langle 4-u^2, u-v, v\rangle$ with $0 \le v \le u \le 2$.

   (a) Find the equation of the tangent plane to the surface at the point $(0,1,1)$.

   (b) Find the surface area of $S$.

   (c) Compute the flux of $\vec{F}(x,y,z) = \langle 2x, y, z\rangle$ across $S$.

2. Find the absolute maximum and minimum of $f(x,y,z) = x + y^2 + 3z$ on the ellipsoid $x^2 + 2y^2 + 3z^2 = 36$.

3. Compute $\int_C (y^2 e^{xy})dx + (e^{xy} + xye^{xy})dy$ where $C$ is the curve consisting of the two line segments from $(0,0)$ to $(2,2)$ and from $(2,2)$ to $(0,5)$.

4. Let $D$ be a region on the $xy$-plane. Let $S$ be the part of the plane $4x-6y+2z = 5$ with $(x,y)$ in $D$. If the area of $S$ is 11, find the area of $D$.

5. Let $E$ be the region which is both inside the sphere $x^2 + y^2 + z^2 = 8$ and above the cone $z = \sqrt{x^2+y^2}$. Let $S$ be the boundary surface of $E$ with inward orientation. Find $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F}(x,y,z) = \langle xz + 5y^2, e^{\cos(xz)}, z^2\rangle$.

6. Find $\int_C \vec{F} \cdot d\vec{r}$ where $C$ is the intersection of the plane $z = 1-2x-3y$ and the cylinder $x^2+y^2 = 4$ oriented clockwise when viewed from above and $\vec{F}(x,y,z) = \langle yz + \cos(x^2), -x^2, 3y\rangle$.

7. Evaluate the surface integral $\iint_S xy\ dS$ where $S$ is the triangular region with vertices $(1,0,0)$, $(0,2,0)$, and $(0,0,2)$.

8. Evaluate the limit or show it does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y^2 - xy^4}{x^5+y^5}$.

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2(y)}{x^2+2y^2}$.

   (c) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{y-2x}{4-xy^2}$.

9. Find all critical points of the function $f$ and determine if each critical point is a local max, local min, or saddle point.

   (a) $f(x, y) = x^3y + 12x^2 - 8y$

   (b) $f(x, y) = e^{4y-x^2-y^2}$

10. Evaluate the following double integrals.

    (a) $\displaystyle\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{y^2}{(x^2+y^2)^{3/2}}\, dydx.$

    (b) $\displaystyle\int_0^1 \int_1^4 x\sqrt{3+x^2/y}\, dydx + \int_1^2 \int_{x^2}^4 x\sqrt{3+x^2/y}\, dydx.$

11. Let $f(x, y, z)$ be differentiable. Suppose $f(1, 3, 5) = 7$ and $\nabla f(1, 3, 5) = \langle 2, -3, 1\rangle$.

    (a) Compute the directional derivative of $f$ at the point $(1, 3, 5)$ in the direction of the point $(-1, 4, 7)$.

    (b) In what direction is the directional derivative at $(1, 3, 5)$ the largest? What is the directional derivative in that direction?

    (c) Find the equation of the tangent plane to the surface $f(x, y, z) = 7$ at the point $(1, 3, 5)$.

    (d) Use linear approximation to estimate $f(.9, 3.2, 5.1)$.

    (e) Compute $\nabla g(3, 2)$ where $g(x, y) = f(2y - x, xy - 3, x + y)$.

12. Let $w = x\sqrt{y} - x - y$. Find the maximum and minimum values of $w$ and where they occur on the triangular region bounded by the $x$-axis, the $y$-axis, and the line $x + y = 12$.

13. Find $\int_C y^2(e^x + 1)dx + 2y(e^x + 1)dy$ where $C$ is the closed path formed of three parts: the curve $y = x^2$ from $(0, 0)$ to $(2, 4)$, the line segment from $(2, 4)$ to $(0, 2)$ and the line segment from $(0, 2)$ to $(0, 0)$.

14. A particle is moved in the plane from the origin to the point $(1, 1)$. While it is moving, it is acted on by the force $\vec{F} = \langle y^2 - ye^x + xy, 2xy - e^x + x^2\rangle$. This experiment is done twice. The first time the particle is moved in a straight line and the second time is it moved along the curve $y = x^3$. The work done by the force the first time is $W_1$ and the second time it is $W_2$. Determine which of $W_1$ and $W_2$ is bigger and by how much.

15. (a) Find a number $c$ such that the force field $\vec{F} = \langle ye^x + 3x^2 + 3y^2, e^x + cxy + 3y^2 \rangle$ is conservative.

    (b) Suppose the constant $c$ has the value found in part a. Find a function $f(x, y)$ such that $\vec{F} = \nabla f$.

    (c) Continuing to assume that $c$ has the value found in part a, find the work done by $\vec{F}$ on a particle moving from $(1, 0)$ to $(0, 1)$ along the circle of radius 1 centered at the origin.

16. Let $S$ be the union of the three surfaces $S_1, S_2, S_3$ where $S_1$ is the part of the cylinder $x^2 + y^2 = 16$ with $0 \leq z \leq 4$ oriented outwards, $S_2$ is the disk $x^2 + y^2 \leq 16$ on the plane $z = 4$ oriented up, and $S_3$ is the hemisphere $z = \sqrt{16 - x^2 - y^2}$ oriented down. Find $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \langle e^{\cos(z)}, 2y + 3x, 1/(x^2 + y^2) \rangle$.

17. Let $E$ be the region inside the cylinder $x^2 + y^2 = 4$, below the cone $z = \sqrt{3x^2 + 3y^2}$, and above the $xy$-plane. Set up the integral $\iiint_E z\, dV$ in 3 coordinate systems: rectangular, cylindrical, and spherical. Pick one and evaluate it.

18. Evaluate $\int_C x^2 y dx + \frac{1}{3} x^3 dy + xy dz$ where $C$ is the curve of intersection of the hyperbolic paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$ oriented counterclockwise when viewed from above.

19. Let $S$ be the part of the sphere $x^2 + y^2 + z^2 = 4$ which is above the plane $z = 1$ oriented upwards. Let $\vec{F}(x, y, z) = \langle yz, -2z, 4z^2 - e^{x^2+y^2} \rangle$. Compute $\iint_S \vec{F} \cdot d\vec{S}$ and $\iint_S \operatorname{curl}(\vec{F}) \cdot d\vec{S}$.